Jimma University

College of Natural Sciences

Department of Mathematics

Math1011 (Mathematics for natural sciences) Worksheet I

Chapter I: Propositional Logic and Set Theory

1) Let p: 387 is a composite number. T

a. q: 123 is a prime number. F

Write each of the following in words, and determine the truth value of each.

$a. q \vee \neg p$ = F $b. \neg q \wedge p$ = T $c. \neg p \Rightarrow \neg q$ = T $d. \neg q \Leftrightarrow \neg p$ = F

2) Suppose $p, q, and\ r$ are propositions. Use truth table to show that each of the following pairs of propositions is logically equivalent.

a. $a.\ (p \vee q) \Rightarrow r\ and\ (p \Rightarrow q) \wedge (q \Rightarrow r)$

$b.\ p \Rightarrow (q \vee r)\ and\ (\neg r) \Rightarrow (p \Rightarrow q)$

$c.\ p \Rightarrow (q \vee r)\ and\ ((\neg r) \wedge p) \Rightarrow q$

3) Let $P(x)$: x is an integer, $Q(x)$: x is a rational number,

$R(x)$: x is an irrational number and $S(x)$: x is a real number.

Express each of the following in symbolic form.

(a) x is an irrational number if x is a real number

(b) x is not an integer or x is not a rational number.

(c) x is a real sufficient condition for x is an irrational number.

(d) x is an irrational number only if x is a real number.

4) Let U = the set of real numbers. Determine the truth value of the following.

(a) $(\forall x)(x^2 + x + 1 > 0)$

(b) $(\exists x)(x^3 + x = 0)$

(c) $(\forall x)(\forall y)\ [x^2 - y^2 = (x + y)(x - y)]$

(d) $(\forall x)(\exists y)\ (xy = 1)$

(e) $(\exists x)(\forall y)(y < x)$

(f) $(\exists y)(\exists x)\ (y < x)$

(g) $(\exists y)\ (\forall x)\ (xy = x)$

(h) $(\forall x)\ (\forall y)\ (x < y \vee x \geq y)$

(i) c $(\forall x)\ (\exists y)\ (x + y = 0)$

(j) d) $(\forall x)\ (\forall y)\ (\exists z)\ (x + y = z)$

5) **With and without using truth table** show that the following pairs of sentences are equivalent:

(a) $p \Rightarrow (q \vee r);\ \neg r \Rightarrow (p \Rightarrow q)$

(b) $\neg[p \vee (\neg p \wedge q)];\ \neg p \wedge \neg q$

(c) $(p \Rightarrow q) \wedge (p \Rightarrow r);\ p \Rightarrow (q \wedge r)$

6) **Use both methods (truth table & formal proof)** to show that the following argument forms are valid.

a) $\neg p \wedge q, r \Rightarrow p, \neg r \Rightarrow s, s \Rightarrow t \mid -t$

b) $\neg p \Rightarrow r,\ p \Rightarrow q,\ \neg q,\ r \Rightarrow s \mid -s \vee t$

c) $(u \vee \neg q) \Rightarrow (\neg s \vee t),\ u,\ \neg s \Rightarrow l,\ (t \vee \neg r) \Rightarrow p \mid -p$

d) $q \Rightarrow r,\ \neg s \Rightarrow (t \Rightarrow p),\ s \vee (q \vee t),\ \neg s \mid -r \vee p$

7) Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$. = 12

8) If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, , then find $n(B)$. = 27

9) In a group of 60 people, 27 like coffee and 42 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?

10) In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

11) If $A, B, and\ C$ are any three sets, then show that,

(a) $(B - A) \cup (C - A) = (B \cup C) - A$,

(b) $(A - B) - (B - C) = A - B$

(c) $P(A) \subseteq P(B) \Leftrightarrow A \subseteq B$

(d) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$

11. Determine whether each of the following arguments is valid or not:

(a) If the moon is made of bananas, then there are monkeys on the moon. The moon is not made of bananas. Therefore, there are no monkeys on the moon.

(b) A sufficient condition for this year to be 2020 is that last year was 2019. This year is 2020. Therefore last year was 2019.

