ADDIS ABABA CITY GOVERNMENT OF EDUCATION BUREAU YEKA SUB–CITY EDUCATION OFFICE PREPARATORY SCHOOLS MATHEMATICS MODEL EXAMINATION FOR GRADE 12TH **NATURAL** SCIENCE STREAM NEHASE 2012 E.C / AUGUST 2020 G.C

GENERAL INSTRUCTIONS:

THIS BOOKLET CONTAINS **MATHEMATICS** EXAMINATION. THE EXAMINATION CONTAINS **65** QUESTIONS **FOR NATURAL SCIENCE STUDENTS**. ATTEMPT ALL THE QUESTIONS. FOLLOW THE INSTRUCTIONS ON THE ANSWER SHEET AND THE EXAMINATION PAPER EXACTLY. USE ONLY **PENCIL** TO MARK YOUR ANSWERS.

THERE IS ONLY **ONE BEST** ANSWER FOR EACH QUESTION. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET. YOUR ANSWER MARK SHOULD BE HEAVY AND DARK, COVERING THE ANSWER SPACE COMPLETELY. PLEASE ERASE ALL UNNECESSARY PENCIL/PEN MARKS COMPLETELY FROM YOUR ANSWER SHEET.

YOU WILL BE LOWED TO WORK FOR **3:00** (OR 3 HOURS). IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY GO BACK AND REVISED. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, LAY YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMPT TO CHEAT IN THE EXAMINATION HALL WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE(S).

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

DIRECTIONS: For each of the following questions, choose the best answers from the given alternatives and carefully **blacken** the letter of your best choice on the separate answer sheet provided.

1. If $f(x) = 2x^2 - 4x - 8$. Then for $h \neq 0$, what is the expression $\frac{f(x+h)-f(x)}{h}$?

   A/ $4x + 2h - 4$ B/ $4h - 4$ C/ $2x - 3h$ D/ $x - h$

2. Let $f(x) = -x^2 - x - 2$, then what is $4f(2) + f(2)\,f(2)$?

   A. -32 B. 32 C. -96 D. 96

3. Let a function $f(x) = 4x^4 - 2x^2 + 8$, then which one of the following is **true** about f.
   A. f is odd C. f is even
   B. f is neither even nor odd D. f is both even and odd

4. If $g(x) = x - 1$ and $(fog)(x) = x^2 - x + 6$, then what is the equivalent expression for $f(x)$?
   A. $x^2 - x + 6$ B. $-x^2 - x + 6$ C. $x^2 + x + 6$ D. $x^2 - x - 6$

5. What is the value of $\lim_{x\to -1} \frac{\sqrt{1+\sqrt{x+5}}-\sqrt{3}}{x+1}$?

   A/ $\frac{\sqrt{3}}{8}$ B/ $\frac{\sqrt{3}}{16}$ C/ $\frac{\sqrt{3}}{12}$ D/ $\frac{\sqrt{3}}{24}$

6. Which one of the following is **not** one-to one correspondence?
   A. $f: (0, \infty) \to (0, \infty)$ given by $f(x) = x^4 \quad \forall x \in \mathbb{R}$
   B. $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \frac{6x-10}{x^2+1}, \quad \forall x \in \mathbb{R}$
   C. $f: \mathbb{R} \to \mathbb{Q}$ given by $f(x) = 3^x, \quad \forall x \in \mathbb{R}$
   D. $f: [5, \infty) \to (-2, \infty)$ given by $f(x) = -2 + \sqrt{x-5}, \quad \forall x \in \mathbb{R}$

7. What is the distance d from the point $p\,(3, 2)$ to the line L: $3x + 4y + 13$?
   A.5 B.6 C.8 D.10

8. What is the equation of the tangent line at point $(3, -3)$ on the circle $x^2 + y^2 + 6x + 10y - 9 = 0$?
   A. $x - 3y - 4 = 0$ C. $x - y + 3 = 0$
   B. $-5x + y + 7 = 0$ D. $x + 3y - 4 = 0$

9. Which one of the following is **not** true about the parabola $x^2 + 12\,y - 2x - 11 = 0$?
   A. its directrix is $y = 4$ C. its focus is $(0, -2)$
   B. its vertex is $(1,1)$ D. its axis is y-axis

10. Which one of the following is **not true** about the relation $R=\left\{(x, y) : y = \frac{x-8}{9-x}\right\}$
    A. Domain of R is $(9, \infty) \cup (-\infty, 9)$
    B. The range of R is $(-1, \infty) \cup (-\infty, -1)$
    C. The inverse of R is $\left\{(y, x) : y = \frac{9x+8}{1-x}\right\}$
    D. R can be a function

11. Which one of the following sentences is a proposition?

A. long live Ethiopia! C. America has good states

B. Please give me some money! D. Man is mortal

12. If $(\neg p \Leftrightarrow q) \Rightarrow (p \vee \neg r)$ is **False**, then which of the following has a truth value **True**?

