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HYPERACOUSTICS

BY JOHN L. DUNK



DIVISION II

SUCCESSIVE
TONALITY

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For names of Authorities to be consulted, the reader is directed to the list at the end of Division I, "Simultaneous Tonality," 1916.

It has not been found practicable to extend same up to the date of present publication, but the Author hopes to remedy this in a later edition.

PREFACE

It will be generally admitted that the phenomena of Tonality show certain determinate characteristics which persist in all the varied works of music.

It may be a matter of opinion as to whether this "determinance" is real or apparent, and whether it is amenable to scientific treament.

The only way to decide the question is to write a book about it and ask the public to judge.

The first division of the subject, "SIMULTANEOUS TONALITY," was published in 1916; we now wish to examine the Successive aspect.

It is a difficult and controversial matter to attempt to separate the transient from the enduring factors of modern musical practice, hence the scope of the discussion has to be limited to fairly well-established cases.

Our aim is to sweep over the works of music somewhat as the astronomer scans the sky with his telescope, and as he does not interfere with the feelings evoked by the glories of the heavens, and cannot alter the working of the subject he studies, so our scientific scrutiny, and the methods adopted for its expression, has no concern whatever with music as an art.

It does not aim at any æsthetic views, and it is hoped that the reader will close this book only with an increased desire to become more familiar with the great works of the art.

The book does not teach harmony or technique, plenty of works and teachers abound for that purpose. It attempts a purely scientific scrutiny, avoiding as far as possible the detached dryness of a more rigidly logical treatment.

In order to avoid disappointment it is necessary to limit expectations (which are possibly awakened by works of a similar character) as to prediction, and increased facilities in the art.

There is no royal road to musical knowledge, although many aids exist.

It must be remembered that a theory can only be constructed from observations of persistent experiences of a general nature, hence is limited to retrospect.

Consequently the reader may expect to find that many individual phenomena are not dealt with, and very many questions necessarily left unanswered.

Tonality is one of the factors in music that particularly appeals to the intellect; the hyperacoustic view attempts to be a mirror in which the deepest mental principles of all ages are depicted.

The structural and tectonic development of Tonality strikingly resembles the organic and logical processes of physiology and psychology with which many readers will doubtless be familiar. Indeed, hyperacoustics appears as a microcosm of Universal Representation in the particular medium of sound.

All possible successions of tones come under the two heads of either absolute chance, or determinate.

The latter may be divided into those which appear merely "correct," and those which may be considered as "significant," and though Significance may elude definition it has to be recognised in a scientific scrutiny.

Determinance may be defined as procedure which can be comprehended outside its effect. A certain amount of success has always attended the examination of tonal phenomena from an outside standpoint, in which an appeal to reason takes the place of mere sense acceptance.

Hyperacoustics attempts to fill the gap between Acoustic conditions (which are limited to the Physical, Physiological, and Psychological aspects) and the descriptive experiences of what is agreed upon as music in general.

A language has grown up round the latter, resulting in a very useful, if somewhat loose, empiric theory.

After mature consideration, it has been decided to strike out a somewhat new basis of thought from a purely scientific point of view, in which both non-musician and musician may join in comprehending and judging.

The reason for commencing with the simultaneous aspect is that much has been done in this field under the name of

Harmonic Theory. Possibly the horizontal or melodic view may be considered as the primary aspect, at least from the point of view of history. The chordal or vertical aspect, however, provides much that can be immediately grasped from an acoustic standpoint.

The method of examination is abstraction, with frequent reference to actual conditions.

Any abstraction is admittedly ideal; a complete survey of a subject must finally include all those apparently negligible factors which are laid aside in making the abstraction. This is an obvious necessity in scientific scrutiny.

The process of abstraction is governed by the relative predominance of factors on some common basis of judgment.

Simplicity forms the starting-point, the most simple cases of graded conditions being taken as the zero in a range of percepts and concepts.

The complicating factor in the case of tonality is that tones in practice are not Euclidian points in pitch.

They are subject to a small libration of position, which is known in practice as " tempering." This makes a mathematical treatment somewhat forbidding in appearance.

If we tighten up libration we lose generality, and if we slacken we get complexity, so that an average is necessary to progress.

An important factor in the historical development is the Principle of Synergy, which states that if a number of associated but unconnected factors are treated as a whole, they become coupled up to form an entity of perception and conception. In this way association does something more than give a transient form to tonal procedure.

From this it is obvious that it is not easy to predict the course of development until the artist has struck out and tested a line of advance. We cannot appreciate an entity until it has become synergised by effective employment as a whole.

Thus the empiric theory of music has always followed practice, and has generally failed when it assumes the rôle of dictator or prophet.

The writer has frequently been asked the reason for discussing the subject, and the argument has been advanced that neither

the art of music nor knowledge in general is benefited by attempted scientific investigation.

To this we may reply that if the proper study of mankind is man, it is surely interesting to study one of the fields of man's activities.

One may be well assured that investigators will never be content to leave the subject alone; a glance at the enormous, though regrettably scattered, literature on the subject confirms that.

The present attempt is to promote clear thinking, definite methods, and above all, a common language among all who are interested in the subject, which is in its nature somewhat abstruse and which covers a wide and ill-defined field. The success that attends these efforts is a matter for each reader's judgment.

For the benefit of those readers who have not seen Division I., Simultaneous Tonality (1916), I have prefaced the present division with a short summary which will be found to contain all the essential features of the subject.

Some slight alterations, suggested by later study, have been made in the arrangement, but the substance of the argument and method appears to me to hold good.

One or two points may be mentioned:—

First, as to musical illustrations. Some of these are given in Tonic Solfa symbols, others in Staff Notation, with which the reader is assumed to be familiar.

Tonic Solfa is easier to print and read, but where a structural aspect has to be pointed out, the semi-graphical Staff Notation is most valuable.

The term TONE generally is used without restriction to denote a musical sound of single pitch.

The word "Cyclon" may indicate a simple harmonic tone, or sinusoidal vibration.

The term AUDENTITY stands for "hearing identity" irrespective of how effected. The term must purposely be left to a somewhat elastic application.

Certain points, perhaps not clear at first, will explain themselves during the exposition.

The redundancy of argument about single points is included in order to appeal to a wide circle of readers, and may be skipped if desired.

INTRODUCTION

Scheme of Symbols, Nomenclature and Terminology

The system of symbols employed in this book is explained as the necessity arises for each new expression.

A glossary is given herewith for convenience.

It would be very good if symbols and formulæ could be altogether dispensed with. The jargon of each particular science is a real obstacle, not only to the acquisition of knowledge, but to the sympathetic understanding by workers in parallel fields, who, occupied with their own formulæ, tend to be repelled when confronted with the hieroglyphics of a sister science.

After considerable thought, it has been found desirable to restrict the symbols to the letters of the present alphabet, and to resist the tendency to sport in Greek or other foreign characters.

Capital letters stand for states and terms, small letters for operations and relations.

Mathematical terminology has been freely borrowed in order to indicate abstract relationship where required.

The glossary should not be considered apart from the text, and even in the latter, want of space precludes much that is important.

Achrome. The "colourless" aspect of the Octave in comparison with that of other intervals.

Adduct. Leading to. Restoration of permuted terms.

Adherence. The relationship of adjacent tones.

Alteration. The change or motion of a Nominant.

Alternative. The two values of the Determinator.

Antinominant. The tone of maximum contrast in harmony, *i. e.* that a semitone away from a Nominant.

Applement. The difference between an Interval and an Octave.

Audentity. Hearing Identity of any tonal expression.

Autophor. Tones which carry themselves independently of the harmony, viz. Passing notes, Suspensions, etc.
Axis. The member of constant pitch in a chordal progression.

Binomial. Two tones considered in any relationship other than chordal.
Black. The "colour" symbol of a Unison.
Blue. The "colour" symbol for the Perfect Fifth.
Bundle. The intervals of a chord reckoned from the lowest tone to each note in succession.

Cacaphonic. A bad sounding progression.
Cacatonic. A "discord" of non-musical elements.
Carminal. The attractive features in a tonal passage.
Catatonic. Progression towards resolution.
Centron. The term of a group which acts as the Centroid.
Change. Alteration of tone name, without movement in pitch.
Chiral. Intervals of First Order are "handed" in direction to distinguish them from those of other Orders.
Chrome. Intervals according to the system of colour symbolisation, when appreciable as a whole.
Coheremce. The relationship of tones in the same chord or series.
Coincidental. The tone, and the Series below it, of primes whose successive harmonics coincide.
Colligation. Associated, but not related.
Comma. The Liminal interval 81 : 80.
Commute. The relation of opposite Species to each other.
Complement. The difference between a smaller interval and one of second order.
Concession. Permutation of determinator in the direction of progression.
Concomitant. The duality of species peculiar to the tones of the Hemicyclic Matrix.
Concordant. Chords of the Triad and parts.
Conservation. Retention of Matrix (Non-modulation of key).
Consonant. Euphonic progression.
Contradeterminator. The seventh Serial member, which "determines" the "contra-triad" 6 : 7 : 8.
Contra-Impellant. The interval (fluent) 15 : 14.

Contra-suboscillant. The practically liminant "fluent" 49 : 48.
Core. A tone or group, enveloped by another group.
Cycle. A chain of relationships closed by a recurrence, *i. e.* an Octave.

Degree. Grading values by specific group relation, *e. g.* intervals, into coherents, adherents, and liminants.
Determinance. Logically explicable relations, as contrasted with chance or arbitrary colligation.
Determinator. The mediant of a triad, which determines its Species.
Diacentron. The diametrical of a centron, *e. g.* notes Fe, and Ta.
Diametrical. The member immediately opposite to any Nominant on a cycle, viz. half an Octave distant.
Diphony. Two part progression.
Discord. Restricted to the static case, a regular chord other than a concord.
Disjunct Fluent. The interval between the "ends" of the Septomial Scale, *i. e.* LD : TD.
Dissonance. Restricted to the progressional sense; a succession which is not consonant.
Distortion. Stretching pitch or time values of a recognised structure without loss of identity.
Domain. The region over which a Matrix can be translated, *i. e.* the possible modulants of key.
Dyad. A chord of two tones.

Educt. Leading from. The permute second order component.
Equigrade. A succession of equal steps in pitch.
Envelope. The tones not belonging to a chord which is regarded as a "Core"; usually refers to the Tetrad about a Triad.
Exchange. Replacement of a tone by another liminally near it, or by another tone in the same definite chord.

Extramur. "Outside the wall." The adjacent but definitely not a liminal or permutable alternative of a tone, *i. e.* adjoining scale note.

Flexion. A tone continuously varied in pitch.
Fluent. The intervals by which Chromes differ. Regarded as of Second Degree, and the basis of determinate Scales.



Fundamental. The Prime tone, and the Series projected upwards in pitch, *i. e.* the harmonic column.

General Matrix. The Septomial of Triad Core, and Tetrad Envelope.

Grade. A step round a cycle.

Green. The colour symbol for the Major Third.

Hemichrome. Half a chrome, such as W/2, B/2, etc.

Hemicycle. Half a cycle, particularly referring to the Pythagorean dextral, which contains the seven "white" notes.

Heptad. Chord of seven tones. The practical limit of chordal aggregation. That of the Tensor best known.

Hexad. Chord of six tones, particularly that formed by the Tetrad Envelope plus the two Axes.

Hypo-violet. The "umbral" interval 7 : 6.

Image (*Mirror*). The tone as far away on the other side of Ray (J).

Impellant. The fluent interval 15 : 16 or its representative.

Indeterminator. The mean tone between F and C D. Of theoretical interest only, at present.

Index of Order. The number of the Octave in which tones first appear in the Series; a power of 2.

Infra. As opposed to "Ultra," the nearest in of any pair of similar terms (*e. g.* the Infra and Ultra oscillants of 9 : 8 and 10 : 9 respectively).

Inherence. Belonging to a determinate limited group or Matrix.

Intraversance. The phase neutrality of the quarter Octave.

Invelope. The antinominant aspect of Envelope, as shown by the Chords of the Augmented Sixth.

Læval. Left-handed in a cyclic or polarised relationship; thus FR is regarded as læval with respect to FB.

Latent. The implied but not sounded tones.

Laxator. The tone opposite in polarity to the Tensor, *i. e.* in F Species; the Subdominant.

Limen. The threshold interval of temperability or librational possibility.

Link. The chord TD : TT : L which links the two concomitant species.

Locus. The position of the Matrix in a determinate tonal Domain; typified by that of some representative tone.

Matrix. A conditionally limited group of tones selected from a Domain.

Mauve. The Colour symbol for a Minor Sixth.

Mode. The phase of a scale with respect to original termini.

Monad. The chordal aspect of a single tone.

Motion. A tone is said to move to another in the same Phonon.

Nomial. A term in relationship other than chordal.

Nominant. A named term in a system.

Nonomial. Group of nine nomials.

Opponents. The polarised or signed half-octaves, as presented by the two First Order intervals.

Order. The classification of Serial components according to the Octave of the Prime in which they first appear.

Oscillant. The Whole Tone interval; the intervals formed by successive addition "oscillate" between con- and dis-cordance.

Paraphony. Parallel (Real or Tonal) motion of phona.

Parasyntony. (Acoustic). The affected region about a point of Resonance.

(Tonal). The regions of Liminance, Antinominance, and Externominance, about a Nominant Tone.

Permutation. Change of a determinator to its alternative Species.

Phase. The argument or "angular" position of a term or space with respect to a complete period.

Phonon. A single "part" in polyphony.

Phonality. The aspect of linear identity persistence in succession.

Polarity. The "hand" or "sign" of First Order terms.

Prime. The first term and its achromes, in a Series.

Primary Axis. A tone which remains exact in pitch in a progression.

Pseudo. Imperfect, and augmented forms of exact relations.

Pythagorean. System of tones related by First Order Intervals.

Quadrant. Quarter of an Octave cycle, approximated by the intervals known as Minor Thirds.

Quadrinomial. A group of four nomials, such as L : P : T : J.
Quasi-triad. A pseudo-triad formed by upper serials, as 7 : 9 : 11.
Quinomial. A group of five nomials.
Quintriad. The group of five Pythagorean triads, note-identical with the Tertriad or Heptad.

Radical chord. That formed by the Series, including the Prime tone.
Recessive. The alternative aspect to a Predominant in audentity.
Red. The colour symbol of the Perfect Fourth.
Replacement. Substitution of a term or statement by another.
Riemann's equation. 9×7 approximates 2^6 *i.e.* 64.

Scale. (1) A system of steps in pitch.
(2) A recurrent pattern in that system.
Secondary Axis. The tone which moves over a liminal interval, *i. e.* is tempered, in a progression.
Seminomial. A halfway stage in a chordal nomial progression.
Serials. The tones and intervals of a Series.
Series. The tones whose frequencies form consecutive whole numbers, and the reciprocal arrangement of same.
Seriopolar. The Matrix consisting of serials of the Polar tones.
Sesqui-order. Intervals such as fluents connecting first and second order, *e. g.* the Impellant.
Skeleton. The ultimate of a reduced form, that of the Series consisting of the Prime and one other serial tone.
Statement. An individual musical passage of not less than two tones.
Sub-oscillant. The Second Order fluent connecting the determinators, *i. e.* the interval 25 : 24.
Super-oscillant. The interval 8 : 7.
Supplement. The difference between a First Order interval and one smaller, *e. g.* V is the supplement of G.

Tensor. The third Serial member, *i. e.* the one which extends the Series beyond mere achromatic relations.
Ternary. Division of the Octave or any interval into three equal parts, Initiator, Continuant, and Consummator.
Tetrad. Chord of Four members.

Tone. The general term for a musical sound appreciated as a single value in pitch.

Translation. Alteration of Matrix within a Domain, *i. e.* modulation of key.

Triad. The Serial chord of three tones, 1 : 3 : 5.

Trinomial. Three related terms, such as the group L : P : T.

Ultra. The further out of a pair of similarly named values.

Umbral. Prime number members of a Series just outside the chromal boundary, practically the Contradeterminator.

Unitor. The phase aspect of an Octave.

Vertication. The invariant direction of the Harmonic Series conventionally considered as "upward."

Violet. The colour symbol for the Minor Third.

White. The colour symbol for the Achrome or Octave.

Yellow. The colour symbol for the Major Third.

Yoke (*Jugator*). The tone Ray, which yokes the concomitant species' in the Hemicyclic Matrix, being Bi-tensor in both.

Zero. The relative Order of the Prime tone and the Octave interval, in comparison with other tones and intervals.

NOTES ON
"SIMULTANEOUS TONALITY"

These are put in because many readers may not have seen "Simultaneous Tonality," published 1916.

Definition of Hyperacoustics.—Science is not concerned with music as an art, but only as an act of man.

On scrutiny of musical works, one perceives evidence of persistent determinate procedure amid considerable individual diversity.

To examine this is the object of a conditional science filling the gap between the known facts of acoustics and the experiences of musical method.

This science may be termed "Hyperacoustics," and for convenience of study divided into Simultaneous and Successive Tonality, Rhythm, Organisation and Significance.

Definition of Tonality.—The science of the determinate use of tones as opposed to arbitrary procedure may be known as Tonality.

For acoustic reasons it is convenient to start from the Simultaneous aspect.

Our brief space precludes detailed exposition, and reasons for adopting a special course. These are dealt with elsewhere.

We are not able to discuss either acoustics or empirical musical theory. Books on both subjects are generally accessible, and our present aim is to fill the gap between.

The object of a special system of names and symbols is to condense space and to afford a mental grip.

Among the many acoustic conditions involved in Tonality, four stand out in particular, and become the basis of our theory of Tonality.

These are—

(1) *The Linear Space Concept of the Pitch Range.*—The pitch range extends continuously over some seven to eleven Octaves, and a definite tone on same appears as a point on a linear space. This may be attributed largely to an approximate coincidence between the external and internal auditory conditions.

(*a*) The expression of tone position in pitch by the frequency

of the vibration, is usually a reciprocal of mechanical and dimensional measurements of its generator (*e. g.* the lengths of strings, pipes, etc.).

(*b*) Pitch as a sensation is not numerical, but the interval between two tones is comprehensible as a "space."

The auditory mechanism is presumed to be, and certainly acts like, a linear system.

An interval is a ratio of frequency and is thus independent of the time unit; a pure number, and transposible as an identity in pitch. Its repetition steps out a Scale of pitch measurement. Therefore an equigrade system as given by logarithms of frequency is most convenient. As a "base" the most predominant interval would be chosen, and if this is divided so as to present aliquot intervals, 2, 3, 4, 5, etc., the common units would be respectively 2, 6, 12, 60, etc. Note the rapid growth.

(2) The *Harmonic Series* forms the basis of Tonal relationship and is discussed in detail in works on Acoustics.

Its importance is due to—

(*a*) The fact that the types of sounds of most use in music are best generated by the vibration of definitely bounded homolinear and homogeneous bodies.

Owing to the mechanical conditions of equilibrium such vibrators can only split into aliquot sections, the most general form of vibration being the sum of such elements.

(*b*) The theorem named after Fourier, and experiments, show that the elements are the simplest form of vibration possible (Cyclons), and they are the only type remaining unchanged in form on passing through any complex mechanical system. In the case of the small amplitudes involved in auditory cases, the Principle of Co-existence of Small Motions practically holds, *i. e.* their effects may be algebraically added.

(*c*) By the principle of Resonance or Syntony, which is presumed to occur in the ear, each element in a range of tuned vibrators responds to the greatest extent to the component of its own (undamped) frequency. Analysis of a sound into its Cyclons thus takes place physiologically.

The actual process in mechanical and electrical cases supports the "Helmholtz" theory. It should be mentioned that there are several rival theories of audition.

(*d*) In contrast to the effect of smooth tone is the jar of "anti-tone" due to beats. These are a minimum with tones in Serial relationship, and the sensation is not an arithmetical measure, but appears to be dependent upon arbitrary factors.

The phenomena of Combinational tones follow the Series relationship. For full discussion see works on Acoustics.

The Hàrmonic Series is a well-known acoustical phenomenon which provides the basis of classification of Intervals, and thus marks out *positions* on the continuous range of Pitch.

The direction in which it is projected from a prime tone is conventionally known as "upwards," hence it establishes a concept of "Vertication."

It should be noticed, however, that if we take a resonator of fixed pitch and listen to a compound note which is continuously lowered in pitch, the resonator will sing out as each harmonic of the prime tone passes its own pitch. The prime tone at these points marks a series of *reciprocal* values to the ordinary or Fundamental Series.

(3) The particularly "*recurrent*" *character of the Octave* and its multiples, of which other intervals show practically no trace, is well known, even if its cause is obscure.

Acoustics shows the Octave tone as the most persistent companion of a Prime, both in Series, Combinational, and right-angle forcing conditions. There is no simpler interval than the Octave to which it can be compared, so that this interval becomes the natural basis of consideration.

The effect of this recurrence is to present the series as a helix projected upon the pitch range, and to enable intervals smaller than the Octave to be considered by their phase on the cycle thus established.

(4) A local disturbance on a connected range of syntonisers, such as we presume the selective mechanism of the ear to be, *is not confined to the actual point of resonance but affects the whole system.*

The general form of the graph of such effect is a "hump," with the characteristics of—

(*a*) More or less blunted summit.

(*b*) Sloping sides.

(*c*) Base tapering off to flatness in both directions.

These present three somewhat ill-defined "tracts" of Parasyntony about the actual point of the Nominant tone. Such curves can be drawn from the resonance formulæ of elementary text-books on mechanics, acoustics, electricity, etc., and can be seen on a sensitive Vibration Tacheometer and heard on a wireless receiver.

Many other factors beside simple resonance are probably involved in the act of hearing, but experiment and general observation point to just such three regions about a Nominant tone, viz.—

(X) A small region of Liminal libration adjacent to the apex representing the Nominant tone.

(A) A region of beat antitone maximum, which may be traced by experiment with two tones.

(E) An externominant region beyond this where the effects of antitone tend to vanish.

These experienced regions converge with rise of pitch, and the mass effect of the tone (remanent tone tint of a Cyclon) changes similarly to an acute quality, in a very similar manner to "width" of the hump mentioned above.

If we simplify our view by taking the effect of beat antitone as a criterion, a very good analogy is afforded by the usual leaky cycle pump. If this is worked too slowly, air escapes; if too fast, the air is simply compressed and expanded quicker than it can pass into the tyre, so that there is a region of maximum efficiency in the speed of pumping bounded by regions of liminance and externominance.

To sum up, we have as four principle acoustic conditions—

(1) The *linear* concept of pitch as a variable continuity.

(2) The *fixed converging* Series of tones, associated with an irreversible *direction* of projection and of *change* of remanent tone-quality from massive to acute.

(3) The *recurrent* (cyclic) character of the Octave.

(4) The Liminal, Antinominant, and Externominant *tracts* about any tone.

Starting from these data, we now proceed to a review of the phenomena of Simultaneous Tonality.

BRIEF RÉSUMÉ OF "SIMULTANEOUS TONALITY"

SECTION I

0. The range of frequencies affecting the ear extends over some ten Octaves, generally understood as from about 16 to 32,000 vibrations per second.

The useful range, however, is restricted to about seven Octaves, the extent of the pianoforte keyboard.

For the purpose of convenient discussion we are expressing all values in what is known as "theoretical" pitch. This is actually slightly lower than the conflicting values in use.

It is based upon powers of two, and for distinction we may take the zero tone as being of one vibration per second, the first Octave tone being double this frequency.

Using the symbol W as indicating the Octave, the powers of two then give the relative pitch position; thus W 8 of 256 vibrations per second is practically the "middle C" which is taken as the centre of the useful range.

There are many ways of indicating pitch, but this method enables easy calculations of frequency by doubling, and is not difficult to remember.

The keyboard represents a rough table of logarithms of frequency, the base being the Octave ratio 2, which is divided into twelve Equally Tempered Units.

1. Let there be given an arithmetical series of Frequencies—

$$1 : 2 : 3 : 4 : 5 : 6 : 7 :$$

which we call the FUNDAMENTAL SERIES—

$$F_1 : F_2 : F_3 : F_4 : F_5 : F_6 :$$

Together with its corresponding reciprocal—

$$1 : 1/2 : 1/3 : 1/4 : 1/5 : 1/6 :$$

which we call the COINCIDENTAL SERIES—

$$C_1 : C_2 : C_3 : C_4 : C_5 : C_6 : C_7 :$$

These two conversities mark out the two distinct extremes of a range of possible arrangements which we term SPECIES. Any form intermediate to the above being regarded as of COMPOSITE SPECIES.

2. For many purposes it is required to express tones not by their mechanical "frequency," but somewhat as "space steps" on the continuous RANGE of PITCH.

Taking the corresponding logarithms of frequency to an arbitrary Base (which is most conveniently the Octave ratio 2 or $\frac{1}{2}$), we obtain a series of Pitches, of positive and negative sign respectively, for the F and C Series.

We are not restricted to the base 2 or $\frac{1}{2}$, but for many reasons it is by far the most suitable value to adopt.

If this be done, all Intervals between tones, and the pitch "spacing" of the position of tones, is expressed in terms of the Octave.

3. The successive ratios of frequency formed by the tones of the Series are pure numbers, *i. e.* independent of the prime of the Series.

Their Pitch differences are expressed in terms of the Base selected.

If the Base be expressed as a length they form a converging series of lengths; each length is transposable in position as a recognisable entity.

This corresponds with the auditory perception of Intervals within ordinary limits.

The Intervals of the Series may be classed into ORDER, which is determined by the *number of the Octave in which such Interval first occurs.*

Thus the—

Octave	2 : 1	is of ZERO ORDER,
Fifth & Fourth	3 : 2 and 4 : 3	are of FIRST ORDER,
The Thirds	5 : 4 and 6 : 5	} are of SECOND ORDER.
as well as	7 : 6 and 8 : 7	

The first and second pairs of Second Order being distinguished as Infra and Ultra respectively.

The Order of the Interval is expressed by the integer term in the Pitch logarithm: *i. e.* the index of the numerical base $2^{\pm 1}$ and the successive Orders form an arithmetical series in pitch.

4. The successive differences in pitch, or ratios in frequency, constitute the System of DEGREES OF DETERMINANCE, and are symbolised by $K_0\ K_1\ K_2\ K_3$.

Their importance in the theory depends upon the distinctive functions of each Degree in the application of Tonality.

K_0 (Zero Degree) represents the series of pitch spaces from a given Nominant Unit or Prime of Frequency, 1.

It stands for the absolute "stretch" from Prime to any member of a Series.

It is conveniently expressed in terms of the first serial space, the Octave, or according to convenience by subdivisions of same such as 1/12 (Equally Tempered Semitone), 1/1200 (Cent of A. J. Ellis), 1/1000 (Unit of Jonquière), etc.

K_1 (First Degree) presents the abstractable successive Intervals as elements which cohere in the Series. These pure transposable ratios can be recognised as entities or static elements of the Series.

Regarded as components of the Timbre or Tone tint of a Note they may be termed "CHROMES." Their convergence in pitch space is evident.

K_2 (Second Degree) presents the differences of the latter.

Presuming any pair of Series Intervals to be moved so as to have one tone coincident or Axial, this difference represents the operator which converts one into another.

Elements of this degree thus imply succession in time as contrasted to the static "Chromes." From this aspect they receive the name of "FLUENTS."

K_3 (Third Degree) presents the Discriminants of the Fluents from each other.

For reasons which will be discussed, we do not proceed beyond examination of the third degree of successive differences.

All these elements of Degree appear in the objective form of Intervals, and it is mathematically obvious that they all appear in intermediate succession in the Series.

As Intervals they are transposable entities, being ratios of frequency. As pitch differences they are expressible as lengths or spaces in terms of the particular logarithmic "base" chosen as unit, the Octave being naturally selected on account of its properties.

We here note the essential difference between Tones and Intervals, *i. e.* that they are nominated by the choice of unit Tone and basic Interval respectively. We do not maintain that the effect of an Interval does not change on transposition of pitch, but that an Interval can always be recognised and named, whereas the gift of absolute pitch estimation is not common.

5. We will now examine a set of phenomena involved in all acoustic systems, of which the ear is the one particularly interesting to us.

These are the inferior limits, more or less loosely determinable, of which full particulars are seen in the more extended works on Acoustics.

They are named as follows :—

E. The Externominant : the inferior limit of concordance.

A. The Antinominant : the mean interval of maximum discordance.

X. The Limen : the maximum extent of libration of a tone possible without loss of identity, *i. e.* extent of slight deviation from exact pitch.

We cannot here discuss the acoustical observations, experiments, and theories, upon which an average value of each is based. It is a somewhat controversial matter; but observation and experiment will convince us that—

E approximates to one-third or one-fourth of an Octave. Natural Coherent.

A approximates to one-third of E, about one-twelfth of an Octave. Natural Fluent.

X approximates to about one-third or one-fourth of A. Natural Discriminant.

In the practice of music it becomes almost a necessity to agree upon some fixed system of interval measurement in order to clearly and economically define the notation, and the claviature

of instruments. This confers an appearance of false simplicity upon tonal procedure, but it enables (in the same way as squared paper does with curves) the highly variable system of tonality to be represented in great generality with a limited number of fixed tones.

Systems of tuning are innumerable, also the literature dealing with them. In general, they are based upon a least common unit to represent the first few "Chromes," and it is found that one-twelfth of an Octave does this with the greatest economy.

Such a unit is called the Equally Tempered Semitone (E.T.S.), and its dimensions are roughly those of an Antinominant. The application of this system will be seen as we proceed.

On applying these three "natural" limits to the respective series of Degrees of Determinance to which they are applicable we obtain a general "boundary" to the Coherent or Concordant portion of the Series at about P_6.

This will be evident upon scrutiny of the following table.

TABLE OF SERIAL INTERVALS AND DIFFERENCES

Expressed in Equally Tempered Semitones, or units of one-twelfth of an Octave, as the nearest whole numbers to the actual values.

Ratios of Frequency.	K_1 (Chromes).	E.T.A.	K_2 (Fluents).	E.T.A.	K_3 (Limina).	E.T.A.
2 : 1	12·00000	12				
			4·98045	5		
3 : 2	7·01955	7			2·94135	3
			2·03910	2		
4 : 3	4·98045	5			0·90225	1
			1·13685	1		
5 : 4	3·86314	4			0·43012	
			0·70673	(1)		
6 : 5	3·15641	3			0·21903	
(Average Boundary)			0·48770			
7 : 6	2·66871	(3)			0·15073	
			0·35697			
8 : 7	2·31174	(2)			0·08433	
			0·27264			
9 : 8	2·03910	2				

It is apparent that the E.T.A. unit meets the condition of approximate common measure for the Chromes up to 6 : 5, but that 7 : 6 and 8 : 7 do not fit so well.

The same applies to the Fluent between 5/4 and 6/5.

Further, it is to be noted that the last "Limen" comes definitely below the one-third of an E.T.A., and is thus definitely "sided," whereas 0·43012 is about one-half the unit, and thus indefinite as to which "side" of the interval it inclines to.

