GULELE SUB-CITY EDUCATION OFFICE, ADDIS ABEBA

GRADE 12 MATHEMATICS MODEL EXAMINATION

SENE 2012/MAY 2020

NUMBER OF QUESTIONS: 65 **TIME ALLOWED:- 3 HOURS**

GENERAL DIRECTIONS

THIS BOOKLET CONTAINS **MATHEMATICS (FOR NATURAL SCIENCE STUDENTS)** EXAMINATION. IN THIS EXAMINATION, THERE ARE A TOTAL OF 65 **MULTIPLE CHOICE QUESTIONS**. CAREFULLY SELECT THE BEST ANSWER AND **BLACKEN** ONLY THE LETTER OF YOUR CHOICE ON THE SEPARATE ANSWER SHEET PROVIDED. FOLLOW THE INSTRUCTIONS ON THE ANSWER SHEET AND THE EXAMINATION PAPER CAREFULLY. USE ONLY **PENCIL** TO MARK YOUR ANSWERS. YOUR ANSWER MARK SHOULD BE **HEAVY** AND **DARK**, COVERING THE ANSWER SPACE COMPLETELY. PLEASE ERASE ALL UNNECESSARY MARKS COMPLETELY FROM YOUR ANSWER SHEET.

YOU ARE ALLOWED TO WORK ON THE EXAM FOR **3 HOURS**. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, PUT YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMPT TO CHEAT IN THE EXAMINATION WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE (S).

PLEASE MAKE SURE THAT YOU HAVE WRITTEN ALL THE REQUIRED INFORMATION ON THE ANSWER SHEET BEFORE YOU START TO WORK ON THE EXAMINATION.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

DIRECTION: Each of the following questions is followed by four possible alternatives. Read each question carefully and BLACKEN the letter of your choice on the answer sheet provided.

1. Which one of the following is a one to one function?

   A. $f = \{(1,6),(2,7),(3,6),(4,8)\}$

   B. $k : \Re \to \Re$ given by $h(x) = |x-3|$

   C. $k : (0,\infty) \to \Re$ given by $h(x) = \log x$

   D. $g = \{(x, y): x \text{ is a student and } y \text{ is his / her rank}\}$

2. The domain of the function f defined by $f(x) = \dfrac{1}{\sqrt{x-|x|}}$ is:

   A. $\mathcal{R}$ B. $(-\infty, 0)$ C. $(0, \infty)$ D. f is not defined for any $x \in \Re$

3. If $f(x) = \ln\left(\dfrac{x}{x-1} + 2\right)$, for $x > 1$, then which one of the following is the inverse of f ?

   A. $g(x) = \dfrac{e^x - 2}{e^x - 3}$ C. $g(x) = \dfrac{e^x - 2}{e^x + 1}$

   B. $g(x) = \dfrac{e^x}{e^x + 1} - 2$ D. $g(x) = e^{\frac{x}{x-1}} - 2$

4. Given the functions $f(x) = x^2 - 2x + 8$ and $g(x) = 0.5x - 3$, then $(fog^{-1})(2)$ is equal to:

   A. 88 B. 100 C. 20 D. 68

5. The simplified form of $\dfrac{x^{n+2} + yx^{n+1} + x^n y^2}{x^{3-n} - x^{-n} y^3} \div \dfrac{x^{2n}}{3}$, *for* $x \neq y$ and $x \neq 0$

   A. $\dfrac{x+y}{3}$ B. $\dfrac{x-y}{3}$ C. $\dfrac{3}{x-y}$ D. $\dfrac{3}{x-y}$

6. The solution set of the equation $\dfrac{x^2}{x+3} + \dfrac{3x}{x^2+5x+6} = \dfrac{2x}{x+3}$ is:

   A. {0, 1} B. {–1, 1} C. {–1, 0, 1} D. {–1, 0}

7. At what value(s) of x does the graph of $f(x) = \dfrac{x^4+1}{x^4+x^2}$ crosses its horizontal asymptote?

   A. $x = 0$ *and* $x = 1$ B. $x = -1$ *and* $x = 1$ C. $x = 1$ D. $x = -1$

8. Consider the sequence $\{a_n\}_{n\geq 1}$, where the first term is $a_1 = 2$ *and* $a_{n+1} = a_n + 4$ for all $n \geq 1$. Then the sum $a_3 + a_4 + a_5 + \ldots + a_{35}$ is equal to:

