EQUILIBRIUM UNIT SEVEN

INTRODUCTION

Equilibrium of one object is defined by the balance of forces and moments. For a particle, force balance tells all. But for an extended object, moment balance is also useful. There are special shortcuts for an objects that has exactly two or exactly three forces acting on it. If friction forces are relevant the possibility of motion needs to be taken into account. Many structures in our everyday lives such as ladders, hanging signs, seesaws, buildings, bridges, etc. are rigid frameworks which have a variety of forces acting on them. The fact that these extended bodies remain stationary, both translational and rotational, tells us something about the sum of the forces acting on them. In this unit, we will see how moments, torque and equilibrium are linked.

7.1 Equilibrium of a particle

By the end of this section you should be able to:
•Find the resultant of two or more concurrent forces acting at a point.
•Define the term equilibrium.
•State the first condition of equilibrium.
•Identify and label the forces and torques acting in problems related to equilibrium.
•Apply the first condition of equilibrium to solve equilibrium problems.

A body is said to be in equilibrium when it is at rest (not just for an instant, but permanently or at least for a finite period of time) or moving with constant velocity. According to Newton's first law, there is no force on a body in equilibrium.

The simplest example of equilibrium would be a particle in the outer reaches of the solar system, sufficiently far from the sun and the planets so as to be subject to negligible gravitational forces. This example is not very interesting since no forces act on the particle. The more interesting examples of equilibrium, as encountered in everyday life, are situations in which the net (or "resultant") force on a body is zero, but several forces are acting on the body; thus the equilibrium of the body results from the fact that the vector sum of all the forces acting on the body is zero.

- $\sum_{i=1}^{n} \vec{F}_x = 0$
- $\sum_{i=1}^{n} \vec{F}_y = 0$
- $\sum_{i=1}^{n} \vec{F}_z = 0$

Consider a situation where three forces are acting on a body, as shown in Figure 7.1. The forces are concurrent, but is the body in equilibrium? For the body to be in equilibrium the net force must be zero.

We can resolve the vectors into their components and then add the components. If the force to the left is equal to the force to the right, and the force up is equal to the force down, then the net force is zero and the body is in equilibrium.



Figure 7.1: A set of forces acting concurrently on an object. All force lines of action intersect at one point.

We can also use vector addition to find the net force. In Unit 2, you learnt how to add vectors by drawing them head to tail. If we draw the force vectors in Figure 7.1 head to tail and they form a triangle, then the net force is zero, as shown in Figure 7.2. This also applies to any number of concurrent forces – if they form a closed shape when drawn, the net force is zero.

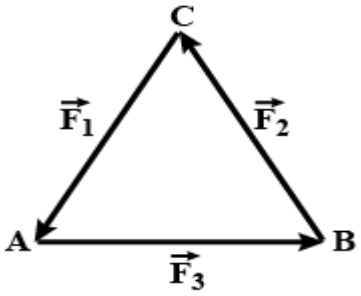


Figure 7.2 When a set of concurrent forces form a closed shape, the net force is zero.

In three dimension of space if the forces acting on the particle are resolved into their respective i, j, k components the equilibrium equation is written as,

$$\sum_{i=1}^{n} \vec{F}_x + \sum_{i=1}^{n} \vec{F}_y + \sum_{i=1}^{n} \vec{F}_z = 0$$

The equation for equilibrium of a particle in space is,

$$\sum_{i=1}^{n} \vec{F}_x = 0; \qquad \sum_{i=1}^{n} \vec{F}_y = 0; \qquad \sum_{i=1}^{n} \vec{F}_z = 0$$

Worked Example: 7.1

Three concurrent forces are shown in Figure 7.3.

Are they in equilibrium?

If the forces are in equilibrium, then $\Sigma F_x = 0$ and $\Sigma F_y = 0$

First resolve each of the forces into its vertical and horizontal components:

	horizontal	vertical
$\mathbf{F_A}$	5.39N(cos21.8°)=5.00N	-5.39N(sin21.8°)=-2.00N
$\mathbf{F_B}$	-6.32N(cos18.4°)=-6.00	-6.32N(sin18.4°)=-2.00N
$\mathbf{F_C}$	+4.21N(cos76°)=1.00N	4.21sin76°=4.00N

Add the horizontal components: 5.00 N + – 6.00 N + 1.00 N = 0 N

Add the vertical components: –2.00 N + –2.00 N + 4.00 N = 0 N

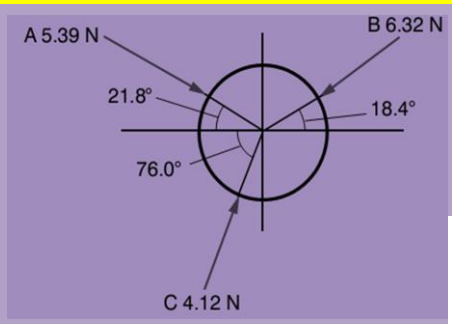


Figure 7.3

The net force both horizontally and vertically is zero so the body is in equilibrium.

Activity 7.1

You are going to investigate forces in equilibrium. Attach three newton meters together. Attach the other end of each newton meter to a heavy block (Figure 7.4).
Record the magnitude of the three forces and the angles of the forces.
Move the blocks and measure the forces and angles again.
Repeat a couple more times.
Resolve each force into its components and add them up. Do they show that the system is in equilibrium?

