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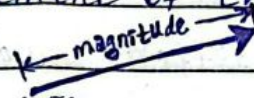
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Grade 11 Unit 5 and Unit 6.

Vector

- Is physical quantity that has both direction & magnitude.
- It describes the movement of the object from one point to another.



- The length of the arrow represents the magnitude of the vector
- The arrow represents the direction of the vector.

Types of Vectors.

There are 10 different types of vectors used in maths & Science.

1. Zero Vector (Null Vector)

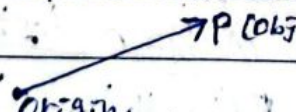
- Is a vector having magnitude zero
- It is represented by a point.
- It has an arbitrary direction or has no specific direction. or indeterminate direction.
- Its initial point of the vector concides with the terminal point

2. Unit Vector

- Is a vector having magnitude of unit length (one unit).

3. Position Vector

- If "O" is taken as reference origin and "P" is an arbitrary point in space, then the Vector $\overrightarrow{OP}$ is called position Vector of the point
- It indicates the position (location) of a point in 3D Cartesian coordinate systems with respect to a reference origin.



4. Co-initial Vectors

- Are vectors having the same starting (initial) point
- 

, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called co-initial vectors

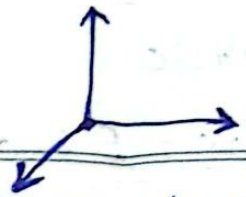
5. Like & Unlike Vectors

- Vectors having the same direction are called Like vectors & Vectors having opposite direction with respect to each other are called Unlike Vectors.

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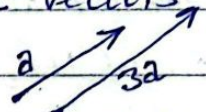
6. Co-planar Vectors



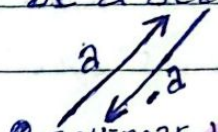
- Are vectors that lie on the same plane (parallel to the same plane) in 3D.

7. Collinear (Parallel) Vectors.

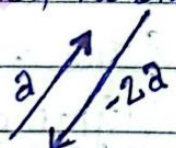
- Are vectors that lie along the same line or parallel lines.
- They may have different magnitude and different direction
- One of the vectors should be a scalar multiple of the other vector



Collinear vectors in the same direction



Collinear vectors in opposite direction



Collinear Vectors in opposite direction

8. Equal Vectors.

- Are vectors having equal magnitude and the same direction.

9. Displacement Vector.

- If a point is displaced from position A to B then the displacement AB represents a vector which is called displacement Vect

10. Negative of a Vector. (a and -a)

- Two vectors having the same magnitude but exactly opposite in direction (They are negative of each other.)

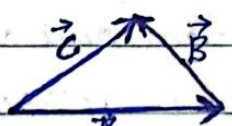
Addition & Substraction of Vectors.

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$



$(A+B) = C$



$(A-B) = C$

$$A+B = |A|^2 + |B|^2 + 2|A||B|\cos\theta$$

$$A-B = |A|^2 + |B|^2 - 2|A||B|\cos\theta$$

$$A \pm B = (A_x \pm B_x)\hat{i} + (A_y \pm B_y)\hat{j}$$

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Multiplication of Vectors

$\vec{A} = A_x i + A_y j + A_z k$
$\vec{B} = B_x i + B_y j + B_z k$

i. Scalar product or inner product or dot product

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta = A_xB_x + A_yB_y + A_zB_z$

- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- $i\cdot i = j\cdot j = k\cdot k = 1$
- $i\cdot j = j\cdot i = j\cdot k = k\cdot i = j\cdot k = k\cdot j = 0$ (b/c $\theta = 90°$)
- $\vec{A}\cdot\vec{B} = |A||B|$, if $\vec{A}$ and $\vec{B}$ are $\parallel$ $(0°)$
- $\vec{A}\cdot\vec{B} = 0$, if $\vec{A}$ and $\vec{B}$ are $\perp$ $(90°)$

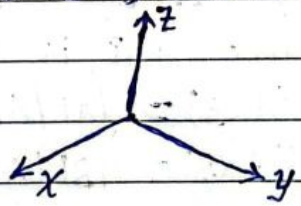
ii. Vector product or cross product

$\vec{A} = A_x i + A_y j + A_z k$
$\vec{B} = B_x i + B_y j + B_z k$

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta$

- $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$
- $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$
- $i\times i = j\times j = k\times k = 0$
- $\vec{A}\times\vec{A} = \vec{B}\times\vec{B} = 0$
- $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|$, if $\vec{A}$ & $\vec{B}$ are $\perp$ $(90°)$
- $\vec{A}\times\vec{B} = 0$, if $\vec{A}$ & $\vec{B}$ are $\parallel$ $(0°)$
- $i\times j = k$ & $j\times i = -k$
- $j\times k = i$ & $k\times j = -i$
- $k\times i = j$ & $i\times k = -j$

$\vec{A} = A_x i + A_y j + A_z k$
$\vec{B} = B_x i + B_y j + B_z k$



$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - B_yA_z)i - (A_xB_z - B_xA_z)j + (A_xB_y - B_xA_y)k$$

- The projection of vector $\vec{A}$ on $\vec{B}$ is
- The component of $\vec{A}$ along $\vec{B}$ is

$= \frac{A\cdot B}{|B|}$ & $\vec{B}$ along $\vec{A}$ is $\frac{A\cdot B}{|A|}$

#1. Find the component of Vector A along Vector B if
$A = 2i + 3j$ and $B = i + j$.

Soln $A\cdot B = (2\times1) + (3\times1) = 5$ // & $|A| = \sqrt{2^2+3^2} = \sqrt{13}$ & $|B| = \sqrt{1^2+1^2} = \sqrt{2}$

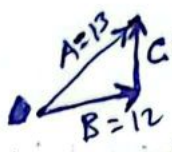
→ The Component of Vector A along Vector $B = \frac{A\cdot B}{|B|} = \frac{5}{\sqrt{2}}$ //

→ The Component of Vector B along $A = \frac{A\cdot B}{|A|} = \frac{5}{\sqrt{13}}$ //

#2. If $\vec{A} = \vec{B} + \vec{C}$ and the values of $A = 13$, $B = 12$ and $C = 5$ then find the angle between AOC

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Sol: 

$A^2 = B^2 + C^2$ → The angle b/n B & C is 90°
$13^2 = 12^2 + 5^2$

The angle between A & C is $\begin{cases} \sin\theta = \frac{12}{13} \Rightarrow \theta = \sin^{-1}\left(\frac{12}{13}\right) \text{ or} \\ \cos\theta = \frac{5}{13} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{13}\right) \\ \tan\theta = \frac{12}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{12}{5}\right) \end{cases}$

#3. If the scalar and Vector products of two Vectors A and B are equal in magnitude then, the angle between A & B is equals to?

Sol: Scalar product = Vector product

$A \cdot B = A \times B$

$|A||B|\cos\theta = |A||B|\sin\theta$ (divide both sides by $|A||B|$)

$\Rightarrow \cos\theta = \sin\theta \Rightarrow 1 = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1) = 45°$

(divide both sides by $\cos\theta$)

#4. If θ is the angle between $\vec{A}$ and $\vec{B}$, then what will be $(B \times A) \cdot A$ and $(B \times A) \cdot B$ or

Sol: $(\vec{B} \times \vec{A}) = -\vec{A} \times \vec{B}$ → differ in direction.

Since $A \times B$ is perpendicular for both $\vec{A}$ and $\vec{B}$

$A \cdot (A \times B) = |A||A \times B| \cos 90° = 0$

$B \cdot (A \times B) = |B||A \times B| \cos 90° = 0$

So, $(B \times A) \cdot A = (B \times A) \cdot B = 0$ //

#5. If $|\vec{A}| = 3$ unit towards east and $|\vec{B}| = 4$ unit towards North find the ratio of $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$.

A. 1:1 B. 1:0 C. 0:12 D. 12:0

Sol: The angle b/n North and east is 90°

$\vec{A} \cdot \vec{B} = |A||B| \cos\theta = 3 \times 4 \cos 90° = 0$

$\vec{A} \times \vec{B} = |A||B| \sin\theta = 3 \times 4 \sin 90° = 12 \Rightarrow A \cdot B : A \times B = 0 : 12$

#6. If $\vec{a} = i - 7j + 7k$ and $\vec{b} = 3i - 2j + 2k$, then which one of the following Vector is (Unit vector) perpendicular to both $\vec{a}$ and $\vec{b}$

a. $\frac{i}{\sqrt{3}} + \frac{j}{\sqrt{3}} + \frac{k}{\sqrt{3}}$

b. $\frac{i}{\sqrt{2}} + \frac{j}{\sqrt{2}}$

c. $\frac{j}{\sqrt{2}} + \frac{k}{\sqrt{2}}$

d. $\frac{j}{\sqrt{2}} + \frac{k}{\sqrt{2}}$

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Solution The vector which is perpendicular to both $\vec{a}$ & $\vec{b}$ is $\vec{a}\times\vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = (-7\times2 - 7\times-2)i - (1\times2 - 7\times3)j + (1\times2 - 3\times-7)k$$

$$(-14+14)i - (-19)j + (19)k$$

$$0i + 19j + 19k$$

$\vec{a}\times\vec{b} = 19j + 19k = 19(j+k)$ → This perpendicular to both $\vec{a}$ & $\vec{b}$

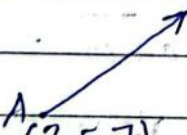
- a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ = $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$|\vec{a}\times\vec{b}| = \sqrt{19^2+19^2} = \sqrt{2\cdot(19)^2} = 19\sqrt{2}$$

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{19j+19k}{19\sqrt{2}} = \frac{j}{\sqrt{2}} + \frac{k}{\sqrt{2}}$$ → This is unit vector ⊥ to both $\vec{a}$ & $\vec{b}$

#7. If A(2,5,7) and B(5,10,6) are the initial and final point of the vector find the component of the vector in coordinate and column form.

