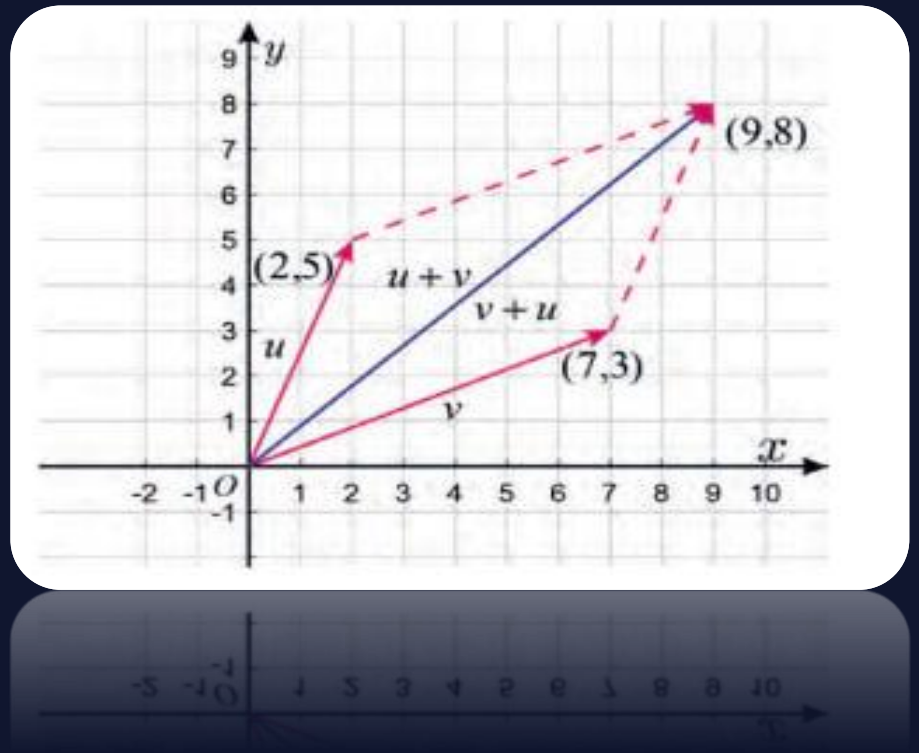
Math's Unit 5

Vectors-Part 1

Exam Based Tutorial



Grade 11 Maths Unit 5 Vectors

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Unit Outcomes

By the end of this unit, you will be able to:

- **Know basic concepts and procedures about vectors and operation on vectors.**
- **Know specific facts about vectors.**
- **Apply principles and theorem about vectors in solving problems involving vectors.**

5.1 Revision on Vectors and Scalars

Scalar:

- **Physical quantities expressed by a certain number associated with a suitable unit without any mention of direction in space is known as scalar.**
- **The number describing the quantity of a particular scalar is known as its magnitude.**
- **For example, time (15 sec), mass (20 kg), length (5 km), area (7 m^2), volume (10 m^3), temperature (37 °C).**

5.1 Revision on Vectors and Scalars

Vectors:

- *A vector is a physical quantity which can be expressed completely by stating both its magnitude with particular unit and direction.*
- *For example, car movement is usually described by giving the speed and the direction, say 60 km/h northeast.*
- *The car speed and car direction together form a vector quantity, velocity of the car.*

1. Which one of the following is a vector quantity? ***(UEE 2014)***

A. Size of a plantation farm.

B. Velocity of a moving car.

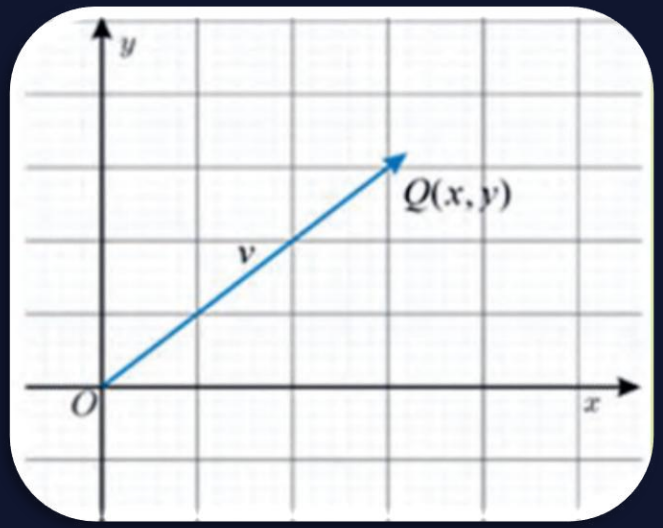
C. Volume of a normal size bottle.

