

1 Foundation Design (Isolated Footing)

An internal column transferring the loads given in table 1 is considered for isolated footing design. The material to be used for are C25/30 and S400. If the column size is 600mm by 600 mm. and the soil has presumptive bearing capacity of 280 Kpa(Allowable), determine the size and reinforcement necessary for the isolated footing to safely transfer the load.

ETABS OUTPUT

Story	Joint Label	Load Case/Combo	Fx	Fy	Fz	Mx	My	Mz
Base	33	Comb1-Ultimate	-7.3925	-2.4641	3587.939	3.9611	-11.7307	-3.0603
Base	33	Comb2-Service	-5.1324	-1.8362	2564.1252	2.9776	-8.1731	-2.1384

Table 1: Joint Reaction from ETABS to an internal column

1.1 Material data

For C25/30 : $f_{ck} = 25 \text{ Mpa}$

For S400: $f_{yk} = 400 \text{ Mpa}$

Concrete design strength

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 3.15}]$$

Where: the recommended value of α_{cc} is 0.85.

$$f_{cd} = \frac{0.85 * 25}{1.5} = 14.17 \text{ Mpa}$$

Steel design strength

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \quad [\text{ES EN 1992 - 1 - 1: 2015, Figure 3.8}]$$

$$f_{yd} = \frac{400}{1.115} = 347.83 \text{ Mpa}$$

1.2 Footing proportioning

Factor of safety (F.S)

$$F.S = \frac{\text{Ultimate load}}{\text{Service load}} = \frac{|3587.939 + 11.73 + 3.96|}{|2564.125 + 2.978 + 8.173|} = 1.399$$

$$\sigma_{all} = 280 \text{ Kpa}$$

$$\sigma_{ult} = 1.399 * 280 \text{ Kpa} = 391.81 \text{ Kpa}$$

Eccentricity at service load

$$e_x = \frac{M_y}{P} = \frac{8.173}{2564.125} = 3.188 \times 10^{-3}$$

$$e_y = \frac{M_x}{P} = \frac{2.978}{2564.125} = 1.161 \times 10^{-3}$$

Now, the stress at the four corners of the footing can be determined from:

$$\sigma = \frac{P}{A} \left(1 \pm \frac{6e_x}{B} \pm \frac{6e_y}{B} \right)$$

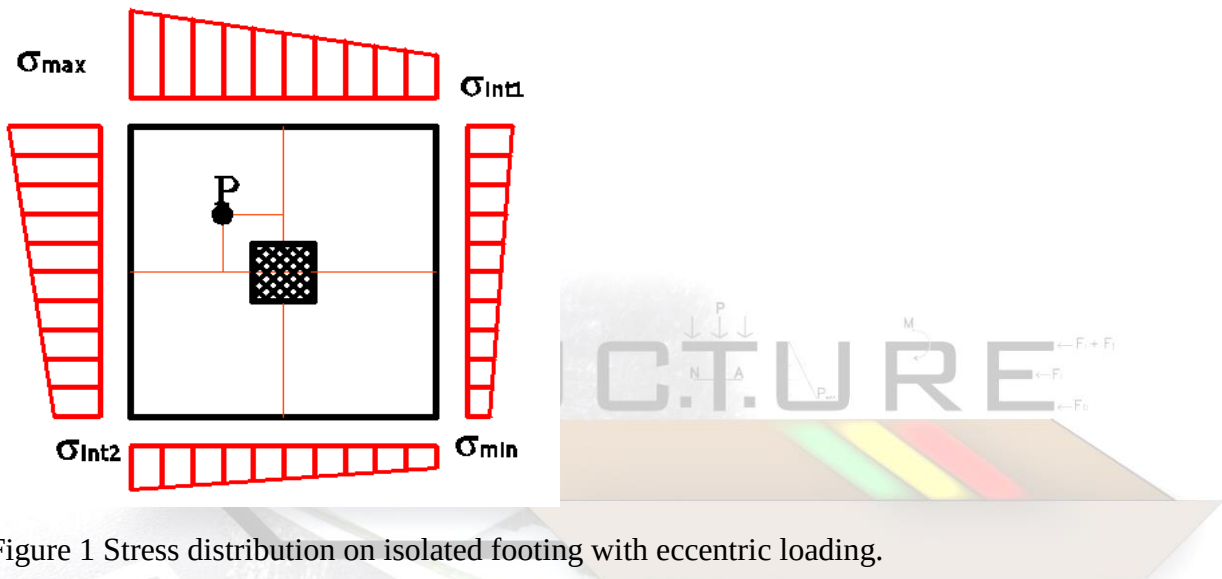


Figure 1 Stress distribution on isolated footing with eccentric loading.