A.2460 B. 2458 C. 2442 D. 2450

9. What is the fourth term of a geometric sequence whose third term is 1 and whose eighth term is $\frac{1}{32}$?

A. 4 B. $\frac{1}{2}$ C. $\frac{1}{8}$ D. $\frac{1}{16}$

10. What is the sum of the series $\sum_{n=1}^{\infty}(-4)\left(\frac{5}{3}\right)^{1-n}$?

A. $-\frac{5}{2}$ B. $-\frac{12}{5}$ C. -4 D. -10

11. Let $f(x) = \begin{cases} 3k - x, & \text{if } x < 1 \\ b + 1, & \text{if } x = 1 \\ kx, & \text{if } x > 1 \end{cases}$. What is the value of b if f is continuous at $x = 1$?

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. -1 D. 3

12. The left hand side limit, $\lim_{x \to 2^+} \frac{x|x-2|}{x-2}$ equals to:

A. 1 B. 2 C. -2 D. Does not exist

13. Let $f : R \to R$ be a continuous function such that $f(1) = \frac{3}{2}$ and $f\left(\frac{1}{\sqrt{2}}\right) = 3$. Then the value of $\lim_{x \to 0} f\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{x}\right)$ is equal to:

A. $\frac{3}{2}$ B. 3 C. $\frac{9}{2}$ D. Can not be determined

14. Which of the following is not true?

A. $\lim_{n\to\infty}\left(\frac{2n-3}{3n+7}\right)^4 = \frac{16}{81}$

B. $\lim_{n\to\infty}\left(\frac{\sqrt{1+9n^2}}{3n+5}\right) = 1$

C. $\lim_{n\to\infty}\left(\frac{1+2\times 10^n}{5+3\times 10^n}\right) = \frac{2}{3}$

D. $\lim_{n\to\infty}\left(\frac{\sqrt{16n^2+9}}{3n-1}\right)^4 = \frac{16}{3}$

15. Evaluate $\lim_{n\to\infty} \frac{(-1)^n n^3 + 1}{2n^3 + 3}$.

A. -1 B. ∞ C. 1 D. does not exist

16. If in a hyperbola, the distance between the foci is 10 and the transverse axis has length 8, than the length of its latus rectum is

A. 9 B. $\frac{9}{2}$ C. $\frac{32}{2}$ D. $\frac{64}{3}$

17. The equation of the directrix of the parabola $y^2 + x + 2y + 1 = 0$ is:

A. $x = \frac{-1}{4}$ B. $y = \frac{-1}{4}$ C. $x = \frac{1}{4}$ D. $y = \frac{1}{4}$

18. What is the equation of an ellipse with centre at (3,2), vertices at (5,2) and (3,5)

A. $9(x-3)^2 + 4(y-2)^2 = 36$ C. $4(x-3)^2 + (y-2)^2 = 1$

B. $4(x-3)^2 + 9(y-2)^2 = 36$ D. $9(x-3)^2 + 4(y-2)^2 = 1$

19. A parabolic arch has a height of 20m and width of 36m at the base. If the vertex of the parabola is at the top of the arch, at what height above the base is it 18m wide?

A. 18m B.15m C.20m D. 24m

20. One of the following is <u>not</u> true about the ellipse whose equation is given by $2x^2 + 8y^2 = 32$

A. the length of the minor axis is 4 C. the foci are $(0, -2\sqrt{3})$ and $(0, 2\sqrt{3})$

B. the major axis is parallel to the x – axis D. (-4, 0) and (0,2) are vertices

21. Which one of the following is equation of a circle whose center is on y-axis and radius is 3?

A. $x^2 + y^2 + 6y = 0$ C. $(x-2)^2 + y^2 = 9$

B. $x^2 + (y-2)^2 = 3$ D. $x^2 - 2x + y^2 = 8$

22. Which of the following is equivalent to $\neg[(p \Rightarrow \neg q) \vee \neg r]$?

A. $(\neg p \vee q) \wedge r$ C. $\neg p \wedge q \wedge r$

B. $p \wedge q \wedge r$ D. $(q \Rightarrow \neg p) \wedge r$

23. Which of the following is **not** true statement on the set of real numbers?

A. $(\forall x)(\exists y)\ (y = x + 2)$ C. $(\exists x)(\exists y)\ (x^2 + y^2 = 5)$

B. $(\exists y)(\forall x)\ (y + x = 3)$ D. $(\exists x)(\exists y)\ (y < x)$

24. If each of the compound propositions $P \wedge \neg Q,\ R \Rightarrow Q,\ \neg R \Leftrightarrow S$ is true, then which one of the following is true?

