

**AYELECH DEGEFU MEMORIAL
GENERAL SECONDARY
AND HIGHER EDUCATION PREPARATORY SCHOOL**



**Mathematics Lecture Notes
For Grade 10**

UNIT-5

TRIGONOMETRIC FUNCTIONS

After completing this unit, you should be able to:

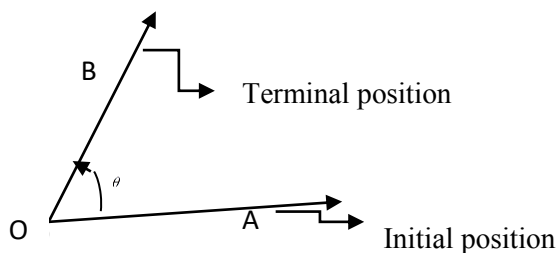
- ❖ know principles and methods for sketching graphs of basic trigonometric functions.
- ❖ understand important facts about reciprocals of basic trigonometric functions.
- ❖ identify trigonometric identities.
- ❖ solve real life problems involving trigonometric functions.

5.1 BASIC TRIGONOMETRIC FUNCTIONS

5.1.1. The Sine, Cosine and Tangent Functions

Basic Terminologies

Angle: An angle is the amount of rotation about a fixed point O from an initial position to new position.



The angle formed by a ray rotating anticlockwise is taken to be a positive angle.

An angle formed by a ray rotating clockwise is taken to be a negative angle.

Angles in standard position

An angle in the coordinate plane is said to be in standard position, if

1. its vertex is at the origin, and
2. its initial side lies on the positive x-axis.

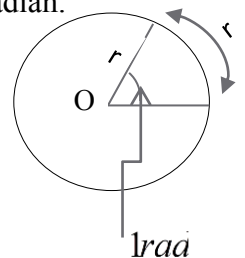
Quadrantal angles

If the terminal side of an angle in standard position lies along the x-axis or the y-axis, then the angle is called a *quadrantal angle*.

Example: $\dots -180^\circ, -90^\circ, 0^\circ, 90^\circ, 180^\circ, \dots$

Radian measure of angles

- The angle θ subtended at the center of a circle by an arc equal in length to the radius is 1 radian.
That is $\theta = r/r = 1$ radian.
- In general, if the length of the arc is s units and the radius is r units, then $\theta = s/r$ radians.



Relationship Between Degrees and Radians

- A circle with radius r units has a circumference of $2\pi r$
- $\theta = \frac{2\pi r}{r} = 2\pi$ radians
 $\Rightarrow 180^\circ = \pi$ radians

To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$

$$\text{i.e., radians} = \text{degrees} \times \frac{\pi}{180^\circ}$$

1.

2. To convert radians to degrees, multiply by $\frac{180^\circ}{\pi}$

$$\text{i.e., degrees} = \text{radians} \times \frac{180^\circ}{\pi}$$

Reference angle(θ_R)

❖ If θ is an angle in standard position whose terminal side does not lie on either coordinate axis, then a reference angle θ_R for θ is the acute angle formed by the terminal side of θ and the x-axis.

❖ In different quadrants we have different formulas for θ_R

1. In first quadrant $\theta = \theta_R$ (If $0^\circ < \theta < 90^\circ$)

2. In second quadrant $\theta_R = 180^\circ - \theta$. In this quadrant,

$$\text{i. } \sin \theta = \sin (180^\circ - \theta) \quad \text{ii. } \cos \theta = -\cos (180^\circ - \theta) \quad \text{iii. } \tan \theta = -\tan (180^\circ - \theta)$$

3. In third quadrant $\theta_R = \theta - 180^\circ$. In this quadrant,

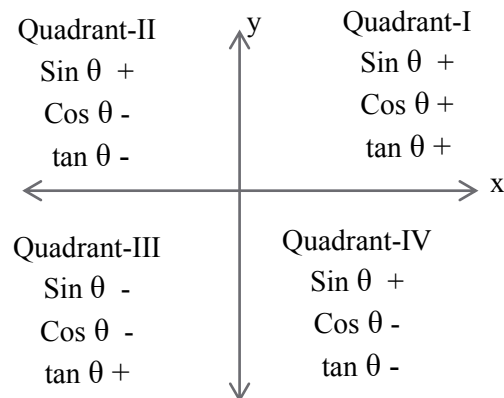
$$\text{i. } \sin \theta = -\sin (\theta - 180^\circ) \quad \text{ii. } \cos \theta = \cos (\theta - 180^\circ) \quad \text{iii. } \tan \theta = \tan (\theta - 180^\circ)$$