Any plan dimension of footing, B and L, must satisfy two things:

- a) The maximum stress must be less than or equal to the allowable bearing capacity.
- b) The minimum stress must be greater than zero.

Condition a)

Since the eccentricity is relatively equal in both axis's, a square footing is ideal.

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e_x}{B} + \frac{6e_y}{B} \right) \leq \sigma_{all}$$

$$\frac{2564.1252}{B^2} \left(1 + \frac{6 \cdot 3.188 \times 10^{-3}}{B} + \frac{6 \cdot 1.161 \times 10^{-3}}{B} \right) \leq \sigma_{all}$$

$$\frac{2564.1252}{B^2} \left(1 + \frac{26.094 \times 10^{-3}}{B} \right) \leq \sigma_{all}$$

$$\frac{2564.1252}{B^2} + \frac{66.91}{B^3} \leq 280$$

$$280B^3 - 2564.1252B - 66.91 = 0$$

Solving the cubic equation:

$$B \geq \begin{cases} 3.039m \\ -0.0261 \\ -3.013 \end{cases}$$

The only possible solution is $B=3.039$ m

To ease setting out during construction take $B=3.1$ m.

Condition b)

$$\sigma_{\min} = \frac{2564.1252}{(3.1)^2} \left(1 - \frac{6 * 3.188 \times 10^{-3}}{3.1} - \frac{6 * 1.161 \times 10^{-3}}{3.1}\right) = 264.57 \text{ kpa} > 0 \quad \text{ok!}$$

$$\sigma_{\max} = \frac{2564.1252}{(3.1)^2} \left(1 + \frac{6 * 3.188 \times 10^{-3}}{3.1} + \frac{6 * 1.161 \times 10^{-3}}{3.1}\right) = 269.06 \text{ kpa} < 280 \quad \text{ok!}$$

Since both conditions are satisfied take $B=L=3.1$ m.

1.3 Depth, D, Determination Eccentricity at ultimate load

$$e_x = \frac{M_y}{P} = \frac{11.7307}{3587.93} = 3.269 \times 10^{-3}$$

$$e_y = \frac{M_x}{P} = \frac{3.9611}{3587.93} = 1.104 \times 10^{-3}$$

The stress at the four corners of the footing:

$$\sigma_{\min} = \frac{3587.93}{(3.1)^2} \left(1 - \frac{6 * 3.269 \times 10^{-3}}{3.1} - \frac{6 * 1.104 \times 10^{-3}}{3.1}\right) = 370.194 \text{ kpa}$$

$$\sigma_{\text{int1}} = \frac{3587.93}{(3.1)^2} \left(1 - \frac{6 * 3.269 \times 10^{-3}}{3.1} + \frac{6 * 1.104 \times 10^{-3}}{3.1}\right) = 371.79 \text{ kpa}$$

$$\sigma_{\text{int2}} = \frac{3587.93}{(3.1)^2} \left(1 + \frac{6 * 3.269 \times 10^{-3}}{3.1} - \frac{6 * 1.104 \times 10^{-3}}{3.1}\right) = 374.92 \text{ kpa}$$

$$\sigma_{\text{int2}} = \frac{3587.93}{(3.1)^2} \left(1 + \frac{6 * 3.269 \times 10^{-3}}{3.1} + \frac{6 * 1.104 \times 10^{-3}}{3.1}\right) = 376.52 \text{ kpa}$$

The average stress:

$$\sigma_{avg} = \frac{374.92 + 371.79 + 376.52 + 370.194}{4} \text{ kpa} = 373.356 \text{ Kpa}$$

Wide beam shear

According to ES EN 1992-1-1:2015, Article 6.2.1(8) the Critical section for shear is at distance d from the face of supports.

