WORK BOOK # 1

DISTANCE & DISPLACEMENT, SPEED & VELOCITY , AVERAGE SPEED & AVERAGE VELOCITY

1. A particle moves in a straight line for 20 seconds with velocity 3 m/s and then moves with velocity 4 m/s for another 20 seconds and finally moves with velocity 5m/s for next 20 seconds. What is the average velocity of the particle?
(A) 3 m/s (B*) 4 m/s (C) 5 m/s (D) zero

Sol. Average velocity $= \dfrac{\text{Total Displacement}}{\text{Total time}}$

$= \dfrac{s_1+s_2+s_3}{t_1+t_2+t_3} = \dfrac{v_1t_1+v_2t_2+v_3t_3}{t_1+t_2+t_3}$

$= \dfrac{3\times20+4\times20+5\times20}{20+20+20} = \dfrac{240}{60} = 4$ m/sec

2. An object travels 10 km at a speed of 100 m/s and another 10 km at 50 m/s. The average speed over the whole distance is:-
(A) 75 m/s (B) 55 m/s (C*) 66.7 m/s (D) 33.3 m/s

Sol. Average speed = $v_{avg} = \dfrac{\text{Total distance covered}}{\text{Total time taken}} = \dfrac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \dfrac{2V_1V_2}{V_1+V_2}$

$= \dfrac{2\times100\times50}{150} = \dfrac{10000}{150} = 66.7$ m/sec

3. The magnitude of average velocity is equal to the average speed when a particle moves :
(A) On a curved path (B*) in the same direction
(C) With constant acceleration (D) with constant retardation

Sol. If the motion of a particle is along a straight line and in same direction then, average velocity = average speed

4. A car runs at constant speed on a circular track of radius 10 m taking 6.28 s on each lap (i.e. round). The average speed and average velocity for each complete lap is :
(A) Velocity 10 m/s, speed 10 m/s (B*) Velocity zero, speed 10 m/s
(C) Velocity zero, speed zero (D) Velocity 10 m/s, speed zero

Sol. speed v is constant

Average speed $v_{avg} = \dfrac{2\pi r}{t} = \dfrac{2\times3.14\times10}{6.28} = 10$ m/sec. (constant)

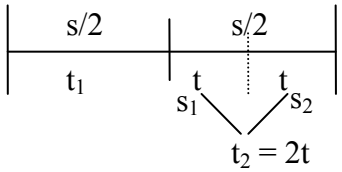
Displacement s = 0

so Average velocity $v_{avg} = 0$

5. A particle moving in a straight line covers half the distance with speed of 12 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is:

(A*) 8.0 m/s (B) 12.0 m/s (C) 10.0 m/s (D) 9.8 m/s

Sol.



Average speed $v_{avg} = \frac{s}{t_1 + t_2} \Rightarrow v_{avg} = \frac{s}{t_1 + 2t}$ .....(1)

$v_0 t_1 = s/2 \Rightarrow t_1 = \frac{s}{2v_0}$ and $s_1 + s_2 = s/2$

$\Rightarrow v_1 t + v_2 t = s/2 \Rightarrow t = \left(\frac{s}{2(v_1 + v_2)}\right)$ ....(2)

from (1) & (2)

$$v_{avg} = \frac{s}{\frac{s}{2v_0} + 2\left(\frac{s}{2(v_1 + v_2)}\right)} = \frac{24}{3} = 8 \text{ m/sec}$$

6. A car travels a distance d km on a straight road in two hours and then returns to the starting point in next three hours. Its average speed is:

(A) $\frac{d}{5}$ km/h (B*) $\frac{2d}{5}$ km/h (C) $\frac{d}{2} + \frac{d}{3}$ km/h (D) none of these

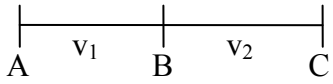
Sol. Average Speed $v_{avg} = \frac{\text{Total Distance}}{\text{Total time}}$

$= \frac{d + d}{2 + 3} = \frac{2d}{5}$ km/h

7. A particle goes from A to B with a speed of 40km/h and B to C with a speed of 60km/h. If AB = 6BC, the average speed in km/h between A and C is ____

Ans. 42 km/hr

Sol.



AB = 6BC

$$\text{Average Speed } v_{avg} = \frac{d}{t} = \frac{AB+BC}{\frac{AB}{v_1}+\frac{BC}{v_1}} = \frac{6BC+BC}{\frac{6BC}{40}+\frac{BC}{60}}$$

$$= \frac{7}{\frac{1}{10}\left(\frac{9+1}{6}\right)} = 7 \times 6 = 42 \text{ m/sec}$$

8. A car moving on a straight road covers one third of a certain distance with 20 km/h and rest with 60 km/h. The average speed is:

(A) 40 km/h (B) 80 km/h (C) $46\frac{2}{3}$ km / h (D*) 36 km/h

Sol. Let Total Distance = d $\quad d_1 = \frac{d}{3} \quad d_2 = \frac{2d}{3}$

$$\text{Average Speed } v_{avg} = \frac{d}{t} = \frac{d_1+d_2}{t_1+t_2} = \frac{d}{\frac{d_1}{v_1}+\frac{d_2}{v_2}} = \frac{d}{\frac{d}{3\times 20}+\frac{2d}{3\times 60}} = 36 \text{ km/h}$$

9. A train covers the first half of the distance between two stations with a speed of 40 km/h and the other half with 60 km/h. Then its average speed is:-

(A) 50 km/h (B*) 48 km/h (C) 52 km/h (D) 100 km/h

Sol. $\text{Average Speed } v_{avg} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 40\times 60}{40+60} = \frac{2\times 4\times 6\times 100}{100} = 48 \text{ Km/h}$

10. A body covers one-third of the distance with a velocity v_1 the second one-third of the distance with a velocity v_2, and the last one-third of the distance with a velocity v_3. The average velocity is:

(A) $\frac{v_1+v_2+v_3}{3}$ (B) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

(C*) $\frac{v_1v_2+v_2v_3+v_3v_1}{3}$ (D) $\frac{v_1v_2v_3}{3}$

Sol. Let Total Displacement = s $\quad s_1=s_2=s_2=\frac{s}{3}$

Average Velocity $v_{avg}=\frac{s}{t_1+t_2+t_3}=\frac{s}{\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}}=\frac{s}{\frac{s}{3v_1}+\frac{s}{3v_2}+\frac{s}{3v_3}}$

$=\frac{3v_1v_2v_3}{\left[v_2v_3+v_1v_3+v_1v_2\right]}$

11. The numerical ratio of displacement to the distance covered is always :-

(A) Less than one (B) Equal to one

(C*) Equal to or less than one (D) Equal to or greater than one

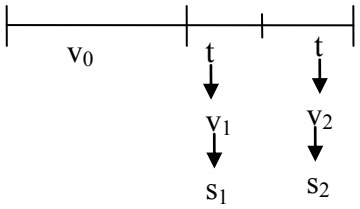
Sol. $\frac{|s|}{d}\le 1$

12. A point object traverses half the distance with velocity v_0. The remaining part of the distance is covered with velocity v_1 for the half time and with velocity v_2 for the rest half. The average velocity of the object for the whole journey is

(A) $2v_1(v_0 + v_2) / (v_0 + 2v_1 + 2v_2)$ (B) $2v (v_0 + v_1)/(v_0 + v_1 + v_2)$

(C*) $2v_0 (v_1 + v_2) / (v_1 + v_2 + 2v_0)$ (D) $2v_2 (v_0 + v_1) / (v_1 + 2v_2 + v_0)$

Sol.



$$s_1 + s_2 = \frac{s}{2} \Rightarrow V_t t + V_2 t = \frac{s}{2}$$

$$t = \frac{s}{2(v_1 + v_2)}$$

Average velocity $v_{avg} = \dfrac{s}{t_1 + 2t} = \dfrac{s}{\dfrac{s}{2v_0} + 2\left(\dfrac{s}{2(v_1 + v_2)}\right)}$

$$v_{avg} = \frac{2v_0(v_1 + v_2)}{(v_1 + v_2 + 2v_0)}$$

13. Select the incorrect statements from the following.

S1: Average velocity is path length divided by time interval.

S2: In general, average speed is greater than the magnitude of the average velocity

S3: A particle moving in a given direction with a non-zero velocity can have zero speed.

S4: The magnitude of average velocity is the average speed.

(A) S2 and S3 (B) S1 and S4 (C*) S1, S3 and S4 (D) All four statements

Sol.

14. An athlete completes one round of a circular track of radius R in 20 seconds. What will be his displacement at the end of 2 minutes 20 seconds?

(A*) Zero (B) 2R (C) $2\pi R$ (D) $7\pi R$

Sol. One Round in 20 sec.

Number of Round in 2 minute 20 sec are $n = \dfrac{140}{20} = 7$ round.

so displacement s = 0

15. If the distance covered is zero, the displacement:

(A*) Must be zero (B) may or may not zero

(C) Cannot be zero (D) depends upon the particle

Sol.

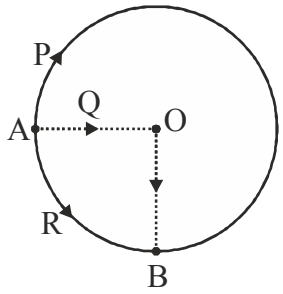
16. If displacement of a particle is zero, the distance covered:

(A) Must be zero (B*) may or may not be zero

(C) Cannot be zero (D) depends upon the particle

Sol.

17. Three particles P, Q and R are situated at point A on the circular path of radius 10 m. All three particles move along different paths and reach point B as shown in figure. Then the ratio of distance traversed by particles P and Q is :



(A) $\frac{3}{4}$ (B) $\frac{1}{3}$ (C*) $\frac{3\pi}{4}$ (D) $\frac{\pi}{3}$

Sol. distance $d_Q = 2R$

$$d_P = 2\pi R \times \frac{3}{4} = \frac{3\pi R}{2}$$

$$\frac{d_P}{d_Q} = \frac{3\pi R}{2} \times \frac{1}{2R} = \frac{3\pi}{4}$$

WORK BOOK # 2

ACCELERATION, AVERAGE ACCLERATION & APPLICATION OF CALCULUS

1. If x denotes displacement in time t and x = acost, then acceleration is :
(A) acost (B*) – acost (C) asint (D) –a sint

Sol. $v = \frac{dx}{dt} = -a\sin t$

$a = \frac{dv}{dt} = -a\cos t$

2. The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to:
(A) $\frac{2a}{3b}$ (B) $\frac{a}{b}$ (C*) $\frac{a}{3b}$ (D) zero

Sol. $x = at^2 - bt^3$

velocity $v = \frac{dx}{dt} = 2at - 3bt^2$

Acceleration $a_0 = \frac{dv}{dt} = 2a - 6bt$

$\Rightarrow 2a - 6bt = 0$

$\Rightarrow t = \frac{a}{3b}$

3. The displacement of a particle starting from rest (at t= 0) is given by $s = 6t^2 - t^3$
The time when the particle will attain zero velocity again, is:
(A*) 4s (B) 8s (C) 12 s (D) 16s

Sol. $v = \frac{ds}{dt} = \frac{d}{dt}(6t^2 - t^3) = 12t - 3t^2$

$12t - 3t^2 = 0 \quad \Rightarrow t = 4s$

4. The velocity of a body depends on time according to the equation, $v = 20 + 0.1t^2$. The body has:
(A) Uniform acceleration (B) uniform retardation
(C*) non-uniform acceleration (D) zero acceleration

Sol. $a = \frac{dv}{dt} = \frac{d}{dt}(20 + 0.1t^2) = 0.2\ t$

Acceleration is a function of time so a is not constant.

5. A body is moving according to the equation $x = at + bt^2 - ct^3$. Then its instantaneous speed is given by:-

(A) $a + 2b + 3ct$ (B*) $a + 2bt - 3ct^2$ (C) $2b - 6t$ (D) None of these

Sol. Instantaneous speed $v = \frac{dx}{dt} = \frac{d}{dt}(at + bt^2 - ct^3) = a + 2bt - 3ct^2$

6. The motion of a particle described by the equation $x = a + bt^2$ where a = 15 cm and b = 3 cm/sec^2. Its instantaneous velocity at time 3 sec will be:-

(A) 36 cm /sec (B*) 18 cm/sec (C) 16 cm/sec (D) 32 cm/sec

Sol. Instantaneous velocity $v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt$

$v_{t=3} = 2 \times 3 \times 3 = 18$ cm/sec

7. If for a particle position $x \propto t^2$ then:-

(A) Velocity is constant (B*) Acceleration is constant

(C) Acceleration is variable (D) none of these

Sol. $x \propto t^2$ so $x = kt^2$

$$v = \frac{dx}{dt} = \frac{d(kt^2)}{dt} = 2kt \text{ (not constant)}$$

$$a = \frac{dv}{dt} = \frac{d(2kt)}{dt} = 2k \text{ (Constant acceleration)}$$

8. The displacement of a particle is represented by the following equation: $s = 3t^3 + 7t^2 + 5t + 8$ Where s is in metres and t in second. The acceleration of the particle at t = 1s is:-

(A) 14 m/s^2 (B) 18 m/s^2 (C*) 32 m/s^2 (D) zero

Sol. $$v = \frac{ds}{dt} = \frac{d(3t^3 + 7t^2 + 5t + 8)}{dt}$$

$$a = \frac{dv}{dt} = \frac{d(9t^2 + 14t + 5)}{dt} = 18t + 14$$

at t =1 $a = 18 + 14 = 32 m/sec^2$

Passage for Ques. 9 to 11

A particle is moving along a straight line and it position is given by $x = t^3 - t^2 + 1$ velocity is given by $\frac{dx}{dt}$ and acceleration is given by $\frac{d^2x}{dt^2}$

9. Velocity of the particle at t = 2s is -

(A) 2 m/s (B) 4 m/s (C) 6 m/s (D*) 8 m/s

Sol. $v = \frac{dx}{dt} = \frac{d(t^3 - t^2 + 1)}{dt} = 3t^2 - 2t$

$\Rightarrow$ at t = 25

$v = 3(2)^2 - 2(2) = 8$m/sec

10. Acceleration of the particle at t = 1s

(A) 2 m/s^2 (B*) 4 m/s^2 (C) 6 m/s^2 (D) zero

Sol. $a = \frac{dv}{dt} = \frac{d(3t^2 - 2t)}{dt} = 6t - 2$

at t = 1s $a = 6(1) - 2 = 4$m/sec

11. Time at which acceleration is zero -

(A) $t = \frac{4}{3}$ sec (B) $t = \frac{2}{3}$ sec (C) t = 1 sec (D*) $t = \frac{1}{3}$ sec

Sol. $6t - 2 = 0$

$t = \frac{2}{6} = \frac{1}{3}$ sec

WORK BOOK # 3

APPLICATION OF CALCULAS

1. The relation $t = \sqrt{x} + 3$ describes the position of a particle where x is in meters and t is in seconds. The position, when velocity is zero, is :-

(A) 2 m (B) 4 m (C) 5 m (D*) zero

Sol. $t = \sqrt{x} + 3$

$x = (t-3)^2$

Velocity $v = \frac{dx}{dt} = \frac{d(t-3)^2}{dt} = 2t - 6$

for $v = 0 \Rightarrow 2t - 6 = 0 \Rightarrow t = \frac{6}{2} = 3$ second

so $x_{t=3} = (3-3)^2 = 0$

2. The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively.

(A) b, – 4d (B) –b, 2c

(C*) b , 2c (D) 2c, –4d

Sol. Velocity $v = \frac{dy}{dt} = \frac{d(a+bt+ct^2-dt^4)}{dt} = b + 2ct - 4dt^3$

initial velocity at t =0 is $V_i = b$

Acceleration $a = \frac{dv}{dt} = \frac{d(b+2ct-4dt^3)}{dt} = 2C - 12dt^2$

Initial acceleration at t = 0 is $a_i = 2C$

3. Which of the following relations representing displacement x of a particle describes motion with constant acceleration ?

(A) $x = 6 - 7t^{-2}$ (B) $x = 3t^2 + 5t^3 + 7$

(C*) $x = 9t^2 + 8$ (D) $x = 4t^{-2} + 3t^{-1}$

Sol. velocity $v = \frac{dx}{dt} = \frac{d(9t^2+8)}{dt} = 18t$

Acceleration $\frac{dv}{dt} = \frac{d(18t)}{dt} = 18$

4. Equation of a particle moving along the x axis is $x = u(t-2) + a(t-2)^2$

(A) the initial velocity of the particle is u (B) the acceleration of the particle is a

(C*) the acceleration of the particle is 2a (D*) at t = 2 particle is at origin

Sol. $x = \mu(t-2)+a(t-2)^2$

$$\text{Velocity } v = \frac{dx}{dt} = \frac{d(u(t-2)+a(t-2)^2)}{dt} = u + 2at - 4a$$

$v_{initial} = u - 4a$

$$\text{Acceleration } a_0 = \frac{dv}{dt} = \frac{d(u+2at-4a)}{dt} = 2a$$

at t = 2, x = 0 (particle is at origin).

5. The intial velocity of a particle is u (at t= 0) and the acceleration is given by f = at. Which of the following relations is valid ?

(A) $v = u + at^2$ (B*) $v = u + \frac{at^2}{2}$ (C) $v = u + at$ (D) $v = u$

Sol. Acceleration $f = \frac{dv}{dt} \Rightarrow dv = fdt$

$$\Rightarrow \int_u^v dv = \int_0^t atdt \Rightarrow v - u = \frac{at^2}{2}$$

$$\Rightarrow v = u + \frac{at^2}{2}$$

6. Starting from rest, the acceleration of a particle is a = 2(t–1). The velocity of the particle at t = 5s is :-

(A*) 15 m/s (B) 25 m/s (C) 5 m/s (D) None of these

Sol. dv = adt

$$\int_0^V dV = \int_0^{t=5} 2(t-1)dt$$

$$V = \left[\frac{2t^2}{2}\right]_0^5 - [2t]_0^5 = 25 - 10 = 15 \text{ m/s}$$

7. A particle moves in a straight line so that $t = \sqrt{x^2-1}$ then a is equal to -

(A*) $\frac{1}{x^3}$ (B*) $\frac{1}{x} - \frac{t^2}{x^3}$ (C) $\frac{t^2}{x^3}$ (D) $\frac{t^2}{x^3} - \frac{1}{x^2}$

Sol.

8. The velocity-time relation of an electron starting from rest is given by u = kt, where k = 2 m/s^2. The distance traversed in 3 sec is :

(A*) 9 m (B) 16 m (C) 27 m (D) 36 m

Sol. dx = udt

$$\int_0^x dx = \int_0^3 ktdt = \frac{k}{2}\left[t^2\right]_0^3$$

$$= \frac{2}{2}\left[(3)^2\right] = 9 \text{ m}$$

Passage for Q.9 to Q.11

A particle is moving along x axis with acceleration a = (6m/s^3)t – 6m/s^2. If particle is initially at the origin and velocity is v_0 = 2m/s.

9. Velocity of particle is given by -

(A) (6t – 6) (B*) $3t^2 - 6t + 2$ (C) $3t^2 - 6t - 2$ (D) $3t^2 - 6t$

Sol. $$\int_{v_0}^{v} dv = \int_0^t adt \Rightarrow \int_{v_0}^{v} dv = \int_0^t (6t-6)\,dt \Rightarrow v - v_0 = \frac{6t^2}{2} - 6t \Rightarrow v = \frac{6t^2}{2} - 6t + v_0$$

$\Rightarrow V = 3t^2 - 6t + 2$

10. Position of particle is given by -

(A) $3t^2 - 6t + 2$ (B*) $t^3 - 3t^2 + 2t$ (C) $3t^2 - 6t$ (D) $3t^2 - 6t + 5$

Sol. $$\int_0^x dx = \int_0^t Vdt$$

$$\Rightarrow x = \int_0^t \left(3t^2 - 6t + 2\right) \Rightarrow x = \frac{3t^3}{3} - \frac{6t^2}{2} + 2t$$

$\Rightarrow x = t^3 - 3t^2 + 2t$

11. Particle comes at x = 0 at -

(A*) t = 1s (B*) t = 2s (C) t = 3s (D) t = 4s

Sol. $t^3 - 3t^2 + 2t = 0$

at $t = 1 \Rightarrow x = 1 - 3 + 2 = 0$

at $t = 2 \Rightarrow x = 8 - 12 + 4 = 0$

WORK BOOK # 4

1. A particle located at x = 0 at time t = 0, starts moving along the positive x-direction with a velocity 'v' which varies as $v = \alpha\sqrt{x}$, then velocity of particle varies with time as : (α is a constant)

(A*) $v \propto t$ (B) $v \propto t^2$ (C) $v \propto \sqrt{t}$ (D) v = constant

Sol. $v = \alpha\sqrt{x}$ ...(1)

$$\frac{dx}{dt} = \alpha\sqrt{x} \quad \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt \quad \Rightarrow \int_0^x x^{-\frac{1}{2}} dx = \int_0^t \alpha dt$$

$$\Rightarrow \frac{x^{\frac{1}{2}}}{1/2} = \alpha t \Rightarrow 2\sqrt{x} = \alpha t$$

$$\Rightarrow \sqrt{x} = \frac{\alpha t}{2} \qquad ...(2)$$

From eq (1) & (2)

$$v = \alpha\left(\frac{\alpha t}{2}\right) = \frac{\alpha^2 t}{2}$$

So $v \alpha t$

2. A particle moves along a straight line such that its displacement at any time t is given by $s = t^3 - 6t^2 + 3t + 4$ metres. The velocity when the acceleration is zero is :

(A) 3 m/s (B) –12 m/s (C) 42 m/s (D*) –9 m/s

Sol. $$v = \frac{ds}{dt} = \frac{d(t^3 - 6t^2 + 3t + 4)}{dt} = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt} = \frac{d(3t^2 - 12t + 3)}{dt} = 6t - 12$$

$$\Rightarrow 6t - 12 = 0 \Rightarrow t = \frac{12}{6} = 2 \text{ sec.}$$

So $v = 3(2)^2 - 12(2) + 3 = -9$ m/s.

3. for a body moving on a straight line –
(A) Average speed can be less than the minimum speed attained by the body.
(B*) Average speed cannot be less than the minimum speed attained by the body
(C*) Magnitude of average velocity can be less than the minimum speed attained
(D) Magnitude of average velocity cannot be less than minimum speed attained.

Sol.

4. An object may have–
(A) Varying speed without having varying velocity
(B*) Varying velocity without having varying speed
(C) Non-zero acceleration without having varying velocity
(D*) Non-zero acceleration without having varying speed

Sol.

5. If the velocity of a body is constant -
(A*) |Velocity| = speed
(B*) |Average velocity| = speed
(C*) Velocity = average velocity
(D*) Speed = average speed

Sol. If velocity of a body is constant means, body moves with constant speed along a straight line.

6. A body moves so that it follows the following relation $\frac{dv}{dt} = -v^2 + 2v - 1$ where v is speed in m/s and t is time in second. If at t = 0, v = 0 then
(A*) Terminal velocity is 1 m/s
(B*) The magnitude of initial acceleration is 1 m/s^2
(C) Instantaneous speed is $v = \frac{-1}{1+t}$
(D*)The speed is 1.5 m/s when acceleration is one fourth of its initial value

Sol. Here $\frac{dV}{dt}$ is acceleration a

(A) Terminal velocity when a = 0 so $-V^2 + 2V - 1 = 0$

$-V^2 + V + V - 1 = 0 \Rightarrow -V(V-1) + 1(V-1)$

$(-V+1)(V-1) = 0$ V = 1 m/sec

(B) When V = 0 a = 1 m/sec

(D) $\frac{1}{4} = -V^2 + 2V - 1 \Rightarrow V = 1.5$

7. Equation of a particle moving along the x axis is :

$x = u(t-2) + a(t-2)^2$

(A) the initial velocity of the particle is u (B) the acceleration of the particle is a

(C*) the acceleration of the particle is 2a (D*) at t = 2 particle is at origin

Sol. $V = \frac{dx}{dt} = \frac{d\left(u(t-2) + a(t-2)^2\right)}{dt} = u + 2at - 4a$ at $t = 0$ $v_{t=0} = u - 4a$

Acceleration $a_0 = \frac{dv}{dt} = \frac{d(u + 2at - 4a)}{dt} = 2a$

at $t = 2$ $x = 0$

8. The position of particle travelling along x-axis is given by $x_t = t^3 - 9t^2 + 6t$ where x_t is in cm and t is in second. Then–

(A*) the body comes to rest firstly at $(3 - \sqrt{7})$s and then at $(3 + \sqrt{7})$ s

(B) the total displacement of the particle in travelling from the first zero of velocity to the second zero of velocity is zero

(C*) the total displacement of the particle in travelling from the first zero of the velocity to the second zero of velocity is –74 cm

(D*) the particle reverses its velocity at $(3 - \sqrt{7})$ s and then at $(3 + \sqrt{7})$s and has a negative velocity for $(3 - \sqrt{7})s < t < (3 + \sqrt{7})s$

Sol.

Passage for Ques. 9 to 11

A particle moves in positive x-direction according to law $s = 12t - t^2$ m. where 't' time in second. (Take + ve x-direction as + ve)

9. Average velocity from t = 0 to t = 8 sec is

(A*) 4 m/s (B) 6 m/s (C) 8 m/s (D) – 4 m/s

Sol. $s = 12t - t^2$

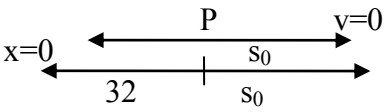
Displacement $s_{t=8} = 12(8) - (8)^2 = 32$

Average velocity $v_{avg} = \frac{32}{8} = 4$m/sec

10. Average speed from t = 0 to t = 8 sec is -

(A) 4 m/s (B*) 5 m/s (C) 6 m/s (D) 8 m/s

Sol. $s = 12t - t^2m$ Average speed t = 0 to t = 8sec



$s = 12 \times 8 - (8)^2 = 32$

$v = \frac{dx}{dt} = 12 - 2t$ $v = 0 = 12 - 2t$

t = 6 sec

$P = 12 \times 6 - 36 = 4m$ so $D = 32 + (2s_0) = 32 + (2 \times 4) = 40$

so $V_{avg} = \frac{40}{8} = 5m/sec$

11. Average acceleration from t = 0 sec to t = 8 sec is-

(A) $-\frac{1}{4}$ m/s^2 (B) $+\frac{1}{4}$ m/s^2 (C) $-\frac{1}{2}$ m/s^2 (D*) – 2 m/s^2

Sol. $v = 12 - 2t$

$v_{t=0} = 12$ $v_{t=8} = 12 - 2(8) = -4m/sec$

$a_{avg} = \frac{V_2 - V_1}{t} = \frac{-4-12}{8} = \frac{-16}{8} = -2$ m/sec^2

WORK BOOK # 5

1. The velocity of a particle is zero at t = 0, then -
(A) the acceleration at t = 0 must be zero
(B*) the acceleration at t = 0 may be zero
(C*) if the acceleration is zero from t = 0 to t = 10 s. the speed is also zero in this interval.
(D*) if the speed is zero from t = 0 to t = 10 sec, then the acceleration is also zero in the interval

Sol.

2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion –
(A) $|\vec{v}|$ must decrease when $\vec{a} < 0$
(B) Speed must increase when $\vec{a} > 0$
(C*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0
(D*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$

Sol. If acceleration is in same direction as velocity, then speed of the particle increases.

If acceleration is in opposite to the velocity then speed decreases, the particle slow down.

3. Pick the correct statements –
(A*) Average speed of a particle in a given time is never less than the magnitude of the average velocity
(B*) It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d}{dt}|\vec{v}| = 0$
(C*) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval
(D) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval (Infinite accelerations are not allowed)

Sol.

4. The displacement (x) of a particle depends on time (t) as :

$$x = \alpha t^2 - \beta t^3$$

(A*) The particle will return to its starting point after time α/β
(B*) The particle will come to rest after time $2\alpha/3\beta$
(C*) The initial velocity of the particle was zero but its initial acceleration was not zero.
(D*) No net force will act on the particle at $t = \alpha/3\beta$

Sol. $x = \alpha t^2 - \beta t^3$

(A) For $t = \alpha/\beta$ $\quad x = \alpha\left(\frac{\alpha}{\beta}\right)^2 - \beta\left(\frac{\alpha}{\beta}\right)^3 \Rightarrow \frac{\alpha^3}{\beta^2} - \frac{\alpha^3}{\beta^2} = 0$

(B) $v = \frac{dx}{dt} = \frac{d\left(at^2 - bt^3\right)}{dt} = 2\alpha t - 3\beta t^2$

Now $2\alpha t - 3\beta t^2 = 0 \Rightarrow 2\alpha - 3\beta t = 0$

$\Rightarrow t = \dfrac{2\alpha}{3\beta}$

$v_{t=0} = 0$

(C) $a = \dfrac{dv}{dt} = \dfrac{d(2\alpha t - 3\beta t^2)}{dt} = 2\alpha - 6\beta t \quad \Rightarrow \quad a_{t=0} = 2\alpha$

(D) At $t = \alpha/3\beta \qquad a = 2\alpha - 6\beta\left(\dfrac{\alpha}{3\beta}\right) = 2\alpha - 2\alpha = 0$

No Net force.