D. Amount of fat in one liter of milk.

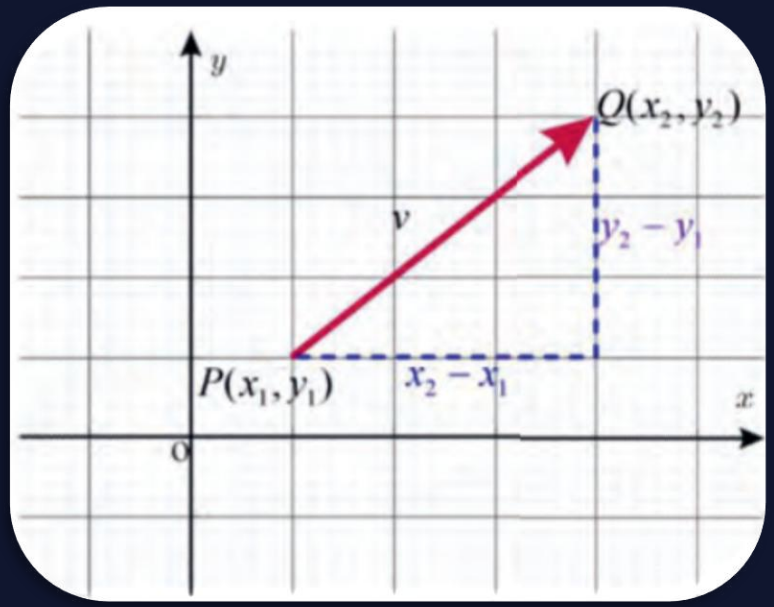
5.2 Representation of Vectors

5.2.1 Components of vectors

- If v is a vector in the plane whose initial point is the origin, O (0, 0) and whose terminal point is Q (x, y) then the coordinate form of a vector v = vector OQ = (x, y) or the column form of a vector v = $\begin{pmatrix} x \\ y \end{pmatrix}$.
- The numbers x and y are components of a vector v.
- The direction of the vector can be given by the slope m = y/x.



- If v is a vector in the plane whose initial point P (x_1, y_1) and whose terminal point Q(x_2, y_2) the coordinate form of a vector v = vector PQ = (x_2-x_1, y_2 - y_1) or the column form of a vector v= $\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \end{pmatrix}$.
- The number X_2-X_1 and Y_2 - Y_1 are called components of a vector v.
- The direction of the vector v can be given by the inclination of the line with slope $m = \frac{y_2 - y_1}{x_2 - x_1}$



- find the coordinate and column forms of the vector v that has initial point at (0, 0) and a terminal point at (3, 6)

5.2.2 Addition and Subtraction of vectors

a. **Addition of Vectors**

i) Graphical method

ii) Analytical method

Note

1. Vector addition obeys commutative property.

$$u + v = v + u$$

2. Vector addition obeys Associative property

$$(u + v) + w = u + (v + w)$$

3. For any three vectors u, v and w, if u = v and v = w, then u = w.

4. The zero vector 0 has the following property:
   For any vector u, u + 0 = 0 + u = u.

5. If $u = (x_1, y_1)$ and $v = (x_2, y_2)$ then $u + v = (x_1 + x_2, y_1 + y_2)$.

5.2.2 Subtraction and addition of vectors

b. Subtraction of Vectors

1. **Subtraction of two vectors u and v is adding the negative of vector v to the vector u. That means u − v = u + (−v).**
2. **You can do subtraction of vectors just like how you can do subtraction of scalers. You subtract the corresponding components of vectors while subtracting vectors.**