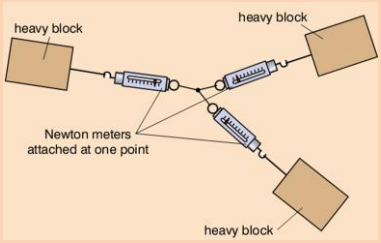


Figure 7.4

Worked Example: 7.2

Five concurrent forces are shown in Figure 7.5.
Are they in equilibrium? (use $sin30° = 0.5, cos30° = 0.86$, $sin45° = cos45° = 0.71$ If the forces are in equilibrium, then ΣFx = 0 and ΣFy = 0
First resolve each of the forces into its vertical and horizontal components:

horizontal	vertical
20N($cos30°$)=17.3N	20N($sin30°$)=10N
25N($cos0°$)=0	25N($sin90°$)=25N
-30N($cos45°$)=-25.98	30N $sin45°$=21.2N
-35N($cos40°$)=-26.8N	35N($sin40°$)=-22.5N

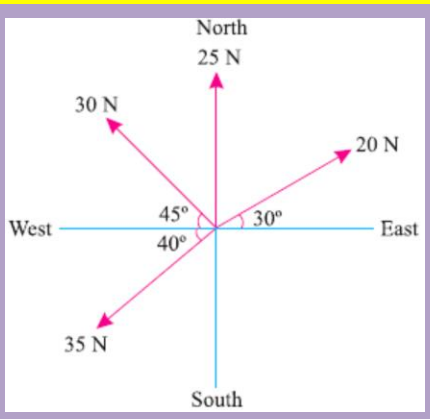


Figure 7.5

Add the horizontal components: 17.3 N + 0-25.98-26.8N=-35.48N
Add the vertical components: 10.00N+25.00N+21.2N-22.5N=33.5N
The net force both horizontally and vertically is not zero so the body is not in equilibrium.

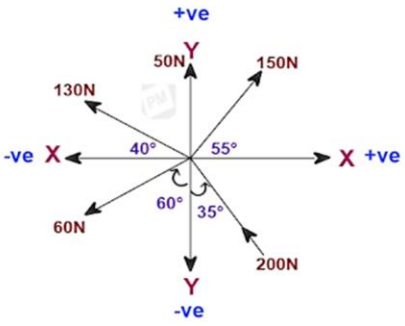
Summary

In this section you have learnt that:

- For a set of concurrent forces acting on a body, if the net force is zero, the body is in equilibrium and this is the first condition of equilibrium.
- Vector addition can be used to find the sum of the forces.

Review questions

1. Find if these set of concurrent forces are in equilibrium
2. by drawing the forces head to tail.
   a) A $\begin{bmatrix}4\\5\end{bmatrix}$N, B $\begin{bmatrix}-3\\-2\end{bmatrix}$N and C $\begin{bmatrix}-2\\-3\end{bmatrix}$N
   b) D $\begin{bmatrix}-6\\1\end{bmatrix}$] N, E $\begin{bmatrix}2\\3\end{bmatrix}$N and F $\begin{bmatrix}4\\-4\end{bmatrix}$ N
3. Are these concurrent vectors in equilibrium?
   Use vector addition to check.
   G $\begin{bmatrix}-6\\2\end{bmatrix}$] N, H $\begin{bmatrix}1\\1\end{bmatrix}$N and I $\begin{bmatrix}5\\-3\end{bmatrix}$ N
4. Are these five concurrent vectors in figure 7.6 is in equilibrium?
   Find their horizontal and vertical components to check.



5. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure 7.7. If the system is in equilibrium determine the forces in the strings AC and BC.

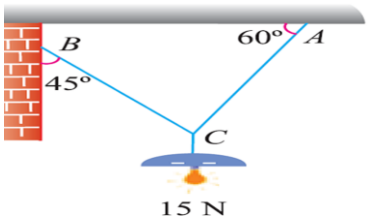


Figure 7.7

6. The system in Fig. 7.8 is in equilibrium with the string in the center exactly horizontal. Find (a) tension T1, (b) tension T2, (c) tension T3 and (d) angle θ

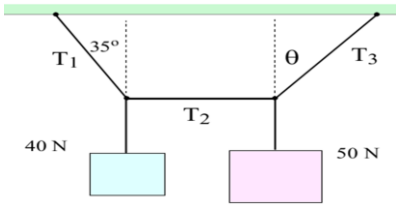


Figure 7.8

7.2 Moment of force or torque

By the end of this section you should be able to:
•Distinguish between coplanar and concurrent forces.
•Draw free body diagrams to show all the forces acting.

In Section 7.1 we looked at concurrent forces. But what happens when a set of forces is not concurrent? Look at the forces shown in Figure 7.9. Forces A, B and C are acting on a rigid body. Forces B and C are equal and opposite. As they are acting on the same point, they balance out. Force A is acting at a point 20 cm from the other two. As the body is rigid, this force will try to turn the body anti-clockwise about the point X. The turning effect is known as a **moment** or torque.

The forces shown in Figure 7.9 are all in the plane of the paper –they are said to be **coplanar**.

You have already been introduced to moments and torque. In Unit 4 you looked at the moments around a see-saw and in Unit 6 you looked at torque about a point. Moments and torque are essentially the same thing, but moments deal with statics and torque is to do with rotational motion.

