

CHAPTER FOUR
Geometric Figures
and
Measurement
SUM OF INTERIOR ANGLES OF
POLYGON
for grade 7

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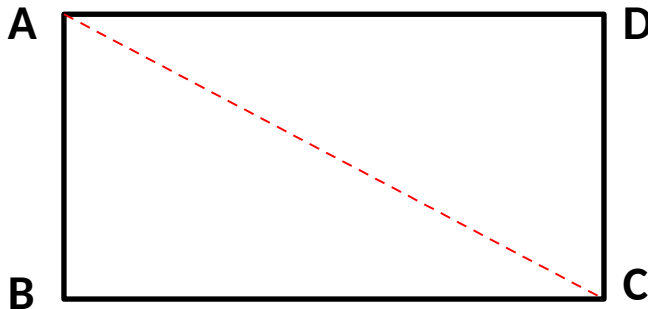
OBJECTIVES:- After completing this lesson you should be able to :-

- ❏ Formulate the sum of all and each interior , exterior angles of polygon.
- ❏ Solve the sum the all and each interior angles of polygon.
- ❏ Formulate and solve the numbers of triangles of the given polygon.
- ❏ Differentiate equilateral , equiangular and regular polygons.

The sum of the interior angles of a convex polygon

- ❖ The measures of all the interior angles of a quadrilateral always add up to 360°
- ❖ You can see this by checking that the angles in this quadrilateral(**tetra**gon) add up to 360° . here we have;

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$$



- ❖ By dividing the quadrilateral in to two triangles so that the measures of the interior angles of the two triangles add up to $180^\circ + 180^\circ = 360^\circ$
- ❖ As shown below the given Table
Number of sides of a polygon and the respective sum of degree measures of all its interior angles.

Sum of Interior Angles in Polygons



quadrilateral



pentagon



hexagon



heptagon



octagon

Convex Polygon	# of Sides	# of Triangles from 1 Vertex	Sum of Interior Angle Measures
Triangle	3	1	$1 * 180 = 180$
Quadrilateral	4	2	$2 * 180 = 360$
Pentagon	5	3	$3 * 180 = 540$
Hexagon	6	4	$4 * 180 = 720$
Heptagon	7	5	$5 * 180 = 900$
Octagon	8	6	$6 * 180 = 1080$
n-gon	n	$n - 2$	$(n - 2) * 180$



- You might have noticed that for an n - sided polygon the number of triangles formed is 2 less than the number of sides n . If that is so you can write the following:
 - ❖ The formula for the number of triangle (T) , determined by diagonals drawn from one vertex of an n - sided polygon is $T = n - 2$
- ✓ Since you have already seen that the sum of the measures of the three angles of a triangles is 180° , you can make the following generalization

➤ The formula for the sum(S) , of the measures of all the interior angles of a polygon of n - sides is given by

$$S = (n - 2) 180^\circ$$

Worked example: Find the numbers of triangles and sum of the measures of all the interior angles in a polygon having 30 sides.

Solution: $n = 30$ sides , $T = n - 2 = 30 - 2 = 28$ 

$$S = (n - 2) \times 180^\circ$$

$$S = (30 - 2) \times 180^\circ ,$$

$$S = 28 \times 180^\circ$$

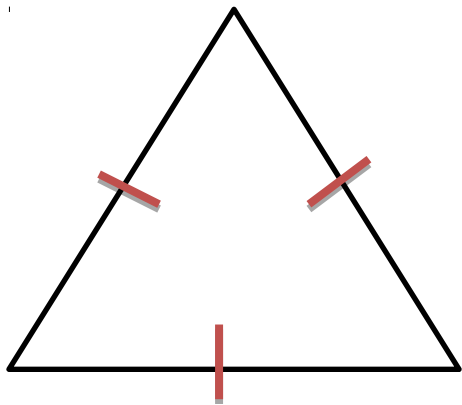
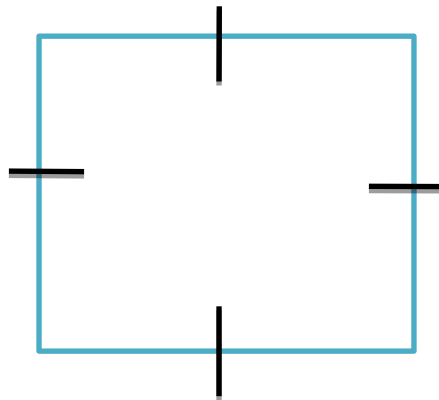
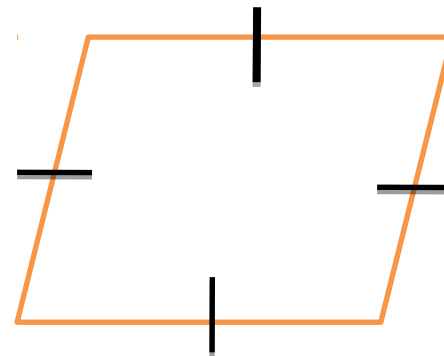
$$S = 5040^\circ$$

Therefore; the sum S , of the measures of all the angles of the polygon is 5040° .