i) Graphical method

ii) Analytical method

Note

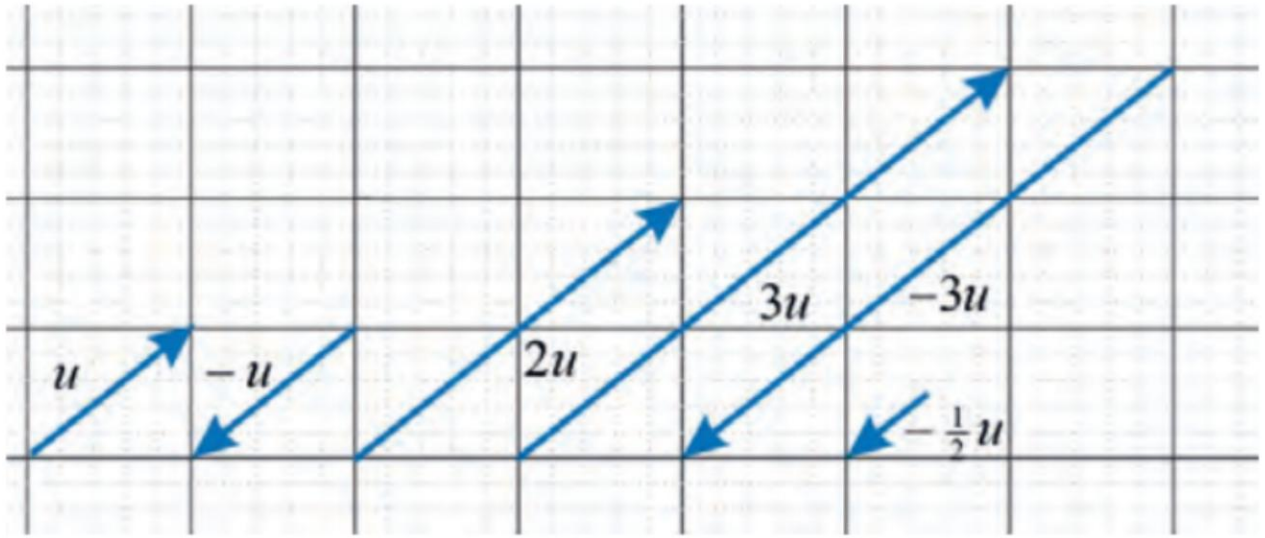
1. If v is any non-zero vector and -v is the negative of v, then v + (-v) = 0.
2. The subtraction of vectors is not commutative.
3. The subtraction of vectors is not associative.

If $\mathbf{u} = (3, 6)$, $\mathbf{v} = (8, 4)$ and $\mathbf{w} = (2, 7)$, then find

a. $\mathbf{u} - \mathbf{v}$ b. $\mathbf{v} - \mathbf{u}$ c. $(\mathbf{u} - \mathbf{v}) - \mathbf{w}$ d. $\mathbf{u} - (\mathbf{v} - \mathbf{w})$

5.2.3 Multiplication of Vectors by a scalar

- The product kv of a vector v and a scalar k is a vector with a magnitude that is k times the magnitude of v, and with a direction that is the same as the direction of v if k > 0, and opposite the direction of v if k < 0. This is called scalar multiplication. If k = 0 or v = 0, then kv = 0.



$$k = -1, 2, 3, -3, \frac{-1}{2}$$

If $\mathbf{u} = (5, -9)$ and $\mathbf{v} = (3, 5)$, then find

a. $2(3\mathbf{u})$ **b.** $(3 + 2)\,\mathbf{u}$ **c.** $3(\mathbf{u} + \mathbf{v})$ **d.** $1 \cdot \mathbf{v}$ **e.** $0 \cdot \mathbf{v}$

2. If $\vec{U}$ = (-3, x) and $\vec{V}$ = (x, y-2) are vectors what is the value of y so that $\vec{U} + \vec{v} = 3\vec{U} - ½\vec{V}$ *(UEE 2005)*

A. 2/3 B. -10/3 C. -4 D. -22/3

3. If ***A = (-2, 3) , B = (3,1) and C is any other point on the plane, then which one of the following is coordinate form of*** $\overrightarrow{AC}$ - $\overrightarrow{BC}$ ***(UEE 2006)***

A. (-5, 2) B. (5, -2) C. (1,4) D. (-1, -4)

4. If **A** = (1, -2), **B = (-3, 2) and** $\vec{V}$ is a position vector such that $2\vec{V} + \overrightarrow{AB} = 0$, then $\vec{V}$ is equal to :***(UEE 2007)***

A. (2,0) B. (-1,0) C. (-2, 2) D. (2, -2)

5. If $\vec{V} = \overrightarrow{AB} + 3\overrightarrow{BA}$ where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$? ***(UEE 2008)***

