

Groups, Rings and Field Theory

1. The order of a and x in a group are respectively 3 and 4. Then the order of $x^{-1}ax$ is
A. 3 B. 4 C. 6 12
2. Which of the following statement is false?
A. Every infinite cyclic group is isomorphic to the additive group of all even integers.
B. All groups of order 4 are isomorphic.
C. All groups of order 7 are isomorphic.
D. All non-commutative groups of order 6 are isomorphic.
3. Consider the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 9 & 6 & 3 & 2 & 1 & 8 & 7 & 5 \end{pmatrix}$. Then the order of σ is (A1)
A. 12 B. 24 C. 6 8
4. Let G be a cyclic group of order n and m be the divisor of n . Then, (A1)
A. G contains a unique subgroup of order n .
B. G may or may not contain any subgroup of order m .
C. G must contain a subgroup of order m , but it may contain more than one subgroups of order m .
D. G cannot have any subgroup of order m .
5. In a non-abelian group, the element a has order 108, then the order of a^{42} is
A. 54 B. 27 C. 18 D. 9
6. In the permutation group S_6 , the number of elements conjugate to $(123)(456)$ is
A. 80 B. 120 C. 60 D. 40
7. If a and a^2 are both generators of a cyclic group of order n , then
A. n must be odd B. n must be even C. n must be prime D. n is not prime
8. The Alternating group A_4 on 4 symbols has a normal subgroup of order
A. 2 B. 3 C. 4 D. 6
9. The number of non abelian groups of order 12 upto isomorphism is
A. 0 B. 2 C. 3 D. 4
10. Two permutations in the symmetric subgroup S_n are conjugate if and only if
A. they fix the same number of elements B. they have the same cycle decomposition
C. they have the same order D. they are odd powers of each other
11. Let G be a finite abelian group of odd order. Let $H = \{x^2 : x \in G\}$. Then
A. H is a subgroup of G only if G is cyclic
B. H is a proper subgroup of G
C. $H = G$
D. H may not be a subgroup of G
12. Upto isomorphism, the number of distinct groups of order 8 is
A. 2 B. 3 C. 5 D. 8
13. The number of quotient groups of S_4 , the symmetric group of degree 4, upto isomorphism is
A. 1 B. 2 C. 3 D. 4
14. In the symmetric group S_n of degree n , $n > 2$, the set of all 3-cycles generates a subgroup of order
A. 3 B. $3n$ C. $n!/2$ D. $n!$
15. The number of generators of a cyclic group of order 8 is
A. 4 B. 1 C. 2 D. 3

16. If G is an abelian group and $a, b \in G$ are such that $o(a) = 5$ and $o(b) = 7$, then $(ab)^{14}$ is equal to
 A. $a^{-1}b$ B. ab^{-1} C. a^{-1} D. b^{-1}
17. In the group S_8 , the order of the permutation $\sigma = (1, 4, 5, 7)(2, 3)$ is
 A. 2 B. 4 C. 6 D. 8
18. The number of normal subgroups of the Klein 4-group is
 A. 2 B. 3 C. 4 D. 5
19. If $f : G \rightarrow H$ is a group homomorphism and K is the kernel of f , then
 A. f is a monomorphism iff $K = \{e\}$ B. f is an isomorphism iff $K = \{e\}$
 C. f is a monomorphism iff $K \neq \{e\}$ D. f is an isomorphism iff $K \neq \{e\}$
20. Let G be a finite group of order n . Which of the following is not true?
 A. If G has a subgroup of order m , then $m|n$.
 B. If G has an element of order m , then $m|n$.
 C. If $m|n$, then G must have an element of order $m|n$.
 D. If $m|n$ and m is a prime, then G must have an element of order m
21. H and K are subgroup of G with indices 3 and 5 in G . Then the index of $H \cap K$ in G is
 A. 3 B. 5 C. a multiple of 15 D. not more than 8
22. Consider the symmetric group S_n of permutations on n symbols. Let A_n be the set of all even permutations on n symbols. Then
 A. A_n is a subgroup of S_n , but not a normal subgroup of S_n .
 B. A_n is a normal subgroup and $\frac{S_n}{A_n}$ is cyclic.
 C. A_n is a normal subgroup, but $\frac{S_n}{A_n}$ is not cyclic.
 D. A_n is not a subgroup of S_n
23. The orders of a and x in group are 3 and 4 respectively. Then the order of $x^{-1}ax$ is
 A. 3 B. 4 C. 6 D. 12
24. How many subgroups does a cyclic subgroup with 10 elements have?
 A. 2 B. 10 C. 4 D. 1
25. Which of the following is a pair of isomorphic groups?
 A. $(\mathbb{Z}, +), (\mathbb{Q}, +)$ B. $(\mathbb{R}, +), (\mathbb{C}, +)$ C. $(\mathbb{R}^\bullet, \cdot), (\mathbb{C}^\bullet, \cdot)$ D. $(\mathbb{R}, +), (\mathbb{R}^+, \cdot)$
26. In a finite group G , let $a, b \in G$ and suppose that the order of b is 4. Let $bab^{-1} = a^3$ and suppose that the order of a is odd, then the order of a
 A. is 9 B. is 7 C. is 5 D. cannot be determined
27. Let G be a group and H be a subgroup of G such that $xyx^{-1}y^{-1} \in H$ for $x, y \in G$, then
 A. H must be abelian B. H must be non-abelian
 C. H must be cyclic D. H must be a normal subgroup of G
28. Let G be a subgroup of the multiplicative group of all non-zero complex numbers. If G is finite, then G is cyclic. Prove
29. The order of the group generated by the matrices $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$, under matrix multiplication is $\dots\dots\dots$.
30. Show that every cyclic group of order n has the unique subgroup of order d , for each $d|n$. Deduce that $\sum_{d|n} \phi(d) = n$, where ϕ is **Euler's phi function**.
31. Let S_3 be the group of all permutation on 3-cycle with identity e . Then the of elements in S_3 that satisfy the equation $x^2 = e$ is
 A. 0 B. 2 C. 3 D. 5

32. Let G be a group of order 15. Then the number of sylow subgroup of G of order 3 is
 A. 0 B. 1 C. 3 D. 5
33. Let $\mathbb{R}$ be the group of real numbers under multiplication and $\mathbb{Z}$ be the subgroup of all integers. Show that $\frac{\mathbb{R}}{\mathbb{Z}} \cong S^1 = \{z \in \mathbb{C} : |z| = 1\}$
34. Let G be the additive group of integers $\mathbb{Z}$ and G' be the multiplicative group of $4th$ root of unity. Let $f : G \rightarrow G'$ be a homomorphism given by $f(n) = i^n$. Then $\text{Ker}(f) =$
 A. ϕ B. $\{4m : m \in \mathbb{Z}\}$ C. $\{(2m)^2 + 1 : m \in \mathbb{Z}\}$ D. $\{2m + 1 : m \in \mathbb{Z}\}$
35. If p is a prime number, then any group of $2p$ has a normal subgroup of order p . Prove
36. Let G be a group of order 49. Then G is $\dots\dots\dots$.
37. Show that $A_n, n \geq 3$ is generated by all cycles of length 3.
38. Let $(\mathbb{Z}, +)$ denote group of integers. The number of automorphism of $(\mathbb{Z}, +)$ is
 A. 1 B. 2 C. 3 D. 4
39. Let G be a finite group of order 200. Then the number of subgroups of G of order 25 is
 A. 1 B. 4 C. 5 D. 10
40. Let G be a cyclic group of order 6. The the number of elements $g \in G$ such that $G = \langle g \rangle$ is
 A. 5 B. 3 C. 4 D. 2
41. The number of elements of order 5 in S_5 is
 A. 5 B. 20 C. 24 D. 12
42. Let G be the group of order 30. Let A and B be a normal subgroup of order 2 and 5 respectively. Then the order of the group $\frac{G}{AB}$ is
 A. 10 B. 3 C. 2 D. 5
43. The number of groups of order n (upto isomorphism) is
 A. finite for values of n B. finite for finitely many values of n
 C. finite for infinitely many values of n D. infinite for some values of n .
44. Upto isomorphism, the number of abelian group of order 10^5 is
 A. 2 B. 5 C. 7 D. 49
45. Any subgroup of $(\mathbb{Q}, +)$ is
 A. cyclic and finitely generated, but not abelian and normal.
 B. cyclic and abelian, but not finitely generated and normal.
 C. abelian and normal, but not cyclic and finitely generated.
 D. finitely generated and normal, but nit cyclic and abelian.
46. The cardinality of the center of $\mathbb{Z}_{12}$ is
 A. 1 B. 2 C. 3 D. 12
47. In $(\mathbb{Z}, +)$, subgroup generated by 2 and 7 is
 A. $\mathbb{Z}$ B. $5\mathbb{Z}$ C. $9\mathbb{Z}$ D. $14\mathbb{Z}$
48. Let G and H be two groups Then $G \times H \cong H \times G$
 A. for any G and H B. only if one of them is cyclic
 C. only if one of them is abelian D. only if $G \cong H$
49. The number of 5-sylow subgroup of $\mathbb{Z}_{20}$ is
 A. 1 B. 4 C. 5 D. 6
50. Let $H = \mathbb{Z}_2 \times \mathbb{Z}_6$ and $K = \mathbb{Z}_2 \times \mathbb{Z}_4$. Then
 A. $H \cong K$ since both are cyclic.
 B. $H \cong K$ since $2|6$ and $(3, 4) = 1$.
 C. $H \not\cong K$ since K is cyclic where as H is not.
 D. $H \not\cong K$ since $\nexists$ an homomorphism from H to K .