A. Q B. S C. $P \wedge \neg S$ D. $S \Rightarrow R$

25. Suppose the following are the premises of an argument.

"He was lazy or he did not like the classroom.

He could not pass the exam, if he was lazy.

He passed the exam."

Which of the following can be a conclusion that makes the argument valid?

A. He did like the classroom

B. If he was not lazy, he did like the classroom

C. He did not like the classroom.

D. He was not lazy and he did like the classroom.

26. If the mean of the frequency distribution given below is 2, then what is the value of x?

V	-7	-2	3	9	13
F	10	6	2	5	x

A. 3 B. 6 C. 4 D. 7

27. Below are the marks of grade 12 students in mathematics out of 100. What is the variance for the frequency distribution given below

Marks	20-40	40-60	60-80	80-100
Number of students	4	10	4	2

A. 107.2 B. 4 C. 304 D. 17.5

28. Which one of the following is the constant term of the expansions of $\left(x^2+\frac{2}{x^3}\right)^5$?

A. 40 C. 80

B. 60 D. 20

29. In a random arrangement of the word "SOCIETY" what is the total number of words in which all the three vowels come together?

A. 120 C. 420

B. 720 D. 840

30. Items produced by a certain company are subject to kinds of defects D_1 and D_2. Out of the total production, if 5% have defectD_1, 10% have defect D_2 and 2% have both defects, then what is the probability that an item to have defect D_2, given that it has defect D_1.

A. 0.4 B. 0.2 C. 0.1 D. 0.5

31. A three digited library identification card is to be printed from the numbers 0, 1, 2, 3, 4 and 5 in such a way that the first is non-zero and **no** number is to be repeated, how many such cards can be printed?

A. 100 B. 120 C. 180 D. 150

32. Consider the experiment of rolling a die whose sample space is {1, 2, 3, 4, 5, 6}. If two dice are rolled simultaneously, what is the probability that a prime number turns up on one of the dice and a composite number on the other?

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{1}{6}$ D. $\frac{5}{6}$

33. Different codes each consisting of five characters are to be generated in each code the first two characters are either A or B and the remaining three characters are any of the digits 0, 1, 2, . . . , 9.how many distinct codes can be generated so?

A. 4000 B. 3600 C. 3000 D. 2400

34. Which of the following matrix is **not** in reduced row echelon form?

A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

35. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & c \end{bmatrix}$. For what value of c is $AB = \begin{bmatrix} 3 & 8 \\ 7 & 18 \end{bmatrix}$?

A. 3 B. 2 C. 4 D. 1

36. Which of the following is the solution set of the system of equation $\begin{cases} x + y + 2z = 1 \\ x + 2y + z = 1 \\ 3x + 4y + 5z = 7 \end{cases}$?

A. $\{(1, 0, 0)\}$ C. $\{(-3k + 1, k,\ k) | k \in \mathcal{R}\}$

B. $\{(k + 1, -k,\ 0) | k \in \mathcal{R}\}$ D. $\emptyset$

37. IF $A=\begin{bmatrix}3 & 3 & 5\\ 0 & -1 & 2\\ 4 & 2 & 1\end{bmatrix}$, then the cofactors of a_{23} *and* a_{31} respectively are:

A. 11 and 6 C. 6 and -11

B. -6 and 11 D. 6 and 11

38. One of the following is false about matrices?

A. If A is a square matrix with order n, then $\frac{(A+A^T)}{3}$ is symmetric matrix.

B. If B is a square matrix with order m, then the matrix $D = 2B - 3B^T$ is skew symmetric.

C. For any two matrices $A = (a_{ij})_{mn}$ and $B = (b_{ij})_{Pq}$, BA exists if and only if $m = q$.

D. For any two none zero matrices A and B their product AB may be zero matrix.

39. Let $A=\begin{bmatrix}2 & 3 & 5\\ 4 & 5 & 7\\ 8 & -3 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 3 & -5\\ 2 & 1 & 7\\ 6 & 5 & 0\end{bmatrix}$. If $C = AB + A^T$, what is the $(2, 3)$ entry of C?