A. $6\overrightarrow{AB}$ B. $-6\overrightarrow{AB}$ C. $12\overrightarrow{AB}$ D. $-12\overrightarrow{AB}$

6. Given two vectors u = (1,3) and v = (-3,5) in the plane then 6u+2v is equal to? ***(UEE 2013/14)***

A. (3,3) B. (0,28) C. (0, 18) D. (2,28)

7. Let u = (-1, 4) and v = (1,-3) be vectors in the plane. Which one of the following equal to u-v? ***(UEE 2014)***

A. (-2,1) B.(0,1) C.(-2,7) D. (0,7)

8. Let a = (1, 3, -1) and b = (2, -1, 1) be vectors in space. Which one of the following is true? ***(UEE 2014)***

A. a-b = (-1, 4, -2)

B. a + b = (3, 4, 0)

C. 3a = (3, 9, 3)

D. a + 2b = (5, -1, 1)

9. Let u = (-1, 6) and v = (2, 4) be position vectors. Then the vector 4u-3v is ***(UEE 2015)***

A. (-10, 12) B. (-10, 11) C.(-4, 24) D.(-15, 18)

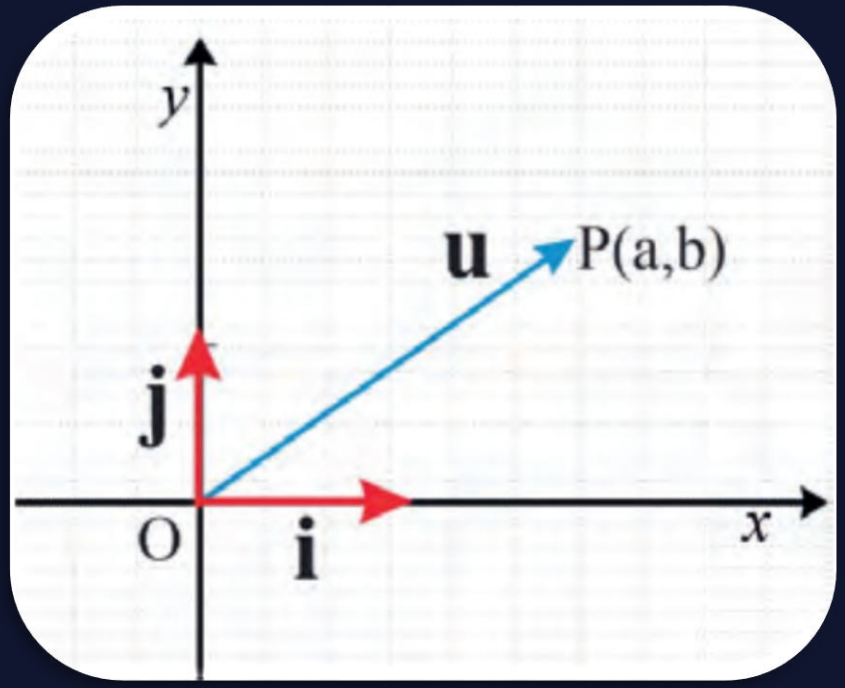
10. Which one of the following is the position vector of the vector with initial point A (1, -1) and terminal point B (2,-3)? **(UEE *2015)***

A.i-4j B. 3i+4j C.i-2j D. 3i-4j.

5.2.4 Unit Vectors

- A unit vector is a vector whose magnitude (or length) is one.
- The *standard unit vectors* are the special unit vectors that are parallel to the coordinate axes, pointing toward positive values of the coordinate.
- **Let A be a vector, the unit vector in the direction of A is given by**

$$\vec{U}_A = \frac{\vec{A}}{|A|}$$



Express the following vectors in terms of the standard unit vectors i and j:

a. (5, -6) b. (-4, 8) c. (−7, 0) d. (0,3)

Express each of the following vectors as a vector in a coordinate:

a. $i + 4j$ **b. $3i-3j$** **c. $-7i+ 8j$**

- If u = (1, 2) and v = (3, 4), then find:

  a. the unit vector in the direction of v.

  b. the unit vector in the direction of u + v.

  c. the unit vector in the direction of u - v.