51. Let $(G = \{-1, 1\}, \bullet)$ be a group and $\mathbb{S}^1 = \{z \in \mathbb{C} : |z| = 1\}$. Let G action on $\mathbb{S}^1$ by complex multiplication. Then cardinality orbit i is
 A. 1 B. 2 C. 4 D. ∞
52. Let S_{10} denote the group of permutations on ten symbols $\{1, 2, 3, \dots, 10\}$. The number of elements S_{10} commuting with element $\sigma = (1 \ 3 \ 5 \ 7 \ 9)$ is
 A. $5!$ B. $5 \cdot 5!$ C. $5! \cdot 5!$ D. $\frac{10!}{5!}$
53. Let $\mathcal{D}_8$ denote the group of symmetries of a square (**Dihedral group**). The minimal number of generators for $\mathcal{D}_8$ is
 A. 1 B. 2 C. 4 D. 8
54. Let G be a cyclic group of order of 8. Then its group of automorphisms has order
 A. 2 B. 4 C. 6 D. 8
55. The number of elements of order 8 in the group $(\mathbb{Z}_{24}, \otimes_{24})$ is
 A. 1 B. 2 C. 3 D. 4
56. Consider the following statements:
 S : Every non-abelian group has anon-trivial abelian subgroup.
 T : Every non-trivial abelian group has a cyclic subgroup.
 Then
 A. both S and T false. B. S is true and T is false.
 C. T is true and S is false D. both S and T are true.
57. Let G be the group with generators a, b given by $G = \langle a, b | a^4 = b^2 = 1, ba = a^{-1}b \rangle$. If $Z(G)$ denotes the center then $\frac{G}{Z(G)} \cong$
 A. $\{e\}$ B. C_2 C. $C_2 \times C_4$ D. C_4
58. Let the bijections f and $g : R \sim \{0, 1\} \rightarrow R \sim \{0, 1\}$ defined by $f(x) = \frac{1}{1-x}$, $g(x) = \frac{x}{1-x}$ and G be the group generated by f and g under the compositions of mappings. It is given that $O(G) = 6$. Then
 A. G and $\text{Aut}(G)$ both are abelian.
 B. G and $\text{Aut}(G)$ both are non-abelian.
 C. G is abelian but not $\text{Aut}(G)$.
 D. $\text{Aut}(G)$ is abelian but not G .
59. The value α for which $G = \{\alpha, 1, 3, 9, 12, 25\}$ is a cyclic group under multiplication mod 56 is
 A. 5 B. 15 C. 25 D. 35
60. Prove that if G is a group with the property that the square of every element is the *identity*, the G is abelian.
61. Let G be a group, and let $a \in G$. Prove that $C(a) = C(a^{-1})$.
62. Let G be a group of order 56. Then
 A. All 7 - Sylow subgroups of G are normal.
 B. All 2 - Sylow subgroups of G are normal.
 C. Either a 7 - Sylow subgroup or a 2 - Sylow subgroup of G is normal.
 D. There is a proper normal subgroup of G .
63. Consider the group $\mathbb{S}_9$ of all the permutations on a set with 9 elements. What is the largest order of a permutation in $\mathbb{S}_9$?
 A. 21 B. 20 C. 30 D. 14
64. Let I_1, I_2 be two ideals of a commutative ring R with identity. Which one of the following is true?
 A. $I_1 + I_2$ and $I_1 \cap I_2$ are ideals of R .
 B. $I_1 + I_2$ is an ideal of R , but $I_1 \cap I_2$ is not an ideal of R .
 C. $I_1 + I_2$ is not an ideal of R , but $I_1 \cap I_2$ is an ideal of R .
 D. Neither $I_1 + I_2$ nor $I_1 \cap I_2$ ia an ideal of R .

65. Let R be the ring of all 2×2 matrices, E be the ring of even integers, T be the ring of integer (mod 10) and S be the ring of multiple of 6, then
- A. only E has no zero divisor B. only R and S have no zero divisors.
 C. only E and S have no zero divisors D. only E and T have zero divisors.
66. Let $R = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \text{ odd}\}$. Then R is a ring such that
- A. R has a unique maximal ideal $I = \{\frac{a}{b} \in R : a \text{ a even}\}$
 B. R has infinitely many maximal ideals
 C. R has finitely many, but more than one maximal ideals
 D. R has no maximal ideal
67. The ring $\mathbb{Z}[X]$ is a
- A. UFD but not PID B. PID ,but not ED
 C. ED, but not a Field D. A Field
68. If R is a commutative ring with 1 and I is an ideal in R such that R/I is a finite ring. Then
- A. I is a prime ideal iff I is a maximal ideal
 B. if I is a maximal ideal, then I is not a prime ideal
 C. if I is a prime ideal, then I is not a maximal ideal
 D. if I is a maximal ideal, then I may or may not be a prime ideal
69. If $f(x)$ is a polynomial over $\mathbb{Z}$, $a \in \mathbb{Z}$ and $f(a) = 30$, then the remainder after dividing $f(x)$ by $x - a$ is
- A. 6 B. 30 C. 15 D. a rational number, but not an integer
70. Let R be a commutative ring with 1 and I be a prime ideal of R . Which of the following are true?
- A. I must be maximal ideal B. If R/I is finite ring, then I is maximal ideal
 C. I is never maximal ideal D. If I is a maximal ideal, then R/I must be a finite ring.
71. If R is the ring of all 2×2 real matrices, E is the ring of all even integers, T is the ring of all integers (mod 10) and V is the ring of the all multiple of 6, then the following has/have no zero divisors
- A. only E B. only R and V
 C. only E and T D. only E and V
72. Which of the following are integral domains with respect to ordinary addition and multiplication?
- $R = \{b\sqrt{2} : b \text{ rational}\}$ $S = \{a + b\sqrt{2} : a, b \text{ rational}\}$
 $T = \{8a : a \text{ rational}\}$ $V = \{a : a \text{ positive integer}\}$
- A. Only R B. Only S and T
 C. Only T D. Only T and V
73. Consider the following statements:
- (1) Every Principal Ideal Domain is an Euclidean domain
 (2) The groups of units in $\mathbb{Z}/37\mathbb{Z}$ is cyclic
 (3) There is a field with 6^5 elements
- A. (1) is true but (2) and (3) false B. (2) is true but (1) and (3) false
 C. (3) is true but (1) and (2) false D. All are False
74. If S is a finite commutative ring with 1, then
- A. each prime ideal is a maximal ideal.
 B. S may have a prime ideal which is not maximal.
 C. S has no non-trivial maximal ideal.
 D. S is a field.
75. There exists zero divisors in the ring of
- A. integers modulo a prime p .
 B. matrices of order p .
 C. polynomial and the field of characteristics.
 D. entire functions

76. Let $\mathbb{Z}_{10}$ denote the ring of integer modulo 10. Then the number of ideals in $\mathbb{Z}_{10}$ is
 A. 2 B. 3 C. 4 D. 5
77. $A = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ is a left Ideal of $\mathfrak{R} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$ but not right ideal. Prove.
78. Let the polynomial $x^2 + 5$ is
 A. irreducible over $\mathbb{C}$ B. irreducible over $\mathbb{R}$
 C. irreducible over $\mathbb{Q}$ D. not irreducible over $\mathbb{Q}$
79. If p is a prime number and $\mathbb{Z}_{p^4}$ denote the ring of integer mod p^4 , then the number of maximal ideal in $\mathbb{Z}_{p^4}$ is
 A. 1 B. 2 C. 3 D. 4
80. Let $\mathcal{R}$ be the commutative PID with $1 \neq 0$. Let P be the non-zero prime ideal of $\mathcal{R}$. Show that P is a maximal ideal of $\mathcal{R}$.
81. The order of the element $(\bar{2}, \bar{2}) \in \mathbb{Z}_4 \times \mathbb{Z}_6$ is
 A. 2 B. 6 C. 4 D. 12
82. Let $(m, n) = 1$. Then Kernel of the ring homomorphism $\phi : \mathbb{Z} \rightarrow \mathbb{Z}_m \times \mathbb{Z}_n$ defined by $\phi(x) = (\bar{x}, \bar{x})$ is
 A. $m\mathbb{Z}$ B. $mn\mathbb{Z}$ C. $n\mathbb{Z}$ D. $\mathbb{Z}$
83. Set of multiples of 4 form an ideal $(\mathbb{Z}, +, \bullet)$. this ideal is
 A. prime ideal but not maximal. B. maximal but not prime ideal.
 C. both prime and maximal ideal D. neither prime ideal nor maximal ideal.
84. Consider $(\mathcal{C}[0, 1], +, \bullet)$, point wise addition and multiplication. Then $\mathcal{C}[0, 1]$ is
 A. a group but not ring.
 B. a ring but not an Integral domain.
 C. a field.
 D. an integral domain but not a field.
85. Let $S = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ be the ring under multiplication and addition. Then the subset $\left\{ \begin{pmatrix} 0 & p \\ 0 & 0 \end{pmatrix} : p \in \mathbb{R} \right\}$ is
 A. not an ideal of S .
 B. an ideal but not prime ideal of S .
 C. is a prime ideal but not maximal ideal of S .
 D. is a maximal of S .
86. Consider $S = \mathbb{C}[x^5]$, complex polynomials in x^5 as a subset of $T = \mathbb{C}[x]$, ring of all complex polynomials. Then
 A. S is an ideal, but not subring of T .
 B. S is neither ideal nor subring of T .
 C. S is a subring, but not ideal of T .
 D. S is both subring and ideal of T .
87. Which of the following is true about $\mathbb{Z}[x] = S$?
 A. S is Euclidean domain since all its ideal are principal.
 B. S is since $\mathbb{Z}$ is Euclidean domain.
 C. S is not Euclidean domain since S is not even Integral domain.
 D. S is not an Euclidean domain since it has non-principal ideal.
88. Let $\frac{\mathbb{Z}}{n\mathbb{Z}}$ denote the ring of integers modulo n under addition and multiplication. Then $\frac{\mathbb{Z}}{9\mathbb{Z}}$ is not a subring of $\frac{\mathbb{Z}}{12\mathbb{Z}}$ since
 A. $\frac{\mathbb{Z}}{9\mathbb{Z}} \not\subseteq \frac{\mathbb{Z}}{12\mathbb{Z}}$. B. $\text{GCD}(9, 12) = 3 \neq 1$.
 C. 12 is not a power of 3. D. $9 \nmid 12$