The cutting up of the Series on the basis of these interdependent approximations of the Degree system to the experienced Auditory limits provides the rational basis for tabulation as follows.

The early Series members may therefore be classed—

CHROMES .	Five Concordant Intervals: (between)	1 : 2 : 3 : 4 : 5 : 6.
INSULANTS .	Two "Intermediate": . . .	6 : 7. 7 : 8.
FLUENTS .	(First Order chrome-differences): .	8 : 9.
(larger) .	(First to Second Order ditto): . .	9 : 10.
	An Intermediate Region: . . .	10 : 11 to 14 : 15.
FLUENTS .	(First to Second Order): . . .	15 : 16.
(smaller)	An Intermediate Region: . . .	16 : 17 to 23 : 24.
	(Second Order):	24 : 25.
	An Intermediate Region: . . .	25 : 26 to 48 : 47.
	Second Order Ultra: . . .	49 : 48.
LIMINAL .	Region of Indiscriminance: . .	49 : 50 and onwards.

6. Practical conditions require the most general representative power combined with economy of tonal material, and this is effected by the choice of the E. T. Antinominant, which enables a great range of tonal values and relations to be fairly well approximated, as will be obvious not only on further perusal of these pages, but by common observation of musical practice with fixed pitches.

In this system all Intervals except the Octaves are inaccurate. But the justly intoned values are more or less closely approximated to by multiples of the unit E.T.A.; 3 and 5 fairly well, 7, 11, 13 less so.

The fact that works of music involving a great range of tonal expression can be satisfactorily rendered in E.T. tones shows that the "approximation" is not a mere makeshift. It must, however, always be remembered that to attain the simplicity of the approximation involves a sacrifice of actual accuracy and tonal purity. As a matter of fact, in most practical cases tones can be slightly corrected by the skilled performer.

It may be noted, in passing, that 10 S approximates to B.

Also that the distinctive "Limen" between O and I is nine-tenths of an E.T. Unit.

This value, which is 128 : 125 (the same as between three G and the Octave), is perhaps the most distinctive and generally representative value. It is exceeded by that between 4 V and W. Reduced to the nearest Serial, the mean Limen would be 42 : 41. Thus any Interval in this neighbourhood could be regarded as a definite Limen in any system based upon the acceptance of the twelve-toned E.T. System.

7. Purely for the purpose of convenience, a colour-name "label" or symbol system has been devised on an easily remembered basis.

There is no metaphysical implication behind this of any connection between Tonal and Colour phenomena, although we have adopted the name "Chromes" in accordance with the convention that images the "Timbre" of a musical sound as "Tone-tint."

TABLE OF COLOUR SYMBOLS FOR INTERVALS

Interval.	Name.	Colour Symbol.
Octave	White	W.
Perfect Fifth	Blue	B.
Perfect Fourth	Red	R.
Major Third	Green	G.
Minor Third	Violet	V.
Unison	Black (Zero)	Z.

These names facilitate the act of thinking of, speaking about, and writing down, of an Interval as a thing apart from its constituent tones.

The basis of the system is the trisection of the Octave by the Serial members 3/4, 4/5, and 5/6, and their representation respectively by the three primary colours Red, Green, and Violet, which are easily remembered.

From the foregoing we derive the compound symbols—

Major Sixth : R plus G equals Yellow : Symbol Y.
Minor Sixth : R plus V equals Mauve : Symbol M.

(The term Purple is not employed, as the symbol P stands for the prime tone of a Series and confusion might arise.)

The Fluents are allocated in order according to that of the Chromes of which they are the converters or differences.

The First Order Fluent is the difference between B and R; its frequency ratio is 9 : 8.

Its complement, with respect to G, is an Inter-order Fluent ratio 10 : 9. These two differ by the Liminal value ratio 81 : 80, which is inappreciable auditorially.

These values are represented in E.T. by two units and known as Whole Tones. For our purpose the special name of OSCILLANT, symbol O, may be used.

This name is chosen because successive additions of this Interval present "oscillation" of discordance and concordance, the odd multiples 1, 3 and 5 presenting the former, and the even multiples 2, 4, and 6 the latter.

It might be noticed that as the addition of the Chromes Y and R gives an interval of an Oscillant and an Octave, the Interval might be denoted by the label—Orange, whose initial corresponds to the already selected symbol.

Again, on adding B to V we get the Applement of an Oscillant to an Octave (the Minor Seventh), which might be labelled Indigo, symbol I.

The colour symbolism might then be extended to—

Major Ninth : W plus O : Orange.
Minor Seventh : W minus O : Indigo.

But I personally think it better to restrict the colour symbols to the early Series Chromes and their concordant compounds.

It is the great theoretical defect of the E.T.A. system that the Semitone unit has to do duty for two "Fluent" values considerably different in just intonation.

The general term "Antinominant," symbol A, of one-twelfth of an Octave, therefore represents both—

(1) The diatonic ("IMPELLANT") of Intermediate (First-Second) Order as the converting difference between R — G and M — B. Its frequency ratio is 16 : 15, and its justly intoned value is 1·13685 of an E.T.A.

It receives the name "Impellant," with the consequent symbol I, from its use as the scalar Interval from the so-called Leading

note Te to the Tonic Doh (and in Coincidental Species from Fah to Me).

It is (or should be) distinguished in staff Notation by a graphical shift upwards or downwards, and in Solfa symbols by the succession of two entirely different tone names.

(2) The "chromatic" SUBOSCILLANT, Symbol S, converting differences of the Second Order Chromes Y — M and G — V of frequency ratio 25 : 24, and its justly intoned value is 0·70673 of an E.T.A. Occupying as it does the same relative position between the Second Order chromes as the Oscillant does to the First Order, it receives the name of "Suboscillant."

It is distinguished from the Impellant in Staff Notation by the prefixion of the symbols to a note, instead of motion, and in Solfa by only partial alteration of name.

The CONTRA-IMPELLANT of frequency ratio 15 : 14 and the CONTRA-SUBOSCILLANT, 49 : 48, may be noted in passing, although their use and position in the tonal theory will be deferred till later.

Values beyond this do not concern us. The Fluents gradually lose their individuality as they approach the indiscriminable Limina.

The Insulating values intermediate to the specific ratios mentioned above serve to keep distinct each type of Fluent. They are, of course, all Fluents, but not normally concerned in Tonal practice.

In the strict sense of the term we should regard R as the "fluent" between W and B, and conversely, B as occupying a similar position with regard to W and R.

Although we shall later have occasion to examine conditions under which R in particular behaves otherwise than a concordant Chrome, and can be "exchanged" for an Oscillant, we will now restrict the general idea of a Fluent to the Interval differences of the three primary Chromes, R, G, and V. This gives us the Oscillant, Impellant, and Suboscillant as standard types.

Limina need not now be specifically described, since their use in Tonality is to mark the accession of indescriminability. They represent merely the vanishing of distinction between Intervals approximating to identity, and while it is difficult to draw a boundary between them and the Fluents, the general average

of determinate conditions corresponding with the E.T. practice may be taken as correct.

8. The librational possibilities of Tones and Intervals, permitting a slight "tempering" without loss of identity, is most conveniently, though not altogether satisfactorily, effected by the E.T. system, whose twelve tones represent the practical minimum.

This system enables quasi-equations between approximating powers of "prime number" frequencies to be established.

Regarding pitch as a "space," the method is akin to the use of squared paper in geometry for plotting the form of continuous curves.

Some of the principal quasi-equations may be noted—

Order Relation.	Interval Relationship.	Quasi-frequency Relations.
Zero : First.	12 B = 7 W	$(3/2)^{12} \equiv 2^7$
	12 R = 5 W	$(4/3)^{12} \equiv 2^5$
First : Second.	4 B = 7 G	$(3/2)^4 \equiv (5/4)^7$
	3 B = 7 V	$(3/2)^3 \equiv (6/5)^7$
Zero : Second.	W = 3 G = 4 V = 6 O = 12 A.	$2 \equiv (5/4)^3 \equiv (6/5)^4 \equiv (9/8)^6 \equiv (19/18)^{12}$

From which we derive—

And—

By accepting the E.T. system, in spite of all its manifold disadvantages, we attain—

(1) A *limit* to the discriminate distinction of Degrees (out of an otherwise infinite extension) in all values when their differences approach the arbitrary liminal value.

(2) *Exchangeability* of an integral multiple of one Interval with another indiscriminable from same, thus permitting transformations of tonal statements, a highly important facility.

(3) The *closing* up of the ends of chains of relationships into cycles, leading to very striking results.

(4) A *common unit* which can be employed for a fixed notation and instrumental claviature, affording a great reduction of complexity and technical difficulty.

As proved by the actual works of music, the E.T. system allows great scope and adequate freedom to the artist. It permits the easy nomination of any component, and it is not too much to say it provides a common language for tonal expression.

The formation of Quasi-equations (Secondary Axes) may be conveniently shown by plotting the Serial and derived intervals as Angles, the Octave being a complete rotation. If the radial lines are thickened out to represent the permissible libration, then, on superimposing the values, the quasi-coincidences are easily seen, and the closures of cycles evident.

The quasi-equations are expressed in the greatest generality by the twelve-termed relationship first quoted, which corresponds also to the number of E.T. units in the Octave.

This group may be known as the DODECANAL CYCLE, as on its completion we return to the Octave of our starting-point.

It may be conveniently represented by a clock dial and its twelve numbers, the steps or GRADES of which may represent either the relationship discussed by Pythagoras (and thus known as PYTHAGOREAN), *i. e.* First Order Intervals, or the ANTI-NOMINANTS.

In this idea the relationship of each component tone to another may be expressed by the *angle* between them, and the position of any tone, relative to a definite starting-place, may be known as its PHASE.

In the cycle we note that every tone and grade has its DIAMETRICAL, which is half an Octave away and may be symbolised by π. These values are the same in both aspects of the Cycle.

It has also its right and left handed (Dextral and Læval) QUADRATES, which are V or Y away, and may be symbolised by $\pi/2$ and $3\pi/2$ respectively.

These values are the same in terms but reversed in the two aspects, and generally every alternate term in one aspect of the cycle is the diametrical of that in the other aspect, thus presenting two sets of relations according to which specific term is affected.

The diametrical ratio of frequencies approximates to 5 : 7, 7 : 10 and 12 : 17, 17 : 24; and this illustrates how approach to a "homochromal or equigrade relationship" contrasting with the distinct Serial individuality of the members below 7 tends to exclude the latter.

It is observed that the resolved direction of projection from any starting-point on the cycle vanishes at the Quadrate and is reversed at the Diametrical.

The Quadrate marks the point where (by approximate equation) the Pythagorean First Order relation, or Antinominant Stepping, becomes equal to and recognised as a Second Order relationship. The Quadrate marks the limit of the Grade type predominance.

The Diametrical marks the attainment of the "mean" Semi-octave, which is neutral to both B and R serials. A practical musician recognises it as the point where, by acceptance of E.T., the "Sharps" meet the "Flats," and from the Octave point of view it represents the transition of projection out from a tone and retraction to its Octave.

The importance of the Phase concept of Tonal relations is that it enables the limits of each Order to be thought of in a convenient and compact manner, which will be found of particular use in the Successive aspect.

9. A set of Chromal relationships, presented by a group of simultaneous tones (or successive tones which could be simultaneous), is known as a CHORD.

The definition of a Chord, in contradistinction to a colligation of tones, is that it may be thought of as a whole, named as a whole, and used as a whole.

Such a definition is bound to depend upon the arbitrary limits of the associated relationships constituting the group, and the evidence of musical history shows that Chords have grown up with the development of practice. At the present day, the older theorists would be sorely puzzled to define some of the groupings employed, but in our scrutiny certain broad lines will be examined which enable any colligation to be classed.

The main basis of Chordance is the absolute and implied Simultaneity of the components, which distinguishes it from another class of compound, to be considered later, in which actual or implication of Successivity is the basis.

This view would exclude the "pure" Fluents from componence therein, and tend to limit the constitution to Chromes.

A Simple Chord is a section of the Series, a Compound Chord is built up of more than one Simple element.

The concept of a Chord is apart from its actual arrangement, inversion, distribution, etc.

The components may be actual, or implied. In the latter case they are known as Latent.

It is sometimes convenient to consider a Floating Chord, or a group of regions of pitch within which the actual tones may be situate.

The typical aspect of a Chord is that known as "close form." Thus the first six members of the Series may be reduced to the compact relationship 4 : 5 : 6, which for brevity may be used when speaking of a Chord apart from its actual arrangement in pitch. We are able to do this by neglecting the alteration due to Achromatic transposition.

The relationship between the members of a Chord is that of Coherence, about which Psychological Acoustics has much to say, and the abstract phenomena pertaining to Chords may be known as CHORDANCE.

On the other hand, the set of relationships presented by Fluents in a succession of "adjoining" tones is known as a SCALE, which represents a train of operating or nominating relations whose approximation to a continuous line of pitch-change presents ADHERENCE.

The relationships of Limina, by vanishing of distinction (Enharmonic Change), which, by the way, are much more easily grasped by graphical representation than by frequency or pitch figures, presents another basis of connection which may be termed LIMINANCE, in that the actual motion of tones in pitch vanishes in Equal Temperament.

To a considerable extent, when Chords are heard, they are comprehended partly at least as Entities; this, however, depends entirely upon the conditions under which they are heard, and so we separate the abstract from the actual tones involved, which is not easy for a beginner to do.

10. The so-called Inversion of a Chord by altering its components to the extent of an Octave or Octaves, may be compared with the addition of the colour white to any tint. It merely alters its "saturation" without modification of the naming tint. In Tonality, by reason of the applicability of this, this may be

known as Achromatic rearrangement, and the Octave values of tones as their ACHROMES.

A similar analogy is afforded by the Logarithm, where, with a base of 2, addition or subtraction of Integers does not affect its Mantissa, which actually represents the phase position of the tone within the period of the Octave.

The property of recurrence of qualities exhibited by the Octave, which makes this procedure rational, is of importance in the Successive aspect.

11. The E.T. Dodecanal system results from the adoption of a convenient unit interval whose whole multiples represent the early series Chromes.

It is based upon the mean interrelation of the system of Determinate Degrees with respect to the experienced limits of minimum, concordance, clear step, and librational extent.

The result presents the portion of the Series below P_6 as a group distinguished from that beyond this limit.

Admitting this limit within the Octave we obtain the

Six concordant intervals Y M B R G V,
of which the Fluents are S I O I S,

the latter involving nothing smaller than can be legitimately represented by the General Antinominant one-twelfth W and its double, the Oscillant.

If the Seventh Series member was admitted to this restricted group, we should have to include the intervals $7:6$ (the Hypo-violet) and $8:7$ (the Superoscillant), whose function is really to act as "Insulators" between the smallest Chrome V and the largest Scalar Fluent O.

We should also have to include the Inversions $7:4$ and $12:7$, etc.

But the important point is that we should have to include the Serial interval $5:7$, which, as we have seen, is practically half an Octave $(5:7)^2 = 25:49$.

Now this Interval is a mean between the two First Order Chromes R and B, appearing as an "Augmented" R and a "Diminished" B. It conflicts in every way with the Serial convergence by which the early Series members are distinguished from each other, and, moreover, the false Octave $49:50$ falls

within the liminal region, but acoustically is a strident discord. The E.T. system itself must preserve the pure justly intoned Octave.

From these considerations it will be obvious that there is some reason for the "boundary" drawn sharply across the Series between P_6 and P_7. The recognition of the limit by both practical musicians as well as theorists from the very earliest ages is evident of some very persistent factors, of which the foregoing is one of the principal.

At the same time we have to recognise that the Coherence properties of the further Series members shows no sharp break. The natural "Blending" of the Cyclon components in a note is evident, and if the observer can get away from convention, he can convince himself that the "Natural Sevenths" exhibit a considerable amount of concordance. Indeed, on the Helmholtz beat criterion, it is seen that this concordance exceeds that of the interval M.

We shall therefore proceed on two lines of development. One, in which the limitation forms the basis of our working method, nomenclature, and symbolism. The other, which will comprehend some of the earlier "odd" Serial members which come beyond the "boundary," especially P_7, and though this way of treating the matter may appear at first incongruous, we shall endeavour to keep to quite logical lines of reasoning and to carefully distinguish which particular aspect happens to be under consideration.

The odd Serials 7, 9, 11, 13, etc., may be known as Umbrals (Shadows).

If we require the Octave as basis of our system of pitch measurement, and further because of free transposablity of intervals it is necessary to use an Equigrade unit, it is seen that the Octave must be divided into either 2, 6, 12, 60, 420, etc., units in order to measure halves, thirds, quarters, fifths, etc. From auditory experience one-twelfth is definitely fluent, and one-sixtieth of an Octave definitely liminal. Hence the E.T. Dodecanal is a most suitable limit, and the fifth part of an Octave is ruled out.

12. The Chord of the chromes formed by the Serials 1 to 6 is known as the TRIAD, since it is reducible to three independent tones whose close arrangement typifies the group

Strictly speaking, any three tones constitute a "Triad," but for many reasons it is convenient to restrict the term to the "Common Chord," Major and Minor, of practice.

There are two species of primary Triad, viz.—

Fundamental: Members $1 : 3 : 5$: symbol Delta $\triangle$.
Coincidental: Members $1^{-1} : 3^{-1} : 5^{-1}$: symbol Nabla ∇.

The triangular symbols were introduced by André and Jonquière.

The three tonal components are known respectively as—

P_1, the PRIME	(F or C),	P.	The originating tone.
P_3, the TENSOR	,,	T.	The extending tone.
P_5, the DETERMINATOR	,,	D.	The Species determinating tone.

The absolute projection of the two species is opposed, each being a mirror image of the other.

In close arrangement the internal chromes are given by $\overset{V}{G}$ and $\overset{G}{V}$ respectively.

The border-line tone P_7 is approximated by two chromal members, situate a minor seventh and a major sixth from the Prime respectively, being nearest the former.

Since it "determines" the species of the Interval R formed by T : P in the same way that the Determinator does with the Chrome B, formed by P : T, it may be called the CONTRADETERMINATOR, symbol ⟨I.

Each component of the Triad is of a particular Order.

The Prime P. is of Zero Order.
,, Tensor T. is of First Order.
,, D (and ⟨I) are of Second Order.

Each term is independent in Species, any such case being known as Partial Species. Primes may be Reversed in direction. Tensors may be Commuted in Polarity or "hand." Determinators (and their Contras) may be Permuted from the original (Educt) species to the Adduct.

The Commutation of the Zero and First Order members simply reverses them in name, leaving the actual tones at the Interval.

The operation swings the Fundamental $\frac{T}{P}$ round P as an Axis to the Coincidental $\frac{P}{T}$, and if there is no other evidence, the First Order Intervals B and R are ambiguous as to Species.

But, as is obvious from the change of internal chromes, $\frac{V}{G}$ to $\frac{G}{V}$, the Determinator actually moves over a Suboscillant inside the First Order Interval, and similarly the Contra over a Contra-suboscillant.

The symbol of Permutation is borrowed from the keyboard, the altered terms being respectively shown by ◗ and ◖.

The theoretical Indeterminator and its Contra are of neutral species, and situate at the mean (actually bisecting the pitch space B or R). It is used in these pages to denote indifference of Second Order Species, and the respective symbols are Ð and Ħ.

The independence of the three Orders is easily comprehended by picturing them as projections in three independent dimensions of space, and their Species change by alteration of sign with respect to the observer, who is imagined to view them from the Zero point.

According to ordinary convention we regard Fundamental Species as projected upwards and Coincidental downwards.

The First Order is imagined to stretch out sideways, the Fundamental Tensor extended to the right hand, and its "Commute" to the left.

The Commute of the Tensor is by analogy known as the LAXATOR, symbol L (the retracting tone).

This leaves the To-and-fro dimension to picture the Determinator, which is imaged as "drawn out" in Educt, and "drawn back" in Adduct Species.

By this means we can get a mental grip on the independent components of a Triad in a way that notation and naming do not obviously provide.

When the species of all the components in a Triad is the same it is a Simple Triad. When the partial species of the components is varied it is known as a COMPOSITE Triad.

It may seem almost ridiculous to talk of a wholly composite Triad in a definite species (*i. e.* one which is actually identical

with the opposite species), but such an independent view is very useful in later analysis.

It may be mentioned that the Triad on the Fundamental Laxator (known as the Major Subdominant Triad) is really composite, because its Tensor is commuted to form the Laxator.

The simple form really has a "flat" mediant. The actual Major and Minor Modes of practice are really both composite, although the latter only looks so.

13. The expression "CHORD" is very loosely used. Actually it is impossible to say where a Chord merges into a Colligation, but we employ the term to denote an associated group of tones connected by some simple system of relationship, which enable the group to be comprehended and treated as a whole.

This persists whether the group is composed of simultaneous tones of equal duration, or is spread out in time to form an Arpeggio.

In order to name a Chord, the method is adopted of neglecting (for the moment) the actual arrangement achromatically and vertically.

According to the number of independent components, a Chord may be known as a Monad, Dyad, Triad, Tetrad, Pentad, Hexad, Heptad, etc., and in contradistinction, the number of tones associated together in any other than chordal-relationship, such as the cyclical Homochrome or Equigrade, is distinguished by the respective names, Unomial, Bi-, Ter-, Quadri-, Quinta-, Sexa-, Septa-nomial.

The idea of my symbolism is to use Greek names to denote passive, and Latin names for active aspects of relationship.

Similarly, capital letters will represent nominative or naming expression, and small letters the "operational" or motive aspects.

(Script letter symbols are also being used in Successive Tonality for a special purpose.)

OUTLINE OF
"SIMULTANEOUS TONALITY"

SECTION II

14. Before proceeding to consider the formation of systems of relationship in Tonality, it will be of some advantage to collect a summary of conditions which will be involved.

These are—

(1) Conditions of—

(*a*) Serial convergence.
(*b*) Tonal region.
(*c*) Octave cyclicality or Achromatism.

(*a*) If tones were, from an auditory point of view, the "Euclidian points" that they appear in physical acoustics they would never touch with the convergence of series.

But they are small region, consequently at some point in the series they touch, and this marks the limit of the series of converging Intermediate Regions.

(*b*) In respective succession, passing out along the Series, the regions E, A, and finally X, touch and mark the limit of the portion of Series regarded as "Concordant."

(*c*) By achromatic reduction it is seen that the most general form of grouping is that of the Triad and its intervening Series members regarded as antithetic, *i. e.* Non-triadal. This is given by the Members—

P		8	10	12	Serial Concordants.
	7	9	11	13	Intermediate Serials.

Hence our consideration will be directed to a System of Tonal Relationship presenting two oppositely qualified portions, one of

which is based upon the Limited Coherence of the early Series, and the other which contrasts absolutely with it, and yet which is closely related to it.

A limited system so constituted forms the group out of which a nominated set of Tones, Chromes, Fluents, Chords, etc., is selected. For this reason it is termed a MATRIX.

THE GENERAL BASIS OF THE MATRICAL THEORY

We pass on to the consideration of a determinate relationship between tones other than that of direct coherence in the same Series.

This is a relation of contrast, and yet unity, the result implying the concept of Non-simultaneity in presentation, which leads to the consideration of Tonality in Succession.

Consider a Series projected upon the continuum known as the pitch range. Each tone of the Series is variable within a small region of nominated identity. Since the Series converges, these regions ultimately touch.

Where these regions are not contiguous, there exist definite intervening spaces, within which any tone is recognisable as differing from the nominating members.

We call the nominated tones and their liminal regions the CORE members (symbol E_o), and their insulating spaces with any tones in same, the ENVELOPE members (symbols E/N or N/E according as they are above or below the tone in question). Each Core tone has thus a Sub and a Super Envelope member, or "Suburb," as it were, and vice versa.

Taking the Triad as a Core, and restricting regions to definite liminal values (say, below a Suboscillant), it is seen that in "close position" we have Four Envelope Regions, viz.—

Two external, P/E and E/T
Two internal, E/P (or D/E) and T/E (or E/D).

The four representative tones in these spaces constitute a TETRAD ENVELOPE about the Core Triad.

The three tones of a given Triad, together with the four tones of its Envelope, constitute a group which we may call a SEPTOMIAL MATRIX, defining the so-called "Key" out of which all chords may be extracted, and to which same originally refer.

The actual constitution of a Matrix will be dealt with in the next paragraph.

(It is interesting to note how, by the original acceptance of a triad of three notes, we derive a Matrix of seven tones, which is variable within a Domain Cycle of twelve members.

There is nothing metaphysical in these numbers, but the importance attributed to these particular values in ancient times cannot fail to interest the student of history and philosophy.)

The pitch position of a Matrix is usually indicated by the one of its members which acts as the equivalent to the mechanical "Centroid" of Mass. This tone is known as the CENTRON, and its relationship to a range of variability is denoted by its LOCUS. The choice of a Centron will be discussed later.

We notice that a Triad is a section of a Series, and thus, by definition, fixed in constitution; but so far we have regarded *any* arbitrary tone in the Envelope region as capable of being part of the Enveloping Tetrad. We shall consider the predominant conditions of tonality which determine the actual forms.

We might reverse the procedure, and start from an assigned TETRAD as core, regarding the enveloping Pentad as arbitrarily varied. In any case the Core and Envelope are to be regarded as individually variable components of the Septomial Matrix.

The practical musician knows the Triad as the maximum concord, and recognises seven tones as the named and noted components of Staff and Solfa Notation, as well as the usual letter notation and the empiric names of Rameau.

He observes that each tone has a triple variability (flat, natural, and sharp) which in the Equal Temperament, by reduction of redundants, give rise of a DOMAIN, within which the Septomial Matrix may vary, of twelve tones, *i. e.* the Equally Tempered Dodecanal.

The idea of a Matrix is that of a Minimum grouping presenting a Core Triad and its contrasting Tetrad Envelope, wherein we note the contrast of two groups united in one Matrix as a factor of Tonality differing from that of the single Coherence of one Series.

The relationship between Core and Envelope members is seen when we consider that the average spacing from one to the other (maximum R/2, minimum V/2) is of the dimension of a Fluent,

which may be looked upon as an operator converting one into the other.

The actual tones of the principal forms may vary, but the general aspect is that the contrasted members, in their alternate occurrence, are spaced near enough to represent the Adherence relationship exhibited by the Scale.

In the absence of any particular determinating condition the Envelope members could be regarded as being arithmetical means of pitch and thus geometrical means in frequency.

Here we note the introduction of a new concept of relationship, other than the Coherence of Series. This is the approximation of a Scale to the Continuous Flexion of a single tone, whereby the identity of the latter becomes drawn out into a line of adherence, or " relationship of contiguity."

The internal relationship of Matrical tones may be known as INHERENCE, and the tones themselves are nominated with respect to an arbitrary or conditional Centron.

The general Matrix appears as a group of seven tones separated by an average Oscillant interval.

If we regard the two external Envelope tones as about the same distance as the mean position of the internal, it is seen that the complementary interval R to the Core interval B is split up into two regions containing two Envelope tones, separated by a "Disjunct" Fluent. The mean extent of the Disjunct is one-third of R or about an Oscillant.

Thus a Septomial is linked with its sub- and super-achromes, and the whole colligation can shift up or down to bring its members in the approximate position of those immediately above or below it. This process is called Step Translation, and is not usual in practice.

When a Matrix alters to another position the process is known as TRANSLATION, and the restricted region in which it can so move is known as its DOMAIN. We cannot examine the conditions of Translation until we know more about the constitution of the Matrix, but if we have adopted the E.T. it is seen that the Matrix can move to eleven other positions.

If, however, we throw over the Dodecanal, and extend our domain as indicated by theorists, the variability of the Matrix is so far extended. Actually, the extension is of little use owing

to Liminance, and it certainly involves great complexity in notation and claviature, which mechanical devices may yet overcome in some practicable manner, allowing the use of some of the finer tone shades and sonority possible with a nearer approximation to just intonation than the E.T. system permits.

Although progression is strictly a matter for Successive Tonality it is not possible to entirely exclude its consideration here. We shall therefore append a few necessary definitions.

Chord changes, or alterations within the Septomial Matrix, which do not tend to move it may be known as CONSERVED since the Centron is invariant thereby.

This covers changes due to mutation of Partial Species, as well as the addition of Contradeterminators remaining appended members.

Motion of the locus of the Matrix (other than achromatic rearrangement) involves variation of the Centron and constitutes Translation.

It is seen that Translation corresponds with what the musical ear *feels* as Modulation, and Conservation corresponds to non-modulatory changes. This is not always obvious with empiric notation; indeed the boundary between the so-called "temporary" and "permanent" modulation is by no means clear. Hence the use of new names in the scientific aspect.

(A reliable test as to whether Translation has really occurred is to *complete any doubtful chord* and note whether the general effect of key is altered thereby. We do not mean the bad harmonic and contrapuntal effects which may be imported by this procedure.

As a simple example, it is possible to test whether modulation into the key of the Dominant or Subdominant has taken place, by making the Triad of the first named into a Dominant Seventh, and that of the latter into a Chord of an Added Sixth.)

When a tone or chord is succeeded by another we may recognise whether it is a mere chance colligation (like the sound of dusting a piano) or a determinate relation exists between the discontinued and succeeding sounds.

In this latter case, even if there be a slight interval or interpolated matter between, we recognise it as a PROGRESSION and are able to say what it is.

If a tone or chrome is altered in actual pitch beyond liminance, it is said to MOVE, although of course it is merely the stopping of one sound and starting another.

If it remains at the same pitch, or only moves over a liminal interval (enharmonic), it is said to CHANGE.

The components which change in a progression are known as the AXES, and the progression is said to be Uniaxial, Binaxial, Triaxial, etc., according to the number.

The components which move are Non-axial. They may be—

(*a*) Arpeggial extensions of a chord.

(*b*) Scalar fluents.

(*c*) The Exchanged or Larger Fluents, which will be discussed in Successive Tonality.

The components of a chord which are sounded are known as ACTUAL.

Those not actually sounded or written, but which are definitely implied, are known as LATENT.

Thus it is possible to have Monad, and often Dyad forms of Triad and larger chords.

Any component which could be added to a particular chord in a particular progression, without modifying the tonal effect, is CO-LATENT.

The Contradeterminator is in general of this type.

A Concord is by definition restricted to some arrangement of either F or C Triad, the limitation being ascribed generally to the Interdetermination of Degrees by the Auditory criticals E, A, and X, but more conveniently referable to the exclusion of the quasi half-octave which would be formed between the Determinator and its contra, were the latter admitted as a Concordant member (and whose exclusion also implies that of those beyond).

A DISCORD is any other form of chord *within the scope of determinance*, which excludes chance and arbitrary colligations.

In Simultaneity a Discord is presented by Core and Envelope members together, even when the actual members are objectively concordant in form.

The terms CONSONANCE and DISSONANCE will be restricted to Progression, while CONCORDANCE and DISCORDANCE apply to static arrangements.