A. 18 B. 42 C. 46 D. 12

40. If z is a complex number such $\frac{-5z}{4+3i} = 3i\left(\sqrt{3}-i\right)$, then $|z|$ is equal to:

A. 6 B. 9 C. 15 D. $\sqrt{15}$

41. The polar form of $2 - 2\sqrt{3}\,i$

A. $4(cos\frac{4}{3}\pi + isin\frac{4}{3}\pi)$ C. $4(cos\frac{5}{6}\pi + isin\frac{5}{6}\pi)$

B. $4(cos\frac{\pi}{3} + isin\frac{\pi}{3})$ D. $4\left(\cos\left(\frac{\pi}{3}\right) - isin\left(\frac{\pi}{3}\right)\right)$

42. If $z = 2 + 3i$, complex number, then $|z|^2 - z^2$ = _____

A. 0 B. $18-12i$ C. $8-12i$ D. $12i$

43. The locus represented by $|z - 1| = |z + i|$ where z = $x + iy$ $x, y \in$R is

A. a circle of radius 1 C. a straight line through the origin

B. an ellipse with foci at (1, 0) and (-1, 0) D. a circle with centre at $\left(\frac{1}{2},\frac{1}{2}\right)$

44. If $y = x^y$, then $\frac{dy}{dx}$ equals: A. $\frac{x^2}{y(1-y\ln x)}$ C. $\frac{y^2}{x(1-x\ln x)}$

B. $\frac{y^2}{x(1-x\ln y)}$ D. $\frac{y^2}{x(1-y\ln x)}$

45. Which of the following is the equation of the tangent line to the graph of $f(x)=\frac{3x}{x-1}$ at $(2, f(2))$?

A. $3y+x=20$ B. $3y-x=16$ C. $y-3x=0$ D. $y+3x=12$

46. Let $f(x)=\ell n(x^2-3x)$. Then, $f''(-1)$ is equal to :

A. $\frac{-17}{16}$ B. $\frac{-7}{16}$ C. $\frac{-5}{16}$ D. $\frac{-19}{16}$

47. If $f(2)=-3,\ f'(2)=4, g(1)=-5,\ g'(1)=1$ and $F(x)=f(2x+2).g(1-x^2)$, then what is the value of $F'(0)$?

A. 19 B. 0 C. -20 D. -40

48. Let $f(x)=x^5-20x$ $on\ [-2, 2]$. Which of the following is the local maximum value of f ?

A. $f(-\sqrt{2})$ B. $f(\sqrt{2})$ C. $f(2)$ D. $f(-1)$

49. A point moves around a circle $x^2+y^2=9$. When the point is at $\left(-\sqrt{3}, \sqrt{6}\right)$ its x-coordinate is increasing at the rate of 20 unit per second. How fast is wits y-coordinate changing in unit per second at that instant?

A. $5\sqrt{2}$ B. $10\sqrt{2}$ C. $10\sqrt{3}$ D. $20\sqrt{3}$

50. Which of the following is false about the graph of $f(x)=\frac{x}{x^2+1}$

A. It is strictly increasing on $[-1, 1]$

B. one of the inflection point is $(0, 0)$

C. It is concave down ward on $\left(-\infty, -\sqrt{3}\right)\cup\left(0, \sqrt{3}\right)$

D. Its relative maximum value is $\frac{-1}{2}$

51. Two concentric circles are such that the radius of one is always three times that of the other. If the radius of the smaller is increasing at the rate of 2m/sec. find the rate at which the area between the circles is increasing when the radius of the larger is 3m.