5. The motion of a particle moving along the y-axis is represented as $y = 3\ (t - 2) + 5\ (t - 2)^2$. Identify the correct statement -
(A) The initial (t = 0) velocity of the particle is 3 ms^{-1}
(B) The acceleration of the particle is 5 ms^{-2}
(C*) The particle is at the origin at t = 2s
(D) All of the above

Sol. $y = 3(t - 2) + 5\ (t - 2)^2$

$V = \dfrac{dy}{dt} - 10t - 17 \qquad V_{t=0} = -17 \qquad (AX)$

$a = \dfrac{dv}{dt} = 10 \qquad (BX)$

At t = 2s $\qquad y = 0$

Passage for Ques. 6 to 8

A particle is moving along a straight line and position of particle is given by

$$x = \left(\frac{1m}{s}\right)t - \left(\frac{1m}{s^2}\right)t^2$$

Where x is meter and t is in second.

6. Distance travelled by the particle in 2 second is -
(A) 2 m (B*) 2.5 m (C) 1.25 m (D) 2.25 m

Sol. $x = \left(\dfrac{1m}{s}\right)t - \left(\dfrac{1m}{s^2}\right)t^2$

$x = t - t^2 \qquad V = 1 - 2t \qquad a = -2$

$V = 0 \ \Rightarrow 1 - 2t = 0 \Rightarrow t = \dfrac{1}{2}$ sec. at t=0 V=1

$s_1 = ut + \frac{1}{2}at^2 = 1\times\frac{1}{2} - \frac{1}{2}\times 2\times\frac{1}{4} = \frac{1}{4}$ m

$s_2 = \frac{1}{2}at^2 = \frac{1}{2}\times 2\times\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$s = s_1 + s_2 = \frac{1}{4} + \frac{9}{4} = \frac{10}{4}$

7. Velocity of particle at t = 2 sec is –

(A*) –3m/s (B) – 2 m/s (C) 1 m/s (D) –1 m/s

Sol. $V = 1 - 2t$ $V_{t=2} = 1 - 2\times 2 = -3$ m/s

8. Acceleration of particle at t = 4s is –

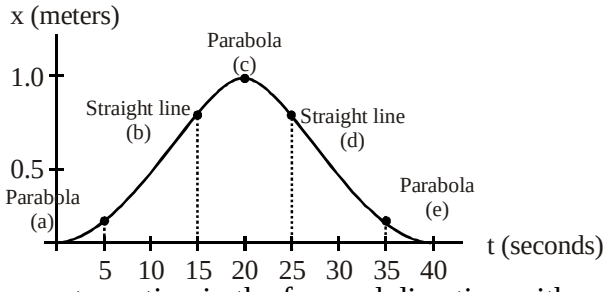
(A) zero (B) –1 m/s^2 (C*) –2m/s^2 (D) –3 m/s^2

Sol. a = –2 Always.

WORK BOOK # 7

KINEMATICS GRAPHS

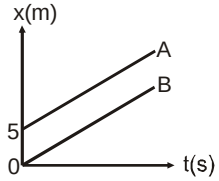
1. The figure below is a displacement vs time plot for the motion of an object, answer questions (i) & (ii) with the letter of appropriate section of the graph.



(i) Which section represents motion in the forward direction with positive acceleration?
(ii) Which section represents uniform motion backwards (–x direction)?

Ans. (i) section (a) as slope = v = $\frac{dx}{dt}$ is positive and increasing. (ii) section (d) as slope = v = $\frac{dx}{dt}$ is negative and constant.

2. Figure shows position-time graph of two cars A and B.

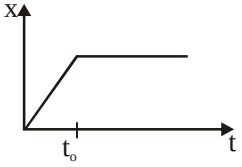


(A) Car A is faster than car B.
(B) Car B is faster than car A.
(C*) Both cars are moving with same velocity.
(D) Both cars have positive acceleration.

Sol. Slope : $\frac{dx}{dt}$ = velocity

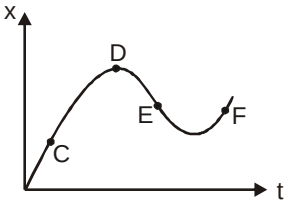
s lope is same so V is same (C)

3. Fig. shows the displacement time graph of a particle moving on the X-axis.



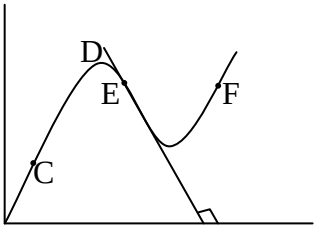
(A) the particle is continuously going in positive x direction
(B) the particle is at rest
(C) the velocity increases up to a time t_o, and then becomes constant.
(D*) the particle moves at a constant velocity up to a time t_o, and then stops.

4. The displacement–time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point:



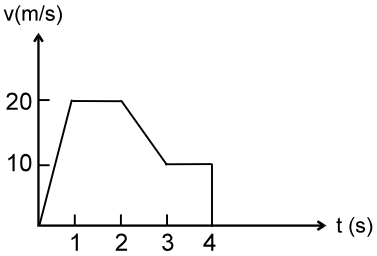
(A) C (B) D (C*) E (D) F

Sol.



(C) Point E, negative velocity

5. The variation of velocity of a particle moving along a straight line is shown in the figure. The distance travelled by the particle in 4 s is :



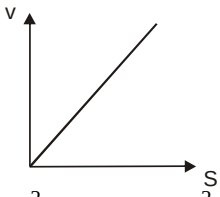
(A) 25 m (B) 30 m (C*) 55 m (D) 60 m

Sol. D = Area under the wave.

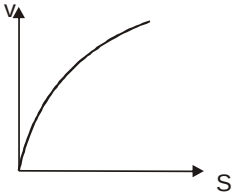
= 10 + 20 + 20 + 5 = 55 m

6. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is :

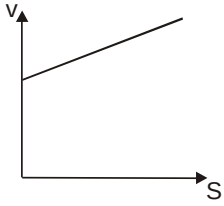
(A)



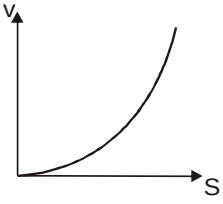
(B*)



(C)



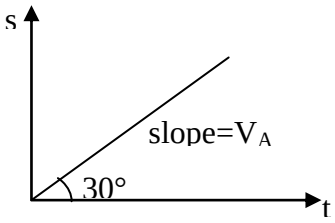
(D)



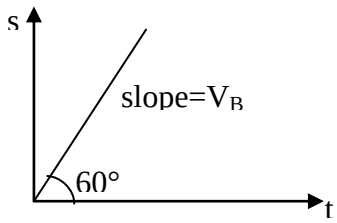
Sol. $V^2 = u^2 + 2as \Rightarrow V^2 = 2as \Rightarrow s \propto V^2$

7. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30^0 and 60^0 with the time axis. If the velocity of A is v_A and that of B is v_B, then the value of $\frac{v_A}{v_B}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D*) $\frac{1}{3}$



A



B

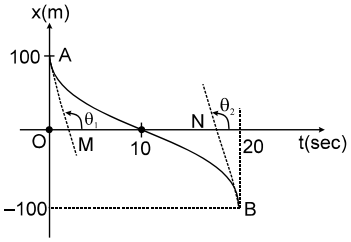
Sol.

$V_A = \tan 30°$ $V_B = \tan 60°$

$$\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

Passage For Ques. 8 to 10

The graph below gives the displacement of a particle travelling along the X-axis as a function of time. AM is the tangent to the curve at the starting moment and BN is tangent at the end moment ($\theta_1 = \theta_2 = 120°$).



8. The average velocity during the first 20 seconds is

(A*) – 10 m/s (B) 10 m/s (C) zero (D) 20 m/s

Sol. $V_{avg} = \frac{\text{Displacement}}{\Delta\text{Time}} = \frac{s_2 - s_1}{\Delta t} = \frac{-100-100}{20} = \frac{-200}{20} = -10\text{m/s}$

so $V_{avg} = -10\text{m/s}$

9. The average acceleration during the first 20 seconds is

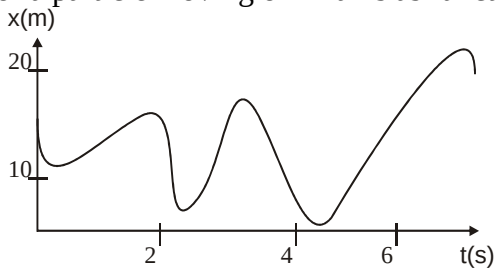
(A) – 10 m/s^2 (B) 10 m/s^2 (C*) zero (D) 20 m/s^2

Sol. $a_{avg} = \frac{V_2 - V_1}{\Delta t} = \frac{\tan 120 - \tan 120}{20} = 0$

10. Time interval during which the motion is retarded.

(A) 0 to 20sec. (B) 10 to 20sec. (C*) 0 to 10sec. (D) None of these

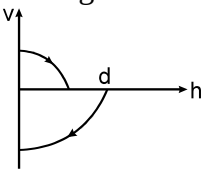
11. Fig. shows the position of a particle moving on X-axis as function of time.

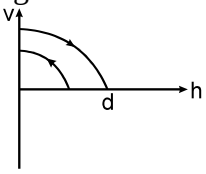


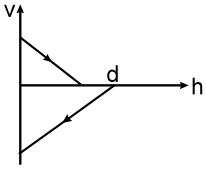
(A*) The particle has come to rest 6 times
(B) The maximum speed is at t = 6 s
(C) The velocity remains positive for t = 0 to t = 6 s
(D) The average velocity for the total period shown is negative.

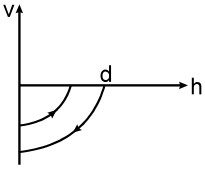
Sol. Slope is zero six time.

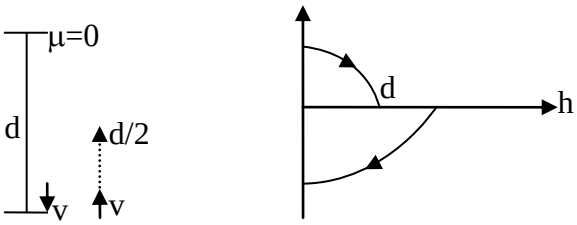
12. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(A*) 

(B) 

(C) 

(D) 

Sol. 

WORK BOOK # 8

MOTION UNDER GRAVITY

1. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by

(A) $T = \frac{2h}{v}$ (B*) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$ (C) $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$ (D) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

Sol. $h = \frac{1}{2}gt^2$, $t_1 = \sqrt{\frac{2h}{g}}$

$t_2 = \frac{h}{V}$ $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{V}$

2. A stone thrown upwards with a speed 'u' from the top of the tower reaches the ground with a velocity '3u'. The height of the tower is:

(A) $\frac{3u^2}{g}$ (B*) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$

Sol. $(-V)^2 = v^2 - 2gh$

$V2 = u^2 - 2gh$

$-2gh = V^2 - u^2 = (3u)^2 - u^2 = 8u^2$

$h = \frac{8u^2}{2g} = \frac{4U^2}{g}$

3. A stone falls from a balloon that is descending at a uniform speed of 12 ms^{-1}. The displacement of the stone from the point of release after 10 seconds is :
($g = 9.8\ m/s^2$)

(A) 490 m (B) 510 m (C*) 610 m (D) 725 m

Sol. U = 12 m/s t = 10 sec

$s = ut = \frac{1}{2}gt^2 = 12\times10 + \frac{1}{2}\times10\times100 = 120 + 500 = 620$

$s = 12 \times 10 + \frac{1}{2}\times9.8\times100 = 120 + 490 = 610$ m

4. A rocket is fired vertically from the ground. It moves upwards with a constant acceleration of 10 m/s^2. After 30 seconds the fuel is finished. After what time from the instant of firing the rocket will it attain the maximum height ? $g = 10\ m/s^2$:

(A) 30 s (B) 45 s (C*) 60 s (D) 75 s

Sol. $V = u + at = 0 + 10 \times 30 = 300$ m/sec

Total time = 30 + 30 = 60 sec

5. A body is released from the top of a tower of height H metres. It takes t time to reach the ground. Where is the body $\frac{t}{2}$ time after the release :

(A) At $\frac{H}{2}$ metres from ground (B*) At $\frac{H}{4}$ metres from ground

(C) At $\frac{3H}{4}$ metres from the ground (D) At $\frac{H}{6}$ metres from the ground

Sol. $H = \frac{1}{2}gt^2$ $t = \sqrt{\frac{2H}{g}}$ $t' = t/2$

so $h = \frac{1}{2}gt'^2 = \frac{1}{2}g\left(\frac{t^2}{4}\right) = \frac{g}{2}\left[\frac{2H}{gH}\right] = \frac{H}{4}$ m from to b

$H - \frac{H}{4} = \frac{3H}{4}$ from ground.

6. A body dropped from the top of a tower covers a distance 7x in the last second of its journey, where x is the distance covered in first second. How much time does it take to reach the ground ?

(A) 3s (B*) 4s (C) 5s (D) 6s

Sol. $s_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(1)^2 = 5$ or $\frac{g}{2} \Rightarrow x = \frac{g}{2}$

$s_2 = \frac{g}{2}(2n-1) \Rightarrow 7x = \frac{g}{2}(2n-1) \Rightarrow \frac{7.g}{2} = \frac{g}{2}(2n-1)$

$\Rightarrow 2n = 7 + 1 = 8$ $n = 4$ sec

7. A body falling from height 'h' takes t_1 time to reach the ground. The time taken to cover the first half of the height is

(A*) $t_2 = \frac{t_1}{\sqrt{2}}$ (B) $t_1 = \frac{t_2}{\sqrt{2}}$ (C) $t_2 = \sqrt{3}\ t_1$ (D) None of these

Sol. $h = \frac{1}{2}gt_1^2$ $\frac{h}{2} = \frac{1}{2}gt_0^2$

$t_0^2\frac{g}{2}\times\frac{2}{gt_1^2} = \frac{h}{2}\times\frac{1}{h}$

$t_0^2 = \frac{t_1^2}{2}$ $t_0 = \frac{t_1}{\sqrt{2}}$

8. If an iron ball and a wooden ball of same radii are released from a height h in vacuum then time taken by both of them to reach ground will be :

(A) unequal (B*) exactly equal (C) roughly equal (D) zero

Sol.

Passage For Ques. 9 to 11

Height of a tower is 80 m. Two balls A and B are thrown simultaneously. Ball A is thrown upwards with speed u from top of tower while ball B is thrown upwards with speed of 50 m/s from the foot of tower

9. Ball A will reaches ground in 8 sec if ball do not collide then u is equal to –

(A) 20 m/s (B) 25 m/s (C*) 30 m/s (D) 35 m/s

Sol. $h = \frac{1}{2}gt_1^2$ $\frac{h}{2} = \frac{1}{2}gt_0^2$

$t_0^2 \frac{g}{2} \times \frac{2}{gt_1^2} = \frac{h}{2} \times \frac{1}{h}$

$t_0^2 = \frac{t_1^2}{2}$ $t_0 = \frac{t_1}{\sqrt{2}}$

10. If the balls meets in air at the height from foot of tower is –

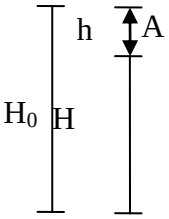
(A) 40 m (B) 80 m (C*) 120 m (D) 160 m

Sol. So $H_0 = 50t - \frac{1}{2}gt^2 = 50 \times 4 - \frac{1}{2} \times 10 \times 16 = 120$ m from base.

11. The time after which balls will meet in air is –

(A) 3 sec (B*) 4 sec (C) 5 sec (D) 6 sec

Sol.



$h = 30t - \frac{1}{2}gt^2$ $(80+h) = 50 - \frac{1}{2}gt^2$

$h - (80+h) = 30t - 50t \Rightarrow$ $80 = 20t \Rightarrow t = 4$ sec

Passage For Ques. 12 to 14

A body is projected vertically upwards with a velocity 98 m/s from ground. Choose upward direction positive y axis.

12. Maximum height attained by the body is

(A*) 490 m (B) 245 m (C) 980 m (D) 122.5 m

Sol. $\uparrow V = 98 m/s$

$$H_{max} = \frac{U^2}{2g} = \frac{98 \times 98}{2 \times 9.8} = 490 \text{ m}$$

13. Velocity with which it will fall to the point of projection is –

(A) 98 m/s (B*) – 98 m/s (C) 49 m/s (D) – 49 m/s

Sol. –98 m/sec $\uparrow V$ $\downarrow V$

14. Time taken to reach the maximum height is

(A) 20 s (B*) 10 s (C) 5 s (D) 15 s

Sol. $V = u - gt \quad \Rightarrow O = u - gt$

$$t = \frac{u}{g} = \frac{98}{9.8} = 10 \text{ s}$$

WORK BOOK # 9

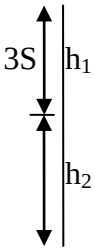
1. If a ball is thrown vertically upwards with 40 m/s. its velocity after two seconds will be
(A) 10 m/s (B*) 20 m/s (C) 30 m/s (D) 40 m/s

Sol. ↑ V = 40 m/s t = 2 sec

$V = u - gt = 40 - 10\times 2 = 20$ m/s

2. A stone is dropped from a certain height which can reach the ground in 5 seconds. It is stopped after 3 seconds of its fall and is again released. The total time taken by the stone to reach the ground will be :
(A) 6 s (B) 6.5 s (C*) 7s (D) 7.5 s

Sol. $H = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times (5)^2 = \frac{10}{2}\times 25 = 125$ m/s



After 2 second it stopped $h_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 9 = 45$ m

So $h_2 = 125 - 45 = 80$ cm $h_2 = \frac{1}{2}gt^2$ {U = 0}

$t_2 = \sqrt{\frac{2h_2}{g}} = \sqrt{\frac{2\times 80}{10}} = \sqrt{16} = 4$ sec

3. With what speed should a body be thrown upwards so that the distances travelled in 5th second and 6th second are equal ?
(A) 58.4 m/s (B*) 49 m/s (C) $\sqrt{98}$ m/s (D) 98 m/s

Sol.

4. Which of the following four statements is false?
(A) A body can have zero velocity and still be accelerated
(B*) A body can have a constant velocity and still have a varying speed
(C) A body can have a constant speed and still have a varying velocity
(D) The direction of the velocity of a body can change when its acceleration is constant

Sol. If velocity is constant then its magnitude must be constant.

5. A body dropped from a tower reaches the ground in 4s. The height of the tower is about :
(A*) 80 m (B) 20 m (C) 160 m (D) 40 m

Sol. $h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 16 = 80$ m.

6. A ball is thrown upward with a velocity of 100 m/s. It will reach the ground after : -
(A) 10 s (B*) 20 s (C) 5 s (D) 40 s

Sol. $\uparrow V = 100$ m/s $V = u - gt \Rightarrow 0 = u - gt \Rightarrow t = \frac{U}{g}$

$t = \frac{100}{10} = 10$ sec

$T = 2T = 20$ sec.

7. If a ball is thrown vertically upwards with speed u, the distance covered during the last 't' second of its ascent is:-
(A) ut (B*) $\frac{1}{2}gt^2$ (C) $ut - \frac{1}{2}gt^2$ (D) $(u + gt)t$

8. A particle is thrown vertical upward. Its velocity at half of the maximum height is 10 m/s. the maximum height attained by it is ($g = 10\ ms^{-2}$) : -
(A) 8 m (B) 20 m (C*) 10 m (D) 16 m

Sol. $V^2 = U^2 - \frac{2gH}{2} \Rightarrow 100 = U^2 - gH$ ....(1)

At max height $V_0 = 0$

$V_0^2 = U^2 - 2gh \Rightarrow o = U^2 - 2gh$ .....(2)

$H = \frac{100}{g} = \frac{100}{10} = 10$ m

9. When a ball is through vertically up with velocity v_0 , it reaches a maximum height 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity –

(A*) $\sqrt{3}v_0$ (B) $3v_0$ (C) $9v_0$ (D) $3/2v_0$

Sol. $H_{max} = \frac{V_0^2}{2g}$ ....(1) $H' = 3H_{max}$

$H' = \frac{V^2}{2g} \Rightarrow 3H_{max} = \frac{U^2}{2g}$ ...(2)

$\frac{U^2}{2g} \times \frac{2g}{V_0^2} = \frac{3H_{max}}{H_{max}} \Rightarrow H_{max} \propto U^2 \Rightarrow \frac{H_2}{H_1} = \frac{U_2^2}{U_1^2}$

10. A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 second it remained in air for :-

(A) 12 s (B*) 13 s (C) 35 s (D) 26 s

Sol. $S_5 = \frac{1}{2}gt^2 = \frac{1}{2} \times g \times (5)^2 = \frac{25g}{2}$

$S_t = 0 + \frac{g}{2}(2n-1) = \frac{g}{2}(2n-1)$ $[s_t = S_5]$

so $\frac{25g}{2} = \frac{g}{2}(2m-1) \Rightarrow 2n-1 = 25 \Rightarrow 2n = 25 + 1 = 26 \Rightarrow$ $n = \frac{26}{2} = 13$ sec

11. An object is dropped vertically down on earth. The change in its speed after falling through a distance d from its highest point is

(A) mgd (B*) $\sqrt{2gd}$ (C) $2\sqrt{g/d}$ (D) $2\sqrt{\frac{mg}{d}}$

Sol. U = 0

$\Delta V = V - 0$

$V^2 = u^2 - 2gd \Rightarrow V = \sqrt{2gd}$

$\Delta V = V - 0 = \sqrt{2gd}$

12. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects on reaching the ground, will be in the ratio of :

(A) $m_1 : m_2 : m_3$ (B) $m_1 : 2m_2 : 3m_3$ (C*) 1 : 1 : 1 (D) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

Passage For Ques. 13 to 14

A particle thrown vertically upwards with velocity u m/s.

13. Time taken by particle to reach at maximum height-

(A) $\frac{2u}{g}$ (B*) $\frac{u}{g}$ (C) $\frac{u^2}{2g}$ (D) $\frac{u^2}{g}$

Sol. $\uparrow V = U$ m/s

At max Height V = 0 so, $t = \frac{U}{g} \Rightarrow \frac{U}{g}$

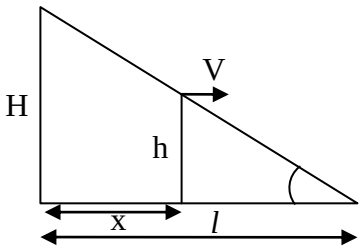
14. Total distance travelled by particle before reach at ground surface-

(A) $\frac{2u}{g}$ (B) $\frac{u}{g}$ (C) $\frac{u^2}{2g}$ (D*) $\frac{u^2}{g}$

Sol. $H_{max} = \frac{U}{2g}$ so total $D = 2H = 2 \times \frac{U^2}{2g} = \frac{U^2}{g}$

15. A man walking with a speed 'v' constant in magnitude and direction passes under a lantern hanging at a height H above the ground. Find the velocity with which the edge of the shadow of the man's head moves over the ground, if his height is 'h'.

Sol.



$\tan\theta(\text{shadow}) = \tan\theta(\text{max})$

$$\frac{H}{\ell} = \frac{h}{(\ell - x)}$$

$$\ell h = H(\ell - x) \Rightarrow H\ell - \ell h = Hx$$

$$\ell(H - h) = Hx \Rightarrow \ell = \frac{Hx}{H-h} \quad ; \quad \frac{d\ell}{dt} = \frac{H}{H-h}\frac{dx}{dt} \qquad V_0 = \frac{HV}{(H-h)}$$

WORK BOOK # 1

(INTRODUCTION OF UNIT VECTOR)

1. A particle is situated at the origin of a coordinate system. The following four forces begin to act on the particle simultaneously.

$\vec{F}_1 = (2\hat{i}+3\hat{j}-\hat{k})\,N, \vec{F}_2 = (3\hat{i}-\hat{j}+\hat{k})\,N; \vec{F}_3 = (2\hat{i}-5\hat{j}+3\hat{k})\,N; \vec{F}_4 = (5\hat{i}+3\hat{j}-3\hat{k})\,N$

The particle will move

(A) along x-axis (B) along y-axis (C) in x-y plane (D) in y-z plane

Sol. Particle is at origin.

$\vec{F}$ (Net force) = $\vec{F}_1+\vec{F}_2+\vec{F}_3+\vec{F}_4$

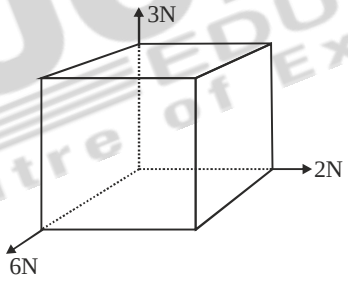
$\vec{F} = (2\hat{i}+3\hat{j}-\hat{k}) + (3\hat{i}-\hat{j}+\hat{k}) + (2\hat{i}-5\hat{j}+3\hat{k}) + (5\hat{i}+3\hat{j}-3\hat{k})$

$\vec{F} = 12\hat{j}+O+O$

$\vec{F} = 12\hat{i}$

So the particle will move along x axis

2. Three forces of magnitudes 2 N, 3 N and 6 N act at a corner of a cube along three edges, as shown in figure. Find the resultant of these forces in N.

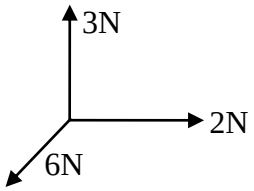


Sol. $\vec{F}_{net} = 2\hat{i}+3\hat{k}+6\hat{j}$

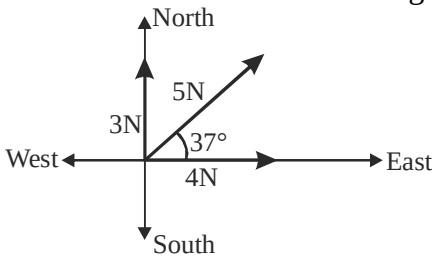
$= 2\hat{i}+6\hat{j}+3\hat{k}$

$|\vec{F}| = \sqrt{(2)^2+(6)^2+(3)^2}$

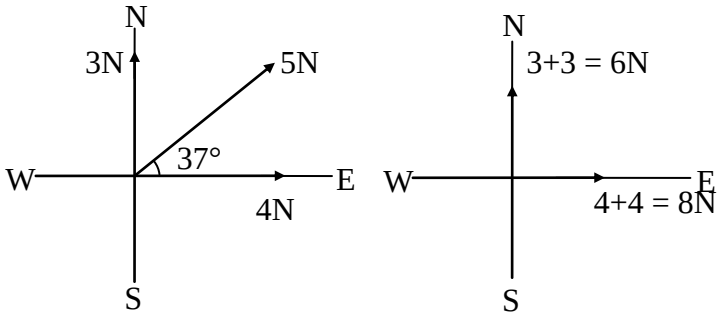
F = 7 N



3. Three forces (shown in figure) act on a body simultaneously. What will be the magnitude (in newton) of smallest additional force to make net force along east direction ?



Sol.



In x $F_x = 4 + 5\cos 37$

$F_x = 8N$

In y $F_y = 3 + 5 \sin 37$

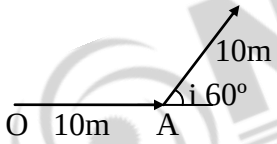
$F_y = 3 + 3 = 6N$

So add 6N towards south to make net force along east direction.

4. A man moves in an open field such that after moving 10 m in a straight line, he makes a sharp turn of 60° to his left. The total displacement of the man just after 7 such turns is

(A) 10 m (B) 20 m (C) 70 m (D) 30 m

Sol.

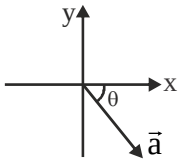
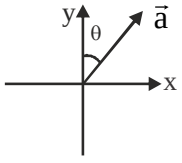
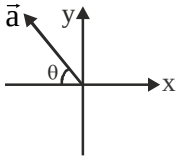


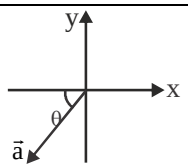
After 6 terms mon is at position O and dis. = O

After 7th term he is at position A and disp. = OA

= 10 m

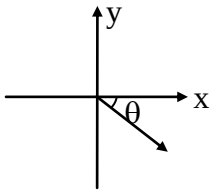
5. Match vector $\vec{a}$ shown in column-I with its unit vector representation in column-II

Column-I		**Column-II**	
(A)		(P)	$\vec{a} = a\sin\theta\hat{i} + a\cos\theta\hat{j}$
(B)		(Q)	$\vec{a} = -a\cos\theta\hat{i} + a\sin\theta\hat{j}$
(C)		(R)	$\vec{a} = -a\sin\theta\hat{i} - a\cos\theta\hat{j}$

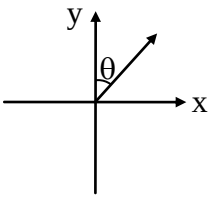
(D) 

(S) $\vec{a} = a\cos\theta\hat{i} - a\sin\theta\hat{j}$

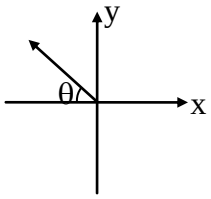
Sol.