CHORDANCE is the abstract factor of Chordal constitution,

and that particular aspect of regarding simultaneous tones; its rationality is based upon Coherence.

FLUENCE is the abstract of Adherence and is based upon the maintainance of Scalar identity.

LIMINANCE is the abstract of Librational variability of tones.

The Skeleton Series is that comprising the Prime and another member. A whole series is regarded as built up of a bundle of such.

The frequencies quoted as illustrations in these pages are the theoretical "powers of two." Actual frequencies in use are (unfortunately) slightly higher than this.

A correcting modulus, or new unit of time, may be used to bring the theoretical values into line with practice.

15. *Definition of a Matrix.*—A Matrix is a limited group of Tones having certain determined relations, whereby the components are nominated in the greatest generality, combined with economy of material.

Definition of a Domain.—A Domain is a limited group of Matrices, forming a region in which a Matrix can be translated as a whole.

Principle of Limitation.—The early occurrence of Secondary Axes, or tones which approximate within liminal regions. These Secondary Axes close the cycle of relations.

The actual constitution of a Matrix may be due to a large number of conditions. In practice, owing to the librational deviation of tones, it is not easy to separate the aspects.

The basis of examination is to direct attention to the predominant cases.

The constitution is due to a system of determinate relationships which form a closed group. Within this group there exists sufficient material for the purpose of musical manifestation, although there is no theoretical restriction to going beyond the group as far as the ear permits.

The most general and economical form is that of the Septomial, already discussed as consisting of the contrasting Core and Envelope. The types to be considered will be those conforming to this plan, and complying with the condition of a closed system, or rather the leading cases of a group of possible closures.

These principal types are two:—

(1) Those in which the Core and its extension form the basis of structure, *i. e.* the Envelope tones being definitely given by relations of the Core.

(2) The converse case, in which the Envelope members are given; the unformulate Core regions can be filled by a Triad of either Species, or by certain substitutes which may at times replace same, such as Augmented and Diminished forms.

Of the first type there are two principal aspects :—

(*a*) That in which extension is made on the basis of the principal intervals, viz. Zero and First Order, *i. e.* the Pythagorean system, which is historically the older.

This is named the HEMICYCLIC SYSTEM.

The relational factor is here the continuance of First Order Step, and its achromatic reduction to any form desired.

The rationale is the close approximation of the quasi-equations

$$12 \begin{matrix} B = 7 \\ R = 5 \end{matrix} W$$

and the fact that the small error is well distributed (*e. g.* in tuning a violin by pure fifths the slight mistuning of the "sixth" between the open G and E strings, is inappreciable).

The closing of the group arises from the less exact but still fair approximation $4\ B = G.\quad 3\ B = Y$ achromatically.

Finally, the Septomial is formulated by the relations of first rank, $L = T/E$, and $TT = E/P$, the additional values following on extension of the process.

(*b*) That in which the extension is made by associating with the Core Triad, its "polar" companions, viz. those directed outwards from P and T respectively. The TERTRIADAL MATRIX.

The closing of the group is determined by the exclusion of further triadal extension, which would introduce a diametrical prime relation, by the extreme terms. The above grouping constitutes a Trinomial of Triads. If it was a Quinomial the importation of tones making second order relations opposed to that of the Core would follow, viz. TTD : LL.

The two pairs of non-axial tones of the "Polar" or "wing" Triads provide four tones for the Tetrad Envelope spaces.

Finally, the extreme tones L and TT are in quasi second-order limitation, thus marking the limit of First Order extension already mentioned in the Hemicyclic case.

The rationale of the second type of Matrix is the coherence

of the Envelope as part of one and the same Series, and the conformation of same to the General Tetrad Form.

The limitation of the group depends upon the method of selection of each particular aspect, but it may be pointed out that the audentity of Series members, *i. e.* their referential relation to a root, falls off so rapidly as to make the effect of serial relationship of little importance beyond P_7 range, and thus is limited to the Tetradial members 2, 3, 5, and 7.

(*c*) Such an aspect may be termed SERIAL, and the principal forms out of the many theoretically possible, are—

(1) The Prime Serial, or Schriabine.
Envelope P (7 : 9 : 11 : 13).
Core P (8 : 10 : 12).
(2) Tensor Serial.
Envelope T (5 : 3 : 7 : 9).
Quasi Core T (11 : 13 : 16).
(3) Laxator Serial.
Envelope L $(11 : \frac{13}{14} : 16 : 20)$.
Quasi Core L (12 : 15 : 18).

(4) The SERIOPOLAR type is a combination of the two latter which obviates some of the weaker features of its components. It is based upon the "Riemann" quasi-equation $L \equiv T_7\ T \equiv L_9$ (error 63 : 64). The Envelope is the same as that in the Tertriadal, one of the polar components being affected by the error above mentioned, and an approximation to the permuted form of Core Determinator is given by the Laxator Contradeterminator.

THE HEMICYCLIC MATRIX

The basis of this aspect is very ancient; it is founded upon Phase relationship.

We take a clock dial, and imagine each figure to represent a Tone member, and the spaces, steps, or "Grades" between same represent a First Order Interval, either B (as usual) or R.

It is found convenient to put the Fundamental Prime (Doh) at "one o'clock," the Tensor (Soh) at two, and the Laxator (Fah) back at twelve o'clock.

Then carrying on this arrangement we get—

The Bitensor (Ray) at III o'clock.
The Tritensor (Lah) at IV o'clock.
The Quadri-tensor (Me) at V o'clock.
The Quin-Tensor (Te) at VI o'clock.

The advantage of this arbitrary arrangement is that we get all the tones represented by the "white notes" of the keyboard on the right half or Hemicycle, which members constitute a Septomial grouping conforming to the general plan. The five "black digitals" come on the Læval or left side.

If we look at the dial in a mirror held above twelve o'clock, we reverse the image about the horizontal diameter drawn from III to IX, and obtain thus the same arrangement representing the Coincidental Species.

This is called the CONCOMITANT COMMUTE relationship.

The Hemicycle divides into two Quadrants symmetrical about the horizontal diameter, whose tone is the mutual Bitensor (Ray). This tone which forms the "Centroid" of the Commute Species may be termed the YOKE, symbol J (Jugator), and is therefore taken as the Basis or Centron of the Ambispecies aspect. (It is also ambiguous in Order since it is in Second Order relationship to both L and TD in both Species.)

Now it is seen that the "Step" intervals B or R exceed and are less, respectively, by one E.T.A., than half an Octave.

Consequently, as we proceed around the Pythagorean Cycle, we gain this Antinominant step over half the Octave at each successive step. When this reaches 3 A it is of the dimensions of V, which is *less* than W by Y, and Y is equal to half an Octave *plus* V.

The result is that we attain the quasi Second Order relationship at three steps, and this is further confirmed by another step which gives us 4 A = G, plus two Octaves.

It is thus evident that the members three and four steps off become equivalent to "Determinators," and this conversity of species applies both ways.

On plotting the pitch extent of the Serial and Matrical intervals as angles, that of the Octave being four right angles—a complete

rotation—any values may be superposed by rotation for comparison.

By using a radius thickened out to the mean value of liminance (say 128 : 125), the early limits of the Concordant Series and Matrices formed therefrom can be seen at a glance.

We can now examine the different aspects in detail.

The Hemicyclic Matrix

System.—The "right-hand" half of the Pythagorean Cycle with the Yoke tone F TT at "three o'clock." The Fundamental Prime is thus at "one o'clock," and the Coincidental ditto at its mirror image in the horizontal diameter, viz. five o'clock.

Centron.—The Yoke, symbol J (Jugator). By making F TT = C TT, actually they differ by 80 : 81, the Comma.

Limitation.—The vertical members on the dial are L and TD, which are in the diametrical relation W/2, thus conflicting with the Step, which is B or R.

Quadrantal limitation is attained by Second Order relationship; thus the "Quadrates" of J are L and TD which form the interval Y or V with same.

The next tones transcending the Vertical limits are LL and T (TD), which are in Second Order and diametrical relationship to the Prime.

Domain.—The Læval half of the Cycle, which closes by the quasi-equation $12 \begin{matrix} B = 7 \\ R = 5 \end{matrix} W$.

The Permutation of the determinators already in Commute relationship.

Thus any Quadrant consisting of the Trinomial of tones L, P, and T has its Commute image in the adjacent Quadrant of LD, PD, and TD, the Yoke being centron, and adjunct to both. The neighbouring Quadrant on the opposite "hand" contains the Permutes LD, PD, and TD.

Finally, the opposite Quadrant contains the Diametricals π (L, P, and T) which are the Contra Determinators of the Commutes.

The Diametrical Yoke is seen to have similar relations to the Yoke. Thus there are Five Læval members in reduced position,

which with the SEVEN of the Dextral half, make up the Dodecanal Domain.

It must be understood that when we speak of "Determinator," there are no such tones in the strictly Pythagorean sense. But the tones which represent them (Tritensor, Quadritensor, and Quintensor respectively) can by borrowing from the Tertriadal aspect be treated as such.

This shows that the Hemicyclic aspect, by itself, is not adequately a complete Matrix.

It will be noted by practical musicians that the right and left-handed "neighbours" to the Nominating Quadrant respectively provide the material of the "Tonic Minor" and "Relative Minor" keys to a given Major key. There have been many heated discussions as to which is the "true Minor" to any Major key (and vice versa).

This controversy is seen to be quite beside the mark, for both can claim the right, in the theoretical sense.

Which is the "practical" rightful heir to the title is a matter of the way it appears in use, and this we leave to the musician. It does not concern the student of Hyperacoustics.

It is thus evident that the Hemicyclic Matrix can represent a duality of Species in the particular relation—

$$F\ P = C\ P\ D$$
$$C\ P = F\ P\ D$$

In practice only one Species is evident at one time. We therefore nominate this species as PREDOMINANT and its Commute as RECESSIVE. Actual commutation involves a liminal "shift" of a Comma, which is inappreciable.

Another way of looking at Hemicyclic members is to group them into RANKS reckoned from the Yoke.

In the first rank we get First Order tones; in the second we have Oscillant discords; in the third, the quasi Second Order "determinators" enter as ultras, and the next rank gives their "infra" form. So progressing we reach closure, when the tones "C-double-sharp and E-double-flat" are attained.

(The Pythagorean is, of course, not the only cycle, *vide* many works on the subject, but it conforms nearest to the general form of Matrix and Domain.)

The Tertriadal Matrix

is based upon the Trinomial of First Order related Tones about the Prime, viz.—

Læval Pole	Centron	Dextral pole
L	P	T

As we have seen, this is limited by the single appearance of the serial chrome relationship B or R. If we go beyond and introduce the Bitensor TT, we at once obtain the quasi Second Order chrome L : TT; similarly with the Bi-laxator, where we get LL : T, a similar liminal importation of another Order.

The Tertriad is a group in which the Triads of the three tones of the trinomial are taken.

If we do this with one Species, we obtain the ordinary form or Composite Specified Tertriad.

But regarding the Laxator as a projection of a Tensor in opposite species, that is, regarding the Trinomial as Vertical in

T L
pitch FP CP it is seen that a Symmetrical Tertriad is com-
L T

posed of two oppositely projected Polar Triads, enveloping an indifferent Core Triad, and thus why the uni-species form is Composite in actuality.

The limitation of the Tertriadal Matrix is that of the Hemicyclic, and, as already mentioned in connection with the General Septomial, a further limitation is introduced by the avoidance of the quasi "Semi-octaves" or Pseudo First Order intervals formed by the additional tones of the Quintriad, *i. e.* LL and TTD, which constitute the interval in question, thus LL : PD, and TTD : P.

The Domain is formulated by taking the Tertriads with the three original Triads as Centrons. This gives us a Heptad, in which the permutable determinators provide the twelve tones of the Dodecanal Domain.

Reverting to the Tertriad, it is seen that we start directly from the Core Triad as Centron, and the two new members of each Polar Triad give us the four Envelope Tones.
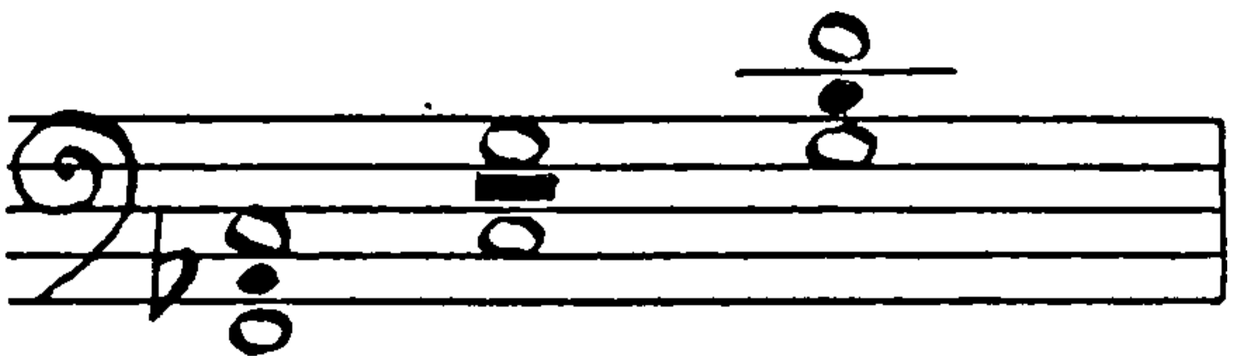
The two tones P and T are the Axes of the group. When

added to the Tetrad Envelope (so as to give the two Polar Triads entire) we get the HEXAD ENVELOPE.

The permutability of the Determinators, which is independent of the "total" species, enables a variety of forms to be obtained, which will be considered later.

The basic form of the Tertriadal Matrix is Symmetrical, thus—

L P T

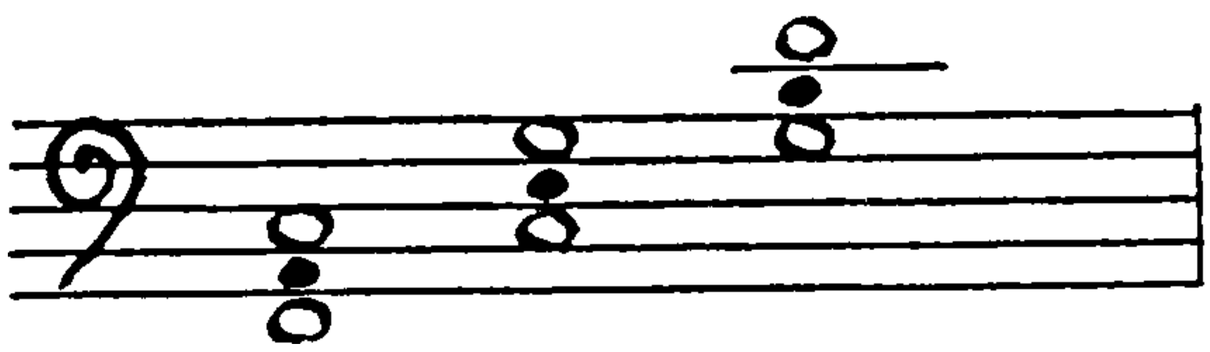


But the specified forms (which correspond to the Hemicyclic) are those of practical application.

These are—

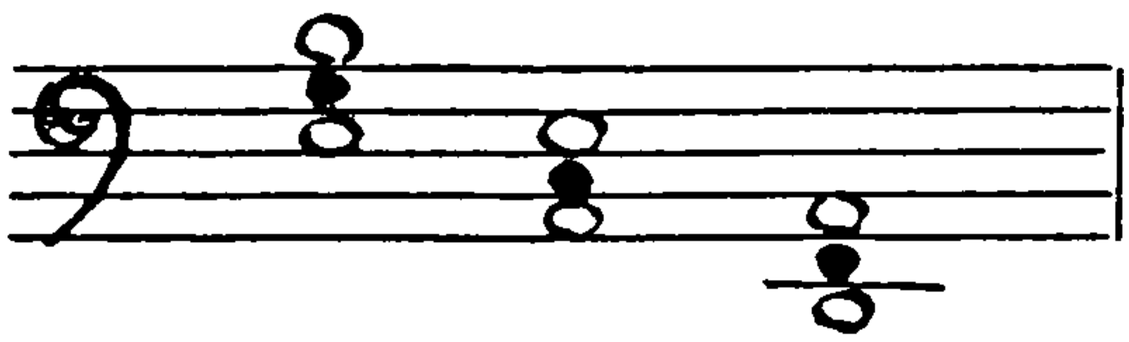
FUNDAMENTAL.

L P T



COINCIDENTAL (Concomitant example).

L P T



It is, of course, impossible in a short summary to go fully into many interesting details. The Division I. of this work should be consulted.

The Tertriadal Matrix can be regarded as arising from a Quadrinominal related by First Order chromes, whose species is respectively Coincidental, Neutral, and Fundamental.

Taking the "horizontal" Centron Triad as of Neutral Species, we note the oppositely directed Polar Triads whose non-axial components constitute the Envelope of the Centron.

The particular point about this aspect is evident upon extending the Polar Triads to their respective Contradeterminators, when it is seen that on neglecting the Riemann Limen 64 : 63 the two extreme members overlap to form Secondary Axes limiting the Matrix. This presents the Riemann Symmetrical Septomial.

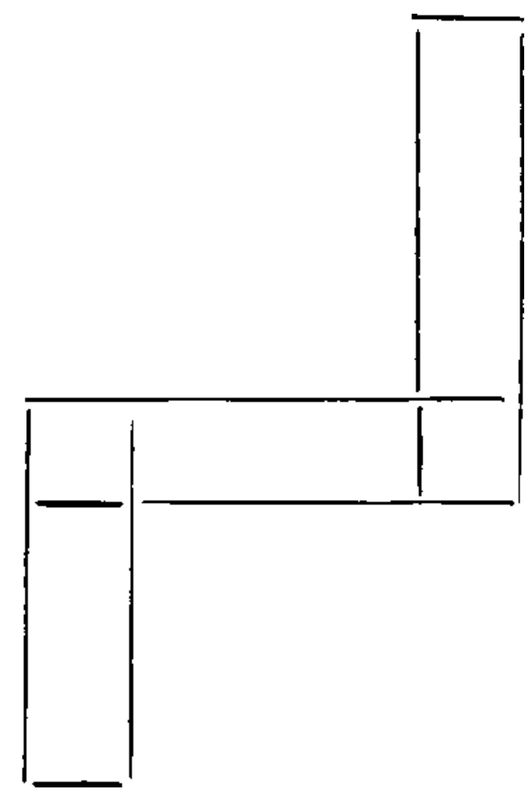
Owing to the permutable alternative determinators, it has a number of forms. The particular form that coincides with, and thus possesses the advantages of, the Hemicyclic Aspect is usually taken as the basis of consideration.

We illustrate the form with tones Doh and Soh as Primary Axes, and the Polar tones Ray and Fah as Secondary couplings and thus limits.

F Polar Triad— FAH / RAY / Te—Ta

Neutral Core— DOH Me / Ma SOH / La—Lah

C. Polar Triad— FAH / RAY



The original limit of the Quadrinomial is FAH : RAY, which is a "Secondary" Second Order Chrome, the mistuning being the definitely liminal Comma 81 : 80.

The Tertriad formed upon a Trinomial whose Poles are diametrically translated is of some interest.

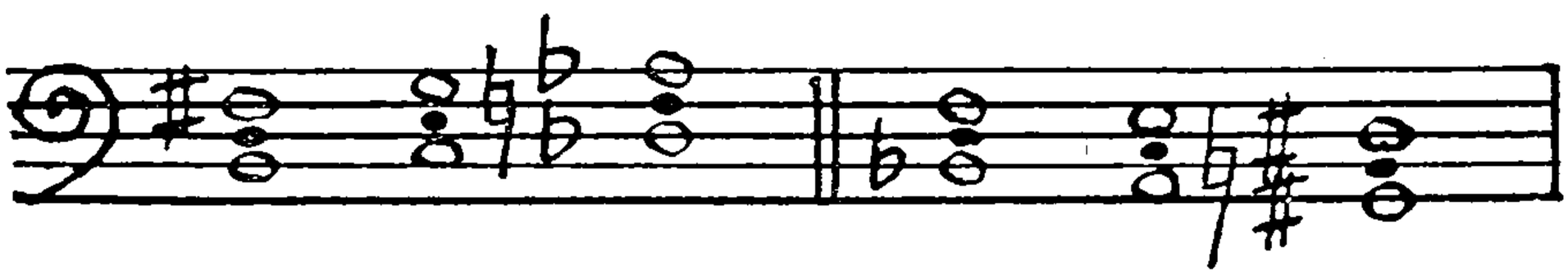
(The corresponding case with diametrical prime and unaltered poles need not be quoted.).

The Trinomial is—

π L P π T

D

and the corresponding Tertriad, with permutable determinators, is—



It will be noted that this presents a Core and Envelope; the "external" Envelope tones are only one E.T.A. out from the Core, resulting in a "Disjunct" of V dimensions.

This disjunct is found in the Symmetrical Tertriad (as well as its converse), and is familiar to musicians in the Coincidental Composite form known as the "Harmonic Minor," where it is recognised as an Augmented Second.

The interval represents a distorted Fluent, but still capable of acting as a Fluent, although of the same E.T. dimensions as the Chrome V.

On expressing Hemicyclic terms with respect to their Tertriadal "Order" we note that a Septomial Matrix contains—
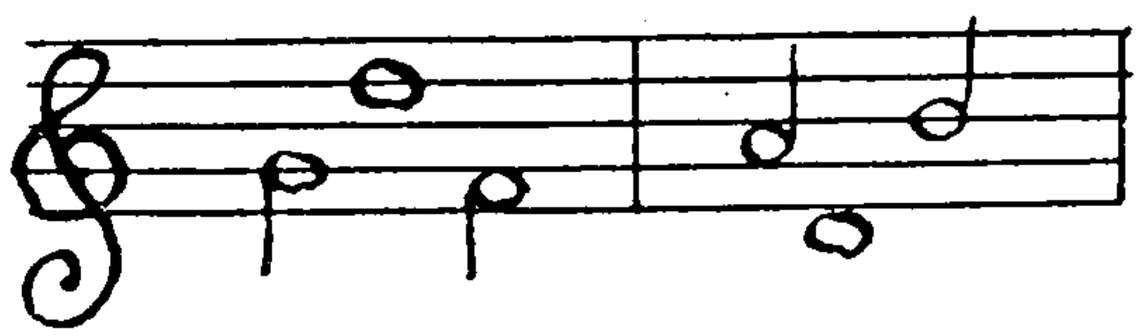
(1) A Yoke of plural Order.
(2) A Trinomial group of Zero-First Axes.
(3) A Trinomial group of permutable Determinators.

By commutation of species (2) and (3) are interchanged and the absolute direction of permutation is settled.

The Yoke J, being of plural Order, may appear either as—

(1) Secondary Axis connecting the Concomitant Species $\overset{F}{C}(TT)$.

(2) Permutable Determinator on the fringe of the system $\overset{F}{C}\left[\dot{L}(LD)\right]$.



In either definite Species the Yoke, as such, vanishes, presenting the Septomial as composed of—

(*a*) A Trinomial of Axes.
(*b*) A Quadrinomial of Determinators.

In any one Species the tone Ray (J) is permutable only in the opposite direction to that of projection in total species; thus—

Ray can be sharpened to Re in C species.
,, ,, ,, flattened to Ra in F Species.

In this particular aspect the Hemicyclic Matrix would include as possible components—

F Species. The Dextral Hemicyclic plus Ma, Ta, La, Ra.
C Species. Ditto, ditto De, Fe, Se, Re.
Both exclude Da, Mee, Fa, Sa, Le, and Tee.

System C.—The Serial type of Matrix is based upon the Envelope, the Core being regarded as the secondary factor.

This is evident from the examples quoted on page 43.

Case (1). The derivation of the Prime Serial from the early Series, by taking that portion of the Series containing the Core Triad and one intermediate member, is obvious.

The musician Schriabine has used this as a basis for his works, and though from an auditory point of view there is more imagination than foundation, yet one would not quarrel with the pioneering venture.

But to attempt such presentations with notation and claviature foreign to same is to defeat the theoretical basis of the process. We shall have more to say on this in Successive Tonality.

Case (2). The Series of the Tensor corresponds approximately with the Hemicyclic Septomial. That is its justification.

The acoustical aspect as derived from trumpet overblowing is entirely conflicted with by the evidence of musical history, which is surprising.

Case (3) is somewhat similar, but does not fit the general form so well. It is noticeable in that the Laxator Contradeterminator approximates to the permuted Prime Determinator.

Case (4). By regarding the Laxator as the Tensor Contradeterminator, and taking Radical Tetrads on both, we get a very fair correspondence with the general case. We retain the

coherence of the two elementary Tetrads, and a quasi Core is formulated by $T_{11\ 13\ 16}$.

The Laxator Contradeterminator is the Tensor Bi-contra-determinator. The importance of this tone is that it forms with the Laxator a non-invertible interval, practically the only one found in tonality. It thus establishes the absolute vertication of Tonal arrangements which we are often able to neglect.

The Laxator components of the Seriopolar Matrix are out of tune by Riemann Limen 64/63.

We can reverse the arrangement and take T as L_9, which gives an identity with the Tertriadal form.

In any case the Seriopolar Matrix is considerably out of tune. Its value lies in the coherence of its parts as coupling up the Envelope.

It is interesting to compare the Frequency numbers of these arrangements, and it is to be observed that the departure from the general form is in no case excessive.

Hence we may fairly claim that Coherence welds the Envelope into something more than "Core outsiders."

Type	P/E	E/P	T/E	E/T
Hemicylic (Aclinears, reverse for decl.)	48	60	72	90
Tertriadal, and Seriopolar with $T = L_9$	45	54	64	80
Ditto with permuted "determinators"	$43\frac{1}{5}$	54	64	$76\frac{4}{5}$
Seriopolar with $L = T_7$	40	48	56	70
Tensor Series	40	48	56	72
Laxator Series. Same as Seriopolar with $T \equiv L_9$				
Serial Prime (Schriabine)	42	54	66	84
Riemann Symmetrical	50	60	70	84
	$49\frac{7}{32}$	60	70	84

We may also note the comparison of Tertriad members on the normal and Schriabine values.

Approximate Tone	Doh	Ray	Me	Fah	Soh	Lah	Te
Normal	96	54	—	64	72	80	90
Schriabine	99	54	—	66	72	81	88

The Symmetrical Tetrad Envelope has another aspect of its own. From the standpoint of Beat-effect, *not serial concordance*, the

Externominant interval is the minimum approach of two tones without much antitone.

Although this interval converges with rise of pitch, yet if we assign a value and pile on each other, we obtain a kind of "beat-free" close form chord.

In the middle of the musical range this is about the Triple-violet chord identified as the Symmetrical Tetrad Envelope, which in this particular aspect is of close form, representable by E.T. tones, and homochromal, which renders it a useful "translative" member.

Consequently we should expect such a colligation to have a kind of acoustic predominance over other arbitrary forms, and given some close approximation to this chord as Envelope, a Triad Core results automatically.

By addition of a polar Axis to the Symmetrical Tetrad, we get the so-called "minor ninth chord" of the theory of harmony.

It will be noticed that the Symmetrical Tertriadal Matrix is not so much used in practice, but forms the variable basis of all systems of Tertriad in actual use.

The foregoing is a sketch of some of the principal aspects of the Septomial Matrix. They are all characterised by—

(1) More or less exact agreement with the General Matrix.

(2) A definite relationship between the members whereby each can be named or symbolised.

(3) The general derivation from certain Primary Axes, and closing of the group by the attainment of Secondary Axes.

The application of these aspects in the scientific view of Tonality is a combination, based upon the general form, of Triad Core and Tetrad Envelope.

(1) The Tertriadal Aspect is the primary basis, because of its foundation on two of the most elementary relationships of Tonality, viz. the Trinomial and the Triad.

Owing to the permutability of its Determinators, it can appear in eight different forms, viz.—

	1	2	3	4	5	6	7	8
Tensor Determinator	D	◗	D	◗	D	◗	D	◗
Prime ,,	D	D	◗	◗	D	D	◗	◗
Laxator ,,	D	D	D	D	◗	◗	◗	◗

Of these, the one corresponding to the Hemicyclic comes first to view.

(2) The Hemicyclic Aspect, since it contains no real Determinators, is of itself of little actual value as a Chordal Matrix; but it determines the particular form of Tertriad which has the properties of its own form, among which may be mentioned—

(*a*) Pythagorean translation, each "grade" of which only alters one tone, leaving six axial.
(*b*) The concomitance of Species by the same seven tones.
(*c*) Five of the classical "Modes" present five of the Tertriadal variants quoted above (the other two involve pseudo-First Order chromes).

For these reasons it upholds the system of notation and clavia-ture that has done such good service (and which seems likely to persist). It is the first aspect of the Tertriad that the student becomes acquainted with, perhaps acquiring an undue prominence thereby.

(3) The Serial Aspect is ancillary to the foregoing, from its power of binding up the Envelope members. Its peculiarities will be seen in dealing with the more complex chords.

Thus we arrive at the most predominant form of Matrix, viz. the Hemicyclic form of the Tertriad which provides the basis of Chordal structure, and from which all other forms may be approached by variation.

Hence when we speak of Hemicyclic Matrix, it shall be understood this is what is meant, as, strictly speaking, to talk of determinators in the Hemicycle is not correct.

The three aspects of the Domain are attained respectively—

(1) Hemicyclic. By inclusion of the five læval members. Limit attained by quasi-equation of the dextral and læval sixth members. This is very close. (Still closer is the mixed form $8\ B \times G \equiv W$.)

(2) Tertriadal. By associating the Tertriads (Heptads) about the two polar and the core Triad, thus obtaining the Tri-tertriad or Tri-heptad, whose reduction (with permutable determinators) presents the twelve tones. Limit attained by the "Secondary" Second Order chrome between LL or TD and TTD.

(3) Serial. By treating each member as a permutable Determinator the same result is attained. The case is not of theoretical importance. In practice it may be noted that the upper Serials 14, 15, 16, 17, 18, 19, 20 form an approximate Chromatic Scale.

If we take the practical attainment of Liminance as 128 : 125 (nearly 42 : 41) as given by the equation 3G = W, it is observed that the Riemann (64 : 63) and Comma (81 : 80) Limina can be regarded as vanishing, and thus the Secondary Axes are established as definitely limiting the Matrix.

16. The Chords employed in Tonality are all selected from the Matrix, nominated thereby, and where they are common to two Matrices present avenues of Translation.

Triads and their parts are Simple Chords. Compound Chords are built up of parts of Triads.

In general (as is obvious) the more complex a chord is the more freely it may be transformed into any other. The simpler forms are more definite and tend to be restricted in practice. At all events it is easier to inadvertently employ them in such a way as to produce Dissonance.

Of the chords possible we have—

(1) Triads, which are concordant parts of the Series.

(2) Chords formed out of the Tertriad, the limiting chord being the Heptad, known to musicians as the chord of the Thirteenth.