If exactly three forces act on a body (2D and 3D) the body is in equilibrium only if

1. the three force vectors are coplanar,

2. and either
   a) have lines of action which intersect at a single point (i.e., they are concurrent), or
   b) they are parallel.

Worked example 7.2

What is the moment about point X in Figure 7.9?
Forces B and C are equal and opposite, so we do not need to consider them.
Moment = force ×perpendicular distance
= 20 N × 0.2 m = 4 N m

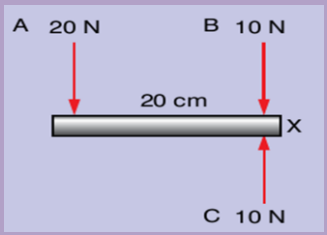


Figure 7.9 Forces acting on a rod

When we analyse moments, we take the moments about an axis, both clockwise and anticlockwise. Anticlockwise is taken to be positive and clockwise negative.
The axis is perpendicular to the plane of the paper. The moment also depends on which axis you choose to take moments about.

Worked example 7.3

What are the moments about P and Q in Figure 7.10?
Use the equation: $\tau = Fd$
Where τ is the momentum of force, F is the component of the force perpendicular to the displacement and d is the displacement from the axis of rotation.
Clockwise moment about P = 8 N × 0.1 m = 0.8 N m
anti-clockwise moment about P = 5 N × 0.05 cm = 0.25 N m
net moment = 0.25 N m – 0.8 N m = –0.55 N m, that is clockwise about P
Anti-clockwise moment about Q = 5 N × 0.25 m + 8 N × 0.1 m
= 1.25 N m + 0. 8 N m = 2.05 N m
There is no clockwise moment about Q, so the net moment is +2.05 N m, that is, anticlockwise about Q.

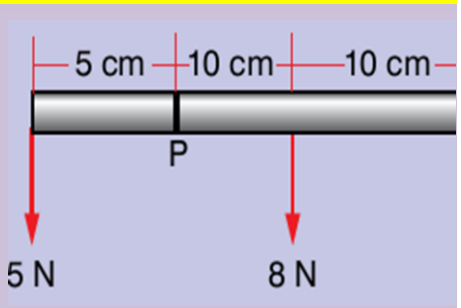


Figure 7.10 Forces acting on a rod

Sometimes forces are not perpendicular to the bar. You should remember from Unit 6 that the moment or torque is given by the vector product of the force and displacement from the axis of rotation:
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = rF\sin\theta\hat{r}$
where θ is the angle between **r** and **F**
This means that we do not need to consider any component of the force that acts in the same direction as the displacement from the axis of rotation. The angle between **r** and **F** is zero, so:
$\tau = rF \sin 0° = rF \times 0 = 0$

Activity 7.3

You are going to investigate the equilibrium of a stack of objects.
Make a stack of objects such as coins or other small flat objects. What happens to the stability of the pile as you increase the number of objects? Explain this in terms of the centre of mass and moments.

Worked example 7.4

What are the moments about R and S in Figure 7.11?

Use the equation $\tau = Fd \sin\theta$

where τ is the moment, F is the force, d is the displacement from the axis of rotation and θ is the angle between the force and the displacement.

Taking moments about R:

Clockwise moment = 0.1m×10N×sin45°+0.2m×15N×sin60°

= 0.71 N m + 2.60 N m = 3.31 N m

Anticlockwise moment = 0.2 m × 12 N × sin 70° = 2.26 N m

Net moment = anticlockwise moment – clockwise moment

=2.26 N m – 3.31 N m

= –0.05 N m, that is clockwise about R

Taking moments about S:

Clockwise moment = 0.45m×10N×sin45°+0.15m×12N×sin70°

= 3.18 N m + 1.69 N m = 4.87 N m

Anticlockwise moment = 0.15 m × 15 N × sin 60° = 1.95 N m

Net moment = 1.95 N m – 4.87 N m = –2.92 N m, that is anticlockwise about S.

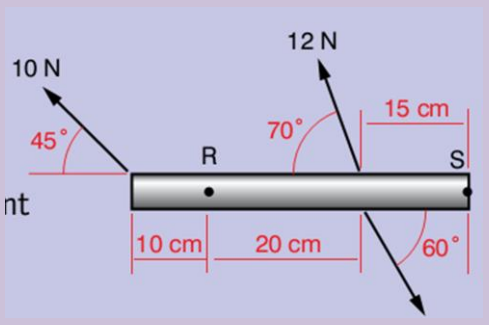


Figure 7.11 Choosing the right axis can help you when solving

When solving problems, you can choose which axis you take moments around. Sometimes you need to choose your axes carefully – it can help to eliminate forces that you do not know.

For example, look at Figure 7.9. You could take moments about axes A, B or C. If you take moments about points A and C, you need to know force *F*, or be able to calculate it. If you take moments about point B, you do not need to know force *F* because it is over the pivot, and hence does not have any turning effect.

Sometime you do need to choose the pivot to include the unknown force. For example, if you were asked to find force *F* from the information you are given, then you need to make sure that you choose a pivot where the moments do involve *F*.

Worked example 7.5

Three forces act on a bar as shown in Figure 7.12. There is a net torque of 2.8 N m anti-clockwise around the axis at A. What is the size of the force at B?