Soln



$\vec{AB} = (B_x - A_x, B_y - A_y, B_z - A_z)$

$\vec{AB} = (5-2, 10-5, 6-5) = (3,5,-1)$ ← coordinate form

$\vec{AB} = \begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix}$ → column form.

#8. If u is a vector what happen to the magnitude and direction of ku if $k>0$ and $k<0$?

Soln If $k>0$, $|ku| = k|u|$ & the direction is the same as u

If $k<0$, $|ku| = |k||u|$ & the direction is the reverse of u

TIPS

$\vec{A} = A_x i + A_y j + A_z k$ → Standard unit vector form of $\vec{A}$

$\vec{A} = (A_x, A_y, A_z)$ → Coordinate form of $\vec{A}$

Norm (Magnitude) of vectors.

→ $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

→ $|a+b| = \sqrt{(a+b)^2} = a^2 + 2a\cdot b + b^2 = \sqrt{|a|^2 + 2a\cdot b + |b|^2}$

→ $|a-b| = \sqrt{(a-b)^2} = a^2 - 2a\cdot b + b^2 = \sqrt{|a^2| - 2a\cdot b + |b|^2}$

→ $\dfrac{\vec{A}}{|A|} = \dfrac{A_x i + A_y j + A_z k}{|A|}$ is a unit vector in the direction of $\vec{A}$

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in the direction of $(\vec{A}+\vec{B})$ and in the opposite direction of $(\vec{A}+\vec{B})$

Option 1 $(\vec{A}+\vec{B}) = (1+2, 2+(-5), 3+1) = (3, -3, 4)$

$|\vec{A}+\vec{B}| = \sqrt{3^2+(-3)^2+4^2} = \sqrt{34}$

(a) $\dfrac{(\vec{A}+\vec{B})}{|\vec{A}+\vec{B}|} = \left(\dfrac{3}{\sqrt{34}}, \dfrac{-3}{\sqrt{34}}, \dfrac{4}{\sqrt{34}}\right)$ → is a unit vector in the direction of $(\vec{A}+\vec{B})$

(b)

Option 2 $|A| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ & $|B| = \sqrt{2^2+(-5)^2+1^2} = \sqrt{30}$

$A \cdot B = (1*2+(2*-5)+3*1) = -5$ → dot product of A & B

$|A+B| = \sqrt{|A|^2+2A\cdot B+|B|^2}$

$= \sqrt{(\sqrt{14})^2+2*(-5)+(\sqrt{30})^2} = \sqrt{14-10+30} = \sqrt{34}$

(a) $\dfrac{(\vec{A}+\vec{B})}{|\vec{A}+\vec{B}|} = \dfrac{3}{\sqrt{34}}, \dfrac{-3}{\sqrt{34}}, \dfrac{4}{\sqrt{34}}$ → is a unit vector in the direction of $(\vec{A}+\vec{B})$

#10. Find a vector in the direction of $5i-j+2k$ which has magnitude of 8 units.

Soln $\vec{a} = 5i-j+2k$ and $|\vec{a}| = \sqrt{5^2+(1)^2+2^2} = \sqrt{30}$

$\dfrac{5}{\sqrt{30}}i - \dfrac{1}{\sqrt{30}}j + \dfrac{2}{\sqrt{30}}$ is a vector in the direction of $\vec{a}$ but it has 1 unit

$8\left[\dfrac{5}{\sqrt{30}}i - \dfrac{1}{\sqrt{30}}j + \dfrac{2}{\sqrt{30}}k\right] = \dfrac{40}{\sqrt{30}}i - \dfrac{8}{\sqrt{30}}j + \dfrac{16}{\sqrt{30}}k$ → Vector in the direction of $\vec{a}$ with $|\vec{a}| = 8$.

Also {Vector in the direction of $\vec{a}$ With $|\vec{a}| = 8$} is =

#11. Find a unit vector in the direction of vector where P(1,2,3) and Q(4,5,6) are points (initial and final) of the vector.

Soln $\vec{PQ} = (4-1)i + (5-2)j + (6-3)k = 3i+3j+3k$

$|\vec{PQ}| = \sqrt{3^2+3^2+3^2} = \sqrt{27} = 3\sqrt{3}$

$\dfrac{\vec{PQ}}{|\vec{PQ}|} = \dfrac{1}{\sqrt{3}}i + \dfrac{1}{\sqrt{3}}j + \dfrac{1}{\sqrt{3}}k$ → Unit vector in the direction of $\vec{PQ}$

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#12. Find the direction cosine of $\vec{A} = i+2j+3k$?

$|A| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ //

$\frac{\vec{A}}{|A|}$ is a unit vector in the direction of $\vec{A} = \frac{1}{\sqrt{14}}i + \frac{2}{\sqrt{14}}j + \frac{3}{\sqrt{14}}k$

$\Rightarrow$ The direction cosine of $\vec{A} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{14}}, \cos\beta = \frac{2}{\sqrt{14}}, \cos\gamma = \frac{3}{\sqrt{14}}$ ($\cos\alpha = \frac{a_x}{|a|}$, $\cos\beta = \frac{a_y}{|a|}$, $\cos\gamma = \frac{a_z}{|a|}$)

#13. Find the position vector of point R which divides two points P and Q whose position vectors are $i+2j-k$ & $-i+j+k$ respectively in the ratio 2:1 internally & externally.

Sol: Given position vector $\vec{OP} = i+2j-k$, $\vec{OQ} = -i+j+k$ } Position vector of the point P & Q

Required position vector of $R = \vec{OR}$

$\rightarrow$ The position vector R dividing the segment joining two points P and Q in the ratio m:n is given by

Internally $\Rightarrow \vec{OR} = \frac{m(\vec{OQ}) + n(\vec{OP})}{(m+n)} = \frac{2(-i+j+k) + 1(i+2j-k)}{(2+1)} = -\frac{1}{3}i + \frac{4}{3}j + \frac{1}{3}k$

Externally $\Rightarrow \vec{OR} = \frac{m(\vec{OQ}) - n(\vec{OP})}{m-n} = \frac{2(-i+j+k) - 1(i+2j-k)}{2-1} = -3i+3k$

#14. If $\vec{a}$ and $\vec{b}$ are two collinear vectors which of the following is false? a. $\vec{b} = t\vec{a}$, for some scalar t

b. $\vec{a} = \pm\vec{b}$

C. The respective components of $\vec{a}$ & $\vec{b}$ are proportional

(D.) $|\vec{a}| \neq |\vec{b}|$ but $\vec{a}$ & $\vec{b}$ have the same direction.

Sol: $\vec{a}$ & $\vec{b}$ can have the same direction or different direction.

#15. Find the projection of the vector $\vec{a} = i+3j+7k$ on $\vec{b} = 7i-j+8k$

$\vec{a}\cdot\vec{b} = (1\times7 + 3\times-1 + 7\times8) = 60$ and $|b| = \sqrt{7^2+(-1)^2+8^2} = \sqrt{114}$

Projection of $\vec{a}$ on $\vec{b}$ or vector component of $\vec{a}$ along $\vec{b}$ is given by

$\frac{\vec{a}\cdot\vec{b}}{|b|} = \frac{60}{\sqrt{114}}$ // $\rightarrow$ The projection.

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#16. If $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$, then find $|\vec{a}|$ & $|\vec{b}|$?

Soln $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 8$

$\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$

$|\vec{a}||\vec{a}| - |\vec{b}||\vec{b}| = |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$63|\vec{b}|^2 = 8 \Rightarrow |\vec{b}|^2 = \frac{8}{63} \Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

$|\vec{a}| = 8|\vec{b}| = \frac{16\sqrt{2}}{3\sqrt{7}}$

#17. Find $|\vec{x}|$, if $(\vec{x}-\vec{a})(\vec{x}+\vec{a}) = 12$ and $\vec{a}$ is unit vector

Soln $|\vec{a}| = 1 \Rightarrow (\vec{x}-\vec{a})(\vec{x}+\vec{a}) = 12$

$|\vec{x}|^2 - |\vec{a}|^2 = 12$

$|\vec{x}|^2 - 1 = 12 \Rightarrow |\vec{x}| = \sqrt{13}$

#18. If $\vec{a}$ is non zero vector of magnitude "a" and "t" is a non zero scalar then $t\vec{a}$ is a unit vector if:

A. $t = 1$ B. $t = -1$ C. $a = |t|$ D. $a = \frac{1}{|t|}$

Soln $|t\vec{a}| = |t||\vec{a}| = 1$

$a = \frac{1}{|t|}$

or $\frac{\vec{a}}{|a|}$ is unit vector also if $t\vec{a}$ is a unit vector $\frac{1}{|a|} = t$

$\Rightarrow \frac{1}{|a|} = t$ or $|a| = \frac{1}{t}$.