A. $36\pi\ m^2/\sec$ B. $24\pi\ m^2/\sec$ **C.** $32\pi\ m^2/\sec$ D. $18\pi\ m^2/\sec$

52. For what values of a and b $h(x)=a+bx-x^2$ have maximum value of 5 at $x=3$

A. -4, 6 B. 4, 6 C. 4, -6 D. -2, 6

53. Which one of the following is equal to $\int \frac{x+\ell n(x+1)}{(x+1)^2}dx$?

A. $\ell n(x+1)+\frac{x}{x+1}+c$ C. $(x+1)^2-\frac{1}{x+1}+c$

B. $(x+1)^2+\frac{1}{x+1}+c$ D. $\frac{x\ell n(x+1)}{x+1}+c$

54. If $F(x)$ is an antiderivative of $f(x)=1-\frac{2}{x^2}$ and $F(1)=0$, then $F(2)$ is equal to:

A. 0 B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 3

55. If $f(x)=\begin{cases}x^2+2, & x\geq 2\\ 5x-4, & x<2\end{cases}$, then $\int_{-1}^{3} f(x)\,dx$ is equal to:

A. $\frac{35}{2}$ B. $\frac{23}{6}$ C. $\frac{6}{23}$ D. $\frac{2}{35}$

56. If $\vec{A}$ = (3, -4) and B = (-1, 2), what is the unit vector in the direction opposite to $\vec{C} = 4\vec{B} + \vec{A}$

A. $\left(\frac{-1}{\sqrt{17}},\frac{4}{\sqrt{17}}\right)$ C. $\left(\frac{1}{\sqrt{17}},\frac{-4}{\sqrt{17}}\right)$

B. $\left(\frac{-1}{\sqrt{17}},\frac{-4}{\sqrt{17}}\right)$ D. $\left(\frac{-4}{\sqrt{17}},\frac{1}{\sqrt{17}}\right)$

57. Let a and b be vectors such that $\|a\|=4$ *and* $\|2a-b\|=\|a+b\|$. Which of the following is equal to $\vec{a}.\vec{b}$?

A. 8 B. 10 C. 12 D. 24

58. A translation takes $(x-1)^2+(y+3)^2=4$ to $(x-3)^2+(y-4)^2=4$ of what is the image of the point (1, -3) under the translation

A. (2, 7) B. (3, 4) C. (4, 3) D. (7, 2)

59. Consider the circle $(x-2)^2+(y-5)^2=8$ *and* the line ℓ given by parametric vector equation (x, y) = (0, -1) + t (1, 1) which of the following is true.

A. ℓ intersects with the circle at two distinct points

B. the line ℓ and the circle have no common point

C. the distance from the centre to the line ℓ is $2\sqrt{2}$

D. ℓ is tangent to the circle at (3, 4)

60. Suppose anticlockwise rotation about the origin through 90^0 is followed by reflection in the x-axis, what is the image of the ellipse given by $\frac{(x-1)^2}{4}+(y+21)^2=1$ under the two transformations?

A. $(x-2)^2+\frac{(y+1)^2}{4}=1$ C. $(y+2)^2+\frac{(x-1)^2}{4}=1$

B. $\frac{(y-1)^2}{4}+(x-2)^2=1$ D. $\frac{(x+1)^2}{4}+(y-2)^2=1$

61. A boat on a sea sailed from its station toward North with a constant speed of 80 km/hr. Another boat from the same station sailed 60° NE(North East) with a constant speed of 100km/hr. If the two boats started sailing at the same time, What is the straight distance between them after they have sailed for just 30 minutes?

A. $10\sqrt{42}$ km B. 90km C. $10\sqrt{41}$ km D. $10\sqrt{21}$ km

62. $\sin^4 x-\cos^4 x$ is equal to:

A. $\sin 2x$ B. $-\sin 2x$ C $\cos 2x$ D. $-\cos 2x$

63. What is the amplitude and period, respectively, of the graph of $(x)=4\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)$?

A. 4, $\frac{\pi}{3}$ B. 2, 3π C. 2, $\frac{2\pi}{3}$ D. 4, 3π

64. $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-1/2\right)\right)$ is equal to:

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. 1

65. When a pilot of an aeroplane was flying horizontally at a height of 500m above stretch of land, he observed a car at an angle of depression of 30^0.How far was the car from the aeroplane?

A. 1000m B. 250m C. $500\sqrt{3}m$ D. $1000\sqrt{3}m$