As the axis is at A, the force of 5 N does not have any effect.

Use the equation: $\tau = Fd$

where τ is the momentum, F is the component of the force perpendicular to the displacement and d is the displacement from the axis of rotation.

Moment about A = 10 N × 0.11 m – F N × 0.05 m = 2.8 N m

So F × 0.05 m = 1.1 N m – 2.8 N m

F = – 1.7 N m/0.05 m = –34 N

The negative sign indicates that the force is in the clockwise direction, which means that it would be 34 N downwards.

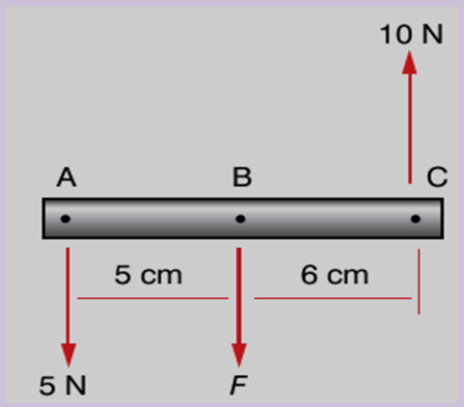


Figure 7.12 Choosing the right axis can help you when solving

Summary

In this section you have learnt that:

• Coplanar forces act in the same plane.

• The moment of a force about an axis is the force multiplied by the perpendicular distance of the force from the axis.

Project work

Research the effects of torque and equilibrium on taps, doors, handlebars and bicycles. Write a report on your findings. You could also research torque in simple electric motors and moving coil meters.

Review questions

1. Two forces act on a bar as shown in Figure 7.13.
   What is the net torque about:
   a) axis A b) axis B?
2. Two forces act on a bar as shown in Figure 7.14.
   a) What is the net torque about:
      i) axis A
      ii) axis B.
   b) What weight would need to be added to A to give a net torque of 3.5 N m anti-clockwise around axis B?

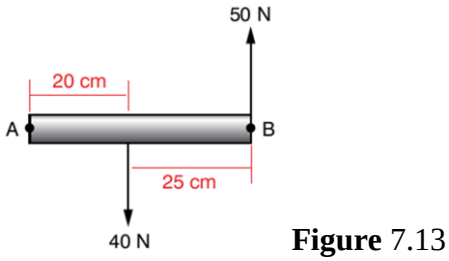


Figure 7.13

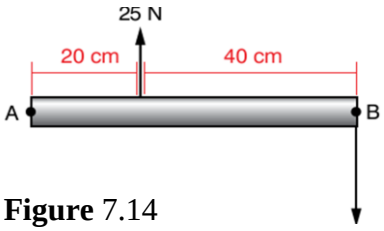


Figure 7.14

3. Three forces act on a bar as shown in Figure 7.15.
What is the net torque about:
a) axis A b) axis B?

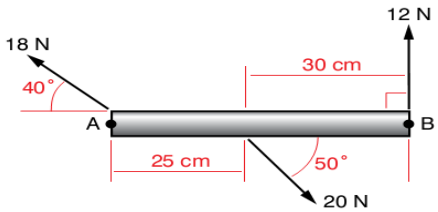


Figure 7.15.

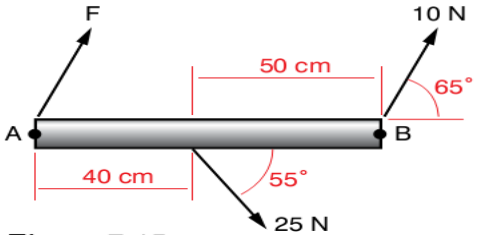


Figure 7.16

4. Three forces act on a bar as shown in Figure 7.16.
a) Calculate the net torque about axis A.
b) The net torque about axis B is zero. Calculate the vertical component of force F.
c) Do you have enough information to calculate the magnitude of force F? Explain your answer.

7.3 Conditions for Equilibrium of a Rigid Object

By the end of this section you should be able to:

• Differentiate static equilibrium from dynamic equilibrium.

• State the second condition for equilibrium.

• Verify the second condition for equilibrium is valid about any arbitrary axis of rotation.

• Describe the difference among the terms stable, unstable and neutral equilibrium.
• Explain why objects are stable, unstable and neutral.
• Explain methods of checking stability, instability and neutrality of rigid bodies.
• Describe the equilibrium conditions for a body acted on by coplanar forces.
• Verify by experiment the conditions necessary for the equilibrium of a set of non-concurrent forces.
• State the conditions for rotational equilibrium.

For a rigid object which is **not** moving at all we have the following conditions:

• The (vector) sum of the external forces on the rigid object must equal zero:

$$\sum \vec{F} = 0$$

When this condition is satisfied we say that the object is in translational equilibrium. (It really only tells us that a Centre of mass is zero, but of course that includes the case where the object is motionless.)

• The sum of the external torques on the rigid object must equal zero.

$$\sum \vec{\tau} = \sum (\vec{r} \times \vec{F}) = 0$$

When this condition is satisfied we say that the object is in rotational equilibrium. (It really only tells us that $\vec{\alpha}$ about the given axis is zero, but —again— that includes the case where the object is motionless.)

When both $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ are satisfied we say that the object is in **static equilibrium**.

