

KINEMATICS,

Problems involving distance, time, speed(velocity) and acceleration are given the name of kinematics (kinema= motion)

UNITS USED:

- Distance travelled- (metres) or (m)
- Time taken- (seconds) or (s)
- Velocity- (metres/second) or (m/s)
- Acceleration- (metres/second/second) or (m/s^2)

NOTE:

- Velocity(speed)= $\frac{\text{distance travelled}}{\text{time taken}}$ m/s
- Acceleration = $\frac{\text{velocity}}{\text{time}}$ m/s^2

GRADIENT GRAPHS

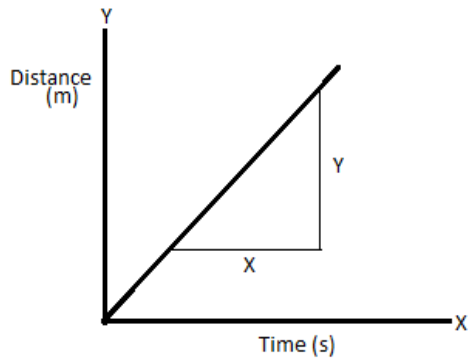
If motion of an object is given in a graph by a straight line then the object travels at constant speed or uniform speed determined by the gradient of the line.

If the graph is a curve, then the object concerned has different speeds at each instant.

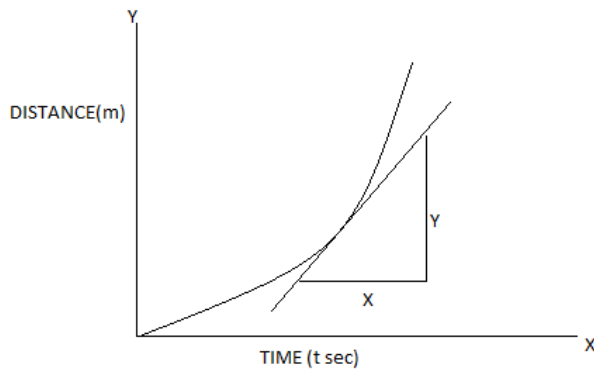
The gradient of the tangent to the curve at that point gives the speed of the object.

TRAVEL GRAPHS

(i) DISTANCE TIME GRAPH

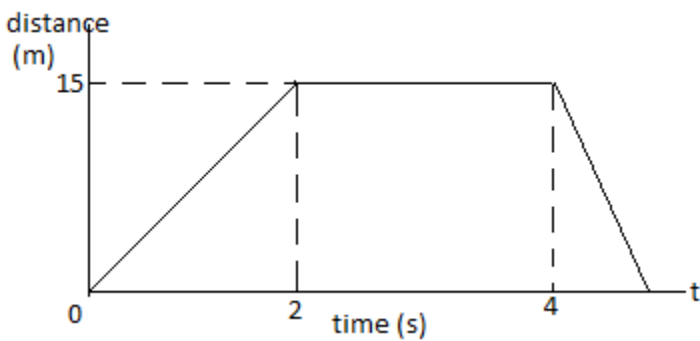


constant rate of change = constant speed.



varying rate of change = varying speed

EXAMPLE 1:



The distance time graph a shows an object starting from a point O, travelling 15m in 2 s, is stationary for another 2s and finally travels back to O in 1s.

Graph can be interpreted as follows:

The total distance travelled is 30 m (going+returning)

From O to A: constant speed = $\frac{15m}{2s} = 7.5 \text{ m/s}$

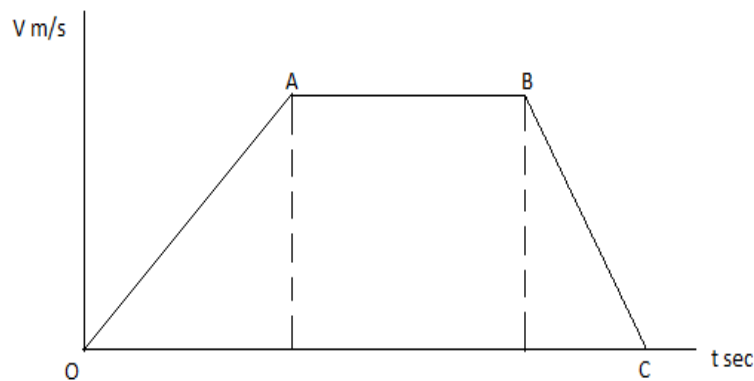
From A to B: The car is stationary, speed = 0 m/s

From B to O: The speed is constant = $\frac{15m}{1s} = 15 \text{ m/s}$

(ii) SPEED (VELOCITY) TIME GRAPH

hints:

- If the graph is a straight line parallel to the time axis (x-axis), then the object is said to travel at constant speed, i.e. there is no acceleration.
- The total distance travelled in t sec is given by the area under speed time graph for that time.

