

$$\Rightarrow x - 2(1) + 1(3) = 7 \Rightarrow x + 1 = 7$$

$$\Rightarrow x = \frac{1}{4} = 7$$

$$\underline{x = 2}$$

$$\Rightarrow x = 29/4. \quad \underline{S.S = \{2, -1, 3\}}$$

What I did in row operation.

* 1st Multiply each entries in 1st row by (-3) and add to 2nd row to make $a_{21} = 0$.

* 2nd Multiply each entries in 1st row by (-2) and add to 3rd row to make $a_{31} = 0$.

* 3rd Multiply each entries of what I got in the above step of 2nd row by (-1) and add to 3rd row to make $a_{32} = 0$.

Note: For the other two cases that is (i) for many solutions see example 7 (page 252) and (ii) for no solution see example 6 (page 251).

2. The Cramer's Rule Consider $AX = B$.

This rule is applied, only when the system has only one solution. And identified by $|A| \neq 0$ when A is the coefficient matrix.

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}.$$

Where $D = |A|$

D_x - Coefficients of x replaced by B .

D_y - Determinant of coefficient matrix once coefficient of y is replaced by the constant B .

1. In how many ways can a committee of 3 members be formed from 7 candidates?
 A. 7
 B. 21
 C. 28
 D. 35
2. Two machines A and B produce respectively 60% and 40% of the total number of items of a factory. The percentages of defective output of these machines are 2% and 5%, respectively. If an item is selected at random, then what is the probability that the item is defective?
 A. 0.032
 B. 0.07
 C. 0.32
 D. 0.426
3. The probability that an electronic device produced by a company does not function properly is equal 0.01. If 2 devices are brought, then what is the probability that at least 1 device function properly?
 A. 0.81
 B. 0.09
 C. 0.18
 D. 0.99
4. A team of 10 researchers consist 4 biologists and 6 chemists. If 3 persons are chosen randomly from the team, what is the probability that at least one is a biologist?
 A. $\frac{2}{3}$
 B. $\frac{2}{5}$
 C. $\frac{5}{6}$
 D. $\frac{7}{10}$
5. A box contains 10 items of which 3 are defective. If 2 items are randomly taken out of the box, what is the probability that both items are not defective?
 A. $\frac{7}{10}$
 B. $\frac{4}{7}$
 C. $\frac{7}{15}$
 D. $\frac{49}{100}$
6. Items produced by a certain company are subjected to two kinds of defects D_1 and D_2 . Out of the total product, 5% have the defects D_1 , 10% have the defect D_2 , and 2% have both defects. What is the probability that a randomly selected item has neither defect D_1 nor defect D_2 ?
 A. 0.13
 B. 0.5
 C. 0.98
 D. 0.87
7. Different codes, each of which consisting of five characters, are to be generated in such a way that the first two characters are any of the English capital letters (A to Z) and the remaining three are any of the digits (0, 1, ..., 9). How many distinct codes can be generated so?
 A. 468,000
 B. 260
 C. 676,000
 D. $26! \times 10!$

8. Suppose that the first 3 letters (A, B, and C) and number digits to be used to form car plates in small town. How many different plates can be formed in total that contain 1, 2, or 3 letters and then followed by 3 digits?
- A. 3000
B. 27,000
C. 39,000
D. 100,000
9. Three persons P_1, P_2 , and P_3 are firing at a target independently and have a probability 0.7, 0.5, and 0.4 respectively, of hitting the target. What is the probability that at least one of them hits the target?
- A. 0.95
B. 0.85
C. 0.91
D. 0.99
10. If distinct codes (words) of eight letters are formed by rearranging the letters in the word 'ABBEBAYE', how many of the codes begin with B or Y?
- A. 840
B. 630
C. 1,680
D. 420
11. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defective of type D_1 , 60 have defective of D_2 and 25 have both types of defectives, what is the probability that a bulb produced by the company has none of the defectives?
- A. 0.95
B. 0.80
C. 0.85
D. 0.20
12. If S is a set with 10 elements and $A \subseteq S$, what is the probability that A has 3 or more elements?
- A. $\frac{7}{10}$
B. $\frac{8}{11}$
C. $\frac{121}{128}$
D. $\frac{7}{128}$
13. Let A and B two events. Suppose that the probability that neither event occurs is $\frac{3}{8}$. What is the probability that at least one of the event occur?
- A. $\frac{1}{8}$
B. $\frac{1}{4}$
C. $\frac{3}{4}$
D. $\frac{5}{8}$
14. How many four digit even numbers can be formed from 1,2,3,4 and 5 if the numbers start with 3?
- A. 40
B. 50
C. 100
D. 120

15. A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls. What is the probability that at least one girl is selected?
- A. $\frac{5}{6}$ C. $\frac{1}{3}$
 B. $\frac{2}{3}$ D. $\frac{1}{6}$
16. A group of six students take their seats at random in a round table for a discussion. What is the probability that two specific students do **not** sit together?
- A. $\frac{3}{5}$ C. $\frac{2}{3}$
 B. $\frac{2}{5}$ D. $\frac{1}{3}$
17. A box contains 5 white, 6 red and 4 blacks of identical size. If 3 balls are randomly taken out of the box one after the other, what is the probability that the first ball is white and both the second and third balls are red?
- A. $\frac{5}{15}$ C. $\frac{4}{75}$
 B. $\frac{2}{15}$ D. $\frac{5}{91}$
18. A three-digit library identification card is to be printed from the numbers 0,1,2,3,4,5 in such a way that the first is non-zero and no-number is to be repeated. How many such cards can be printed?
- A. 100 C. 150
 B. 120 D. 180
19. A student needs to select 3 books from 3 different mathematics, 3 different physics and 1 history book. What is the probability that one of them is mathematics and the other two are either physics or history book?
- A. $\frac{3}{35}$ C. $\frac{15}{35}$
 B. $\frac{9}{35}$ D. $\frac{18}{35}$
20. Let $A = \{1,2,3,4,5,6,7\}$, $B = \{7,8,9\}$ and $C = \{8,9,10\}$. If one of the numbers is deleted randomly from each of these sets, what is the probability that all the three deleted numbers are even or all are multiple of 3?
- A. $\frac{8}{63}$ C. $\frac{1}{9}$
 B. $\frac{2}{21}$ D. $\frac{4}{5}$
21. A private college has 1000 students 60% of these students are males. 45% of these students pay their payment by credit card including 175 females. What is the probability that the student is a male or a credit card user?
- A. 0.225 C. 0.675
 B. 0.775 D. 0.325