(A) 

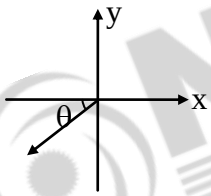
(S) $\vec{a} = a\cos\theta\hat{i} - a\sin\theta\hat{j}$

(B) 

(P) $\vec{a} = a\sin\hat{i} + a\cos\theta\hat{j}$

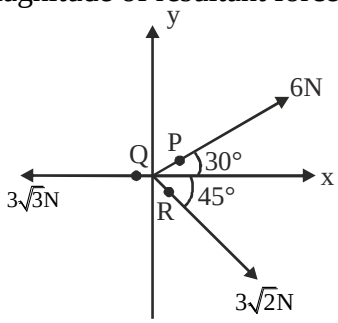
(C) 

(Q) $\vec{a} = -a\cos\theta\hat{i} + a\sin\theta\hat{j}$

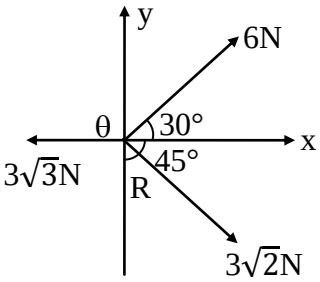
(D) 

(R) $\vec{a} = -a\sin\theta\hat{j} + a\cos\theta$
$\vec{a} = -a\cos\theta\hat{i} + a\sin\theta\hat{j}$

6. Three ants P, Q and R are pulling a grain with forces of magnitude 6N, $3\sqrt{3}$ N and $3\sqrt{2}$ N as shown in the figure. Find the magnitude of resultant force (in N) acting on the grain.

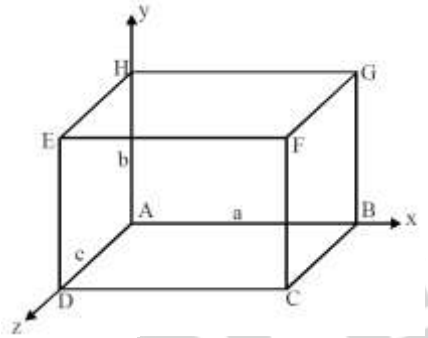


Sol.



$$\vec{F} = \vec{F}_P + \vec{F}_Q + \vec{F}_R$$
$$= (6\cos 30\hat{i}) + 6\sin 30\hat{j}) + (3\sqrt{2}\cos 45\hat{i} - 3\sqrt{2}\sin 45g) + (-3\sqrt{3}\hat{i})$$
$$= 3\sqrt{3}\hat{j} + 3\hat{j} + 3\hat{i} - 3\hat{j} - 3\sqrt{3}\hat{i} - 3\hat{i}\ N$$

7. Figure shows a cuboid of edge length a, b, c respectively. Write coordinates of points B, C, D, E, F, G, H and write position vectors of points F, E, G and C with respect to origin A.



Sol. B = (a, o, o) C = (a, o, c) D = (o,o, c) E = (a, b, c)

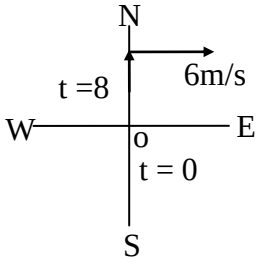
F = (a, b, c) G = (a, b, o) H = (a, b, o)

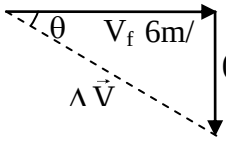
here A is origin.

8. A particle passes a point at the instant t = 0 s, moving with uniform velocity $2\sqrt{3}$ m/s in the north direction. It turns towards east at the instant t = 8 s and moves with uniform velocity 6 m/s. Direction of change in its velocity vector is

(A) 30° east of north (B) 60° south of east (C) 60° east of north (D) 30° south of east

Sol.





$$= -2 \qquad \Delta\vec{V} = \vec{V}_f + (-\vec{V}_i)$$
$$= \vec{V}_f - \vec{V}_i$$

$$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$$

θ = 30° from east (downward)

9. In first and second columns of the following table, unit vectors $\hat{a}$ and $\hat{b}$ of some geographical directions are mentioned. Complete the third column.

Unit Vector $\hat{a}$	Unit Vector $\hat{b}$	Scalar Product $\hat{a}.\hat{b}$
West	Up	----------
South	East	----------
30° North of west	Down	----------
60° South of east	North	----------
Up	North	----------
Down	30° East of up	----------
30° Up of east	West	----------
30° Down of south	Up	----------

Sol.

West	up	$\hat{a}\cdot\hat{b}=0$
South	Eeast	$\hat{a}\cdot\hat{b}=0$
30°N of W	Down	$\hat{a}\cdot\cdot\hat{b}=0$
60° S of E	North	$\hat{a}\cdot\hat{b}=-\sqrt{3}/2$
Up	North	O
Down	30°East of up	$\hat{a}\cdot\hat{b}=-\sqrt{3}/2$
30° up of E	West	$\hat{a}\cdot\hat{b}=-\sqrt{3}/2$
30° Down of S	Up	$\hat{a}\cdot\hat{b}=-\frac{1}{2}$

10. A bird moves from point (1, –2) to (4, 2). If the speed of the bird is 10 m/s, then the velocity vector of the bird is

(A) $5\left(\hat{i}-2\hat{j}\right)$ (B) $5\left(4\hat{i}+2\hat{j}\right)$ (C) $0.6\hat{i}+0.8\hat{j}$ (D) $6\hat{i}+8\hat{j}$

Sol. $\vec{r}_1=(\hat{i},-2\hat{j})$ $\vec{r}_2=(4\hat{i}+2\hat{j})$

speed = 10m/s velocity = ?

we know velocity is along displacement vector.

$\vec{r}=\vec{r}_2-\vec{r}_1=(4\hat{i}+2\hat{j})-(\hat{i}-2\hat{j})$

$=4\hat{i}+2\hat{j}-\hat{i}+2\hat{j}$

$=4\hat{i}-\hat{j}+2\hat{j}+2\hat{j}$

$\vec{r}=3\hat{i}+4\hat{j}$

Unit vector along displacement $\Rightarrow \vec{r}=\frac{3\hat{i}+4\hat{j}}{5}$

Velocity $\vec{V} = 10\left(\frac{3\hat{i} + 4\hat{j}}{5}\right)$

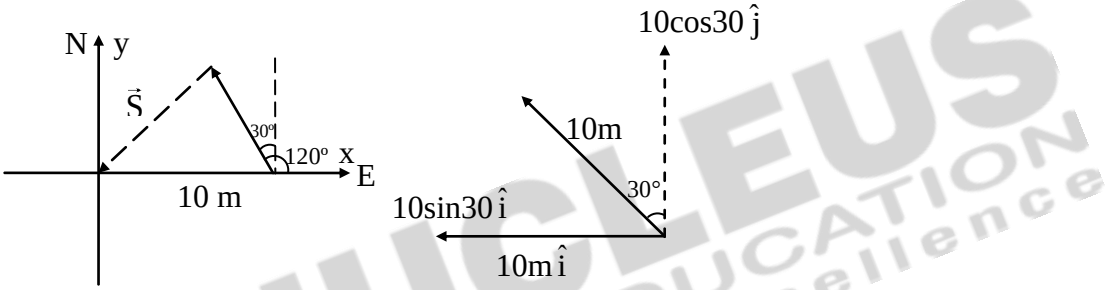
$\vec{V} = 6\hat{i} + 8\hat{j}$

11. Consider east as positive x-axis, north as positive y-axis. A girl walks 10 m east first time and then 10 m in a direction 30° west of north for the second time and then third time in unknown direction and magnitude so as to return to her initial position. What is her third displacement in unit vector notation.

(A) $-5\hat{i} - 5\sqrt{3}\hat{j}$ (B) $5\hat{i} - 5\sqrt{3}\hat{j}$

(C) $-5\hat{i} + 5\sqrt{3}\hat{j}$ (D) She can not return

Sol.



So, $10\hat{i} + 10\cos 30\,\hat{j} - 10\sin 30\hat{i} + \vec{S} = 0$

$10\hat{i} + 5\sqrt{3}\,\hat{j} - 5\hat{i} + \vec{s} = 0$

$5\,\hat{i} + 5\sqrt{3}\,\hat{j} + \vec{s} = 0$

$\vec{s} = -5\hat{i} - 5\sqrt{3}\,\hat{j}$

12. Which of the following statement(s) is/are correct?

(A) The unit vector of velocity and force can be the same.

(B) The angle between two unit vector is always 90°.

(C) The unit vector of velocity is always perpendicular to acceleration.

(D) The difference between magnitudes of two unit vector is equal to magnitude of difference of two unit vectors.

Sol. (A) ✓ Unit vector of velocity and force can be the same

13. The position vector of a point is $(2\hat{i} + 2\hat{j})$ m

(a) Find magnitude of this vector

(b) Find angle with x-axis

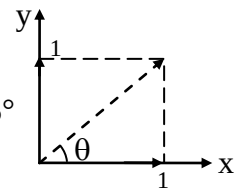
(c) Find the volume of a cone which is generated when a line segment representing this position vector is rotated about x-axis with one end remaining fixed at origin

Sol. Position vector = $(2\hat{i} + 2\hat{j})$ m.

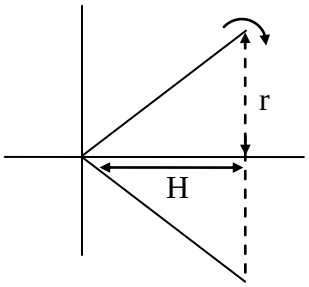
(a) $\vec{P} = (2\hat{i} + 2\hat{j})$

$|\vec{P}| = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}$

(b) $\tan\theta = \frac{2}{2} = 1$ $\theta = 45°$



(c) Volume of cone = $\frac{1}{3}\pi r^2 h$



$H = 2\sqrt{2}\cos 45$
$H = 2m$
$r = 2m$ (r = 2 in y)

Volume $V = \frac{1}{3} \times p \times (2)^2 \times 2$

$V = \frac{8\pi}{3}\ m^3$

WORK BOOK # 2

(PRODUCT OF VECTOR)

1. $\vec{A}+\vec{B}=2\hat{i}$ and $\vec{A}-\vec{B}=4\hat{j}$ then angle between $\vec{A}$ and $\vec{B}$ is

(A) 127° (B) 143° (C) 53° (D) 37°

2. The angle between two vectors $\vec{R}=-\hat{i}+\frac{1}{3}\hat{j}+\hat{k}$ and $\vec{S}=x\hat{i}+3\hat{j}+(x-1)\hat{k}$

(A) Is obtuse angle (B) Is acute angle

(C) Lies between 60° and 120° (D) Depends on x

Sol. $\theta = \frac{\vec{R}\cdot\vec{S}}{RS} = \frac{-x+1+x-1}{\sqrt{(-1)^2+(1/3)^2+1^2}\sqrt{x^2+(3)^2+(x-1)^2}} = 0$

$\theta = 90° = \frac{\pi}{2}$

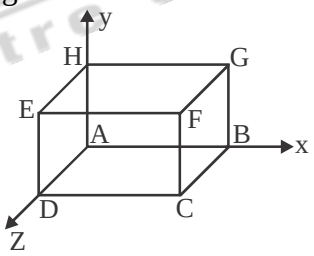
3. The vector having a magnitude of 10 and perpendicular to the vector $3\hat{i}-4\hat{j}$

(A) $4\hat{i}+3\hat{j}$ (B) $5\sqrt{2}\hat{i}-5\sqrt{2}\hat{j}$ (C) $8\hat{i}+6\hat{j}$ (D) $8\hat{i}-6\hat{j}$

Sol. A = 10 perpendicular to vector $3\hat{i}-4\hat{j}$

From options $8\hat{i}+6\hat{j}$ magnitute 10 and product (dot) with $3\hat{i}-4\hat{j}$ give O.

4. Figure shows a cube of edge length α.



Column-I	**Column-II**
(A) The angle between AF and x-axis	(P) 60°
(B) Angle between AF and DG	(Q) $\cos^{-1}\frac{1}{3}$
(C) Angle between AE and AG	(R) $\cos^{-1}\frac{1}{\sqrt{3}}$
	(S) $\cos^{-1}\sqrt{\frac{2}{3}}$

Sol. (A) $\vec{A}=\vec{AF}=AB\hat{i}+AH\hat{j}+Ad\hat{k}$

$AF=\sqrt{\alpha^2+\alpha^2+\alpha^2}=\sqrt{3\alpha}$

AB = AF cos α

$$\cos\alpha = \frac{AB}{AF} = \frac{\alpha}{\sqrt{3}\,\alpha}$$

$$\alpha = \cos^{-1}\left(\frac{\alpha}{\sqrt{3}\alpha}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

(B) Projection of $\overrightarrow{AF}$ on $\overrightarrow{DG} \Rightarrow AF\cos\theta = \overrightarrow{AF}.(DG)$

$$\sqrt{3}\alpha\cos\theta = (\alpha\hat{i}+a\hat{j}+a\hat{k})\cdot(\alpha\hat{i}+\alpha\hat{j}-\alpha\hat{k})$$

$$\cos\theta = \frac{\alpha^2}{3\alpha^2}\ q = \cos^{-1}\left(\frac{1}{3}\right)$$

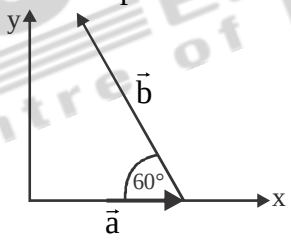
(C) $\overrightarrow{AE} = \alpha\hat{j}+a\hat{k}$ $\quad AE = \sqrt{2}\,\alpha$

$\overrightarrow{AG} = \alpha\hat{i}+\alpha\hat{j}$ $\quad AG = \sqrt{2}\alpha$

$AE\cos\beta = \overrightarrow{AE}\cdot AG \Rightarrow \sqrt{2}\alpha\cos\beta = (a\hat{j}+2\hat{k})\cdot(\alpha\hat{i}+\alpha\hat{j})\ \sqrt{2}\,\alpha$

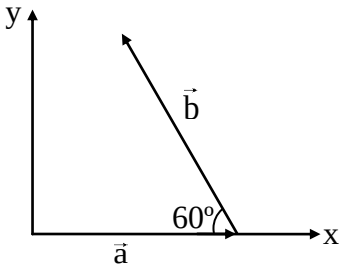
$$\cos\beta = \frac{1}{2} \qquad \beta = 60^\circ$$

5. Two vectors $\vec{a}$ and $\vec{b}$ of magnitudes 2 units and 4 units respectively are shown in the figure. Which of the following mathematical operations is/are correct?



(A) $\vec{a}+\vec{b}=2\sqrt{3}\hat{j}$ (B) $\vec{a}-\vec{b}=4\hat{i}-2\sqrt{3}\hat{j}$ (C) $\vec{a}.\vec{b}=4$ (D) $\left(\vec{b}.\hat{a}\right)=2$

Sol.



$|\vec{a}|=2$ $\qquad |\vec{b}|=2$

$\vec{a} = 2\hat{j}$ $\qquad \vec{b} = 4\hat{i} + 4\hat{j} = -\frac{-4}{2}\hat{j} + \frac{4\sqrt{3}}{2}\hat{j}$ (cos60 sin60)

$\hat{b} = -2\hat{j} + 2\sqrt{3}\hat{j}$

(a) $\vec{a}+\vec{b}=2\hat{i}+(-2\hat{i}+2\sqrt{3}\hat{j})=2\sqrt{3}\hat{j}$

(b) $\vec{a}-\vec{b}=2\hat{i}+(-2\hat{i}+2\sqrt{3}\hat{j})=4\sqrt{3}\hat{j}$

(c) $\vec{a}\cdot\vec{b}=-4+O=-4$

(d) $\vec{b}\cdot\hat{a}=(-2\hat{i}+2\sqrt{3}\hat{j})\cdot\frac{2\hat{i}}{2}=-4+4\sqrt{3}\times 0=\frac{-4}{2}=-2x$

6. The angle between) $(\hat{i}+\hat{j}+\hat{k})$ & $(2\hat{i}+2\hat{j}+2\hat{k})$ is

(A) $\text{Cos}^{-1}\frac{1}{3}$ (B) $\text{Cos}^{-1}\frac{1}{\sqrt{3}}$ (C) $\text{Sin}^{-1}\frac{1}{3}$ (D) None of these.

Sol. $\vec{A}=(\hat{i}+\hat{j}+\hat{k})\,\&\,(2\hat{i}+2\hat{j}+2\hat{k})$

CDS $\theta=\frac{\vec{A}\cdot\vec{B}}{A\cdot B}\frac{=2+2+2}{\sqrt{3}\times 2\times\sqrt{3}}=\frac{6}{6}=1$

$Q=\cos^{-1}(1)=0^{o}$

7. A particle travels with speed 50m/s from the point (3, –7) in a direction $7\hat{i}-24\hat{j}$. Find its position vector after 3 seconds.

Sol. $\vec{P}$, = $(3\hat{i}-7\hat{j})$ V = 50m/s direction vector of speed = $7\hat{i}-24\hat{j}$

so, direction $\hat{a}=\frac{7\hat{i}-24\hat{j}}{25}$

so $\vec{V}=50\,\vec{r}$

$=50(7\hat{i}-24\hat{j})=2(\hat{i}-24\hat{j})$
25

Displacement $\vec{s}=\vec{V}\pm=2(7\hat{i}-24\hat{j})\times 3=42\hat{i}-24\hat{j}$

$\vec{P}_2=\vec{P},+\vec{s}=(3\hat{i}-7\hat{j})+(42\hat{i}-144\hat{j})$

$=45\hat{i}-151\hat{j}$

8. (a) Calculate $\vec{r}=\vec{a}-\vec{b}+\vec{c}$ where $\vec{a}=5\hat{i}+4\hat{j}-6\hat{k},\vec{b}=-2\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{c}=4\hat{i}+3\hat{j}+2\hat{k}$.
(b) Calculate the angle between $\vec{r}$ and the z-axis.
(c) Find the angle between $\vec{a}$ and $\vec{b}$

Sol. (a) $\vec{r}=\vec{a}-\vec{b}+\vec{c}$

$=(5\hat{i}+4\hat{j}-6\hat{k})-(-2\hat{i}+2\hat{j}+3\hat{k})+(4\hat{i}+3\hat{j}+2\hat{k})$

$=11\hat{i}+5\hat{j}-7\hat{k}$

(b) Angle between $\vec{r}$ and z axis

$\cos\theta = \vec{r}\cdot\hat{k} = (11\hat{i}+5\hat{j}-7\hat{k})\cdot\hat{k} = -7$

$|\vec{r}|\cdot|\hat{k}| \quad \sqrt{121+25+49}\times 1 = \frac{-7}{\sqrt{195}}$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$

(c) Angle between $\vec{a}\cdot\vec{b} = (5\hat{i}+4\hat{j}-6\hat{k})\cdot(-2\hat{i}+2\hat{j}+3\hat{k})$

$a\cdot b \quad \sqrt{25+16+36}\cdot\sqrt{4+4+9}$

$= \quad -10+8-18 = -20$

$\sqrt{77}\cdot\sqrt{17} \qquad \sqrt{1309}$

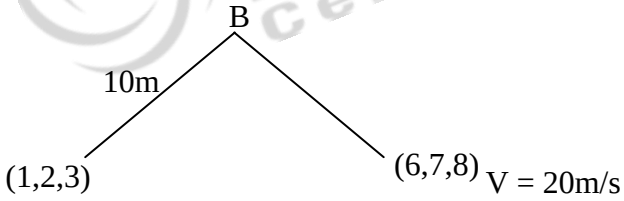
Paragraph for Question Nos. 9 to 11

Madam X is initially at point A(1, 2, 3). In search of a diamond necklace she moves 10 m in the direction $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ and reaches a point B. Then she gets a clue for her diamond necklace and reaches point C(6,7, 8). Her speed is constant in the whole path which is equal to 20 m/s.

9. Her velocity vector in path AB is

(A) $20\left(\hat{i}+\hat{j}\right)+\sqrt{2}\hat{k}$ (B) $10\left(\hat{i}+\hat{j}+\sqrt{2}\hat{k}\right)$ (C) $20\left(\hat{i}+\hat{j}+\hat{k}\right)$ (D) $-\hat{j}+\left(\sqrt{2}-3\right)\hat{k}$

Sol. Paragraph (9–11)



Direction of path AB = $\hat{i}+\hat{j}+\sqrt{2}\,\hat{k}$

So, velocity in path AB = $\vec{V} = V\hat{a} = 20\left(\frac{\hat{i}+\hat{j}+\sqrt{2}\hat{k}}{2}\right)$

$\vec{V} = 10(\hat{i}+\hat{j}+\sqrt{2}\,\hat{k})$

10. The coordinate of point B is

(A) (6, 7, 8) (B) (6, 8, $5\sqrt{2}$) (C) (6,7, $3+5\sqrt{2}$) (D) (11, 22, $10\sqrt{2}+3$)

Sol. $\vec{s}_{AB} = 10\hat{a}\left(\frac{\hat{i}+\hat{j}+\sqrt{2}\,\hat{k}}{2}\right)$

$\vec{s}_{AB} = 5\hat{i}+5\hat{j}+5\sqrt{2}\hat{k}$

$\vec{B} = \vec{A} + \vec{S}_{AB} = (\hat{i}+2\hat{j}+3\hat{k}) + (5\hat{i}+5\hat{j}+5\sqrt{2}\hat{k})$

$\vec{B} = (6\hat{i}+7\hat{j} + (3+5\sqrt{2})\hat{k})$

$B = (6, 7, 3+ 5\sqrt{2})$

11. Her velocity vector in path BC is

(A) $20\hat{k}$ (B) $-20\hat{k}$ (C) $\left(5\sqrt{2}-5\right)\hat{k}$ (D) $\left(10\sqrt{2}-5\right)\hat{k}$

Sol. Velocity vector in path BC.

displacement BC = $\vec{C}-\vec{B}$

$= (6\hat{i}+7\hat{j}+8\hat{k})-(6\hat{i}+7\hat{j}+3\hat{k}+5\sqrt{3}\hat{k})$

$= (8-3-5\sqrt{2})\hat{k} = (5-5\sqrt{2})\hat{k}$

unit vector along BC = $-\hat{k}$

so velocity vector $\vec{V} = -20\hat{k}$

WORK BOOK # 3

(General 2-D Motion & Projectile)

1. Can an object maintain uniform velocity when its acceleration is not zero?

(A) no, impossible

(B) yes, but only if the acceleration is in the direction of the velocity

(C) yes, but only if the acceleration is opposite to the direction of the velocity

(D) yes, if the acceleration is perpendicular to the velocity.

Sol. Object cannot maintain uniform velocity if acceleration. is not zero. Later its direction or magnitude or both may change.

2. Two balls of equal mass are shot upward simultaneously from the same point on the ground with the same initial speed, but at different angles to the horizontal. Which of the following statements must be true?

(A) The ball launched at the larger angle hits the ground first.

(B) The two balls hit the ground at the same time.

(C) The ball launched at the larger angle always has more total mechanical energy.

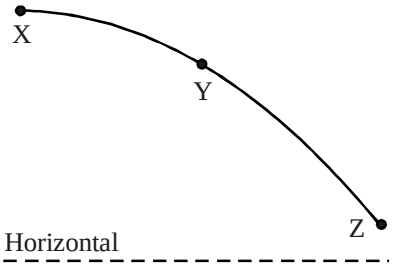
(D) The ball launched at the smaller angle hits the ground first.

Sol. U = same for both ; angle is different from horizontal. The ball which have less time of flight hits the ground firest.

$$T = \frac{2U\sin\theta}{g} \qquad \sin\theta\ (0 \to a)$$

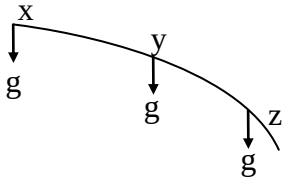
Time of flight depends vertical component of velocity so smaller angle, hits the ground first.

3. A stone is thrown horizontally and follows the path XYZ shown. The direction of the acceleration of the stone at point Y is



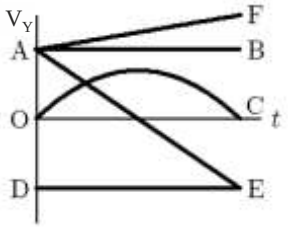
(A) ↓ (B) → (C) ↘ (D) ↙

Sol.



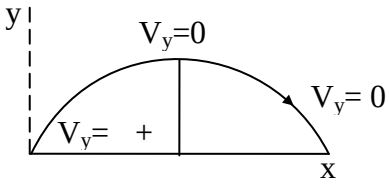
(A) ↓ always

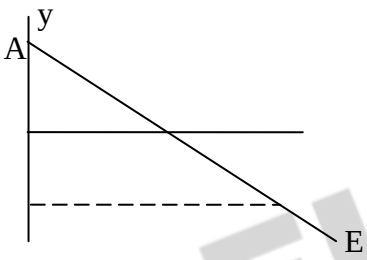
4. Which of the curves on the graph below best represents the vertical component v_y of the velocity versus the time t for a projectile fired at an angle of 45° above the horizontal?



(A) OC (B) DE (C) AB (D) AE (E) AF

Sol.



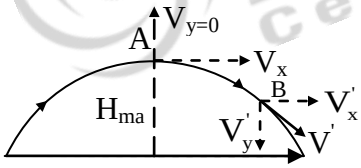


5. A ball is thrown into the air and follows a parabolic trajectory. Point A is the highest point in the trajectory and point B is a point in the path as the ball is falling back to the ground. Choose the correct relationship between the speeds and the magnitudes of the acceleration at the two points.

(A) $v_A > v_B$ and $a_A = a_B$ (B) $v_A < v_B$ and $a_A > a_B$

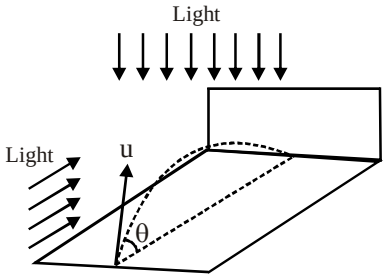
(C) $v_A = v_B$ and $a_A \neq a_B$ (D) $v_A < v_B$ and $a_A = a_B$

Sol.

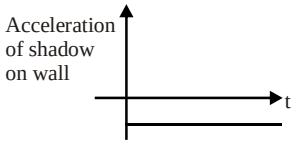


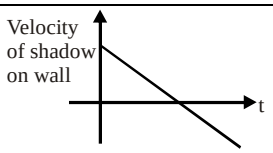
(D) $V_A < V_B$ and $a_A = a_B \downarrow g$

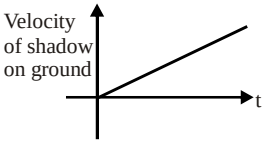
6. A projectile is projected as shown in figure. A proper light arrangement makes a shadow on the wall as well as on the floor?

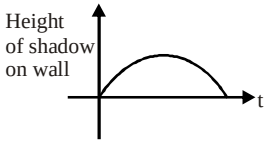


Which of the following graphs is incorrect.

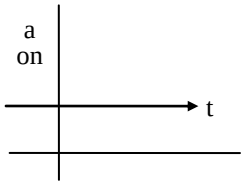
(A) 

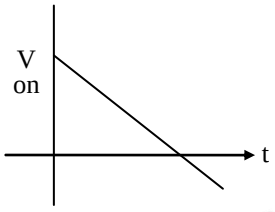
(B) 

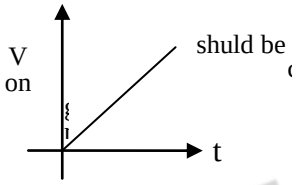
(C) 

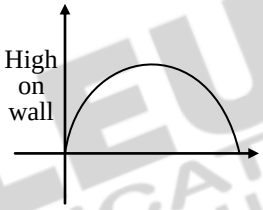
(D) 

Sol.

(A) 

(B) 

(C) 

(B) 

7. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
(A) The one with the farthest range.
(B) The one which reaches maximum height.
(C) The one with the greatest initial velocity.
(D) The one leaving the bat at 45° with respect to the ground.