11. Suppose $\vec{A}$ = 2j - k and $\vec{B}$ = 5i + 15k, where i, j and k are the standard unit vectors in the directions of positive x, y and z axis respectively. Which one of the following is the unit vector in the direction of $\vec{A} + \frac{1}{5}\vec{B}$? *(UEE 2005)*

A. $\frac{3}{5}$ i + $\frac{3}{5}$ k

B. $\frac{1}{3}$ i + $\frac{2}{3}$ j + $\frac{2}{3}$ k

C. $\frac{4}{5}$ j - $\frac{3}{5}$ k

D. $\frac{2}{3}$ i - $\frac{1}{3}$ j + $\frac{2}{3}$ k

12. If u = 2j - k and v = i – 8j + 3k, then what is the unit vector in the direction of 5u+v? ***(UEE 2011)***

A. $\frac{1}{3}$i + $\frac{2}{3}$ j - $\frac{2}{3}$ k

B. $\frac{2}{3}$i + $\frac{1}{3}$ j - $\frac{2}{3}$ k

C. i + 2j – 2k

D. $\frac{1}{3}$i + $\frac{2}{3}$ j + $\frac{2}{3}$ k

13. Let $\vec{u}$ = (2, 0) and $\vec{v}$ = (-1, 3) be two vectors in the plane and $\vec{w}$ = -3$\vec{u}$ +2$\vec{v}$

Which one of the following is the unit vector in the direction of $\vec{w}$ in terms of the standard unit vectors i and j? ***(UEE 2012/13)***

A. $\frac{3}{5}$ i - $\frac{4}{5}$ j B. - $\frac{3}{5}$ i + $\frac{4}{5}$ j C. $\frac{4}{5}$ i - $\frac{3}{5}$ j D. - $\frac{4}{5}$ i + $\frac{3}{5}$ j

14. Which of the following represents the vector (3,-5, 7) in the space using the standard unit vectors i, j and k? ***(UEE 2013/14)***

A. 3i-5j+7k B. -3i-5j-7k. C. 3i-5j-7k D. 3i+5j+7k

5.2.5 Norm of Vectors

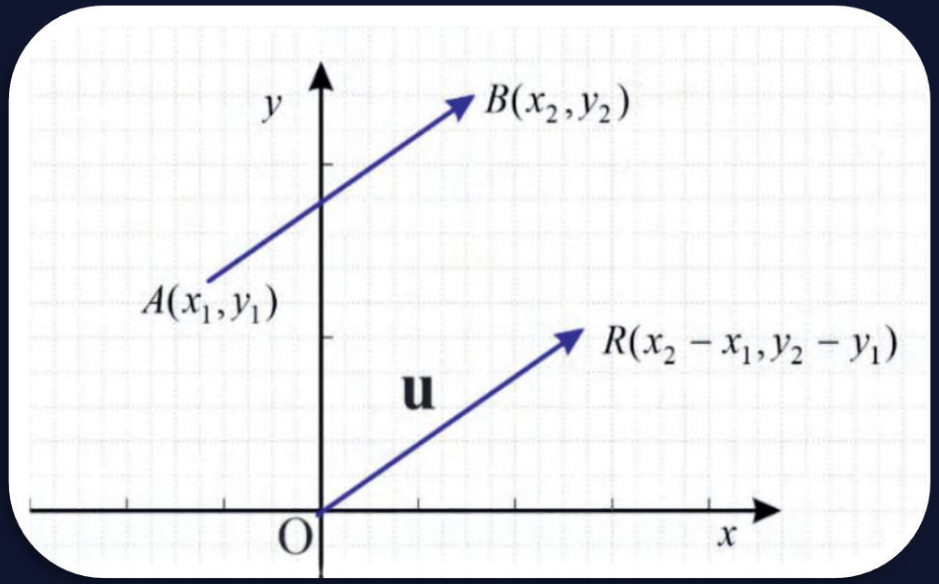
- If $\overrightarrow{AB}$ is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), then its position vector u is determined as

$$u = (x_2 - x_1, y_2 - y_1) = (x_2 - x_1)i + (y_2 - y_1)j$$

Thus, $(x_2 - x_1)$ and $(y_2 - y1)$ are the coordinates of u with respect to the base (i, j)

The norm or magnitude of a vector u
Is given by

$$|\mathbf{u}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



Find the norms of the following vectors:

a. $(3, -3)$ b. $(4, 6)$ c. $(-4, 0)$ d. $(0, 6)$

15. What is the magnitude of the vector whose initial point is at (1, -3) and terminal point is at (4, 1) ? ***(UEE 2013/14)***

A. 1 B. 4 C. 5 D. 6

Note:

- Two vectors u and v are parallel if u = kv for some non-zero k. furthermore, if k> 0 they are actually in the same direction, if k < 0, they are in opposite direction.
- Two vectors $u = u_1i + u_2j$ and $v = V_1i+V_2j$ are equal if and only if their components are equal. i.e., $u_1 = V_1$ and $u_2 = V_2$.
- Two or more vectors are said to be equal if they have the same magnitude and the same direction.