(3) The Tri-tertriad reduces to a Pentriad or Tridecanal, and with its permutable Determinators provides (in E.T.) the twelve tones of the Dodecanal Domain, and thus all chords possible within same. Several of these are of very limited application in practice, but they are theoretically possible.

The most "unrelated" chord to the Centron is its Diametrical, but this does not preclude its use.

We see the real meaning of the term "Matrix" (which is borrowed by analogy from mathematics and mineralogy), for we have attained the group out of which all chords within the E.T. can be extracted, and which indicates those possible transcending the limit. The crystallisation of form conferred by the E.T. makes a general theory of tonality possible, but it certainly

does confer a somewhat artificial appearance on the system, making new departures, such as those of Schriabine, appear somewhat more mystic than they really are.

17. The Contradeterminator splits R in the same way as D does B, but it is considerably out of tune with the E.T. system.

It is therefore regarded as a Discord with respect to Tertriadal terms, being represented, when required, in educt and adduct form by the two tones respectively between which it occurs.

In conjunction with the Triad, it presents the RADICAL TETRAD (symbolised by a square, within which the Prime is indicated).

The Tetrad appears in four forms, the general expression being N (1 : 5 : 3 : 7).

Symbol	*Constitution*
	P : D : T : ◻
	P : ◗ : T : ◻
	P : D : T : ◖
	P : ◗ : T : ◖

Radical Tetrads are thus distinguished from Envelope Tetrads, which are symbolised by a DIAMOND symbol, inscribed with the CORE indication. These also appear in four forms.

The Educt Fundamental Bi-contra-determinator FPCC is identified with the so-called Minor Thirteenth resolving upwards.

The F Contradeterminator (educt) of the Tensor Triad presents the well-known chord of the Dominant Seventh.

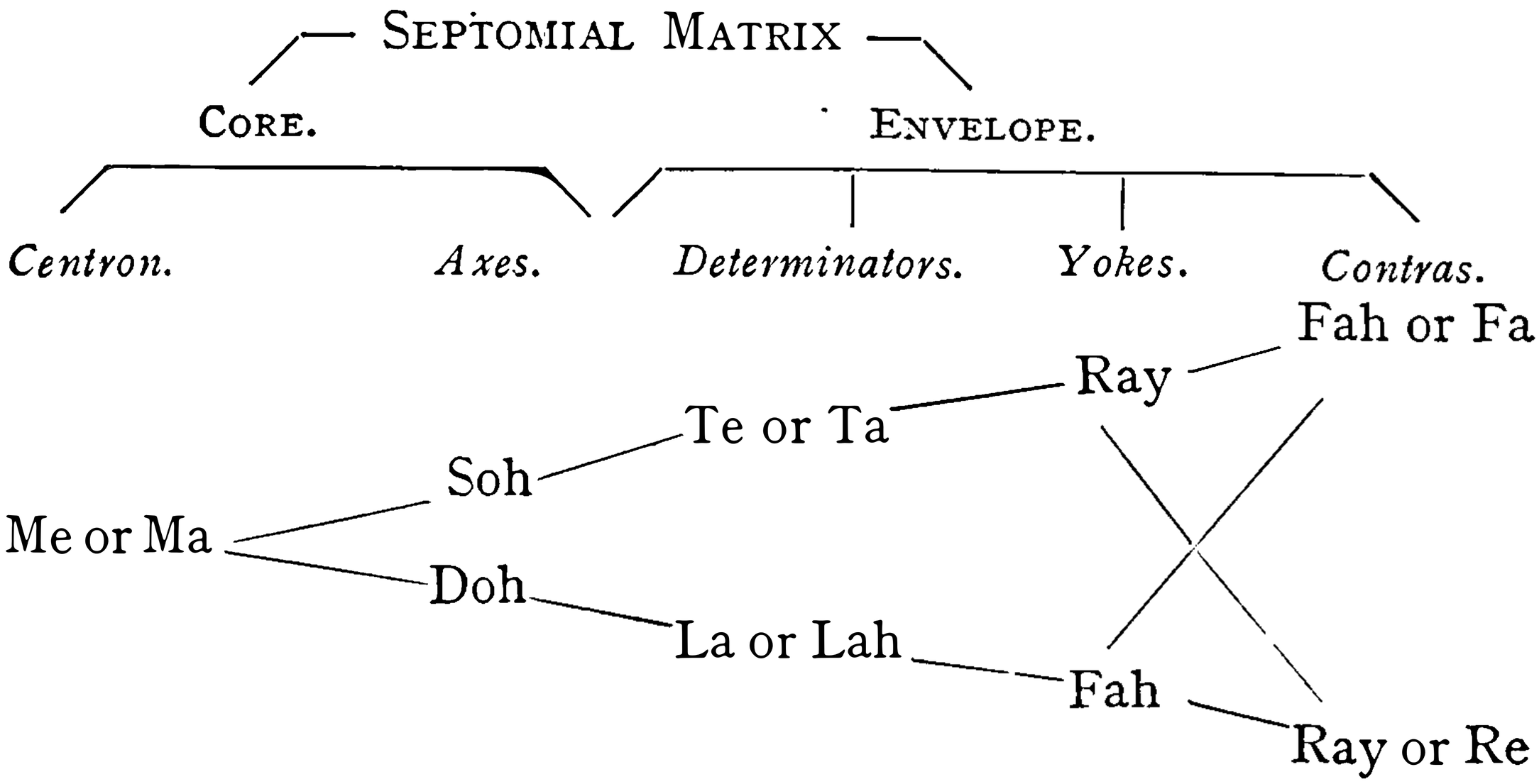
The Adduct form on the Laxator presents the "Chord of the Added Sixth." (The adduct form on the Tensor is rarer owing to its translational character, but is frequently employed.)

18. Hugo Riemann's equation $9 \times 7 = 2^6$ is important as presenting a quasi-cycle of period 2W.

The F and C Contradeterminators differ from exact Second Order relation by the small interval 49 : 50.

The opposed Radical Tetrads exhibit the neutral Core and Hexad Envelope as constituting the earliest "closed" system, or Matrix, as follows.

The examples are given in F Species.



The above diagram shows the basic character of the seven-toned Matrix, and the cross-coupling of the last pairs.

It also exhibits the Species independence of Core and Envelope, and the tonal importance of the Triple Violet Chord, which is perhaps the most consonant arrangement of tones after the Triad.

The Double Violet "diatonic" chord in the Envelope is of importance as the Link of Concomitance.

The closure of the Matrix is effected by the Secondary Second Order relation between Ray and Fah.

The coupling of Core and Envelope is effected primarily by the Axes, Prime and Tensor, and secondarily by the attainment of (approximate) vice versa relation at the respective Contra-determinators.

When the Matrical configuration is reduced to close form we obtain the seven-toned scale, in which definite fluent steps join up the successive alternation of Core and Envelope members.

Out of the varied forms possible with permutable Determinators, one coincides with the Hemicyclic form, which as we have seen can represent two Species (Concomitance). This form thus becomes the basis of Matrical reference.

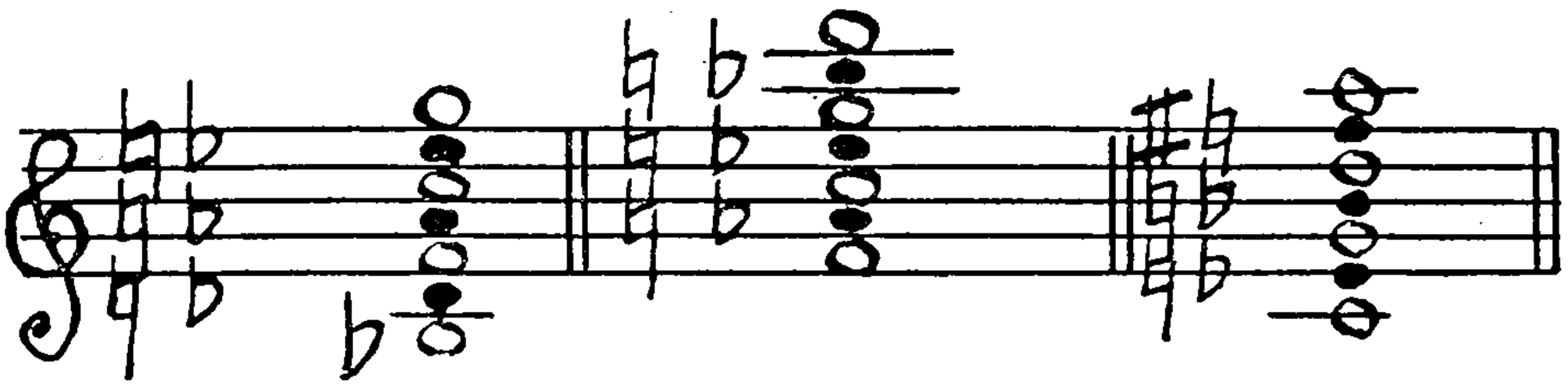
19. Every Triad has its attendant enveloping Tetrad, as well as being capable of extension to a Radical Tetrad itself, and the converse relation also holds.

We can thus have a TER-TETRAD of three enveloping, or three radical Tetrads.

The independent permutability of the Enveloping Tetrad is evidenced by the variants of the Minor Scale, of which three are recognised in practice. The corresponding independence of the Core is shown by the Tierce de Picardie or so-called "Hungarian" Matrix.
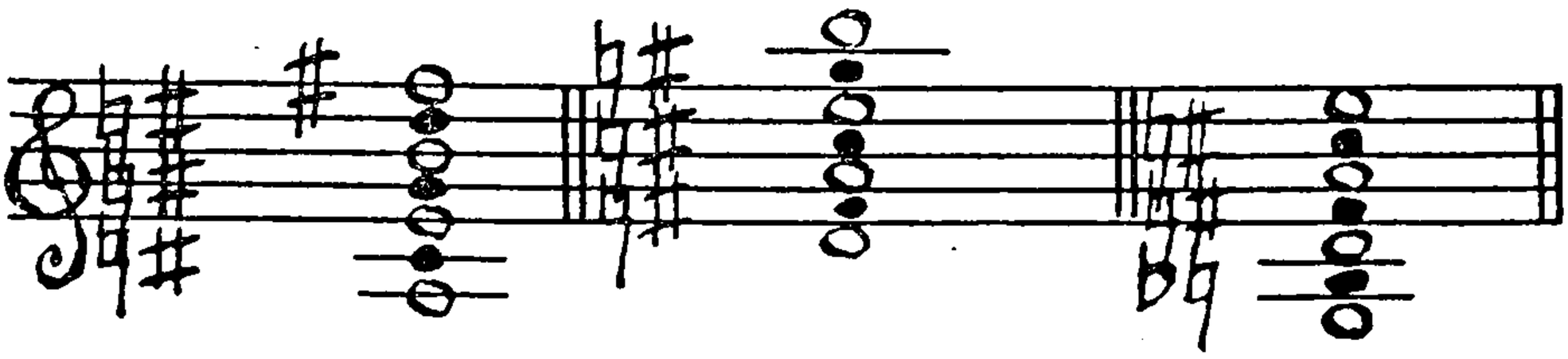
20. The three Tertriads, or Heptads, which constitute the Septomial basis of the Domain, are illustrated in concomitant relationship herewith :—

. Species.



L P T

. Species.



Musicians will easily recognise in them the groups presenting the so-called "root" sevenths, ninths, elevenths and thirteenths, and it is evident also why the "Supertonic," and not the Sub-dominant, becomes the root of the third group, as might have been expected at first.

On putting these three groups together we obtain a PENTRIAD, having five permutable determinators, giving thirty-two different possible forms of chord.

In itself this is an impracticable colligation, but out of it we can abstract those chords known in practice as "Augmented Sixths" and "Chromatic Discords."

21. The twelve tones of the so-called Chromatic Scale form what may be known as an Antinominant Cycle (every tone is the antinominant of its neighbour).

Since First Order intervals are in excess and defect of the Diametrical (half Octave) by one step, it follows that the Antinominant Cycle is the Pythagorean with every alternate member translated half an Octave, and that consequently two arrangements are possible.
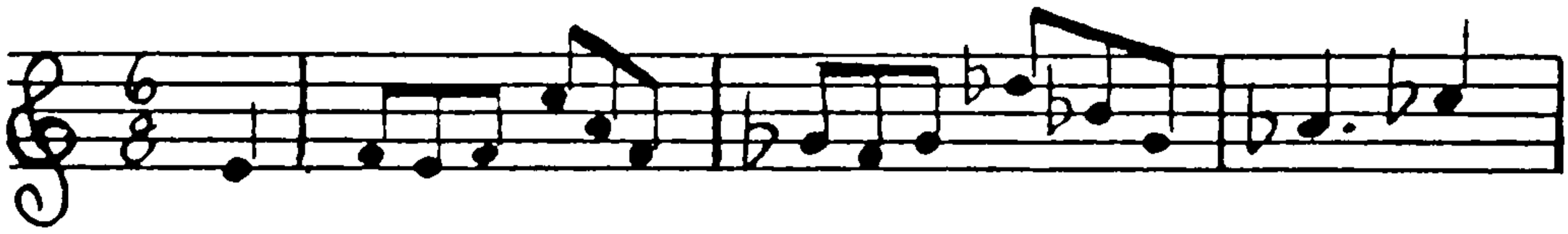
In this arrangement the relationship between the member is no longer Axial, as in the Pythagorean case, but each tone is a near neighbour, in fact is not far outside the limits of liminance. The shift of tone in stepping does not exceed that due to Permutation.

From this Antinominant Matrix a tonal system can be selected in a very similar manner to the Pythagorean case. But, owing to the latter being the basis of our Staff and Sol-fa Notation, the former looks most complicated, although the effect on the ear is nearly as smooth as with the Pythagorean case.
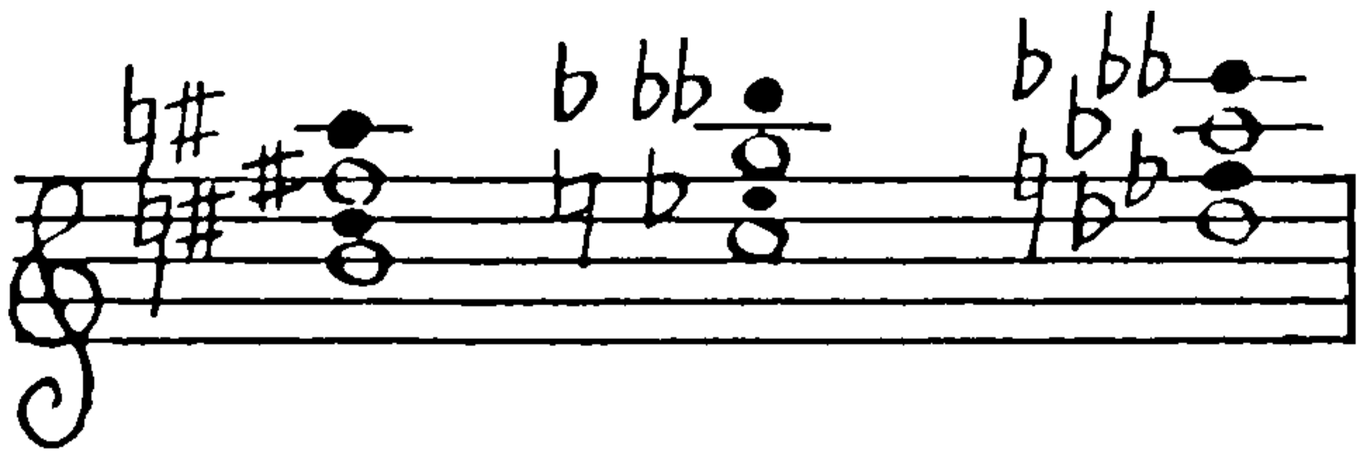
The symmetrical enveloping Tetrad envelope is the same in both Pythagorean and Antinominant forms.

If the Triads of this matrix are extended to Radical Tetrads by adding the Contradeterminator, the diametrical interval W/2 thus formed renders the progressions axial.

Example of an Antinominant Progression.



The three Radical Tetrads of the Antinominant Matrix (F Species).



22. The three compound chords, consisting of the Tensor Radical Tetrads about the three Triads of the Tertriad (and known in Fundamental Species as the Supertonic, Dominant, and Tonic Sevenths respectively), can be translated over a half-octave to the corresponding relations in the Antinominant Matrix.

In this position they are known as the Chords of the Augmented Sixth, on the flat Submediant, flat Supertonic, and (rare) flat Dominant, respectively.

The great point is that they normally resolve on the same Triads in both positions, as may be seen from experience.

There is yet another aspect in which these Chords of the Augmented Sixth may be considered.

Their normal resolution is by antinominant step diverging, and by analogy with the Envelope theory the outline interval tones may be termed the INVELOPES of the Core. This aspect exhibits particularly the difference of procedure in the Antinominant and Pythagorean relations.

Diametrical translation of the so-called "French" Augmented Sixth does not alter the relative constitution of the chord.

23. The so-called "Chromatic Sevenths" are those which progress in the opposite direction to that usual with Pythagoreans.

In this case the Impellant "resolution" 15 to 16, or vice versa, is replaced by the Contra-impellant 14 to 15, and vice versa. A corresponding case is the replacement of the Oscillant 8 : 9 by the Contra Oscillant 8 : 7.

The principle of this discrimination is seen in the different aspects of the "chords of the seventh" respectively as extracts from the Tertriadal (Hexad) Envelope and the Serial form.

In the first case we have the so-called normal "diatonic" Dominant Seventh class; and in the second case, the higher Thirteenth type, and the Chromatic class of discord.

The determination as to whether a tone of a Tetrad is a Contradeterminator or part of a chord compounded of Triad is evident from the context, but in experience it is found that the root form of the chord in F Species usually sounds much better and more distinct than when inverted.

This peculiarity is shared by practically no other chord. In the particular case of the Laxator and its Contradeterminator, in conjunction with parts of the Tensor triad, this non-invertibility of the upward resolving is most marked, and has given rise to much controversy. The question as to whether the note in question is a minor thirteenth or a sharpened fifth is seen to be wholly beside the mark.

The Symmetrical Homochrome chords, viz. Symmetrical Tetrad, Double Green, Double Red, etc., have great translational facilities, being axial in as many matrices as there are tone components.

The use of "quarter-tones of $\frac{1}{2}$ A has been suggested, and indeed implied in parallel Second Order progressions.

Such tones have hitherto not been found practical.

It may be noted that the Violet Splitter Serial II fills this place in the Triad on TT $(11/10)^{24} = .993432$ W.

The interval is too near liminance to be of much use at present.

24. *Vertication.*—Hitherto we have neglected the absolute achromatic arrangement of chord members in pitch, and this abstraction is justified by the many important qualities which are unaffected by actual arrangement.

We now consider the characteristics of Tonality depending upon the absolute direction in pitch, which because of their importance are collected under the name of Vertication.

A Series can be regarded as a *sum* of superimposed intervals, W, B, R, G, V, etc., but it may also be considered as a *bundle* of the skeleton intervals reckoned from the Prime.

This view may be applied to any chord, taking the lowest tone as Axis. Since the F Series is an actual, while the C Series is an artificial arrangement of tones in pitch, only the former is of auditory importance, and the lowest tone in either case, the Bass, is the unit of reckoning.

There are one less in number of bundle elements than notes in any chord, and it is found by experience that these intervals are of greater audentity than the successive serial intervals.

This is illustrated by many cases, among which we may notice—

The chord formed by the bundle of Y and G or M and G is in ordinary arrangement constituted by the superposition of the chromes R plus G and R plus V respectively.

The consecutive succession of such chords sounds more like the first than the second arrangement, for the Second Order chromes are consonant consecutively, while R is dissonant, and the whole "fauxbourdon" of these "first inversions" is definitely consonant.

Again, a consecution of the bundles formed by the super-

position of B on Y or B on M sounds better—although still dissonant—than its inverse with B appearing in the " bundle."

Bundle analysis in the first case yields $\begin{matrix} W + G \\ Y \end{matrix}$ and $\begin{matrix} W + V, \\ M \end{matrix}$ which, neglecting the achrome, are of Second Order.

Again, the chord W/2 superposed on V sounds much less discordant than its inverse, V on W/2; so much so that it is by general consent admitted into Strict Counterpoint, which otherwise bars all discords and directive intervals.

It is seen that in the first case the bundle contains Y and V, two Second Order chromes; while in the second case W/2 appears along with Y.

In order to distinguish clearly between the *superposed* and *bundle* intervals in a chord, we may make use of a little known theorem of George Boole, who considered the algebraic product of magnitudes in the same straight line. According to this strictly logical finding, which we have not space to discuss, we may regard superimposed intervals as successively *added*, and bundle intervals as *multiplied*, and symbolise chords accordingly.

Vertication is particularly evident with the Laxator Contradeterminator, in conjunction with the Tensor triad.

This chord sounds well when the Contradeterminator Re is above the Laxator Fah, but if this arrangement is inverted it loses much of its distinctive agreeability.

These observations, easily proved on the piano, form the basis of the practical way of viewing chords known as Thorough-bass, in which the intervals are reckoned and figured from the bass tone (in the case of Pedals, the local Bass is taken). This practice was based upon auditory experience.

It is probably due to the almost unconscious reference of intervals to the Fundamental Prime, and is possibly affected by the physiological phenomenon of the Membrana Tympani " focussing " itself upon the lowest tones present in a sound, as discussed by Alfred M. Mayer, Lord Rayleigh, and various foreign investigators.

Applying bundle analysis to the Series, it is seen that the Applement R and complementary intervals such as V, Hypo-V, etc., do not appear.

25. It will be noticed that all the regular forms of Chord

(excepting the French Augmented Sixths) appear as a sum of Second Order intervals, or "thirds," thus eliminating closer spacing.

In this case the intermediate tones, which would fill up the "thirds" to form a scale, are definitely recognised as not being members of that particular chord *in that form*, and when passing along a scale we step alternately from chordal to non-chordal tones.

The use of these non-chordal tones as "Autophors," or tones which carry themselves independently of the harmony, is a matter for discussion in Successive Tonality.

26. These notes on Simultaneous Tonality are simply to introduce the next Division, and therefore conclude at this point, fuller details being found in the volume published in 1916, the matter of which was written in 1912.

HYPERACOUSTICS

DIVISION II

SUCCESSIVE TONALITY

PART I

THE INCEPTION OF PHONALITY

CHAPTER I

CONTINUED IDENTITY AS THE BASIS OF A PHONAL ASPECT

1. Prominent Conditions forming Commencement to Study

In any particular aspect of a phenomenon, there are generally to be found certain prominent conditions whose examination forms a natural commencement to study.

In Simultaneous Tonality the harmonic series is obviously the basis of theoretical treatment, and in the present case a commencement may be made by examining a condition of considerable importance relating to tones in succession.

2. The Act of Listening. Continued Identity of Source and Sound

The act of listening to a tone implies attention to its source: thus the persisting identity of the latter is by implication conferred upon the former.

The ear is so sensitive to any change of source (unless very exact jointing is contrived) that a tone, though merely a physical train of vibrations, is perceived and thought of as an entity which

is conveniently defined by its pitch, within wide variations of enunciation, duration, loudness, character (tone-tint), mode of finish, location of source, etc.

There are, of course, certain acoustic limits to the rapidity of a sequence of sounds, but within the ranges employed in music, these have but little formulative effect.

The acoustic importance of pitch as a measurable determined attribute of tone is responsible for its predominance in tonal theory: thus in abstract we commonly define tones by pitch alone.

The identity of a source persists to a great extent even when pitch is varied by sliding up and down the range. This constitutes the portamento or flexion, whose continuity persists even when the sound is intermittent.

Listening to a varying tone becomes an act of following a linear identity, giving rise to the concept of a Phonon.

This term is abstracted from the expressions Polyphony, Cacophony, etc.

The continuity of a Flexion is also associated with the generation of musical sound, as by the muscular sense of stretch in the vocal chords and with the spacing of control on many instruments, with a corresponding implication of Adherence between successive tones forming a Phonon.

Variation of frequency continuously is an acoustical possibility associated with geometrical and mechanical conditions of acoustics, the result being an audible continuity of pitch variation throughout the range of the ear.

3. Definition of Phonality

Phonality is to a considerable extent independent of chordance and of older origin in musical development.

It is probably attributable to the primary interest of man in sound, *i. e.* listening to sources of sound favourable or otherwise to well-being.

In chordance one considers usually only the progression between two chords as two sounds, but the range of phonality is wider, since attention is directed to a line of continuity instead of two connected " points " of tone.

The number of co-progressing phona in a musical passage is denoted by the prefixes Mono, Di, Tri, Tetra, Penta, etc.

It may be recalled that Oettingen uses the term Phonality for what we call the Coincidental Species.

4. Production and Audition of Tone

Man possesses organs of great capability for both producing and hearing sounds, musical or otherwise (though the range of audibility far exceeds that of the voice).

In the sister sense of colour vision there are no specific colour producing or changing organs, thus entirely differentiating the development of the arts of colour and of tone. Possibly the chameleon may possess some rudimentary sense of colour-change continuity: man certainly has none at present. Hence the futility of the many ingenious attempts to develop an art of colour-music.

5. The Different Way in which Sound is employed for Speech and Music

Sounds are principally employed by man for two determinate purposes, Speech and Music.

The former requires a type of sound suitable to the distinct expression of a conventional code and its extensional variants in thought. The latter presumes as a beginning attractive character and definite elements.

The continuous flexion is not generally musical, being naturally associated with cries of pain, wailing, etc., as may be observed with the Dœppler effect; but it serves the flexional variations of a "stem" sound required in the complex secondary process of speech.

The discrete scale of tones, *i. e.* definite notes of definite duration at definite intervals, exhibits that peculiar attractive character which we call "melodiousness" in itself.

We are unable to explain this, any more than we can the agreeability of colour to the eye, but this attractive property is the first qualification of the discrete scale for attention, apart from any such secondary use as implied in speech.

The "stepped" scale retains much of the linear identity of the flexion, and it lends itself to the use of determinate constructive effort.

It preserves the definite interval steps that enable it to be noted down in memory and on paper and reproduced at any subsequent time at any practicable pitch.

Hence sounds fall into two classes, one of which is used for speech, and the other for musical purposes.

Throughout the course of its development musical tone has been influenced by its association with speech, but both have evolved on their own lines.

6. Perception of Pitch and Interval. The Step

The perception of pitch position, other conditions apart, is referable to the alteration in cyclon quality varying from massive to acute on rising.

It is difficult for the average unaided ear to judge pitch position nearer than about half an Octave.

A tone appears as a point on the pitch-range bounded by the regions above and below. When another tone is brought into consideration the interval can be judged very exactly apart from its pitch position, and thus the system of tonality is based upon Interval rather than pitch position.

The predominance of the Octave and its recurrent character practically compel the inclusion of this interval in any system of measurement.

As it is too large to be itself a unit, the problem of pitch measurement turns upon the fraction of the Octave that shall be selected as a unit.

Of course other methods are possible and have been developed, but our investigation is concerned with the general method of widest currency in all ages, namely the most determinate system combined with the greatest convenience.

Such a system would favour the division of the Octave into equal intervals of such a number as to comprise the early aliquotients 2, 3, 4, 5, etc. As we have seen, this implies respectively 2, 6, 12, 60, etc., units, the growth of the number and the corresponding diminution in the size of the unit being rapid.

7. Influence of Metric Rhythm on Stepped Alteration of Pitch

Concurrently with the development of the art of music from tone, there has been the rhythmic side (which is still evident in the practice of so-called uncivilised nations).

Consideration of Rhythm and its metric framework is treated in another Division, but it is interesting to note that the incipient agreeability of a metric impression and its capability of determinate development resemble that of tone.

CHAPTER II

THE DISCRETION OF THE PITCH RANGE

8. The Germ of Melody in the Scale

The succession of discrete tones known as the Scale exhibits the germ of melody (this can be developed by rhythmic variation). This attractive characteristic, combined with its capability as a medium for presenting structural figures and retaining much of the flexionic character of vocalisation, makes it the basis of consideration.

9. The Idea of Scale

The scale is the oldest form of regulated musical structure. It is first inferred as a catalogue of a system of the fixed tones employed in forming particular melodies.

With oral transmission no very wide circle could be expected, so that the condensation of practice necessary to attain generality and economy implies a system of definite scales.

The phonograph now enables the music from all parts of the world to be collected and examined. That of ancient times has largely to be inferred by deciphering notation, tablature, and description, and by reconstructing data from ancient instruments and illustrations.

10. The Two Kinds of Scale. Chromatic and Diatonic

There are two kinds of definite Scale—

(1) An indifferent equigrade, as a Domain of melodic form.

(2) A nominated system not necessarily equigrade, which appears at a very early stage in history.

The relationship between the two provides for the translation of melodic forms unchanged over a limited range, thus tending to a greater precision than the original pitch continuum provides.

Musical history shows that the accession of harmony (effect of Chordance) was slow and indirect in determining the shape of scales. Its inception was the recognition of the Octave and First Order intervals; those of Second Order had but little influence until a much later date.

11. Economic Selection of a Domain

The practical wide and general appeal of music is favoured by the economic selection of a limited number of tones for a "Domain" instead of the range of continuous pitch. Upon this a still more limited system of nominated tones could be projected. This forms the starting-point of a determinate tonality from which an infinite number of individual variants could branch out.

12. The Three-fold Relationship between Tones

The relationship between tones may be regarded as three-fold—

(1) Coherence in a chord.
(2) Adherence in a linear identity (Phonon).
(3) Inherence in a determinate system of nominance (Matrix).

The first refers to the leap from value to value in the blending Series.

The second, to the original flexion of a tone as referred to an identical source.

The third, if not involving the others, is originally based upon the relationship of two tones to a mutual third tone.

Consequently a tone may (1) leap, (2) slide, (3) change its name.

13. The Bias of Absolute Vertication

Tones and intervals are conveniently thought of in abstract as apart from actual pitch, which to a great extent holds in practice.

They are, however, subject to the ineliminatable condition of Absolute Vertication, being reckoned in direction by Species; *i. e.* Fundamental in the direction of the Harmonic Series, and Coincidental in the reverse. Intervals in succession are thus differentiated from the static aspect of simultaneity.

14. Discrete Tones, as opposed to the Continuous Flexion

Discrete tones are those held for a short time (long enough for the auditory mechanism to locate them) and followed with, or without, intervening silence, by other similar tones.

The conditions as to the discretion of the pitch continuum into a scale of small steps is implied by some method of fixing tones in pitch and time.

Neither theoretically nor æsthetically are we limited to a fixed system, but for general determinance such must be the basis.

The maintenance of flexionic continuity is favoured by the use of steps smaller than the arpeggio of simple chords, but not necessarily so small that the discrete definity is lost by mergence into a flexion.

History shows that the primary discretion was not directly that of the upper series of tones such as is given by a trumpet, as might have been supposed from the known early use of such instruments; although harmonics from twelve upwards do form an approximate scale and have been so used.

15. Acoustic Conditions other than Chordal favouring Discretion

We therefore turn to the effect of acoustic conditions other than chordal for a first discussion.