This will only tell you that the body is in equilibrium – it could be moving at a steady velocity or it could be at rest. If a body is not moving and there are no net forces or torque on the body, it is in **static equilibrium**.

If the body is moving and there are no net forces or no net torque acting on the body, there is no net acceleration. The body will continue to move at the same velocity. The body is in **dynamic equilibrium**.

For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.

The necessary and sufficient condition for the static equilibrium of a body are that the resultant force and couple from all external forces form a system equivalent to zero,

$$\sum \vec{F} = 0 \text{ and } \sum \vec{\tau} = \sum (\vec{r} \times \vec{F}) = 0$$

Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\sum \vec{F}_x = 0 \, , \sum \vec{F}_y = 0 \quad \text{and} \sum \vec{F}_z = 0$$
$$\sum \vec{\tau}_x = 0 \, , \sum \vec{\tau}_y = 0 \quad \text{and} \sum \vec{\tau}_z = 0$$

If we know the forces and distances acting on a system, we can take moments about any point to see if the system is in equilibrium.

Two Important Facts for Working Statics Problems

i. The force of gravity acts on all massive objects in our statics problems; its acts on all the individual mass points of the object. One can show that for the purposes of computing the forces and torques on rigid objects in statics problems we can treat the mass of the entire object as being

concentrated at its center of mass; that is, for an object of mass M we can treat gravity as exerting a force Mg downward at the center of mass.
(This result depends on the fact that the acceleration of gravity, g is usually constant over the volume of the object. Otherwise it is not true.)

ii. While there is only one way to write the conditions for the forces on a rigid object summing to zero, we have a choice in the way we write the equation for the total torque.
Eq. 3.3 does not specify the choice of the axis for calculating the torque. In general it matters a great deal which axis we pick! But when the sum of torques about any one axis is zero and the sum of forces is zero (translational equilibrium) then the sum of torques about any axis will give zero; so for statics problems we are free to pick the most convenient axis for computing P. Often this will be the point on the object where several unknown forces are acting, so that the resulting set of equations will be simpler to solve.

Strategy for solving problems in static equilibrium:

- Determine all the forces that are acting on the rigid body. They will come from the other objects with which the body is in contact (supports, walls, floors, weights resting on them) as well as gravity,
- Draw a diagram and put in all the information you have about these forces: The points on the body at which they act, their magnitudes (if known), their directions (if known).
- Write down the equations for static equilibrium. For the torque equation you will have a choice of where to put the axis; in making your choice think of which point would make the resulting equations the simplest.

• Solve the equations! (That's not physics. . . that's math.) If the problem is well–posed you will not have too many or too few equations to find all the unknowns.

Worked example 7.6

Three forces act on a bar as shown in Figure 7.17.
Is the bar in equilibrium?
Work out the net force.
The net force is 80 N–50N–30 N = 0 N
Work out the net torque.
Use $\tau = rF$

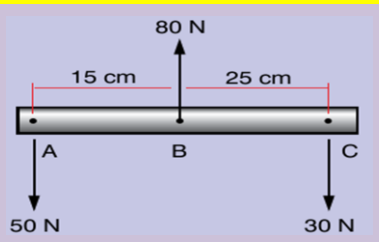


Figure 7.17

Taking moments about A: 80 N × 0.15 m – 30 N × 0.4 m =12 N m – 12 N m = 0 N m
Taking moments about B anticlockwise: 50 N m × 0.15 m =7.5 N m
Taking moments about B clockwise: 30 N × 0.25 m = 7.5 N m
So the net torque about B is zero.
Taking moments about C: 50 N × 0.4 m – 80 N × 0.25 m =20 N m – 20 N m = 0 N.
The net torque is zero.
As the net force and net torque is zero, the system is in equilibrium.

If a system of coplanar forces is in equilibrium, we can take moments about any point and find that the net torque is zero.

Remember that when you take moments about a point, you do not include any forces acting at that point. We could also take moments from somewhere that is not on the bar. For example, if we take moments about a point that is 10 cm to the left of A, the moments are:
80N×0.25m–50N×0.1m–30N×0.5m=20Nm–5Nm–15Nm=0 Nm
The net torque is still zero.

The equilibrium of particle can be of three types :

stable equilibrium	**unstable equilibrium**	**neutral equilibrium.**
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

The bottles in Figure 7.18 show three types of equilibrium. When a bottle is standing upright, it is in stable equilibrium. As long as the centre of mass stays inside a point vertically above the base, it will not fall over. If a small moment acts on the top of the bottle and pushes is slightly, when the force is removed the bottle will fall back into its stable equilibrium position.
When a bottle is lying on its side, it is in neutral equilibrium. If a small moment acts perpendicular to the top surface of the bottle, it will roll. When the moment is removed, the bottle will come to rest in a new position because of the force of friction between the bottle and the surface, still lying on its side. There is an infinite number of positions that the bottle could be in.
When a bottle is standing on its neck, it is in unstable equilibrium.
A small torque acting on the bottle will cause it to fall over because the centre of mass does not have to move very far until it is over a point which is outside what is now the base of the bottle.



Figure 7.18 Stable, neutral and unstable equilibrium of a bottle

Activity 7.6

Figure 7.19 shows a bar that is free to rotate about a hinge at its left-hand end. It is held horizontally by a force.