Sol. One which reaches maximum height has more time of flight and will be in the air for the longest time

8. A particle is projected from a horizontal plane (x-z plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by.

(A) $\frac{2ba}{c}$ (B) $\frac{ba}{c}$ (C) $\frac{3ba}{c}$ (D) None of these

Sol. $\vec{V} = a\hat{i} + (b - ct)\hat{j}$

$\int ds = \int v dt = at\hat{i} + (bt - ct_2^2)\hat{j}$

When particle reaches ground y component (displacement in y) is O

so $S_y = bt - \frac{ct^2}{2} \Rightarrow O = bt - \frac{ct^2}{2}$

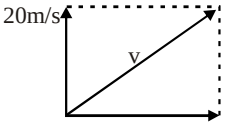
$$b - \frac{ct}{2} = O \quad \Rightarrow t = \frac{2b}{c}$$

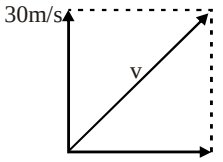
so time of flight $= \frac{2ab}{2}$

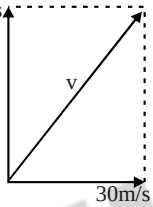
$R = V_x \times T$

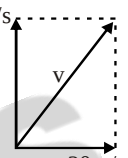
$= a \times \frac{2b}{c} = \frac{2ab}{c}$

9. Out of the following four cases of projection where components along horizontal and vertical are given the maximum range is for:

(A) 

(B) 

(C) 

(D) 

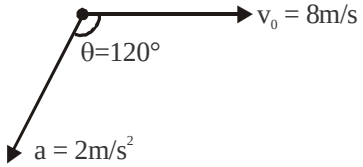
Sol. $R = \frac{2U_xU_y}{g}$

(A) $R = \frac{2\times30\times20}{10} = 120$ (B) $R = \frac{2\times30\times30}{10} = 180$

(C) $R = \frac{2\times30\times40}{10} = 240$ (D) $\frac{2\times20\times30}{10} = 120$ m.

Paragraph for Question Nos. 10 to 12

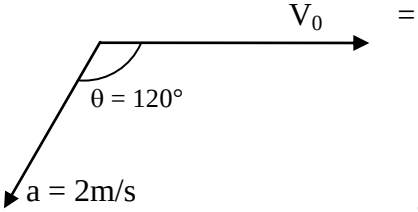
The figure shows the velocity and acceleration of a particle at the initial moment t = 0 of its motion. The acceleration vector of the particle remains constant.



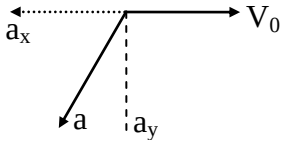
10. The instant of time after t = 0 at which the speed of the particle is 8 m/s again is

(A) 2 s (B) 8 s (C) 16 s (D) 4 s

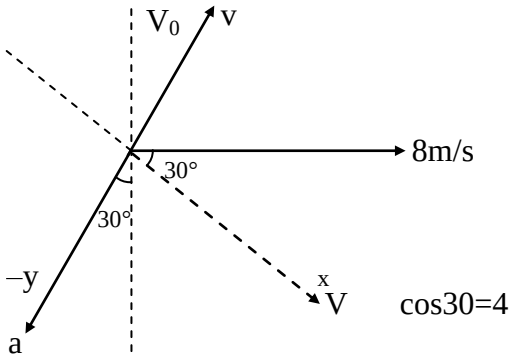
Sol. Paragraph 10 to 12



a remain constant.



Component a_x reduced the velocity and get to 0 and then reverse it direction (as consider an x)



component of velocity perpendicular to acceleration is not change. But component of velocity antiparallel to acceleration is change and comes to zero and again

Velocity should be –4 in y axes to make speed 8 m/s again.

$-4' = v\cdot -at$ $\quad <V_0 = 4>$

$-t = \underset{a}{V_0 - V^1} \Rightarrow = t = \underset{a}{V' - V_0}$

$t = \dfrac{4+4}{2} = 4$ sec

11. The instant of time at which the acceleration and velocity vector of the particle are mutually perpendicular is

(A) 4 s (B) 8 s (C) 2 s (D) 16 s

Sol. acceleration and velocity will be perpendicular if component along y axis of velocity become O and only component of velocity along x axis remains which is constant

so $O = V_{sin30} - at$

$O = 4 - 2t$

$t = 2$ sec.

12. The magnitude of displacement of the particle in time interval t = 0 to t = 4 seconds is

(A) $16\sqrt{3}$ m (B) 32 m (C) 30 m (D) $32\sqrt{3}$ m

Sol. y displacement of particle in 4 sec is S_y

$S_y = 4 \times 4 - \dfrac{1}{2} a t^2 = 16 - \dfrac{1}{2} \times 2 \times 16 = O$

x displacement of particle in 4 sec in S_x

$S_x = 4\sqrt{3} \times 4 = 16\sqrt{3}$ Ans.

13. The co-ordinate of the particle in x-y plane are given as $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$. The motion of the particle is

(A) along a straight line (B) uniformly accelerated

(C) along a parabolic path (D) non uniformly accelerated

Sol. Coordinate $x = 2 + 2t + 4t^2$ $y = 4t + 8t^2$

$V_x = 2 + 8t$ $a_x = 8$

$V_y\ 4 + 16t$ $a_y = 16$ uniformly accelerated motion.

$x = 2 + 2t + 4t^2 = 2 + \frac{4}{2}$

$y = 2x - 4$ Along the straight line

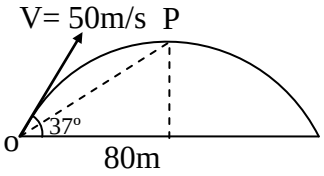
WORK BOOK # 4

(Projectile Motion)

1. A body is thrown from a point with speed 50 m/s at an angle 37º with horizontal. When it has moved a horizontal distance of 80 m then it's distance from point of projection -

(A) 40 m (B) $40\sqrt{2}$ (C) $40\sqrt{5}$ (D) none of these

Sol.



x = 80m $U_x = 50\cos 37 = 50\times \frac{4}{5} = 40$ m/s

$U_y = 50\sin 37 = 50 \times \frac{3}{5} = 30$ m/s

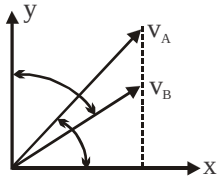
Let check its Range $R = \frac{2U_xU_y}{g} = \frac{2\times 40\times 30}{g} = \frac{2\times 40\times 30}{10} = 240$ m

So to move horizontal x time $t = \frac{x}{U_x} = \frac{80}{40} = 2$ second

So $y = U_yt - \frac{1}{2}gt^2 = 30\times 2 - \frac{1}{2}\times 10\times 4^2 = 60 - 20 = 40$ m.

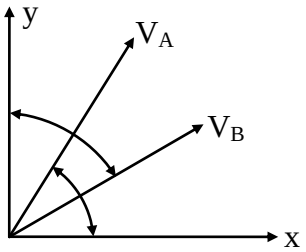
$OP = \sqrt{(80)^2+(40)^2} = \sqrt{1600\times 5} = 40\sqrt{5}$

2. Two projectiles are projected with velocity v_A, v_B at angles θ_A (from horizontal) and θ_B (from vertical) as shown in the figure below, such that $v_A > v_B$ but having same horizontal component of velocity. Which of the following is correct ?



(A) $T_A > T_B$ (B) $H_A > H_B$ (C) $R_A > R_B$ (D) $R_B > R_A$

Sol.



$V_{Ax} = V_{Bx}$

with horizontal $Q_A > Q_B$

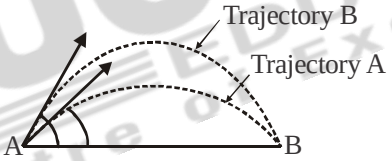
$A_{Ay}^2 + V_{Ax}^2 > V_{By} + V_{Bx}^2$

so $V_{Ay} > V_{By}$

$R = \frac{2V_x V_y}{g}$ $R_A > R_B$ $T = \frac{2V_y}{g}$ $\therefore T_A > T_B$

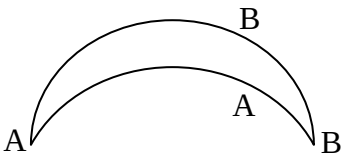
$T = \frac{Vg^2}{2g}$ $\therefore$ $H_A > H_B$

3. A man wishes to throw two darts one by one at the target at B with same speed so that they arrive at the same time. Mark the correct statements about the two projections.



(A) Projectile that travels along trajectory A was projected earlier
(B) Projectile that travels along trajectory B was projected earlier
(C) Second dart must be projected at angle θ_B such that $\theta_A + \theta_B = 90°$
(D) Second dart must be projected at angle $\theta_A > \theta_B$

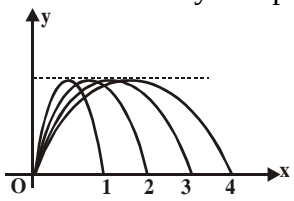
Sol.



(B) Projectile that travels along trajectory B was projected earlier

(C) Second dart must be projected at Q_B such that $Q_A + Q_B = 90°$

4. Figure shows four possible trajectories of a kicked football. Ignoring air resistance, rank the curves according to the initial horizontal velocity component the highest first



(A) 1, 2, 3, 4 **(B) 1, 3, 2, 4** **(C) 3, 4, 1, 2** **(D) 4, 3, 2, 1**

Sol. 4, 3, 2, 1

5. A particle is projected with a velocity $\vec{v}_0 = a\hat{i} + b\hat{j}$ from ground with y-axis as vertical and it lands on ground.

	Column-I		**Column-II**
(A)	$\frac{v_x}{v_y}$ at $t = \frac{b}{2g}$	(P)	$\frac{a}{\sqrt{a^2+b^2}}$
(B)	h_{max}	(Q)	$\frac{2a}{b}$
(C)	R (Horizontal)	(R)	$\frac{b^2}{2g}$
(D)	$\frac{v_{av}}{v_0}$ where v_{av} = Average velocity for entire journey	(S)	$\frac{2ab}{g}$
		(T)	$\frac{a}{b}$

Sol. $\vec{v}_0 = a\hat{i} + b\hat{j}$ from ground.

(A) $U_n = a$

$U_y = b$ $\quad V_y = U_y - gt = b - g\left(\frac{b}{2g}\right) \Rightarrow v_y = \frac{b}{2}$

so at $\frac{b}{2g}$ $\quad \frac{v_x}{V_y} = \frac{2a}{b}$ (Q)

(B) $H_{man} = \frac{U_y^2}{2g} = \frac{b^2}{2g}$ (R)

(C) $R = \frac{2U_xU_y}{g} = \frac{2ab}{g}$ (S)

(D) $V_aV = a$ (as U_x a constant for while jurney)

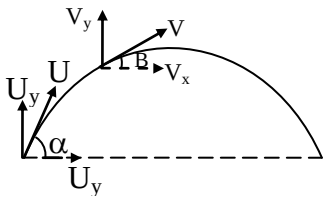
(in y displaceemnt is zero so $V_yV_g = 0$)

$\frac{V_{av}}{V_0} = \frac{a}{\sqrt{a^2+b^2}}$

6. A ball is projected with velocity u at an angle α with horizontal plane. Its speed when it makes an angle β with the horizontal is -

(A) $u\cos\alpha$ (B) $\frac{u}{\cos\beta}$ (C) $u\cos\alpha\cos\beta$ (D) $\frac{u\cos\alpha}{\cos\beta}$

Sol.



Horizontal velocity is constant.

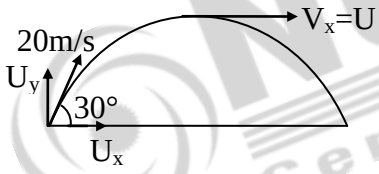
$V\cos\beta = U\cos\alpha$

$V = \frac{U\cos\alpha}{\cos\beta}$

7. A particle is projected with speed 20m/s at an angle 30° with horizontal. After how much time the angle between velocity and acceleration will be 90° –

(A) 1 sec (B) 2 sec (C) 1.5 sec (D) Never

Sol.



$a \perp V$ when particle is not heighest point

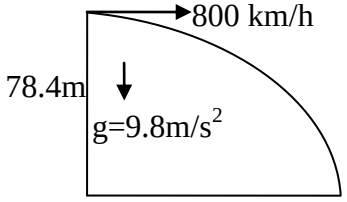
At Hman $V_y = O$

$O = U\sin30 - gt$

$t = \frac{U\sin30}{g} = \frac{20}{2\times10} = 1$ sec.

8 An Indian fighter plane flying horizontally with speed 800 km/hr releases a bomb (on Pakistani bunker) at a height of 78.4 m from the ground, when will the bomb strike the ground? Give your answer in seconds. [$g = 9.8\ m/s^2$]

Sol.



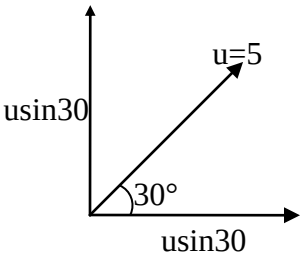
$U_y = 0$ $U_x = 800$ Km/h

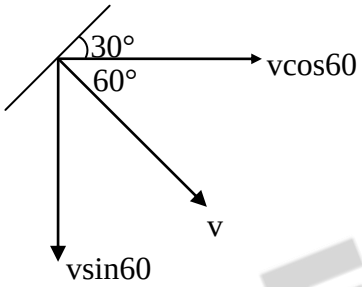
$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 78.4}{9.8}}$

t = 4s

9. A projectile at any instant during its flight has velocity 5 m/s at 30° above the horizontal. How long after this instant, will it be moving at right angle to the given direction?

Sol.





vcos60= ucos30

$V = \frac{u\cos 30}{\cos 60} = \frac{5\times\sqrt{3}}{2\times 1}\times 2 = 5\sqrt{3}$

$V_y = 5\sqrt{3}\sin 60 = 5\sqrt{3}.\frac{\sqrt{3}}{2} = \frac{15}{2}$ m/s

So $\frac{-15}{2} = 5\sin 30 - gt$

$\Rightarrow \frac{-15}{2} = \frac{5}{2} - 10t \Rightarrow 10t = \frac{20}{2} = 10$

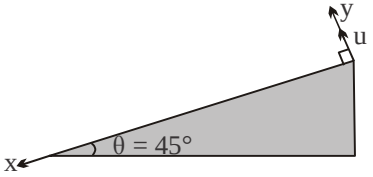
$\Rightarrow$ t = 1 sec

WORK BOOK # 5

(Projectile motion)

Paragraph for Question nos. 1 to 3

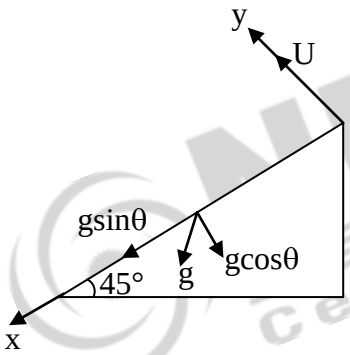
An inclined plane makes an angle θ = 45° with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 (as shown in figure). x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction.



1. The instant of time at which velocity of stone is parallel to x-axis

(A) $\frac{2\sqrt{2}u}{g}$ (B) $\frac{2u}{g}$ (C) $\frac{\sqrt{2}u}{g}$ (D) $\frac{u}{\sqrt{2}g}$

Sol. 1.



$U_x = 0$ $U_y = U$

$a_x = g\sin\theta$ $a_y = -g\cos\theta$

When stone is parallel to x axis = $V_y = 0$

$V_y = U_y + g_y t$

$t = \frac{U}{g\cos 45} = \frac{\sqrt{2}U}{g}$

2. The instant of time at which velocity of stone makes an angle θ = 45° with positive x-axis

(A) $\frac{2\sqrt{2}u}{g}$ (B) $\frac{2u}{g}$ (C) $\frac{\sqrt{2}u}{g}$ (D) $\frac{u}{\sqrt{2}g}$

Sol. When stone make angle with θ = 45° with x axis means $V_y = V_x$ .....(1)

$V_x = 0 + g\sin 45t'$

$V_x = \frac{g}{\sqrt{2}}t'$

$V_y = U - g\cos\theta t'$

$V_y = U - \frac{g}{\sqrt{2}}t'$

From $\frac{g}{\sqrt{2}}t^1 = U - \frac{g}{\sqrt{2}}t'$

$\Rightarrow \sqrt{2}gt' = U \Rightarrow t' = \frac{U}{\sqrt{2}g}$

3. The instant of time at which (starting from t = 0) component of displacement along x-axis is half the range on inclined plane is

(A) $\frac{2\sqrt{2}u}{g}$ (B) $\frac{2u}{g}$ (C) $\frac{\sqrt{2}u}{g}$ (D) $\frac{u}{\sqrt{2}g}$

Sol. At man x mean Range (at stirring point) y = 0

$y = U_y t + \frac{1}{2}g_y t^2$

$0 = Ut - \frac{1}{2}g\cos 45 t^2 \Rightarrow t = \frac{2\sqrt{2}U}{g}$

Rang $R = U_x t + \frac{1}{2}g_x t^2 \Rightarrow Uxt + \frac{1}{2}g\sin 45 t^2$

$R = \frac{g}{2\sqrt{2}} \times \frac{8U^2}{g^2} = \frac{4}{\sqrt{2}}\frac{U^2}{g}$

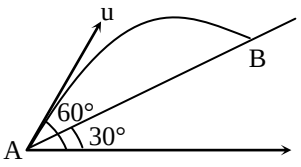
So $\frac{R}{2} = U_x t_0 + \frac{1}{2}g_x t_0^2$

$\frac{1}{2}\left(\frac{4}{\sqrt{2}}\frac{U^2}{g}\right) = 0 + \frac{1}{2}g\sin 45 t_0^2$

$t_0^2 = \frac{4U^2}{g^2}$

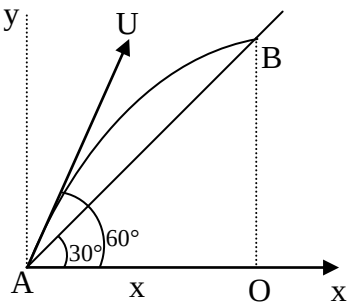
$t_0 = \frac{2U}{g}$ Ans.

4. Time taken by the projectile to reach A to B is t. Then the distance AB is equal to –



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}\,ut$ (D) 2 ut

Sol.



$x = AO = U\cos 60t$

$x = \frac{Ut}{2}$

$\cos\theta = \frac{x}{AB}$

$AB = \frac{x}{\cos 30} = \frac{x\sqrt{3}}{2} = \frac{Ut}{2} \times \sqrt{2}$

$AB = \frac{x.2}{\sqrt{3}} = \frac{Ut}{2} \times \frac{2}{\sqrt{3}}$

$AB = \frac{Ut}{\sqrt{3}}$

5. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4s. Which of the following are correct ?

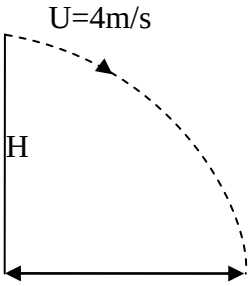
(A) The height of the table is 0.8 m

(B) It hits the ground at an angle of 60° with the vertical

(C) It covers a horizontal distance 1.6 m from the table

(D) It hits the ground with vertical velocity 4 m/s

Sol.



$U_x = 4m/s$

$U_y = 0$

$t = 0.4s$

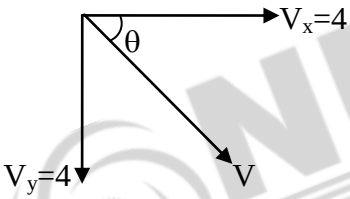
So $H = U_y t + \frac{1}{2}gt^2 = oxt + \frac{1}{2}\times 10(0.4)^2$

$H = 0.8$ m (A)

$R = U_x t = 4 \times 0.4 = 1.6$ m from table (C)

At instant of Hitting ground $V_x = 4m/s$

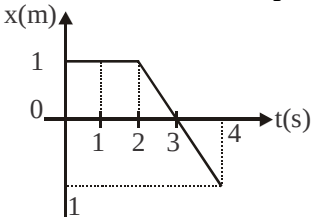
$V_y = U_y + gt = 0 + 4 = 4$ m/s (D)



$\tan\theta = \frac{4}{4} = 1$

$\theta = 45°$ from horizontal

6. The position-time graph of a particle is shown. If the particle was at x = 1m :



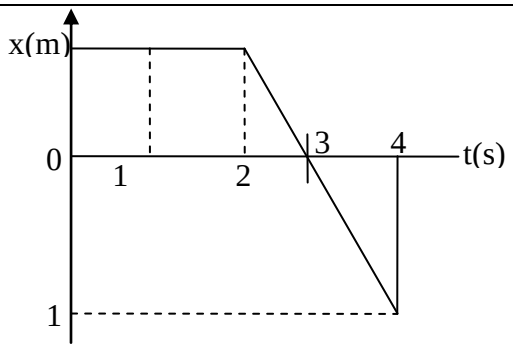
(A) The particle was at rest for 2s

(B) The particle moves forward with uniform velocity of 1m/s in + x-axis and then moves in – x-axis with same speed

(C) The particle will be at distance of 1m from the origin after 4s

(D) The average speed of the particle is equal to its initial speed during last two seconds.

Sol.



Particle was at x = 1m.

(A) Particle was at rest from 2s, slope of graph between 0 to 2 second is 0. so velocity is 0.

(C) At moves towards the origin and then in -x.

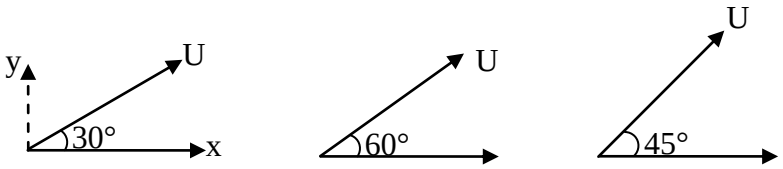
that t = 3 sec the particle is at origin at t = 4 sec particle is at –1 m from origin. it is at 1m from 0.

(D) Speed of Particle is 0 for first two second. It start to move after 2 second. So average speed of Particle is equal to initial speed during last two second.

7. Three particles are projected with same speed simultaneously from same point with angle of projection with the horizontal 30°, 45° and 60°. Their times of flights are T_{30}, T_{45} and T_{60} Respectively, horizontal range R_{30}, R_{45} and R_{60} respectively and maximum vertical heights as H_{30}, H_{45} and H_{60} respectively.
Column-I contains ratio of the above quantities, match them with their values given in column-II.

Column-I	**Column-II**
(A) $\frac{T_{60}}{T_{30}}$	(P) 1
(B) $\frac{8H_{30}}{R_{45}}$	(Q) $\sqrt{3}$
(C) $\frac{4H_{60}}{R_{60}}$	(R) $\frac{\sqrt{3}}{4}$
	(S) $4\sqrt{3}$

Sol.



(A) $\frac{T_{60}}{T_{30}}\ \frac{2U\sin\theta}{g}\times\frac{g}{2U\sin\theta_2}=\frac{\sin 60}{\sin 30}=\frac{\sqrt{3}}{2}\times 2\ =\ \sqrt{3}$

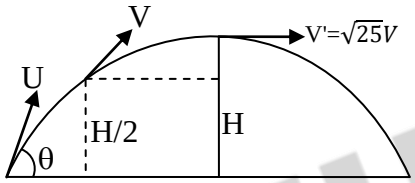
(B) $\frac{8H_{30}}{R_{45}}\ \frac{(8)\,U_y^2\sin^2\theta}{2g}\times\frac{g}{2U_xU_y}$

$$= \frac{(8)U^2\sin^2 30}{2} \times \frac{1}{2U^2\cos 45.\sin 45} = 1$$

(C) $\frac{4H_{60}}{R_{60}} = \frac{(4)U^2\sin^2 60}{2g} \times \frac{g}{2U^2\cos 60\sin 60} = \frac{4\times 3}{4\times 2} \times \frac{1}{2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}} = \sqrt{3}$

8. A projectile is thrown with some initial velocity u at an angle θ to the horizontal. Its speed when it is at the highest point is $\sqrt{\frac{2}{5}}$ times the speed v when it is at height half of the maximum height. Then ratio $\frac{v}{u} = \sqrt{\frac{n}{8}}$. Find the value of n.

Sol.



$V^2 = U^2 - 2g\left(\frac{H}{2}\right) \Rightarrow V^2 = U^2 - gH$

And. $V'^2 = U^2 - 2gH = V'^2 = U^2 - 2gH \Rightarrow \left(\sqrt{\frac{2}{5}}V\right)^2 = U^2 - 2gH$

$\frac{2}{5}V^2 = U^2 - 2gH$

From these equation $V = \sqrt{\frac{5}{8}}U$ so n =5 Ans.

WORK BOOK # 6
(Projectile Motion)

1. The equations of motion of a projectile thrown in x–y plane from origin are x = 8t, $y = 6t - 10t^2$ then the angle of projection is –

(A) $\tan^{-1}(3/4)$ (B) $\tan^{-1}(4/3)$ (C) $\sin^{-1}(3/4)$ (D) $\cos^{-1}(3/4)$

Sol. $x = 8t$ $y = 6t - 10t^2$

$V_x = 8$ $V_y = 6 - 20\,t$

$U_x = 8$ $U_y = 6$

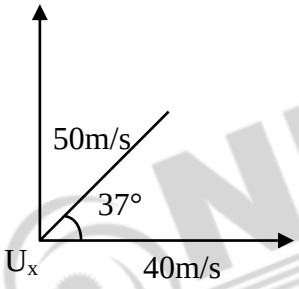
$\tan\theta = \frac{6}{8}$ $\boxed{\theta = \tan^{-1}(3/4)}$ (A) Ans

2. A body is thrown from a point with speed 50 m/s at an angle 37º with horizontal. When it has moved a horizontal distance of 80 m then it's distance from point of projection is -

(A) 40 m (B) $40\sqrt{2}$ m (C) $40\sqrt{5}$ m (D) none of these

Sol.

$U_y = 30$m/s



$S_x = 80$m

$U_x \times t = 80$ $\boxed{t = 2\text{sec}}$

$Sy = 30 \times 2 - \frac{1}{2} g \times 4$

$= 60 - 20 = 40$ m

Total distance $S = \sqrt{Sx^2 + fy^2}$

$= \sqrt{1600 + 6400} = 40\sqrt{5}$

3. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacements x and y vary with time t in second as : $x = 10\sqrt{2}\,t$; $y = 10t - t^2$. The maximum height attained by the ball is-

(A) 100 m (B) 75 m (C) 50 m (D) 25 m

Sol. $X = 10\sqrt{3}\,t$ $y = 10t - t^2$

$V_x = 10\sqrt{3}$ $V_y = 10 - 2t$

$U_x = 10\sqrt{3}$ $U_y = 10$

At maximum height $0 = 10 - 2t$

$\boxed{t = 5\text{sec}}$

Now $\boxed{y = 50 - 25 = 25m}$

4. A projectile is given an initial velocity of $\hat{i} + 2\hat{j}$. The Cartesian equation of its path is - ($g = 10\ m/s^2$)

(A) $y = 2x - 5x^2$ (B) $y = x - 5x^2$ (C) $4y = 2x - 5x^2$ (D) $y = 2x - 25x^2$

Sol. $U = \hat{i} + 2\hat{j}$

$U_x = 2\hat{i}$

$U_y = 2\hat{j}$

$\tan\theta = 2$ $U = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$

$\because$ $y = x\tan\theta - \frac{g}{2} - \frac{x^2}{U^2}(1 + \tan^2\theta)$

$= x2 - 5\frac{x^2}{5}(1 + 4)$

$\boxed{y = 2x - 5x^2}$

5. The equation of motion of a projectile is, $y = 12x - \frac{3}{4}x^2$ what is range of the projectile ?

(A) 12.4 m (B) 16 m (C) 30.6 m (D) 36.0 m

Sol. $y = 12x - \frac{3}{4}x^2$

Compare with

$y = x\tan\theta = \frac{g}{2}\frac{x^2}{U^2\cos^2\theta}$

$\tan\theta = 12$

$\frac{g}{2U^2\cos^2\theta} = \frac{3}{4}$

$U^2\cos^2\theta = \frac{2g}{3}$

$U = \frac{1}{100}\sqrt{\frac{2g}{3}}$

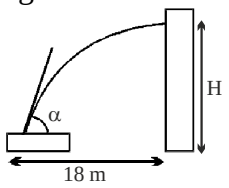
$U = \sqrt{\frac{145 \times 20}{3}}$

Or Ranyel means $y = 0$.

$12x - \frac{3}{9}x^2 = 0$

$\boxed{x = 16m}$

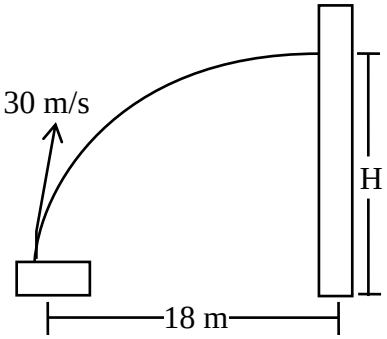
6. A fire nozzle situated at a distance 18m from the building discharges water with an initial velocity of 30 m/s as shown in the figure. Find :



(i) the maximum height H that can be reached by water.

(ii) the corresponding angle α.

Sol.