(1) Theory of Parasyntony.—When we experiment with two tones either by direct or indirect methods, we note that the "antitonal" jar of beat effect rises from a zero at unison to a region of maximum efficiency beyond which it gradually fades out with widening interval.

We therefore recognise three "Regions" on each side of the nominant tone of (1) Liminance, (2) Antinominance, (3) Extern-nominance, each somewhat indefinite as to boundary, but which converge with rise of pitch.

These may be compared with the physical effect of a tone on a range of tuned syntonisers. On such an instrument as a vibration tacheometer, for instance, there appears a "hump" curve about the "nominant" periodicity, with a blunt top and sloping sides.

The boundaries of these three regions are not in themselves sufficiently determinate as to form the basis of a discrete scale.

(2) Psycho-physical aspect.—The transposability of an interval as an identity recalls the general basis of measurement as developed in the Psycho-physics of Fechner and Weber, now recognised as an important division of experimental psychology.

It is interesting to notice that the general ratio of stimulus to

perceptible limen was found to be about 1 : 3, which is the approximate pitch ratio between the minimum chrome, dodecanal step, and extreme liminant.

The pyscho-physic method is based upon the estimation of actual pitch distance. If an arbitrary unit interval be chosen it can be repeated as a recognisable entity successively higher and lower in pitch, which presents an equigrade scale as the simplest basis of determinance.

This measures pitch by a constant ratio of frequencies, *i. e.* by logarithmic spacing.

We do not maintain that the actual sense is logarithmic : it is certainly not parallel with frequency change, and of itself is more akin to a thinning out of mass rather than a spacial extension.

On the basis of interval identity, which is a primary experience, pitch is naturally judged as a linear extension, so that by the logarithmic relation we are able to link up the visual dimensions of sound generators (*e. g.* harp strings, organ-pipes, etc.) with pitch position on the linear pitch range, and thus are able to use the equigrade system for shaping melodies somewhat like plotting curves on squared paper.

We must always remember that the " Pitch " of audition and that of calculation are not necessarily the same thing, and that they are only connected by the capability of grasping an interval as a thing in itself.

The Octave becomes the most natural base of such a logarithmic system, as we have seen in Simultaneous Tonality.

16. The Establishment of Scale apart from Chordance

From these two conditions we see that the establishment of a scale is not effected apart from tonal factors.

Parasyntony tends to a limitation in the size of steps, and the logarithmic view implies an equigrade domain replacing the free-pitch range as a generalised basis upon which individuals can pool their experiences of mutually recognised intervals.

But the establishment of the scale itself is a matter involving tonal conditions, and the way in which these come into action quite independently of any recognised system of chordance is discussed in the next chapter.

17. The Effect of Practical Conditions of Performance

Out of these acoustic conditions the practical requirements of performance—claviature of instruments and convention of notation—tend to crystallise a limited system.

Although the vocalist (and certain instruments partly) is independent of claviature, experiments show that singers tend to the spacing rather than the chromal aspect of intervals.

The limen of larynx control (about W/48) does not seem to affect the general problem.

The musician seeks extension in order to obtain desired effects, but the wide diffusion of music in the world implies that limitations such as those of the fingers, etc., shall control practice (a very interesting aspect of the history of musical development that repays study).

Most young students find notation and claviature quite complicated enough, and the average performer soon finds his limitations. Hence the general desideratum is the smallest range of sounds as domain, and as nominated system, that is capable of doing what is required.

Thus such an economic generality obtains predominance over the many possible, proposed (and freak) systems of notation and claviature, which tend to quietly die out in spite of some excellent sponsors.

CHAPTER III

THE *A, E, I, O, U* METHOD OF CLASSIFYING INTERVALS ACCORDING TO PHASE

18. The Peculiar Recurrent Property of the Octave

The Octave is an interval of predominating importance from the acoustical point of view.

It is the first interval of the Series, therefore the first to spring into being from any condition that gives rise to non-cyclon vibration. It is also the relationship due to "side-forcing" on the toggle joint or Melde-Faraday basis.

Since it is the simplest interval it cannot be referred to anything simpler for comparison. Thus, like the sensations of colour-white, tastelessness, blood-heat, inodour, etc., it forms the natural standard of interval comparsion in the graded system of spaces.

Although its acoustical predominance is obvious, we are quite unable to explain its recurrent character (Synergy is an hypothesis), but this is a definite fact of experience which is shared by no other interval, at least to any extent. Like ferro-magnetism as an outstanding property in the para-magnetism of certain substances, it is a predominant phenomenon.

It may also be noted that any number of Octaves sound much the same as one, except being a little "hollow." The same cannot be maintained of any other interval.

19. The Effect in relation to Smaller Intervals

The recurrent character of the Octave is not perfect, but it is so pronounced that for many purposes the interval may be treated as if it were a unison, as is shown by the invertibility of chords, etc.

Starting from this standpoint we wish to examine all other intervals, whether given by serial relationship or approximating thereto.

If we are prepared to admit that the sense of pitch extension is sufficiently similar to that of the projection of magnitude in space, we are able to deduce certain important characteristics in the fractional parts of the Octave, based upon the aspect of Phase which arises from this characteristic recurrence.

20. The Cycle as the Geometrical Analogue of Octave Recurrence

We may regard pitch space as extending vertically and twisted by Octave recurrence into a Helix.

The abstract Octave space may be represented by a plane circle and its passage in pitch by a Traverse, which, starting from a nominant tone, returns after the completion of a "Unitor."

When a periodic variable approximates to regular form it may be treated as if it were circular, with confidence that the results will more or less closely approximate to a true state of affairs.

21. The Opponence of Sign exhibited by Intervals near Half an Octave

Upon halving the cycle we obtain two directed "Opponents," respectively Out and Home (project and retract), with regard to the nominant tone.

The character of these half-octaves and the Serial intervals nearest to them is difference of sign or Polarity.

The First Order serial intervals B and R are found to exhibit these directed characteristics, which are entirely different to those of the Octave. Their analogy to concave and convex semicircles is seen to be rational.

Other analogies are easy to find in physics, chemistry, mathematics, and the sciences generally.

22. The Quadrantal "Neutrality" of Quarter-octaves

Halving an Opponent gives rise to a "quadrantal" neutrality directed neither out nor home, but *across* the implied direction of the Opponent.

This resembles the mathematical relation of orthogonality as expressed by $\sqrt{-1}$, which is not a minus quantity but a value of independent sign.

The Serials G and V, and their applements M and Y, are of this type; they are of Second Order, and to express their "in between reversal" character we term them Intraversants. It is to be noted that the limit of chromality is reached with same.

This characteristic independence of opponent polarity is seen to be a reversal of that of Opponence, in the same way as this latter reverses the Unitor qualities of the Octave; so that the result of each successive split is to a great extent contrary to the specific phase character of the original interval concerned.

23. Half-quadrants, Octants, or "Extramural" Intervals

The halves of Intraversants (the octants in phase) are of an intermediate character, being half-way between orthogonal neutrality and identity in phase argument.

A term laying off in this manner may be regarded as adjacent but different to a nominant term, and as such the expression "Extramural" perhaps conveys the idea.

In dimensions such an interval is practically an Oscillant. It is therefore definitely not a chrome, nor yet small enough to be a limen.

24. The Interval of Alternative Values of Scale Tones

On halving an Extramur, we approach very near but do not quite attain identity. The phase argument is now small but not indiscriminate.

The effect is rather that of a slight deviation in the term itself to an alternative value of the nominant.

If we exceed this, we pass outside the identity, *i. e.* to an Extramur. If, on the contrary, we do not reach it, the effect of deviation is practically negligible; thus nothing is gained by further halving.

25. Table of the *A, E, I, O, U* Classification of Intervals

These five classes of interval may be remembered by the initial symbols *A, E, I, O, U*, which should always be written

in script to distinguish them from the letter symbols already in use.

It is to be noted that the following relations hold :—

Name.	*Symbol.*	*Order.*	*Degree.*	*E.T.A.*
Unitor	*U*	Zero		12
Opponent	*O*	First	First (Chromes)	6
Intraversant	*I*	Second		3
Extramur	*E*	Third	Scalar Fluent	1½
Alternative	*A*	Fourth	Permuting Fluent	¾

The idea of this special view is to ascertain the *phase* characteristics of the Octave fractions apart from their *serial* qualities on which the basis of chordance rests.

We are thus able to note the character of a discrete scalar construction apart from the bias of harmony; that is to say, we find out the properties of the pitch space before applying the conditions already discussed in Simultaneous Tonality, and this method enables us to approach the subject of Successive Tonality with an open mind.

26. Examination of the Properties of these Classes of Interval

It is to be noted that according to the assumptions made at the commencement of this chapter, we can deduce certain properties of intervals less than an Octave from those of the Octave itself, and these deductions would hold good even if the Series did not extend beyond the Achrome.

It remains to be seen whether this is at all a fact of experience.

The "Unitor" character of the Octave is taken for granted.

The opponence of the First Order intervals, and of the exact half-octaves when regarded as augmented and diminished, *i. e.* distinguished in relative place within the Octave, is the foundation of not only Tonality, but much that determines Organisation, as shown by the influence of fugue-like forms in practice, for example.

The neutral combinability of Intraversants is seen in the abundance of consecutive thirds and sixths found in practice.

The neighbouring non-identity of the Extramurs is evident in the average dimensions of the "named" scale. Finally, the alternative permutability of the determinator within the triad whose general species is constant, is an important principle of both theory and practice.

Without claiming too much for the new way of looking at intervals, it will probably be found helpful in understanding the course of history in which a system of scales suitable to chordance developed long before the chord itself.

27. Ternary, and other Divisions of the Octave

When the Octave is divided into three portions, the case is analogous to that of the three cube roots of unity; two of which are polarised in sign, the other being neutral.

On the traverse basis such divisions resemble the sides of an equilateral triangle, which, starting from a point, are successively Out, Neutral, and Home.

The boundaries between these are of Second Order, and the neutral division contains the First Order relations.

We may compare these relations with the concomitance of Species and direct Second Order relationship of the Out and Home intervals, as contrasted with the neutrality between the bounding tones when expressed in Pythagorean form (as keys of 4 sharps and 4 flats respectively).

These theoretical distinctions, of course, vanish in equal temperament.

As a matter of fact, the Second Order liminability of these G intervals is not so far removed from the true quadrant Intraversants. The Ternary view is thus somewhat in the background.

The Octave may also be divided into 5, 7, etc., parts, but nothing is gained thereby, since the variation from the halving system is small.

28. Implication of a Unit of Pitch Measurement One-twelfth of an Octave

It is interesting to note that the intervals due to bisection of Frequency (B and R) and of Pitch (W/2) differ by an interval approximating to one-twelfth of an Octave, which thus appears

as the polarising factor when added to or deducted from the neutral W/2 (and whose accumulation to the minimum chrome of Second Order forms the practical limit to Pythagorean extension).

Compare this fact with that of the division of the Octave into twelve units in order to obtain aliquotients 2, 3, 4, and 6, but not 5, 7, etc. (which involve much smaller units). We see that there is a strong presumption in favour of Dodecanal division apart from the conditions adduced by inter-determinate Chordance.

CHAPTER IV

THE APPLICATION OF THE *A, E, I, O, U* METHOD TO THE TONAL MATRIX

29. THE CONVERGENCE OF THE SERIES; AN APPROXIMATE RULE

WHEN the Series is plotted out in pitch it is seen that the values converge. But the rate of convergence also converges, so that sections approximate to equigrade for short stretches.

In mathematics this fact is shown by the value of the following expression, where x equals the Serial number.

Starting from unity, the value of $\left(\frac{x+1}{x}\right)^x$ creeps out from 2 to the Napierian e, a transcendental value approximating 2·71828.

This means that if we take an interval between an Octave and an Octave plus a sharp fourth, and chop it up into x equal parts, these latter will be closely (exactly, if we hit the right initial value) Serials of the same membership number x.

The importance of this well-known theorem to Tonality is in showing that representation of Serials by Equigrades is not so inaccurate as to be an absurdity.

30. COMPARISON OF SERIALS WITH THE EQUIGRADE SYSTEM

The most convenient way to compare the early Serials with an Equigrade system is either by—

(1) Tables of comparison (usually with Semitones, Cents or 1000ths of the Octave).

(2) Graphs of the hyperbolic curve on squared paper, which can be plotted from a table of Logs or with the slide-rule. This is perhaps the most convincing method.

By either method we are able to show side by side the Equigrade system and the Serial relations classed in Order, Degree, and Phase, as well as the derived Matrical systems from which we get our tonal scales.

We also note the important fact that the relative error of mistuning is not excessive up to P6.

31. Chromes and Fluents merging into Liminants

From the tonal point of view the Series is divisible into regions according to the size of the interval, which are known respectively as Chromal, Fluent, and Liminal.

Chromes are formed by the early series intervals of "Externominant" dimensions, up to an elastic boundary whose average is P6, so that 6 : 5 is recognised as a Chrome, and 7 : 6 as beyond.

Fluents come next and gradually merge into limina as they converge. These latter are intervals, but too small to be generally effective.

32. Steps of Scale are of Fluent Dimensions

The steps of the Scale are of Fluent dimensions. If larger, they are liable to be confused with arpeggial leaps of chromal dimensions. If smaller, they become limina, and approximate to flexionic continuity which is not generally determinate.

The determinate basis of the limit is the inter-relation between the three types of region.

It is evident that average boundaries can be estimated, and by the system of Degrees, these inter-determinate a general system.

But there is another basis depending upon the maintenance of distinct opposition of phase exhibited by the Serial Opponents B and R.

The Serial interval 7 : 5 is closely approximate to a mean between same (its double is a false Octave, error 50 : 49).

Consequently this interval falls outside the criterion of concordance, and thus P7 is regarded as an "Umbral."

The rationality of this view is based upon the predominance of Polar Opponence as a phenomenon observable in all types of musical structure.

The approximate mean is one of a general class $\frac{2n+1}{2n-1}$ formed by adding the respective adjacent numerators and denominators, the error from the true mean diminishing with rise of membership.

33. The Series of Fluents

It may be generally noted that the rate of convergence of the function $\frac{1}{x}$ is $-\frac{1}{x^2}$.

For the Serial intervals we get the second Series of Fluents or Discriminants as interpolated in the row of frequency ratios below, with their symbols written above.

W	B	R	G	V	Hypo V	Super O	O infra	O ultra
$\frac{2}{1}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	$\frac{8}{7}$	$\frac{9}{8}$	$\frac{10}{9}$

R	O	I	S	Inter 2	2	Riemann	Comma
$\frac{4}{3}$	$\frac{9}{8}$	$\frac{16}{15}$	$\frac{25}{24}$	$\frac{36}{35}$	$\frac{49}{48}$	$\frac{64}{63}$	$\frac{81}{80}$

The Serial Chromes are given in frequency by $\frac{n+1}{n}$, and the converting Fluents between same $\frac{n^2}{n^2-1}$ by the ratios.

If Fluents are subject to the same criteria of boundary as their generating Chromes we then get—

2/1 to 6/5 as Chromes; 7/6 and beyond as Umbrals.
4/3 to 25/24 as Fluents; 36/35 and beyond as quasi-merging into actual Limina.

The leading Fluent 4/3 is identical with the "Retracting" Chrome R. The next Fluent, the Oscillant 9/8, is definitely beyond the Chromal limit, and the intermediate "Contrachromes" form Insulators between definite Chromes and Fluents.

34. View of a Scale

In the Successive aspect of Tonality we are particularly concerned with Scale systems as limited by Arpeggio and Flexion.

A Scale appears as—

(1) A measure system of pitch space.
(2) A nominated system of tonal determinants.

The *E* and *A* class of Equigrade scale are seen to respectively approximate to the Tonal and Domain forms 2 and 1 above.

35. The Selection of a Unit

The E.T. system, in taking a unit approximating to B — W/2 — R, does an act which may look unscientific from a theoretical standpoint, but which practice accepts without much sense of trammelling, as actual works prove.

On striking an average between the Serial and Cyclic Properties of tonal relationship we obtain a tonal system of the greatest generality and economy, depending upon the relative accuracy of the intervals of greatest Audentity, *i. e.* the early Serials. This enables melodies to be constructed and the power of harmony to be employed to a great extent.

The E.T. unit is practically a mean of the E and A class of Equigrade, and in using two different symbols for the same thing, our conventional notation makes use of an abstract concept.

36. The Criteria of Degree and Order

If we admit the necessity of an Equigrade system, and limitation by economy to the least division of the Octave, as well as the recognition that any integral division of the Octave must be measured in units which also measure the lower integral divisions, we see that the Equigrades will be fractions of the Octave, either 2, 6, 12, 60, 420, etc.

Now one-twelfth an Octave is definitely fluent, one-sixtieth is practically liminal. Hence there is a strong predisposition to the Dodecanal system.

The comparison with the criteria of Order and Degree show in detail the deviations, as well as the fact that W/12 measures fairly well the chromes up to P6, but that P7 is somewhat considerably out of tune, thus limiting consideration to three Orders of Interval.

In Just Intonation the largest deviation involved is that between the Second and Zero Orders, which gives the average liminal mean boundary at between 36/35 and 49/48; so that Matrices based on these principles can be fairly used to form nominated scale systems.

Here again graphs of the harmonic curve show the relative error of tunings involved very clearly to the eye.

37. The General Septomial Matrix and Scale

The general Matrix is the Septomial, constituted of a Core Triad and its Enveloping Tetrad.

This gives us a seven-toned scale as the nominated minimum, and the aspect of most general applicability is the Hemicyclic form of the Tertriadal.

If we plot the "just" values of Tertriadal, Hemicyclic and Serial forms of Matrix we find that they agree fairly closely, hence the Septomial Scale can be regarded as well established from a theoretical standpoint.

Its most general form is in either species—

$$\begin{pmatrix} & P & & PD & T & \\ TD & & TT & & L & LD \end{pmatrix} \begin{matrix} \text{Core} \\ \text{Envelope} \end{matrix}$$

Since the disjunct fluent TD : LD between the extremes in achrome is of the dimensions of an Oscillant, the addition of this interval completes the Octave Unitor Cycle.

Such a scale represents the minimum adequate closed system, starting from the primary axials P and T, and coupled up in extremes by various secondary or quasi-axial members, which will be discussed later.

The requirements of music are so varied as to invite a large number of possible nominate values.

But the general desideratum is the retention of the early predominant relationships, and this is provided for by the Matrix which corresponds to the abstract idea of what is called "Key" in practice.

The comparison of Equigrade and Serial systems may be extended to methods of tuning, etc., other than the Dodecanal.

There is a vast literature dealing with this question, but the ear remains arbiter in practice, although it is not always recognised how determinate its judgments are.

CHAPTER V

THE SYSTEM OF SCALES

38. The Two Meanings of the Expression "Scale"

The term Scale actually means a stairway, but when we speak of a Scale generally there are really two meanings implied.

(1) The pattern of the mode of division of the pitch range throughout, *i. e.* a system.

(2) The particular section, which can be repeated at higher or lower pitch. This appears as a unit recurrent pattern between two definite terminal tones, *i. e.* a thing.

The context shows which meaning is to be understood.

39. Classification of Scales

Scales in general may be divided into Equigrade, Convergent and Mixed systems. We might include arbitrary systems such as are presumed to be used by so-called uncivilised nations. But are these any more out of tune than the average household piano or amateur orchestra?

As we have seen, the tendency of a general requirement is met by an Equigrade basis, on which a nominated scale of mixed appearance is formulated.

We see that although the Equigrade basis is the most general and persistent by reason of its determinance, it does not bar the use of any other.

The difficulty with Convergent scales is that no one interval is an exact multiple of others; thus rearrangement in different pitch position is impeded. This is a distinct drawback to general use.

40. The Septomial Scale, and its Continuation by Octave Recurrence

The Septomial Scale pattern is representative of the Matrical system as regards locatability and nominance of components. As we have seen, this is capable of appearing in varied aspects.

The actual differences disappear in E.T. terms, showing as permuted alternatives in some tones, but the implied differences upon which specific aspects depend can be enhanced by special methods so as to produce a wide range of effects.

The fact that this can be done successfully by E.T. tones is evidence of its capability in the hands of a master.

Upon adding the Disjunct Fluent TD : LD to the Septomial we complete the Octave pattern, consisting of alternate Core and Envelope members. This pattern is then achromatically repeated throughout the pitch range of some eight useful octaves out of the eleven or twelve audible.

The actual termini of an Octave range are arbitrary, but it is usual to take as such the F and C Primes in the Tertriadal aspect, and the mutual Yoke in the particular Hemicyclic view.

It is then possible to nominate each member of the scale according to its predominant relationship to the termini or centron of reference, so that each tone has a definite name which may be fixed or movable according to an arbitrary standard of pitch.

The facility of sliding liminally from one matrix to another is possible; this somewhat masks the determinate outline in practice.

The acceptance of these naming systems does not in any way limit the nomination of members to any one basis, but it affords a starting-point, arbitrary as it may appear, for considering any extension.

The crystallisation of the general system (Hemicyclic form of Tertriadal type) by claviature and notation follows, and the persistence of the systems in use is distinct evidence of the predominance of determinance over arbitrary choice.

Any arbitrary system sooner or later has to give way, but although our staff and symbolic notations and the plan of claviatures have many obvious drawbacks, they may be regarded as survival of the fittest without implying that further extension cannot take place.

The study of the pedigree of our conventional scale system is full of interest and information, and is recommended.

41. The Symmetrical Semi-scale

It is to be observed that the scale between J termini (Ray Mode) is symmetrical in both Species, and recurs at the Semi-scales, whereas if any other termini are chosen the pattern recurs first at the Octave.

If we were restricted to the J range, possibly some other view might arise, but in practice all termini are involved.

Hence the generic term "Scale" in the limited sense, implies an Octave range, and any pattern is conveniently named as the "Mode" of its termini.

42. The Convergent Scale

The convergent type of scale is given by the upper serials of fluent dimensions. Although this type is acoustically so obvious, it has not been developed to any extent owing to the restriction as to inter-transposability.

The composer Schriabin has, however, developed a scale (or a quasi E.T. modification of it) as given by the Serials P7 to 14. Whether this is anything more than a transient phase in the history of tonality remains to be seen. The instrumental difficulties are obvious.

43. The Three Forms of Septomial, and their Close Agreement

Subject to the general relationship of Core and Envelope which is seen to involve non-simultaneity or swing about axis, *i. e.* fluent, the principal aspects of the Septomial are—

(1) The conventional Tertriadal, with permutable determinators.

(2) The Hemicyclic, or Half-Pythagorean.

(3) The varieties of Serial, which presents the Third-piling figure so familiar in chordance.

44. Tabulation of Tertriadal Aspects, and Particularisation of the Hemicyclic Form of same as General Representative

As regards the Tertriadal aspect, owing to the division of the Septomial Matrix and consequent scale into two interwoven

elements of Core and Envelope (the latter subdividing into polar elements), and the fact that the part species of each is independently variable, we obtain the following eight variants :—

Læval Polar Element of Envelope. LD	Core (Centron) Element. PD	Dextral Polar Element of Envelope. TD	Tonal Name.	Practical Scale Name (Fundamental).
◗	D	D	Symmetric Educt	Picardian Harmonic
◗	◗	D	,, Adduct	Ordinary Minor
◗	D	◗	Permuted Educt	Unnamed
◗	◗	◗	(Envelope) Adduct	Descending Melodic
D	D	D	HEMICYCLIC	NATURAL SCALE
D	◗	D	Adduced Core Hemicyclic	Ascending Melodic
D	D	◗	Antisymmetric Educt	Unnamed
D	◗	◗	,, Adduct	Unnamed

Of these variants, the one which coincides with the Hemicyclic is predominant for the following principal reasons :—

(1) The Concomitant commutability of the members in Species, each tone having dual conserved values.

(2) Pythagorean relationship, with its audental and axial advantages.

(3) Capability of presenting the "Modes" by Composite Species intermediate to the extremes F and C.

The Serial aspect is concerned with the Coherence of the Envelope independently of the Core.

The generalised form of scale thus represents either Species, and any tone may be a determinator in some aspect; consequently each member has three potential aternatives known respectively as Natural, Sharp, and Flat. This is shown by prefixed symbols on the graphical staff, and by name variation in the symbolic Sol-fa notations.

45. COMPOSITE FORMS OF SCALE

From the simple forms, Composite scales can be built up on certain determinate lines, the principal of which is the independent variability of each order in partial species.

It may be noticed that the scales usually recognised in practice are all Composite, the pure Symmetric Tertriad (which implies a neutral "indeterminator" to the Centron Triad) is the theoretical origin, but remains decidedly in the background in practice, where determinance of Species is a necessity.

46. Concomitance of Opposite Species

The concomitance about J, whereby each scale member has a commute recessive aspect (and is thus dual), can be extended to an invariance of Envelope, or such part of same that remains Hemicyclic, viz. the Link chord TD : TT : L.

This and the Symmetrical Tetrad (triple Violet) are important in the application of Concomitance.

There is an Orthogonal Concomitance about L when all three determinators are permuted; thus each Scale has two closely related concomitant relations, which are recognised in the Tonal and Relative Minors to a given Major Key, and vice versa.

47. Liminance, Secondary Axes, and Umbral Members

The function of Liminance, which appeared in the Serial aspect as a failure of the ear to discriminate between adjacent intervals, is now seen as a simplifying agency, which, by permitting quasi-equation, formulates secondary axes linking up the loose ends of a system.

Thus definite groups of relations are completed as the starting-points for tonal development.

This interdeterminance involves fixing apparently arbitrary limits, but as the resulting reactions reduce the average to a practical definity, it is seen that the transcendence of a boundary *if regarded in the same way* upsets the whole basis.

Thus the inclusion of the Contradeterminator practically affects the whole system.

On the other hand, we know that the actual boundaries of concordance are not hard and fast, so recognise the members just over the line as "Umbrals" or "shadows," different from the systemal members, yet capable of being brought into use in special aspects, for which special arrangements and modifications of the system may have to be temporarily made.

In the general case, Umbrals appear as insulating members between intervals of different degree, *i. e.* separating the minimum chrome V from the oscillant fluent.

This is supported by the "out of tune" effect in E.T. of the seventh member of the Series.

48. The Helical Symbol System of Serials

We pictured the range of pitch as twisted into a "Helix" by the cyclic recurrence shown by the Octave.

This method gives us a possible set of shorthand symbols for the serials by taking the corresponding parts of the plane projection, *i. e.* a sinuous curve.

It is obvious that conventional assumptions have to be made, so, following the usual arbitrary method, we regard Pitch as vertical rising from the Bass, and polarity as dextral for First Order Fundamental.

The permutation of the determinator is regarded as from the observer, and drawn toward on reversal.

The three Orders are thus pictured as in the three dimensions of space.

The combination as shown in the Scale is rather complex, since we have—

(1) Actual direction of pitch progression.
(2) Species direction with respect to a nominant.
(3) Polar direction of the Semi-scales.
(4) Permutable direction of the determinators (and their contras), with consequent combinations giving quarter-scale variants.

The unalterable direction (vertication) of actual species may be compared to the flow of a river, up or down which a person may swim, cross, or dive into.

49. Distorted Triads and their Use

Two similar First Order Semi-scales either overlap or fall short by an oscillant. The First Order interval may be either B (Dextral F, Laeval C) or R (Dextral C, Laeval F), or both be represented in distorted form by the neutral W/2 (Augmented R, Diminished B).

These are definitely shown in correct notation, but are presented by the same tones in E.T. claviature.

These distorted forms come into practical use in the Sequence, and in the various regular and irregular "Modes" of the Scale.

Their original almost surreptitious use, the devices used to normalise them (out of which sprang the equal temperament), and final recognition as important factors in chordance, form an interesting story in the history of music.

The variants of Semi-scales and their naming from the scale in which they normally occur may be compared with the Greek "Tetrads," which represent the first really scientific basis of practice known to history—they were probably not the first actually.

50. The Modes and Tropes of a Scale

We will now examine the forms arising from the arrangement of the general Scale pattern.

If we take the extended Scale formed by achromatic repetition of the basic pattern, it is seen that we can either—

(1) Shift the termini, so as to include the same extent, and employ the same members in a rearranged form,

(2) Retain the termini and rearrange the tones inside same; this process can give the same patterns, but without alteration of mean pitch of the whole.

The first process presents the Regular Modes of the Scale, of which there are consequently seven.

The second process, when carried out to produce the same patterns, gives us the Regular Tropes, which thus appear as Translated Modes.

Further, we may cut up our original Scale into sections and transpose these bodily to form new arrangements.

These can present, in addition to the Regular, Irregular Modes, and the same process and forms can be similarly arrived at as Irregular Tropes.

In this aspect the Modes appear as Phase relates of the original Scale form which is taken as zero or an Antinominant Cycle.

51. The Regular Modes. Imperfect Forms

We may denote the Modes by their Greek or more conventional ecclesiastical names; or number them in the conventional manner; or follow Helmholtz in naming them from the new termini.

The essential factor in practicability is that the tones and intervals are altered to such a slight extent as to be capable of being regarded and used in much the same way as in the original Matrix and Scale.

Each objective tone varies in turn in Species and Order, so that at one time it may be a permutable determinator and thus be flattened or sharpened without losing its relative Scale nominance.

As we shift the termini in pitch the Second Order intervals will in some cases suffer permutation. The First Order intervals remain unchanged in five of the Regular Modes which are thus named Perfect.

The remaining two Imperfect Modes were the bugbear of the ancient theorists, and constitute natural bounds to Modal and Tropic translation.

All sorts of artifices were used in the past to reconcile practice and "theory." The modern musican rightly does not worry over these, the ear and intelligence are his criteria—or should be.

It is to be observed that modern methods involve much partial use of Modal translation, thus adding the resources of the past to those of modern chordance.

52. Semi-scalar Rearrangement and Composite Species Aspect

It is observed that the partial species variability of the Composite Tertriadal aspects enables the five "perfect" Modes to be presented.

There is, however, another important aspect to consider.

This is Semi-scalar Rearrangement, which is equivalent to a First Order Modal Translation.

Its derivation from the original Greek Tetrads (R Semi-scales) and its practical bearing on vocal range and distribution cannot be discussed here, but in practical modal working we recognise

Authentic or Ortho Modes, and their Plagal or Hypo colleagues, which latter are even numbered in the conventional scheme.

Theoretically every Authentic Mode has two **Plagals**, and vice versa, according to the polarity, but in practice only the "forward" Pythagorean one is recognised (F.T. Polar), the other being regarded as the "Ortho" of the original Authentic in the Plagal aspect, somewhat as the Laxator is regarded as a reversed Tensor.

Regarded in the aspect as Composite Species, Modes appear in the following aspects :—

(1) Direct Species (Ortho or Authentic).

(*a*) Fundamental (Composite in consequence of	Lah)	Range	Doh.
(*b*) Coincidental (,, ,,	Soh)	,,	Me
(*c*) Ambi-species (ditto) or Yoke Mode			Ray

(2) Indirect Species (Hypo or Plagal).