A. ¬p ⇔r B. q⇒p C. q∧ ¬r D. ¬q⇔r

13. "If 3 is a factor of 12, then 12 is an even number" be a statement. Then what is the inverse of the statement?

A. If 3 is not a factor of 12, then 12 is an odd number

B. If 12 is an even number, and then 3 is a factor of 12.

C. If 12 is an odd number, then 3 is not a factor of 12

D. If 12 is an odd number, then 3 is a factor of 12.

14. Let U = Natural numbers, P(x) = x is divisible by 1, q(x) = x is prime, r(x) = x is divisible by 6. Then which one of the following quantifier has a truth value True.

A. $(\forall_x) r(x)$ B. $(\exists_x)[p(x) \Leftrightarrow q(x)]$ $C.(\exists_x)[r(x) \wedge q(x)]$ $D.(\forall_x)p(x)$

15. If A = $\begin{pmatrix} -1 & 2 & 3 \\ -4 & -5 & 6 \\ -1 & 0 & -2 \end{pmatrix}$, what is the matrix B such that, A+B is a unit matrix?

A. $\begin{pmatrix} 2 & -2 & -3 \\ 4 & 6 & -6 \\ 1 & 0 & 3 \end{pmatrix}$ B. $\begin{pmatrix} 2 & 3 & 0 \\ 1 & -1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ C. $\begin{pmatrix} -7 & 0 & -1 \\ 2 & 0 & 3 \\ 1 & -1 & -2 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

16. If A = $\begin{pmatrix} 1 & -1 & 2 \\ 3 & -2 & 4 \\ 2 & 3 & -4 \end{pmatrix}$ then what is $|A|$? A. 2 B. 4 C. 3 D. 1

17. What is the value of x, if $\begin{vmatrix} 1-x & 0 & 0 \\ 0 & x-1 & 0 \\ 0 & 0 & -x+1 \end{vmatrix}$ = 64?

A. -3 B. 3 C.5 $D.-5$

18. Suppose A is a $3x3$ matrix. If $det(A) = 4$, then what is the value of $det(5A)$?

A. -500 B. 1000 C. -1000 D. 500

19. How many distinct permutations can be formed from all the letters of the word "SMART"?

A. 240 B. 120 C. 180 D. 200

20. Let = $\left(\frac{1+i}{1-i}\right)^{351}$, then what is the value of z? A. i B. 1 C. $-i$ D. -1

21. Let A and B be two events. Suppose that the probability that neither event occurs is $\frac{1}{8}$. What is the probability that at least one of the events occurs?

A. $\frac{3}{8}$ B. $\frac{7}{8}$ C. $\frac{5}{8}$ D. $\frac{1}{8}$

22. What is the solution set of the rational equation $\frac{6}{x} - \frac{3x-6}{x^2-2x} = 3 - \frac{6x-6}{3x-2}$?

A.$\{-1\}$ B.$\{1\}$ C.$\{2,-1\}$ D.$\{1,2\}$

23. Let $p(x) = x^2 - x$ is positive, then what is the equivalent expression of $\neg\big((\exists_x)p(x)\big)$?

A. $(\exists_x)(x^2 - x < 0)$ C. $(\forall_x)(x^2 - x \geq 0)$

B. $(\forall_x)(x^2 - x \leq 0)$ D. $(\exists_x)\neg(x^2 - x > 0)$

24. Let $2x^2 + 4x + 10 = 0$. Then what is the solution set of the equation in set of complex number?

A. $\{2+i, 2-i\}$ C. $\{2+4i, 2-4i\}$

B. $\{-1+2i, -1-2i\}$ D. $\{1-2i, -1-2i\}$

25. Let $Z = \left(\frac{1+i}{1+\sqrt{3}i}\right)^{19}$. Then which one of the following is not true about Z?

A. $Argz = \frac{5\pi}{12}$ C. The modulus of Z is $\sqrt{2^{19}}$

B. The polar form of Z is $\frac{\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}}{\sqrt{2^{19}}}$ D. $Argz = 19Arg\left(\frac{1+i}{1+\sqrt{3}i}\right)$