Equation of trajectory

x = 18m U = 20m/s

y = H

Now

$$H = x\tan\theta - \frac{g}{2}\frac{x^2}{y^2}(1+\tan 2\theta) \quad \ldots(i)$$

$$H = 18\tan\theta - \frac{g}{2}\frac{18^2}{30^2} - \frac{g}{2}\frac{18^2}{U^2}\tan^2\theta$$

$$\text{for } H_{max} = \frac{dH}{d\theta} = 0$$

$$18\sec^2\theta = \frac{5\times 18\times 18}{900}\times 2\tan\theta\sec^2\theta = 0$$

$$18 = \frac{18}{5}\tan\theta$$

$$\tan\theta = 5 \qquad \boxed{\theta = \tan^{-1}(5)}$$

Put it in (i)

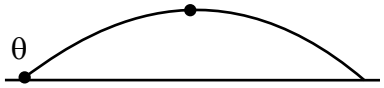
$$H = 18\times 5 - \frac{18\times 18}{900}(1+25)$$

$$= 90 - \frac{9\times 26}{5} = \frac{450-234}{5} = \frac{216}{5}$$

$$H_{max} = 43.2$$

7. A particle P is projected, from a point O on horizontal ground, with speed v at an angle θ above the horizontal, where tanθ = 1/3.The particle passes through the point with coordinates (3a, 3a/4) relative to horizontal and vertical axes at O in the plane of motion. (a)Determine the value of v in terms of a and g. A particle Q is projected from O at the instant when P is moving horizontally. It strikes the ground at the same place and at the same instant as P. (b) Calculate the speed of projection of Q and find the tangent of the angle of projection. Express all the answers in terms of a and g.

Sol.



given $\tan\theta = \frac{1}{3}$

(a) given x = 3a $\quad y = \frac{3a}{4}$

using equation of trajectory for parabola.

(b) $y = x\tan\theta - \frac{g}{2}\frac{x^2}{v^2}(1+\tan^2\theta)$

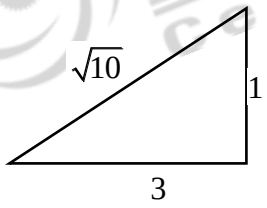
$$\frac{3a}{4} = 3a\times\frac{1}{3} - \frac{g}{2}\frac{ga^2}{v^2}\left(1+\frac{1}{9}\right)$$

$$\frac{3}{4} = 1 - \frac{2ga}{2v^2} \quad \frac{10}{9}$$

$$\frac{3}{4} = 1 - \frac{5ga}{v^2}$$

$$\boxed{v = 2\sqrt{5ga}}$$

$\sqrt{18g}$



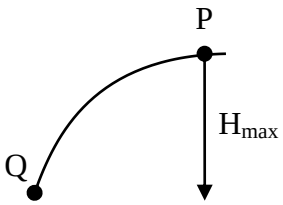
$\text{Sin}\theta = \frac{1}{\sqrt{10}}$

$\text{Cos}\theta = \frac{3}{\sqrt{10}}$

$V_y = V\sin\theta = 2\sqrt{g} = \sqrt{2g}$

$V_x = 6.\sqrt{ga} = \sqrt{18g}$

(b)



$$T_{risep} = \frac{V_y}{g} = \frac{\sqrt{2ga}}{g} = \sqrt{\frac{2ga}{g}} = \sqrt{\frac{2a}{g}}$$

$$T_{fallp} = \sqrt{\frac{2a}{g}} = T_{fligh'p'} = \sqrt{\frac{2a}{g}}$$

$$T_{fligh\theta} = \sqrt{\frac{2a}{g}} = T_{fallp}$$

$$\frac{2U_y}{g} = \sqrt{\frac{2a}{g}}$$

$$\boxed{U_y = \sqrt{\frac{g2a}{4g}} = \sqrt{\frac{ga}{2}}}$$

$R_P = V_x \times t_p$ Q $R_p = R_o$

$Ru = U_x \times t_b$

$$\sqrt{18ga} \times \sqrt{\frac{8a}{g}} = U_x \times \sqrt{\frac{2a}{g}}$$

$$\sqrt{\frac{18ga \times g \times 8}{g \times 2a}} = U_x \qquad U_x = \sqrt{72ag}$$

$$\boxed{U = \sqrt{72ag} + \frac{ga}{2} = \sqrt{\frac{145ga}{2}}}$$

$$\tan\theta = \sqrt{\frac{ga}{2 \times 12ag}}$$

$$\boxed{\theta = \tan^{-1}\frac{1}{12}}$$

8. The position vector of a particle moving on x-y plane is given by $\vec{r} = t^2\hat{i} + \left(t - t^2\right)\hat{j}$. Find (a) the trajectory of the particle (b) the time at which its speed is minimum.

Sol. $\vec{r} = t^2\hat{i} + (t - t^2)\hat{j}$

(a) $\because$ $x = t^2$ $t = \sqrt{x}$

$y = t - t^2$

$y = \sqrt{x} - x$

$$\boxed{(x+y)^{1/2} = x}$$

(b) diff. $\vec{r}$ w.r.t

$$\vec{V} = 2t\hat{i} + (1-2t)\hat{j}$$

$$|\vec{V}| = \sqrt{(2t)^2 + (1-2t)^2}$$

$$V = \sqrt{4t^2 + 1 + 4t^2 - 4t}$$

$$V = \sqrt{8t^2 - 4t + 1}$$

S minimum value

$$\frac{dv}{dt} = 0$$

$$\frac{1}{2}(8t^2 - 4t + 1)(16t - 4) = 0$$

$$(2t^2 - t + \frac{1}{4})\,(4t - 1) = 0$$

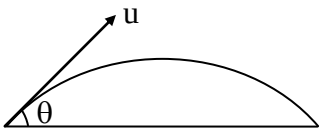
$$8t^3 - 4t^2 + t - 2t^2 + t - \frac{1}{4} = 0$$

Hit & fail method at $t = \frac{1}{4} = 0$

at $= \frac{1}{4}$ sec min.

9. The maximum range of a particle with a certain speed on a horizontal plane is R. Find its maximum range when projected up on an inclined plane with inclination 30°.

Sol. For horizontal plane–



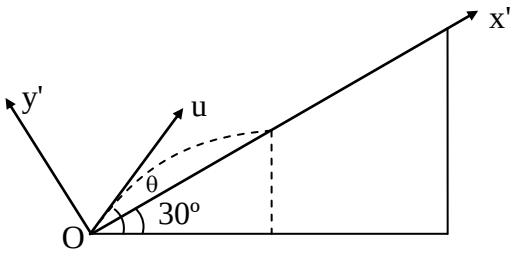
$$R = \frac{u^2 \sin 2\theta}{g}$$

R = max at $\theta = 45^o$

$$R = \frac{u^2 \sin 2(45^o)}{g}$$

$$R = \frac{u^2}{g}$$

for inclined plane :



In y' direction

$$S_{y'} = u_y T + \frac{1}{2} a_y T^2$$

$$0 = u\sin(\theta - 30^o)\, T - \frac{1}{2} g \cos 30^o\, T^2$$

$$T = \frac{2u \sin(\theta - 30^o)}{g \cos 30^o} \qquad \ldots..(1)$$

in x' direction

$R = u_{x'} T + \frac{1}{2} a_{x'} T^2$

$R = u\cos(\theta - 30^\circ)T - \frac{1}{2} g (\sin 30^\circ)T^2$

$\therefore \quad R = \frac{2u^2 \sin(\theta - 30^\circ)\cos\theta}{g\cos^2\theta}$

For R_{max} ; $\frac{dR}{d\theta} = 0$

On solving $\theta = 60^\circ$

$R_{max} = \frac{u^2}{g\left(1+\frac{1}{2}\right)} = \frac{2}{3}\frac{u^2}{g} = \frac{2}{3}R$

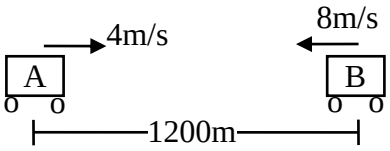
WORK BOOK–7

1. Two cars are moving towards each other with velocity 4m/s & 8 m/s respectively from two different points in straight line, 1200 m apart simultaneously, find time after which both cars meet :-

(A) 300 sec (B) 150 sec (C) 100 sec (D) None of these

Ans. **(C)**

Sol.



$V_{A/B} = 12$ m/s

$$t = \frac{1200}{12}$$

$$\boxed{t = 100 \text{ sec}}$$

2. In the previous question find difference of distance travelled by each car in others frame :-

(A) 400 m (B) 0 m (C) 800 m (D) None of these

Ans. **(B)**

Sol. In each other frame :

They travel 1200 m so their different e

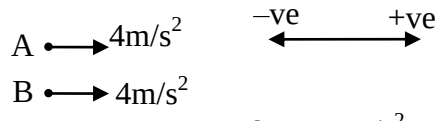
$1200 - 1200 = 0$

3. Two cars A & B start from rest (from the same point) in same direction with acceleration 8 m/s^2 & 4 m/s^2 respectively then acceleration of car B in frame of A (Take direction of motion of car is positive):-

(A) 4 m/s^2 (B) – 4 m/s^2 (C) 12 m/s^2 (D) None of these

Ans. **(B)**

Sol.



$a_{A/B} = a_B - a_A = 4 - 8 = -4$ m/s^2

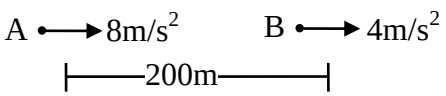
4. In the previous question when distance between both cars becomes 200 m, find distance travelled by the

car A in ground frame :-

(A) 200 m (B) 600 m (C) 800 m (D) 400 m

Ans. **(D)**

Sol. $S_A = \frac{1}{2} \times 8 \times t^2$ …(i)

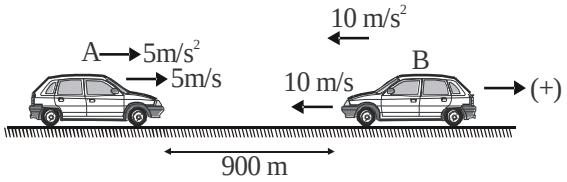


$= \frac{1}{2} \times 8 \times 100$

$S_{A/B} = \frac{1}{2} \times 4 \times t^2$

S_A = 400 m $\qquad \frac{200}{2} = t^2$ $\qquad$ Put it into (i)

5. Velocity and acceleration of both the cars is given in the figure find sum of displacement of each car in ground frame when they finally meet :-



(A) –300 m (B) 300 m (C) 600 m (D) –600 m

Ans. **(A)**

Sol. Time when they are meet

$V_{A/B}^2 = U_{A/B}^2 + 2a_{A/B}S$

$V_{A/B}^2 = 225 + 27000$

$V_{A/B} = \sqrt{27225}$

$\boxed{V_{A/B} = 165}$

$V_{A/B} = U_{A/B} + at$

165 = 15 + 15t

In groomed frame disp.

$S_A = \frac{1}{2} \times 5 \times 100 + 5 \times 10 = 300$

$S_B = -10 \times 10 - \frac{1}{2} \times 10 \times 100 = -600$

Sum of displace

= – 600 + 300 = –300

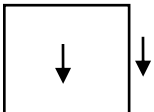
Ans. (A)

6. If a lift is moving downwards with constant velocity 5 m/s. A coin is dropped from the lift, find velocity of coin at the same instant when it is dropped in frame of lift and in frame of ground :-

(A) 5 m/s (downward), & 0 (B) 0 & 5 m/s (downward)

(C) 5 m/s (upward) & 0 (D) 0 & 5 m/s (upward)

Ans. **(B)**

Sol. An frame of  5m/s cos4. an ground frame

Dift.

$V_{C/\ell} = V_C - V_\ell$ $\qquad V_C = 5m/s\downarrow$

= 5 – 5 = 0

7. A lift is moving downward with constant velocity 10 m/s. A coin is dropped at height 5m from the floor of lift, find the time taken by coin to reach the floor :-

(A) 1 s (B) 0.5 s (C) 2 s (D) 1.5 s

Ans. **(A)**

Sol. In field frame initial $V_{C/\ell} = 0$ $a = -g$

$$5 = 15t + \frac{1}{2}at^2$$

$$5 = \frac{1}{5}gt^2$$

$$\boxed{t = 1 \text{ sec}}$$

8. In the previous question, find distance travelled by coin in ground frame :-

(A) 10m (B) 5 m (C) 20 m (D) 15 m

Ans. **(D)**

Sol. In grand frame $S = 15t + \frac{1}{2}at^2$

$$S_G = -10 \times 1 + \frac{1}{2}(-g) \times 1$$

$$= -10 - 5 = \text{(down ward) } 15$$

$$\boxed{S_G = 15\text{m}}$$

9. A lift is moving with velocity 10 m/s upward with acceleration 5 m/s^2 upwards. At the same instant of time, a coin is dropped from 7.5 m above the floor of lift, find distance travelled by the coin in ground frame before it strikes the floor :-

(A) 5 m (upward) (B) 10 m (downward) (C) 10 m (upward) (D) 5 m (downward)

Ans. **(A)**

Sol. In lift frame tome to drop the win floor

$$7.5 = \frac{1}{2}g \times t^2 \qquad a = g + 5 = 15$$

$$\boxed{t = 1\text{sec}}$$

In ground frame total travel distance

$= 12.5 - 7.5 = 5\text{m}$ VP word

10. A lift is moving vertically downwards with speed 10 m/s and acceleration 5 m/s^2 downwards. A coin is released 10 m from floor of lift, find distance travelled by lift when coin strikes the floor of lift :-

(A) 20 m (B) 10 m (C) 25 m (D) 30 m

Ans. **(D)**

Sol. Time when coin strike in lift frame

$a_{c/\ell} = 5 \text{ m/s}\downarrow$

$U_{C/\ell} = 0$

$S = \frac{1}{2}at^2$

$10 = \frac{1}{2}\times 5t^2$ $\qquad \boxed{t = 2\text{sec}}$

Distance travel by lift in time inter

$U = 10\ m/s\downarrow \quad a = 5m/s^2\downarrow$

$S = 10 \times 2 + \frac{1}{2} \times 4$

$S = 30\ m$

11 A person walks up a stationary escalator in t_1 second. If he remains stationary on the escalator, then it can take him up in t_2 second. If the length of the escalator is L, then

(i) determine the speed of man with respect to the escalator

(ii) determine the speed of the escalator.

(iii) how much time would it take him to walk up the moving escalator ?

Ans. (i) $v_{me} = \frac{L}{t_1}$, (ii) $v_e = \frac{L}{t_2}$, (iii) $t = \frac{t_1 t_2}{t_1 + t_2}$

Sol. (i) $V_{m/L} = \frac{L}{t_1}$

(ii) $V_{L/m} = \frac{L}{t_2}$

(iii) Let t be the total time.

No $V = V_{m/L} + V_{L/m}$

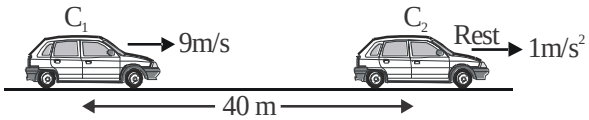
$\frac{L}{t} = \frac{L}{t_1} + \frac{L}{t_2}$

$\boxed{t = \frac{t_1 t_2}{t_1 + t_2}}$

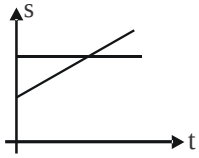
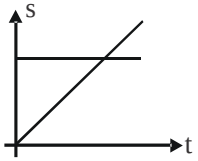
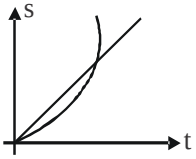
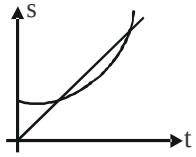
WORK BOOK–8

Paragraph for question nos. 1 to 3

Two cars travelling on a straight road. The car C_1 is going at a constant speed of 9 m/s. 40 m ahead of C_1 is car C_2 at rest, but starts moving with an acceleration of 1 m/s^2.

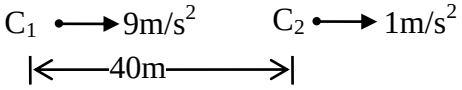


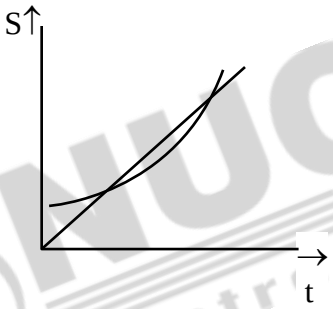
1. The graph correctly representing the position of two cars with time w.r.t. initial position of C_1.

(A)  (B)  (C)  (D) 

Ans. **(D)**

Sol.



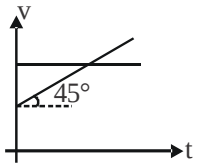
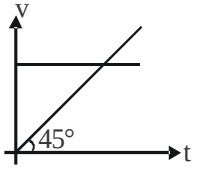
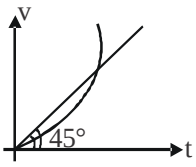
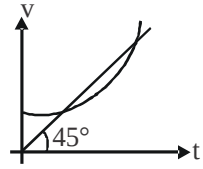


w.r.t to initial position S C_2 = 40 m aheall with C1 and moves with lonst acceleration

$S = \frac{1}{2} \times 1t^2$

C_1 = initial pasis moves with can speed straight line $\boxed{S = Vt}$

2. The graph correctly representing the velocity of the two cars with time

(A)  (B)  (C)  (D) 

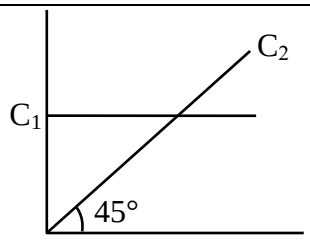
Ans. **(B)**

Sol. C_1 moves with constant speed with gm/s and C_2 moves with constant acceleration so initial C_2 Rest $\quad \therefore \quad U = 0$

$V = \cancel{U}^{0} + at$

$V = at \qquad a = \tan\theta$

$1 = \tan\theta \qquad \boxed{\theta = 45°}$



3. During their motion, the car C_1 over takes the car C_2. But soon C_2 overtakes C_1 so determine the maximum lead that C_1 can have from C_2.

(A) 0.5m (B) 9.5m (C) 81m (D) 90m

Ans. **(A)**

Sol. In frame of Reference of C_2

Initial $V_{C_1/C_2} = 9$ m/s

Initial $a_{C_1/C_2} = -1$ m/s^2

$S = 40$ m

Time other they meet.

$$S = ut + \frac{1}{2}at^2$$

$$40 = 9t + \frac{1}{2} \times (-1)t^2$$

$$80 = 18t - t^2$$

$$t^2 - 18t + 80 = 0$$

$$t^2 - 10t - 8t + 80 = 0$$

$$t(t - 10) - 8(t - 10) = 0$$

$$\boxed{t = 8,10}$$

means care, cross the carat t = 8sec and after 2 sec C_2 cross the C_1

So at t = 9 distance travel by cari C_1 & car C_2

$Sc_1 = 9 \times 9 = 81$m. (before 40m)

$Sc_2 = \cancel{Ut^0} + \frac{1}{2}at^2$ (after 40m)

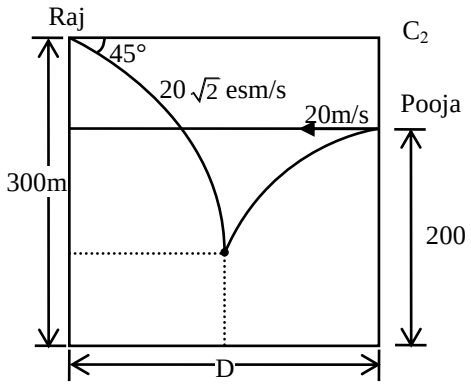
$$= \frac{1}{2} \times 1 \times 81 = 40.5$$

$Sc_1 = 81 - 40 = 41$ [after 40 m]

Now the maximum lead that C_1 from C_2 is = 41 – 40.5 = 0.5m.

Paragraph for Question Nos. 4 to 6

Two friends Raj & Pooja playing a game of collision of balls and throwing balls from the top of the tower simultaneously as shown in the figure. If the balls collide in air at point P and point O is treated as origin ($g = 10$ m/s^2)

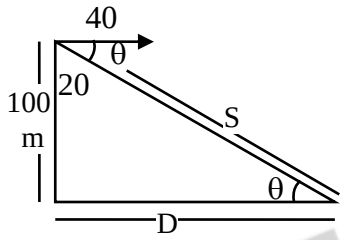


4. Distance D between the towers is -

(A) 100 m (B) 200 m (C) 400 m (D) 800 m

Ans. **(B)**

Sol. In For Pooja



$$\tan\theta = \frac{20}{40} = \frac{1}{2}$$

$$\tan\theta = \frac{100}{D} = \frac{1}{2} \qquad D = 200m$$

when they meet,

5. Co-ordinate of point P is-

(A) (100,75) (B) (100,125) (C) (75,100) (D) (175,100)

Ans. **(A)**

Sol. $S^2 = (200 - 40t)^2 + (100 - 20t)$

$$\frac{ds^2}{dt} = 0$$

$0 = 2(200 - 40t) - 40 + 2(100 - 20t) - 2$

$= -[(200 - 40t)40 + (100 - 20t)20]$

$= 400 - 80t + 100 - 20t$

$100t = 500$

$T = 5$ sec

Now Co–ordinate

$x = 20 \times = 100 \Rightarrow U\cos\theta \times t$

$y\cos d = 300 - (+20\times5 + \frac{1}{2} \times 10 \times 25)$

$= 300 - [+100 + 125]$

$= 300 - 125$

$y\cos = 75$

6. If wind starts blowing horizontally along positive x-direction, due to which a horizontal acceleration of magnitude $a_x = 2m/s^2$ is imparted to the balls, then co-ordinates of point of collision will be-

(A) (125, 100) (B) (75, 100) (C) (125, 75) (D) (100, 125)

Ans. **(C)**

Sol. y – co-ordinate are same

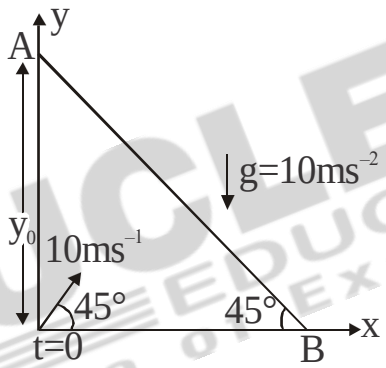
Chang l in x-cordinate

$$x = 100 + \frac{1}{2} \times 2 \times 25$$

$$\boxed{x = 125}$$

Paragraph for Question Nos. 7 to 9

AB is an inclined roof and a particle is projected from origin towards the roof such that path of the particle will just touch the roof once.



7. Particle will touch roof at t = ...

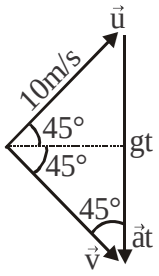
(A) 2s (B) $\sqrt{2}$s (C) $\frac{1}{\sqrt{2}}$s (D) $2\sqrt{2}$s

Ans. **(B)**

Sol. At the time of touching velocity makes an angle 45° with horizontal in downward direction.

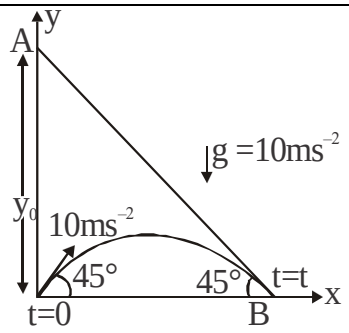
From $+ \ \vec{v} = \vec{u} + \vec{a}t$

$$\frac{10}{gt} = \sin 45° \Rightarrow t = \sqrt{2}s$$



OR

Particle will touch roof at time of landing.



So t = $\frac{2u\sin\theta}{g} = \frac{2(10)\sin 45°}{10} = \sqrt{2}s$

8. Maximum height attained by projectile is

(A) 5m (B) $\frac{15}{2}$m (C) $\frac{7}{2}$m (D) $\frac{5}{2}$m

Ans. **(D)**

Sol. $H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{(10)^2\left(\frac{1}{\sqrt{2}}\right)^2}{2(10)} = \frac{5}{2}$m

9. The value of y_0 is

(A) 5m (B) 15 m (C) 20 m (D) 10 m

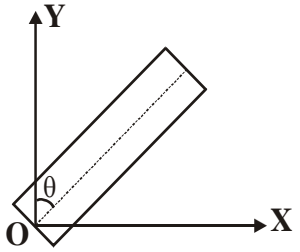
Ans. **(D)**

Sol. $y_0 = R = \frac{u^2\sin 2\theta}{g} = \frac{(10)^2(1)}{10} = 10m$

10. The axis of a cylindrical vessel is inclined at an angle θ with y-axis. The magnitude of velocity of falling rain is v. Origin is selected at O. The rate of filling the vessel will be maximum, if

(A) $\vec{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$ (B) $\vec{v} = -v\sin\theta\hat{i} - v\cos\theta\hat{j}$

(C) $\vec{v} = v\cos\theta\hat{i} - v\sin\theta\hat{j}$ (D) $\vec{v} = v\sin\theta\hat{i} - v\cos\theta\hat{j}$



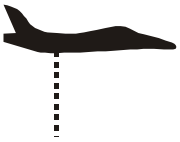
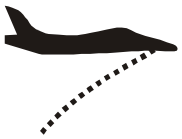
Ans. (B)

11. An observer on ground sees a boat cross a river of width 800 m perpendicular to its stream in 200 seconds. He also finds a man on a raft floating at speed of 3 m/s with river. The distance travelled by boat as seen by man on the raft in crossing the river is -

(A) 800 m (B) 1000m (C) 1200m (D) 1600m

Ans. (B)

12. Figure shows a still photograph from a war movie. Bombs have been dropped from the plane B-52 at regular intervals. Air plane moves with constant speed in horizontal direction. Which figure may be true still photograph. [Assume that there is no wind.]

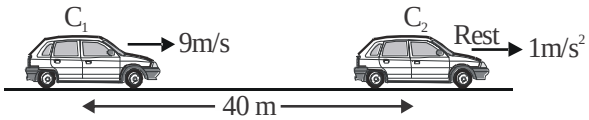
(A)  (B)  (C) (D)

Ans. **(A)**

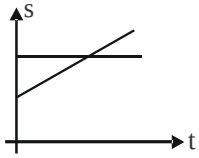
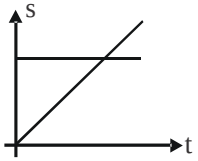
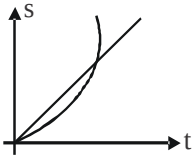
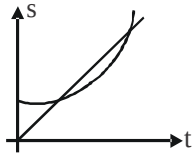
WORK BOOK–8

Paragraph for question nos. 1 to 3

Two cars travelling on a straight road. The car C_1 is going at a constant speed of 9 m/s. 40 m ahead of C_1 is car C_2 at rest, but starts moving with an acceleration of 1 m/s^2.

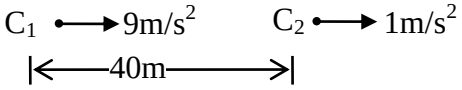


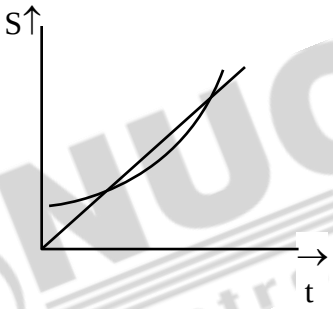
1. The graph correctly representing the position of two cars with time w.r.t. initial position of C_1.

(A)  (B)  (C)  (D) 

Ans. **(D)**

Sol.



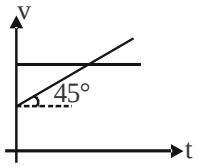
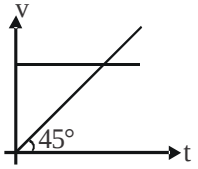
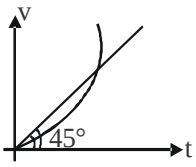
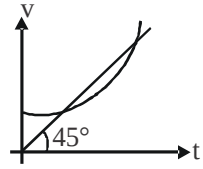


w.r.t to initial position S C_2 = 40 m aheall with C1 and moves with lonst acceleration

$S = \frac{1}{2} \times 1t^2$

C_1 = initial pasis moves with can speed straight line $\boxed{S = Vt}$

2. The graph correctly representing the velocity of the two cars with time

(A)  (B)  (C)  (D) 

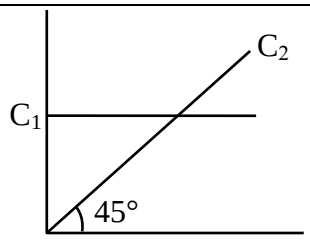
Ans. **(B)**

Sol. C_1 moves with constant speed with gm/s and C_2 moves with constant acceleration so initial C_2 Rest $\quad \therefore \quad U = 0$

$V = \cancel{U}^{0} + at$

$V = at \qquad a = \tan\theta$

$1 = \tan\theta \qquad \boxed{\theta = 45°}$



3. During their motion, the car C_1 over takes the car C_2. But soon C_2 overtakes C_1 so determine the maximum lead that C_1 can have from C_2.