#19. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = 3i+2j+2k$ & $\vec{b} = i+2j-2k$

Soln $(\vec{a}+\vec{b}) = 4i+4j$ and $(\vec{a}-\vec{b}) = 2i+4k$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16i - 16j - 8k$$

$$|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = \sqrt{16^2 + (-16)^2 + 8^2} = \sqrt{576} = 24$$

$$\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|} = \pm\frac{(16i-16j-8k)}{24} = \pm\left(\frac{2}{3}i - \frac{2}{3}j - \frac{1}{3}k\right)$$

$\frac{2}{3}i - \frac{2}{3}j - \frac{1}{3}k$ & $-\frac{2}{3}j + \frac{2}{3}j + \frac{1}{3}k$

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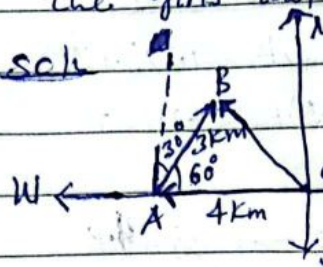
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#20. What is the unit vector in xy-plane making 30° with +ve X-axis.

Sol: $\vec{u} = \cos\theta\,i + \sin\theta\,j = \cos 30°\,i + \sin 30°\,j = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$

#21. A girl walks 4km towards west and then she walks 3km in a direction of 30° east of North and stops. Determine the girls displacement from her initial departure?

Sol:



$\vec{AB} = |\vec{AB}|\cos 60°\,i + |\vec{AB}|\sin 60°\,j = \frac{3}{2}i + \frac{3\sqrt{3}}{2}j$

$\vec{OA} = -4i$

$\vec{OB} = \vec{OA} + \vec{AB} = \left(-4+\frac{3}{2}\right)i + \frac{3\sqrt{3}}{2}j = -\frac{5}{2}i + \frac{3\sqrt{3}}{2}j$

#22. For what value of x, $x(i+j+k)$ is a unit vector?

Sol: $\sqrt{x^2+x^2+x^2} = \sqrt{3x^2} = 1$, $x = \pm\frac{1}{\sqrt{3}}$

#23. If $\vec{a} = i+j+k$, $\vec{b} = 2i-j+3k$, $\vec{c} = i-2j+k$, find a vector of magnitude 2, parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$?

Sol: $2\vec{a}-\vec{b}+3\vec{c} = 2(i+j+k)-(2i-j+3k)+3(i-2j+k) = 3i-3j+2k$

$|2\vec{a}-\vec{b}+3\vec{c}| = \sqrt{3^2+(-3)^2+2^2} = \sqrt{22}$

$\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|} = \left(\frac{3}{\sqrt{22}}i - \frac{3}{\sqrt{22}}j + \frac{2}{\sqrt{22}}k\right)$ → is unit vector in the direction of $2\vec{a}-\vec{b}+3\vec{c}$

$\left(\frac{-3}{\sqrt{22}}i + \frac{3}{\sqrt{22}}j - \frac{2}{\sqrt{22}}k\right)$ → is unit vector in the opposite direction of $2\vec{a}-\vec{b}+3\vec{c}$

Since opposite vectors are parallel

$\pm 2\left(\frac{3}{\sqrt{22}}i - \frac{3}{\sqrt{22}}j + \frac{2}{\sqrt{22}}k\right) = \pm\left(\frac{6}{\sqrt{22}} - \frac{6}{\sqrt{22}}j + \frac{4k}{\sqrt{22}}\right)$

This is a vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$ and magnitude 2.

#24. If $\vec{a}$ and $\vec{b}$ are two unit vectors, and "θ" is the angle b/n $\vec{a}$ & $\vec{b}$, for what value of "θ" $\vec{a}+\vec{b}$ is a unit vector?

A. $\theta = \pi/4$ B. $\theta = \pi/3$ C. $\theta = \pi/2$ (D) $\theta = \frac{2\pi}{3}$

Sol: $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2} = 1 \Rightarrow \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} = 1$

$|a|^2 + 2\vec{a}\cdot\vec{b} + |b|^2 = 1$

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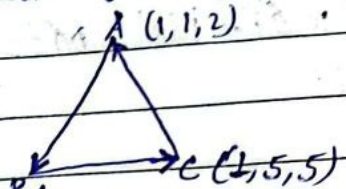
$1^2 + 2|a||b|\cos\theta + 1 = 1$ $\cos\theta$ is $-ve$ in 2nd quadrant

$\Rightarrow 2\cos\theta = -1$ & $\cos\theta = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}(-\frac{1}{2}) = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$

#25. Find the Area of triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5)

Two adjacent sides in vector form

Soln



$\vec{AB} = (2-1)i + (3-1)j + (5-2)k = i + 2j + 3k$

$\vec{BC} = (1-2)i + (5-3)j + (5-5)k = -i + 2j$

$(\vec{AB}) \times (\vec{BC}) = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = -6i - 3j + 4k$

$|\vec{AB} \times \vec{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{61}$

Area of triangle $= \frac{1}{2} * |\vec{AB} \times \vec{BC}| = \frac{\sqrt{61}}{2}$ //

26. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, what can we conclude about $\vec{a}$ & $\vec{b}$?

Soln $\vec{a} \cdot \vec{b} = |a||b|\cos\theta = 0 \Rightarrow$ either $|a| = 0$ or $|b| = 0$ or $\theta = 90°$

$\vec{a} \times \vec{b} = |a||b|\sin\theta = 0 \Rightarrow$ either $|a| = 0$ or $|b| = 0$ or $\theta = 0°$

case1: If $\theta = 90°$, $\vec{a} \cdot \vec{b} = 0$ but $\vec{a} \times \vec{b} \neq 0$

Case2: If $\theta = 0°$, $\vec{a} \times \vec{b} = 0$ but $\vec{a} \cdot \vec{b} \neq 0$

case3: either $|a| = 0$ or $|b| = 0$, both $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b} = 0$

Conclusion Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

#27. If $|a| = 4$ & $|b| = 7$ and the angle between a & b is $\frac{\pi}{3}$, find the cosine of angle b/w $(3a - 2b)$ and a

$(3a - 2b) \cdot a = |3a - 2b||a|\cos\theta$

$\cos\theta = \frac{(3a - 2b) \cdot a}{|3a - 2b||a|} = \frac{3a \cdot a - 2a \cdot b}{\sqrt{(3a - 2b)^2} * |a|}$

$(3a - 2b)(3a - 2b) \rightarrow 9a^2 - 12ab + 4b^2$

$= \frac{3|a||a| - 2|a||b|\cos\theta}{\sqrt{9a \cdot a - 12\,a \cdot b + 4\,b \cdot b} * |a|}$

$\cos\theta = \frac{3(4)^2 - 2(4)(7)\cos\frac{\pi}{3}}{\sqrt{9*16 - 12*4*7\cos\frac{\pi}{3} + 4(7)^2} * 4}$

$\cos\theta = \frac{20}{\sqrt{172} * 4} = \frac{20}{8\sqrt{43}} = \frac{5}{2\sqrt{43}} = \frac{5\sqrt{43}}{86}$ //

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θ is acute angle if $\vec{a}\cdot\vec{b}>0$
θ is 90° if $\vec{a}\cdot\vec{b}=0$
θ is obtuse angle if $\vec{a}\cdot\vec{b}<0$

#28. Determine the value of k, so that the angle b/n the vectors $u=(k,1)$ and $v=(-2,3)$ is acute and obtuse?

Soln Acute $u\cdot v>0$

$-2k+3>0 \Rightarrow -2k>-3 \Rightarrow k<\frac{3}{2}$ //

Obtuse $u\cdot v<0$

$-2k+3<0 \Rightarrow -2k<-3 \Rightarrow k>\frac{3}{2}$ //

Exercise 5.7 #2

#29. $u=(8-x, 6-y)$ & $v=(x-4, y+2)$, If $u=v$, find u & v

If two vectors are equal their corresponding components are equal

$8-x=x+4$ and $6-y=y+2$

$x=6$ // and $y=2$ // $\Rightarrow u=(2,4)$ & $v=(2,4)$

Exercise 5.13

#30. Find the area of parallelogram defind by the vector

$u=2i+j$ and $v=i+3j$

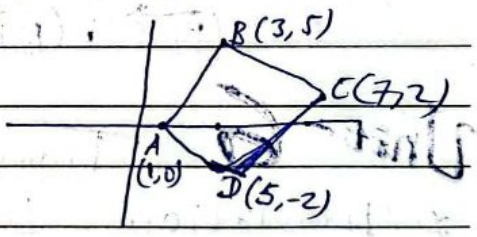
$(u\times v)=(2\times3-1\times1)k=5k$ & $|u\times v|=\sqrt{5^2}=5$ //