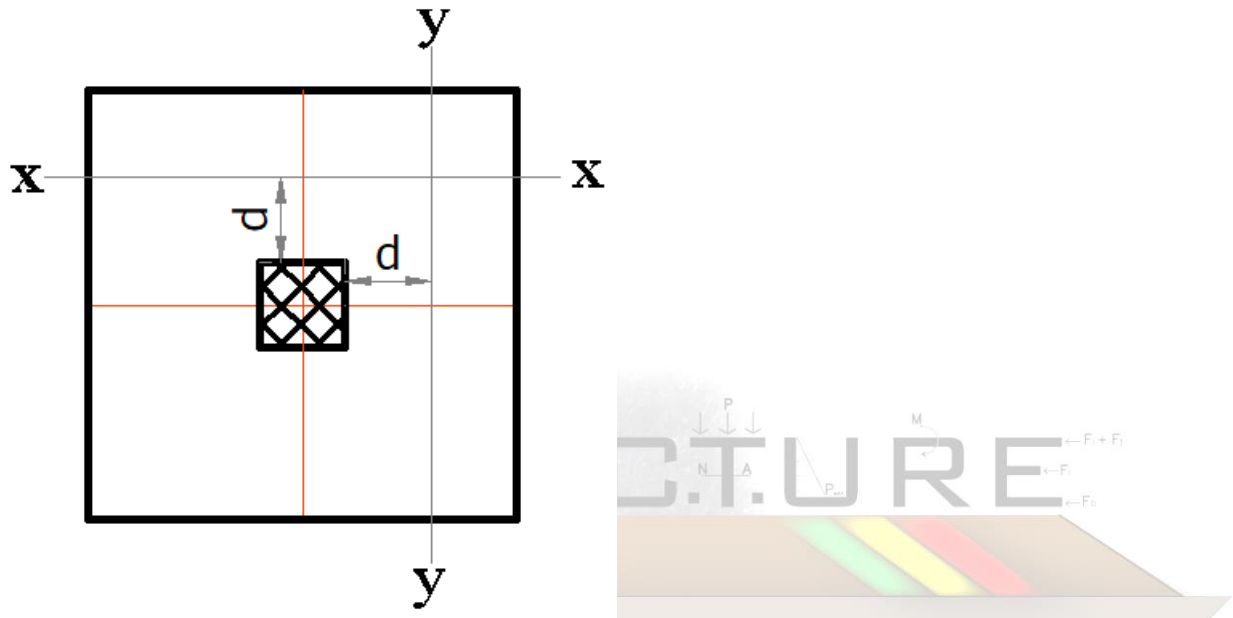


Figure 2: Critical section for wide beam shear, plan.

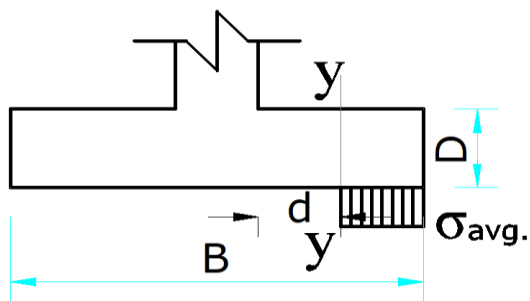


Figure 3: Critical section for wide beam shear, Section.

The design shear force at section y-y is:

$$V_{y-y} = \sigma_{avg} B * \left(\frac{B}{2} - \frac{C_x}{2} - d \right)$$

$$V_{y-y} = (372.356)(3.1) * \left(\frac{3.1}{2} - \frac{0.6}{2} - d \right) = 1154.304 * (1.25 - d) = 1442.88 - 1154.304d$$

Wide beam shear resistance (the design value of shear resistance for members not requiring shear reinforcements) is given by:

$$V_{rd,c} = \left[C_{rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} \right] b_w d \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.2 a}]$$

$$C_{rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$\rho_1 = \rho_{min} = 0.26 \frac{f_{ctm}}{f_{yk}} = 0.26 * \frac{0.3 f_{ck}^{\frac{2}{3}}}{f_{yk}} = 0.26 * \frac{0.3 * (25)^{\frac{2}{3}}}{400} = 0.0017 < 0.02$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{rd,c} = \left[0.12 \left(1 + \sqrt{\frac{200}{d}} \right) (100 * 0.0017 * 25)^{\frac{1}{3}} \right] 3100 * d \text{ KN} = 602.57 * d * \left(1 + \sqrt{\frac{200}{d}} \right)$$