(A) 0.5m (B) 9.5m (C) 81m (D) 90m

Ans. **(A)**

Sol. In frame of Reference of C_2

Initial $V_{C_1/C_2} = 9$ m/s

Initial $a_{C_1/C_2} = -1$ m/s^2

$S = 40$ m

Time other they meet.

$$S = ut + \frac{1}{2}at^2$$

$$40 = 9t + \frac{1}{2} \times (-1)t^2$$

$$80 = 18t - t^2$$

$$t^2 - 18t + 80 = 0$$

$$t^2 - 10t - 8t + 80 = 0$$

$$t(t - 10) - 8(t - 10) = 0$$

$$\boxed{t = 8,10}$$

means care, cross the carat t = 8sec and after 2 sec C_2 cross the C_1

So at t = 9 distance travel by cari C_1 & car C_2

$Sc_1 = 9 \times 9 = 81$m. (before 40m)

$Sc_2 = \cancel{Ut^0} + \frac{1}{2}at^2$ (after 40m)

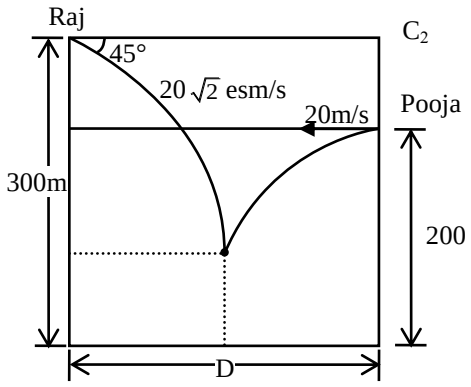
$$= \frac{1}{2} \times 1 \times 81 = 40.5$$

$Sc_1 = 81 - 40 = 41$ [after 40 m]

Now the maximum lead that C_1 from C_2 is = 41 – 40.5 = 0.5m.

Paragraph for Question Nos. 4 to 6

Two friends Raj & Pooja playing a game of collision of balls and throwing balls from the top of the tower simultaneously as shown in the figure. If the balls collide in air at point P and point O is treated as origin ($g = 10$ m/s^2)

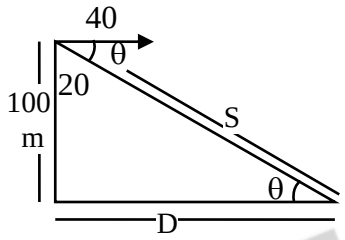


4. Distance D between the towers is -

(A) 100 m (B) 200 m (C) 400 m (D) 800 m

Ans. **(B)**

Sol. In For Pooja



$$\tan\theta = \frac{20}{40} = \frac{1}{2}$$

$$\tan\theta = \frac{100}{D} = \frac{1}{2} \qquad D = 200m$$

when they meet,

5. Co-ordinate of point P is-

(A) (100,75) (B) (100,125) (C) (75,100) (D) (175,100)

Ans. **(A)**

Sol. $S^2 = (200 - 40t)^2 + (100 - 20t)$

$$\frac{ds^2}{dt} = 0$$

$0 = 2(200 - 40t) - 40 + 2(100 - 20t) - 2$

$= -[(200 - 40t)40 + (100 - 20t)20]$

$= 400 - 80t + 100 - 20t$

$100t = 500$

$T = 5$ sec

Now Co–ordinate

$x = 20 \times = 100 \Rightarrow U\cos\theta \times t$

$y\cos d = 300 - (+20\times5 + \frac{1}{2} \times 10 \times 25)$

$= 300 - [+100 + 125]$

$= 300 - 125$

$y\cos = 75$

6. If wind starts blowing horizontally along positive x-direction, due to which a horizontal acceleration of magnitude $a_x = 2m/s^2$ is imparted to the balls, then co-ordinates of point of collision will be-

(A) (125, 100) (B) (75, 100) (C) (125, 75) (D) (100, 125)

Ans. **(C)**

Sol. y – co-ordinate are same

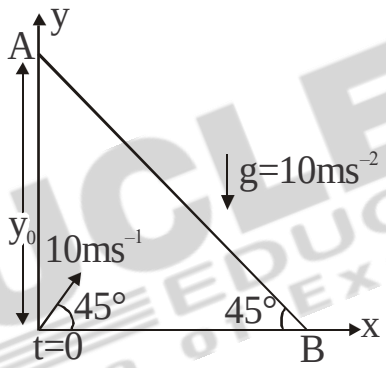
Chang l in x-cordinate

$$x = 100 + \frac{1}{2} \times 2 \times 25$$

$$\boxed{x = 125}$$

Paragraph for Question Nos. 7 to 9

AB is an inclined roof and a particle is projected from origin towards the roof such that path of the particle will just touch the roof once.



7. Particle will touch roof at t = ...

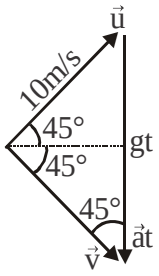
(A) 2s (B) $\sqrt{2}$s (C) $\frac{1}{\sqrt{2}}$s (D) $2\sqrt{2}$s

Ans. **(B)**

Sol. At the time of touching velocity makes an angle 45° with horizontal in downward direction.

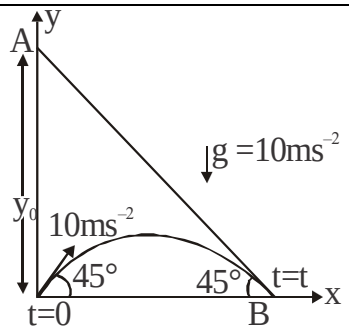
From $+ \ \vec{v} = \vec{u} + \vec{a}t$

$$\frac{10}{gt} = \sin 45° \Rightarrow t = \sqrt{2}s$$



OR

Particle will touch roof at time of landing.



So t = $\frac{2u\sin\theta}{g} = \frac{2(10)\sin 45°}{10} = \sqrt{2}s$

8. Maximum height attained by projectile is

(A) 5m (B) $\frac{15}{2}$m (C) $\frac{7}{2}$m (D) $\frac{5}{2}$m

Ans. **(D)**

Sol. $H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{(10)^2\left(\frac{1}{\sqrt{2}}\right)^2}{2(10)} = \frac{5}{2}$m

9. The value of y_0 is

(A) 5m (B) 15 m (C) 20 m (D) 10 m

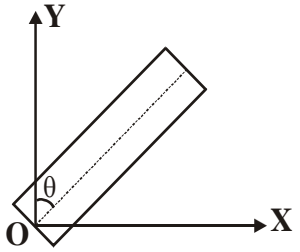
Ans. **(D)**

Sol. $y_0 = R = \frac{u^2\sin 2\theta}{g} = \frac{(10)^2(1)}{10} = 10m$

10. The axis of a cylindrical vessel is inclined at an angle θ with y-axis. The magnitude of velocity of falling rain is v. Origin is selected at O. The rate of filling the vessel will be maximum, if

(A) $\vec{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$ (B) $\vec{v} = -v\sin\theta\hat{i} - v\cos\theta\hat{j}$

(C) $\vec{v} = v\cos\theta\hat{i} - v\sin\theta\hat{j}$ (D) $\vec{v} = v\sin\theta\hat{i} - v\cos\theta\hat{j}$



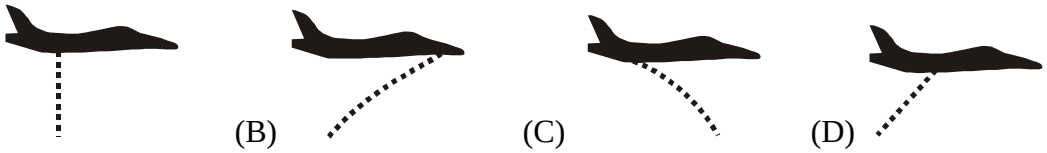
Ans. (B)

11. An observer on ground sees a boat cross a river of width 800 m perpendicular to its stream in 200 seconds. He also finds a man on a raft floating at speed of 3 m/s with river. The distance travelled by boat as seen by man on the raft in crossing the river is -

(A) 800 m (B) 1000m (C) 1200m (D) 1600m

Ans. (B)

12. Figure shows a still photograph from a war movie. Bombs have been dropped from the plane B-52 at regular intervals. Air plane moves with constant speed in horizontal direction. Which figure may be true still photograph. [Assume that there is no wind.]



(A) (B) (C) (D)

Ans. **(A)**

WORK BOOK–9

MINIMUM DISTANCE OF APPROACH

Paragraph for question nos. 1 to 3

Ram and Shyam are walking on two perpendicular tracks with speed 3 ms^{-1} and 4 ms^{-1} respectively. At a certain moment (say t = 0 sec) Ram and Shyam are at 20 m and 40 m away from the intersection of tracks respectively and moving towards the intersection of the tracks.

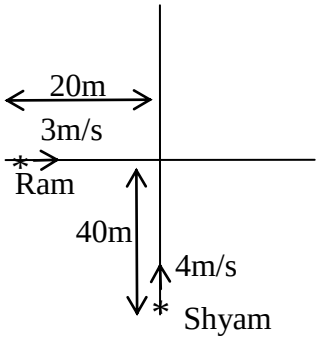
1. During the motion the magnitude of velocity of Ram with respect to Shyam, is -

(A) 1 ms^{-1} (B) 4 ms^{-1} (C) 5 ms^{-1} (D) 7 ms^{-1}

Ans. **(C)**

Sol. Given $\vec{V}_R = 3\hat{i}$

$\vec{V}_S = 4\hat{j}$



In frame of reference of Shyam

$\vec{V}_{RS} = \vec{V}_R - \vec{V}_S$

$\Rightarrow 3\hat{i} - 4\hat{j}$

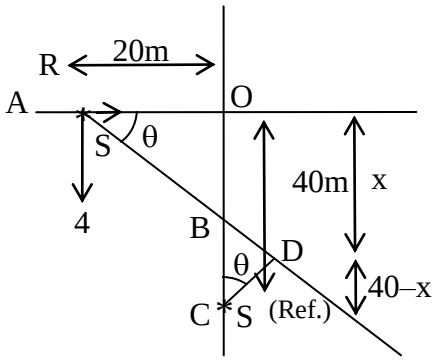
$|\vec{V}_{RS}| = \sqrt{(3)^2 + (4)^2} = 5$ m/s

2. Shortest distance between them subsequently, is -

(A) 18 m (B) 15 m (C) 25 m (D) 8 m

Ans. **(D)**

Sol. w.r.t Shyam



and $BC = 40 - x = 40 - \frac{80}{3}$

$\Rightarrow \frac{40}{3}m$

and shortest distance

$CD = BC\cos\theta$

$\Rightarrow \frac{40}{3} \times \frac{3}{5} \Rightarrow 8$ m

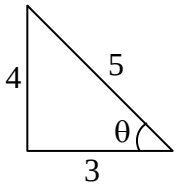
$\tan\theta = \frac{4}{3}$

CD = shortest distance

Now from ΔAOB

$\frac{x}{20} = \frac{4}{3}$

$\Rightarrow x = \frac{80}{3}m$



3. The time t when they are at shortest distance from each other subsequently, is -

(A) 8.8 sec (B) 12 sec (C) 15 sec (D) 44 sec

Ans. **(A)**

Sol. From ΔAOB

$\frac{OA}{AB} = \cos\theta$

$\Rightarrow \frac{20}{AB} = \frac{3}{5}$

$\Rightarrow AB = \frac{100}{3} m$

and AD = AB + BD

$\Rightarrow \frac{100}{3} + BC\sin\theta$

$\Rightarrow \frac{100}{3} + \frac{40}{3} \times \frac{4}{5}$

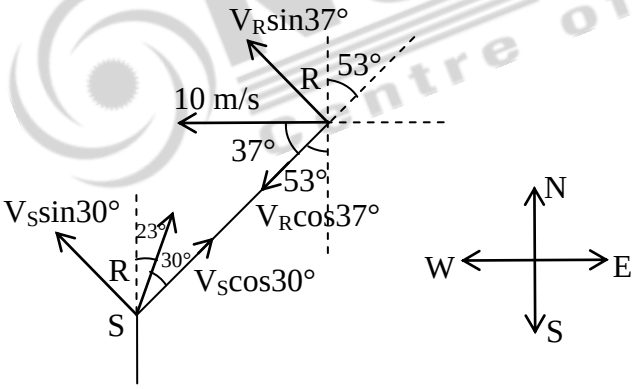
$\Rightarrow \frac{132}{3} m$

4. Ram moves with a velocity of 10 m/s in west direction. Shyam moves a direction 23° East of North. Ram is 100 m away from Shyam in direction 53° East of North of him. What should be speed of Shyam
so that he collides with Ram :-

(A) $4\sqrt{3}$m/s (B)10 m/s (C) 12 m/s (D) none of these

Ans. (C)

Sol. Given = $V_R = 10$



By figure

If born are collides then only they has velocity along SR so

$V_S \sin30° = V_R\sin37°$

$\Rightarrow V_S \frac{1}{2} = 10 \times \frac{3}{5}$

$\Rightarrow V_S = 12$ m/s

5. Two particles are projected from the same point, with the same speed, in the same vertical plane, at different angles with the horizontal. A frame of reference is fixed to one particle. The position vector of
the other particle, as observed from this frame, is $\vec{r}$. Which of the following statements are correct?

(A) $\vec{r}$ is a constant vector

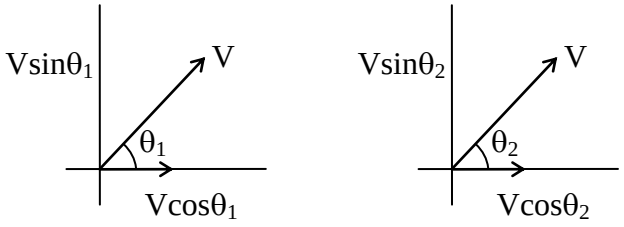
(B) $\vec{r}$ changes in magnitude and direction with time

(C) The magnitude of $\vec{r}$ increases linearly with time; its direction does not change

(D) The direction of $\vec{r}$ changes with time; its magnitude may or may not change, depending on the angles of projection

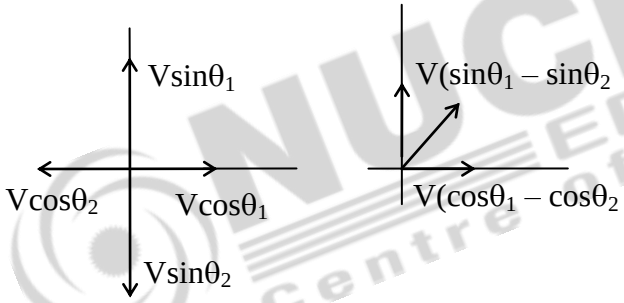
Ans. **(C)**

Sol.



(a) (b)

w.r.t. frame of reference of (b)

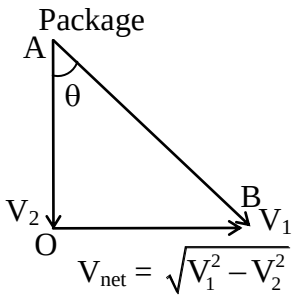


6. At a given instant, an observer stationary on the ground sees a package falling with speed v_1 at certain angle with the vertical. A pilot flying at a constant horizontal velocity relative to the ground sees the package falling vertically with a speed v_2 at the same instant. What is the speed of the pilot relative to the
ground?

(A) $v_1 + v_2$ (B) $v_1 - v_2$ (C) $\sqrt{v_1^2 - v_2^2}$ (D) $\sqrt{v_1^2 + v_2^2}$

Ans. **(C)**

Sol.



$V_{net} = \sqrt{V_1^2 - V_2^2}$

V_2 = Speed of package sees by pilot

V_1 = Speed of package w.r.t. ground

So speed of pilot is from ΔAOB

$V_P = \sqrt{V_1^2 - V_2^2}$

WORK BOOK–10

1. **Statement-1 :** The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank. They may reach opposite bank simultaneously moving along different straight line paths.

Statement-2 : For above boats to cross the river in same time, the component of their velocity relative to river in direction normal to flow should be same.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Ans. **(A)**

2. Two boats were going down stream with different velocities. When one overtook the other a plastic ball floating in water was seen in line with two boats. Some time later both boats turned back simultaneously

& went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first ?

(A) the boat which has greater velocity (relative to water)

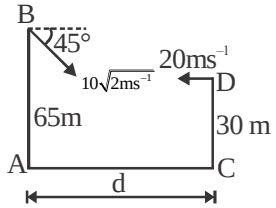
(B) the boat which has lesser velocity (relative to water)

(C) both will reach the ball simultaneously

(D) cannot be decided unless we know the actual values of the velocities and the time after which they turned around.

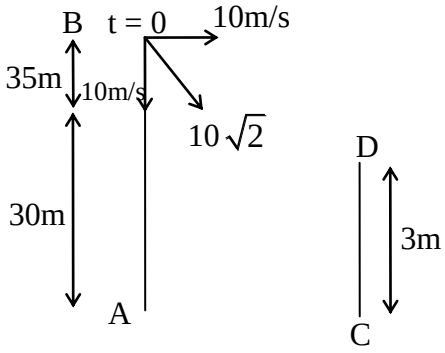
Ans. **(C)**

3. A ball of mass m is thrown at an angle of 45° to the horizontal from top of a 65m high tower AB as shown is Fig. Another identical ball is thrown with velocity 20 ms^{-1} horizontally towards AB from top of a 30m high tower CD one second after the projection of first ball. Both the 20 ms^{-1} balls move in same vertical plane. If they collide in mid air. Calculate distance AC.



Ans. **40m**

Sol.



For Ball B

In x-direction

X = vt = 10 m

In y-direction

$S = ut - \frac{1}{2}gt^2$

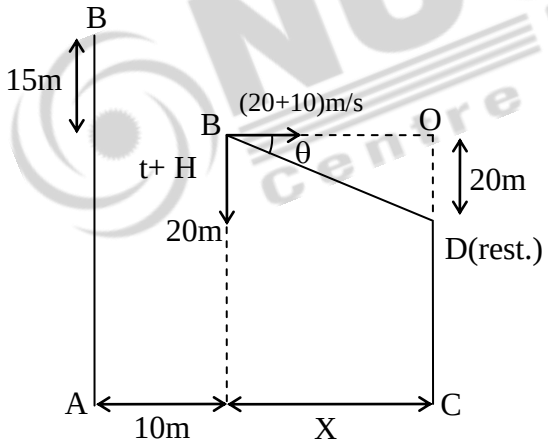
= 10 – 5 = 15m

And after t = 1 sec. velocity of B ball in y-direction

$V_y = u + at$

= –10 – 10 = –20 m/sec.

Now the situation after t = 1 sec. w.r.t. D ball frame



Condition for collides with ball D is its move along B'D So

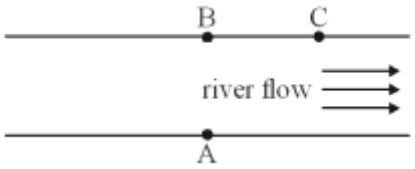
ΔB'OD

$\tan\theta = \frac{20}{30} = \frac{20}{x}$

so X = 30 m

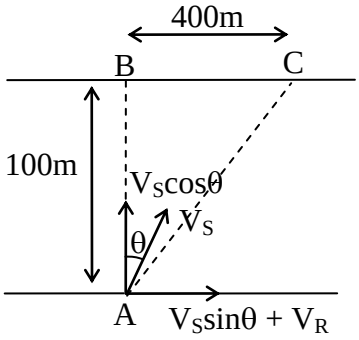
Now AD = 10 + X = 40m

4. A river is flowing with a speed of 1 km/hr. A swimmer wants to go to point C starting from A. He swims with a speed of 5 km/hr, at an angle θ w.r.t. the river flow. If AB = BC = 400 m. Then find the value of θ (in degree).



Ans. **53**

Sol.



Given V_S = 5 km/h

V_R = 1 km/h

Now time to cross $t = \frac{400}{V_S\cos\theta} = \frac{400}{5\cos\theta}$

and drift. BC = $(V_S\sin\theta + V_R)t$

$\Rightarrow 400 = (5\sin\theta + 1)\frac{400}{5\cos\theta}$

$\Rightarrow 5\cos\theta = 5\sin\theta + 1$ .....(1)

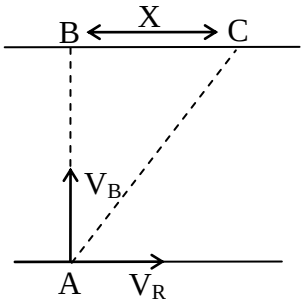
By solve this equation q = 37°

So from horizontal it is $\phi = 90° - \theta = 53°$

5. A boat can move at a speed of 6 km/h in still water. It has to cross a river in minimum time of ½ hr. Calculate the distance (in km) it drifts along river, if river flows at a speed of 4 km/h.

Ans. **2**

Sol.



Given V_B = 6 km/h

$t_{min} = \frac{1}{2}$ hour

$V_R = 4$ km/h

We know for minimum time crossing is flow $\perp$ to the river flow of

Now drift $X = V_R\, t_{min.}$

$\Rightarrow 4\times\frac{1}{2} = 2$km

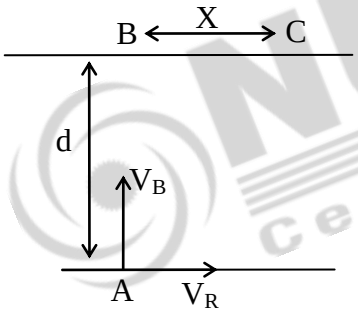
6. A mass crosses a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift
120 m. If he crosses the river taking shortest path, he takes 12.5 minute, find
(i) width of the river,
(ii) velocity of the boat with respect to water,
(iii) speed of the current.
Assume $v_{b/r} > v_r$

Ans. **200 m, 20 m/min, 12 m/min**

Sol. Given $t_{min.} = 120$ m

$x = 120$ m

C-1 for minimum time



$t_{min.} = \frac{d}{V_B}$

$\Rightarrow 10\times60 = \frac{d}{V_B}$ .....(1)

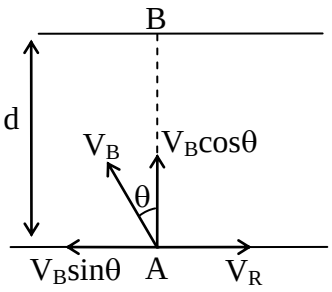
and $X = V_B\, t_{min.}$

$\Rightarrow X = 600\, V_R$

$\Rightarrow 120 = 600\, V_R$

$\Rightarrow V_R = \frac{1}{5}$ m/sec.

C-2 for shortest path



$V_B\sin\theta = V_R$

and $t = \dfrac{d}{V_B\cos\theta}$ .....(2)

$\Rightarrow 12.5\times60\ \dfrac{V_B 600}{V_B\cos\theta}$ [from eq$_n$. (1)]

$\Rightarrow \cos\theta = \dfrac{4}{5}$

$\theta = 37°$

From (1)

$d = \dfrac{1}{3}\times600 = 200m$

From (2)

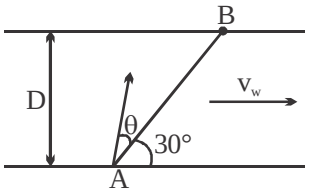
$V_B = \dfrac{V_R}{\sin\theta} = \dfrac{1\times5}{5\times3} = \dfrac{1}{3}$ m/sec.

(i) width of the river d = 200m

(ii) $\vec{V}_B = \dfrac{1}{3}\times60$ = 20m/min.

(iii) $\vec{V}_R = \dfrac{1}{5}\times60$ = 12 m/min.

7. Two ports A and B are separated by a river of width D. Water in the river flows with speed V_w. A boat
crosses the river from port A to port B. The speed of the boat relative to water is V_B. Given
$V_w = \sqrt{3}V_B$

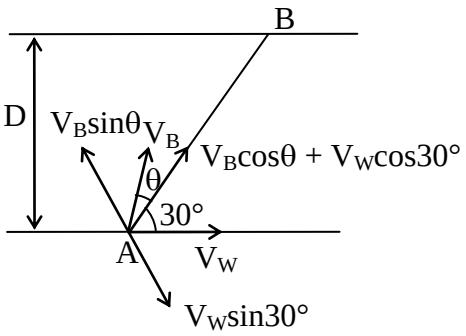


(a) Find the angle "θ" with AB in which the boat should start relative to water so that it moves along AB.

(b) Find the time taken by the boat to reach the port B in terms of D and V_B.

Ans. **(a) 60°, 120° (b)** $\frac{D}{V_B}, \frac{2D}{V_B}$

Sol.



Given $V_w = \sqrt{3}\, V_B$

and AB = $\frac{D}{\cos 60°}$ = 2D

(a) If heat move along AB then

$V_B\sin\theta = V_W\sin30°$

$\Rightarrow V_B\sin\theta = \frac{\sqrt{3}V_B}{2}$

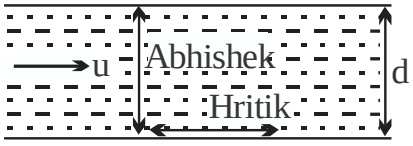
$\Rightarrow \theta = 60°$

(b) time t = $\frac{AB}{V_B\cos\theta + V_w\cos30°} = \frac{2D}{V_B\cos60° + \sqrt{3}V_B\frac{\sqrt{3}}{2}}$

$\Rightarrow \frac{2D}{\frac{V_B}{2} + \frac{3V_B}{2}} = \frac{4D}{4V_B}$

t = $\frac{D}{V_B}$

8. Abhishek and Hritik have identical motorboats whose speed relative to the water is v. They begin a race at the same point on the bank of a river of width d. The river flows at a constant and uniform speed u, which is less than v. Abhishek must cross the river to the point on the opposite bank directly across, and then return to the starting point. Hritik must travel downstream a distance d and then upstream to the starting point. (Assume that the boats are travelling at speed v relative to the water at all times. That is neglect the time necessary to accelerate from the start or to turn around.)



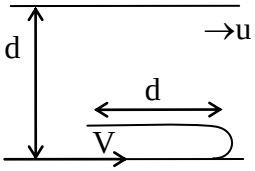
(a) What is the time for Hritik to make the round trip?

(b) What is the time for Abhishek to make the round trip?

(c) Which person takes more time for the round trip?

Ans. (a) $T_1 = \left[\frac{2vd}{v^2-u^2}\right]$ (b) $\frac{2d}{\sqrt{v^2-u^2}}$ (c) Hritik takes more time

Sol.



Hritik

(a) For Hritik

Down stream time

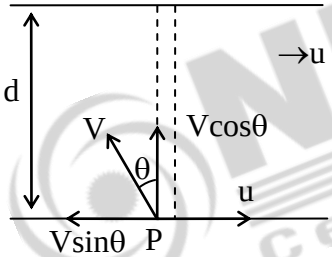
$t_1 = \frac{d}{v+u}$

Upstream time $t_2 = \frac{d}{v+u}$

Total time $t = t_1 + t_2$

$\Rightarrow \frac{2vd}{v^2-u^2}$

(b) for Abhishek



Abhishek

Condition for ⊥ move

$v\sin\theta = u$

$\sin\theta = \frac{u}{v}$

Now time $t = \frac{d}{v\cos\theta}$

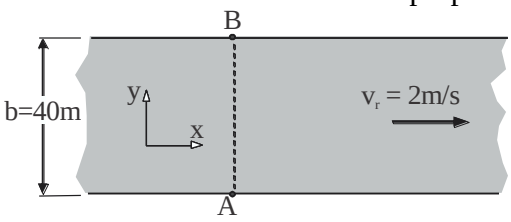
$t = \frac{d}{v\sqrt{1-\sin^2\theta}}$

so $t = \frac{d}{\sqrt{v^2-u^2}}$

Total time = 2t

$= \frac{2d}{\sqrt{v^2-u^2}}$

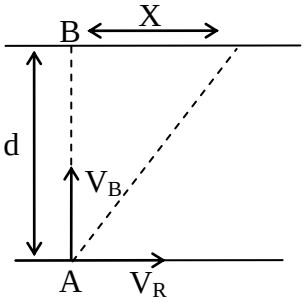
9. Boat moves with velocity 5m/s on still water. It is steered perpendicular to the river current.



(A) Will it reach point B or somewhere else on the other bank ?
(B) How long will it take to cross the river ?
(C) How far down stream, will it reach the other bank?
(D) Does it take minimum time in this way ?

Ans. (a) Somewhere down stream (b) 8 s (c) 16 m (d) Yes

Sol.



V_R = 2m/s and V_B = 5 m/s

time to cross river

$t = \frac{40}{5}$ = 8 sec.

and $X = V_R t = 2\times 8 = 16$ m

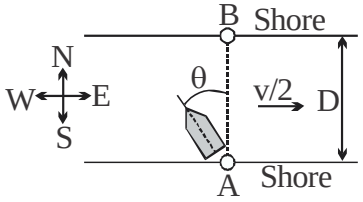
(A) No, some where at down stream

(B) t =8 sec.

(C) X = 16 m

(D) Yes

10. Two ports, A and B on a North-South line are separated by a river of width D. A boat cross the river from port A to port B. The speed of the boat relative to the water is v & that river is v/2 towards east. State all your answer in terms of v and D.