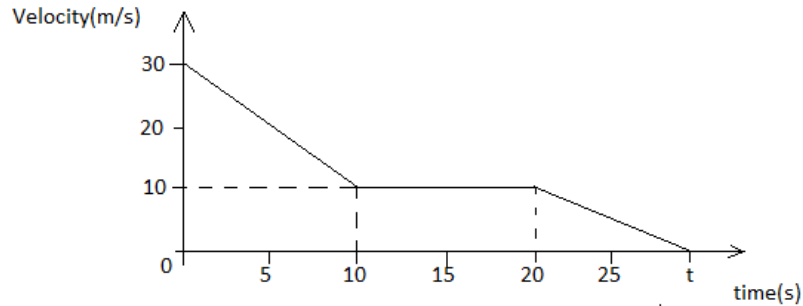


NOTE:

- From O to A, Speed is constantly changing, hence there is constant(uniform) acceleration.
- From A to B, Speed is constant (not changing), hence there is no acceleration.
- From B to C, Speed is decreasing uniformly, hence there is deceleration.

EXAMPLE 2

The diagram shows the velocity - time graph of a particle during a period of t seconds.



Calculate

- the acceleration of the particle in the first 10 seconds,
- the value of t , if it travelled 50m from the 20th second,
- the average speed of the particle for the whole journey.

EXPECTED ANSWER:

(a) $a = \frac{v-u}{t}$, But $u = 30 \text{ m/s}$, $v = 10 \text{ m/s}$, time $t = 10 \text{ s}$

$$a = \frac{10-30}{2}$$

$$a = -2 \text{ m/s}^2$$

(b) Distance traveled is equal to the area under the graph.

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$50 = \frac{1}{2} \times (t-20) \times 10$$

$$50 = 5t - 100$$

$$5t = 50 + 100$$

$$\frac{5t}{5} = \frac{150}{5}$$

$$\therefore t = 30 \text{ s}$$

$$(c) \text{ average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Distance = area under the graph

$$\therefore D = \text{area A} + \text{area B} + \text{area C}.$$

$$D = \frac{1}{2} \times 10 \times 20 + 10 \times 20 + \frac{1}{2} \times 10 \times 10$$

$$D = 100 + 200 + 50$$

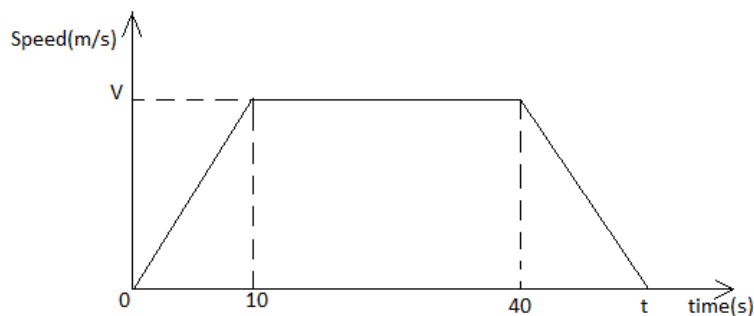
$$D = 350 \text{ m} \wedge \text{time } t = 30 \text{ s}$$

$$\therefore \text{Av. Speed} = \frac{350 \text{ m}}{30 \text{ s}} = \frac{35}{3} \text{ m/s} = 11 \frac{2}{3} \text{ m/s}.$$

EXAMPLE 3

The diagram below is the speed-time graph of a bus which leaves a bus stop and accelerates uniformly for 10 seconds over a distance of 100m.

It then maintains the speed it has attained for 30 seconds and finally retards uniformly to rest at the next bus stop. The whole journey takes t seconds.



If the two bus stops are 1 kilometre apart, find

- (i) the value of V ,
- (ii) the acceleration in the first 10 seconds,
- (iii) the total time (t) taken for the whole journey.

EXPECTED ANSWER

(i) distance $\therefore$ area under the graph

$$\therefore \frac{1}{2} \times \text{base} \times \text{height}$$

$$100 \text{ m} = \frac{1}{2} \times 10 \text{ s} \times V \text{ m/s}$$

$$\therefore V = \frac{100}{5} \text{ m/s}$$

$$\therefore V = 20 \text{ m/s}$$

(ii) $\text{acceleration} = \frac{\text{speed}}{\text{time}} \text{ m/s}^2$

$$\therefore \text{acceleration} = \frac{20 \text{ m/s}}{10 \text{ s}}$$

$$\therefore \text{acceleration} = 2 \text{ m/s}^2$$

(iii) note: $1 \text{ km} = 1000 \text{ m}$,

$$\therefore \text{total distance} = \frac{1}{2} (a+b)h$$

$$\text{but dist} = 1000 \text{ m}; a = 40 - 10 = 30 \text{ s}; b = t; h = \frac{20 \text{ m}}{\text{s}},$$

$$\therefore 1000 = \frac{1}{2} (40+t)20$$

$$\therefore 100 = (40+t)$$

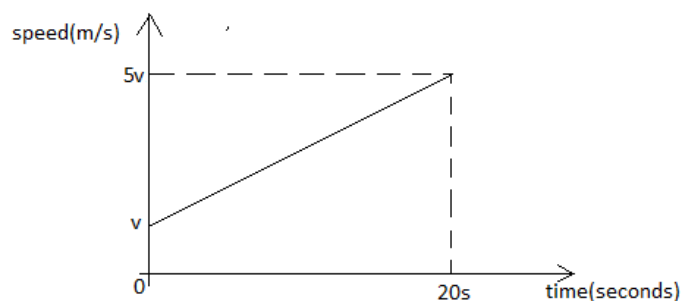
$$\therefore t = 100 - 40$$

$$\therefore t = 60 \text{ seconds}$$

$$\therefore \text{total time taken} = 60 \text{ seconds}$$

LEARNER ACTIVITY:

The diagram below is the speed-time graph of a particle. The particle accelerates uniformly from a speed of $v \text{ m/s}$ to a speed of $5v \text{ m/s}$ in 20 seconds.