And the minimum $V_{rd,c}$

$$V_{rd,c}(\min) = (v_{min}) b_w d \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.2 b}]$$

$$v_{min} = 0.035 k^{\frac{3}{2}} f_{ck}^{\frac{1}{2}} \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.3 N}]$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$v_{min} = 0.035 k^{\frac{3}{2}} f_{ck}^{\frac{1}{2}} = 0.035 * \left(1 + \sqrt{\frac{200}{d}} \right)^{\frac{3}{2}} * (25)^{\frac{1}{2}} = 0.175 \left(1 + \sqrt{\frac{200}{d}} \right)^{\frac{3}{2}}$$

$$V_{rd,c}(\min) = 0.175 \left(1 + \sqrt{\frac{200}{d}} \right)^{\frac{3}{2}} * 3100 * d = 542.5 \left(1 + \sqrt{\frac{200}{d}} \right)^{\frac{3}{2}} d$$

Equating the shear capacity and the design shear force:

$$602.57 * d * \left(1 + \sqrt{\frac{200}{d}} \right) = 1442.88 - 1154.304d$$

Solving the nonlinear equation using excels goal seek

$$d = 0.69354 \text{ m} = 693.54 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{693.54}} = 1.53 < 2 \text{ ok!}$$

$$V_{rd,c} = 602.57 * d * \left(1 + \sqrt{\frac{200}{d}}\right) = 602.57 * 0.69354 * \left(1 + \sqrt{\frac{200}{693.54}}\right) = 642.32 \text{ KN}$$

And from using the minimum $V_{rd,c}$ and equating it the shear force:

$$1442.88 - 1154.304d = 542.5 \left(1 + \sqrt{\frac{200}{d}}\right)^{\frac{3}{2}} d$$

Solving the nonlinear equation using excels goal seek

$$d = 0.65468 \text{ m} = 654.68 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{654.68}} = 1.55 < 2 \text{ ok!}$$

$$V_{rd,c}(\text{min}) = 542.5 \left(1 + \sqrt{\frac{200}{d}}\right)^{\frac{3}{2}} d = 542.5 * 0.65468 * \left(1 + \sqrt{\frac{200}{654.68}}\right)^{\frac{3}{2}} = 687.17 \text{ KN}$$

Since $V_{rd,c} < V_{rd,c}(\text{min})$ take depth from $V_{rd,c}(\text{min})$ which is 654.68 mm

The overall depth is:

$$\text{Cover} = 50 \text{ mm} \quad \text{XC2 EXPOSURE CLASS}$$

$$d = 654.68 + 50 + \frac{12}{2} = 710.68 \text{ mm}$$

Take D=720 mm.

Revise d:

$$d = 720 - 50 - 12/2 = 660 \text{ mm.}$$

Punching Shear

1. The punching resistance of **column bases should be** verified at control perimeters **within** 2d from the periphery of the column [ES EN 1992-1-1:2015. Article 6.4.4(2)].
2. Control perimeters at a **distance less than 2d** should be considered where the concentrated force is opposed by a high pressure (e.g. soil pressure on a base) [ES EN 1992-1-1:2015. Article 6.4.2(2)].