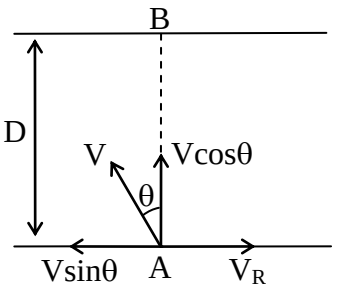


(A) What is the direction of the rowing of the boat so that it crosses directly on a line from A to B ? How long does the trip take?

(B) Suppose the boat wants to cross the river from A to the other side in the shortest possible time. In what direction should he row the boat? How long does the trip take?

Ans. (a) $\theta = 30°$, $t = \dfrac{2d}{v\sqrt{3}}$, (b) $t = D/v$, $x = D/2$

Sol.



$V_R = \dfrac{V}{2}$

(A) For move along AB

$v\sin\theta = V_R$

$\Rightarrow v\sin\theta = \dfrac{v}{2}$

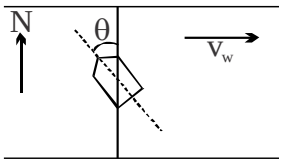
$\theta = 30°$

time $t = \dfrac{D}{v\cos 30°} = \dfrac{2D}{\sqrt{3}V}$

(B) For move to minimum time boat move $\perp$ to the river and $t_{min.} = \dfrac{D}{V}$

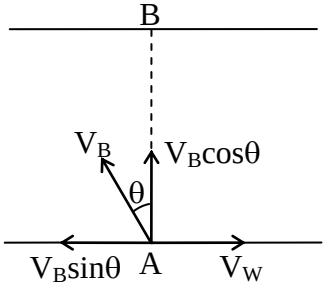
and drift (X) = $\dfrac{V}{2} t_{min.} = \dfrac{D}{2}$

11. A boat is to go straight across a stream that is flowing at 5.0 km/h to east. The boatman knows that his speed with respect to water is 10 km/h. The angle at which he must head the boat, and his speed relative to the shore will be :-



(A) 10 km/h at 0°
(B) $5\sqrt{3}$ km/h at 60°
(C) $10\sqrt{3}$ km/h at 30°
(D*) $5\sqrt{3}$ km/h at 30°

Sol.



$V_W = 5$ km/h

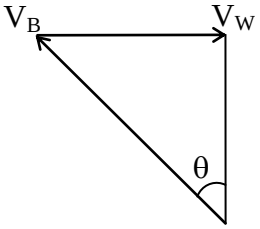
V_B = 10 km/h

To move along AB

$V_B \sin\theta = V_W$

$\Rightarrow \sin\theta = \frac{5}{10}$

$\Rightarrow \theta = 30°$



Relative speed to the shod is = $V_B \cos\theta$

$= 10\cos 30°$

$= 5\sqrt{3}$ km/h

12. **Statement-1 :** Relative velocity of body A with respect to another body B is independent of the reference frame R from which velocity of A $\left(\vec{V}_{A/R}\right)$ and velocity of B $\left(\vec{V}_{B/R}\right)$ are observed.

Statement-2 : Velocity of body A with respect to another body B is $\vec{V}_{A/R} - \vec{V}_{B/R}$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Ans. **(A)**

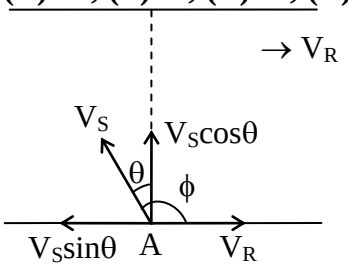
13.

	Column-I	**Column-II**
(A)	If swimmer can swim at 5m/sec in still water and if velocity of water flow is 4m/sec then angle between direction of swimming and direction of river flow to minimize drift.	(P) 53°
(B)	If swimmer can swim at 5m/sec in still water and velocity of flow is 3m/sec then angle between direction of velocity of swimmer with respect to river and the direction of river flow if swimmer crosses the river in minimum time .	(Q) 127°
(C)	If swimmer can swim at 4 m/sec and velocity of flow is 3m/sec then angle of resultant velocity (w.r.t. ground) with the direction of river flow if swimmer heads perpendicular to flow of river.	(R) 143°

(D) Angle between direction of fluttering of flag and north if wind blows towards south west direction with a velocity $3\sqrt{2}$ m/sec. Man moves with a velocity 7m/sec along west, holding flag in his hand. (S) 90°

Ans. **(A)-R ; (B)-S ; (C)-P ; (D)-Q**

Sol.



Given $V_R = 4$ m/s

$V_S = 5$ m/s

(A) For minimum drift

Condition $V_S\sin\theta = V_R$

$\Rightarrow \theta = \sin^{-1}\left(\frac{4}{5}\right)$

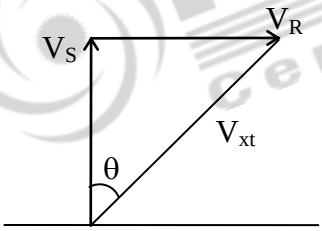
$\Rightarrow \theta = 53°$

$\phi = 90 + 53 = 143°$

(B) For minimum time the direction of swimmer is $\perp$ to the river flow.

So $\theta = 90°$

(C)



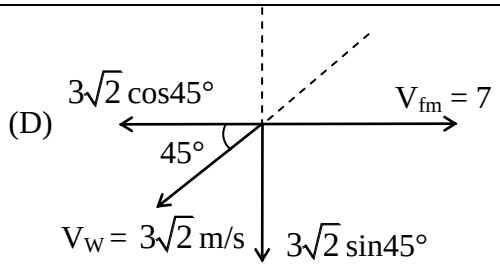
Given $V_S = 4$ m/s

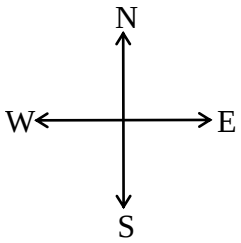
$V_R = 3$ m/s

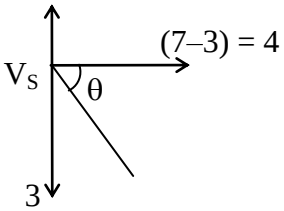
$\tan\theta = \frac{V_S}{V_R} = \frac{4}{3}$

$\Rightarrow \theta = 53°$

(D)







So $\tan\theta = \frac{3}{4} = 37°$

From, north $\phi = 90 + \phi = 90 + 37 = 127°$

WORK BOOK–11 (Relative Motion)

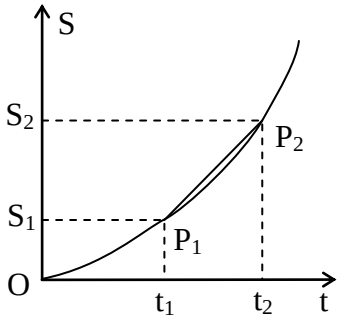
1. **Statement-1 :** The instantaneous velocity is always tangential to path.
and
Statement-2 : The slope of position-time graphs represent the velocity.
(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.
(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.
(C) Statement–1 is True, Statement–2 is False.
(D) Statement–1 is False, Statement–2 is True.

Ans. **(B)**

Sol. Instantaneous velocity

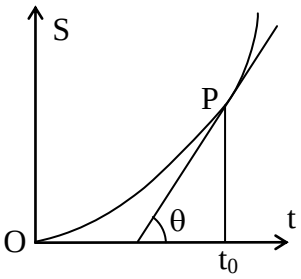
$$V = \lim_{t\to 0}\frac{\Delta S}{\Delta t} = \frac{ds}{dt}$$

In curve if $\Delta t \to 0$, the point P_2 comes very close to point P,



Instantaneous velocity can be found by determining the slope of the tangent of the displacement time graph at that instant velocity at point P at time t_0

$V = \frac{ds}{dt} = \tan\theta$



2. **Statement–1 :** For motion in parabolic path, $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d|\vec{v}|}{dt}$ may be zero.

and

Statement–2 : Acceleration in a body may be due to change in direction of velocity only.

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

Ans. **(A)**

Sol. $\left|\frac{d\vec{V}}{dt}\right|$ represents magnitude of rate of change of velocity means acceleration.

Velocity is vector quantity, and change when even only direction of motion changes

$\frac{d|\vec{V}|}{dt}$ represents rate of change of speed. It may be zero even acceleration may not zero, for example uniform circular motion.

3. **Statement-1 :** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.

and

Statement-2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories must intersect at some point except projection point.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Ans. **(C)**

4. **Statement -1 :** Two projectiles having same range must have the same time of flight.
Statement -2 : Horizontal component of velocity is constant in projectile motion under gravity.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Ans. **(D)**

Sol. $R = \frac{2U_x U_y}{g}$ and $T = \frac{2U_y}{g}$

R depends on horizontal and vertical component of initial velocity but T depends only on vertical component of initial velocity.
In projectile motion under gravity acceleration due to gravity is always perpendicular to horizontal component of velocity.

5. **Statement-1 :** When a particle is observed from two different inertial reference frames the general shape of the trajectory of particle is same.
Statement-2 : The position vector of a particle and its velocity are relative quantities.
(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

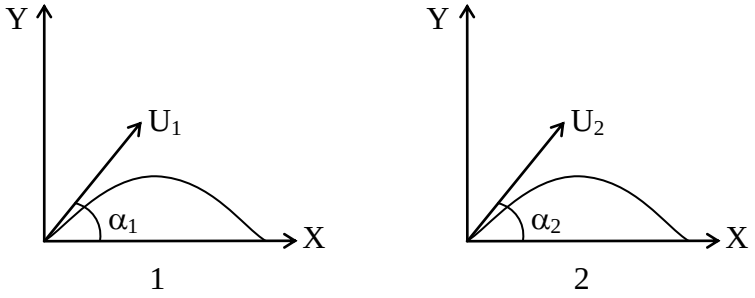
Ans. **(D)**

Sol. Velocity of particle may different as observed from two different inertial frame. Hence horizontal or vertical component of velocity may also different wrt. different frame.
If horizontal component of velocity is zero wrt. a inertial frame, motion may be straight line.
Position vector and velocity of particle are relative quantities.

6. **Statement 1 :** The trajectory of a projectile w.r.t. another projectile is a straight line .
and
Statement 2 : The relative velocity of a projectile w.r.t. another projectile is constant.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Ans. **(A)**

Sol.



$a_{12} = g - g = 0$

Relative acceleration is zero

Relative motion between two particle is uniform.

$(U_{12})_x = U_1 \cos\alpha_1 - U_2 \cos\alpha_2$

$(U_{12})_y = U_1 \sin\alpha_1 - U_2 \sin\alpha_2$

The relative velocity of a projectile wrt. another projectile is constant.

Hence relative motion of 1 with respect to 2 is a straight line at an angle $q = \tan^{-1}\left[\frac{(U_{12})_y}{(U_{12})_x}\right]$ with positive X axis.

7. **Statement-1 :** If separation between two particles does not change then their relative velocity will be zero.

Statement-2 : Relative velocity is the rate of change of position of one particle with respect to another.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

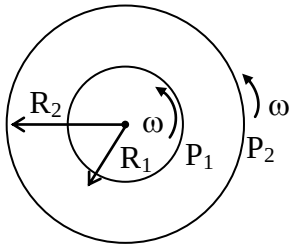
(D) Statement-1 is false, statement-2 is true.

Ans. **(D)**

Sol. When two particle moving in concentric circles with same angular velocity

$V_{P_1} = \omega R_1$ and $V_{P_2} = \omega R_2$

$V_{P_1} - V_{P_2} = \omega R_1 - \omega R_2$



8. **Statement-1 :** Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

Statement-2 : Relative acceleration between any of the pair of projectiles is zero.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

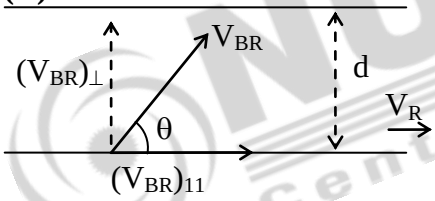
Ans. **(A)**

Sol. Acceleration of all particles is g downwards. So relative acceleration between any of the pair of projectile is zero.
So relative motion between any of two pair is uniform.

9. **Statement-1 :** The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank. They may reach opposite bank simultaneously moving along different straight line paths.
Statement-2 : For above boats to cross the river in same time, the component of their velocity relative to river in direction normal to flow should be same.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Ans. **(A)**

Sol.



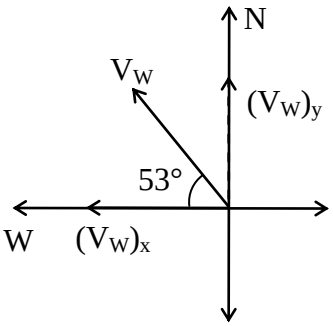
Time to cross river $T = \frac{d}{(V_{BR})_\perp}$

It depends on component of velocity of boat relative to river in direction normal to flow.

10. A boy holding a flag when is standstill, the flag flutters in 53° north of west and when he starts running westwards with velocity 6 m/s, the flag flutters in the north. Wind velocity is closest to
(A) 8 m/s, 53° north of west
(B) 8 m/s, 53° east of south
(C) 10 m/s, 53° north of west
(D) 10 m/s, 53° east of south

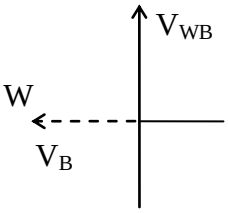
Ans. **(C)**

Sol. When boy stand still
$V_B = 0$



When $V_B = 6$ m/s

So $(V_{WB})_X = 0$ $\Rightarrow (V_W)_X = V_B = 6$m/s



So $V_W \cos 53° = (V_W)_X$

$\Rightarrow V_W\left(\frac{3}{5}\right) = 6$

$\Rightarrow V_W = 10$ m/s

$\Rightarrow \vec{V}_W = 10$ m/s, 53° north of west

WORK BOOK–12 (Relative Motion)

Paragraph for Question 1 to 3

Rain drops are falling with a velocity $10\sqrt{2}$m/sec making an angle of 45° with vertical. The drops appear to be falling vertically to a man running with constant velocity. The velocity of rain drops change

such that the rain drops appear to be falling vertically with times the velocity it appeared earlier to the same person running with same velocity-

1. The magnitude of velocity of man w.r.t. ground is

(A) $10\sqrt{2}$ m/s (B) $10\sqrt{3}$ m/s (C) 10m/s (D) 20 m/s

2. After the velocity of rain drops change, the magnitude of velocity of raindrops w.r.t. ground is

(A) 20 3 m/s (B) 10 m/s (C) 10 3m/s (D) 20 m/s

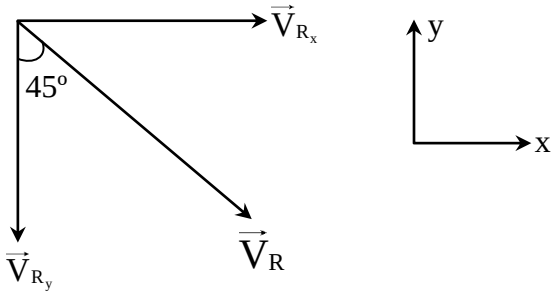
3. The angle between initial and final velocity vectors of the rain drops w.r.t. to the ground is

(A) 15° (B) 8° (C) 22.5° (D) 37°

1. Ans. (C)

Sol. $\vec{V}_R = V_{R_x}\hat{i} + V_{R_y}(-\hat{j})$

$\Rightarrow \quad \vec{V}_R = 10\hat{i} - 10\hat{j}$



$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$

$\Rightarrow \quad -10\hat{j} = (10\hat{i} - 10\hat{j}) - \vec{V}_M$

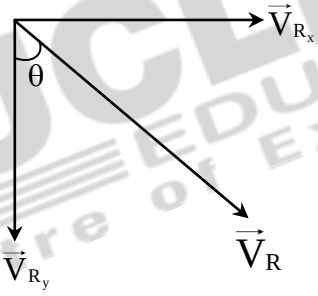
$\Rightarrow \quad \vec{V}_M = 10\hat{i}$

2. Ans. (D)

Sol. Again rain appears to falling verticle so

$\vec{V}_{R_x} = \vec{V}_M$

$= 10\hat{i}$



And

$\vec{V}_{R_y} = -10\sqrt{3}\hat{j}$

$V_R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20$ m/s

3. Ans. (A)

Sol. $\tan\theta = \dfrac{10}{10\sqrt{3}} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

Angle between initial and final velocity vector $\Delta\theta = 45^\circ - 30^\circ = 45^\circ$

Comprehension: Rain and man

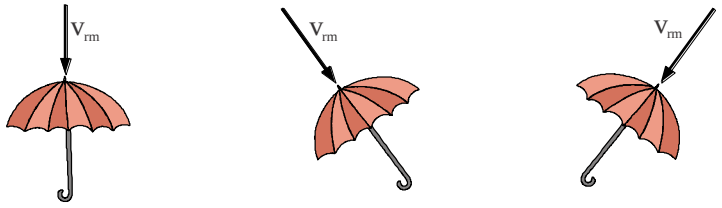
By the term velocity of rain, we mean velocity with which raindrops fall relative to the ground. In absence of wind, raindrops fall vertically and in presence of wind raindrops fall obliquely. Moreover raindrops acquire a constant terminal velocity due to air resistance very quickly as they fall toward the earth.

A moving man relative to himself observes an altered velocity of raindrops, which is known as velocity of rain relative to the man. It is given by the following equation.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

A standstill man relative to himself observes rain falling with velocity, which is equal to velocity of the raindrops relative to the ground.

To protect himself a man should hold his umbrella against velocity of raindrops relative to himself as shown in the following figure.



4. Rain is falling vertically with velocity 80 cm/s.

(a) How should you hold your umbrella?

(b) You start walking towards the east with velocity 60 cm/s. How should you hold your umbrella?

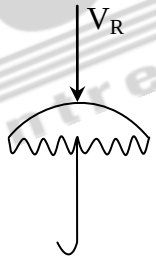
(c) You are walking towards the west with velocity 60 cm/s. How should you hold your umbrella?

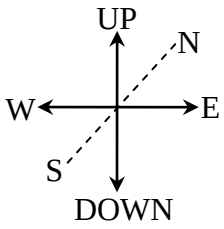
(d) You are walking towards the north with velocity 60 cm/s. How should you hold your umbrella?

(e) You are walking towards the south with velocity 80 cm/s. How should you hold your umbrella?

Ans. (a) Vertically (b) 53° above east (c) 53° above west (d) 53° above north (e) 45° above south

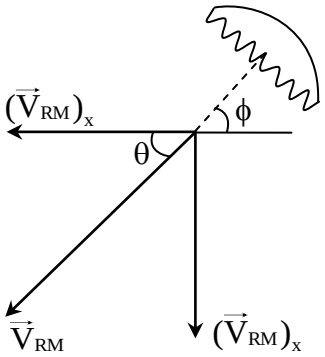
Sol. (a) $V_M = 0$ $\quad \vec{V}_R = -80\hat{j}$

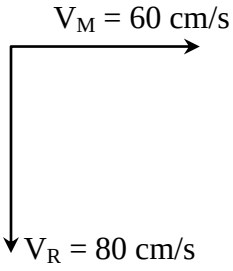




Hold umbrella vertically

(b) $\tan\theta = \dfrac{80}{60} = \dfrac{4}{3}$



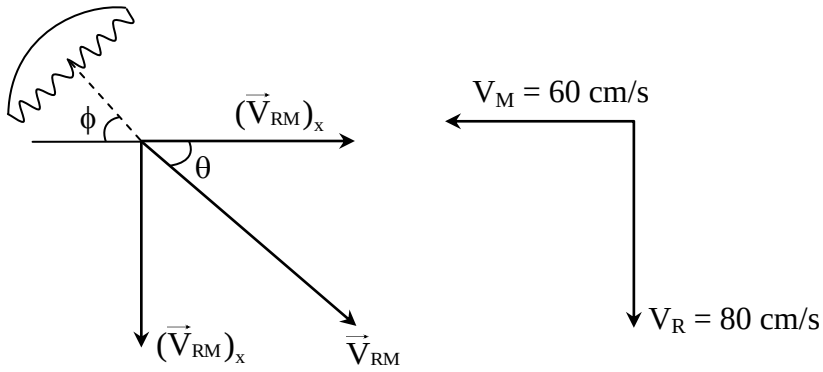


$\Rightarrow \quad \theta = 53^o$

So $\quad \phi = 53^o$

Ambrella at 53º above east

(c) $\tan\theta = \frac{80}{60} = \frac{4}{3}$



$\Rightarrow \quad \theta = 53^\circ$

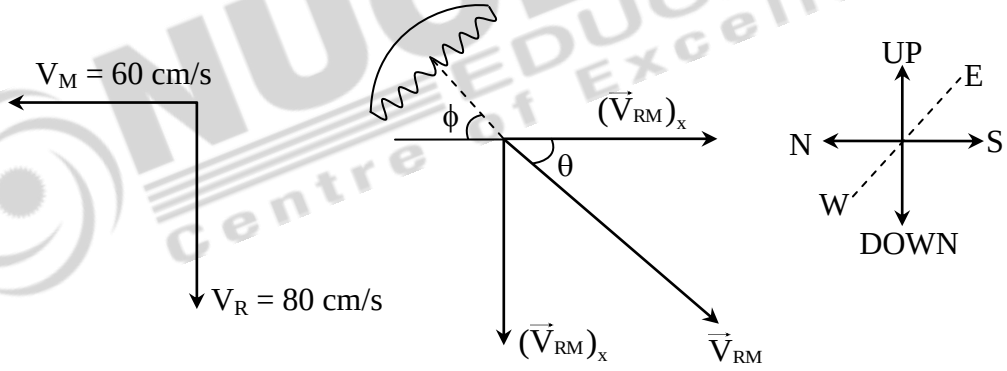
so $\quad \phi = 53^\circ$

Ambrella at 53º above west

(d) $\Rightarrow \quad \theta = \frac{80}{60} = \frac{4}{3}$

$\Rightarrow \quad \theta = 53^\circ$

so $\phi = 53^\circ$

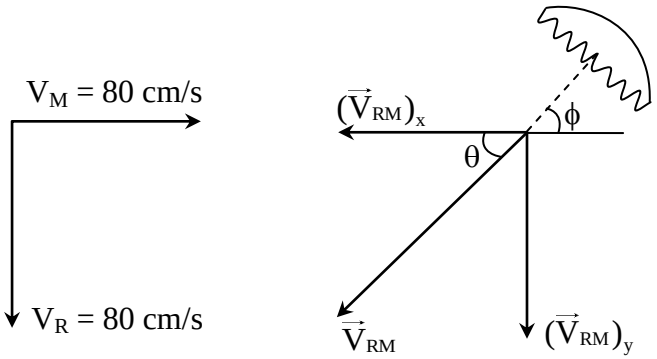


Ambrella at 53º above north

(e) $\tan\theta = \frac{80}{80}$

$\Rightarrow \theta = 45^\circ$

so $\phi = 45^\circ$



Ambrella at 45º above south

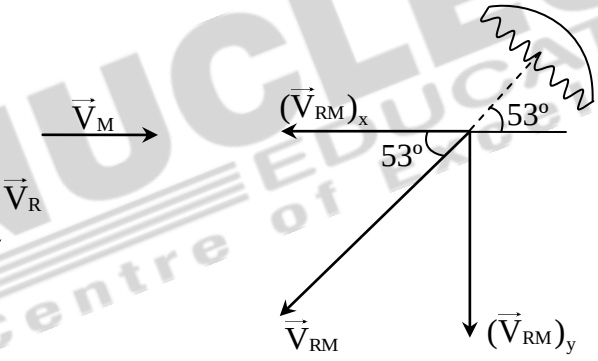
5. When you are standstill in rain, you have to hold your umbrella vertically to protect yourself.
(a) When you walk with velocity 90 cm/s, you have to hold your umbrella at 53° above the horizontal.

What is velocity of the raindrops relative to the ground and relative to you?

(b) If you walk with speed 160 cm/s, how should you hold your umbrella?

Ans. (a) 120 cm/s vertically150 cm/s 53° above horizontal (b) 37° above the horizontal.

Sol. (a)



$(\vec{V}_{RM})_x = 90\,\text{cm/s}$

$\tan 53^\circ = \dfrac{(\vec{V}_{RM})_y}{(V_{RM})_x}$

$\Rightarrow \quad \dfrac{4}{3} = \dfrac{(\vec{V}_{RM})_y}{90} \Rightarrow (\vec{V}_{RM})_y = 120\,\text{cm/s}$

$\vec{V}_{RM} = -90\hat{i} - 120\hat{j}$

$\vec{V}_{RM} = \sqrt{90^2 + 120^2}$ = 150 cm/s 53º above horizontal

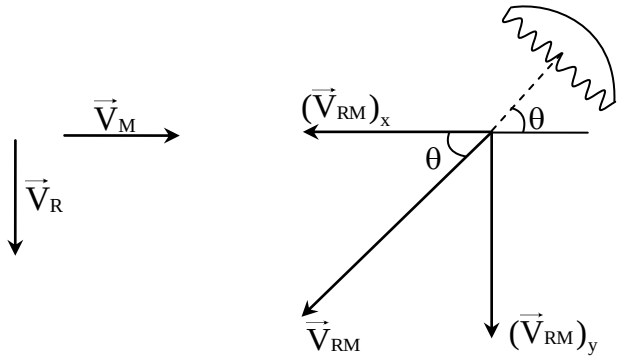
$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$

$(-90\hat{i} - 120\hat{j}) = \vec{V}_R - 90\hat{i}$

$\Rightarrow \vec{V}_R = -120\hat{j}$

$\Rightarrow \vec{V}_R = 120\,\text{cm/s}$ vertically

(b)



$(\vec{V}_{RM})x = 160$ m/s

$(\vec{V}_{RM})y = V_R = 120$ cm/s

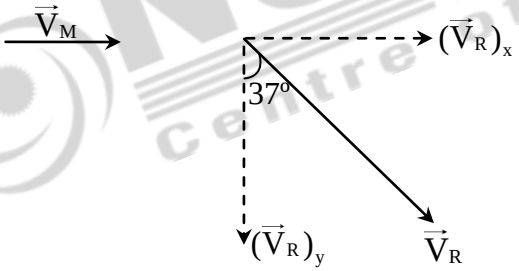
$\tan\theta = \frac{120}{160} = \frac{3}{4} \Rightarrow \theta = 37^o$

Umbrella should be hold at $\theta = 37^o$ above horizontal

6. A man walks in rain at 72 cm/s due east and observes the rain falling vertically. When he stops, rain appears to strike his back at 37° from the vertical. Find velocity of raindrops relative to the ground.

Ans. 120 cm/s

Sol.



$(\vec{V}_R)_x = \vec{V}_M = 72\ \text{cm/s}\,\hat{i}$

$\vec{V}_R \sin 37^o = (\vec{V}_R)_x$

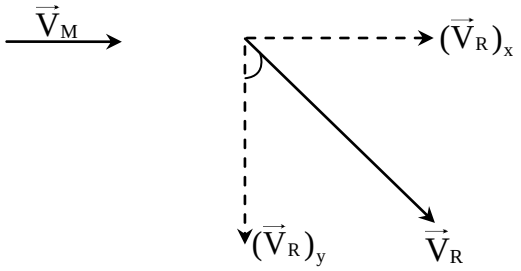
$\Rightarrow \quad \vec{V}_R\left(\frac{3}{5}\right) = 72$

$\Rightarrow \quad V_R = 120$ cm/s

7. When you walk in rain at 75 cm/s, you have to hold your umbrella vertically and when you double your
speed in the same direction, you have to hold your umbrella at 53° above the horizontal. What is the rain
velocity?

Ans. 125 cm/s at 37° from the vertical

Sol.

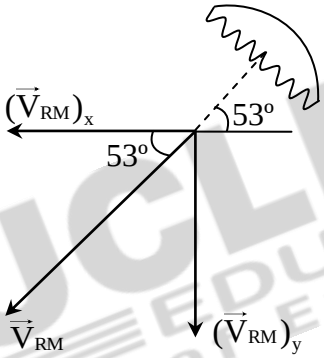


$\vec{V}_M$ = 75 cm/s rain appear vertically, so

$(\vec{V}_R)_x = \vec{V}_M$ = 75 cm/s $\hat{i}$

Now $\vec{V}_M$ = 150 cm/s $\hat{i}$ $\quad$ $(\vec{V}_R)_x$ = 75 cm/s $\hat{i}$

$(\vec{V}_{RM})_x$ = 75 cm/s $(-\hat{i})$



$$\tan 53^\circ = \frac{(\vec{V}_{RM})_y}{(\vec{V}_{RM})_x}$$

$$\Rightarrow \frac{4}{3} = \frac{(\vec{V}_{RM})_y}{75}$$

$\Rightarrow (\vec{V}_{RM})_y$ = 100 cm/s $\quad$ $\Rightarrow (\vec{V}_{RM})_y$ = 100 cm/s $(-\hat{j})$

So $\quad (\vec{V}_R)_y$ = 100 cm/s $(-\hat{j})$

$V_R = \sqrt{75^2 + 100^2}$ = 120 cm/s

Comprehension: Flag in wind

When you are standstill holding a flag, the flag flutters in the direction of wind. When you start running the direction of fluttering of the flag changes in to the direction of the wind relative to you. In all case a flag flutters in the direction of the wind relative to the flag.