The area of parallelogram $=5$ units

Exercise 5.14 #5

#31. Find the area of quadrilateral whose vertices are

$A(1,0)$, $B(3,5)$, $C(7,2)$ and $D(5,-2)$



Vector form of two adjacent sides

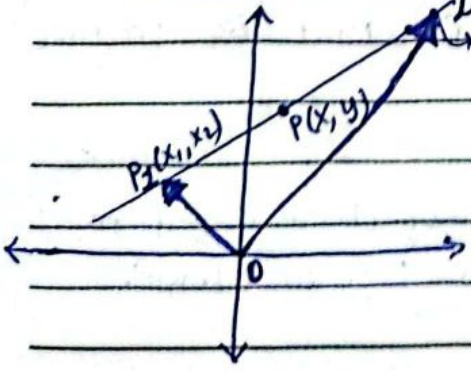
$\vec{AB}=(3-1)i+(5-0)j=2i+5j$

$\vec{AD}=(5-1)i+(-2-0)j=4i-2j$

$(\vec{AB}\times\vec{AD})=\begin{vmatrix} i & j & k \\ 2 & 5 & 0 \\ 4 & -2 & 0 \end{vmatrix}=-24k$

$|\vec{AB}\times\vec{AD}|=\sqrt{(-24)^2}=24$ → Area of the quadrilateral

Vector & lines



⊗ $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are two points

⊗ $\vec{OP_1}$ is position vector of $P_1=\vec{P_1}$

⊗ $\vec{OP_2}$ is position vector of $P_2=\vec{P_2}$

⊗ $V=\vec{P_1P_2}$ is is vector from P_1 to $P_2=(x_2-x_1, y_2-y_1)$

$V=(x_2-x_1, y_2-y_1)$ → Direction Vector of the Line.

$V=(a,b)$

If $\lambda \in R$ and $V \neq 0$ ($a \neq 0$ & $b \neq 0$) (λ → parameter)

1. $P = P_1 + \lambda V \Rightarrow (x, y) = (x_1, y_1) + \lambda(a, b) = (x_1 + \lambda a, y_1 + \lambda b)$
The vector equation of the line

2. $x = x_1 + \lambda a$
$y = y_1 + \lambda b$ } parametric equation of the line.

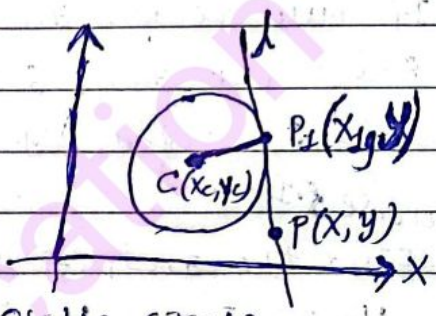
3. $\frac{x - x_1}{a} = \frac{y - y_1}{b}$ or

$\frac{1}{a}x - \frac{1}{a}x_1 = \frac{1}{b}y - \frac{1}{b}y_1 \Rightarrow$

$\frac{1}{a}x - \frac{1}{b}y + \left(\frac{1}{b}y_1 - \frac{1}{a}x_1\right) = 0$

$Ax + By + C$ → Standard equation of the line.

Vectors & Circles



Let { $C(x_c, y_c)$ be the center of the circle
$P_1(x_1, y_1)$ be the point of tangency of the circle
$P(x, y)$ be any arbitrary point on the line "l"

⇒ The line "l" is tangent to the circle if and only if $(\overrightarrow{P_1C}) \cdot (\overrightarrow{P_1P}) = 0$ (dot product is zero)

Unit 6. Transformation of the plane

could be 1. Rigid – size or shape of the preimage is not changed
2. Non rigid – size is changed but shape remains the same.

1. Translation – Defined as moving the object in space by keeping its size, shape or orientation constant
- Each point of the shape must be moved in the same direction and for the same distance.

2. Reflection – Flipping the object across a line by keeping its shape or size constant to produce mirror image.

3. Rotation – Rotating an object about a fixed point with out changing its size or shape.

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1. Translation

I) If $V=(a,b)$ be translation vector and $P(x,y)$ is a point then $P(x,y)$ will be translated to $P'(x+a, y+b)$

II) Equation can be translated by substituting $(x-a)$ in place of "x" and $(y-b)$ in place of "y".

2. Reflection

I) Reflection in the line $y=mx$, where $m=\tan\theta$ → line of reflection

If $P(x,y)$ is a point then $P'[(x\cos 2\theta + y\sin 2\theta), (x\sin 2\theta - y\cos 2\theta)]$ → Image of P

II) Reflection in the line $y=mx+b$ → line of reflection.

Case 1: If the line to be reflected is ⊥ to the line of reflection then the image of the given line is it self.

Case 2: If the line to be reflected is // to the line of reflection then the image of the given line is also parallel to the line of reflection and then find the value of the constant by drawing a line ⊥ to the line of reflection.

- The y-intercepts of the pre image and the image are at equal distance from the y-intercept of the line of reflection in opposite direction with respect to the line of reflection.

Case 3:

If the line to be reflected is non ⊥ and non // to the line of reflection then the line is intersecting line to the line of reflection, Then the image of such lines can be found by drawing a ⊥ line to the line of reflection and take any arbitrary point on the given line. The mid point of the arbitrary point on (l) and the image of the arbitrary point on (l') is located on the line of reflection (l_r). Find the intersection of the given line and the line of reflection finally find a line which pass through the intersection point and the image of the arbitrary point.

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3. Rotation Let $\begin{cases} O(x_0, y_0) - \text{be center of rotation} \\ \theta \text{ or } R_\theta - \text{be angle of rotation} \end{cases}$

Case 1 :- If $P(x,y)$ is apoint $R_\theta^{\circ}(x,y) \Rightarrow$ rotation of $P(x,y)$ about θ

$$R_\theta(x,y) = \left[\underbrace{x_0 + (x-x_0)\cos\theta - (y-y_0)\sin\theta}_{x'},\ \underbrace{y_0 + (x-x_0)\sin\theta + (y-y_0)\cos\theta}_{y'}\right]$$

Case 2:

Rotation of line

Take any two arbitrary points on the line and then rotate these points about the given "θ" and center of rotations finaly find a line which pass through $R_\theta(x_1, y_1)$ and $R_\theta(x_2, y_2)$

Solved Problems

#32. The line passes through the point (1,4) and (2,2), then find the vector equation, parametric equations and standard equations of the line.

Soln $\Rightarrow V = (2-1, 2-4) = (1,-2) \rightarrow$ direction vector of the line.

$P = P_1 + \lambda V = (1,4) + \lambda(1,-2)$ for $\lambda \in R$

$(x,y) = (1+\lambda, 4-2\lambda)$ $\rightarrow$ Vector equation of the line

$\rightarrow x = 1+\lambda$; $y = 4-2\lambda$ $\xrightarrow{\text{for } \lambda \in R}$ Parametric equation of the line.

$x-1 = \frac{y-4}{-2}$ or

$-2x-y+6 = 0$, or $\}$ Solving for λ. Standard equation of the line

$2x+y-6 = 0$

Exercise 5.15

#33. Find the vector equation of the line passing through the point having vector $5i+4j$ and having direction vector $(3,-4)$.

Soln Vector equation of the line passing through $(5i+4j)$ and parallel to V is $(5i+4j) + \lambda(3i-4j)$.

$\rightarrow$ Vector equation of the line

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#34. Find the vector equation of the line, parametric and standard equation of the line through (1, 3) and (−1, −1)

Soh $P_1 = (1, 3)$ $P_2(-1, -1)$

→ direction vector, $V = \overrightarrow{P_1P_2} = (-1-1, 1-3) = (-2, -4)$

a) $P_1 + \lambda V = (1, 3) + \lambda(-2, -4)$ or $(x, y) = (1-2\lambda, 3-4\lambda)$ } Vector equation of the line

b. $x = 1-2\lambda$, $y = 3-4\lambda$ } parametric equation of the line.

c. solving for λ → parameter

$\frac{x-1}{-2} = \frac{y-3}{-4}$ or $(-4x+2y-2=0)$ or $(-2x+y-1=0)$ } standard equation of the line

#35. Find the vector equation of the line passing through the points (2, 3) and (−1, 1).

Soh $P_1 = (2, 3)$, $P_2 = (-1, 1)$

direction vector, $V = (-1-2, 1-3) = (-3, -2) = \overrightarrow{P_1P_2}$

$V = (2-(-1), 3-1) = (3, 2) = \overrightarrow{P_2P_1}$

- $(2, 3) + \lambda(-3, -2)$ or $(x, y) = (2-3\lambda, 3-2\lambda)$
- $(-1, 1) + \lambda(-3, -2)$ or $(x, y) = (-1-3\lambda, 1-2\lambda)$
- $(2, 3) + \lambda(3, 2)$ or $(x, y) = (2+3\lambda, 3+2\lambda)$
- $(-1, 1) + \lambda(3, 2)$ or $(x, y) = (-1+3\lambda, 1+2\lambda)$

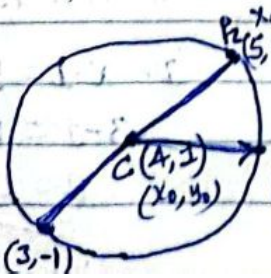
} All of them are the vector equation of the line.

Exercise 5.16 #2.