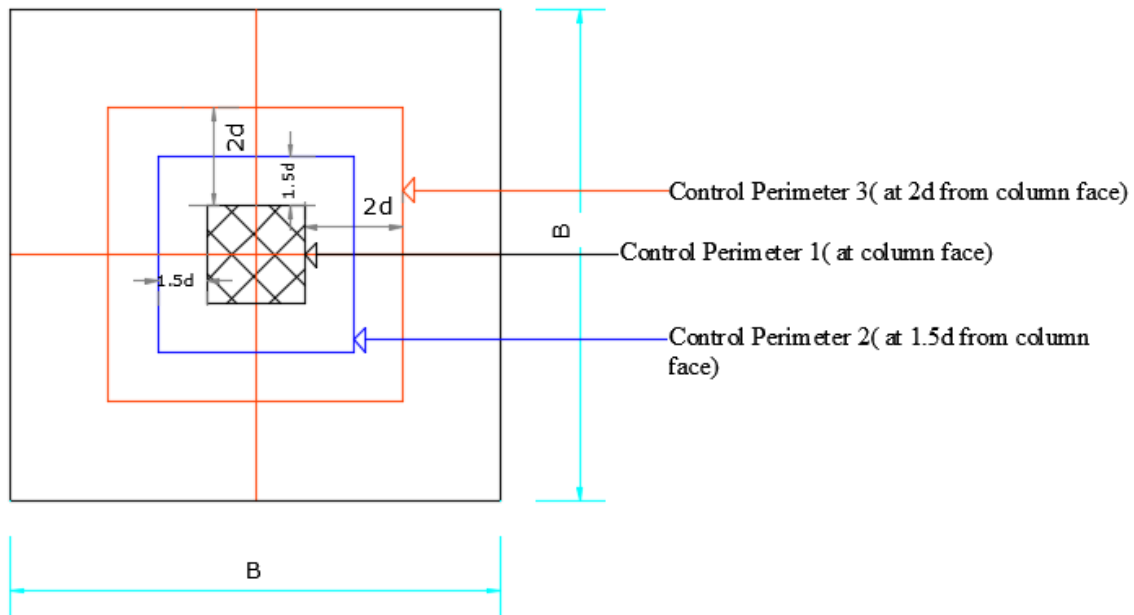


Figure 4: Critical section for punching shear.

Check punching shear at critical perimeter 1(column face)

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed} \quad [ES EN 1992 - 1 - 1: 2105, Eqn. 6.48]$$

$$V_{Ed} = \frac{V_{Ed,red}}{u d} \quad [ES EN 1992 - 1 - 1: 2105: Eqn. 6.49]$$

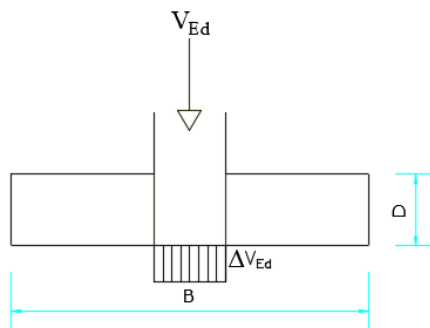


Figure 5: pictorial representation of V_{Ed} and ΔV_{Ed} for control perimeter 1

$$V_p = V_{Ed,red} = P_d(V_{Ed}) - [\sigma_{avg}(C_x) * (C_y)](\Delta V_{Ed}) = 3587.939 - 373.36 * (0.6)(0.6) = 3453.53 \text{ KN}$$

Punching shear resistance is given by (since ES EN is not clear about this verification Euro code is used):

With $\theta=22^\circ$

$$V_{rd,max} = \frac{\alpha_{cw} u_o z v_1 f_{cd}}{\cot(\theta) + \tan(\theta)} \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.9 (with } b \text{ replaced with } u)]$$

$$v_1 = v = 0.6 \left[1 - \frac{f_{ck}}{250} \right] = 0.6 \left[1 - \frac{25}{250} \right] = 0.54$$

$$z = 0.9d = 0.9 * 660 = 594 \text{ mm} \quad [0.9d \text{ is assumed for value for } z]$$

$$\alpha_{cw} = 1$$

$$u_o = 2 * 0.6 + 2 * 0.6 = 2.4 \text{ m (perimeter of column)}$$

$$\cot(22) = 2.47 \approx 2.5 \text{ and } \tan(22) = 0.404 \approx 0.4$$

$$V_{rd,max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot(\theta) + \tan(\theta)} = \frac{1 * 2400 * 594 * 0.54 * 14.17}{2.5 + 0.4} * 10^{-3} = 3761.52 \text{ KN}$$

$$V_{rd,max} > V_p(V_{Ed,red}) \theta=22^\circ \text{ and control perimeter 1 is ok!}$$

Control perimeter 2 (at 1.5d distance from the column face)

This control perimeter is following ES EN 1992-1-1:2015, Article 6.4.2(2)]. And some books use d instead of '1.5d' which is very conservative.

Shear

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed} \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.48}]$$

$$V_p = V_{Ed,red} = P_d(V_{Ed}) - [\sigma_{avg} (C_x + 3d) * (C_y + 3d)] (\Delta V_{Ed})$$

$$V_p = 3587.939 - 373.36 * (3 * 0.66 + 0.6) * (3 * 0.66 + 0.6) = 1102.705 \text{ KN}$$