89. The number of homomorphisms $\phi : \mathbb{Z}_5 \rightarrow \mathbb{Z}_{20}$ is (as rings)
 A. 1 B. 2 C. 4 D. 5
90. Match the following in an Integral domain.
 U : The only idempotent element(s) (a) 0
 V : The only idempotent element(s) (b) 1
 W : The only idempotent and unit element(s) (c) 0, 1
 A. U -a V -b W -c.
 B. U -b V -c W -a.
 C. U -c V -a W -b.
 D. U -a V -c W -b.
91. Let $(\mathbb{Z}, +, \bullet)$ be the ring of integers. Then every non-trivial ring homomorphism $f : \mathbb{Z} \rightarrow \mathbb{Z}$ is
 A. both $1 - 1$ and onto B. $1 - 1$ but not onto
 C. onto but not $1 - 1$ D. neither $1 - 1$ nor onto
92. Let $M_3(\mathbb{R})$ be the ring of all 3×3 . If $I, J \subseteq M_3(\mathbb{R})$ are defined as
 $I = \left\{ \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ and $J = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ then
 A. I is a right ideal and J is a left ideal.
 B. I and J are both left ideals.
 C. I and J are both right ideals.
 D. I is left and J is right.
93. Let $\mathcal{R} = \{\alpha_0 + \alpha_1 i + \alpha_2 j + \alpha_3 k : \alpha_i \in \mathbb{Z}_3\}$ be **the ring of quaternion** over $\mathbb{Z}_3$. Then
 A. $\mathcal{R}$ is a field. B. $\mathcal{R}$ is a division ring.
 C. $\mathcal{R}$ has zero divisor D. None.
94. Let $\mathcal{R}[x]$ be the polynomial ring in x with real coefficients and let $I = \langle x^2 + 1 \rangle$ be the ideal generated by the polynomial $x^2 + 1$. Then
 A. I is a maximal ideal. B. I is a prime ideal but not maximal.
 C. I is not a prime ideal. D. $\frac{\mathcal{R}[x]}{I}$ has zero divisors.
95. Consider $\mathbb{Q}$ and $\mathbb{Z}_2$ as fields. Let $f(x) = x^3 - 9x^2 + 9x + 3$. Then $f(x)$ is
 A. irreducible over $\mathbb{Q}$ but reducible over $\mathbb{Z}_2$.
 B. irreducible over both $\mathbb{Q}$ and $\mathbb{Z}_2$.
 C. reducible over $\mathbb{Q}$ but irreducible over $\mathbb{Z}_2$
 D. reducible over both $\mathbb{Q}$ and $\mathbb{Z}_2$.
96. In $\mathbb{Z}_5$, let $f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$. Then the zero of $f(x)$ over $\mathbb{Z}_5$ are 1 and 3 with respective multiplicity
 A. 1 and 4 B. 2 and 3 C. 2 and 2 D. 1 and 2
97. Any maximal ideal commutative ring $F[x]$ of polynomial over a field F is the principal ideal generated by an irreducible polynomial . Prove.
98. A finite dimensional inner product space always has
 A. an orthogonal basis, but may not have orthonormal basis
 B. an orthonormal basis
 C. a basis, but may not have an orthogonal basis
 D. a generating set, but no basis
99. Let F, K be fields and suppose there exists a non zero homomorphism $f : F \rightarrow K$. Then F must be
 A. a field isomorphic to a subfield of K B. an infinite field is isomorphic to a subfield of K
 C. a finite field is isomorphic to a subfield of K D. a field isomorphic to K
100. Which of the following is true?
 A. There exists a field of order 36, but not of order 49

- B. There exists a field of order 36 and a field of order 49
 C. There neither exist a field of order 36 nor a field of order 49
 D. There does not exist a field of order 36, but there exists a field of order 49
101. Vector space V properly contains subspaces W and T . Dimensions of W and T are 3 and 5. What is the minimum possible dimension of V ?
 A. 5 B. 6 C. 8 D. 15
102. Let I be the ideal generated by $x^4 + x^3 + x^2 + x + 1$ in $\mathbb{Z}_2[x]$ and $F = \frac{\mathbb{Z}_2[x]}{I}$. Then
 A. F is an infinite field.
 B. F is a Finite field of 4 elements.
 C. F is a Finite field of 8 elements.
 D. F is a Finite field of 16 elements.

Linear Algebra

1. Let V be the vector space of 2×2 matrices over $\mathbb{R}$. Which of the following **are not** linear transformations? (LA)
 (I) $T([a_{ij}]) = a_{11}$ (II) $T(A) = A + I$ (III) $T(A) = \text{Trace} A$ (IV)
 $T([a_{ij}]) = \sum a_{ii}$
 A. only (I) and (II) B. only (II) C. only (III) D. only (IV)
2. Let S be the solution space of a set of an homogeneous linear equations with real coefficients in n unknowns. If A is the matrix of this system of equations, then (LA)
 A. dimension $S = n - \text{rank } A$ B. dimension S is always n
 C. dimension S is infinite D. dimension $S = n + \text{rank } A$
3. Let A be an $n \times n$ matrix over $\mathbb{R}$. Consider the following statements: (LA)
 (I) $\text{rank } A = n$. (II) $\det A \neq 0$.
 A. I implies II, but II does not imply I
 B. II implies I, but I does not imply II
 C. $I \Leftrightarrow II$
 D. There is no relation between the two statements.
4. If the characteristic roots of $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$ are λ_1 and λ_2 , then the characteristic roots of $\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$ are (LA)
 A. $\lambda_1 + \lambda_2, \lambda_1 - \lambda_2$ B. λ_1 and λ_2
 C. $1/\lambda_1$ and $1/\lambda_2$ D. None of the above.
5. Let V be the vector space of real polynomial of degree not exceeding 2. Let $f(x) = x - 1, g(x) = x + 1, h(x) = x^2 - 1, j(x) = x^2 + 1$. Then the set $\{f, g, h, j\}$ is (LA)
 A. linearly independent
 B. linearly dependent because $fg = h$
 C. linearly dependent because $f + g + h = 0$
 D. linearly dependent because $f - g - h + j = 0$
6. If A, B are two $n \times n$ matrices over $\mathbb{R}$ and $\alpha \neq 0$, a scalar, then (LA)
 A. $\det(\alpha A + B) = \alpha \det A + \det B$ B. $\det(\alpha A - B) = \alpha \det A - \det B$
 C. $\det(\alpha AB) = \alpha \det A \det B$ D. $\det(\alpha AB) = \alpha^n \det A \det B$
7. If $T : V \rightarrow V$ is a linear operator with $\dim V = n$ and T has n distinct eigenvalues, then (LA)
 A. T must be invertible. B. T must be diagonalizable.
 C. T must be invertible as well as diagonalizable D. T is not diagonalizable.
8. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation on V defined by $Y(x, y) = (x + y, y)$. Then the characteristic polynomial of T is (LA)
 A. $1 - 3x + x^2$ B. $2 - 2x$ C. $1 - 2x + x^2$ D. $1 + x^2$

9. Let W be the subspace spanned by $\mathbb{R}^2$ spanned by $(1, 2)$. Which of the following pair represent the same elements of the quotient space $\mathbb{R}^2/W$? (LA)
 A. $W, (1/2, 3) + W$ B. $(1, 1/2) + W, (6, 21/2) + W$
 C. $(1, 1/2) + W, (2, 3/2) + W$ D. $(\sqrt{2}, 1) + W, (\sqrt{2}, 2) + W$
10. Suppose a, b, c, d are linearly independent vectors of the real vector space. Consider the two set of vectors $S_1 = \{a + b, a + c, a + d\}$, $S_2 = \{a + b, b + c, c + d, a + d\}$. Which of the following is true? (LA)
 A. Both S_1 and S_2 are linearly independent.
 B. S_1 is linearly independent but not S_2 .
 C. S_2 is linearly independent but not S_1 .
 D. Neither S_1 nor S_2 is linearly independent.
11. Which of the is not a subspace of $\mathbb{R}^3$? (LA)
 A. $\{(a, b, c) : a, b, c \text{ rational}\}$ B. $\{(0, 0, 0)\}$
 C. $\{(a, a + b, a + 2b) : a, b \text{ real}\}$ D. $\{(a, a - b, b) : a, b \text{ real}\}$.
12. Let A be a 3×3 matrix over $\mathbb{R}$. Suppose $a, b, a \neq b$ are the only eigenvalues of A in $\mathbb{R}$. A is diagonal over $\mathbb{R}$ iff the minimum polynomial of A (LA)
 A. equal to the characteristic polynomial of A B. $(x - a)(x - b)$
 C. $(x - a)^2$ D. $(x - b)^2$
13. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ be the linear transformation by $T(w, x, y, z) = (2w - x, 3y - 4z)$. By fundamental theorem $\mathbb{R}^4/W$ is isomorphic to $\mathbb{R}^2$, where W is the subspace generated by (LA)
 A. $\{(1, -2, 0, 0), (0, 0, 4, -3)\}$ B. $\{(1, -2, 0, 0), (0, 0, 4, 3)\}$
 C. $\{(1, 2, 0, 0), (0, 0, 4, -3)\}$ D. $\{(1, 2, 0, 0), (4, 3, 0, 0)\}$
14. A is a 5×5 complex matrix whose characteristic polynomial is $(x + 1)^5$ and whose minimal polynomial is $(x + 1)^3$. Then (LA)
 A. There is a unique Jordan canonical form for A .
 B. There are two possible Jordan canonical forms for A .
 C. There are three possible Jordan canonical forms for A .
 D. A does not have a Jordan canonical form.
15. Let V, W be the subspaces of $\mathbb{R}^n$ with $V \cap W = \{0\}$. Let B_1, B_2 be the bases of V and W respectively. Then, (LA)
 A. $B_1 \cup B_2$ is a basis of $V + W$.
 B. $\{x + y : x \in B_1, y \in B_2\}$ is a basis of $V + W$.
 C. $\{x - y : x \in B_1, y \in B_2\}$ is a basis of $V + W$.
 D. $V + W$ has no finite basis.
16. Let V be the space of $n \times n$ matrices over $\mathbb{R}$ and W be the subspace of symmetric matrices. Then the dimension of W is (LA)
 A. $\frac{n^2}{2}$ B. $\frac{n(n-1)}{2}$ C. $\frac{n(n+1)}{2}$ D. $\frac{n}{2}$
17. Let T, U be linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. Then, (LA)
 A. $U \circ T = 0 \Rightarrow T = 0$ B. $U \circ T$ is one-one $\Rightarrow T$ is one-one
 C. T is one-one $\Rightarrow U \circ T$ is one-one D. T is onto $\Rightarrow U \circ T$ is onto
18. Suppose A and B are 2×2 matrices with same trace and same determinant. Then, (LA)
 A. A must be equal to B .
 B. There must exist an invertible matrix P such that $P^{-1}AP = B$.
 C. A^2 must be equal to B^2 D. A and B have the same characteristic polynomial.
19. Consider the set of equations: (LA)