#36. Find the equation of circle whose diameter is segment from (5, 3) to (3, −1).

Soh $D = \sqrt{(5-3)^2 + (3-(-1))^2} = \sqrt{20} = 2\sqrt{5}$ & radius = $\sqrt{5}$

Center $= \left[\frac{5+3}{2}, \frac{3+(-1)}{2}\right] = (4, 1)$ → the mid point of the segment



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radius $= |\vec{CP}| = \sqrt{(x-4)^2+(y-1)^2}$, $\left[\vec{CP} = (x-4, y-1)\right]$

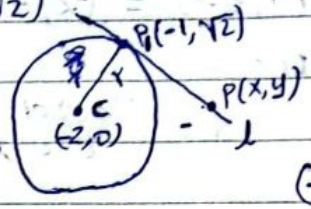
$r^2 = |\vec{CP}|^2 = (x-4)^2+(y-1)^2$

$\sqrt{5}^2 = x^2-8x+16+y^2-2y+1$

$5 = x^2+y^2-8x-2y+17 \Rightarrow x^2+y^2-8x-2y+12=0$

#37. Find the equation of the tangent line to the circle $(x+2)^2+y^2=3$ at $(-1,\sqrt{2})$

Soln



The point of the tangency is on the circle, so $(-1+2)^2+(\sqrt{2})^2 = 3$

$1+2=3 \Rightarrow 3=3$

$(-1,\sqrt{2})$ can be point of tangency

At the point of tangency the radius is $\perp$ to the line.

$(\vec{radius}) = (\vec{CP}) = (-1-(-2), \sqrt{2}-0) = (1,\sqrt{2})$

The line $= (\vec{l}) = (x-(-1), y-\sqrt{2}) = (x+1, y-\sqrt{2})$

$\vec{r}\cdot\vec{l} = (\vec{CP_1})\cdot(\vec{P_1P}) = (1,\sqrt{2})\cdot(x+1, y-\sqrt{2}) = 0$

$= (x+1)+\sqrt{2}y-2=0$

$x+\sqrt{2}y-1=0$ or $y = \frac{-1}{\sqrt{2}}x+\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}(x-1)$

#38. The vector equation of the line "l" is given by $P=(1,0)+t(2,2)$ $t\in R$, which of the following points lie in the line?

A. (1,0) B. (2,2) C. (-5,-6) D. (3,0) E. A D

Soln option 1 $P_1=(1,0)$ & $V=(2,2) = \vec{P_1P_2} = (x-1, y-0)$

$P_2=(x,y) \Rightarrow x-1=2$ & $2=y-0$

$x=3$ $y=2$

$P_2=(3,2)$ → the line which pass through P_1 & P_2

$y-y_1 = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$

$y-0 = \frac{(2-0)}{(3-1)}(x-1) \Rightarrow y = 1(x-1) \Rightarrow y = x-1$

Option 2: From the vector equation $(x,y) = (1+2t, 0+2t)$

$x = 1+2t$ and $y = 2t \Rightarrow$ Parametric equation of the line.

$\frac{x-1}{2} = \frac{y-2}{2}$ → standard equation of the line.

$x-1=y$

A. $(x-1)=y$ $(1-1)=0$ $0=0$ ✓

B. $x-1=y$ $2-1=2$ $-1=2$ ✗

C. $x-1=y$ $-5-1=-6$ $-6=-6$ ✓

D. $x-1=y$ $3-1=0$ $2=0$ ✗

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#39. Which one of the following points A, B and C are collinear?

A. A(1, -4), B(-2, -3), C(11, -11) C. A(1, 2), B(2, 4), C(3, 9)

B. A(-2, -3), B(4, 9), C(-11, -21) D. A(2, 2), B(3, 6), C(1, 4)

Soln Any points are collinear if thy have the same slope

A. Slope b/n points A & B $\Rightarrow \frac{y_B - y_A}{x_B - x_A} = \frac{-3-(-4)}{-2-1} = \frac{-1}{3}$

Slope b/n B & C $\Rightarrow \frac{y_C - y_B}{x_C - x_B} = \frac{-11-(-3)}{11-(-2)} = \frac{-8}{13}$

A, B, C → are not collinear points

(B.) Slope b/n A & B $\Rightarrow \frac{y_B - y_A}{x_B - x_A} = \frac{9-(-3)}{4-(-2)} = \frac{12}{6} = 2$

Slope b/n B & C $\Rightarrow \frac{y_C - y_B}{x_C - x_A} = \frac{-21-9}{-11-4} = \frac{-30}{-15} = 2$

A, B, C → are collinear points

C. Slope b/n A & B $\Rightarrow \frac{4-2}{2-1} = 2$ & Slope b/n B & C $\Rightarrow \frac{9-4}{3-2} = 5$

$2 \neq 5$, so A, B, C → are not collinear

D. Slope b/n A & B $\Rightarrow \frac{6-2}{3-2} = 4$ & Slope b/n B & C $\Rightarrow \frac{4-6}{1-3} = \frac{-2}{-2} = 1$

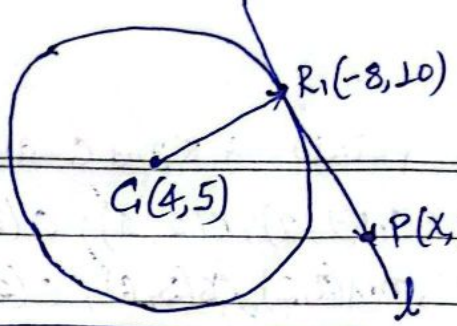
$4 \neq 1$, so A, B, C are not collinear.

#40. Find the equation of the tangent line to the circle $x^2 + y^2 - 8x - 10y - 128 = 0$, at the point R(-8, 10).

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Exercise 5.17 #2



$(\vec{CR}) \cdot (\vec{R_1P}) = 0$

$(-8-4, 10-5) \cdot (x+8, y-10) = 0$

$(-12, 5) \cdot (x+8, y-10) = 0$

$-12(x+8) + 5(y-10) = 0$

$-12x - 96 + 5y - 50 = 0$

$-12x + 5y - 146 = 0$

$\boxed{-12x+5y-146=0 \text{ or } 12x-5y+146=0}$

Exercise 5.17 #1 and #2 are the same problem

#41. If $2x-4y+40=0$ is tangent line to the circle $x^2+y^2+4x-8y=0$, find the point of tangency.

Exercise 5.17 #1 The answer to exercise 5.17 (#1) is $2x-4y+40=0$

Soln $2x-4y+40=0 \Rightarrow 4y = 2x+40 \Rightarrow \boxed{y = \frac{1}{2}x+10}$

$x^2+y^2+4x-8y=0$

$x^2+\left(\frac{1}{2}x+10\right)^2+4x-8\left(\frac{1}{2}x+10\right)=0$

$x^2+\frac{1}{4}x^2+10x+100+4x-4x-80=0$

$\frac{5}{4}x^2+10x+20=0$ → multiply by $\frac{4}{5}$

$x^2+8x+16=0$

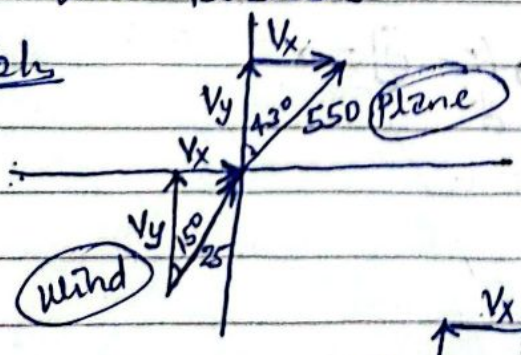
$(x+4)^2=0 \Rightarrow x=-4 \quad y=\frac{1}{2}(-4)+10=8$

$(-4,8)$ is the point of tangency.

Exercise 5.18 #4

#42. An airplane is flying in the direction of 43° East of North (also abbreviated as N43E) at a speed of 550 m/h. A wind with speed 25 m/h comes from the South West at a bearing of N15E. What is the ground speed and new direction of the plane?