Equilateral, equiangular and regular polygons

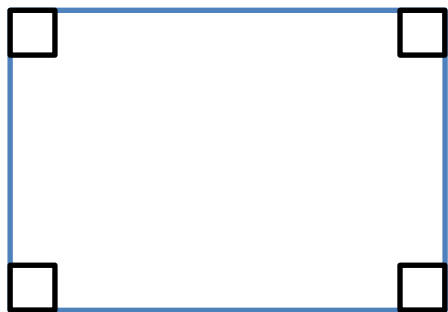
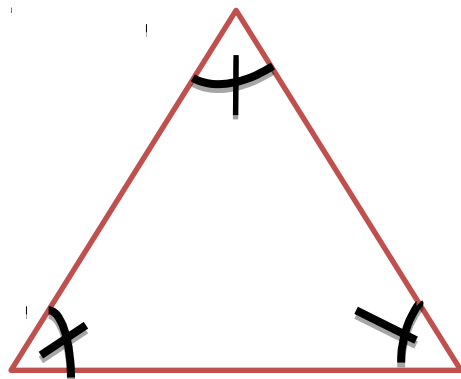
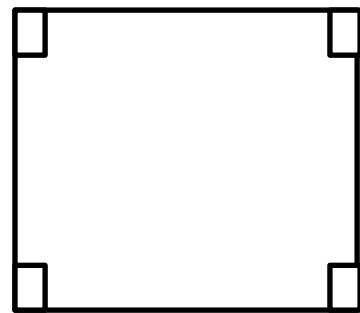
Definition: A polygon whose all sides are congruent is called an **Equilateral polygon**.

An Equilateral triangle, square and a rhombus are examples of equilateral polygons.



Definition : A polygon whose **all angles are congruent** (of the same size angle) is called **an equiangular polygon**.

A rectangle , **an Equilateral triangle** and **square** is an example of equiangular polygon.



Definition : A polygon which is **both equilateral** and **equiangular** is called a **regular polygon**.

Equilateral triangle and **square** are examples of regular polygon.

Worked example: The angles of a hexagon are

x , $2x$, $3x$, $3x$, x and $2x$. What is the value of **x** ?

Solution: the sum of the measure of interior angles of hexagon ($n=6$) ;

hence $S = (n-2)180^\circ = (6-2)180^\circ = 720^\circ$

Thus , $x + 2x + 3x + 3x + x + 2x = 720^\circ$

$12x = 720^\circ$ /divided both sides by
12/

$x = 720^\circ / 12$

$x = 60^\circ$

- ❖ The measure of each interior angle of an n-sided regular polygon is $\frac{(n-2) \times 180^\circ}{n}$
- ❖ The measure of each exterior angle is $\frac{360^\circ}{n}$
- ❖ The sum of each interior angle and each exterior angle of regular polygon is 180°

Worked example: Find the degree measure of each interior and exterior angle of 18 sided regular polygon ?

Solution: For $n = 18$ sided regular polygon

The degree measure of each interior is

$$\frac{(n - 2) \times 180^\circ}{n}$$

$$= (18 - 2) \times 180^\circ / 18$$

$$= 16 \times 10^\circ$$

$$= 160^\circ$$

- The degree measure each exterior angle is

$$\frac{360^\circ}{n}$$

$$= 360 / 18$$

$$= 20^\circ$$

Summary

- ❖ The measures of all the interior angles of a quadrilateral always add up to 360° .
- ❖ The formula for the number of triangle (T) , determined by diagonals drawn from one vertex of an n - sided polygon is $T = n - 2$
- ❖ The formula for the sum(S) , of the measures of all the interior angles of a polygon of n sides is given by $S = (N - 2) 180$
- ❖ ◦ A polygon whose all sides are congruent is called an **Equilateral polygon**.

- ❖ A polygon whose all angles are congruent (of the same size or measure) is called **an equiangular polygon**.
- ❖ A polygon which is **both equilateral** and **equiangular** is called **a regular polygon**.
- ❖ The measure of each interior angle of an n-sided regular polygon is $\frac{(n - 2) \times 180^\circ}{n}$
- ❖ *The measure of each exterior angle is $\frac{360^\circ}{n}$*
- ❖ *The sum of each interior angle and each exterior angle of regular polygon is 180° .*



ACTIVITY

1 In a **8 sided** polygon; find

- a) The numbers of triangle from a vertex
- b) Sum of all interior angles
- c) Sum of each interior and exterior angles

2. If the sum of the measures of all the interior angles of a polygon is 1080° , how many sides does the polygon have?