8. When you are standstill holding a flag the flag flutters in the north and when you run at 8 m/s due east, the flag flutters in direction 37° north of west. Find the wind velocity.

Ans. 6 m/s due north

Sol. When $V_M = 0$ flag flutters in the north so wind is in north direction

$(\vec{V}_{WM})_x$ = 8 m/s

$\tan 37^\circ = \dfrac{(\vec{V}_{WM})_y}{(\vec{V}_{WM})_x}$

$\Rightarrow \dfrac{3}{4} = \dfrac{(\vec{V}_{WM})_y}{8} \Rightarrow (\vec{V}_{WM})_y = 6$ m/s

$(\vec{V}_{WM}) = \vec{V}_W - \vec{V}_M$

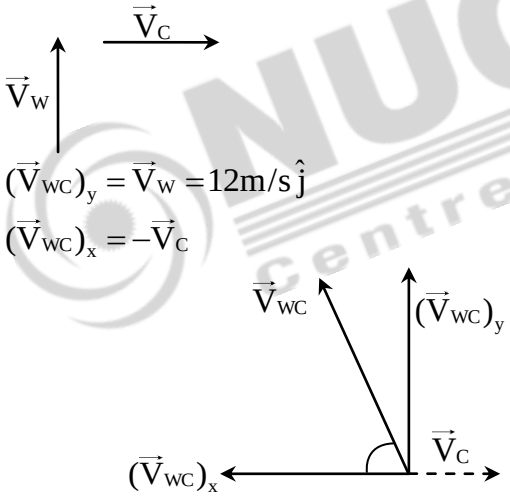
$\Rightarrow \quad (-8\hat{i} + 6\hat{j}) = \vec{V}_W - 8\hat{i}$

$\Rightarrow \quad \vec{V}_W = 6$m/s due north

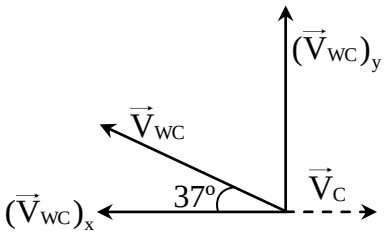
9. Wind is blowing uniformly due north everywhere with velocity 12 m/s. A car mounted with a flag starts
running towards east. After 9 s from start the flag flutters in 53° north of west and after 16 s from the start
the flag flutters in 37° north of west.
(a) Find velocity of the car 9 s after it starts.
(b) Find velocity of the car 16 s after it starts.
(c) If the car maintains uniform acceleration, find acceleration of the car.

Ans. (a) 9 m/s (b) 16 m/s (c) 1 m/s^2

Sol.



At t = 9 second



At t = 16 second

(a) at t = 9 second

at 53 $= \dfrac{12}{(\vec{V}_{WC})_x} \Rightarrow (\vec{V}_{WC})_x = 9$ cm/s

So $V_C = 9$ m/s

(b) at t = 16 second

$\tan 37^\circ = \dfrac{12}{(\vec{V}_{WC})_x} \Rightarrow (\vec{V}_{WC})_x = 16$ m/s

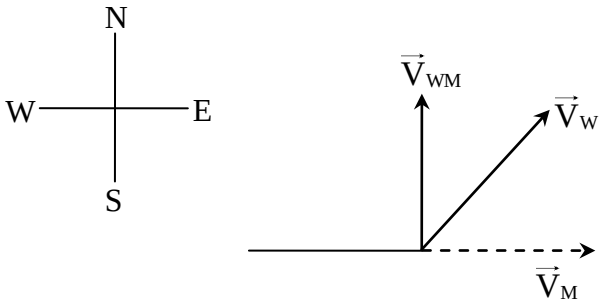
so $V_C = 16$ m/s

(c) Acceleration a = $\frac{16-9}{7}$ = 1 m/s^2

10. Holding a flag, when you run at 8 m/s due east, the flag flutters in the north and when you run at 2 m/s due south, the flag flutters in the northeast. If the wind velocity is uniform and remain constant, find the wind velocity.

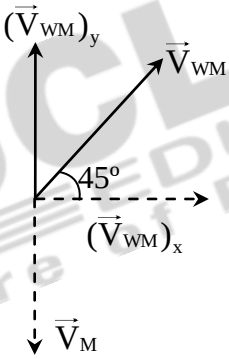
Ans. 10 m/s, 37° north of east

Sol.



When $\vec{V}_M$ = 8 m/s flag flutter in north so $\vec{V}_{WM}$ is along north

So $(\vec{V}_W)_x = \vec{V}_M = 8\text{m/s}\hat{i}$

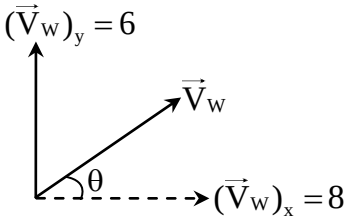


Now $(\vec{V}_{WM})_x = (\vec{V}_W)_x$ = 8m/s$\hat{i}$

And from figure it is clear that $(\vec{V}_{WM})_y = (\vec{V}_{WM})_x$

$(\vec{V}_W)_y - (-2) = 8$

$(\vec{V}_W)_y$ = 6m/$\hat{j}$



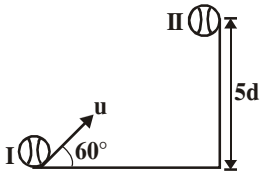
So $\vec{V}_W = \sqrt{8^2+6^2}$ = 10 m/s

$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 37^\circ$

So $\vec{V}_W$ = 10 m/s, 37° north of east

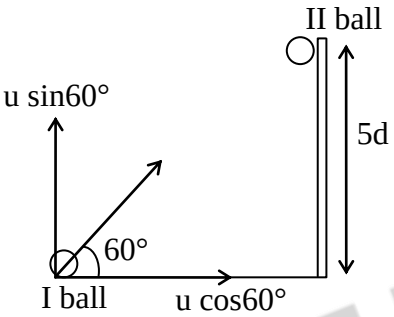
ADVANCE PROBLEMS

1. Ball I is thrown towards a tower at an angle of 60° with the horizontal with unknown speed (u). At the same moment ball II is released from the top of tower as shown. Balls collide after two seconds and at the moment of collision, velocity of ball I is horizontal. If height of the tower is 5d meter, find the value of d.



Ans 8

Sol.



Given t = 2sec.

At the time of collision I ball have only horizontal velocity so it is a maximum height then for I ball

$1/y = uy + ayt$

$\Rightarrow 0 = u\sin60° - 10xt$

$\Rightarrow \frac{\sqrt{3}u}{2} = 20$

$\Rightarrow u = \frac{40}{\sqrt{3}}$ m/s .....(1)

Now In frame of Ref. of II ball

$a_{I/II} = 0$

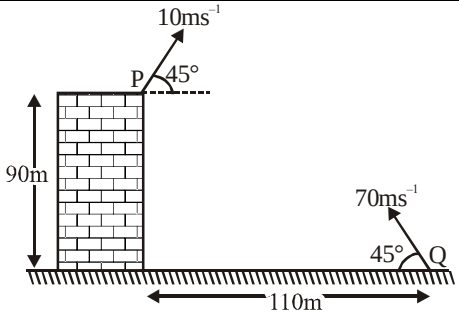
Initial velocity of I ball $U_{I/II} = u\sin60°$

So In case of collision

$5d = 4\sin60°xt = \frac{40}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \times 2$

$\Rightarrow d = 8$

2. Two particles P and Q are launched simultaneously as shown in figure. Find the minimum distance between particles in meters.



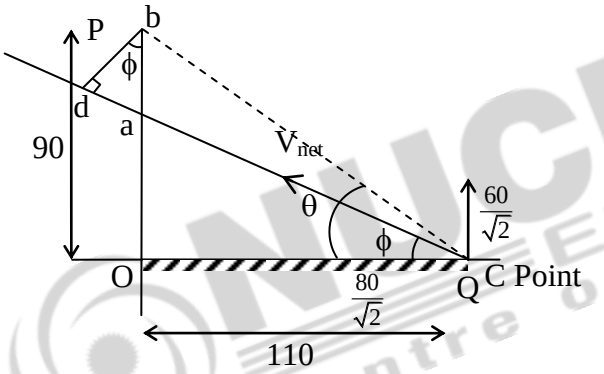
Ans. 6

Sol. In frame of P particle

$\vec{V}_{QP} = \vec{V}_Q - \vec{V}_P$

$\Rightarrow \frac{-70}{\sqrt{2}}\hat{i} + \frac{70}{\sqrt{2}}\hat{j} - \frac{10}{\sqrt{2}}\hat{i} - \frac{10}{\sqrt{2}}\hat{j}$

$\Rightarrow \frac{-80}{\sqrt{2}}\hat{i} + \frac{60}{\sqrt{2}}\hat{j}$



$\tan\theta = \frac{90}{110}$

and $\tan\phi = \left(\frac{\frac{60}{\sqrt{2}}}{\frac{80}{\sqrt{2}}}\right) = \frac{3}{4}$

$\Rightarrow \phi = 37°$

Now In ΔaOC

$Oa = OC \tan\phi$

$= 110 \times \frac{3}{4} = \frac{330}{4}$ m

Now $ab = 90 - \frac{330}{4}$

So minimum distance $bd = ab\cos\phi$

$\Rightarrow \frac{30}{4} \times \frac{4}{5} = 6$ m

3. A motor boat set out at 11 a.m. from a position $-6\hat{i}-2\hat{j}$ and travels at a steady speed of magnitude $\sqrt{53}$ on a direct course to intercept a ship. The ship maintains a steady velocity vector $3\hat{i}+4\hat{j}$ and at 12 noon is at a position $3\hat{i}-\hat{j}$. Find

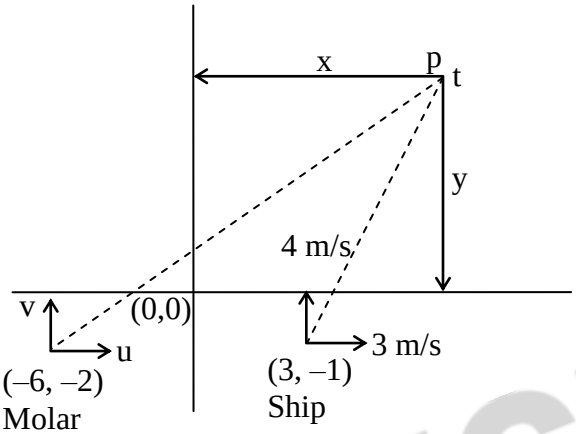
(a) the velocity vector of the motor boat,

(b) the time of interception and

(c) the position vector of point of interception. If distances are measured in kilometres and speeds in kilometres per hour.

Ans. (a) 7i + 2j (b) 12:30 pm (c) 9/2i + j

Sol.



Let both molar and slip intercept at point P in t sec.

and velocity of motor in x-direction is u and in y-direction is v.

So $v^2 + u^2 = 53$ ....(5)

For motor

$x + 6 = ut$ .....(1)

and $y + 2 = vt$ .....(2)

For slip start after 1 hour.

$x - 3 = 3(t - 1)$ .....(3)

and $y + 1 = 4(t - 1)$ .....(4)

from (3) $x = 3t$

from (4) $y = 4t - 5$

Now from (1), (2) and (3)

$$\left(\frac{x+6}{t}\right)^2+\left(\frac{y+2}{t}\right)^2 = 53$$

$$\Rightarrow \left(\frac{3t+6}{t}\right)^2+\left(\frac{4y+2}{t}\right)^2 = 53$$

By solving equation $t = \frac{3}{2}$

then $u = \frac{3t+6}{t}$ and $v = \frac{4t-3}{t}$

Put time

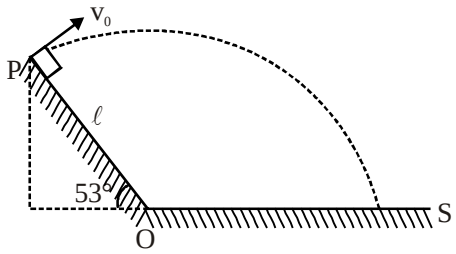
$\Rightarrow u = 7$ and $v = 2$

(a) Velocity of motor = $7\hat{i}+2\hat{j}$

(b) Time of Interception = 11 a.m + 1.5 hours = 12:30 P.m

(c) Position x = $3\times\frac{3}{2}=\frac{9}{2}$; y = 1

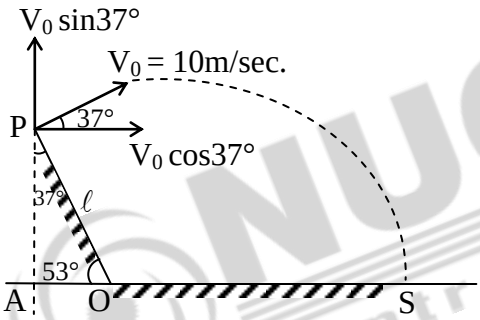
4. A stone is projected from point P on the inclined plane with velocity v_0 = 10 m/s directed perpendicular to the plane. The time taken by the stone to strike the horizontal ground S is (Given PO = ℓ = 10 meter)



(A) 1.5 sec **(B) 1.4 sec** **(C) 2 sec** **(D) 2.3 sec**

Ans. **(C)**

Sol.



Given V_0 = 10 m/s

ℓ = PA = 10 m

Now AP = $\ell\cos37° = 10\times\frac{4}{5}$ = 8 m

So In y-direction

$S_y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow -8\ 10\sin37°\times t - \frac{1}{2}\times10\times t^2$

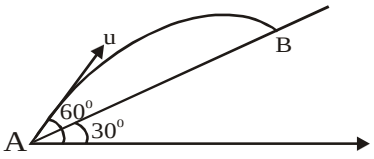
$\Rightarrow 5t^2 - 6t - 8 = 0$

$t = \frac{6\pm\sqrt{365+4\times5\times8}}{10}$

$\Rightarrow \frac{6\pm\sqrt{196}}{10}$

Considered +ve time t = 2sec.

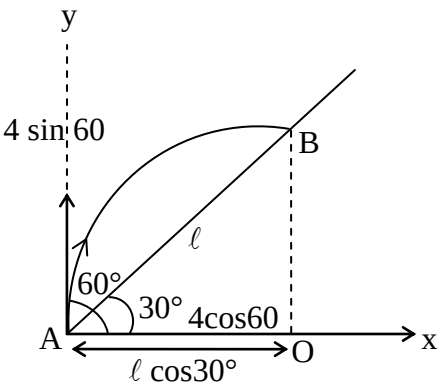
5. Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}\,ut$ (D) 2 ut

Ans. **(A)**

Sol.

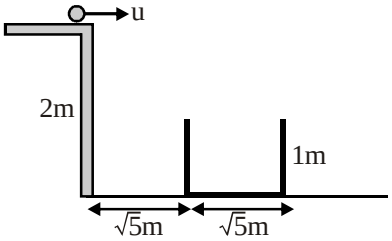


Now OA = ucos60°t

$\Rightarrow \ell\cos30° = 4\cos60°t$

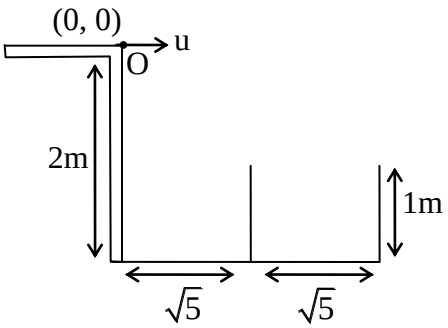
$\Rightarrow \ell = AB = \frac{ut}{\sqrt{3}}$

6. An open box of height 1 m is placed near a 2m high table as shown. Marble rolling on the table, leave the table and are collected in the box. u_{max} and u_{min} are the maximum and minimum values of velocities of marbles to fall into the box. Find the value of $u_{max} - u_{min}$ (in m/s).



Ans. **5**

Sol.



Let O point as origin to

Trajectory of projectile motion is

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ .....(1)

For maximum velocity coordinate is

$y = -1$; $\theta = 0$

$x = 2\sqrt{5}$

Now from(1)

$+1 = 0 + \frac{10+\left(2\sqrt{5}\right)^2}{2\times u^2\times 1}$

$U_{max.} = 10$ m/sec.

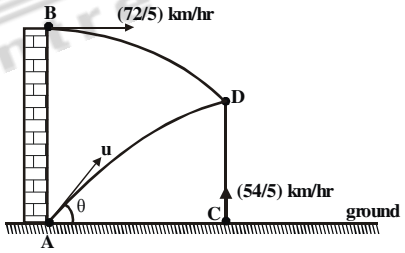
For minimum velocity coordinate is $\left(\sqrt{5}, -1\right)$ from (1)

$-1 = 0 - \frac{10\times\left(\sqrt{5}\right)^2}{2\times u^2}$

$U_{min} = 5$ m/sec.

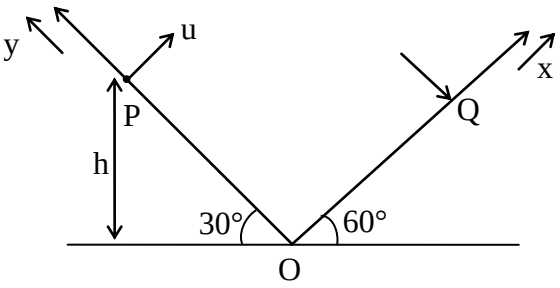
$U_{max.} - U_{min.} = 5$ m/sec.

7. In the given figure points A and C are on the horizontal ground & A and B are in same vertical plane. Simultaneously bullets are fired from A, B and C and they collide at D. The bullet at B is fired horizontally with speed of $\frac{72}{5}$ km/hr and the bullet at C is projected vertically upward at velocity of $\frac{54}{5}$ km/hr. Find velocity of the bullet projected from A in m/s.



Ans. **5**

Sol.



Given

$V_B = \frac{72}{5}\times\frac{5}{18} = 4$ m/sec.

$V_C = \frac{54}{5} \times \frac{5}{18}$ = 3 m/sec.

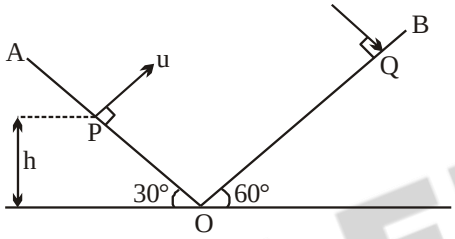
Condition for collide A, B & C are

$U_{AX} = V_B$ = 4m/sec.

$U_{AY} = V_C$ = 3m/sec.

So $U_A = \sqrt{(3)^2 + (4)^2}$ = 5 m/sec.

8. Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity u = $10\sqrt{3}$ m\s along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q. Calculate



(i) Time of flight,

(ii) Velocity with which particle strikes the plane OB,

(iii) Vertical height h of P from O,

(iv) Maximum height from O attained by the particle and

(v) Distance PQ

Ans. (i) 2 s; (ii) 10 m/s; (iii) 5m;'(iv) 16.25 m; (v) 20m

Sol. (i) For time of flight

$V_x^0 = U_x = a_x t$ $\qquad a_x = -g\sin60°$

$\Rightarrow 0 = 10\sqrt{3} - g\sin60°t$ $\qquad U_x = 10\sqrt{3}$ m/sec.

$\Rightarrow t = \frac{10\sqrt{3} \times 2}{10\sqrt{3}}$ $\qquad V_x = 0$

$\Rightarrow t$ = 2 sec.

(ii) $V_y = U_y^0 = a_y t$ $\qquad a_y = -g\cos60°$

$\Rightarrow V_y = -g\cos60° \times 2$

$= -10$ m/sec.

(iii) distance OP is

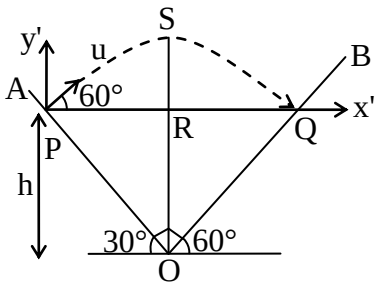
$Sy = ugt + \frac{1}{2} qyt^2$

$\Rightarrow OP = \frac{1}{2} \times g\cos60 \times (2)^2$

= 10m.

More height h = OP sin30 = 5m

(iv) max. height from O



Now we chose x' – y' plane and then

RS $= \frac{u^2 \sin^2 60}{2g} = \frac{100\times3\times3}{2\times10\times4} = 11.25$ m

Then max. height from O = OR + RS

= h + RS = 5 + 11.5 = 16.25 m

(v) distance OQ $S_x = u_x t + \frac{1}{2} a_x t^2$

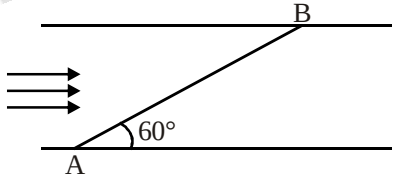
$\Rightarrow$ $OQ = 10\sqrt{3}\times2 - \frac{1}{2}\times10\times\frac{\sqrt{3}}{2}\times4$

$\Rightarrow$ $OQ = 10\sqrt{3}$

and $PQ = \sqrt{(OQ)^2 + (OP)^2}$

$\Rightarrow$ $\sqrt{300+100} = 20$ m.

9. A swimmer starts to swim from point A to cross a river. He wants to reach point B on the opposite side of the river. The line AB makes an angle 60° with the river flow as shown. The velocity of the swimmer in still water is same as that of the water

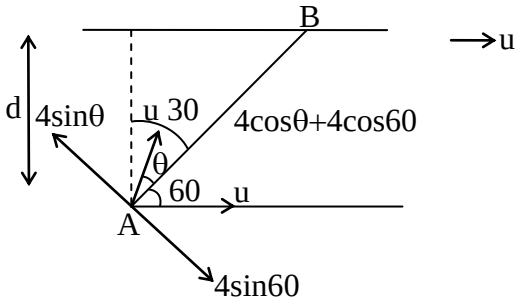


(i) In what direction should he try to direct his velocity ? Calculate angle between his velocity and river velocity.

(ii) Find the ratio of the time taken to cross the river in this situation to the minimum time in which he can cross this river.

Ans. (i) 120°; (ii) $\frac{2}{\sqrt{3}}$

Sol.



Given-

$V_S = u$

$V_R = u$

(a) If swimmer move along AB then

$u \sin\theta = u \sin 60$

$Q = 60$

Then from horizontal it is

$\phi = 60 + Q = 120°$

(b) $t_{min} = \dfrac{d}{u}$

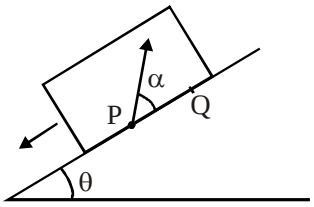
and time taken in (a) condition is

$$t = \frac{AB}{V_{net}} = \frac{d}{\frac{\cos 30}{4\cos 60 + 4\cos 60}} = \frac{d}{24\cos 60\cos 30}$$

$$t = \frac{2d}{\sqrt{3}u}$$

so $$\frac{t}{t\,min.} = \frac{2}{\sqrt{3}}$$

10. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to box is u and the direction of projection makes an angle α with the bottom as shown in figure.

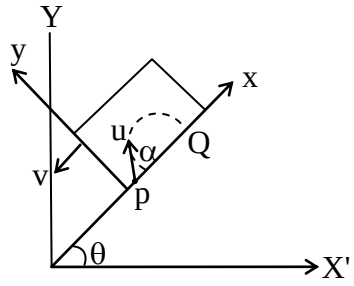


(i) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).

(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

Ans. (i) $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$; (ii) $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

Sol.



Initial speed of particle with respect to box is u so box is as ref. frame

$\vec{u}_{Pb} = u\cos\propto \hat{i} + 4\sin\propto \hat{j}$

and acc. with respect to box

$a_{Pb} = \vec{a}_p - \vec{a}_b$

$\Rightarrow \quad -\sin\theta\hat{i} - g\cos\theta\hat{j} - (-g\sin\theta\hat{i})$

$= \quad -g\cos\theta\hat{j}$

So in X direction there is no. acc.

So PQ = (4 cos ∝) T ……..(1)

and In y – direction

$Sy = 4y\,T - \frac{1}{2}ay\,T^2$

$T = \frac{24\sin\alpha}{g\cos\theta}$

From (1)

$PQ = \frac{u^2\sin 2\alpha}{g\cos\theta}$

(b) Let box has V velocity along inclined plane if we take X' – Y' plane then

Velocity of box $\vec{V}_{bG} = -V\cos\theta\hat{i} - V\sin\theta\hat{j}$

Velocity of particle or t box $\vec{V}_{Pb} = 4\cos(\alpha+\theta)\hat{i} + 4\sin(\alpha+\theta)\hat{j}$

Now $\vec{V}_P = \vec{V}_{Pb} + \vec{V}_{PG}$

$= [u\cos(\alpha+\theta) - V\cos\theta]\hat{i} + [4\sin(\alpha+\theta) - V\sin\theta)\hat{j}$

no displacement in x' axis so $V_x' = 0$

$4\cos(\alpha+\theta) - V\cos\theta = 0$

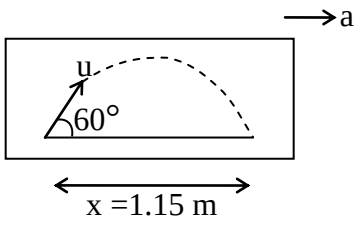
$\Rightarrow \quad V = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

EXERCISE (J)

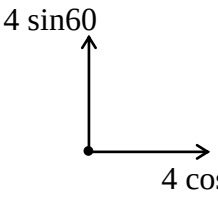
1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s², is **[IIT JEE 2011]**

Ans. 5

Sol.



Given u = 10 m/s
x = 1.15 m



Time of flight for particle

$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times10\times\sqrt{3}}{2\times10}$$

$$= \sqrt{3}\ \pi$$

In time T distance covered by particle

$$\delta_x = u_x T + \frac{1}{2}\ 9 \times T^2 + 1.15$$

$$\Rightarrow \quad \delta_x = 1.15 + \frac{1}{2} \times a \times 3$$

This is range of particle

$$R = \frac{3}{2}a + 1.15$$

$$\Rightarrow \quad \frac{u^2\sin2\theta}{g} = \frac{3}{2}a + 1.15$$

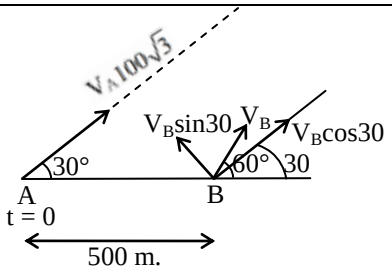
$$\Rightarrow \quad \frac{100\times\sqrt{3}}{2\times10} = 1.15 = \frac{3}{2}a$$

$$\Rightarrow \quad 2 = 5 m/sec^2$$

2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}\,ms^{-1}$. At time t = 0s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **[IIT JEE -2014]**

Ans. **5**

Sol.



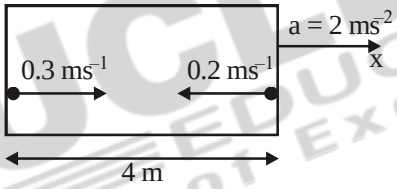
In forms of A, B move ⊥ to line of motion of means A sec only ⊥ velocity of B to

$V_B \cos 30 = 100\sqrt{3}$

$\Rightarrow$ $V_B = 200$m/sec

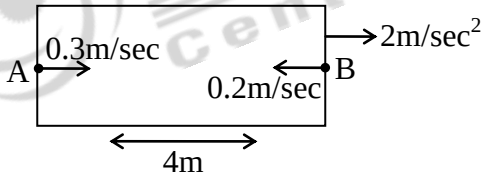
Then time $t = \dfrac{d}{V_B \sin 30} = \dfrac{500}{100} = 5$ sec.

3. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[IIT JEE-2014]**



Ans. **2 or 8**

Sol.



C-1 There is two cosec in first case a ball does not collide with chamber then

In forms of B

$\vec{V}_{AB} = 0.3 + 0.2 = 0.5$

Relative = 4m

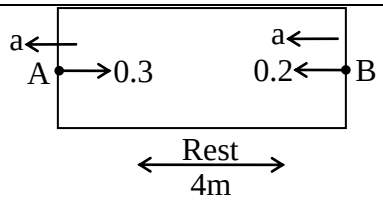
so $t = \dfrac{S}{\vec{V}_{AB}} = \dfrac{4}{0.5} = 8$ sec.

C-2 In closed chamber forms of chamber to displacement covered by A is

$U_A^2 = U_A^2 - 2a\sqrt{A}$

$\Rightarrow$ $O = (0.3)^2 - 2 \times 2 \times \sqrt{A}$

$\Rightarrow$ $S_A = 0.0225$ m



This displacement S_A negligible with respect to chamber length so for B ball

$$S_B = 4_B T + \frac{1}{2} at^2$$

$$\Rightarrow \quad 4 = 0.2T + \frac{1}{2}(2)\, T^2$$

$$\Rightarrow \quad t \approx 2 \text{ sec.}$$