$$\begin{aligned}x + y + z &= 2 \\2x + 2y + 2z &= 4 \\3x + 3y + kz &= 5\end{aligned}$$

- A. For any $k \neq 3$, there are at least two linearly independent solutions.
 B. For any $k \neq 3$, there is a unique solution.
 C. If $k = 3$, the space of solutions is 1-dimensional.
 D. If $k = 3$, there is a unique solution.
20. Suppose A is a 2×2 matrix such that $A^{10} = 0$. Then, (LA)
 A. A must be zero B. The rank of A must be 1
 C. The rank A must be 0 D. $A^2 = 0$
21. Let f be a quadratic form on $\mathbb{R}^n$ and $x \in \mathbb{R}^n$, $a \in \mathbb{R}$. Then $f(ax) =$ (LA)
 A. $a^2 f(x)$ B. $af(x)$ C. $-af(x)$ D. $|a|f(x)$
22. U is a subset of $\mathbb{R}^3$ given by $U = \{(x, y, z) : x + y = -y + z = 0\}$, then (LA)
 A. U is a subspace of $\mathbb{R}^3$ of dimension 1 B. U is a subspace of $\mathbb{R}^3$ of dimension 2
 C. U is a subspace of $\mathbb{R}^3$ of dimension 3 D. U is not a subspace of $\mathbb{R}^3$
23. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the function defined as $T(x, y, z) = (x + y, y + z)$. Then (LA)
 A. T is not linear
 B. T is linear and the corresponding matrix referred to the standard basis is $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
 C. T is linear and the corresponding matrix referred to the standard basis is $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$
 D. the corresponding matrix referred to the standard basis is $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$
24. A basis for the eigenspace of the matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ consists of (LA)
 A. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ B. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ C. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ D. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
25. The system of equation $x + 2y - 3z = 4$ $3x - y + 5z = 2$ $4x + y + (a^2 - 14)z = a + 2$ (LA)
 has an infinite number of solutions, if a is equals to
 A. 0 B. 1 C. 4 D. -4
26. The eigenvalues of the matrix of the quadratic form associated with the quadratic equation $5x^2 - 4xy + 8y^2 = 36$ are (LA)
 A. 1,2 B. 2,3 C. 4,9 D. -4,9
27. A and B are 3×3 matrices and $AB = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix} \neq BA$. (LA)
 Which of the following is true?
 A. both A and B have inverses B. neither A nor B can have an inverse
 C. A^{-1} may exist but B^{-1} cannot exist D. B^{-1} may exist but A^{-1} cannot exist
28. A linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}$ is non-null, i.e., $Tx \neq 0, \forall x \in \mathbb{R}^n$. The dimension of the null-space of T is (LA)
 A. 0 B. 1 C. $n - 1$ D. n
29. If the matrix representation of a linear transformation on a vector space has the Jordan canonical form consisting of exactly k nontrivial matrices arranged along the main diagonal, then k is (LA)

- A. the number of eigen vectors
C. the number of invariant subspaces
- B. the number of minimal invariant subspaces
D. the number of distinct eigen values
30. If all the characteristic roots of a matrix A in $M_n(\mathbb{R})$ are zero, then (LA)
A. there is a $B \in M_n(\mathbb{R})$ such that $AB = I \neq BA$
B. A is a permutation matrix
C. $A^n = 0$
D. $\det A = 1$
31. Let U, V, W be three real vector spaces such that $W \subset V \subset U$. Let $\dim(U/W/V/W) = r$. If $\dim U = m$, $\dim V = n$, $\dim W = p$, then (LA)
A. $r = n - p$, $(U/W/V/W) \cong U/W$
B. $r = m - p$, $(U/W/V/W) \cong U/V$
C. $r = m - n$, $(U/W/V/W) \cong U/V$
D. $r = m - n$, $(U/W/V/W) \cong V/W$
32. If V is an inner product space and $T : V \rightarrow V$ is a linear transformation with adjoint T^* . Then T is unitary if (LA)
A. $TT^* = I$
B. $TT^* = T^*T$
C. $T = T^*$
D. $T + T^* = 0$
33. If V is an inner product space and $u, v \in V$, then $\langle v + v, iv \rangle =$ (LA)
A. $\|v\|^2 + i \langle u, v \rangle$
B. $\|u\|^2 + i \langle u, v \rangle$
C. $\|v\|^2 - i \langle u, v \rangle$
D. $\|u\|^2 - i \langle u, v \rangle$
34. Let A be an $n \times n$ matrix over $\mathbb{R}$ such that $A^3 = A$. Then (LA)
A. A must be the identity matrix
B. A^2 must be the identity matrix
C. A must be a null matrix
D. A must be diagonalizable
35. If V^* is the dual space of the vector space V and $\dim V = n$. Then $\dim V^* =$ (LA)
A. 1
B. n
C. n^2
D. $n - 1$
36. If $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined by $T(x, y, z) = (0, x, y)$, then the nullity of T^2 is (LA)
A. 0
B. 1
C. 2
D. 3
37. Let U and V be vector spaces over K of dimension m and dimension n respectively. If $T : U \rightarrow V$ is a linear transformation of rank r , then the dimension of the Kernel T is (LA)
A. $m - r$
B. $n - r$
C. $m + n - r$
D. $mn - r$
38. Let U and V be the vector spaces over a field K , $\dim U = m = \dim V$ and $T : U \rightarrow V$ be a linear transformation. Then (LA)
A. T must be both one-one and onto
B. T is one-one iff T is onto
C. T must be one-one, but not onto
D. T must be onto, but not one-one.
39. The vectors $(1, 0, 1), (1, 1, 0), (a, b, c)$ of $\mathbb{R}^3$ are linearly dependent if and only if (LA)
A. $a = 0$
B. $a = b = c$
C. $a = b - c$
D. $a = b + c$
40. The set of all 2×2 matrices of the form $\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix}$, $a, b \in \mathbb{R}$ with usual matrix addition and scalar multiplications (LA)
A. forms a vector space
B. forms a vector space only when $a \geq 0, b \leq 0$
C. forms a vector space only when $a \leq 0, b \geq 0$
D. does not form a vector space
41. A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is given by $T(1, 0, 0) = (1, 2, 0)$, $T(0, 1, 0) = (1, 0, 2)$, $T(0, 0, 1) = (0, 1, 2)$. Then the **kernel** of T (LA)
A. has dimension 2
B. has dimension 1
C. has dimension 0
D. is generated by $(1, 1, -1)$
42. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Then (LA)
A. A is diagonalizable over $\mathbb{R}$
B. A is not diagonalizable over $\mathbb{C}$
C. A is diagonalizable over $\mathbb{R}$, not over $\mathbb{C}$
D. A is diagonalizable over $\mathbb{C}$, not over $\mathbb{R}$