26. If $Z = cos(\frac{\pi}{60}) + i\ sin(\frac{\pi}{60})$, then what is the value of Z^{20}?

A. $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ B. $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ C. 1+i D. $\frac{\pi}{2} + \frac{\pi}{2}i$

27. Which one of the following is **not true** about the graph of $f(x) = \frac{x^3-6x^2+11x-6}{x^4-5x^2+4}$?

A The graph has vertical asymptotes at $x = -1$ & $x = -2$

B The graph of f crosses the $x-axis$ at $x = \pm 1, \pm 2$

C The graph has a hole at $x = 1$ & 2

D The graph of f has a horizontal asymptote at $y = 0$

28. If $\{A_n\}$ is an arithmetic sequence with $A_1 = 10$ & $A_5 = 18$, *then what is* $\sum_{n=1}^{10} A_n$?

A. 190 B.160 C.180 D.170

29. What is the sum of the series $\sum_{n=0}^{\infty} 4^n\ 5^{2-n}$?

A.∞ B. 101 C. 110 D. 100

30. What is the sum of the 1st 100th term of the sequence 5, 10, 15, 20, 25…?

A.2525 B. 22,750 C. 25,250 D. 2275

31. If 4-geometric means are inserted between 3 and 96, what are the inserted numbers?

A. {6,18,54,90} B. {9,18,36,72} C. {3,6,12,24} D. {6,12,24,48}

32. What is the value of the $6\lim_{x\to 0}\frac{cos^2x-1}{x^2cos2x}$?

A. -6 B. 6 C. $does$ not exist D. ∞

33. Let the function f be defined by $\begin{cases} \frac{tanmx}{x}, x > 0 \\ 3x - 2m^2, x \le 0 \end{cases}$, If $m \neq 0$, and then what is the value of m that make the function f is continuous at $x = 0$?

A. $\frac{1}{2}$ B. -1 C. $\frac{-1}{2}$ D. 1

34. What is the value of $\lim_{x\to\overline{0}}\frac{-|x|+2xe^x}{x}$?

A. doesn't exist B. 0 C. -3 D. 3

35. Let $f(x) = x^3 - 3x$. Then what is value of the $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$?

A. -9 B. 9 C. 3 D. -3

36. Let $f(x) = e^{3x+2}$ and $g(x) = tan\,x$. If $F(x) = xf(x) - f(g(x))$ .Then what is the value of $F'(0)$?

A/ $\frac{1}{e}\left(1-\frac{1}{e}\right)$ B/ $-2e^2$ C/ $4e^2$ $D/\frac{1}{e}\left(1-\frac{4}{e}\right)$