If $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$, which of the vectors below are parallel to $\mathbf{u}$ or $\mathbf{v}$?

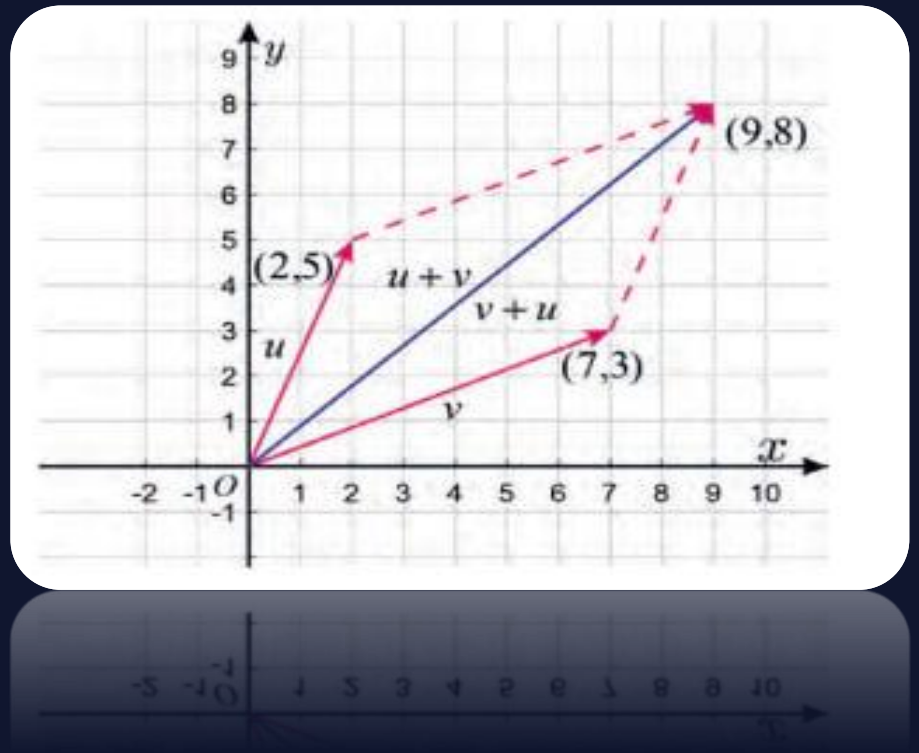
1. $a = -2\mathbf{i} - \mathbf{j}$
2. $b = 5\mathbf{i} - 10\mathbf{j}$
3. $c = 4\mathbf{i} - 8\mathbf{j}$
4. $d = 6\mathbf{i} + 3\mathbf{j}$
5. $e = -2\mathbf{i} + 4\mathbf{j}$
6. $f = -4\mathbf{i} - 2\mathbf{j}$

Thank you!!!

Math's Unit 5

Vectors-Part 2

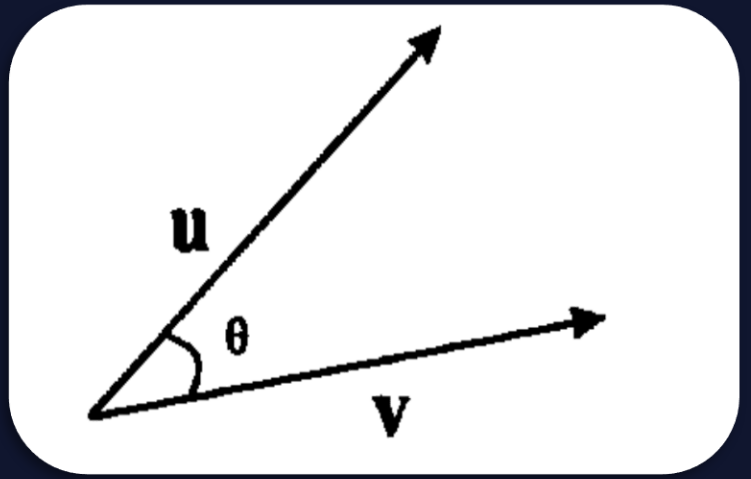
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Vector Product

1. Scalar (inner or dot) product

- If u and v are vectors and is the angle between u and v, then the dot product of u and v, denoted by u. v is given by $u \cdot v = |u||v| \cos\theta$
- If $u = (u_1, u_2)$ and $v = (V_1, V_2)$ are vectors, then $u \cdot v = U_1V_1 + U_2V_2$.