43. How many linear transformations $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ satisfy $T(1, 1, 0) = 7$, $T(0, 1, 1) = -1$, $T(1, 0, 1) = -2$ and $T(2, 3, 5) = 7$? (LA)
 A. infinitely many B. Two C. One D. Zero
44. Let A be an $n \times n$ nilpotent matrix over a field K . Then $A + I$ (LA)
 A. must be nilpotent B. must be a zero matrix
 C. must be invertible D. must have at least one eigenvalue which is 0
45. For $(x_1, y_1, z_1), (x_2, y_2, z_2) \in \mathbb{R}^3$, define $\langle (x_1, y_1, z_1), (x_2, y_2, z_2) \rangle = ax_1x_2 + by_1y_2 + cz_1z_2$, where a, b, c are real numbers. Then $\langle, \rangle$ is an inner product on $\mathbb{R}^3$ if (LA)
 A. a, b, c are all strictly positive B. a, b, c are all non-negative
 C. $abc > 0$ D. $abc \geq 0$
46. If W is a 3-dimensional subspace of a finite dimensional vector space V . If the quotient space V/W has dimension 8, the dimensional of V is (LA)
 A. 3 B. 8 C. 11 D. 5
47. Which of the following sets of functions is linearly dependent in the vector space $\mathcal{C}[0, 1]$ of real continuous functions over $[0, 1]$? (LFA)
 A. $\{1, x, x^2 + 1\}$ B. $\{\sqrt{2}, x^2, x^3, 3x^2 + \sqrt{3}\}$
 C. $\{x + 1, x^2 + 1\}$ D. $\{x^2 + 1, x^2 + 5\}$
48. Let V be the space of all polynomials of degree not exceeding 3 over reals. Let $\mathcal{T}$ be the linear transformation from V to V defined by $\mathcal{T}(f(x)) = f'(x)$, where f' is the derivative of f . The matrix representation of $\mathcal{T}$ corresponding to the basis $1, x, x^2, x^3$ of V is (LA)
 A. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ B. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$ C. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
49. T_1 and T_2 are linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$ and λ is a common eigen value of T_1 and T_2 . Then λ will be an eigen value of (LA)
 A. $T_1 \circ T_2$ B. $T_1 + T_2$ C. $2T_1$ D. $T_1 - T_2$
50. Let V and W be subspaces of $\mathbb{R}^n$. Then (LA)
 A. $\dim(V + W) = \dim(V) + \dim(W)$ B. $\dim(V + W) > \dim(V) + \dim(W)$
 C. $\dim(V + W) < \dim(V) + \dim(W)$ D. $\dim(V + W) \geq \max\{\dim(V), \dim(W)\}$
51. Let A, B, C be an $m \times n$ matrices. Then $\text{rank} \begin{pmatrix} A & B \\ C & O \end{pmatrix}$ is (LA)
 A. equal to $\text{rank } A + \text{rank } B + \text{rank } C$
 B. equal to $\text{rank } A + \text{rank } B - \text{rank } C$
 C. equal to $\text{rank } A + \text{rank } C - \text{rank } B$
 D. at least $\text{rank } A + \text{rank } C$
52. Let A be an $m \times n$ matrix, $b \in \mathbb{R}^m$, then the system of equations $AX = b$ has a solution if and only if (LA)
 A. $b \in \text{column space of } A$ B. $b = 0$
 C. $b \in \text{row space of } A$ D. $\text{rank}[A|b] > \text{rank } A$
53. If A is a 3×3 matrix over R and $a, b, c \neq 0$, are the only characteristic roots of A in $\mathbb{R}$, then the characteristic polynomial of A is (LA)
 A. $(x - a)(x - b)$ B. $(x - a)(x - b)(x - c)$ for some $c \neq a, b$
 C. $(x - a)^3 + (x - b)^3$ D. $(x - a)^2(x - b)$ or $(x - a)(x - b)^2$

54. For a 3×3 real numbers, let $C(A)$ denote the set of the real characteristic roots of A . Suppose $C(B) = C(B^{-1})$ for some non-singular matrix B with no repeated eigen values, then (LA)
 A. either 1 or -1 must be an eigen value of B
 B. B^2 must be I
 C. 1 and -1 are the only possible real eigen values of B
 D. 1 must be an eigen value of B
55. Let R be the set of matrices $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ with real entries under usual matrix multiplication and matrix addition. Then R is (LA)
 A. a field
 B. a commutative ring but not an integral domain
 C. a non-commutative ring
 D. an integral domain, but not a field
56. A is a 3×3 matrix, whose minimal polynomial as well characteristic polynomial are equal to $(x - a)^2(x - b)$, $a \neq b$. Then the Jordan canonical form of A is (LA)
 A. $\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}$ B. $\begin{pmatrix} a & 0 & 0 \\ 1 & a & 0 \\ 0 & 1 & b \end{pmatrix}$ C. $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 1 & b \end{pmatrix}$ D. $\begin{pmatrix} a & 0 & 0 \\ 1 & a & 0 \\ 0 & 0 & b \end{pmatrix}$
57. Let A be a 5×5 complex matrix of finite order i.e., there exists a positive integer n such that $A^n = I$, then (LA)
 A. there exists a non-singular matrix P such that $P^{-1}AP$ is diagonal
 B. A must have all distinct eigen values
 C. The characteristic polynomial of A must be $x^5 = 1$
 D. A must be a diagonal matrix.
58. Let $V = \mathbb{R}^4$ and let W be the subspace generated by $(1, 0, 0, 0), (1, 0, 1, 0)$, then the cosets (LA)
 A. $(1, 1, 1, 0) + W$ and $(0, 1, 0, 0) + W$ form a basis of V/W
 B. $(0, 1, 0, 0) + W$ and $(2, 1, 1, 0) + W$ form a basis of V/W
 C. $(0, 1, 0, 1) + W$ and $(2, 1, 1, 1) + W$ form a basis of V/W
 D. $(1, 1, 0, 0) + W$ and $(0, 0, 1, 1) + W$ form a basis of V/W
59. Let $M_1 = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, $M_2 = \begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix}$, $M_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. (LA)
 Then the minimal polynomial of the matrix $\begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$ is
 A. $x^2(x - 3)^2(x + 3)^2$
 B. $x^2(x^2 - 9)$
 C. $x^2(x^2 - 9^2)$
 D. $x(x - 3)^2(x + 3)^2$

Real Analysis

1. $\lim_{n \rightarrow \infty} \sum_{k=1}^{\infty} \frac{1}{k(k+1)} =$ (RA)
 A. 0 B. 1 C. $\log_e 2$ D. ∞
2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 \sin(\frac{1}{x})$, $x \neq 0$, $f(0) = 0$. (RA)
 A. f is differentiable everywhere with continuous derivative.
 B. f is differentiable everywhere other than the point 0.
 C. f is nowhere differentiable.
 D. f is differentiable but its derivative is not continuous.
3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = [x^2]$. The points of discontinuity of f are (RA)
 A. only the integral points B. all rational points
 C. $\{\pm\sqrt{n} : n \text{ is a nonnegative integer}\}$ D. all real numbers.

4. The function $f(x) = |x| + 3$ is (RA)
 A. continuous as well as differentiable on $\mathbb{R}$.
 B. continuous on $\mathbb{R}$ but not differentiable anywhere on $\mathbb{R}$.
 C. continuous of $\mathbb{R}$ nut not differentiable anywhere on $\mathbb{R}$.
 D. not continuous on $\mathbb{R}$.
5. Let $\{a_n\}$ be a sequence of positive numbers. Then, (RA)
 A. $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1 \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$
 B. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1 \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1$
 C. $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1 \Leftrightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$
 D. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ may exist, but $\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$ may not
6. A monotonic sequence (RA)
 A. is always continuous.
 B. is continuous only if it satisfies intermediate value property.
 C. can be nowhere continuous.
 D. can be discontinuous at uncountably many points.
7. Let $f(x) = |\sin \pi x|$, $x \in \mathbb{R}$. Then, (RA)
 A. f is continuous everywhere.
 B. f is continuous everywhere and differentiable nowhere.
 C. f is continuous everywhere and differentiable everywhere except at integral values of x .
 D. f is differentiable everywhere.
8. Let $A = \{m + n\sqrt{n} : m, n \in \mathbb{Z}\}$. Then, (T)
 A. A is dense in $\mathbb{R}$.
 B. A has only countably many limit points in $\mathbb{R}$.
 C. A has no limit point in $\mathbb{R}$.
 D. Only irrational numbers can be limit points of A .
9. The series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is (RA)
 A. Uniformly convergent.
 B. Point wise convergent, but not uniformly convergent.
 C. Convergent nowhere.
 D. Convergent only at points which are not integral multiple of π .
10. Let $a_n = \begin{cases} \frac{1}{n^2} & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$. Then $\sum_{n=1}^{\infty} a_n$ is (RA)
 A. convergent
 B. divergent
 C. oscillatory
 D. none
11. Let $a_n = \begin{cases} 1 + (1/n) & \text{if } n \text{ is even} \\ -1 - (1/n) & \text{if } n \text{ is odd} \end{cases}$. Then (RA)
 A. $\limsup a_n = \liminf a_n = 1$
 B. $\limsup a_n = \liminf a_n = -1$
 C. $\limsup a_n = 1, \liminf a_n = -1$
 D. $\limsup a_n = -1, \liminf a_n = 1$
12. Let f be a continuous real valued function on $[0, 1]$ such that $f(0) = -1$ and $f(1) = 1/2$, then there (RA)
 always exists a $t \in [0, 1]$ such that
 A. $f(t) = -2$
 B. $f(t) = 1$
 C. $f(t) = 3/2$
 D. $f(t) = -1/2$
13. The function $f(x) = \exp(-\frac{1}{x^2})$, $x \neq 0$ and $f(0) = 0$ (RA)
 A. is not differentiable at $x = 0$
 B. $f^{(n)}(0) = 0$ for all n
 C. $f^{(n)}(0) \neq 0$ for all n
 D. $f^{(n)}(0)$ does not exist for some n
14. The function $f(x) = 1/x$, $x \geq 1$ is (RA)
 A. uniformly convergent
 B. not continuous at $x = 1$
 C. not uniformly continuous
 D. not differentiable

