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IIT JEE MATHEMATICS

Greatest Integer Function

$$f(x) = [x]$$



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Greatest Integer Function $[x]$: $f(x)=[x]$

In this article, we are going to learn in depth about definition of Greatest Integer function, Others Name of Greatest Integer Function, Symbol of Representation of Greatest Integer Function, Derivation for Greatest Integer Function, The Properties of the Greatest Integer Function, Domain and Range of Greatest Integer Function, Graph of Greatest Integer Function, and its formula. We will also solve some MCQ based problems on the Greatest Integer Function for a depth knowledge and understanding of the concept

Greatest Integer Function $[x]$: Others Name

Others Name of Greatest Integer Function is

- (1) Step function or Stepwise function
- (2) Round Down function
- (3) Floor of x function
- (4) Int Function

Greatest Integer Function also know as Step Function Because It Convert any function into step function.

Greatest Integer Function also known as Round Down function It round off any real number to its nearest integer.

Greatest Integer Function known as Int Function in Computer programming

Western Mathematician use Name "Floor of x function" in Place of "Greatest Integer Function."

Greatest Integer Function $[x]$: Symbol of Representation

We can use any of these symbols for representation of Greatest Integer of any function

- (1) $[x]$
- (2) $\lfloor x \rfloor$

Greatest Integer Function of x , $f(x) = [x]$

Greatest Integer Function of $\sin x$, $f(x) = [\sin x]$

Greatest Integer Function of 4.23, $f(x) = [4.23]$

Greatest Integer Function of $\{x\}$, $f(x) = [\{x\}]$

Greatest Integer Function of $[x]$, $f(x) = [[x]]$

Greatest Integer Function of x^2 , $f(x) = [x^2]$

Greatest Integer Function of $x^2 + 4x + 3$, $f(x) = [x^2 + 4x + 3]$

Greatest Integer Function of Zero, $f(x) = [0]$

NOTE:

Sometime Greatest Integer Function of x can be represented as $f(x) = \text{gif}(x)$



If Concept of Greatest Integer Function is used in any problem of JEE Main and JEE Advance they write $[\cdot]$ - Denotes Greatest Integer Function less than or equal to x .

Greatest Integer Function $[x]$: Definition

$F(x) = [x]$ $\rightarrow$ Give Greatest Integer among all Integers those are less than or equal to x

From above definition we can conclude two points

- (1) Greatest Integer Function always give an Integral Value i.e., $[x]$ give an Integer
- (2) $[x] \leq x$

From Point (2)

- (A) $[x] < x$, when $x \notin \mathbb{Z}$, Here $\mathbb{Z}$ belongs to Set of Integers
- (B) $[x] = x$, when $x \in \mathbb{Z}$, Here $\mathbb{Z}$ belongs to Set of Integers

Example1: Calculate the Value of $[3.01]$?

Write all Integers Which is less than $3.01 = 3, 2, 1, 0, \dots$

Choose Greatest Integer among $3, 2, 1, 0, \dots = 3$

So, $[3.01] = 3$ **Ans**

Example2: Calculate the Value of $[3.99]$?

Write all Integers Which is less than $3.99 = 3, 2, 1, 0, \dots$

Choose Greatest Integer among $3, 2, 1, 0, \dots = 3$

So, $[3.99] = 3$ **Ans**

Example3: Calculate the Value of $[-7.01]$?

Write all Integers Which is less than $-7.01 = -8, -9, -10, -11, \dots$

Choose Greatest Integer among $-8, -9, -10, -11, \dots = -8$

So, $[-7.01] = -8$ **Ans**

Example4: Calculate the Value of $[-7.99]$?

Write all Integers Which is less than $-7.99 = -8, -9, -10, -11, \dots$

Choose Greatest Integer among $-8, -9, -10, -11, \dots = -8$

So, $[-7.99] = -8$ **Ans**



Example5: Calculate the Value of $[17]$?

Write all Integers Which is less than or equal to $17 = 17, 16, 15, \dots$

Choose Greatest Integer among $17, 16, 15, \dots = 17$

So, $[17] = 17$ **Ans**

Example6: Calculate the Value of $[-17]$?

Write all Integers Which is less than or equal to $-17 = -17, -16, -15, -14, \dots$

Choose Greatest Integer among $-17, -16, -15, -14, \dots = -17$

So, $[-17] = -17$ **Ans**

Example7: Calculate the Value of $[.99]$?

Write all Integers Which is less than or equal to $.99 = 0, -1, -2, -3, \dots$

Choose Greatest Integer among $0, -1, -2, -3, \dots = 0$

So, $[.99] = 0$ **Ans**

Example8: Calculate the Value of $[0]$?

Write all Integers Which is less than or equal to $0 = 0, -1, -2, -3, \dots$

Choose Greatest Integer among $0, -1, -2, -3, \dots = 0$

So, $[0] = 0$ **Ans**

Example9: Calculate the Value of $[\sqrt{2}]$?