(*a*) Hypo-plagal	F Species	,,	Soh
(*b*) Ditto	C ,, ...	,,	Lah
(*c*) Ditto	J ,, ...	,,	Lah / Soh

(3) Diametrical or Pseudo-modes.

These appear in two principal aspects, viz. Anti-direct or Vertical (Hemicyclic) diametrical range.

(*a*) Upwards or Anti-fundamental (*b*) Downwards or Anti-coincidental	} Isotonic	Range ,,	Fah Te

We also note—

(*c*) Their Orthogonal-læval types, such as	Isotones	Range	La / Soh

which are not in the diatonic General Scale, and the aspect of those of which the Ortho-authentic Direct Species is Hypo-plagal, *i. e.* Contra.

(*a*) Contra or Læval Hypo,	Authentic Direct	F	} Isotonic	Range	Fah
(*b*) Ditto	ditto	C		,,	Te
(*c*) Ditto	ditto	J		,,	Lah / Soh

The last is isotonic with the Hypo-plagal Modes.

We thus obtain the fourteen historical Modes from Parti-species colligation, an aspect that would enable us to dispense with the term "Mode" if desired.

But it is advisable to keep in touch with the observational sources of tonal arrangement.

53. The Hypo, or Plagal, Aspect of the Modes

With respect to direct species relation, it is noted that absolute direction is associated with the Plagal Cadence.

It is observed that—

(1) The Hypo-plagal Fundamental Mode, reading downwards, is the Coincidental Scale with all determinators permuted, and vice versa in the opposite species.

The Hypo-plagal Modes are seen to be a rearrangement of Semi-scales about a Centron, instead of between a Centron and its achrome as termini.

This turning inside out is merely a case of the general re-arrangeability of the Trinomial Term : Relation : Term as Relation : Term : Relation.

This Composite Species view enables us to identify the Regular Modes and their Hypo-plagals with the different forms of Tertriad. By including also the distorts of First Order Chromes, the Imperfect Modes are brought in.

The dual translatability giving a trinomial of interrelated Modes is an extension of practice by means of the Semi-scalar individuality. It incidentally settles the old controversy as to whether the "Tonic" or "Relative" is the true Minor of a Major Key.

Apart from practical conditions, which may settle the question according to requirements, it is seen that both concomitants are equally to be regarded, and that both "Dominant" and Sub-dominant" Plagals are equally related in theory, the one forward and the other backward in respect to a given "Authentic" scale.

54. Ternary Rearrangement. The Oscillant or Hexatonic Scale

The rearrangeability of Ternary and Quarter Scales presents also a method of composite construction.

From the former we derive the obviously composite form of the "harmonic" Minor Scale, which when carried out through the Octave gives us the Equigrade Oscillant or Hexatonic Scale (whose triangular illustration we have already noted). This has a peculiar, though restricted, use in practice.

In notation and claviature it appears more complex than it

really is. From the standpoint of the Hexatonic Scale the ordinary Septomial looks like two portions united at two points by Impellants, which are respectively projective and retractive.

55. The Approximation of a Convergent Scale

The Serial (convergent) Scale 7, 8, 9, 10, 11, 12, 13, 14 may be approximated by the Fah Mode ascending and the Te Mode descending.

The member Pii is practically an Indeterminator, and is conveniently represented by the nearest E.T. tone above and below according to the direction of progression.

In the natural scale the tones De and Ma would thus be used. The rest of the members are near enough to E.T. members to be tuned or faked as desired in practice.

This Schriabin Scale has a peculiar character of its own, but, as used by the composer, it is implied more by the harmonic environment which is adequate for the purpose.

56. Aspect of the Scale in the Past. The Ever-present Question of Tuning

It is interesting to trace the rise of the Scale from classical times, together with the inception of determinate effect of Second Order intervals by the interaction of ecclesiastic and secular methods in the Middle Ages.

The process was by no means direct, nor restricted to Greek sources.

Many attempts have been made to obtain better tuning than that of the E.T. Dodecanal, but the claviature, instrumental construction and maintenance, and notation are most complex. Although at present there seems to be no opening for such, yet the progress of mechanism leads one to hope that the great work accomplished in this field (see literature and experiments) may not have been altogether in vain.

PART II

THE RELATIONAL ASPECT OF PHONALITY

CHAPTER VI

MONOPHONY

57. The Monophonon

One would naturally commence the study of Successive Tonality apart from Chordance with the Monophonon or single "voice," in which all the grades of experience are shown.

Although these are not always expressible in outside terms they can be regarded as critical factors.

There is no objective adherence in a succession of tones, this being conferred by reference to an actual or conceived source-identity.

58. Different Kinds of Phona

We recognise phona as ranging from the absolute chaotic (such as might be produced by dusting a piano) to the determinative, which carries evidence of human thought.

This latter class includes the "significant" phona in which independence and personality of expression is shown to an extent that the conventional term "Melody" only indefinitely represents.

Determinance in practice is largely a matter of reference to origins, hence the differences of individual judgment that abound.

Whatever its form, we recognise that a determinate monophonon has grown from something else; it is not an isolated expression in the musical universe.

59. Range. Aphonality.

As pure sense expressions—apart from the intelligence—we judge phona on a range between the extremes of euphony and cacophony.

It appears that a phonon is free to go how it likes, but there are certain conditions which tend toward, but not necessarily give rise to, that loss of identity which we may term Aphonality.

Our inquiry is directed toward the conditions of maintenance in determinance: in the simplest case identity and continuity under all the vicissitudes of environment; together with conditional initiation, persistence, and discontinuance (whereby the elements of Rhythm are formulated).

A monophonon is an absolute unit of experience, but is only intelligible in terms of its parts, *i. e.* the tones employed. Although in practice regarded as perfectly free, in general use it is advisedly distributed equably about a certain allotted band of pitch, which in practice represents the effective range of its source.

Consequently unjustifiable intermissions, extreme variation of character, transposition to extremes of pitch, wide and awkward leaps, persistent lack of balance in distribution, and, in general, unjustified asymmetry of structure, tend to impose a strain on the act of listening.

One must be careful to distinguish between ranges determined largely by extraneous conditions (such as what is easy to sing or reach on an instrument) and the abstract case.

There is always a tendency to run in grooves in all branches of art, but the ear has a far wider range than the voice or practically any instrument. The older text-books are not always clear in their rules on this point, and of course modern technique has greatly extended the average scope and form of melodic passages.

Historically, the desire for euphony and avoidance of cacophony, combined with a discipline recognising the danger of a surrender to mere agreeability, has tended to preserve a number of short melodious formulæ of which common melody is constructed.

60. False Relation

We shall consider the maintenance of phonic identity amid other phona in the next chapter, but there is one principle (discussed in detail in Chapter VIII) that may be noticed here.

This is Kinetic False Relation, which shows how Octave recurrence affects the monophonon. The observation is that alternative substitution at the Octave is in general dissonant.

Alternative progression depends upon flexionic continuity, which is a characteristic not lending itself to achromatic translation from a phonal point of view.

61. Concession of Determinator

We may also note that determinators are naturally permutable to their alternative values in the direction of their actual movement in pitch. This is familiar in the ascending and descending variants of the melodic minor scale, being due to the tendency of partial species to follow the implied direction of progression, and to formulate Impellants agreeable thereto.

Thus a rising minor scale implies a fundamental species aspect.

62. Borderland Conditions

Borderland conditions are those in which liminal libration is stretched to its limits.

It is easy to see that by means of such processes translation may take place, and practically anything be done.

The unconscious variation in pitch of untrained choirs is a case in point, and a clever composer can, by processes analogous to tonal sleight-of-hand, almost hypnotise the auditors into a deviation of determinate procedure far from the original basis of fixed values. In the same way it is possible to carry out changes under a cloud of sound that would not otherwise appear acceptable.

The extent to which this quasi-deception of the ear may be legitimately carried is a matter of practical æsthetics: we may always remember that the average listener only hears a proportion of what is written in the score.

Determinance stands on the maintenance of distinct phonality, and rational initiation and termination of each element involved.

63. Bass Phonon Characteristics

It may be noted that in the majority of cases examined Bass Phona move by step, and when they leap, the tones of the interval are axial with either or both of the preceding or following chords.

Also that in polyphony one of the phona at any one time is usually acting as a "Stem" by its direct scalar or arpeggial motion, or that a phonon of this type may be added without disturbance.

These observations are not necessary conditions from a theoretical point of view.

CHAPTER VII

POLYPHONY

64. Distinction between Polyphony and Chordance

Polyphony and Chordance are two distinct things.

The simplest distinction is that the former is a co-progression of "linear" phona, and the latter a succession of chords, although identical forms may arise from both methods of construction, and in practice it is often impossible to eliminate either aspect altogether.

These two aspects are often pictured as if at "right angles" to each other; phonality depending upon horizontal Adherence, and chordance on vertical Coherence.

It is convenient for the moment to defer consideration of the non-harmonic tones which play a large part in actual polyphony.

65. Conditions of Phonal Individuality

The determinal basis of polyphony is the persistence of individuality of each separate element, and this is the foundation of all rules of part-writing. The act of listening is thus divided up into lines of attention, which are affected by—

(1) The number of phona: a very large number tending to swamp individuality.

(2) The clear range of each in pitch, which is imperilled by Crossing (by either "cut" or "mesh"), Touching, or Overlapping in succession.

The ambiguity of the issuing phona is the reason of this, since, other conditions apart, it is only difference of pitch that sustains phonal individuality.

The obvious way of getting over this ambiguity in practice is by making each phonon distinctive in some manner, either by the character of the tones employed or by the form and figure of the passage.

(3) The increase or decrease of the number of phona, which is

determined by the conditions of initiation and discontinuance of any one phonon. The criterion of phonality is based upon the rationale of initiation, persistence, and discontinuance as from a source, in relation to the whole work.

The general basis of polyphony is therefore weaving without tangling, and this art reached a high stage of development at the time of Palestrina.

We have no independent basis, nor is any required, of determinative phonality, so therefore use the expressions of chordance with appropriate distinctions, leaving the consideration of non-chordal colligation for convenience until Part III.

66. The Tetraphonic Standard

The first problem is that of the general basis of phonal number and distribution.

The primary discrimination of phona is based upon the relative pitch position, and phona are usually numbered from the Bass upwards.

Chordal construction in its simplest (not reduced) generality consists of progressions between Core and Envelope.

The former is a triad, the latter a tetrad which thus sets the pattern Tetraphony.

This is the standard, and is confirmed in practical working by the convenient distribution of the different classes of human voices, and to a lesser extent by certain exigencies of instrumental construction, such as the keyboard whereon each hand takes two phona in charge.

The distribution follows probably from the acoustic sonority of the serial arrangement of the first six harmonic tones over a range of two and a half Octaves. The average range of each phonon being about one Octave, and the average spacing about half.

Vertication and the experiences of the Parasyntonic regions set a general convergent form with rise of pitch.

Thick part-writing is quite possible, but presents a more chordal effect, since it does not favour phonal clearness as a rule. The beating of overtones, etc., is an acoustical factor not to be neglected in practice.

67. The Natural Melodic Implication of Highest Phonon

The natural melodic implication of the highest phonon is an observed phenomenon of which no very satisfactory explanation has as yet been given.

It may possibly be due to the fact that the highest phonon represents the arbitrary limit of an otherwise infinite series at a definite member, whereas downwards all harmonic extension is naturally terminated by the actual or latent prime tone or tones.

Classical practice, as we know, starts from a Canto Fermo which was originally the tenor voice, but the experience of a melody floating on the top cannot be explained away by scholastic niceties.

The standards mentioned above are not rules of practice, which is infinitely varied by reason of the references possible, but they represent the foundation of polyphonic method in all ages.

68. Classification of Relative Motion

The relative motions of phona are classed into three groups, viz. Parallel (Paraphony), Oblique, and Contrary (Enantiophony).

Each exhibit certain peculiarities. In particular it may be observed that Enantiophony lends itself best to balanced distribution, also that Paraphony tends to reduce forms and figures to one leading type of chordal motion.

These motions may be exactly in pitch step, in which case they cannot altogether remain restricted to one Septomial system or key. This is called Real Motion.

Or they may be conserved to one Matrix; thus parallel motion is of the nature of approximation, but shows regular form in notation and name. This is Tonal Motion.

This latter forms the basis of theoretical treatment, since the deviations do not exceed the alternative values of the tones employed.

69. The Scale of Naturals

The diatonic, general, or Natural Scale of the Hemicyclic form of Tertriadal aspect preserves the greatest number of determinative characteristics, among which we may note the Pythagorean

cyclic relations with their multiaxial relationship; concomitant duality of conserved Species; and the Regular Modal forms of Composite Species.

70. The Influence of Phase

The influence of the respective orders is on the basis of phase. The achrome introduces no new nominance, but simply extends the pitch spacing. If this latter is found to be of negligible effect, achromatic inversion, rearrangement and transposition may be regarded as practically identical, as we assumed in studying Chordance.

But such changes alter the "Bundle." Thus the actual extension of pitch space may have important effects, as already noted in the case of False Relation. These matters are considered in the next chapter.

The Opponents exhibit a different phase character, viz. polar direction or "hand." Consequently the Semi-scale is a factor of considerable importance in polyphony and organisation.

The Quadrants (Second Order and ternaries) are more or less neutral as regards direction. Their function is largely correlative of an established direction, so that one of the phona becomes the leader, whose nominance is reinforced by its companion.

Lesser intervals are of fluent type and do not influence phase. When fluents are presented as simultaneous, they appear simply as discords with implied resolution at some subsequent epoch.

Hence the mean polyphonic range (apart from paraphonic conditions discussed later) tends towards semi-scalar dimensions preserving polarity.

71. Pairs of Parallel Moving Tones

The clue to the general form of Tetraphony is seen in the parallel motion of the "free" pairs of tones in the progressions between Core and Envelope.

In the case of the "imperfect" Modes, Pythagorean distinction of number of scale members in each symmetrical semi-scale prevents ambiguity.

Each R Semi-scale may be regarded as comprising three intervals acting respectively as Initiator, Continuant, and

Consummator; thus following the Ternary basis of local phase distinction.

These general principles underlie the application of Polyphony in practice, but it is necessary to turn to the effects obtained in order to gain knowledge of the detail principles involved.

The most general effect is the variable quality whose extremes are Consonance and Dissonance (in the sense in which we use these terms), which can best be discussed from the Chordance aspect, where it appears to a great extent as a Functional property.

CHAPTER VIII

CONSONANCE AND DISSONANCE

72. The Concept of "Sonance"

The concept of Sonance is abstracted from the range of phenomena whose extremes are Con- and Dis-sonance.

Since these are in general associated with certain types of progression, Sonance may be regarded as a function of condition, and the dissonant conditions may be catalogued.

Dissonance is distinguished from cacophony in that it may be the outcome of determinate procedure, and thus have a place among the effects capable of employment in music.

Sonance appears both as a guide in practice and a dependent phenomenon in theory.

73. Types of Dissonant Progressions

Dissonant Progressions may be classed into four types, under two groups.

(There are several other milder forms of dissonance which depend for their effect upon similar conditions, and which usually appear from inadvertence and want of constructive skill.)

(A) Relational: (1) Kinetic False Relation.
(2) False Relation of the Tritone.
(B) Chromal: (3) Consecutive Octaves and Fifths.
(4) "Exposed" or "Hidden" ditto.

The rationale of practice, upon which composer, performer, and auditor agree, is Determinance.

Each of the above classes of progression presents dissonance as a certain function of nominance-audentity of the tones. If this latter be altered the dissonance may be modified even to vanishing.

From the practical aspect of many text-books these latter cases are treated as "exceptions" or "licences" (as if there were exceptions or licences in natural phenomena?), and bans the student from the particular progressions as obstacles to be

avoided, which generally sets him to work experimenting with them. The scientific aspect is only concerned with the examination of dissonance as functional phenomena.

74. The Two Methods of using Associated Tones

We have to notice two entirely different tonal effects indistinguishable in notation and actual performance, which are both used freely in practice. These are, the use of tones as—

(1) Serial components building up the tone tint, *i. e.* thickening the mass of the prime tone, as is done with mixtures and ranks on the organ, as well as frequently in modern orchestral and choral work.

We know that to effect this object successfully we have to satisfy a very strict ear which notes the slightest deviation. (The writer had great hopes for Dr. Cahill's Telharmonium in this direction.) The rigid serial coupling may be approximated artificially with fair success.

(2) Members of individual phona.

It is seen that the first case aims at *blending* the tones, making all except the lowest to vanish as notes; while in Case (2) the aim is diametrically opposite to this, viz. the preservation of phonal identity under all circumstances, like strands of wire in a cable.

The particular type of dissonance in which these come into conflict is presented by the so-called Consecutive Octaves and Fifths respectively.

Case (1) is perfectly natural and agreeable in effect; actual analysis of most of the notes used in music, which may be carried as far as the drum of the ear, show the Octaves and Fifths generally well pronounced.

In Case (2) the effect is a brazen harshness, which if it occurs inadvertently is distinctly unmusical.

It will be seen, therefore, that the mere appearance of "consecutives" in notation is no criterion of the effect produced, which may be good or bad according to whether either case is aimed at and more or less successfully achieved.

It is the nominance of relative phonality that is involved, and the phenomenon is a striking proof of the necessity for an

abstract theory of tonality corresponding to what a musician actually feels and hears, rather than what strikes the eye in notation.

75. Kinetic False Relation

It is in general dissonant to sound a tone with, or in succession to, an Octave of its Alternative.

The helical illustration enables us to picture the case of an Alternative, which is flexionally (phonally) a near relation, transposed an Octave, which is chordally a near relation.

The two together represents a "skewed" perpendicular.

The Octave is such an independently striking interval that the slightest deviation from it is apparent.

In cases of contrary motion with determinators permuted in the respective directions of progression (as with the variants of the melodic minor scale) the effect is not so dissonant.

Also, if the chordance is locally translative the effect is shaded off. Again, if the rhythmic dimensions of the phona are different the false relation is not so strident and may be quite agreeable.

We see, therefore, that the effect is variable. It is not merely swamped by the favourable conditions, but the dissonance is, as it were, diminished in relative scale of proportion to the other determinative factors of considerable audentity.

76. False Relation of the Tritone

This effect arises when the tones L and TD occur in different phona successively, the tones moving by step. It is most evident in Diphony.

The anomaly arises from the respective colligations PD : TD as "arpeggial," and similarly L : TD in Septomial nominance.

The Tritone L : TD practically settles the nominance of the two tones, particularly the second order implication of TD; that of PD is implied by the preceding L.

But the intrinsic audentity of the First Order interval PD : TD implies direct relationship, *i. e.* inherence in the same Series, so that either the progression L to PD is not an Impellant or TD is not a Determinator.

There is thus a determinant conflict between the individual

coherence of the respective tones in their triads and their mutual coherence.

If a third phonon is added the actual determinance of the passage is rendered more evident, and these local effects retreat into the background.

Traces of the effect are seen between TT and LD and when the permuted laxator determinator appears as a Concomitant, *i. e.* both as Se and La.

In the Hemicyclic view TD is a Quintensor, but the tritone is associated with a polar envelope, which implies it as a determinator "leading note" to P.

Hence the anomaly and dissonance, which tends to disappear upon a modification of nominance and reduction of audentity of the secondary First Order interval PD : TD.

This dissonance renders the Triad on the Mediant the most difficult of any to deal with in Fundamental Tonality.

In other Modes it is not so evident because of its polar association.

The dissonance, like Kinetic False Relation, may be minified in relation to environment; it may be swamped, or evaded by rhythmic alterations. But at the same time the effect is capable of being used when its particular striking effect is required.

77. Paraphony of Zero Order

Consecutive Unisons or Octaves may be viewed from the general condition of Polyphony, *i. e.* the determinate start and stop of individual phona.

If a phonon be doubled either over a definite extent or in accordance with a rhythmic schema the case does not arise.

But inadvertent parallelism destroys the individuality of either of the phona involved, and the resumption of independent movement is nearly as bad.

The main effect is that of the arbitrary termination of one of the phona, as when a violin string breaks.

The sensation is somewhat like the sticking together of two independent parts of mechanism, the jamming of cogs, for instance.

The use, however, of transient doubling is a valuable accentual device.

78. Paraphony of First Order Intervals

Every one knows the dissonance of consecutive fifths, and of bare fourths, and even real thirds. It is fatally easy to stumble into such progressions through inadvertence in part-writing and when mechanically filling up harmony.

The student is often puzzled to see that what the text-books ban is often done by the masters.

The dissonance arises from anomaly of phase progression by the opponent semi-scales.

On proceeding from a tone to its achrome we pass successively over the two opposed semi-scales, the second reversing the sign of the first. Thus we traverse an out and home journey from a tone to its cyclic achrome, which is a perfectly conceivable operation.

Now if we take the out and home semi-scales and force them to proceed in parallel we get into conflict with the referential nomination of the opponents.

We cannot conceive or perform a simultaneous journey out and home.

It is a logical absurdity that any one object at any one time can be oppositely predicated. We must either abandon the predication or we must get round the idea of simultaneity and collocation.

This can be effected in Tonality if, by environment or a superior audentity, the phase opponence is put into the background.

If the First Order intervals involved are secondary members of composite chords (such as occur in the higher discords), then the phase opponence is not so evident in relation to the primary case. This may occur with Pentads, etc., in which the total chordance effect is clearly determined.

Again, the phonic individuality may be affected by Alternative progressions (chromatic swamping) or by ambiguity of the issuing phona from a strongly audental chordal collocation.

The ear has limitations in the perception of phonal clearness which may be taken advantage of. In general, the dissonance may be magnified or diminished locally.

The scientist is not concerned with expedients of practice, but only observes logical anomaly where it occurs.

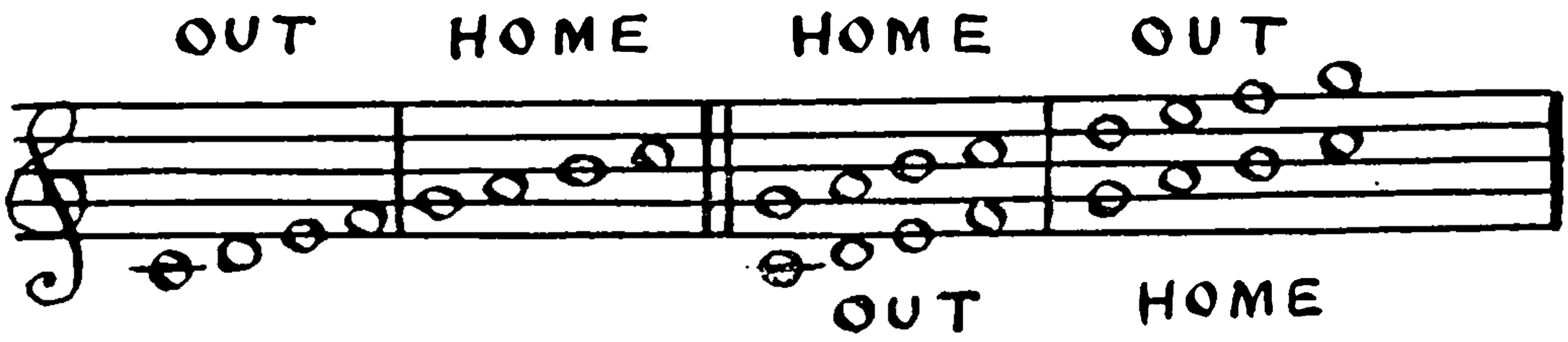
The dissonance may be contrasted with the consonance arising from the *contrary motion* of two out or two home progressions which agree in eventually meeting.

The reversal of direction has reversed the phase anomaly in this case.

The phenomenon may be compared with the experiment of taking a well-recognised chordal progression, cutting each chord into two, and transposing the parts; which in general produces the most horrible discord and dissonance to both ear and eye.

This is simply due to the anomaly with respect to the original determinate progression, and consecutive fifths are exactly similar in effect and logical implication.

The expert musician, whose æsthetic perception ranges over a wide field, finds no difficulty in handling the material. The student, who works with a more limited field of vision, cannot lightly indulge in extremes without risking indeterminance and chaos.



79. Hidden or Exposed Zero and First Order Dissonances

In this case the effect is due to the implication of Consecutives by approach to and termination at these intervals of considerable serial audentity.

The character of the dissonance is similar though not quite so strident, and it is more easily stumbled into owing to the non-appearance of actual consecutives on paper.

As may be expected, it is strongest when between extreme phona, and it is much diminished by strong chordal progression (such as direct Core to Envelope and vice versa), and by step motion of the upper part.

It is not particularly noticeable with R as terminus.

Observation shows that—

(1) The effect is due to the fixing of relative nominance of the tones by the audentity of the terminal interval, and thus the phase destination of the motion leading to same.

(2) That the extent of the approach range is negligible in comparison with the actual direction of the approach.

(3) That distortion of chordal progression in duration does not greatly alter the effect.

We see that the predominating factors are the direction in pitch and the nominance of terminal tones.

Whether the actual effect is due to an implied filling up the gaps in approach by continuous scalar latent tones, or whether the whole progression is viewed as a distorted "consecutive," is practically a matter of indifference. The dissonance does not arise until we arrive at the phase fixing tones, when the sensation is as if we had arrived from a "consecutive." Hence the anomaly of the names Hidden and Exposed, which refer to the two ways of viewing the phenomenon.

The case may be compared with the dissonance of progression from sevenths and ninths to octaves by similar motion.

The practical effect is to limit the number of chordal components which can be employed at any one time in specific progressions. This limitation vanishes to a great extent with "thick" part-writing.

It may be pointed out that the whole phenomenon of dissonance supports the theory of individual linearity which we term Phonality.

80. Weakly Dissonant Cases

With regard to several cases of weakly dissonant effect, such as are mentioned in many text-books, the effect may generally be traced to indeterminance of either chordance or of phonal continuity.

It is unnecessary to go into details which concern practice and are allied with the æsthetic conditions under which they appear.

As a rule, such progressions should be confined to subordinate parts of the scheme of presentation, unless it is the deliberate intention of the composer to portray such negative characteristics.

CHAPTER IX

SPECIFIC CONSONANCE IN POLYPHONY

81. CONSONANCE OF SECOND ORDER PARAPHONY

CONSONANCE of Second Order paradiphony is an observed fact which might have been inferred from a reversal of First Order characteristics.

Hauptmann pointed out this, but his philosophy is unsuitable for scientific purposes. What he tried to explain from a philosophy of reason we attempt to view from a science of phase.

Examination is therefore directed to—

(1) The reason for Consonance in this Second Order case.

(2) The place of paraphony of this type in Tonality and its relationship to the Axial theory.

It is to be noted that the conditions holding with Second Order intervals are essentially different from those of First Order Opponence.

The rational explanation is based upon the "quadrant relationship" in phase.

82. THE SECOND ORDER DIPHONON

A Second Order diphonon resembles what mathematicians call a complex quantity, *i. e.* one made up of two independent coexistent simple values. This we illustrate geometrically by two magnitudes at right angles, and symbolically by an expression such as $a \pm \sqrt{-1}\, b$.

The resultant of this expression represents a directed quantity or vector, which in many cases may be treated in the same way as a simple quantity.

The Intraversant does not affect the polarity or phase direction of a tone, simply combining with it to form an extended similarity, and thus does not introduce any anomaly or contradiction of pre-established predicates.

We must distinguish between the real (exact) and tonal (scalar)

cases. In the first named, blending undoubtedly occurs, and if the phona are otherwise individualised there is a slight effect of consecutive dissonance, associated with matrical anomaly.

The second case is pure concomitance, agreeable even to the verge of sweetness, the continued change of local species differentiating the effect entirely from that of blending.

The Ternary interval is similarly consonant in succession.

We note that the Initiator and Consummator divisions are of Second Order type, and thus harmonically quadrantal, but that the Continuant or middle section involves First Order relations.

83. Hemicyclic Aspect of Second Order Consonance

It is important to note the Hemicyclic aspect.

In the duality of Concomitance, a pair of tones in Second Order relationship may be of either species, but in each case one of the tones is definitely a determinator in respect to the other. Upon commutation of species, this nominated relationship is only reversed in pitch. It cannot be regarded as a case of dual species progression, but the interval relationship is invariant in order, although inverted.

84. The Diphon as an Entity

As a consonance the diphony presents a peculiar unity of character due to co-polarity of direction, so that the diphonon can replace a monophonon for many practical purposes.

The Diphonon is thus a unit aspect as contrasted with the diphony of two independent elements; but, unlike the Octave case, the chordance is definite. Thus the phona do not disappear individually, but are differentiated in importance, one phonon taking the lead and the other backing it up.

There is a certain amount of fusion, however, hence the rule of Strict Counterpoint is to avoid more than three consecutive Second Order intervals; the object being to inculcate freedom of phonal treatment in the student.

Scalar Second Order diphona appear in the two forms of Infra (V and G) and Ultra (M and Y), each being the applement of the other.

With these two forms any harmonic basis can be outlined. This involves switching over at certain points from one to the other, by means of the interval R (which thus appears in a fluent aspect).

85. The Triphonon

In combination the two forms present Triphony, the most consonant form being that in which only Second Order intervals appear on bundle analysis, this being direct evidence of Vertication.

This succession of "triads in first inversion" is known as the Fauxbourdon (the name comes from a naïve process of doctoring so-called theory to agree with practice, so dear to mediæval musical pedantry), and the Triphonon so formed may for many purposes replace the Monophonon.

It is found in all kinds of compositions, being used, and abused, to a considerable extent.

Second inversions, with R in the bundle, and those with B between upper phona, are also known in practice; the dissonance with these latter forms is always evident.

It is observed that the G and M tones in these chords are in concomitant relationship to a nominant tone. The remaining tone fixes the actual species.

86. Second Order Relationship of Chord Spacing

Second Order relationship is important from the fact that it is the spacing of both Core and Envelope.

If one tone of a triad is axial, then parallel motion of the other two, as in the simplest progressions, is diphony, as may be noted with the inter Core-envelope progressions about P and T as Axes.

87. Contrary Motion or Enantiophony

Contrary motion of phona, typified in the simplest case by simultaneous scale steps, is known as Enantiophony, and it is generally consonant.

In the diphonic case, converging or diverging (odd or even achrome periods), we note two types defined by the manner

in which the phona cross: either by "cutting" on the same tone, or "meshing" across two tones by vice-versa stepping, the locus being the centre of all the intervals involved.

From the fact that the paradiphony of semi-scales is dissonant, we should expect its reverse, contrary motion, to be highly consonant.

Starting from the Octave Primes we mesh across the Polar tones L and T, and thus divide each phonon into semi-scales.

Conversely, starting at L and T spaced a seventh apart, we "cut" on the Prime tone, and this note appears as a monad surrounded by a dyad.

The terminal by unison or Octave "cut" thus determines the local Mode of the progression.