15. Let $f : [0, 2] \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/2 & \text{if } x \text{ is rational} \end{cases}$. (RA)
- Let A and B be the upper Riemann integral and the lower Riemann integral respectively of f on $[0, 2]$. Then,
- A. $A = B = 0$ B. $A = 1$ and $B = 0$ C. $A = 1/2$ and $B = 0$
D. $A = B = 1/2$
16. The total variation of the function $\sin x$ on the interval $[0, 2\pi]$ is (RA)
- A. 1 B. 2 C. 2π D. 4
17. If (a_n) is a sequence of real numbers such that $a_n a_{n+1} < 0$ for all $n \in \mathbb{N}$. Then it follows that, (RA)
- A. (a_n) cannot converge.
B. If (a_n) converges, then the limit is 0.
C. (a_n) is unbounded.
D. (a_n) converges to a negative number.
18. If a series $\sum a_n$ of non-ve terms is divergent, it is necessary that (RA)
- A. (a_n) is unbounded B. (na_n) is unbounded
C. $(n^2 a_n)$ is unbounded D. (a_n) is bounded
19. The radius of convergence of the power series $\sum n z^n$ is (RA)
- A. 1 B. ∞ C. 0 D. 2
20. Let $f(x) = x^2 + x$ for all $x \in \mathbb{R}$. It follows that for any two real numbers $a < b$ then, (RA)
- A. f takes a value c between a and b B. $f'(c) = a + b + 1$, for some c between a and b
C. $f'(c) = f(c)$ for some c between a and b D. $f(c) = 0$ for some c between a and b
21. Let $f(x) = \begin{cases} 1 & 0 \leq x < \frac{1}{3} \\ 2 & \frac{1}{3} \leq x < \frac{2}{3} \\ 3 & \frac{2}{3} \leq x < 1 \end{cases}$ (RA)
- Then the Riemann integral of f over $[0, 1]$
- A. does not exist B. is exactly 2 C. is exactly 3 D. a number $c \in (2, 3)$
22. If $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are functions such that $f + g, f - g$ are continuous on $\mathbb{R}$. Then (RA)
- A. f may not be continuous of $\mathbb{R}$ B. g may not be continuous of $\mathbb{R}$
C. one of f and g may not be continuous on $\mathbb{R}$ D. both f and g are continuous on $\mathbb{R}$
23. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ takes rationals to irrationals and irrationals to rationals. Then (RA)
- A. f cannot be continuous B. f cannot have countable range
C. f must be constant D. f must be bounded.
24. Every countable subset of $\mathbb{R}$ (RA)
- A. is bounded B. is unbounded
C. is finite D. has its complement id unbounded.
25. If $a, b \in \mathbb{R}$ and $ca \leq cb$ whatever be $c \in \mathbb{R}$, then (RA)
- A. $a = b$ B. $a < b$ C. $a > b$ D. $a \neq b$
26. If f is defined on $(0, 1)$ by $f(x) = \frac{1}{n}$, $\frac{1}{n+1} < x \leq \frac{1}{n}, n = 1, 2, 3 \dots$, then (RA)
- A. $\lim_{x \rightarrow 0} f(x)$ does not exist B. $\lim_{x \rightarrow 0} f(x) = 0$
C. $\limsup_{x \rightarrow 0} = 1, \liminf_{x \rightarrow 0} = 0$ D. f is continuous of $(0, 1]$
27. In the space $X = [0, 1]$, d is defined by $d(x, y) = \min(x, y), x, y \in X$ is (RA)
- A. a metric on X
B. not a metric on X since $d(x, y) \neq d(y, x)$

- C. not a metric on X since $d(x, z) \leq d(x, y) + d(y, z)$ fails
D. not a metric on X since $d(x, y) \not\geq 0$
28. If $\sum a_n$, $a_n > 0$ is convergent, then (RA)
A. $\sum a_n^2$ is divergent to $+\infty$ B. $\sum a_n^2$ is divergent to $-\infty$
C. $\sum a_n^2$ is convergent D. $\sum a_n^2$ oscillates infinitely
29. Let $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational in } [0, 1] \\ -1 & \text{if } x \text{ is irrational in } [0, 1] \end{cases}$. Then, in $[0, 1]$ (RA)
A. $f(x)$ is continuous everywhere B. $f(x)$ is Riemann integrable
C. $f(x)$ is not Riemann integrable D. $f(x)$ is continuous only at the irrationals
30. Let $f(x) = x^2 \cos(1/x)$, $x > 0$ and $f(0) = 0$. Then in $[0, 1]$ (RA)
A. $f(x)$ is of bounded variation and $f'(x)$ exists
B. $f(x)$ is of bounded variation and $f'(1/2)$ does not exist
C. $f(x)$ is not of bounded variation and $f(x)$ is continuous
D. $f(x)$ is discontinuous at $x = 2/\pi$
31. The set of all rational number p/q where p does not assume the values 3 and -3 is (RA)
A. uncountable B. bounded and finite C. countable D. countable and bounded
32. Let $f(x) = \begin{cases} \sin(\frac{1}{x}) & x > 0 \\ x & x \leq 0 \end{cases}$. Then, (RA)
A. $f(x)$ is continuous everywhere
B. $f(x)$ has a discontinuity of the first kind at $x = 0$
C. $f(x)$ has a discontinuity of the second type at $x = 0$
D. $f(x)$ is monotone
33. A countable set of irrationals which is dense in the set $\mathbb{R}$ of all real numbers is (RA)
A. the set I of all irrational numbers B. $\{\sqrt{p} : p \text{ is prime}\}$
C. $\mathbb{Q} \cup \{\sqrt{p} : p \text{ is prime}\}$ D. $\{\sqrt{2} - q : q \in \mathbb{Q}\}$
34. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be a function. Suppose A is the statement " g is continuous at 0"; B is the statement " $g(x_n) \rightarrow g(0)$, whenever $x_n \rightarrow 0$." Then (RA)
A. A implies B but B does not imply A B. B implies A but A does not imply B
C. A implies B and B implies A D. A does not imply B and B does not imply A
35. Let $x_n = (-1)^n + \frac{1}{n}$, $n = 1, 2, 3, \dots$. Then (RA)
A. $\limsup x_n = 1$ B. $\limsup x_n = -1$ C. $\limsup x_n = \infty$
D. $\limsup x_n = 0$
36. Which of the following statements is correct? (RA)
A. $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ both converge B. $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ both diverge
C. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ converges D. $\sum_{n=1}^{\infty} \frac{1}{n}$ converges, but $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ diverges
37. Let $f(x) = 1 + \sqrt{x}$, $x \geq 0$. Then $E = \{x \geq 0 : f(x) \text{ is an integer}\}$ is a set which is (RA)
A. null B. finite and non-empty
C. countably infinite D. uncountable
38. Let $h(x) = 1 + x^2$, $x \in \mathbb{R}$. Then (RA)
A. h is continuous on $\mathbb{R}$, but not monotonic on $\mathbb{R}$
B. h is monotonic on $\mathbb{R}$, but not continuous on $\mathbb{R}$
C. h is both monotonic and continuous on $\mathbb{R}$
D. h is neither continuous on $\mathbb{R}$ nor monotonic on $\mathbb{R}$

39. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions such that $f + g$ and $f - g$ are continuous. Then (RA)
 A. neither f nor g need not be continuous
 B. both f and g must be continuous
 C. g must be continuous but f may not be
 D. f must be continuous but g may not be
40. Let $f : [0, 1] \rightarrow \mathbb{R}$. Then (RA)
 A. f is of bounded variation $\Rightarrow f$ is increasing on $[0, 1]$
 B. f is bounded on $[0, 1] \Rightarrow f$ is of bounded variation
 C. f is of bounded variation $\Rightarrow f$ is continuous on $[0, 1]$
 D. f is of bounded variation $\Rightarrow f$ is bounded on $[0, 1]$
41. If A is a square matrix and A^T is its transpose. Then $A - A^T$ is a (RA)
 A. symmetric matrix
 B. skew-symmetric matrix
 C. singular matrix
 D. non-singular matrix
42. Let $S = \{(a_1, a_2, a_3, \dots, a_n, \dots) : a_i \in \mathbb{R}, \forall n\}$. S is a vector space over $\mathbb{R}$ with componentwise addition and componentwise scalar multiplication. (FA)
 Let $C_0 = \{(a_1, a_2, a_3, \dots, a_n, \dots) \in S : \lim a_n = 0\}$. Then C_0 is a subspace of S , if
 A. $0 < \alpha < 1$
 B. $\alpha = 0$
 C. $\alpha = 1$
 D. α -any real number
43. $\lim_{n \rightarrow \infty} \frac{(\log n)^4}{n} =$ (RA)
 A. tends to ∞
 B. is 1
 C. is 0
 D. does not exist
44. If $f(x) = 1/(x+1), x \geq 0$ (RA)
 A. continuous on $(0, 1)$, but not uniformly continuous on $(0, 1)$
 B. uniformly continuous on $(0, 1)$, but not on $[0, 1]$
 C. not continuous on $(0, 1)$
 D. uniformly continuous on $[0, 1]$
45. Let $f_n(x) = x^n, x \geq 0$. Then (RA)
 A. $\{f_n\}$ does not converge point wise anywhere
 B. $\{f_n\}$ converges point wise on $[0, 1]$, but not uniformly on $[0, 1]$
 C. $\{f_n\}$ converges uniformly on $[0, 1]$
 D. $\{f_n\}$ converges at $x = 0$ only
46. An example of a which is not uniformly continuous in $(0, 1)$ is (RA)
 A. $f(x) = x^2$
 B. $f(x) = 1/x^2$
 C. $f(x) = \sin x$
 D. $f(x) = e^x$
47. If f is a continuous on $[a, b]$ and $\int_a^b f(x)dx = 0$, then (RA)
 A. there exists on (a, b) such that $f(c) = 0$
 B. $f \equiv 0$ on $[a, b]$
 C. $f(x) = 0$ for finitely many x in $[a, b]$
 D. there exists c in (a, b) such that $f \geq 0$ on $[a, c]$ and $f \leq 0$ on $[c, b]$
48. Which one of the following is true for all functions f from $\mathbb{R}$ to $\mathbb{R}$? (RA)
 A. If f is surjective and increasing, then f is continuous
 B. If f is continuous and surjective, then f is monotonic
 C. If f is continuous and monotonic, then f is surjective
 D. If f is increasing, then f is either continuous or surjective
49. $\bigcup_{n=3}^{\infty} \left[\frac{1}{n}, 1 - \frac{1}{n} \right] =$ (RA)
 A. $(0, 1)$
 B. $(0, 1)$
 C. $[0, 1]$
 D. $[0, 1]$
50. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function and suppose $A = \{x \in \mathbb{R} : x > 0\}$ and $B = \{x \in \mathbb{R} : x \leq 0\}$, then (RA)
 A. f is continuous $\Rightarrow f|_A$ and $f|_B$ are continuous, but the converse is false.

- B. $f|_A$ and $f|_B$ are continuous $\Rightarrow f$ is continuous, but the converse is false.
 C. f is continuous $\Leftrightarrow f|_A$ and $f|_B$ are continuous.
 D. There is no relation between continuity of f on $\mathbb{R}$ and continuity of $f|_A$ and $f|_B$.