Resistance

$$V_{rd,c} = C_{rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} u d \geq V_{min} u d \quad [\text{ES EN 1992 - 1 - 1: 2015, Eqn. 6.47 (with } b = u)]$$

$$C_{rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{660}} = 1.55 < 2 \text{ ok!}$$

$$\rho_1 = \rho_{min} = 0.26 \frac{f_{ctm}}{f_{yk}} = 0.26 * \frac{0.3 f_{ck}^{\frac{2}{3}}}{f_{yk}} = 0.26 * \frac{0.3 * (25)^{\frac{2}{3}}}{400} = 0.0017 < 0.02$$

$$u = 2 * (3d + c_x) + 2 * (3d + c_y)$$

$$u = 2 * (3 * 0.66 + 0.6) + 2 * (3 * 0.66 + 0.6) = 7.68 \text{ m} = 10,320 \text{ mm}$$

$$V_{rd,c} = \left[0.12 * 1.55 * (100 * 0.0017 * 25)^{\frac{1}{3}} \right] * 10,320 * 660 * 10^{-3} \text{KN} = 2052.105 \text{ KN}$$

And the minimum $V_{rd,c}$

$$V_{rd,c} = (v_{min})ud \quad [\text{ES EN 1992 – 1 – 1: 2015, Eqn. 6.2 b(with } U = b)]$$

$$v_{min} = 0.035k^{\frac{3}{2}}f_{ck}^{\frac{1}{2}} \quad [\text{ES EN 1992 – 1 – 1: 2015, Eqn. 6.3 N}]$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$k = 1 + \sqrt{\frac{200}{404}} = 1.703 \leq 2 \text{ ok!}$$

$$v_{min} = 0.035k^{\frac{3}{2}}f_{ck}^{\frac{1}{2}} = 0.035 * (1.703)^{\frac{3}{2}} * (25)^{\frac{1}{2}} = 0.389$$

$$V_{rd,c} = 0.389 * 10,320 * 660 * 10^{-3} = 2649.55 \text{ KN}$$

Control perimeter 3(at 2d distance from the column face)

Since 2d (1.32) is greater than $(B-C_x)/2$, this critical section lies outside the footing and no verification is required.

Therefore, 720 mm depth from shear is satisfactory.

1.4 Reinforcement Calculation

The critical section for flexure is at the face of column.

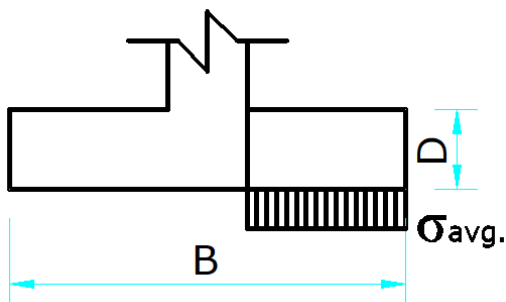


Figure 1.1 Critical section for moment.

$$M_{sd} = \sigma_{avg} \frac{B}{2} * \left(\frac{B}{2} - \frac{C_x}{2} \right)^2 = \frac{\sigma_{avg} B}{8} (B - C_x)^2$$

$$M_{sd} = \frac{372.356 * 3.2}{8} (3.1 - 0.6)^2 = 930.89 \text{ KN-m}$$

Design moment per meter width

$$M_{sd} = \frac{930.89 \text{ KN} - m}{3.1} = 300.288 \text{ KN} - m / m$$

Reinforcement calculation:

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{300.288 * 10^6}{14.17 * 1000 * 660^2} = 0.047$$

$K_z = 0.975$ (from design chart)

$$A_s = \frac{M_{sd}}{K_z d f_{yd}} = \frac{300.288 * 10^6}{0.975 * 660 * 347.83} = 1348.51 \text{ mm}^2$$

Check minimum reinforcement:

$$\rho_{min} = 0.26 \frac{f_{ctm}}{f_{yk}} = 0.26 * \frac{0.3 f_{ck}^{\frac{2}{3}}}{f_{yk}} = 0.26 * \frac{0.3 * (25)^{\frac{2}{3}}}{400} = 0.0017 < 0.02$$

$$A_{s,min} = 0.0017 * 1000 * 660 = 1122 \text{ mm}^2$$

$A_s > A_{s,min}$ Provide A_s .

Using diameter 14 bar:

$$A_{s,min} = \frac{a_s}{A_s} * b = \frac{154}{1348.51} * 1000 = 114.15 \text{ mm} \approx 110 \text{ mm}$$

Check maximum spacing of reinforcement.

$$\text{From ES EN 1992-1-1:2015, Article 9.3.1.1(3): } S_{max} \leq \begin{cases} 3d = 3 * 660 = 1980 \\ 400 \text{ mm} \end{cases}$$

$S_{max} = 400 \text{ mm} > 110 \text{ mm}$, provide 110 mm.

Provide Ø14 c/c 110mm.