(a) find the expression in terms of v, for acceleration.

(b) the distance travelled by the object from 0 seconds to 20 seconds is 80m. Find the value of v.

(c) Find the speed at t = 15 seconds.

EXPECTED ANSWERS

(a) acceleration $\frac{v-u}{t}$

but $u=v$ and $v=5v$

$\therefore$ acceleration $\frac{5v-v}{20} \text{ m/s}^2 = \frac{4v}{20} \text{ m/s}^2$

$\therefore$ acceleration $\frac{1}{5}v \text{ m/s}^2$

(b) $\text{total distance} = \frac{1}{2}(a+b)h$

but $a=v$, $b=5v$, $h=20$ and $\text{dist}=80\text{m}$

$\therefore 80 = \frac{1}{2}(v+5v)20$

$\therefore 6v = 8$

$\therefore v = \frac{8}{6} \text{ m/s}$

$\therefore v = \frac{4}{3} \text{ m/s}$

(c) $\therefore$ acceleration $\frac{1}{5} \times \frac{8}{6} \text{ m/s}^2 = \frac{4}{15} \text{ m/s}^2$,

$\therefore$ find speed at $t=15\text{ s}$, we use $\text{acceleration} = \frac{v-u}{t}$

let speed at $t=15\text{ s}$ be $y = \text{final velocity}$, $v = \frac{4}{3} \text{ m/s}$

be initial velocity.

$$\therefore \frac{4}{15} \text{ m/s}^2 = \frac{y - \frac{4}{3}}{15}$$

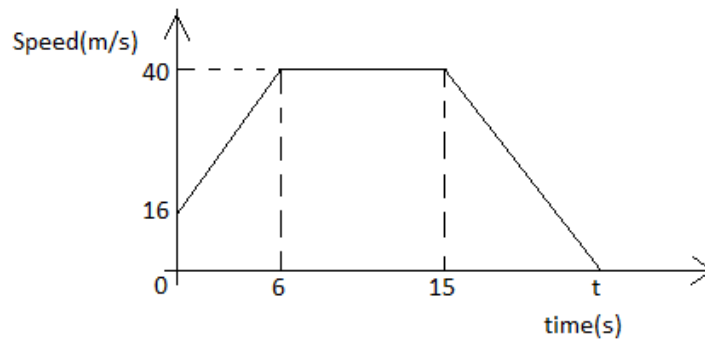
$$\therefore 15 \times 4 = 15 \times y - 15 \times \frac{4}{3}$$

$$\therefore 4 = y - \frac{4}{3}$$

$$\therefore y = 4 + \frac{4}{3}$$

$$\therefore y = \frac{16}{3} \text{ m/s} = \text{speed at } t = 15\text{s}$$

LEARNER ACTIVITY:



The diagram represents the speed-time graph of a moving object.

- (i) Calculate the speed of the object when $t = 4\text{s}$.
- (ii) Calculate the distance travelled in the first 15 seconds.
- (iii) Given that the rate at which the object slows down after $t = 15$ is equal to half the rate at which the object accelerates during the first 6 seconds, calculate the time at which it stops.

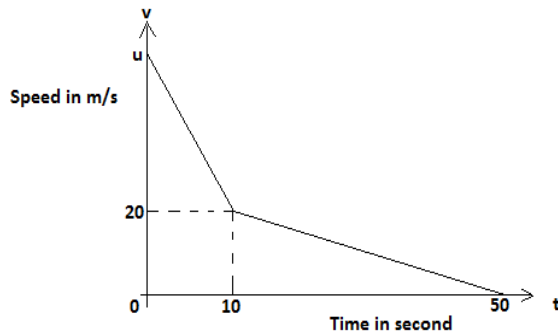
EXPECTED ANSWERS:

- (i) speed at $t = 4\text{s}$ is equal to **32m/s**,
- (ii) distance travelled = **528m**,
- (iii) time at which it stops = $t = \mathbf{35\text{s}}$.

PRACTICE QUESTION:

(Q) The diagram is the speed- time graph of a car which is uniformly retarded from u m/s to 20 m/s in 10 seconds.

The car is then uniformly retarded at a different rate until it finally comes to rest after a further 40 seconds.



Calculate

(a) the speed of the car after 20 seconds,

(b) the retardation during the final 40 seconds of its motion,

(c) the value of u , if the distance travelled in the first 10 seconds is 275 metres.

ANSWERS

(a) 15 m/s, (b) $\frac{1}{2}$ m/s², (c) 35 m/s .