Value of $\sqrt{2} = 1.4142$

Write all Integers Which is less than or equal to $1.4142 = 1, 0, -1, -2, -3, \dots$

Choose Greatest Integer among $1, 0, -1, -2, -3, \dots = 1$

So, $[\sqrt{2}] = 1$ **Ans**

Example10: Calculate the Value of $[\sqrt{3} + 1]$?

Value of $\sqrt{3} + 1 = 1.71 + 1 = 2.71$

Write all Integers Which is less than or equal to $2.71 = 2, 1, 0, -1, -2, -3, \dots$

Choose Greatest Integer among $2, 1, 0, -1, -2, -3, \dots = 2$

So, $[\sqrt{3} + 1] = 2$ **Ans**



Example11: Calculate the Value of $[e]$?

Value of $e = 2.71$

Write all Integers Which is less than or equal to $2.71 = 2, 1, 0, -1, -2, -3, \dots$

Choose Greatest Integer among $2, 1, 0, -1, -2, -3 \dots \dots = 2$

So, $[e] = 2$ **Ans**

Example12: Calculate the Value of $2^{[\sqrt{2}]}$?

Value of $\sqrt{2} = 1.41$

Write all Integers Which is less than or equal to $1.41 = 1, 0, -1, -2, -3, \dots$

Choose Greatest Integer among $1, 0, -1, -2, -3 \dots \dots = 1$

So, $[\sqrt{2}] = 1$

$2^{[\sqrt{2}]} = 2^1 = 2$ **ANS**

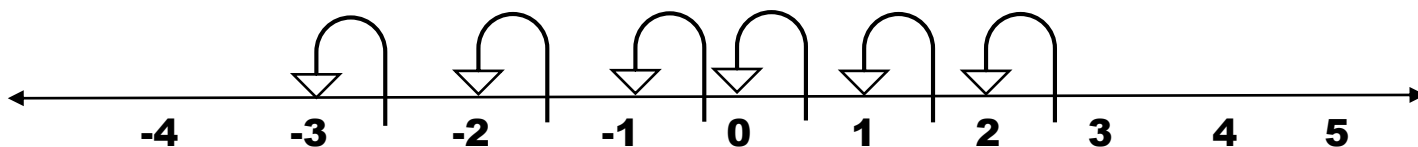
Greatest Integer Function $[x]$: Graph

In this article, we are going to sketch the graph of Greatest Integer Function $f(x) = [x]$

Domain of Greatest integer function is all real number i.e., $x \in (-\infty, \infty)$

Range of Greatest integer Function is set of Integers (z) i.e., $f(x) \in \mathbb{Z}$

Real Number Line:



From above real number Line

If

$$0 \leq x < 1 \quad [x] = 0$$

$$1 \leq x < 2 \quad [x] = 1$$

$$3 \leq x < 4 \quad [x] = 3$$

$$2 \leq x < 3 \quad [x] = 2$$

$$-1 \leq x < 0 \quad [x] = -1$$



$$-2 \leq x < -3 \quad [x] = -2$$

.....

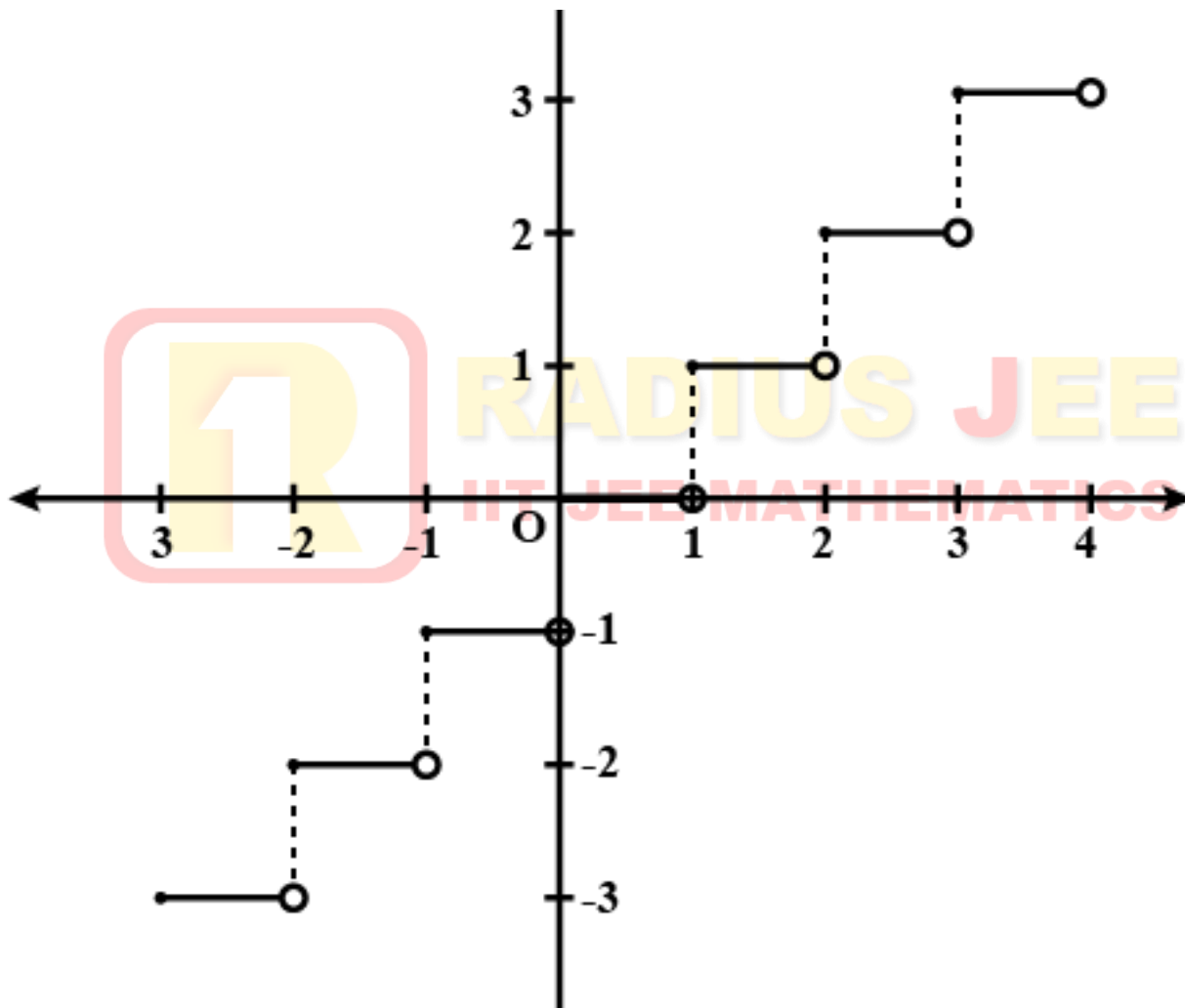
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$$n \leq x < n + 1 \quad [x] = n$$