Soln



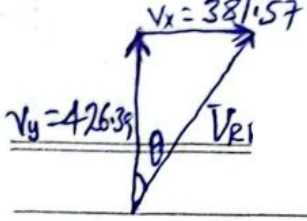
Aeroplane	Wind
$V_x = 550 \times \sin 43°$	$V_x = 25 \times \sin 15°$
$= 375.1$ m/h	$= 6.47$ m/h
$V_y = 550 \times \cos 43°$	$V_y = 25 \times \cos 15°$
$= 402.24$ m/h	$= 24.148$ m/h



$V_x = 381.57$ m/h

$V_y = 402.24 + 24.148 = 426.39$ m/h

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$\tan\theta = \frac{V_x}{V_y} = \frac{381.57}{426.39} \Rightarrow \tan\theta = 0.89488$

$\theta = \tan^{-1}(0.89488) = 41.82^\circ$ → direction of the plane

$V_R = \sqrt{(426.39)^2 + (381.57)^2}$

$= 572.19$ m/h

or $\cos\theta = \frac{V_y}{V_R} \Rightarrow V_R = \frac{V_y}{\cos(41.82^\circ)} = \frac{426.39 \text{ m/h}}{0.74524} = 572.19$ m/h

or $\sin\theta = \frac{V_x}{V_R} \Rightarrow V_R = \frac{V_x}{\sin(41.82^\circ)} = \frac{381.57 \text{ m/h}}{0.66679} = 572.19$ m/h

1/ Translation

#43. If the point A(3, -2) is translated to the point A'(7, 10) then find the image of the following equations.

a/ Ellipse $4x^2+3y^2-2x+6y=0$

b/ Hyperbola $xy = 1$

c/ parabola $y^2 = 4x$

D/ Function $f(x) = x^3 - 3x^2 + 4$

Solution The translation vector is, $V = (a,b) = (4, 12)$ because

$A(3,-2) + V(a,b) = A'(7,10)$

$(3+a, -2+b) = A'(7,10) \Rightarrow \begin{cases} 3+a=7 \Rightarrow a=4 \\ -2+b=10 \Rightarrow b=12 \end{cases}$

$\Rightarrow$ If (a,b) is a translation vector then the image of any equations having variable x & y ; can be found by substituting $(x-a)$ in place of "x" and $(y-b)$ in place of "y"

a/ $4(x-4)^2+3(y-12)^2-2x+6y=0$

$4(x^2-8x+16)+3(y^2-24y+144)-2(x-4)+6(y-12)$

$4x^2-32x+64+3y^2-72y+432-2x+8+6y-72$

$4x^2+3y^2-34x-66y+432=0$

b/ $(x-4)(y-12)=1 \Rightarrow$

$x-4$

$y-12$

$-12x+48$

$xy-4y$

$xy-12x+4y+48$

$xy-12x-4y+48=1$

$xy-12x-4y+47=0$

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C) $y^2 = 4x \Rightarrow (y-12)^2 = 4(x-4)$ or

$y^2 - 24y + 144 = 4x - 16$ or

$y^2 - 24y - 4x + 160 = 0$ V

D) $f(x) = x^3 - 3x^2 + 4$

The translation shift the graph of the function 4 units to the positive x-axis and 12 units to the positive y-axis

$f(x-4) - 12 = (x-4)^3 - 3(x-4)^2 + 4 = (x-4)^2(x-4) - 3(x-4)^2 + 4$

$= (x-4)^2[(x-4) - 3] + 4$

$= (x-4)^2(x-7) + 4$

$= (x^2 - 8x + 16)(x-7) + 4$

$= x^3 - 8x^2 + 16x - 7x^2 + 56x - 112 + 4$

$f(x-4) - 12 = x^3 - 15x^2 + 72x - 108$

$f(x-4) = (x^3 - 15x^2 + 72x - 108) + 12$ → shift the graph 12 units upward

$f(x-4) = x^3 - 15x^2 + 72x - 96$

#44. If a translation T takes (1,3) to (3,-2) and the second translation takes (-1,0) to (2,2). Find where T followed by S takes (5,-2) and also find where S followed by T takes (5,-2).

Soln $S(T(5,-2))$ ⇒ Translation Vector $T = (2, -5)$ $\overset{(3-1)}{\nearrow}$ $\overset{(-2-3)}{\nearrow}$

⇒ Translation Vector $S = (3, 2)$ $\overset{2-(-1)}{\nearrow}$ $\overset{2-0}{\nearrow}$

$S(T(5,-2)) = S(5+2, -2+(-5)) = S(7,-7) = (7+3, -7+2) =$

S followed by T — T followed by S ← $S(T(5,-2)) = (10, -5)$

$T(S(5,-2)) = T(5+3, -2+2) = T(8,0) = (8+2, 0+(-5)) = (10, -5)$

#45. From question (44) find T followed by S of the translation of the line $2x - y = 0$.

$S(T(2x-y=0)) = S(2(x-2) - (y-(-5))) = S(2x-4-y-5) = S(2x-y-9=0)$

$2(x-3) - (y-2) - 9 = 0 \Rightarrow 2x - 6 - y + 2 - 9 = 0$

T followed by S: $S(T(2x-y=0)) = 2x - y - 13 = 0$

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#46/ If a translation T takes the origin to (-1, 1) then find the image of the point (1,3) and (-3,6), Triangle with vertices A(2,-2), B(-3,2), C(4,1) and also the image of the circle whose equation is $x^2+y^2=4$

Soln Translation Vector is, $V = (-1, 1)$

→ The image of (1,3) is $(1+(-1), 3+1) = (0, 4)$
→ The image of (-3,6) is $(-3+(-1), 6+1) = (-4, 7)$
} The image of the two points

$T(2,-2) = (2+(-1), -2+1) = (1, -1)$
$T(-3,2) = (-3+(-1), 2+1) = (-4, 3)$
$T(4,1) = (4+(-1), 1+1) = (3, 2)$
} The image of the 3 vertices of the triangle.

$T(x^2+y^2=4) = [x+(-1)]^2 + (y-1)^2 = 4 \Rightarrow$

$(x+1)^2 + (y-1)^2 = 4$ → The image of the circle.

#47/ If translation takes the point (-1,3) to the point (4,2), then what is the image of the lines $l: y=2x-3$ and $l: 5y+x=1$?

Soln Translation Vector, $V = (4-(-1), 2-3) = (5, -1)$

→ $T(y=2x-3) = y-(-1) = 2(x-5)-3 \Rightarrow y+1 = 2x-13$
$= y = 2x-14$ → The image of the line

→ $T(5y+x=1) = 5(y-(-1)) + (x-5) = 1 \Rightarrow 5y+5+(x-5) = 1$
$= 5y+x=1$ → The image of the line which it itself

#48/ Determine the equation of the curve $2x^2+3y^2-8x+6y=7$, when the origin is translated to (2,-1).

Soln Translation Vector, $V = (2-0, -1-0) = (2, -1)$

$T(2x^2+3y^2-8x+6y=7) = 2(x-2)^2 + 3(y-(-1))^2 - 8(x-2) + 6(y-(-1)) = 7$
$= 2(x^2-4x+4) + 3(y^2+2y+1) - 8x+16+6y+6 = 7$
$= 2x^2+3y^2-16x+12y+(8+3+16+6-7) = 0$
$= \underline{2x^2+3y^2-16x+12y+26=0}$

#49/ Find the image of the circle $(x+1)^2+(y-3)^2 = 5$, when translated by the vector $\vec{PQ}$, where P(1,-1) and Q(-4,3)

Soln Translation Vector, $V = (-4-1, 3-(-1)) = (-5, 4)$

$T((x+1)^2+(y-3)^2=5) = [(x+5)+1]^2 + [(y-4)-3]^2 = 5$
$= \underline{(x+6)^2+(y-7)^2 = 5}$

#50/ A translation T takes the origin to (3,-2). A second translation S takes the origin to (-2,-1). Find where T followed by S takes the origin, and where S followed by T takes the origin.

Soln Translation Vector $T = (3,-2)$
Translation Vector $S = (-2-0, -1-0) = (-2, -1)$

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→ T followed by S = $S(T(0,0)] = S(3,-2) = (3+(-2), -2+(-1)) = (1,-3)$ //

→ S followed by T = $T[S(0,0)] = T(-2,-1) = (-2+(3), -1+(-2)] = (1,-3)$

#51/ If the image of point (-2,3) under translation T is (3,-1). What is the image of the point (4,2) under the same translation.

Sol: Let the Translation vector be $V = (x,y)$

$(-2+x, 3+y) = (3,-1) \Rightarrow \begin{cases} -2+x = 3 \Rightarrow x = 5 \\ 3+y = -1 \Rightarrow y = -4 \end{cases}$

The translation vector is, $V = (5,-4)$

The image of (4,2) is $(4+5, 2+(-4)) = (9,-2)$

Exercise 6.1, #2

#52/ If a translation takes (1,-4) to (-3,2); find the image of the line $l: 5x+4y+9=0$

Sol: Translation Vector, $V = (-3-1, 2-(-4)) = (-4,6)$

$T(5x+4y+9=0) = 5(x-(-4)) + 4(y-6) + 9 = 0$

$= 5x+4y+5=0$

Exercise 6.2 #2

#53/ If the image of the line $2x-3y=7$ is $2x-3y=0$, what is the translation vector which is parallel to x-axis or y-axis?

Sols: Let $V = (a,b)$ be the translation vector.