The torque due to the force of gravity is balanced by the force *F*.

Consider what happens to the force *F* and the force H_{vert} as the force *F* is moved along the rod in Figure 7.19a.

In Figure 7.19b, the force is fixed at the opposite end of the rod to the hinge. Consider what happens to the forces F, H_{vert} and H_{hor} as the angle of *F* is varied.

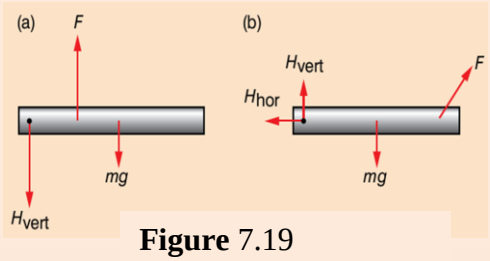


Figure 7.19

Worked example 7.7

A ladder rests against a wall at an angle of 60° to the horizontal. The ladder is 8 m long and has a mass of 35 kg. The wall is considered to be frictionless.

Find the force that the floor and wall exert against the ladder.

There is no vertical component of force from the wall because it is frictionless, so the force from the wall is horizontal. There are two components to the force from the floor: a horizontal force from friction with the floor and a vertical normal component.

Draw a diagram to show the forces (Figure 7.20).

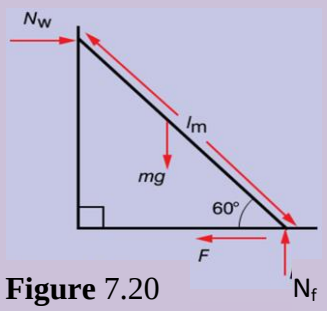


Figure 7.20

As the ladder is in equilibrium, the sum of the forces is zero.

$\Sigma \mathbf{F} = 0$

Vertical component: $N_f - mg = 0$

where N_f is the normal force from the floor.

Horizontal component: $N_w - F = 0$

where N_w is the normal force from the wall.

Similarly, the moments taken around any axis will be zero $\Sigma \boldsymbol{\tau} = 0$.

Use the base of the ladder and taking the anticlockwise direction to be positive:

$mg \times l/2 \times \cos 60° - N_w \times l \times \sin 60° = 0$

From the force equations we can see that $N_f = mg$ and $N_w = F$

Substituting for N_w and rearranging the torque equation gives:

$F = mg \cos 60°/2 \sin 60° = mg/2 \tan 60°$

$= (35 \text{ kg} \times 9.8 \text{ m/s2})/(2 \times 1.732) = 99 \text{ N}$

$N_f = mg = 35 \text{ kg} \times 9.8 \text{ m/s2} = 343 \text{ N}$

So total force from the floor is [–99] N

Or: $F = \sqrt{(992 + 3432)}$ at an angle of $\tan^{-1}(343/99)$

= 357 N at an angle of 73.9° to the horizontal.

Rotational equilibrium

In Unit 6 you learnt that you can apply the equations and laws of linear motion to rotational motion. In the same way, the conditions of equilibrium can be applied to rotational motion. If a body is in rotational equilibrium, the sum of all the external torques acting on the body must be zero

Worked example 7.8

Is the body shown in Figure 7.21 in rotational equilibrium?
The anticlockwise torque is *Fx*.
The clockwise torque is *Fx*.
So the net torque is 0 and the
body is in equilibrium.

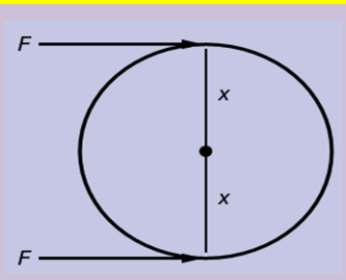


Figure 7.21

Activity 7.7

Design a simple children's toy that will not fall over. When you push this toy over, it should bounce back up to the vertical.
What are the characteristics it needs for it to bounce back up when it is pushed over?
You will need to use what you have learnt about equilibria, moments and centre of mass.

Summary

In this section you have learnt that:
• The conditions for a system of coplanar forces to be in equilibrium are that the net force and net torque must be zero.
$\Sigma \mathbf{F} = 0$ and $\Sigma \boldsymbol{\tau} = 0$
• When a system is in equilibrium, it does not matter which axis you take moments about.
• In rotational equilibrium, the sum of all the external torques acting on the body must be zero.

Review questions

1. A bar has a 20 N weight at one end, as shown in Figure 7.22. You have a weight of 15 N to hang somewhere on the bar so that the bar is in equilibrium. Where would you hang the 15 N weight on each of these bars? Consider the bar to have no mass.

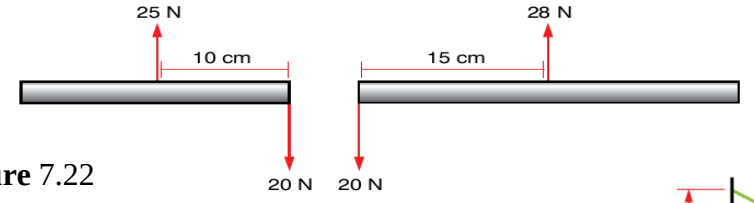


Figure 7.22

2. Figure 7.23 shows a sign that is hanging from a horizontal bar. The bar is fixed to the wall at P, and is free to rotate.
There is a wire strung from the outer end of the bar to a point 60 cm above the pivot. The sign and the bar have a mass of 2 kg. Assume that the wire is massless. The system is in equilibrium. What is the size of the force in the wire?