This aspect emphasises the phonal implication of Core and Envelope and the particular importance of the Envelope members L and T which form part of the Tensor Radical Tetrad (in F Species this is identified with the Dominant Seventh).

In the progression we also pass through the concomitant recessive triad.

Upon substituting Diphona and Triphona for the Monophonon in these progressions we obtain complete formulæ of the Core-Envelope type in all Modes.

The double Triphonon (Hexaphonon) is practically the limit.

It is obvious from observation that progressions in contrary motion are less harsh than in oblique or parallel, particularly when they are not simple Core-Envelope cases.

The case where two phona approach and retreat, thus simulating crossing, is discussed elsewhere.

88. The Introduction of the Contradeterminator into Contrary Motion

If we start at two tones in B relationship, we cut on D and mesh across the Octave of the position of the Contradeterminator.

The case is reversed with R as interval.

On taking a Second Order interval as commencement, we cut on tones of Third Order, and so on.

If, however, we wish to arrive at the Octave simultaneously it is necessary to interpolate another tone.

This makes the number of steps in R equal to those in B (viz. four), and we are not restricted to the choice of the third tone, but the most simple tonal relations are satisfied by that which stands for the Contradeterminator, *i. e.* the Bi-laxator or Permuted Tensor Determinator.

This is not at all unusual in simple practice, the actual tones employed being the Prime and Tensor Contradeterminators; the latter, being already represented by the Laxator, implies its local "impellant tone" (leading note) as stranger.

We thus introduce two tones which are obviously not ordinary Permuted Determinators (although identical in note), since they are used successively with the notes they would otherwise be the Alternatives of, as scale members.

The tones in question are Ta and Fe, the former being PD (in F Species), and the latter the complementary leading note—local TD—with regard to the Tensor, the respective positions being reversed in C Species.

Such an extension of the ordinary scale constitutes an Octomial—the concomitant pair of such presenting a Nonomial. The rationale of its existence is seen to depend upon the agreement of a simple arrangement in Enantiophony with a simple chordal progression and its reverse.

Many other forms are of course possible, and not confined to simple Enantiophony.

This case illustrates one way in which Umbral tones become admitted into practical tonality.

The Contradeterminator is the only one of this class that is of much use, since the 9th Serial is already tonally recognised as a Bitensor (nine being factorisable).

The 11th Serial practically bisects the E.T. unit, and the 13th, etc., are getting diminished in audentity.

89. The Contra-Triad

Another aspect is to regard R as a pantagraphically diminished B in which the Contradeterminator completes a "Contra-triad."

However, it is not so used ordinarily, so we are confined to the aspect of contrary motion in practice.

It is observed that the Prime Contradeterminator can either

converge to LD or diverge to TD. This is the normal progression, *i. e.* to an infra Second Order tone.

In the first case the progression is identical with that of L to PD in the Tertriadal aspect, but whereas the latter would imply local or general translation, to a Laxator Prime (Sub-dominant key) the former remains conserved in the Prime Matrix.

The second case presents the step of a Contra-impellant 14 to 15 and vice versa, which may be regarded as a Serial reversal of the usual 15 to 16 and vice versa.

Empiric theory has long recognised these Chromatic Discords and resolutions which seem to go the opposite direction to the normal cases, but it is not until we gain a clear concept of the Contradeterminator that we see why tones such as Ta and, by reversal of contra-impellant, Fe, can be used as Scale Tones without inducing local translation.

They are evidently not Permutes (correct notation should show this, although for convenience of printing and reading they are often wrongly indicated), as we see from their treatment in the case of Enantiophony from Second to Zero Order intervals with simultaneous steps.

It is unnecessary to develop other cases such as may be investigated by the reader.

90. Core Tetrads

If the Contradeterminator be admitted as an essential tone of the Scale (although restricted to special purposes), it is possible to consider the extension of the Core to a Tetrad form, such chord being distinguished from the Composite forms already familiar in Envelope configurations.

This practice supports the Tetraphonic basis of polyphony.

These Concordant Tetrads are Serials, but in E.T. the ear has to be trained (the enantio case does this) to accept the representative tones Ta and Fe, which are more out of tune than any other component of the chord.

That the ear can get used to grasping these distorted tones is proved by the case of the Augmented and Diminished First Order chromes, and indeed by the use of the E.T. itself. Once

this assimilation is effected we obtain many of the apparent complexes of modern harmony, which appear as Core Tetrads.

The process is one of great power, because each core can be directed, but as a general rule of practice it should not be overdone, or an effect of sloppiness is only too certain.

91. Axial Function of Core Tetrads in the Antinominant Matrix

One particular aspect of importance is seen in the case of the Antinominant Tertriad, in which the relationship of the trinomial of triads is due to contiguity, and thus not axial.

Since the Contradeterminators are the diametricals of their respective determinators, the Antinominant is thus linked with the corresponding Pythagorean arrangement (admitting the quasi-equation $7:5 = W/2$), thus permitting axial relations from which progressional forms comparable with those already considered may be derived.

92. The Octomial Scale

We observe that much that looks complex in modern practice may be reduced to simple forms by viewing one of the four tones in a Tetrad as Contradeterminator in an Octomial Scale.

It is practically not necessary to distinguish it in notation, but it would be of great assistance to theoretical examination if this could be done.

The celebrated controversy as to the Augmented Fifth versus the Diminished Thirteenth—discussed by Day and others—is a case in point.

CHAPTER X

CHORDAL ASPECT. AXIAL THEORY

93. The Two General Forms of Scale

Scales, considered as a succession of steps from one tone to another, may be of either of the general forms—

(1) Nominate, of which the Septomial is typical.
(2) Connective, such as the Dodecanal.

The components in this latter become named only in respect to the former. Of itself the Domain is non-matrical, the Dodecanal representing the least system in which nominance of the Septomial extension vanishes.

The former is the basis of a determinate system of tonality, and the latter its flexible vehicle, itself impartial, and only nominated by considering alternatives present.

The use of the purely impartial chromatic scale in practice is rare, but it is always recognisable in passages that merely connect, swoop up to, or lead, and, in general, behave much as the portamento would under the same circumstances.

Tertriadal nomination with its three orders of components forms the usual basis. The Hemicyclic form of same, because of its concomitant duality, Pythagorean form, and modal composite species capability, is taken as the starting-point, or Scale of "Naturals," both in general consideration and as the plan of Staff and Solfa Notation. Thus arises the apparent anomaly of Phonal expression in the terms of Chordance.

The primary axes of the Tertriad, connecting Core and Envelope, are P and T. Those coupling the concomitant Predominant and Recessive Species in the Hemicycle are J with TD and L as orthogons.

The variants of the purely Tertriadal form are shown in Chapter V.

94. Progression described from the Chordal Standpoint. The Axis Coefficient

Progression of any kind is most definitely described from a chordal standpoint.

If tonal manifestation, regarded as a phenomenon, was based on something like the chance configurations of the kaleidoscope, nominance would not be important. But the first condition of determinance is knowledge of terms and relations in definite aspects; consequently we proceed to describe succession in chordal terms.

94*a*.—Definition of Terms relating to Chordal Progression

(*a*) A Chord Alters, by either motion or change of its components.

(*b*) A Component Moves when its actual or E.T. Pitch alters.

(*c*) A Component Changes when its nomination alters with respect of an Axis or Conserved system.

Although the Core-Envelope basis is the simplest aspect, it is not the only one.

The concept of an Axis, although only a mode of expression, enables us to grasp and describe progression in a compact manner, which can be employed with both Primary (exact) and Secondary (liminal) Axes.

Progressions are known as Uniaxial, Binaxial, etc., according to the number of simultaneous axes. A Chrome axis is of course binaxial in tones.

A convenient method of describing an axial progression is by the use of the Axis Coefficient, which is in the form of a fraction prefixed to the axial tone, of which the denominator denotes the old, and the numerator the new value.

Example:—(LD/T) Fah shows the Matrix Prime of the first chord as Ta, and the progression changes it to La.

It may be urged that this is merely a device, but it is convenient.

95. Specification of Chord

The specification of a Chord involves statement of—

(1) Its Constitution with respect to the Matrix, and its Species.
(2) The locus of the Matrix in the Domain (Key).
(3) The absolute position in Pitch. This is negligible in the simple theory, but it affects practice.
(4) The actual achromatic arrangement of the tones. This also is negligible in the first abstraction, but must later be taken into account.

96. Definitions of Chord and Colligation. Tests

The criterion of Chordance is the coherence of members on some recognised basis, originally Serial.

Any other collection of tones, although objectively resembling a chord, is merely a colligation.

This definition of Chordance practically includes all colligations as composite forms of chord, since all intervals may represent some Serial value.

But as the audentity of Serial Coherence diminishes, so does the entity aspect of the resulting chord form.

The practical test of Coherence is the possibility of Chordal Exchange, which is discussed shortly.

97. Clue to the Basis of Chordal Relationship. Types of Progression

In the parallel case, practically any progression is possible, although some sound more definitive than others.

The clue to the criterion is seen in the comparative rarity of the diametrical progression, which evidences the necessity of some connective agency which is a minimum in this particular case, and this is found to be actual or latent Axiality.

Practically we judge progressions as Congruous or Incongruous, which premises an abstract "Gruence" as the range between the two extremes.

We are able to define progressions as—

(A) Conserved to one Matrix, which is reducible to Core and Polar Envelope members.

(B) Translation between two Matrices, in which case the "centroid point" of conservation is extended into a "line" of relation in the Bi-matrix.

The forms of progression may be classed as belonging to either or both of the following—

(X) Cyclic, in the same species.
(Y) Commutative, between the species.

The first type is exhibited in the most general manner by the Pythagorean Cycle (and its alternative-diametrical colleague, the Antinominant Cycle), wherein each step involves actual alteration of one tone out of seven, thus retaining six axes. Moreover, the binaxial Second Order progressions are expressible as Ternary or Quadrantal in terms of the same.

The second type refers to two primary kinds—

(1) Concommutation, possible in the Hemicycle owing to the conserved dual validity of the components.

(2) Permutation, which enables a relation of the same kind to be established at right angles and opposite hand.

98. Predominance of Hemicyclic Form of Tertriadal Scale

It is evident from these considerations that the Hemicyclic "form" of the Tertriad complies most generally with the conditions required; also that by means of Permutation any Domain tone can be brought in; J having a dual validity of alternatives, so that the dodecanal is arrived at with a minimum of material.

It is not possible to find any other system of which this can be said, so that it is obvious that the persistent survival of our so-called Diatonic Scale of Naturals is due to something more than the mere arbitrary choice of the numberless composers, performers, and audiences, even if one allows for the convenience of a fixed notation and claviature.

99. Preparation and Resolution of Chords

In the usual terminology of practice it is customary to regard Discords as originally "prepared," and eventually "resolving" on a Centron chord, although referential practice has for long permitted preparation to be inferred only.

From our standpoint we note the Anatonic progression from Core to Envelope by direct or successive stages.

The reverse simplification of Envelope chords towards the Core may be termed Catatonic.

The umbral extension of Triads to Tetrads has already been discussed.

Experience shows that many of the old axioms of practice are referential only, but the basis on which they stand is still determinative, however much practice may have branched out therefrom.

The Core Triad remains the basic representative of Centron simplicity and of terminal possibility in the final Cadence, but owing to the elastic variability in both pitch and time possible in practice some very strange-looking colligations may be accepted by the ear.

100. The Principle of Chordal Exchange

The principle of Chordal Exchange is an important factor in Successive Tonality.

Its value is seen by the manner in which phonal continuity of original flexionic implication can be maintained across a leap by means of Coherence.

If a definite phonon arrives at a tone in a well-established chord it may issue from another tone of the same chord without impaired continuity of effect.

The process may be illustrated by a person walking along a corridor and ascending or descending to another floor by means of a staircase or lift.

The principle has, however, an opposite aspect due to the possible ambiguity of any one of several phona in such a case.

If a number of phona at a certain epoch formulate a chord of definite type there is always some uncertainty as to the identity of each issuing phonon.

This case is shown when two phona approach and retreat to and from a common tone of meeting, in which case the natural discrimination of pitch is affected by the apparent persistence of direction which would arise on actual crossing.

Unless the two phona are distinguished by tone characteristics or local figuration, the result, which occurs with both unisons and achromes, tends not only to ambiguity, but "hollowness" of tone mass. Hence touching is normally aphonal unless managed with care.

The avoidance of doubling of Leading Note or Mediant is pointed out in elementary text-books, and the ambiguity about J, as symmetrical between P and PD, is noticeable.

101. The Character of the Bass Phonon

We have already noticed an observation with regard to the Bass Phonon, which is found to either move by step in scale, or if it leaps the leading or following tone is axial with the succeeding or preceding chord.

In a minority of cases this is found not to hold, but the second condition is secured by definite chordal exchange.

The writer has examined Bass Phona in a large number of cases and found the above conditions to hold to an extent almost warranting its reference as the "Bass Law," but he recommends his readers to search for contradictions observable in the case of real Basses, that is, those that sound satisfactory.

CHAPTER XI

PHONAL ASPECT. FLUENT THEORY

102. Pitch and Interval. Chrome Conversion

It is evident that the expression of tones in frequencies, *i. e.* by multiples of an arbitrary unit taken as base, does not correspond with any auditory perception.

Until this physical measure was first ascertained by Father Mersenne there was no definite knowledge of frequency as such, only its reciprocal measure in terms of pipe and string lengths, etc., which was the basis of the earliest tonal determinance.

On the other hand, Pitch is a definite perception.

We possess no natural space measure in our ears; this arises from the recognition of intervals, leading to the stepping out of equigrade values, in which the frequency of the tones is represented by its logarithm to an arbitrary base.

The conversion of one length (Chrome) into another, to be independent of measure-scale, should be a Ratio, so that the fluent should strictly be expressed as a logarithm of pitch or lo-logarithm of frequency. This, however, does not correspond to any auditory perception, so is not employed.

Our ears do not perceive any difference beyond the relative dimensions in intervals regarded as chromes, fluents, or limina. It is only when we come to regard the special *use* of each degree in tonal procedure that we can conceive fluents differing entirely from chromes on the ground of connoting a time succession.

This is not easy to do, but there is reason for it.

103. The Function of the Fluent

Fluents are, or can be chordally exchanged for, small intervals of the dimensions of a scale step, their functions being—

(1) To represent the phonic continuity and thus the adherence of the flexion by means of the scale.

(2) By reference to an Axis, to nominate each scale tone with

respect to the group, and to represent the tonal "operation" performed in passing from any one tone to any other.

As a matter of fact, axes may be many; we refer to any one selected in a particular aspect.

The position of the fluent as a scale step is obvious from the primary relations of the Matrix.

If we leave out the large fluent 4/3 for the moment, it is seen that the remainder of the group are smaller than the minimum Chrome V, and thus a scale is not liable to be mistaken for a concordant arpeggio. The steps are far enough apart to prevent merging into a liminal continuity, but not so large as to prevent the flexionic implication of adherence being effective.

104. The Nomination of Scale Steps

A fluent appears as something which nominates each scale step by reference to an Axis connecting two Chromes.

This way of looking at it implies a concept of non-simultaneity.

A Chrome is an experience in itself, abstracted from the blending Series, and can be heard, recognised, and thought of both simultaneously and successively (as an arpeggio) at will.

We have endeavoured to symbolise this concept by the term "Chrome" and the use of colour-labels, which are quite inapplicable to the non-simultaneous fluent.

Spectral shift is about the nearest analogy to the latter, but it is best to cut quite loose from the colour symbol system when it ceases to be helpful.

105. A Fluent implies Reference to a Principal Axis

The necessary reference to one Axis at least out of the many possible, definitely bars simultaneous conception.

If the tones are heard together, they merely sound a discord whose obvious implication is towards resolution by one or both tones moving to a concord.

We have thus no immediate perception of the fluent. Notation does not distinguish the degree, but only the magnitude from which the degree is inferred.

It is not so much the antitonal jarring of beats that drives

the two tones apart, although this is a factor, as the realisation that these tones are axially related, and an axial concept implies movement, *i. e.* non-simultaneity.

The importance of the fluent concept is that it introduces the notion of Time into tonality, in a way that the chordance aspect does not.

A fluent, considered as an interval, refers to a series of Axes. Again, in the E.T. system the limited actual intervals available represent several such Series.

The reader may note the analogy to mathematical equations which may have several "roots."

In the present view we confine attention to the earliest cases, which, according to the property of Series, are the only factors of much audentity.

Essentially, however, the expression of a fluent, as correctly shown by its coefficient, embraces the two nominated tones which it connects.

The importance of this view is that each scale fluent has a definite place in the scale, relative to the termini.

106. The Numerics of Fluents and the Series

The numerical theory of Tonal Degree shows fluents as Pitch differences or Frequency Ratios of Chromes, and as such they refer to a calculable series of possible Axes.

To maintain the distinction between certain small intervals (and their applements and achromes) regarded as fluents and as Serials respectively, we nominate the Order of a fluent by that of the Chromes it connects.

As a Serial, the same objective interval has its own order.

To make this clear we may refer to the origin of fluents.

Let $^{P}{}_{1\ 2\ 3\ 4\ 5\ 6\ 7}$, etc., be a Series whose general term is $^{P}{}_{n}$.

Its Serial chromes are given by the successive ratios $P\left(\frac{n-1}{n}\right)$.

The Series of fluents is given in terms of the *larger* Serial Chrome by $P\left(\frac{n^2}{n^2-1}\right)$.

Thus we get the series of Fluents—

4/3 9/8 16/15 25/24 36/35 49/48 64/63 81/80

as converters between

W B R G V Hyp. V Sup. O O_i O_u

and name them as follows—

	Fluents of:	
4/3	R.	The Sesqui or Inter-Z-First Order.
9/8	O.	The First Order.
16/15	I.	The Inter-First-Second Order.
25/24	S.	The Second Order.
36/35		The Sesqui Second Order.
49/48	2.	The Contra Second Order.
64/63	(Riemann).	The Inter-Second-Third Order.
81/80	(Comma).	The leading Third Order, etc.

The ratio of each of these to their sub-adjacent chrome is an increasing value.

The superior limit of the "intrinsic" fluent is the Oscillant.

R is indeed a fluent, but it is also a Chrome, depending on its use as to which aspect predominates.

Usually the chromal audentity of R prevails, so that it is difficult to think of such a large interval as a Step in the same way as other fluents.

As a converter of W to B it is one-sided, appearing as a Semi-scale range rather than a step; but its exchangeability (by deduction of a Second Order Chrome) for O or I can be effected in many cases without altering the tonal implication of a progression, which is proof of its dual Degree.

Every musician knows that R possesses certain distinctive peculiarities.

While not going so far as the semi-humorous theory of Mr. James Glover, it is obviously an important factor in simple melody, and as such may often be regarded as its fluent rather than arpeggial-chromal aspect.

107. The Oscillant, Impellant, Suboscillant, and Umbral Fluents

The "whole tone" of the Scale represents a multitude of fluents, of which the leading types are the infra First Order Oscillant (strictly the only fluent of the pure Hemicyclic Matrix) and the ultra form 10/9 which converts R : V and Y : B, being of "Sesqui" Order.

To these may be added the neutral disjunct fluent which, by linking up the ends of the tetrad envelope, completes the scale of Octave range.

The latter, owing to the permutability of both its tones, has four values, two being oscillants, one Antinominant, and the augmented Oscillant identical with the Chrome V. The last is familiar in the so-called harmonic minor Mode.

The Impellant 16/15 is an important factor in the Scale, as may be inferred from the tone named Leading Note.

It is of Sesqui Order and represents the smallest scalar fluent.

The Suboscillant 25/24 of Second Order is not a scalar fluent, but an Alternative permutor.

We then enter upon the "umbral" region containing the Inter 36/35, and Contra-suboscillant 49/48, which verge upon liminance, the latter stage being definitely attained with the Riemann converter 64/63 and the Comma 81/80.

108. The Fluents of the Scale

In passing along the Septomial Scale we alternate between Core and Envelope tones until we complete the Unitor by the wholly-envelope disjunct fluent.

Each of the regular fluents of the Scale connects a Second Order tone with one of Zero or First Order, and is affected by the permutability of the former tone, thus presenting the same fluent in two alternative forms.

The essential "fluent" aspect of these steps lies in their Locatable nominance in the Scale as stages in a cyclic progression between Achrome termini.

The dimensions of these fluents do not exceed the acoustical region of parasyntony about each tone, which is the determinating basis of Adherence.

109. The "Invelope"

When an interval is regarded as Core it is sometimes convenient to regard the internal adjacent tones as forming an Invelope. This aspect is particularly concerned with the Antinominant Matrix in which chords such as those known as "Augmented Sixths" are involved. It may be recalled that the interval of the seventh normally converges to its Core, but when diametrically translated to form an augmented Sixth it diverges, hence the distinction.

110. Practical View of Liminance

We have regarded Liminance as of Third Degree in Tonal Determinance, but it is practically the auditory limit of distinction between both chromes and fluents.

Beyond V the open serial chromes begin to be confounded with their neighbours, as one may see by experiment. In fact, the multiplicity of Second Order tuning is probably one of the reasons why such intervals were so late in appearing as determinative factors in tonal history (they practically came into effective vogue with solo-chordal instruments of keyboard type).

Again, fluents become ineffective as they approach liminal dimensions, so that really Liminance is not a Degree of Determinance, but a general indeterminance. It is, however, convenient to approach it from that aspect, and possibly some intrinsic determinance peculiar to the Third Degree may be developed in future, although the physiological constants of an ear that gets on very well with the Equal Temperament cannot be normally altered.

111. The Extensive Possibility of a few Representative Fluents in Practice

Examination of actual works of music composed during the last forty years shows what can be done in the way of building up composite scale forms with the few fluents available. It is not the actual number of objective intervals, but the variety of their axial reference that enables the modern musician to say so much with such a restricted vocabulary.

The gradual development of the fluent as something rationally incomprehensible, except in succession, has followed the concept of Chromality. We can perceive chromes apart from their tones in abstract and think and talk about them.

Although it is not so easy, we can similarly abstract the notion of a fluent from the actual steps of a scale or melody as something having a place relative to the Matrix or Key.

One of the characteristics of an admittedly significant melody is its power of standing alone. We may attribute the manner in which its fluents imply the axiality independently of harmony as a manifestation of the powers of composition and agreed perception of the auditor.

112. Extramural and Alternative Classes of Fluent

We have noted that the Extramural class of intervals are definitely associated with Scale steps and the Alternative class with permutation.

In passing along the Scale we definitely change position (as shown on Staff Notation) or a name (as indicated by the Solfa symbols) for a contiguous but non-identical value.

A permuted tone is regarded not as a step value but as an alternative identity, as indicated by the Staff prefix or the Solfa name modification.

The lesser audentity of the Second Order tones in relation to those of simpler orders is evidenced by this procedure.

113. The Directive Property of Alternative Fluents

The directive property of Alternatives is to be noted.

Determinators naturally tend to permutation in the absolute direction of progression, as evidenced by the dual forms of the so-called Melodic minor mode, and the possibility of combining these latter in contrary motion without an excessive effect of false relationship.

This is not invariant, but it persists in practice, and is thus based on something more than mere arbitrary choice or convention.

114. Multi-fluents. The Quarter-tone Step

In polyphony the simultaneous occurrence of fluents enables us to consider "chords" of fluents, *i. e.* multi-fluents. These are of interest in the cases where the Diphonon and Fauxbourdon Triphonon replace the Monophonon as already discussed.

The case is of interest as leading to the logical implication of a quarter-tone, where an impellant parallels a two-stage oscillant. So far, however, practice has been content to hold one of the notes for a half stage, but both string players and singers are heard sometimes unconsciously to use the quarter-tone or A/2 step in this case, and one can hardly object to it.

115. The Umbral or Contra-fluents

The Contra-fluents, *i. e.* 36/35, 49/48, and 64/63, are involved, theoretically at least, when we admit umbral tones.

The Contra-suboscillant 49/48 is the permutor of the Contra Second Order umbral chromes known as the Hypo-violet and Super-oscillant respectively.

Practically it is a Limen, but since these intervals, if employed, are represented by the somewhat extremely out of tune E.T. tones F (Ta and Lah), C (Fe and Soh), the small step becomes the much larger E.T. unit.

This condition arises, for instance, when such chords as the Added Sixth and the Seventh are regarded as adduct and educt forms of Radical Tetrads.

116. The Mean Value of the Liminant in Ordinary Practice

The two E.T. fluents have to do duty for a large number of values even in restricted tonality. The Oscillant, as we have seen, represents at least three important fluents, and the Impellant represents not only the widely different Extramur and Alternative steps, but their Contra values.

There is a very real difference of space between I and S, viz. the Dieses 128/125, which is practically of the same dimensions as a Contra-suboscillant.

The fact that this 25 per cent. libration is not only put up with, but also practically negligible, shows the importance of the Liminal factor in Tonality.

117. The Implication of Succession contained in the Fluent Concept

To recapitulate, the fluent is seen to be the important factor in tonality which definitely implies Succession, and thus introduces the consideration of different epochs into Tonality. It links up the progressive flexion with the static chord, and its positional value is seen to be the basis of the sandwich-like form of Core Envelope scale, on which determinate tonality originally rests.

CHAPTER XII

MATRICAL AND DOMAIN ASPECT. INHERENCE THEORY

118. The Secondary Axis in the Matrix

The foundation of Matrical Inherence is the secondary axis, which is a result of liminance.

The secondary axis provides another path of connection between a pair of tones already related by a primary.

This is evident in the Serial aspect of the Matrix in which the Core Triad is approximated by the higher polar serial tones.

The Hemicyclic relationship provides the most striking illustration in the case of the Yoke J, which has a dual validity. J as 81 is a Bitensor or Terprime of extended First Order. J as 80 is of Second Order in derivation. By eliminating the Comma 81 : 80 we obtain the Secondary Axial path between P and D, so that each is the Quadritensor of the other. (Isotopes!)

119. The Liminal Error involved in the Secondary Axes

Secondary Axes involve errors of liminal extent which are valued in respect to those involved in the mutual accommodation of tunings. The largest error in practice is that involved in using the same interval for the Impellant and the Suboscillant which is the "Dieses" between 3 G : W (128 : 125), approximating to the Serial mean (42 : 41) of the Umbral Fluents 36 : 35 and 49 : 48.

When a Secondary Axis is taken as exact, it logically implies error in the original primary axis.

120. The Closing of Groups of Relationship by Secondary Axes

Without Secondary Axes, matrical inherence would cease to have any meaning, each tone being the centre of a set of relationships extending to infinity, but never coinciding with the value of any other system beyond recurrences of the primary relation.

But in a closed system two tones are related by inherence: we say they are in the same key, and similarly with the Domain in which the "key" may modulate. Thus the relations of the two tones are always determinative, and the fact that liminance allows practically any tone to be so regarded does not affect the theoretical importance.

Indeterminance arises from anomaly of predicates, as we have seen in the case of Dissonances. For instance, there is one tone that by Matrical inherence is placed in definite contrast to a Nominant, that is the Diametrical.

It is not indeterminative in itself, but only when the relationship is brought into conflict with the phase opposition. Thus in ordinary practice it is comparatively rare to find diametrical translation, but by no means uncommon for an Envelope to swing over diametrically.

In Triadal chordance a diametrical represents the furthest cyclic departure, and ambiguity of polarity. In Tetradial systems, admitting the Contradeterminator, the relationship is definite.

It may be noted that the three least frequently direct translations (modulations) are to the "diametricals" of the sub-cycles G, which are the "Continuants" of the Ternary divisions of the Octave scale, *i. e.* from Doh to Ray, Fe, and Ta as keynotes respectively.

121. Transcendence of Matrical Limits

Matrical inherence is thus associated with a system of limitation: a ring fence around a limited group of tones. But this does not prevent transcendence.

The strength of the limitation is in the audentity of the conditions by which it is formulated. There are, however, lines of lesser audentity along which it is possible to proceed with extension in a manner that practicians show has apparently no end.

We have already seen how the Umbral Contradeterminator (which is barred by the primary equation $7 : 5 \equiv W/2$) is used with effect, and the complex effects of modern harmonic methods which the ear actually accepts is evidence that the "fence" of a matrix is not unscalable.

122. Flexionic Connection and the Antinominant Matrix

We must never forget that in the background of convention we have the original flexionic slide, and its resultant, the Antinominant Matrix and its Scale, all capable of parallel extension to the Pythagorean methods.

This is more in use than is generally supposed, but since it has no specific form in notational basis it does not strike the eye in the same way as the more evident Pythagorean foundation, and up to the present I have not come across any text-book treating it adequately from an empiric point of view.

It now wants some one to suggest a simple replacing symbol for temporary and permanent modulation, which shall do for Staff what the modulant does for Solfa, and which shall satisfy writer, printer, and reader, and eliminate some of the clusters of "accidentals" that encumber our pages. The really wonderful results obtained in modern days with much the same material and script that did duty over two centuries ago is evidence of the vitality of old methods, and the marvellous power of mental reference, but improvements could be made.

123. Translation

Translation is change of matrical position. Mutation is change of Species.

These are so involved in practice as to make separate examination somewhat difficult.

Both operations can be effected in two ways, viz.—

(1) Slide of all members along pitch.

(2) Axial swing about a constant tone which changes its name but retains, actually or liminally, its pitch.

The principal representatives of these processes are seen in the Antinominant and Pythagorean cycles respectively.

124. Concomitance as Quadrantal Relationship

Concomitance is the particular commute relation of Species in the Hemicycle about an axial J and the intervals symmetrically disposed thereabout, viz. P : PD, etc.

Quadri-concomitance occurs about L, the tones remaining axial if the determinators have previously been permuted.

Every "Quadrant" of the cycle (as a Quadrinomial of Zero First Order Axials, or a Trichromal of First Order Intervals), has thus two companion quadrants, respectively to the right and left on the dial, the direction being reversed on commutation.

The remaining quadrant of diametricals is related concomitantly and quadri-concomitantly to these two companions respectively. It also contains the ultra-concomitants (contra-determinators), and so is linked up by the Tetradial extension of the Tertriad, the whole presenting the Domain of twelve tones.

If the original quadrant is regarded as of Predominant Species, its two companions are considered Recessive. Up to the present, practice only recognises one Species at any one time, but it is impossible to say whether combinations of both may not become the art material of the future.

125. The Two "Handed" Quadrantal Relationship

With respect to these two "handed" companions, musicians are familiar with the Relative and Tonic Minors to a given Major Key, and vice versa, and much controversy and ink have been expended in discussing which is the true Minor.

With the practical side of the question as affected by conditions of performance we have nothing to do, and every practician and teacher has a right to his honest opinion.

All that theory recognises is that every Nominant has two neighbours of different species, concomitant in tones with the unpermuted and permuted variants of the Septomial, so that both views of the "Minor" are covered.

There are thus two aspects to every case of Translation, since a Commute matrix may be regarded as an original species with permuted (alternative) determinators.

In the same way, a monophonon scale is indefinite in both Species and Mode until harmony or accentuation settles the respective order of its component tones.