$2(x-a) - 3(y-b) - 7 = 2x-3y+0=0$

$2x-2a-3y+3b-7 = 2x-3y+0$

$2x-3y+(3b-2a-7) = 2x-3y+0$

$\Rightarrow 3b-2a-7=0 \Rightarrow \boxed{3b-2a=7}$

If $a=0, (0, \frac{7}{3})$ is the translation vector // to the y-axis

If $b=0, (-\frac{7}{2}, 0)$ is the translation vector // to the x-axis

Exercise 6.2, #3

#54/ Find the equation of the image of the circle $(x+1)^2+(y+2)^2=7$ when translated by the vector $\overrightarrow{AB}$, where A=(2,3) and B=(-5,4)

Sols: Translation Vector, $(\overrightarrow{AB}) = (-5-2, 4-3] = (-7,1)$

$T((x+1)^2+(y+2)^2=7] = ((x-7+1)^2 + (y-1+2)^2 = 7$

$= (x+11)^2 + (y+1)^2 = 7$

Exercise 6.3 #3

2 Reflection

#55/ Find the image of (3,2) when reflected about the line $y=mx$, where $m=\tan\theta$ and $\theta = \frac{\pi}{4}$

Sol: $x' = x\cos 2\theta + y\sin 2\theta = 3\cos\frac{\pi}{2} + 2\sin\frac{\pi}{2} = 2$ //

$y' = x\sin 2\theta - y\cos 2\theta = 3\sin\frac{\pi}{2} - 2\cos\frac{\pi}{2} = 3$ //

The image of (3,2) is (2,3)

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#56/ Find the image of P(3,2), Q(0,1) and (-5,7) when reflected about the line $y = \frac{1}{\sqrt{3}}x$

Soln Slope $= \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ or $30°$

• $(3,2) \Rightarrow \begin{cases} x' = x\cos 2\theta + y\sin 2\theta = 3\cos\left(\frac{\pi}{3}\right) + 2\sin\left(\frac{\pi}{3}\right) = \frac{3}{2} + \frac{2\sqrt{3}}{2} = \frac{3+2\sqrt{3}}{2} \\ y' = x\sin 2\theta - y\cos 2\theta = 3\sin\left(\frac{\pi}{3}\right) - 2\cos\frac{\pi}{3} = \frac{3\sqrt{3}}{2} - 1 = \frac{3\sqrt{3}-2}{2} \end{cases}$

The image of P(3,2) is $P'\left(\frac{3+2\sqrt{3}}{2}, \frac{3\sqrt{3}-2}{2}\right)$

• $(0,1) \Rightarrow \begin{cases} x' = x\cos 2\theta + y\sin 2\theta = 0 + 1\cdot\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} // \\ y' = x\sin 2\theta - y\cos 2\theta = 0 - 1\cos\frac{\pi}{3} = -\frac{1}{2} // \end{cases}$

The image of Q(0,1) is $Q'\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

• $(-5,7) \Rightarrow \begin{cases} x' = x\cos 2\theta + y\sin 2\theta = -5\cdot\frac{1}{2} + 7\cdot\frac{\sqrt{3}}{2} = \frac{-5+7\sqrt{3}}{2} // \\ y' = x\sin 2\theta - y\cos 2\theta = -5\cdot\frac{\sqrt{3}}{2} - 7\cdot\frac{1}{2} = \frac{-7-5\sqrt{3}}{2} \end{cases}$

The image of (-5,7) is $\left(\frac{-5+7\sqrt{3}}{2}, -\frac{7-5\sqrt{3}}{2}\right)$

#57/ Find the image of A=(1,-2) after it has been reflected in the line $y = 2x$

Soln



Given $y = 2x \Rightarrow \tan\theta = 2 \Rightarrow$

$\frac{y}{x} = \frac{2}{1} \Rightarrow x = 1$ & $y = 2$ //

$x^2 + y^2 = r^2$
$2^2 + 1^2 = r^2$
$5 = r^2$
So $r = \sqrt{5}$

$\rightarrow \sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}}$ & $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} = \frac{4}{5}$ //

$\rightarrow \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}}$ & $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2 = -\frac{3}{5}$ //

• $(1,-2) \Rightarrow \begin{cases} x' = x\cos 2\theta + y\sin 2\theta = 1\cdot\left(-\frac{3}{5}\right) + (-2)\cdot\frac{4}{5} = -\frac{11}{5} // \\ y' = x\sin 2\theta - y\cos 2\theta = 1\cdot\frac{4}{5} - (-2)\cdot\left(-\frac{3}{5}\right) = -\frac{2}{5} // \end{cases}$

The image of (1,-2) about $y = 2x$ is $\left(-\frac{11}{5}, -\frac{2}{5}\right)$

#58/ Find the image of the point P(x,y) after the reflection in x-axis and y-axis, $y = x$ and $y = -x$

Soln From $y = mx$,

Case 1. When $y = 0$ or $\tan\theta = \frac{0}{x} \Rightarrow \theta = 0°$, $y = 0$ (reflection in the x-axis)

$P(x,y) = P'(x\cos 2\theta + y\sin 2\theta,\ x\sin 2\theta - y\cos 2\theta)$
$= P'(x\cos 0° + y\sin 0°,\ x\sin 0° - y\cos 0°)$
$= P'(x\cdot 1, -y\cdot 1) = P'(x, -y)$

$P(x,y) = P'(x,-y)$ → Reflection in the line $y = 0$ (slope $= m = 0$) or in x-axis.

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Case 2. From $y=mx$,

reflection in the line $x=0$ $(y\text{-axis})$, $\tan\theta = m = \frac{y}{0}$ → undefined

$\theta = 90°$ and $2\theta = 180°$

$P(x,y) = P'(x\cos 2\theta + y\sin 2\theta,\ x\sin 2\theta - y\cos 2\theta)$

$= P'(x\cos 180° + y\sin 180°,\ x\sin 180° - y\cos 180°)$

$= P'[x*(-1) + y*0,\ x*0 - y*(-1)]$

$P(x,y) = P'(-x, y)$ → reflection in the line $x=0$ $(y\text{-axis})$

Case 3. From $y = mx$

reflection in the line $y = x$ $(m = \tan\theta = 1) \Rightarrow \theta = 45°$ & $2\theta = 90°$

$P(x,y) = P'(x\cos 2\theta + y\sin 2\theta,\ x\sin 2\theta - y\cos 2\theta)$

$= P'(x\cos 90° + y\sin 90°,\ x\sin 90° - y\cos 90°)$

$= P'(y\sin 90°,\ x\sin 90°)$

$P(x,y) = P'(y, x)$ → reflection in the line $y = x$ $(\theta = 45°)$

Case 4 From $y = mx$

reflection in the line $y = -x$ $(m = \tan\theta = -1)$ $\theta = \frac{3\pi}{4}$ or $135°$

$2\theta = 270°$

$P(x,y) = P'(x\cos 2\theta + y\sin 2\theta,\ x\sin 2\theta - y\cos 2\theta)$

$= P'(x\cos 270° + y\sin 270°,\ x\sin 270° - y\cos 270°)$

$= P'(x\sin 270°,\ x\sin 270°)$

$= P'(y*(-1),\ x*(-1))$

$P(x,y) = P'(-y, -x)$ → reflection in the line $y = -x$ $(\theta = 135°)$

59/ Find the image of $(0,1)$ when reflected about the line $y = \sqrt{3}x$

Soln Slope $= m = \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$ & $2\theta = 120°$

$P'(x',y') = (x\cos 120° + y\sin 120°,\ x\sin 120° - y\cos 120°)$ Exercise 6.5 #1

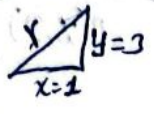
$= (0*\cos 120° + y\sin 120°,\ 0*\sin 120° - y\cos 120°)$

$= (y\sin 120°,\ -y\cos 120°) = \left(y*\frac{\sqrt{3}}{2},\ -y*\left(-\frac{1}{2}\right)\right) =$

$P'(x',y') = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

60/ Find the image of $(2,3)$ after reflection about $y = 3x$

Soln Slope $= m = \tan\theta = 3 \Rightarrow \tan\theta = \frac{y}{x} = 3 \Rightarrow y = 3,\ x = 1$



$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{3^2+1^2} = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{10}}$ and $\sin 2\theta = 2\sin\theta\cos\theta = 2*\frac{3}{\sqrt{10}}*\frac{1}{\sqrt{10}} = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{10}}$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2 = \frac{1}{10} - \frac{9}{10} = \frac{-8}{10}$

$\Rightarrow \sin 2\theta = \frac{3}{5}$ & $\cos 2\theta = -\frac{4}{5}$

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$P'(x,y) = P'(x\cos 2\theta + y\sin 2\theta,\ x\sin 2\theta - y\cos 2\theta)$

$P'(2,3) = P'\left[2\left(-\frac{4}{5}\right)+3\times\frac{3}{5},\ 2\times\frac{3}{5}-3\left(-\frac{4}{5}\right)\right] = \left(\frac{-8}{5}+\frac{9}{5},\ \frac{6}{5}+\frac{12}{5}\right)$

$P'(2,3) = P'\left(\frac{1}{5},\ \frac{18}{5}\right) = \left(\frac{1}{5},\ \frac{18}{5}\right)$

#61/ Find the image of the line $2y+x=1$ after reflection in the line $y=2x-3$

Soln l_r = line of reflection, $y=2x-3$, slope $= 2$.

l = line to be reflected, $2y+x=1 \Rightarrow y=-\frac{1}{2}x+\frac{1}{2}$, slope $= -\frac{1}{2}$

$(\text{Slope})_1 * (\text{Slope}_2) = -\frac{1}{2}\times 2 = -1 \rightarrow l$ and l_r are $\perp$ lines