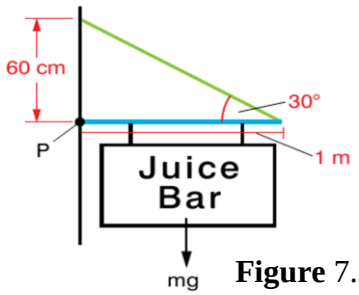


Figure 7.23

3. A ladder, of length 3 m and mass 20 kg, leans against a smooth, vertical wall wall so that the angle between the horizontal ground and the ladder is 60° Find the magnitude of the friction and normal forces that act on the ladder if it is in equilibrium.
4. A uniform rod of length 2 m and mass 5 kg is connected to a vertical wall by a smooth hinge at A and a wire CB, as shown in Figure 7.24. A 10 kg mass is attached to D. Find:
   a) the tension in the wire
   b) the magnitude of the force at the hinge A. (Hint: think about the position of the centre of mass.)
5. The system in Fig. 7.25 is in equilibrium. A mass of 225 kg hangs from the end of the uniform strut whose mass is 45.0kg. Find (a) the tension T in the cable and the (b) horizontal and (c) vertical force components exerted on the strut by

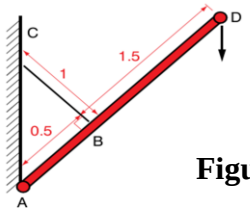


Figure 7.24

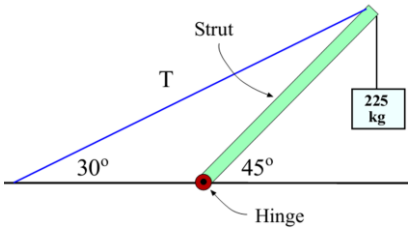


Figure 7.25

7.4 Couples

•By the end of this section you should be able to:
•Define the term couples.
•Describe the rotational effects of couples on the rigid body.
•Solve problems involving the equilibrium of coplanar forces.

A pair of two equal and unlike parallel forces (*i.e.* forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) , as shown in Figure 7.26 is known as a couple. As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

Arm of a couple

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple as shown in Figure.

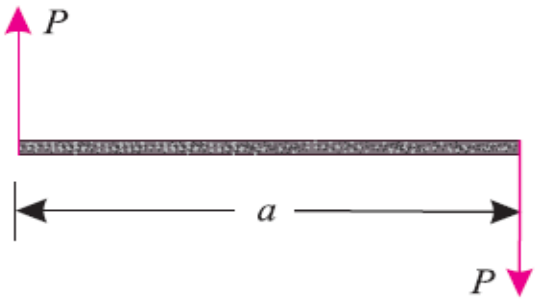


Figure 7.26

MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple.

Mathematically:

Moment of a couple = P × a

Where, *P = Magnitude of the force, and a = Arm of the couple.*

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts: ***Classification of Couples***

1) Clockwise couple, and

2) Anticlockwise couple

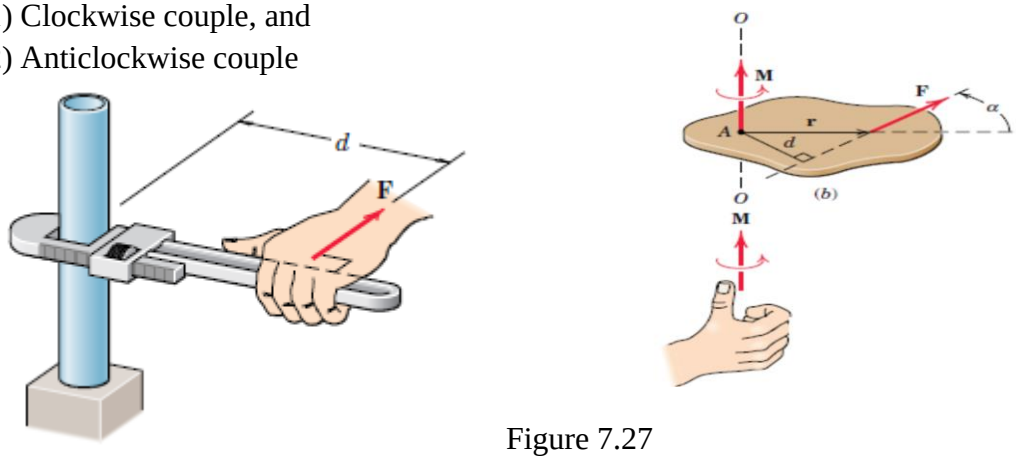
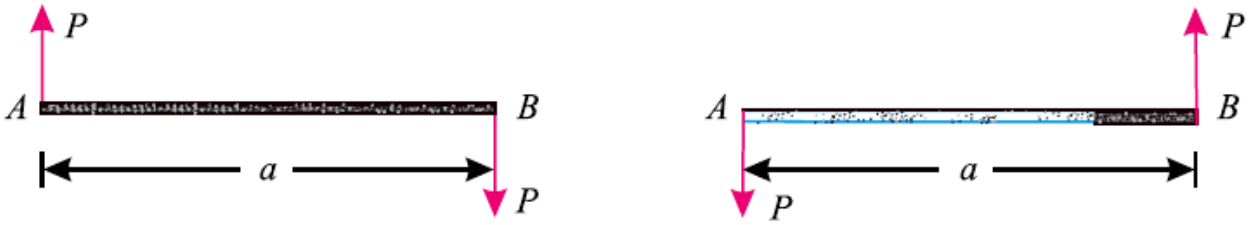


Figure 7.27

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a **clockwise couple** as shown in Figure (a). Such a couple is also called positive couple.