12. Let $U = \{2, 3, 6, 8, 9, 10, 11, 13, 15, 17, 18, 19, 22, 23, 25, 27, 29, 32, 33, 37, 39\}$

$A = \{x : x\ is\ a\ positive\ prime\ factor\ of\ 78\}$

$B = \{x \in U : x\ is\ a\ composite\ number\}$

$C = \{x \in U : x - 6 \in U\}$

$D = \{x \in U : x + 5 \in U\}$ Find

$(a) A \cup B \quad (b) C \cap D \quad (c) B - D \quad (d)(A \cap C)' \quad (e) B \Delta D \quad (f)(B \cup C) \setminus D \quad (g) \mathcal{P}(A)$

13. Let $A \cup B = \{a, b, c, d, e, x, y, z\}$ and $A \cap B = \{b, e, y\}$.

(a) If $B - A = \{x, z\}$, then $A =$ acd.

(b) If $A - B = \phi$, then $B =$ b e y x.

(c) If $B = \{b, e, y, z\}$, then $A - B =$ acdx.

(d) $\mathcal{P}(A \cap B) =$ _____.

1) Mr X has three classes. Each class has 36, 48 and 84 students respectively. Mr X wants to divide each class into groups so that every group in every class has the same number of students and there are no students left over. What is the maximum number of students he can put into each group?
2) Using the prime factorization of the integers, find the GCF and LCM of the given numbers.
   a. a) $m = 420, n = 1540\ q = 6600$
   b) $m = 840, n = 1890, q = 5250, t = 14170$
3) Find the value of
   a. an odd natural number x if LCM(x,40)=1400.
   b. an even natural number x if LCM(x,40)=1400
4) Using the principle of mathematical induction prove each of the following:
   (a) $n^3 - 7n + 3$ is divisible by 3 ,for all natural numbers n.
   (b) $x^n - y^n$ is divisible by $x - y$, where x and y are integers with $x \neq y$ and n any natural number.
   (c) $n^2 < 2^n$ for all natural numbers $n \geq 5$.
   (d) $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$ for all natural numbers n.
5) Prove or disprove that $\frac{3}{\sqrt{2}}$ is not rational number.

6) In each of the following determine (i) upper bounds (ii)lub(sup) (iii) lower bounds (iv)glb(inf)
   a)$A = (-2, \infty)$ (b) $A = \{x | x^3 - x > 0\}$ (c)$A = \{2 - \frac{n}{n+1} | n \in N\}$
   d)$A = \{\frac{3+5n^2}{n+n^2} | n \in N\}$
7) Express each of the following as fractions, i.e, in the form$\{\frac{a}{b} | a, b\ integers\}$:
   (a) $-5.32\overline{456}$ (b) $1.00\overline{79}$ (c) $10.020\overline{1}$ (d) $-1.\overline{23456789}$
8) Simplify each of the following:
   (a) $[-11, -8) \cup (-9, -1) \cup [-6, -2]$ (b) $(-3.5, 6) \cap (-5, 2.\overline{43}) \cap (0.\overline{3}, 11.2\overline{36})$
9) Express each of the following in polar form:
   a) $2 + 2\sqrt{3}i$ b) $-5 + 5i$ c) $-\sqrt{6} - \sqrt{2}i$ d) $1 - i$ e) $-3i$ f) -10

10) Write each of the following in rectangular form (i.e. x+yi).

a) $(\sqrt{3}+i)^7$ b) $\frac{3i^{30}-i^{19}}{2i-1}$ c) $(-\sqrt{3}-i)^5$ d) $(\sqrt{3}-i)^6$

12) Find the modulus and principal argument of $(1+i)^{10}$

13) Find all values of z such that: a) $z^5=-32$ b) $z^2=1+i$ c) $z^2+(2i-3)z+5-i=0$

d) $(z+1)^4=1-i$ e) $e^z=1+\sqrt{3}i$ f) $z^2(2-z^2)=2$

14) Find the values of $(\sqrt{3}+i)^{1/2}$ and express each in rectangular form.