22. In how many ways can 4 people be arranged in a row than if they were arranged in a Circle?
A. 1
B. 6
C. 18
D. 12
23. Two machines A and B work independently. The probability that both machines A and B work is 0.4. If the conditional probability that machines B works given that machine A works is 0.5, then the conditional probability that machines A works given that Machine B work is
A. 0.8
B. 0.3
C. 0.5
D. 0.7
24. Items produced by a certain company are subjected to two kinds of defects D_1 and D_2 . Out of the total product, if 5% have the defects D_1 , 10% have the defect D_2 , and 2% have both defects, then what is the probability for an item have defect D_2 given that it has defect D_1 ?
A. 0.2
B. 0.05
C. 0.1
D. 0.4
25. In how many ways can 6 boys and 6 girls sit around a table of 12 seats, if no two boys are to sit together?
A. 120
B. 720
C. 86,400
D. 6,400
26. What is the total number of natural numbers that can be formed using the digits 1, 2, 3 and 4, if no digit is repeated in any of the numbers formed?
A. 64
B. 32
C. 24
D. 16
27. If in a 3- digit number the last digit must be zero and repetition are not allowed, then which of the following is the number of 3-digit numbers that can be formed with the digits 0, 1, 2, ..., 9?
A. 81
B. 90
C. 720
D. 810

28. Let S be the set of four digit numbers formed by using 0, 1, 2, 3, 4 and 45. What is the probability that an element of S is divisible by 5?

A. $\frac{1}{3}$

C. $\frac{1}{9}$

B. $\frac{2}{3}$

D. $\frac{2}{5}$

29. Cards are numbered consecutively from 1 to 20. They are placed in a box and two cards are with drawn at random. Assume that each of the twenty cards are equally likely to be withdrawn. What is the probability that the sum of the numbers on the two withdrawn cards is 10?

A. The probability that are defective is $\frac{1}{60}$

B. The probability that non is defective is $\frac{5}{24}$

C. The probability that exactly one is defective is $\frac{7}{20}$

D. The probability that at least two are non-defective is $\frac{49}{60}$

30. If $C(n, 2) = 66$, then n is:

A. 12

C. 11

B. 13

D. 20

31. Which of the following is the middle of $(2x + \frac{1}{x})^6$?

A. 8

C. 160

B. 64

D. 20

32. In the expansion of $(x^2 - \frac{2}{x})^5$, What is the fourth term?

A. $10x$

C. $80x^{-2}$

B. $-80x$

D. $-40x^4$

33. If the six terms in the binomial expansion on $(\sqrt{x} + \frac{1}{x})^n$ is a constant, then what is the value of n ?

A. 15

C. 5

B. 18

D. 10

Solution For the above problems:

1) Find the derivative of each of the following:

b) $f(x) = 2x \cdot |x|$, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$f(x) = 2x \cdot \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}, \quad f(x) = 2 \cdot \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

$$f'(x) = 2 \cdot \begin{cases} 2x, & x > 0 \\ 0, & x = 0 \\ -2x, & x < 0 \end{cases}$$

Ex: Show that $f'(0) = 0$? or $f(x)$ is differentiable at $x=0$?

$$f'(x) = 4 \cdot \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$f'(x) = 4|x|$$

Exercise: Find the derivative of:

a): $f(x) = x \cdot |x|$

b) $f(x) = x^2 \cdot |x|$

c) $f(x) = x^3 \cdot |x|$

d): $f(x) = |x|$

Ex: d):

$f(x) = |x| = \sqrt{x^2}$, Use chain rule to find the derivative?

2) Find the maximum interval on which f is differentiable

a) $f(x) = |x-2| + |x|$

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Intersection
~~Intersection~~

$|x-2| \wedge |x|$, for

✓ $x \geq 0$ and $x \geq 2$ take: $x \geq 2$ which is $x-2 \wedge x$

✓ $x < 0$ and $x < 2$, take $x < 0$, $-x \wedge -(x-2)$

✓ $0 < x < 2$, take: $x \geq 0$ and $x < 2$, $x \wedge -(x-2)$

$$f(x) = |x-2| + |x| = \begin{cases} x+x-2 & x \geq 2 \\ x-x+2 & 0 < x < 2 \\ -x-x+2 & x \leq 0 \end{cases}$$

$$f(x) = \begin{cases} 2x-2, & x \geq 2 \\ 2, & 0 < x < 2 \\ -2x+2, & x \leq 0 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x \geq 2 \\ 0, & 0 < x < 2 \\ -2, & x \leq 0 \end{cases}, \text{ thus}$$

$f(x)$ is diff ~~at~~
 $(-\infty, 0) \wedge (0, 2) \wedge (2, \infty)$

Not diff:

at $x = 0 \wedge x = 2$

↳ Sharp corner at these points. Plot the curve.