37. Let $h(x) = x\,g\,(3x+2)$, $g\,(-4) = 12$ and $g'(-4) = 10$, then what is the value of $h'(-2)$?

A. -48 B. 48 C. 72 D. -72

38. Let $f(x) = x^5 - 15x^3 + 6$, in $[-1, 1]$. then which one of the following is **not** true about the function f ?

A. the minimum value of f is -156 & maximum value of f is 162.

B. the function is decreasing on $(-\infty, -3]$ & increasing on $[3, \infty)$

C. one of the inflection point f is $(0, 6)$

D. the graph concave down on $\left(\frac{3\sqrt{2}}{2}, 0\right)$

39. What is the equation of the tangent line to the graph of $f(x) = \frac{8}{x^2} - 2x + 4$ at $(-2,\ f\,(-2))$

A. $6x + y + 22 = 0$ C. $6x - y - 22 = 0$

C. $6x - y + 22 = 0$ D. $6x + y - 22 = 0$

40. Let $f(x) = -3x^4 + 4x^3 + 12x^2 + 24$. Then what is the local minimum value of the graph of f ?

A. -56 B. 56 C. 24 D. -24

41. What is the second derivative of f if $f(x) = ln(x^2 + 2)$?

A. $\frac{4-2x^2}{(x^2+2)^2}$ B. $\frac{4+6x^2}{(x^2+2)^2}$ $C.\frac{2-2x^2}{(x^2+2)^2}$ $D.\frac{4-x^2}{(x^2+2)^2}$

42. Which one of the following expression is an anti- derivative of $f(x) = \frac{-12}{x\sqrt{x}}$?

A. $\frac{-12\sqrt{x}}{x} + c$ $B.\frac{24\sqrt{x}}{x} + c$ $C.\frac{-24\sqrt{x}}{x} + c$ $D.\frac{12\sqrt{x}}{x} + c$

43. If $h(x) = \frac{x-6}{\sqrt{x-6}}$ and $f(x) = \frac{9}{x} + 6$, $then\ what\ is\ h\big(f(x)\big)$?

A. $\frac{\sqrt{x}}{9}$ $B.\frac{\sqrt{x}}{3}$ C. $\frac{3}{\sqrt{x}}$ D. $\frac{9}{\sqrt{x}}$

44. What is $\int_0^1 \frac{(x-1)^2}{x+1}$? A/ $\ln 2 + \frac{1}{2}$ $B.\,4ln2 - \frac{5}{2}$ C/$\ln 3 + 5$ $D.\ln 3 + \frac{1}{5}$

45. What is the value of $\int_0^2 x^2 \ln(1 + x^3)\mathrm{d}\,x$?

A. $18\ln 3 - 10$ $B.\,6\ln 3 - 10$ $C.\,6\ln 3 - 8$ $D.\,18\ln 3 - 8$

46. What is the area of the region bounded by the graph of $y = x^2 - 4$ and $y = 4 - x^2$?

A. $\frac{16}{3}$ B. $\frac{64}{3}$ $C.\frac{32}{3}$ $D.\ \frac{48}{3}$

47. Let a particle P is moving from point A towards point B with acceleration given by $a(t) = 2t^3 - 8t^2 + 10t + 6$ for time t≥ 0 and a velocity of 12m/s at $t = 2sec$. Then what is velocity of the particle just at $t = 3sec$ (in m/s)?

A. 14 B. 26 C. 16 D. 24

48. Which one of the following is **incorrect** about the application of **1st** and **2nd** derivative test on monotonicity, extreme values and concavity?

A. If $f``(c) > 0$, then the graph concave upward & $f``(c) < 0$, the graph concave downward on the interval I and $f``(c)$ exist

B. If $f``(c) > 0$ and $f`(c) = 0$, then $f$ has local minimum & If $f``(c) < 0$ and $f`(c) = 0$, then f has local maximum at $x = c$

C. If $f`(c)$ change its sign from" $-$ " $to$ " $+$ ". Then $f$ has local maximum & If $f`(c)$ change its sign from" $+$ " to " $-$ ". Then f has local minimum at $x = c$.

D. If $f`(x) \geq 0, \forall x$, in the interior of I, then $f$ is decreasing on I & If $f`(x) \leq 0, \forall x$, in the interior of I, then f is increasing on I

49. Let $f(x) = \frac{1}{e^{-x+10}}$. Then what is the n^{th} derivative of f?

A. $f^{(n)}(x) = e^{x-10}$ C. $f^{(n)}(x) = -e^{-x-10}$

B. $f^{(n)}(x) = e^{-x+10}$ D. $f^{(n)}(x) = -e^{x-10}$

50. Which of the following is **not** a convergent sequence?

A. $\left\{7^{1/n}\right\}$ B. $\left\{\left(\frac{\pi}{3}\right)^n\right\}$ C. $\left\{1 + \frac{(-1)}{9^n}\right\}$ D. $\left\{\left(\sqrt{3}\right)^{-n}\right\}$

51. What is the maximum possible interval for which the function f is continuous for $f(x) = \frac{x^2+9}{x^4-81}$?

A $(-\infty, -3] \cup [-3, 3] \cup [3, \infty)$ C $(-\infty, -3] \cup [3, \infty)$

B $(-\infty, -3) \cup (3, \infty)$ D $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

52. Let $f(3) = ln\,7$. Then what is $\int \frac{5x^2-8x+5}{x^3-3x^2+3x-2} dx$?

A. $3ln|x-2| + ln|x^2-x+1| + ln\,7$ C. $3ln|x+2| + ln|x^2+x+1| + ln\,7$

B. $3ln|x-2| + ln|x^2+x+1| + ln\,7$ D. $3ln|x-2| + ln|x^2+x+1| + ln\,7$

53. If the region bounded by the curve $y = 5x^2$ and the line $y = 20$ rotates about the x-axis. Then what is the volume of the solid of revolution?