51. Let $f_n(x) = \frac{nx}{1+n^2x^2}, x \in \mathbb{R}, n = 0, 1, 2, \dots$, then (RA)

- A. $\{f_n\}$ converges pointwise at $x = 0$ only.
 B. $\{f_n\}$ converges pointwise at precisely non-zero values of x .
 C. $\{f_n\}$ converges pointwise not uniformly.
 D. $\{f_n\}$ converges uniformly.

52. $\lim_{n \rightarrow \infty} [\sqrt{n^2 + 1} - n] =$ (RA)
 A. ∞ B. 1 C. 0 D. does not exist.

53. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \text{distance of } x \text{ from its nearest integer}$, then (RA)
 A. f is continuous everywhere, differentiable nowhere.
 B. f is neither continuous nor differentiable.
 C. f is continuous everywhere, but not differentiable only at the points $\{\frac{n}{2} : n \in \mathbb{Z}\}$.
 D. f is continuous everywhere and differentiable everywhere.

54. If f is differentiable and $f(a) = f(b) = 0$ for $a < b$, then (RA)
 A. $f\left(\frac{a+b}{2}\right)$ B. $f'(\xi) = 0$ for some $\xi, a < \xi < b$
 C. $f(x) = 0$ for all $x \in [a, b]$ D. $f(x) = 0$ for some $x \in [a, b]$

55. Let a, b, c be positive real numbers. The minimum value that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ can assume is (RA)
 A. 6 B. 3 C. 1/2 D. 4

56. Let $(a_n)_{n=1}^{\infty}$ be the sequence defined by (RA)

$$a_n = \begin{cases} (-1)^k \left(k + \frac{1}{k}\right) & \text{if } n = 2^k \text{ for some positive integer } k \\ 0 & \text{otherwise} \end{cases}$$

then $\limsup_{n \rightarrow \infty} a_n$ is

- A. 1 B. -1 C. 0 D. e

57. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x & \text{if } x \text{ is a rational number} \\ -x & \text{if } x \text{ is an irrational number} \end{cases}$ (RA)
 A. f is continuous nowhere B. f is continuous exactly at one point.
 C. f is continuous everywhere D. f is continuous only at the irrational points

58. The equation $\sin x = \frac{x}{2}, x \in \mathbb{R}$ has (RA)
 A. No roots in $[-2, 2]$ B. Exactly one root in $[-2, 2]$ and some root outside $[-2, 2]$
 C. All roots in $[-2, 2]$ D. Infinitely many roots in $[-2, 2]$

59. Let $A = \{x \in \mathbb{R} : x^2 < 2\}$. (RA)
 A. This set is bounded above but not bounded below
 B. This set is bounded above and below
 C. This set is neither bounded above nor bounded below
 D. This set is bounded below but not bounded above

60. A function of bounded variation (RA)
 A. is necessarily continuous everywhere
 B. can have at most countably many points of discontinuity
 C. can have uncountably many points of discontinuity
 D. can never be continuous

61. Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers satisfying $a_{n+1}^2 = 2a_n - 1$, then (RA)
 A. $\lim_{n \rightarrow \infty} a_n = 2$ B. $\lim_{n \rightarrow \infty} a_n = 1$ C. $\lim_{n \rightarrow \infty}$ does not exist D. (a_n) oscillates boundedly
62. If f is a differentiable function on the real line such that $f(0) = 5$ and $f(x) < 5$ for $x \neq 0$, then (RA)
 A. $f'(0) = 5$ B. $f'(0) = -5$ C. $f'(0) = 0$ D. $f'(0) = 25$
63. The series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ is (RA)
 A. converges uniformly on $[-2, 2]$ B. converges uniformly on $[-1, 1]$
 C. diverges everywhere D. not uniformly convergent in any interval of the real line
64. The limit $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n^2}$ exists and is the Riemann integral of (RA)
 A. $f(x) = x^2, 0 \leq x \leq 1$ B. $f(x) = k, 0 \leq x \leq 1$
 C. $f(x) = 3x, 0 \leq x \leq 1/3$ D. $f(x) = x, 0 \leq x \leq 1$
65. Let (a_n) be a sequence of real numbers such that $\liminf a_n = 0$ and $\limsup a_n = 1$, then (RA)
 A. all the terms should lie between 0 and 1
 B. there must be infinitely many terms between 0 and 1
 C. there are only finitely many terms outside $[-1, 2]$
 D. there are only finitely many terms > 0
66. Let $\mathbb{Q}$ be the set of rational numbers. Then (RA)
 A. there is no one-one function from $[0, 1]$ to $\mathbb{Q}$
 B. there is no onto function from $[0, 1]$ to $\mathbb{Q}$
 C. there is no one-one function from $\mathbb{Q}$ to $[0, 1]$
 D. there is a bijection between $\mathbb{Q}$ and $[0, 1]$
67. Let $f : [0, 1] \rightarrow \mathbb{R}$ be a discontinuous monotonic function, then the range of f is (RA)
 A. a bounded set that is not an interval
 B. a bounded interval
 C. an unbounded connected set
 D. a set that is neither bounded nor connected

Complex Analysis

1. Let $A = \{z \in \mathbb{C} : |z - 2| + |z + 1| \geq 3\}$. Then (CA)
 A. A is a closed and bounded set of $\mathbb{C}$
 B. A is unbounded, proper subset of $\mathbb{C}$
 C. $A = \mathbb{C}$
 D. A is a bounded set which is not closed.
2. Let $E = \{z \in \mathbb{C} : |z| \geq 1\} \cup \{i\}$. Then, (CA,T)
 A. E is an open subset (of $\mathbb{C}$), which is not closed.
 B. E is closed subset (of $\mathbb{C}$), which is not open.
 C. E is both open and closed.
 D. E is neither open nor closed.
3. The radius of convergence of the power series $1 + az + \frac{z^2}{a^2} + a^3 z^3 + \frac{z^4}{a^4} \cdots$, where $|a| < 1, a \neq 0$ is (CA)
 A. $\frac{1}{|a|}$ B. $|a|$ C. 0 D. ∞
4. The transformation $z \rightarrow \frac{1}{z}$ maps a straight line passing through the origin into (CA)
 A. a straight line not passing the origin.
 B. a straight line passing through the origin.
 C. a circle not passing through the origin.
 D. a circle passing through the origin.

5. if $f : D \rightarrow D$, where D is the open disc with radius 2 and if f is analytic over D , then (CA)
 A. $f'(0) = 3$ B. $|f'(0)| > 3$ C. $|f'(0)| \leq 2$ D. $f'(0)$ does not exist.
6. If f is an entire function such that $f(iy) = \exp(iy)$, where $0 \leq y \leq 1$, then (CA)
 A. $f(x + iy) = \exp(iy)$, for every x and y . B. $f(x + iy) = \exp(x + iy)$, for every x and y .
 C. $f(x + iy) = \exp(x + iy)$, only for $0 \leq y \leq 1$. D. $f(x + iy) = \exp(iy)$, whenever $0 \leq y \leq 1$.
7. Suppose that f is nonconstant entire function and let $A_r = \max\{|f(z)| : |z| = r\}$. Then the function (CA)
 $r \mapsto A_r$ is
 A. strictly increasing. B. increasing but not strictly.
 C. bounded and increasing. D. decreasing.
8. Let $f(z) = z^n \cos(1/z)$, $z \neq 0$. Then f is (CA)
 A. is entire. B. has removable singularity at $z = 0$.
 C. has a pole at $z = 0$. D. has an essential singularity at $z = 0$.
9. If f is a meromorphic function on $\mathbb{C}$. Then (CA)
 A. poles of f are isolated but no zeros.
 B. zeros of f are isolated, but poles of f need not be isolated.
 C. zeros and poles of f are isolated.
 D. neither poles nor zeros of f are isolated.
10. There is a non constant entire function f such that (CA)
 A. f has zeros at $\{n : n \in \mathbb{N}\}$ B. f has zeros at $\{\frac{1}{n} : n \in \mathbb{N}\}$
 C. f has zeros at $\{\frac{1}{2^n} : n \in \mathbb{N}\}$ D. f has uncountable number of zeros.
11. The real part of the square of a complex number is equal to (CA)
 A. the square of its real part.
 B. the square of its imaginary part.
 C. the sum of the square of the real and imaginary parts.
 D. the difference of the square of the real and imaginary parts.
12. The poles of the function $\frac{\cos z}{z^2 + 1}$ are (CA)
 A. integral multiples of 2π B. 0 only C. i and $-i$ D. ± 1
13. Let r be the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$. Let $b_n = a_{n+2}$ for all n . Then the radius of convergence (CA)
 of $\sum b_n z^n$ is
 A. $r + 2$ B. r C. r^2 D. $r - 2$
14. In the Laurent's series of $\frac{\sin z}{z^2}$ at $z = 0$, the coefficient of $\frac{1}{z}$ is (CA)
 A. 0 B. π C. -1 D. 1
15. If $f(z) = \frac{\sin(z^2)}{z^2}$, then $\int_{|z|=1} f(z) dz =$ (CA)
 A. $1/2$ B. 0 C. 2 D. 4
16. Let $f(z) = \frac{\cos 2\pi z}{z^2 + 1}$ and $\Gamma = \left\{ z : \left| z - \frac{i}{2} \right| = \frac{1}{4} \right\}$. Then $\int_{\Gamma} f(z) dz =$ (CA)
 A. 0 B. πi C. $-\pi i$ D. $2\pi i$
17. If $f(z) = u(x, y) + ixy$ is analytic then (CA)
 A. $u(x, y) = x^2 + y^2$ B. $u(x, y) = x^2 - y^2$
 C. $u(x, y) = \frac{1}{2}(x^2 + y^2)$ D. $u(x, y) = \frac{1}{2}(x^2 - y^2)$
18. An example of a function $f(z)$ such that $\int_c f(z) dz = 1$, where c is the circle $|z| = 1$ is (CA)
 A. $\frac{2\pi i}{z}$ B. $\frac{1}{2\pi i z}$ C. z D. constant function 1.