Note

- i. j = 0, since $\cos(\frac{\pi}{2}) = 0$.
- i· i = j · j = 1, since cos(0) = 1.
- If either u or v is 0 then u▪ v = 0.
- u · u > 0 if u # 0 and u • u= 0 if u = 0.
- dot product of vector is commutative.

Note

- If the vectors u and v are parallel and pointing to the opposite direction, then $u \cdot v = -|u||v|$.
- In particular, for any vector u, you have $u \cdot u = |u|^2$, you write u^2 to mean $|u|^2$.
- If the vectors u and v are perpendicular or orthogonal, then u.v = 0 since $\cos(\frac{\pi}{2})$.
- If u and v are nonzero vectors, then the cosine of the angle between u and v is given by $\cos\theta = \frac{u.v}{|u||v|}$
- If u, v and w are vectors and k is a scalar then

  1. $k(u \cdot v) = (ku) \cdot v = u \cdot (kv)$
  2. Distributive property

     i) $u \cdot (v + w) = u \cdot v + u \cdot w$

     ii) $(v + w) \cdot u = v \cdot u + w \bullet u$

Find the dot product of the vectors $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = 7\mathbf{i} - 2\mathbf{j}$.

Find the cosine of the angle formed by two vectors $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$.

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors $|\mathbf{u}| = 3$ and $|\mathbf{v}| = 6$ and $\theta = \frac{\pi}{3}$.

a. Find $|3\mathbf{u} - 2\mathbf{v}|$.

b. Find the cosine of angle between $\mathbf{3u - 2v}$ and $\mathbf{u}$.

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors $|\mathbf{u}| = 3$ and $|\mathbf{v}| = 6$ and $\theta = \frac{\pi}{3}$.

a. Find $|3\mathbf{u} - 2\mathbf{v}|$.

b. Find the cosine of angle between $\mathbf{3u - 2v}$ and $\mathbf{u}$.

Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors $|\mathbf{u}| = 3$ and $|\mathbf{v}| = 6$ and $\theta = \frac{\pi}{3}$.

a. Find $|3\mathbf{u} - 2\mathbf{v}|$.

b. Find the cosine of angle between $\mathbf{3u - 2v}$ and $\mathbf{u}$.

16. Let $\vec{a}$ = 2i + (x-1)j + k and $\vec{c}$ = i - j + yk be vectors. If $\vec{a}$. $\vec{c}$ = 0 and |a| =3 Then the possible value of y? ***(UEE 2005)***

A. -4 B. -1 C. 3 D. 4

17. Which one of the following is necessarily true? **(UEE 2005)**

A. If $|\vec{A}| = |\vec{B}|$, then $\vec{A} = \vec{B}$

B. If $|k\vec{A}| = k|\vec{A}|$ for any real number K

C. If $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A}.\vec{B} = 0$

D. If $\vec{U}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A}.\vec{U} = |\vec{A}|$

18. Suppose p(1,2,1) and Q(1,0,2) are points in space and $\vec{A} = \overrightarrow{PQ}$. If $\vec{B}$ is parallel to $\overrightarrow{PQ}$ and $\vec{A} \cdot \vec{B}$ = -10 then which of the following is true? ***(UEE 2005)***

A. $\vec{A}$ and $\vec{B}$ has the same direction.

B. $|\vec{B}| = 10|\vec{A}|$

C. $|\vec{B}| = 1/10\ |\vec{A}|$

D. $|\vec{B}| = 2\ |\vec{A}|$

19. If $\vec{A}$ is perpendicular to $\vec{B}$. what is the cosine of the angle between $\vec{A}$ and $\vec{A}$ - $\vec{B}$ ***(UEE 2005)***

A. $\frac{|\vec{A}-\vec{B}|}{|\vec{A}|}$

B. $\frac{|\vec{A}|}{|\vec{A}-\vec{B}|}$

C. $\frac{|\vec{A}-\vec{B}|}{|\vec{B}|}$

D. $\frac{|\vec{B}|}{|\vec{A}-\vec{B}|}$

20. Suppose $\vec{A}$ = 3i – 4j and **B** is a vector in the xy- plane such that the angle between A and B is $\frac{\pi}{3}$. If $\vec{U}$ is the unit vector in the direction of B, then $\vec{A}$ **.(**$\vec{A}$ **-2u) is equal to** ? ***(UEE 2006)***