So, the image of l after reflection in l_r it itself $\Rightarrow l' = 2y+x=1$

#62/ Find the image of the line $y=2x+1$ after reflection in the line $y=2x-3$

Soln Line of reflection: $l_r = y=2x-3$, slope $= 2$

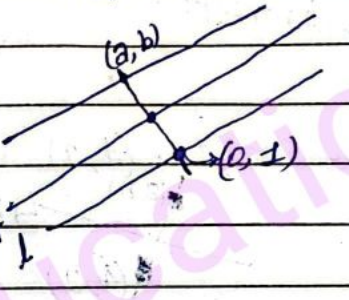
line to be reflected: l: $y=2x+1$, slope $= 2$

l and l_r are parallel, so the line, after reflection it also parallel to both l & l_r

So, l': $y=2x+b$ & find b

→ Take any point on l, let $(0,1)$

→ Take the image of $(0,1)$ be (a,b) on l'



(l, l_r, l')

② The line passing through $(0,1)$ & (a,b) is $\perp$ to all of the 3 lines

$\frac{(b-1)}{a-0} = -\frac{1}{2} \Rightarrow 2(b-1) = -a \Rightarrow \boxed{a+2b=2}$ → eqn (1)

③ The mid point of the point $(0,1)$ & (a,b) is on l_r

mid point of $(0,1)$ & (a,b) is $\left(\frac{a+0}{2}, \frac{b+1}{2}\right)$ → This point is on l_r

l_r: $y=2x-3$

$\frac{(b+1)}{2} = 2\left(\frac{a}{2}\right)-3 \Rightarrow \frac{(b+1)}{2} = a-3 \Rightarrow b+1 = 2a-6$

$\Rightarrow \boxed{2a-b=7}$ → eqn (2)

find "a" and "b" using the two eqns

$a+2b=2$ }
$2a-b=7$ }

$a+2b=2$
$4a-2b=14$ ← multiply by 2 using elimination method

$5a=16,\ a=\frac{16}{5}$

$2a-b=7 \Rightarrow 2\left(\frac{16}{5}\right)-7=b$

$\Rightarrow \frac{32}{5}-7=b \Rightarrow b=\frac{-3}{5}$ //

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The point $(a,b) = \left(\frac{16}{5}, \frac{-3}{5}\right)$ lie on l'

$l': y = 2x + b$

$-\frac{3}{5} = 2\left(\frac{16}{5}\right) + b$

$-\frac{3}{5} - \frac{32}{5} = b$, so $b = -7$, since $y = 2x + b$

$y = 2x - 7$

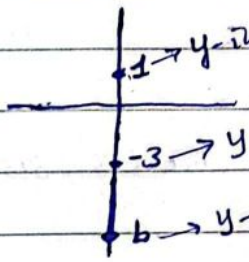
Finaly the image of the line $y = 2x + 1$ after reflected about the line $y = 2x - 3$ is $2x - 7$

Short Cut Method (Worknew. A)

y-intercept of $l: y = 2x + 1$ is 1

y-intercept of $l_r: y = 2x - 3$ is -3

The y intercepts of l & l' are located at equal distance from the y-intercept of l_r but in opposite direction



→ The distance b/n 1 & -3 = 4

∴ So, go down 4 unit from -3 we get -7, so $b = -7$

image is $y = 2x + b = 2x - 7$

#63/ Find the image of (4,2) when reflected about the line $y = 2x - 1$

Soln



Exercise 6.6 #1

Let (a,b) be the image of $(4,2)$

(1) • The line which pass through (a,b) and $(4,2)$ is $\perp$ to $l_r: y = 2x - 1$

$\frac{b-2}{a-4} = \frac{-1}{2}$ → slope of the perpendicular line to $y = 2x - 1$

$2(b-2) = -a+4 \Rightarrow 2b - 4 = -a + 4$

$\boxed{a + 2b = 8}$ → equ (1)

(2) The mid point of the points (a,b) and $(4,2)$ lie on $l: y = 2x - 1$

Mid point of the points $= \left(\frac{a+4}{2}, \frac{b+2}{2}\right)$ lie on $y = 2x - 1$

$y = 2x - 1$

$\frac{b+2}{2} = 2\left(\frac{a+4}{2}\right) - 1$

$\frac{b+2}{2} = a + 3 \Rightarrow b + 2 = 2a + 6$

$\boxed{2a - b = -4}$ → equ (2)

$\begin{pmatrix} a + 2b = 8 \\ 2a - b = -4 \end{pmatrix} \Rightarrow a + 2b = 8$

$(4a - 2b = -8) \Rightarrow a = 0$

$b = 4 \Rightarrow (0, 4)$

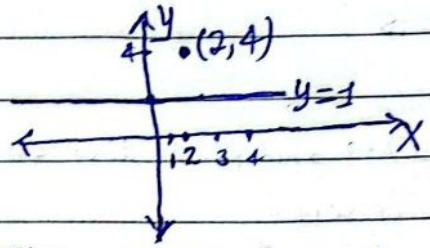
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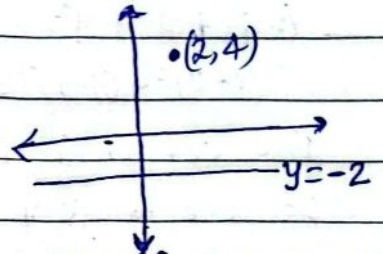
Exercise 6.6 #3

#64. Find the image of (2,4) when reflected about the line $y=1$ and $x=-2$

Soh



Exercise 6.6 #3



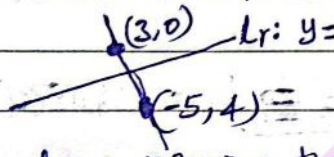
The x-value is unchanged
The y-value is 3 units down ($4-1=3$) from $y=1$ $\Rightarrow P(2,4)=P'(2,-2)$

The value of x is unchanged
The value of y is 6 units down ($4-(-2)=6$) from $y=-2$
$\therefore P(2,4)=P'(2,-8)$

#65. If the image of the point $(-5,4)$ under reflection is $(3,0)$, find the line of reflection.

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Exercise 6.6 #4



① The line passing through the point $(-5,4)$ and its image $(3,0)$ is $\perp$ to the line of reflection

$\frac{0-4}{3-(-5)} = \frac{-4}{8} = \frac{-1}{2}$ → slope of the line passing through $(-5,4)$ & $(3,0)$

So, the line of reflection has slope $=2$, so $m=2$

② The mid point of $(-5,4)$ and $(3,0)$ lie on the line of reflection

mid Point $= \left(\frac{-5+3}{2}, \frac{4+0}{2}\right) = (-1,2)$ ∈ Lr

$Lr: y = 2x+B$

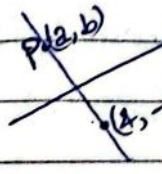
$2 = 2(-1)+B \Rightarrow B=4$

$Lr: y=2x+4$ is the line of reflection.

#66. If P is the reflection of point $(4,-4)$ in the line $2y=x+1$ find the coordinate of P.

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Exercise 6.6 #5



$Lr: 2y=x+1 \Rightarrow y=\frac{1}{2}x+\frac{1}{2}$

So the line passing through (a,b) and $(4,-4)$ has slope $=-2$

$\frac{b-(-4)}{a-4} = \frac{b+4}{a-4} = -2 \Rightarrow b+4 = -2a+8 \Rightarrow \boxed{2a+b=4}$

mid point is on Lr $\Rightarrow \frac{(b-4)}{2} = \frac{1}{2}\left(\frac{a+4}{2}\right)+\frac{1}{2} \Rightarrow \boxed{a-2b=-14}$

$P=(a,b)=\left(\frac{-6}{5}, \frac{32}{5}\right)$

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Example 6.9

#67. Find the image of the circle $x^2+y^2-4x-2y+4=0$ after reflection in the line $l: y=x+1$

Soln $x^2-4x+4+y^2-2y+1=1$

$(x-2)^2+(y-1)^2=1^2$

The center of circle is $(2,1)$ and $(2,1)$ lies on l_r

So the image of the circle is itself.

#68. Find the equation of the image of the circle $(x-4)^2+(y-1)^2=9$ after reflection about the line $y=x+1$

Sol: The center of circle $=(4,1)$

$l_r: y=x+1$

Slope of the $l_r = 1$

Slope of the line passing through (a,b) & $(4,1) = -1$

$\frac{(b-1)}{(a-4)} = -1 \Rightarrow (b-1) = -a+4$

$a+b=5$ → equ ①

Exercise 6.8 #2

$\frac{(b+1)}{2} = \frac{(a+4)}{2} + 1$

$b+1 = a+4+2$

$a-b=-1$ → equ ②

solving equ ① & equ ②

$a+b=5$

$a-b=-1$ $\Rightarrow a=2, b=3$

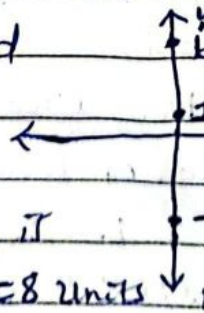
$(x-2)^2+(y-3)^2=9$

#69. Find the image of the line $y=-3x-7$ after reflection about the line $y=-3x+9$

Soln The given line is $\parallel$ to the line of reflection (have the same slope)

The image of the line after reflection of l is parallel to the line of reflection and line required to be reflected

Using Short cut Method



$l': y=-3x+b$

The y intercept of l' is located at $|1-(-7)|=8$ units upward from $y=1 \Rightarrow 8$ units above $y=1$ is $y=9$

So $y=-3x+9$