A. 120π $B. 160\pi$ $C. 1280\pi$ $D. 1260\pi$

54. An object is moving along the parabola $y = 2\sqrt{x}$ in xy-plane. At what point on its path does the object become closest to the point $(5, 0)$?

A. $(2, 2\sqrt{2})$ B. (2, 1) C. (2, 6) D. $(3, 2\sqrt{3})$

55. The radius of a right circular cylinder of height 5 cm increases at a rate of 4cm/min. what is the rate of change of the volume of the cylinder when the radius is 3cm?

A. $120\pi cm^3/min$ $B. 180\pi cm^3/\min$ $C. 60\pi cm^3/\min$ $D. 40\pi cm^3/min$

56. Let an equation of a circle $(x-2)^2 + (y-3)^2 = 25$. then what is the equation of the image of circle after a reflection about the line $y = x + 3$?

A. $x^2 + (y-5)^2 = 25$ C. $(x-5)^2 + y^2 = 25$

B. $(x+2)^2 + (y+3)^2 = 25$ D. $(x-2)^2 + (y-3)^2 = 50$

57. Let $\vec{a} = 3i + 4j$ and $\vec{b} = 4i + 2j$ and $\vec{c} = 2i + 3j$. Then which of the following is the value of $\vec{a}.(\vec{b} - 3\vec{c})$?

A. -16 B. -34 **C**. 16 D. 34

58. Let $\vec{U}$ and $\vec{V}$ are vectors such that $|\vec{U}| = 15$, $|\vec{V}| = 5$ and $\vec{U} \cdot \vec{V} = 75$, then what is the magnitude of $\vec{U} - \vec{V}$?

A. 100 B. 110 C. 10 D. 11

59. What is the translation vector V= (h, k) so that the equation $x^2+y^2+8x-10y+5=0$ is transformed to an equation of $x^2 + y^2 - 2x + 6y - 26=0$?

A. $V = (-8, 5)$ B. $V = (-8, -5)$ C. $V = (-5, -8)$ D. $V = (5, -8)$

60. Let f(x) $= -10\sin(3x - \frac{\pi}{3}) + 2$. Then which one of the following is **not** true about the graph of f?

A. The amplitude is 10 and the period is 8π

B. The four equal intervals are $\left\{\frac{4\pi}{3}, \frac{10\pi}{3}, \frac{16\pi}{3}, \frac{22\pi}{3}, \frac{28\pi}{3}\right\}$

C. The phase shift is $\frac{4\pi}{3}$

D. The length interval is $\frac{4\pi}{3}$

61. What is the general solution of $sin^2x - sinxcosx = 0$?

A. $\left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$ C. $\left\{0, \frac{\pi}{4}, -\pi, \frac{5\pi}{4}, -2\pi\right\}$

B. $\left\{0, -\frac{\pi}{4}, \pi, -\frac{5\pi}{4}, 2\pi\right\}$ D. $\left\{0, -\frac{\pi}{4}, -\pi, -\frac{5\pi}{4}, -2\pi\right\}$

62. Let $siny=\frac{9}{15}$ and $cosx=\frac{5}{13}$. Then what is the value of $tan(x + y)$?

A. $\frac{63}{16}$ B. $-\frac{63}{16}$ C. 1 D. -1

63. A flying airplane is sighted in a line from two observation stations A and B. The angle of elevation of the airplane from A is 30° and from B is 60°. A & B are on the same side of the airplane. If the distance between A and B is 1200m. Then what is the altitude of the airplane?

A. $600\sqrt{3}$m B. $900\sqrt{3}$m C. $300\sqrt{3}$m D. $400\sqrt{3}$m

64. Let $\theta = \cot^{-1}(\frac{12}{5})$. Then what is the value of $\csc(2\theta)$?

A. $\frac{120}{169}$ B. $\frac{13}{5}$ C. $\frac{169}{120}$ D. $\frac{5}{13}$

65. Which one of the following points lies outside the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z = 11$?

A. $(2, -2, -2)$ B. $(-1, 0, 1)$ C. $(0, -1, 1)$ D. (1, 1, 1)