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an **anticlockwise couple** as shown in Figure (b). Such a couple is also called a negative couple.



a) Clockwise couple b) anticlockwise couple

Fig. 7.28 Classification of Couples

Worked example 7.9

Let us consider the moments about three separate axes, as shown in Figure 7.29.

about A = $-Fx + F(x + d) = Fd$

Moment about B = $Fy + F(d - y) = Fd$

Moment about C = $-F(d + z) - Fz = Fd$

So we can see (from the worked example) that the moment of a couple is independent of the axis we

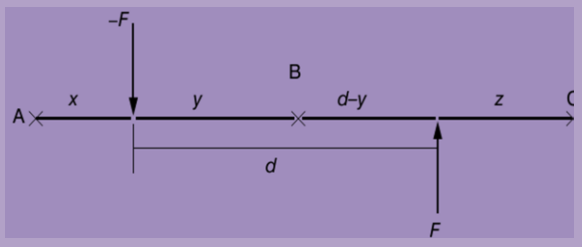


Figure 7.29

take the moment around, as long as the axis is perpendicular to the plane of the couple.
The properties of a couple are:
• the linear resultant of a couple is zero (e.g. F + –F = 0)
• the moment of a couple is not zero and has the same magnitude irrespective of the position of the perpendicular axis chosen.
If a set of coplanar forces satisfies these two conditions, it is said to be a couple.

Worked example 7.10

Two forces of 20 N act as a couple. The forces act at a distance of 10 cm from the centre of rotation. What is the torque produced by the couple?

Forces act at 10 cm from centre of rotation, so they are 20 cm apart.
Torque = Fd = 20 N × 0.2 m
= 4 N m

Summary

In this section you have learnt that:

- A couple is a set of forces with a resultant moment but no net force.
- The torque of a couple is the force multiplied by the distance between the forces.

Review question

1. Calculate the torque for the following couples:
   a. 50 N, 40 cm distance perpendicular to axis of rotation.
   b. 120 N, 5 cm distance perpendicular to axis of rotation.

End of unit questions

1. Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–6.
2. What is torque?
3. a) What are the conditions for equilibrium?
   b) If an object is in equilibrium, is it also in static equilibrium? Explain your answer.
4. A 20 kg box is suspended from the ceiling by a rope that weighs 1 kg. Find the tension at the top of the rope.
5. A particle is acted on by the forces as shown in Figure 7.30. Resolve the forces horizontally and vertically to find the magnitude of the forces P and θ.

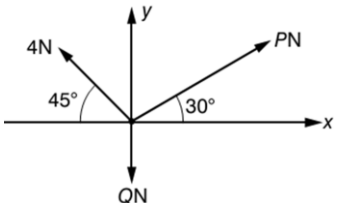
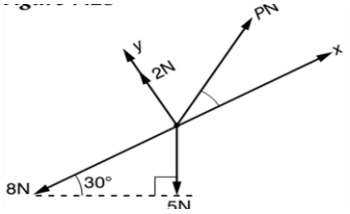


Figure 7.31



6. A particle is at equilibrium on inclined plane under the forces shown in Figure 7.31. Find
   a) the magnitude of the force *P*
   b) the magnitude of the angle θ.
7. A particle of mass 3 kg is held in equilibrium by two light unextensible strings. One string is horizontal, as shown in Figure 7.32. The tension in the horizontal string is P_N and the tension in the other string is θ N. Find
   a) the value of θ
   b) the value of *P*.

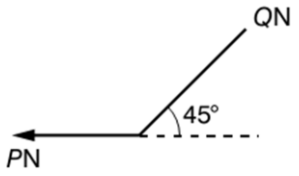


Figure 7.32

8. What are the conditions for there to be no rotation of a body?
9. What are the differences between concurrent and coplanar forces?
10. What are the differences between static and dynamic equilibrium?
11. A car is driving along a road at a speed of 20 m/s.
    a) What forces will be acting on the car?
    b) What additional information do you need to know to show that the car is in a dynamic equilibrium?

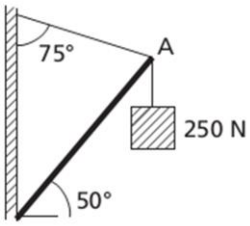


Figure 7.33

12. A rigid rod is hinged to a vertical support in the Figure 7.33 and held at 50° to the horizontal by means of a cable when a weight of 250N is suspended as shown in the figure. Determine the tension in the cable and the compression in the rod, ignoring the weight of the rod.
13. A ladder having a uniform density and a mass m rests against a frictionless vertical wall at an angle of 60°. The lower end rests on a flat surface where the coefficient of static friction is μ_s=0.40. A student with a mass M = 2m attempts to climb the ladder. What fraction of the length L of the ladder will the student have reached when the ladder begins to slip?