19. If $a, b \in \mathbb{C}$, $a, b \neq 0$, $az + b\bar{z} = c$, $c \in \mathbb{C} - \{0\}$ is a straight line in the tangent plane iff (CA)
 A. $b = \bar{a}$ and $c = \bar{c}$ B. $\operatorname{Re}(a) = \operatorname{Re}(b)$ C. $b = \bar{a}$ and $\operatorname{Re}(c) = 0$ D. $a, b \neq 0$
20. The map $z \rightarrow |z| : \mathbb{C} \rightarrow \mathbb{C}$ is not linear transformation because (CA)
 A. it does not transform the straight line $z + i\bar{z} = 0$ onto a straight line.
 B. the cross ratio formed by the points 0, 1, 2, 3 is not preserved.
 C. the cross ratio formed by the points 0, 1, 2, ∞ is not preserved
 D. it is continuous on the whole of $\mathbb{C}$
21. If $f(z)$ is analytic in a simply connected domain D and γ is a simple closed curve lying inside D , (CA)
 then for $a \in D$, we have $\frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{z-a} dz =$
 A. $f'(a)$
 B. 0
 C. $f'(a)$ if the winding number of γ with reference to a is 1
 D. 0 if the winding number of γ with reference to a is 0
22. If $A = \begin{pmatrix} a & b+ic \\ b-ic & d \end{pmatrix}$, then $\bar{x}A^T x$, for every choice of $x \in \mathbb{C}^2$ is (CA)
 A. pure imaginary B. complex number C. 0 D. real number
23. If $f(z) = \frac{e^z}{z-1}$, $z \neq 1$, then (CA)
 A. f has a simple pole at $z = 1$ and its residue at $z = 0$ is -1
 B. f has a simple pole at $z = 1$ and $f(z) - e/(z-1)$ is the derivative of an analytic function is a nbd of 1
 C. f has a simple pole at $z = 1$ where its residue is 1
 D. for no complex number c is the function $f(z) - c/(z-1)$ the derivative of an analytic function in any nbd of 1
24. A Möbius transformation which transforms the upper half plane into the lower half plane is (CA)
 A. $w = \bar{z}$ B. $w = \frac{z-i}{z+i}$ C. $w = \frac{1}{z}$ D. $w = \frac{z+i}{z-i}$
25. Let $f(z) = |z|/\operatorname{Re}(z)$ or 0 according as $\operatorname{Re}(z) \neq 0$ or $\operatorname{Re}(z) = 0$. Then, at $z = 0$ the function $f(z)$ is (CA)
 A. having non-zero limit B. differentiable
 C. continuous but not differentiable D. neither continuous nor differentiable
26. The integral $\int_{|z|=2} \frac{\cos z}{z^3} dz =$ (CA)
 A. πi B. $-\pi i$ C. $2\pi i$ D. $-2\pi i$
27. If $f(z) = \frac{1}{e^z - 1}$, $zn \neq 2\pi i, n = 0, 1, 2, 3, \dots$, then (CA)
 A. f has removable singularities at the omitted points
 B. f has essential singularities at the omitted points
 C. f has simple pole at the omitted points
 D. f has double poles at the omitted points at which the residue is 0
28. Let $f(z) = \frac{e^z}{(z-1)(z+3)^2}$ and Γ be the circle $|z| = 3/2$ described in the positive sense. Then (CA)
 $\int_{\Gamma} f(z) dz =$
 A. 0 B. $\pi e i / 8$ C. $-\pi e i / 8$ D. $\pi i (e - 5e^3) / 8$
29. At $z = 2$, the function $\frac{\exp(z-2)}{z-2}$, has (CA)
 A. no singularity B. a simple pole C. a pole of order 2 D. an essential singularity

30. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function such that $f(1/n) = 1/n^2, \forall n \in \mathbb{N}$. Then (CA)
 A. such a function does not exist B. f should be bounded
 C. f has a pole at zero D. $f(z) = z^2$ for all $z \in \mathbb{C}$
31. For a circle in the Riemann sphere passing through the point at infinity N , the image under the stereographic projection is (CA)
 A. a straight line in the complex plane B. a circle in the complex plane
 C. an ellipse in the complex plane D. a line segment in the complex plane
32. The Möbius transformation that fixes the three points 1, 2, and 3 is (CA)
 A. the function $\frac{z-1}{2-3z}$ B. the identity function z
 C. the real part function $\operatorname{Re}(z)$ D. the conjugate function $\bar{z}$
33. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be entire function and let $f(i) \neq f(0)$. It follows that (CA)
 A. f is onto B. The derivative f' is bounded
 C. The range of f is an unbounded set D. f' is constant
34. The radius of convergence of $\sum a_n z^n$ is given as R . Then the radius of convergence of $\sum b_n z^n$, where $b_n = (n+1)a_{n+1}, \forall n$ is (CA)
 A. can be anything between $R/2$ and $2R$ B. at least R , can be sometimes $> R$
 C. at most R , and can be sometimes $< R$ D. exactly equal to R
35. $\int_c \frac{1}{z} dz$, where c is the circle centered at 2 with radius 1 (CA)
 A. has value 0 B. is equal to $2\pi i$ C. is equal to 1
 D. cannot be evaluated since the integral does not exist
36. Let $E = \{z \in \mathbb{C} : |z-2| + |z-1| < 3\}$. Then E describes (CA)
 A. the whole complex plane B. the empty set
 C. interior of the disc D. interior of an ellipse
37. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be defined as $f(z) = \frac{1}{z^2+1}$. Then (CA)
 A. f is discontinuous everywhere
 B. f is continuous everywhere, but differentiable nowhere
 C. f is continuous at every $z \in \{i, -i\}$, but differentiable nowhere
 D. f is differentiable at every $z \notin \{i, -i\}$
38. If $f(z) = \frac{1}{(z-1)^2} + \frac{1}{(z-1)^3}$, the residue of f at $z = 1$ is (CA)
 A. 0 B. 1 C. 2 D. is not defined
39. The function $f(z) = \frac{1}{z-1}$ (CA)
 A. does not have a Taylor expansion in powers of $f(z-2)$
 B. is an entire function
 C. has a Laurent expansion in powers of $(z-1)$ with infinite number of nonzero coefficients
 D. has a Laurent expansion in powers of $(z-1)$ consisting of a single term with non zero coefficient
40. If $f(z)$ has the Laurent expansion $\frac{a_{-2}}{z^2} + \frac{a_{-1}}{z} + a_0 + a_1 z + \dots$ in the unit disc, then the residue f^2 at 0 is (CA)
 A. a_{-1} B. $(a_{-})^2$ C. $2(a_1 a_{-2} + a_{-1} a_0)$ D. $2(a_0 a_{-1} + a_1 a_{-2})$
41. The residue of $\frac{\sin z}{z^4}$ at its singularity is (CA)
 A. 0 B. 1 C. $-1/6$ D. 4
42. If $\gamma(t) = 2 \exp(it)$, where $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, then $\int_{\gamma} z^2 dz$ is (CA)
 A. $-i$ B. 0 C. i D. $1/2$

43. If $f : \mathbb{C} \rightarrow \mathbb{C}$ is analytic and real valued, then (CA)
 A. f is non-constant B. f maps open sets to open sets
 C. f is constant D. $f(z) = z^n$ for some $n > 0$
44. If f is entire and $|f(z)| \leq A|z|^n$ for some positive integer n and $A > 0$, then (CA)
 A. $f(z) = z^n$ B. $f(z) = \exp(nz)$
 C. $f(z) = n \exp(z)$ D. $f(z)$ is a polynomial of degree less than or equal to n
45. If f and g are analytic on the same region D and if $f(z)g(z) = 0$ for all $z \in D$, then (CA)
 A. at least one of f and g vanishes identically on D
 B. f and g both vanish everywhere on D
 C. f vanishes only at finitely many points of D and g never vanishes
 D. f and g vanish only at finitely many points of D
46. Suppose $f(z) = (z-1)(z-2)(z-3)^{-1}$ and $\gamma(t) = 4 \exp(it)$, $0 \leq t \leq 4$, then the argument principle (CA)
 implies that $\int_{\gamma} f'(z)f(z)^{-1} dz$ is
 A. $2\pi i$ B. $-2\pi i$ C. 1 D. 0
47. The residue of $\exp(1/z)$ at $z = 0$ is (CA)
 A. 2 B. ∞ C. 1 D. -2
48. Every non constant Mobius transformation of $\mathbb{C}$ (CA)
 A. has always a pole B. is a conformal map in its domain of definition
 C. is an analytic function of $\mathbb{C}$ D. is a continuous map of $\mathbb{C}$
49. One of the values of i^i , where $i = \sqrt{-1}$, is (CA)
 A. $\exp(\pi/2)$ B. $\exp(-\pi/2)$ C. $\exp(i\pi/2)$ D. $\exp(-i\pi/2)$
50. The function $\frac{\sin z}{z^4}$ has (CA)
 A. pole of order 2 at the origin with residue 1
 B. pole of order 1 at the origin with residue 1
 C. pole of order 1 at the origin with residue 2
 D. an essential singularity at the origin with the residue 1
51. Let $f(z) = \begin{cases} \frac{z}{|z|} & z \neq 0 \\ 0 & z = 0 \end{cases}$ (CA)
 A. f is discontinuous at 0 B. f is not analytic at 0, but it is continuous at 0
 C. f is differentiable at 0 D. f is not differentiable at 0, but it is continuous at 0
52. If $f(z) = e^z \sin(1/z)$, then f has (CA)
 A. no singularities B. an essential singularity at $z = 0$
 C. a removable singularity at $z = 0$ D. a pole of order 1 of at $z = 0$