We therefore encounter at times the paradoxical case of Translation neutralised by Commutation and vice versa, and when the principle is applied to partial and mixed cases it is seen how complex the problems which confront the analyst may be. It is here that the ear comes in as judge.

126. Conditions of Interworking with Major and Minor Modes

The practical bearing of these theoretical relations is evident in the close interworking of the Major and Minor Modes, which differ from the abstract case in being always subject to Vertication, *i. e.* directed upwards as Fundamental.

It is evident that if this were not the case, the resources of practical Tonality would be cut down by one-half, and intermodal working could not be successfully developed without extra complications due to reversal of Species direction.

Both Hemicyclic Fundamental (Major Mode) and the variants of Coincidental forms (Minor Modes) are composite in species, and are referable to the original Symmetrical Tertriads which are neutral in species and consequently rarely used.

We see that the conventional Tonic tone and the Coincidental Prime are aspectively different.

The fact that the Neapolitan Sixth progression sounds so much better in the Minor than the Major Mode is one testimony that the Minor Tonic or so-called Keynote is actually the Coincidental Tensor, round which such a progression is normal.

The whole question turns upon the variety of aspects presented by the theory of Tonality as compared with the more limited view (accepted for economic reasons) of simple empirics, which is supported by the conventional system of notation based upon a Fundamental standpoint.

We can readily (and with profit) abstract Vertication conditions in theory, but they cannot be eliminated in practice. The conversity of the Species' remains true, but the musician does not necessarily stand on his head to view it, and therefore works with the apparent system of Mode rather than the theoretic mirror image.

127. The Centron of a Matrix

The "centron" of a matrical system is illustrated by the mechanical centroid of a mass, whose motion typifies that of the latter.

The centron tone is the theoretical centre of symmetry about which the other members of the system are in equilibrium. In

the Tertriadal aspect the Prime Determinator of either species occupies the position, and the Yoke is the natural centron of the concomitant Hemicycle.

In practice, however, it is usual to choose the Fundamental Prime and the Coincidental Tensor as "keynotes," and this is generally a convenient method, since Triads *on* a given tone are then read upwards conformably with the theory of permutation. These should carefully be distinguished (as regards C. Species) from the Triads *of* a given tone.

128. The Principle of Musical Notation

The basis of notation is the Hemicyclic form of the Tertriadal aspect, giving a Septomial scale of "naturals," each of which can be sharpened or flattened by prescription of symbol or alteration of stem name. Translation is then indicated by the signature.

Commutation and Mode are not specifically indicated.

The gradual development of Translation may be traced in musical history. In earlier days elaborate rules attempted to regulate procedure. The improvement of instruments and technique of practice, together with the international exchange of ideas and idiom, led to the great development now observable, which is somewhat bewildering on the old basis of judgment.

Practical Translation takes place axially or by scalar step, and the circle of twelve keys is completed liminally by the equivalence of six sharps and flats.

Liminal creep can be used to actually modify pitch, but it is not yet recognised as a practical method.

129. Local and Complete Translation

Translation may be either local (temporary modulation) or complete. The boundary between the two is difficult to define, and notation in this matter usually follows the line of convenience rather than theoretical strictness.

The test of local translation is the capability of adding a Pedal note, and for short passages, completion of chord; thus, if a Subdominant Triad can be made into an "Added Sixth" and a Dominant Triad extended into a Seventh, without alteration of effect, the original key has not been quitted.

For examples of practical methods of modulation see text-books and actual works. The process generally involves either or both, (1) addition to Core, (2) modification of Core.

130. The Cyclic Domain and its Theoretical Extension

The cyclic Domain follows from Pythagorean extension. Thus the Tertriad may be extended into a Tri-tertriad (Triheptad) embracing five Triads, whose five permutable determinators provide the læval hemicyclic tones.

The Domain may theoretically be widened indefinitely along recognised lines of extension by—

(1) Dividing up the flexion into smaller steps than W/12.
(2) Extending chains of relationships and neglecting the earlier secondary axes.

The ear, however, limits general theoretical extension, for what is gained in membership is lost to liminance.

The basis of limitation thus tends to remain with the general twelve-toned scale, and without predicting that it will always do so, there seems no reason at present for scrapping the E.T. reference and its early secondary axes.

131. Inherence as a Binding Element in Time

An important point about Matrical Conservation or simple Translation is that tones, intervals and progressions throughout a lengthy passage are all related either directly, or by simple transformations, to the same basis of judgment.

This persistence of "Key" throughout an extended portion of musical work is a binding element in Time.

Its simplest manifestation is seen in the departure from, and return to, a given tone, by a pair of complementary progressions, at Achrome points.

This is the Tonal basis of the Cadence.

FREE TONES

PART III

THE OUTCOME OF PHONALITY

CHAPTER XIII

THE AUTOPHOR

132. Tones outside Direct Chordal Relationship

We now consider tones which are outside the pale of direct chordal relationship, although nothing distinguishes them as such in notation.

We have considered Chordal progression, whose elements are primarily regarded as tones simultaneously moving to tones, and so far, the influence of one progression upon its successor has been neglected, as our consideration has been entirely restricted to a passage involving two tones.

In Phonal progression we have extended the scope on the Scale basis, which consists of a system of steps between termini, the relative durations, etc., being neglected and only the direction and termini considered.

In practice we are at an early stage brought into contact with more extended conditions than these, viz. non-simultaneity, and progressions which take place by evident stages, in which certain tones appear as if they were detached from the chordal scheme.

133. Recognition of Intermediate Tones as Self-carrying

It is evident that the combined view of Chordance and Phonality will involve consideration of two classes of tones, viz.—

(1) Those cohering in chordal relationship, referring originally to the Series.

(2) Those definitely intermediate to or recognisably outside same.

These two sets of relationships are capable of a considerable amount of independent freedom.

Referring to the original Core-Envelope basis, if we take either a definite Core or a definite Envelope, the tones of the "not-core" or "not-envelope" are detached"; in fact, there is considerable freedom of choice in filling up the gaps of chordance.

The chordal tones, actual or latent, constitute the anchors of the phonon; they couple up the melody and harmony of the particular passage and chordance.

The "free" or intermediate tones carry themselves between the anchors, subject only to the condition of phonal continuity, and for this reason they may be known as Autophors or self-carrying tones.

The Extramur class of relation, as already discussed in Chapter III., are "neighbours" of a given nominant.

The Septomial shows each tone of the Core to have a sub- and super-Envelope tone, or suburb, which is definitely not of the Nominant or any alternative form of same. (Notation is not always correct on this point.)

Similarly with the Envelope, only here we have two extreme polar tones adjacent which are mutually negative to each other in consequence of their polarity.

The criterion of an Autophor is its necessary (eventual) progression to an anchor tone. It is not determinative to stop on an Autophor, that is, tonally. Tempo variation in Rhythm does not affect the case.

134. Non-simultaneity in Progression

A different aspect of the same thing is afforded by the condition of Time succession, which was first introduced by the prolation of a tone into a Flexion (Phonal aspect) and by the Fluent (Chordal aspect).

The time factor is still further extended by the possibility of non-simultaneity of the progression of the different tones in a chord, by the use of differing durations, so that intermediate forms appear between the initial and final stages.

These may generally be called Internomials (Seminomials, when restricted to one intermediate to two Nomials, and a Bidodecanal Cycle of Pythagoreans can be constructed, in which the intermediate triads appear as recessive in species, as well as binaxially related to their neighbours).

Such intermediate tone colligations may be identical with or differ from recognised simple or composite chords, but their rationale is recognition as stages in a process which must sooner or later, directly or indirectly, arrive at the terminal anchor chord.

It is possible to imply the initial Anchor and drop it in actual cases, and when this was first done it raised a terrific storm of indignation from the classical theorists, but the ear accepts such implications. The final stage is not so easily eliminated, but we will not maintain that it cannot be done.

135. Practical Conditions of Autophors

The practical use of Autophors greatly extends the scope of Tonality, since it only involves recognition by the auditor, and certain practical conditions as to carrying out effectively.

Practically anything can be interpolated into the passage between anchors, and the initial and final stages of a definite progression may be widely separated by intermediate matter so long as the phonality and chordance are determinate.

As every practician knows, to carry this out effectively requires considerable skill and knowledge, since it is fatally easy to submerge the phonality or swamp the chordance.

It is obvious that the conditions of melodic formulation would be much restricted if it was absolutely necessary to always keep to chordal progression. On the other hand, absolutely free phonality would be indeterminate, there being in this case no basis for regarding any given configuration as more determinate than any other.

136. The Two Classes of Autophor, and Two Aspects of Each

We thus recognise two classes of Autophor, viz.—

(1) Tones Interpolated between two chordal members. Of

these the latter may be identical, as in the case of Auxiliary or Grace notes, ornaments, etc., or they may be different tones of the same chord, as with Passing and Change notes.

The initial anchor may be actual or latent, but the final is always bound to appear.

(2) Passages Substituted for the tones of the second chord of a progression, as Suspensions, Anticipations, Retardations, and the inverse case of Pedal points.

Combination of both forms of Autophor gives rise to some very complex forms, and it is astonishing to note how prolation may be extended.

We must also include elision as a process of reduction, which is the source of much that looks complicated.

One typical case is the deletion of the middle stage in a suspension. If this has an independent chordal implication it is usually very effective, corresponding to ellipsis in speech.

It is possible to view the two classes in another aspect. Thus Class 1 (Interpolants) may be regarded as phonal substitutes for bare gaps, and this process may of course be reversed.

Class 2 (Replacements) may be also regarded as chordal progressions distorted obliquely in time.

137. The Classification of Counterpoint

Our musical readers will probably be familiar with academic Counterpoint, that valuable training school which no one can afford to neglect, and others who are interested in tonal problems will probably have read text-books on same.

It will be recalled that five so-called "Species" are delineated. For theoretical purposes these can be reduced to three, viz.—

(1) Chordal. Note against note.

(2) Autophor Interpolants, to which arpeggials are added in practice.

(3) Autophor Substitutions.

The rules of Strict and Free Counterpoint are concerned with the inculcation of good practice, and with this we are not concerned. Our duty is simply to point out the logical basis of the phenomena, and this is the maintenance of phonal and chordal determinance.

138. Conditions as to Effective Use

In the Interpolant class the Autophors must be phonal or arpeggial in form; not necessarily restricted to simple cases, but the actual tones must flow with continued identity, and must couple up with the chordance scheme at some recognised anchor point or its extension.

The Substitutional class are subject to the same dual determinance. The substituted passage must be phonal in itself, and since the time distortion of a chord simply extends the harmony into an arpeggio, the connection of initial and final stages is thus dual.

Although it is not determinate to dispense with the terminal chord, this latter may have itself progressed, leaving its implication as the actual anchor.

139. Pedal Points

An inverse process is involved in what used to be termed "Pedal points."

These matrical anchors are tones, chords, or passages, held throughout a free co-progression for the purpose of maintaining Conservation or a definite line of Translation among a maze of tones and chords which may be of the most diverse appearance.

Originally only the Tertriadal Primary Axes P and T were so employed, but subject to the condition of clear determinance, practically any tone, chord, or formula may act the part successfully.

There is no theoretical limit to the composite nature of the forms resulting, provided always that the effect is determinate.

Amateur efforts usually result in somewhat lumpy effects from a desire to get all that is possible into the score, and the Autophor may easily submerge the chordance schema.

But if the ear be satisfied, it is not for theory to attempt arbitration.

140. The Change-note Passage as a Sequence

An important case arises with the form of Interpolant known as Change Note, which we may anticipate from a later chapter.

Here a scalar " filling " is turned into a simple pattern swinging from side to side of the line of progression.

In this case the formula is made up of a repeated figure on a scale or arpeggio, giving rise to the simple form of Sequence, which is found to be of considerable importance in basic structure.

141. Parcels of Tones as Entities

The Autophor view enables us to comprehend passages of successive tones as unit Parcels, and thus to advance towards our next division of Hyperacoustics, viz. Rhythm.

This view is based on the fact that the tones of the parcel are bound together by their common destination, and we can thus think of a musical passage as a whole somewhat as we do with a chord or arpeggio in Tonality.

Further, the arrival at an Anchor may be a transient or a terminal stage. In the latter case a definite epoch is marked out, and the final note or chord in conjunction with the progression leading up to it becomes conceptionally differentiated from its surroundings, although it may not tonally differ from any other employed progression.

Here we have to recognise something beyond the primary acoustic concepts from which we started, which implies phenomena of which Tonality is the medium, but which transcends same; somewhat as Tonality does the purely acoustic factors mentioned in the commencement.

From a practical point of view the reader will doubtless have come to somewhat the same conclusion, that there is a factor in Tonality which refers to Rhythm.

In contradistinction to the scholastic method of Counterpoint we may refer to the broad evidence of history, in which Discant developed a polyphonic form consisting broadly of Autophors anchored more or less satisfactorily at intervals. The chordance audentity of the anchors then became prominent, and derived phonal idioms arose which almost swamped the original free polyphony.

Modern practice neglects none of these factors, and successfully employs them in parts without the water-tight compartment methods formerly insisted on or implied.

142. Observation of Tonal Factors Implying Rhythm

Among the noticeable cases of Rhythm implication in Tonality the following may be mentioned.

Second inversions of Triads are most satisfactory as precedents in Cadences.

When W or B occur respectively on identical accents in successive bars, the effect tends to dissonance, especially between extreme phona.

Change of simple harmony most frequently takes place on an accented beat.

In strict counterpoint of the second species, the accented tones form concords, the unaccented tones are optionally either concordant or autophors.

Crossing of phona on a weak accent is less evident than otherwise.

The repetition of a tone in a change-note sequence tends to arrest the flow of the progression.

Autophor tones, if not changing and in the same direction, are representative of the Flexion.

A change-note autophor anticipating the first note of a new harmony is in general weak in effect.

Subdivisions of notes within a measure are best on weak accents.

Rapid change of harmony, or subharmonies on autophors, or excessively florid development, as a rule outweighs the effect of the principal tempo. Autophoric polyphony should be used somewhat sparingly unless heavy effects are required; in the latter case an effect of interpolated independent harmony is noted. Compare the general practice of Beethoven with that of Mozart.

CHAPTER XIV

EXTENSION AND DEVELOPMENT

143. The Processes of Extension and Reduction, and the Conditions of Use

We have seen that the actual tonality of practice is developed from and referable to certain basic forms along determinate lines, and we have to inquire into the system of these.

The process of Extension consists in developing from a primary basis of Phonal Scale and simple Chordal Progression.

The reverse process, Reduction, consists essentially in analysis or simplification to scale-form passages and harmonic framework. On the other hand, by elision of intermediate stages more complex forms can arise.

The essential condition (not always attained in practice) is the persistence of some factor of recognisability, so that the identity can be definitely maintained.

The student is trained in the Variation form, but it is fatally easy to vary a subject beyond recognisability.

Further, there is recognisability both by eye and ear, and every musician knows that the two are not always the same, and when one considers the extraneous factors involved in performance and the many details that militate against persistence of identity, it is obvious that reading a score through quietly before performance and the function of the properly written analytical programme are very helpful.

144. Extension in the Light of History

The best source of information on Extension is musical history, where it is obvious that the principle of Genetic Descent is the foundation, and the biologist will be struck with the comparison, which is not surprising if one considers the psychologic aspect of the process of composition.

History shows, of course, stages of rapid alteration, almost amounting to the introduction of new material and modes. Also there are always accidental discoveries and secondary products, etc., but progress of musical evolution is mutual agreement

between composer, performer, and auditor on the basic points. As a rule the composer is well ahead in this partnership, and the most extreme complexes are not always clear as to determinate ancestry.

It is difficult to trace the result of intuition and intense concentration, and composers as a rule cannot describe their methods.

145. Methods of Extension

The processes of Extension and Reduction are twofold.

(1) Inter and Extrapolation between original matter pulled apart or closed up.

(2) Substitution of more or less extended matter.

The necessary condition of Interpolation is a satisfactory joint at ends combined with maintained determinance.

That of Substitution is the mutual inherence of Substitute and Replaced matter in some common system.

The latter may be compared with similar processes in chemistry, whereby a class characteristic is maintained over a considerable range of substances (modern investigation is providing a numerical basis to this), and mathematical substitution and interpolation may be cited for the benefit of those familiar with them.

We have already noted one important case of substitutional possibility in Paraphony.

The closure of cycles of relationship always provides two connecting paths at least, one via the Primary and the other via the Secondary Axis (although the distinction between the two vanishes in the E.T. Domain, being replaced by "near" and "far" paths).

The variable range from exact diatonic to most free chromatic effect is evidence of what can be done; it is comparable with the modification of primary geometrical figures by rounding off corners, and the changes of form familiar in crystallography.

146. Pseudo-substitution, Elasticity of Practice, Difficulties of Judgment

One somewhat important point is the possibility of Pseudo-substitution.

It is obvious that one can get near substitutes without actually attaining same. Such "weak points" are known to every musician. The great masters, working on broad outlines, are sometimes indifferent to niceties of detail, and lesser lights occasionally lapse into expedient courses not always strictly determinate.

With Tonal morality and æsthetics we have nothing to do, and indeed we are barred from judgment owing to our whole tonal system being based upon approximation.

The extent to which this elasticity can be rightly or conveniently stretched to fit some particular dramatic or associated exigency is no matter for the scientist.

It is evident that if the liberty of practice be abused it may pass with some people for some time, but not with all people all the time. Consequently pseudo methods tend to quietly disappear.

It may be added that it is not at all easy to distinguish between brilliant sleight of hand and real progress along unorthodox lines. Critics do their best, but it is only Time that arbitrates decisively.

Music, like language and thought, lives by growth, and judgment is based upon retrospection.

147. The Sequence as an Element of Extension

As already mentioned, the basic material of Tonality is the Scale and Arpeggio of chord.

From these the simplest formula that can be developed is the Sequence, which is as important a factor in modern as in ancient work.

The essence of a Sequence is the repetition of a real or tonal pattern along a Scale or Arpeggio. It is thus of wave-like form, but must not be thought to imply metre.

The range of the Sequence is that of the Scale or Arpeggio, and is subject to the same conditions of recurrence.

In Septomial formulæ the triadal sequence (if as usual tonal) imports the diminished and augmented triads, which in this aspect can be treated as if pure. The scalar conservation is responsible for this.

Once these distorted forms have been synergetically associated in the ear, they may be abstracted and used as factors of

great power in tone colour, which indicates another line of Extension.

The hemicyclic forms occur at Opponent intervals, and thus introduce "singular points" twice in the Octave. These frequently form the limits of such a passage.

With Composite scales we get other augmented and diminished triads, many of which are familiar to the reader.

With certain arpeggial sequences, of which that on the Seriopolar chord is best known, and with "real" sequences, the pattern gets further away from its tonal origin as we proceed.

Such forms are frequently closed by a slight liminal modification or aspective change, so as to somewhat unexpectedly bring back the extension to the original matrix.

Plenty of instances of this are found in classical works.

148. The Twofold Development of Sequence

Sequences may be derived in two basic ways—

(1) The impression of a pattern upon a plain scale or arpeggio.

(2) Time-distortion of a chordal scalar passage, *i. e.* by suspension or anticipation.

Associated with such progressions is a predication of accentual interpretation, which in some cases is strong.

One may recall many old rules (still useful, in spite of modern freedom), and though these may now be regarded as out of date, there is no doubt that many sequences give an impression of incipient accentual differentiation.

Practice shows this is not invariant, but a primary implication, which serves to introduce recognition of that metrical factor already noted with Cadences.

If a given key holds, the polar progression to the Centron Triad naturally implies cadence. We merely note that such an implied differentiation shows there is something beyond Tonality in Hyperacoustics, of which Tonality is the vehicle.

CHAPTER XV

TYPES OF PASSAGES, AND THEIR ASSOCIATION MOULDS

149. Recognition of the "Passage" as an Element of Something beyond Tonality

We have now reached a stage when we recognise the capability of the determinate systems known as Tonality to become, as it were, the plastic material of some determinative agency other than itself.

Although this is no place to discuss Rhythm, it is advantageous to consider the accession of such factors as a suitable termination to this Division.

Tonal material is employed in practice in more or less obviously articulated passages. Experience shows that there is no invariable restriction of tonal material as regards this articulation, although certain forms are common. We are now concerned with the progression between two notes, and the scale between two termini, irrespective of any accentual difference or articulate division, or indeed anything beyond pitch distinction within the ranges of the other variables accepted by the ear. The point is now that there are such things as "passages" of tones which can be thought of and grasped as a whole, and whose performatorial emphasis as entities is a general improvement on a mere successive presentation.

150. Classification of Passages

We may therefore define a Passage as a group of successive tones, chords, etc., whether continuous or intermitted, which for any reason we wish to consider as a whole.

(1) Monophonic, which may be classed as either Adherent (Scales), Coherent (Chords), or Inherent (in respect to a definite matrix or translation line).

(2) Polyphonic. A significant succession (definite melody) leaping from one to another phonon.

This form is developed on the general principle of satisfactory effect, viz. floridity exchanged between parts, some of which rest for a time on that useful device, the Pedal note.

From a vocal point of view the effect is to give each singer a turn, and an opportunity to breathe.

(3) Polytonic.

The same thing may be done with different instruments in the orchestra (compare the fourth movement of Tschaikowsky's 6th Symphony). In this case the units change their relative pitch position.

(4) Combinations and variations of the abŏve not directly classifiable, which are used for special purposes.

151. The Four Types of Passage, and Table of Associated Conditions

Passages are infinitely varied, but four different associative bases are predominant. In general, passages partake of all four types, but specialisation is common.

Withŏut wishing to cripple the general view, we can tabulate these four leading types in conjunction with their principal association conditions.

These four types are—

(1) Formal.
(2) Vocative.
(3) Carminal.
(4) Personal.

The following Table shows the musical types alongside certain non-musical acts of man with which they have been particularly associated.

It is obvious that the association has had considerable effect in determining the way in which musical passages have developed.

TYPE	ASSOCIATION
(1) Formal. *Material.* Figures traced by— (*a*) Pitch outline. (*b*) Tonal determinance, which are capable of relational classification as— (*w*) *Imitation*, generating a parallelism with the origin which implies a right-angled "dimension" to the progression. (*x*) *Contrast*, generating with the originator a wave-form as part of a cyclic, and thus introducing period and phase conditions. (*y*) *Neutrality*, which may extend, spread out, pull apart, and insulate the elements of (*w*) and (*x*), thus converting "peaks" into plateaux and parallels into "bulb" forms. (*z*) *Continuation*, which either reinforces or weakens the argument. Associated therewith are— *Singular points.* (*t*) Commencement. (*u*) Centron or mean axis. (*v*) Terminus (Cadence).	Operations of a formal or prescriptive nature, known generally as— *Ceremonies*: Religious, State, Military, Civil, Economic, Social, Recreative, which resolve into— *Elements*: Rectilinear, Curvilinear, Rotative. *Epochs*: Stops, Singularities, Gesticulations. The traverse of such operations constitutes a Structure, of which the rational forms are one type and the architecture of the whole another. The *Significance* of the ceremony is the agreement of designer, performer and beholder as to the *Aim* of the actual acts or what they represent. The *Determinance* is the maintenance of a Plan of Procedure and definite details as against rambling incoherence.
(2) Vocative. *Material.* Statements or Utterances which are traced by the inflect pitch shapes apart from tonal determinance. The formulation is Articulate, which originates from— (*c*) The Logic of thinking in propositional elements and constructs, which further refers to the forms of thought, and is shown by the device of mathematical expression (the Bracket system). (*d*) The Vocalisation of Syllabic structure, and compound vocal tone groups, articulated by Cæsuræ and Clausulæ, constituting Phrases, which are relatively associated into Sentences. Larger associations are dealt with in the Division treating of Organisation. The general statement of a Proposition connotes a Retort or Answer which may be prescribed or merel Interjectional in t e	*Speech.* The expression of thought and emotion by means of sound, by the use of a Code of convention (as a basis), and the use of the inflectional possibilities. The vocal tones of speech, which are capable of great variation, are always bombarding the ear, but, as in the similar case of the Harmonic Series, are not individually noted unless attention is directed to them. The formulation of speech is developed— (*e*) By the logical method of thought which is common to all languages. (*f*) By the syntax and prosody of the Code and its Grammar defining Languages and Dialects. (*g*) By the special rules of Rhetoric, Oratory, Versification and definite methods of determinate expression. The *Significance* is simply what is agreed upon by composer, speaker and hearer.

TYPE	ASSOCIATION
(3) CARMINAL (Latin, *carmen*, a song). *Material.*[1] The fundamental reason for the existence of music is because it sounds nice. The elementary tone, chord, progression, etc., possess this quality. The natural tendency is to search for beautiful material, and when found or invented, to exhibit it in the manner best calculated to show it to advantage, together with choice of suitable associative factors. It also leads to contrast and inhibition, restraining the pure gratification and the development of complete and ideal cases. The weak point of the Carminal factor is its liability to abuse by both gratification and inhibition. Manifestations of this type do not persist, but grow up sporadically in all ages. *Method.* The elements are absolutely given and amenable to no theory. The triad and symmetrical tetrad are cases in point.	Intrinsic beauty as an attractive element, including the primary elements affecting the senses, and the higher beauty appealing to the mind as much by its restraint as its exuberance. The treatment of the factors involved follows— (*h*) Acquisition. (*i*) Exhibition. (*j*) Mounting in a suitable environment or aggregate. (*k*) Contrast to avoid satiation, fatigue, etc. (*l*) Contemplation of complete and ideal case. The danger of surrender to gratification is obvious. *Significance* comparable with the function of the blossom in biology. *Determinance* is mainly shown in disposition versus mere display.
(4) PERSONALITY. *Material.* A factor akin to Anthropomorphism, the individual persistence throughout a lengthy manifestation which comprises both the definite "leit-motiv" and the "atmospheric impression" of a tone, chord, passage, etc. *Character*, referable to descriptive terms applied to persons or objects, *e.g.* bold and hard to soft and clinging, etc. Working out by modes of Extension and Reduction. *Closing*, by peroration or complete view in relation to an implied predication. *Mode.* Appelative; the matter is treated to call attention to the individuality rather than the Form, Utterance, or Carminality. Variability is limited to persistence of prehensibility.	The Drama, in the widest sense in the use of language and other methods of "telling a story," which involves characters and backgrounds, together with the subjective ideas and their working out. The technique of the drama is highly developed and hinges principally upon the interaction of persons and their environment. The economics of the case tend to concentrate the picture about— (*m*) A principal. (*n*) A foil or contrast. (*o*) Colleagues (the two genders provide groupings). *Significance* is exhibited in behaviour relative to characteristic motive. *Determinance* by agreed convention as against nonsense or total caprice.

152. The Four Types of Appreciating Mind

We have to consider four general types of mind that, as composers, performers, and auditors, tend to be attracted to, concentrate upon, and more or less successfully develop either of the four classes we have tabulated.

Intellectual effort is largely concerned with Formulation and Organisation of Dramatic effect.

The Poetic and Oratoric faculty seizes upon the Vocative capability.

The average ordinary person, and indeed every one in their heart of hearts, likes and seeks Carminality. The artist seeks the refined aspects because from his wider standpoint he realises how much gratification in music, as in all arts, may degenerate into non-musical manifestation.

The frontiersman of music is the mind which admits limitations, but only as functions. He is not concerned with trammels, laws, and precedents, but only with the capability of his art to unfold and develop under genius and ability in marvellously flexible means of expression. If he achieves success it is because his almost subconscious grasp of fundamental truth ensures the outpouring being always determinate. Inhibition strengthens the persistence of effort by lopping off all that is putatively sterile, even if it might excite glamour at first, and he is able to train without crippling his methods with inapplicable rules.

If we examine an admitted musical work it is found to be always determinate. On the other hand, the weak points of works which may have at one time attracted by their superficial brilliance or sentimentality are found to be falsehoods introduced into music by the methods of deception only too possible under the actual limitations of presentation.

153. General Characteristics of the Four Types

It is admittedly difficult to describe the four different classes of Passage from a purely scientific point of view. That being so, we shall diverge to experience of the general type.

Formulative passages are well marked, often showing structural balance, and most frequently dividing up into comparative chromal sections, B, R, G, V.

Such passages are well suited for the construction of works appealing largely to the intellect by their structure, such as the Fugues and Sonata-form type of composition, and, as one may see on studying the methods of the great masters such as Bach and Beethoven, careful choice of "subject," "theme," etc., always preceded actual work.

Vocative passages are seen best in the simple form of the Song, where, in general, the expression in a speech-like form is aimed at. The question of intonation similarity between the tones of speech and recitative is a disputable question.

The germ of the Carminal basis is selection of the empiric effect from that exhibited in the metrically spaced tonal forms, the scale and the arpeggio. In this connection it is interesting to note how entertainers at the piano will improvise tunes on scales, and how buglers can blow skeletons of well-known airs, etc.

This selection is carried into the fertile region of chordance, and the cases in which certain forms are worked almost to death points the line of advance.

The higher Carminality certainly steers a cautious course, but the same objective is seen to be the basis, whether it is the poignant and pungent beauty of the higher discords and composite forms of chord.

The Personal type is associated with a dramatic or ideational consequence. For examples the reader cannot do better than refer to Wagner and all that has been done since in different schools.

The range varies from the broad association of a passage with a person, which follows its anthropomorph through all the vicissitudes of life ("leit-motiv"), to the underlying idea or emotion, which can best be expressed by the art of music. This latter is more a psychic element or state, and the success of the process depends upon the automatic agreement of composer, performer, and auditor. The "leit-motiv," programme, *idée fixe* type does not always come off successfully, as may be tested by questioning any intelligent audience who are not in the know as to the composer's intentions, but the true "impression" being of a deeper and less conventional nature is remarkably in tune with the mood of the triple personality involved—that is, in the majority of cases.

154. Closing Remarks. The Boundary of Tonality

It would be possible to continue discussion on these lines to a considerable length, but having reached the boundaries of another Division of Hyperacoustics it is as well to close.

It is to be regretted that the exigencies of publication do not admit of illustrating the points mentioned by actual examples.

Sufficient information has, however, been given to enable the reader to look up many actual cases in which the conditions more or less closely apply, and possibly it may be found feasible to publish a fuller discourse on these interesting subjects.

155. Retrospect to Primary Acoustic Conditions

It is interesting to look back to the primary Acoustic conditions from which we started, and from which we have endeavoured to consistently draw logical conclusions.

These are—

(1) The linearity of Pitch both physically and auditorally.
(2) The fixed interval relations given by the Harmonic Series.
(3) The theory of parasyntonic "regions" about each tone.
(4) The predominant recurrent character of the Octave, which, although associated with the phenomena of the Harmonic Series, is not, as a phenomenon, implicit in it.

It is a far cry from these simple foundations to the complex developments of Successive Tonality exhibited in actual musical works.

The extent to which the logical connection has been established in these pages is a matter that the writer must leave to the sympathetic judgment of his readers.

THE END

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