A. 20 B. 5 C. 15 D. 30

21. Suppose **A** = 2i – j + 2k and **B** is a vector in space such that $|\vec{B}| = \vec{A} \cdot \vec{B}$. If $\vec{u}$ is the unit vector in the direction of $\vec{B}$, then $|\vec{A} + \vec{U}|^2$ is equal to:***(UEE 2006)***

A. 16 B. 12 C. 10 D. 14

22. If **A** = 4i – 3j and **U** is a unit vector such that $|\vec{A} + \vec{U}|^2 = 27$, then the cosine of the angle between $\vec{A}$ and $\vec{U}$ is equal to:***(UEE 2007)***

A. 0.1 B. 0.2 C. 0.3 D. 0.4

23. Let the angle between $\vec{V}$ = -2i – j + 2k and $\overrightarrow{PQ}$ **be 60° , where P and Q are points in space. If** $\vec{V}$ **.** $\overrightarrow{PQ}$ **= 2, then what is the distance between P and Q?*(UEE 2008)***

A. 3/4 B. 4/5 C. 4/3 D. 5/4

24. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$ and $|\vec{A}| = 4$, then $\vec{A} \cdot \vec{u}$ **is equal to ?** ***(UEE 2008)***

A. $\frac{1}{4}$ B. 4 C. $\frac{1}{4}$ D. 2

25. What is the value k, for which the two vectors $\vec{u} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2k \\ -5 \\ 4 \end{pmatrix}$ are perpendicular? ***(UEE 2008)***

A. 4 B. -4 C. 3 D. -3

26. For two nonzero vectors $\vec{a}$ and $\vec{b}$ if |a + b|= |a|, then which of the following is true? ***(UEE 2009)***

A. $2\vec{a}.\vec{b} = \vec{b}.\vec{b}$

B. $2\vec{a} + \vec{b}$ and $\vec{b}$ are perpendicular

C. $2\vec{a} + \vec{b}$ and $2\vec{a} + \vec{b}$ are parallel

D. $\vec{a}.\vec{b} - \vec{b}.\vec{b} = 0$

27. Let $\overrightarrow{PQ}$ be a vector with initial point P = (1, 5) and terminal point Q = (4, 0). If $\vec{V}$ = xi+2j is parallel to $\overrightarrow{PQ}$, then what is the value of x? ***(UEE 2009)***

A.-6/5 B. -2/5 C. -3 D. 3

28. Let $\vec{A}$ and $\vec{B}$ be vectors in space such that $\vec{A}$. $\vec{B}$ = -2 and $\vec{B}$= 6i-7j+√15k. If θ is the angle between $\vec{A}$ and $\vec{B}$, then what is the value of | $\vec{A}$ |? ***(UEE 2009)***

A. $\frac{1}{5}\cos\theta$ B. $\frac{1}{5\cos\theta}$ C. $-\frac{1}{5}\cos\theta$ D. $\frac{-1}{5\cos\theta}$

29. If the dot products of a vector $\vec{A}$ with the vectors i-j+k, 2i+j-3k and i+j+k are 4, 0 and 2, respectively, what is $\vec{A}$? ***(UEE 2010)***

A. $\vec{A}$ = (2,1,1) B. $\vec{A}$ = (-2,1,-1) C. $\vec{A}$ = (-2,-1,1) D. $\vec{A}$ = (2,-1,1)

30. If the angle between the vectors $\vec{A}$ = (2, -1,1) and $\vec{B}$ = (1,1,α) is $\frac{\pi}{3}$ then what is the value of α ? ***(UEE 2011)***

A. 1 B. 2 C. -1 D. -2

31. Which of the following is NOT true about Scalar product of vectors? *(UEE 2012/13)*

A. The scalar product of vector is distributive over addition of vectors.

B. The scalar product of perpendicular vectors is zero.

C. The scalar product is commutative.

D. The scalar product of non-zero parallel vectors is zero.

32. Which one of the following is the scalar (dot) product of the vectors (2,-4, 6) and (-2, -1, 0)? ***(UEE 2013/14)***

A. 8 B. 6 C. 0 D. -8

33. Let u= (3,-5) and v = (-1,-1). What is u . v? ***(UEE 2014)***

A. -8 B. 2 C.-4 D. 4

Thank You !!!!!