Here n Represents an Integer

So, greatest integer function will always give an integral value

Range of Greatest Integer Function will be Set of Integers



Graph of Greatest Integer Function

Greatest Integer Function $[x]$: Limit, Continuity and Differentiability

Limit:

Limit Does not exist at all $x \in I$ because

$$\text{LHL} \neq \text{RHL}$$

$$\lim_{x \rightarrow I^-} f(x) \neq \lim_{x \rightarrow I^+} f(x)$$

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**Continuity:**

Greatest Integer function $f(x) = [x]$ is not continuous at all $x \in I$ because

Left hand limit is not equal to Right hand limit

$$\lim_{x \rightarrow I^-} f(x) \neq \lim_{x \rightarrow I^+} f(x)$$

Differentiability

Greatest Integer function $f(x) = [x]$ is not differentiable at all $x \in I$ because

Left hand Derivative is not equal to right hand limit

NOTE

Greatest Integer function $f(x) = [x]$ will continuous, differentiable and Limit exist for $x \in (-\infty, \infty) - \{I\}$ where I represent integers

Greatest Integer Function $[x]$: Properties

- $f(x) = [x]$ = Give always an Integral value
- $[x] \leq x$ for all $x \in R$
- $[x] < x$ for all $x \notin z$ (Integer)
- $[x] = x$ for all $x \in z$ (Integer)
- $x - 1 < [x] \leq x$ for all $x \in R$
- $[[[x]]] = [x]$
- $[x + I] = [x] + I$, where $I \in \text{Set of Integers}$
- $[x + [x]] = [x] + [x]$
- If $[x] = n$, then $n \leq x < n + 1$
- $[x] + \{x\} = x$, where $\{x\} = \text{Fractional Part of } x \text{ Function}$
- $[x] + [-x] = \begin{cases} 0, & \text{when } x \in z \\ -1, & \text{when } x \notin z \end{cases}$

- If $[x] \geq k$, then $x \geq k$
- If $[x] \leq k$, then $x < k + 1$
- If $[x] > k$, then $x \geq k + 1$
- If $[x] < k$, then $x < k$
- $[nx] \neq n[x]$, Where $n \in \text{Integer}$
- $[nx] = \sum_{k=0}^{n-1} \left(x + \frac{k}{n}\right)$
- $[2x] = [x] + \left[x + \frac{1}{2}\right]$
- $[3x] = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right]$
- $[x] = \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right]$
- $x - 1 < [x] \leq x < [x] + 1$
- $[x] - [-x] = 2[x] + 1$ when $x \notin z$ (Integer)
- $[x] - [-x] = 2[x]$ when $x \in z$ (Integer)



Greatest Integer Function $[x]$: Problems and their Solution

Example 13. Solve for x ? If $[x + 1] = 4$

- (A) $x \in [3, 4]$
- (B) $x \in [3, 4)$
- (C) $x \in [4, 4)$
- (D) None

We Know that

$$\text{If } [x] = n$$

Then

$$n \leq x < n + 1$$

So, if $[x + 1] = 4$

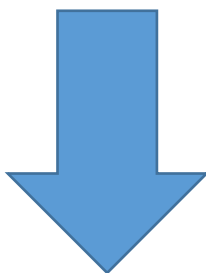
$$4 \leq (x + 1) < 5$$

$$3 \leq x < 4$$

$$x \in [3, 4) \text{ Ans}$$

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Greatest Integer Function

$$F(x) = [x]$$

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