4. In fourth quadrant $\theta_R = 360^\circ - \theta$. In this quadrant,

$$\text{i. } \sin \theta = -\sin (360^\circ - \theta) \quad \text{ii. } \cos \theta = \cos (360^\circ - \theta) \quad \text{iii. } \tan \theta = -\tan (360^\circ - \theta)$$

Note: The above formula for θ_R in all quadrants is for rotation to anticlockwise direction and $0^\circ < \theta < 360^\circ$

Question: How do you find θ_R for angles greater than 360° ? For negative angles?



Note: 1. If the sum of the measures of two acute angles is 90° , then the two angles are said to be Complementary angles.

Example: 2° & 88° , 31° & 59°

2. For two complementary angles α & β ,

$$\sin \alpha = \cos \beta, \cos \alpha = \sin \beta \text{ \& } \tan \alpha = \frac{1}{\tan \beta}$$

3. If the sum of the measures of two angles is 180° , then the two angles are said to be Supplementary angles.

Example: 129° & 51° , 170° & 10°

Co-terminal angles

- Co-terminal angles are angles in standard position that have a common terminal side.

Example: The four angles $60^\circ, -300^\circ, 420^\circ, 780^\circ$ are co-terminal angles.

- Given an angle θ , all angles which are co-terminal with θ are given by the formula $\theta \pm n(360^\circ)$, where $n = 1, 2, 3, \dots$

Example: Find two co-terminal angles with

- a. -95° b. 230° c. -126°

Solution:

- a. -95°
If $n = 2$, $-95^\circ + 2(360^\circ) = -95^\circ + 720^\circ = 625^\circ$ or
 $-95^\circ - 2(360^\circ) = -95^\circ - 720^\circ = -815^\circ$
b. & c ---- exercise

Note: Co-terminal angles have the same trigonometric values for the same trigonometric function.

Example: $\sin 30^\circ = \sin 390^\circ = \sin(-330^\circ) = 0.5$

5.1.2. Graphs of the Sine, Cosine and Tangent Functions

- ❖ A function that repeats its values at regular intervals is called a periodic function.

1. Graph of the sine function ($y = \sin \theta$)

- The sine function repeats after every 360° or 2π radians. Therefore the period of sine function is 360° or 2π
- The domain of sine function is $\mathbb{R}$.
- The range of sine function is $-1 \leq y \leq 1$

Reading assignment: Read graphs of the sine function from your text book on page 194-196 and study its properties.

2. Graph of the cosine function ($y = \cos \theta$)

- The cosine function repeats after every 360° or 2π radians. Therefore the period of cosine function is 360° or 2π
- The domain of cosine function is $\mathbb{R}$.
- The range of cosine function is $-1 \leq y \leq 1$

Reading assignment: Read graphs of the cosine function from your text book on page 196-197 and study its properties.

3. Graph of the tangent function

a. The tangent function repeats after every 180° or π radians. Therefore the period of tangent function is 180° or π

b. The domain of tangent function = $\left\{ \theta : \theta \neq n\frac{\pi}{2}, \text{ where } n \text{ is an odd integer} \right\}$

i.e when $\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \text{ etc}$

c. The range tangent function is $\mathbb{R}$

Reading assignment: Read graphs of the tangent function from your text book on page 197-199 and study its properties.

5.2. SIMPLE TRIGONOMETRIC IDENTITIES

Let θ be an angle in standard position and P(x, y) be a point on the terminal side of θ and r is the distance of p from O(0,0).

1. Pythagorean Identities

$$\text{i. } (\cos\theta)^2 + (\sin\theta)^2 = 1 \quad \text{ii. } (\tan\theta)^2 + 1 = (\sec\theta)^2 \quad \text{iii. } (\cot\theta)^2 + 1 = (\csc\theta)^2$$

Note: $(\sin\theta)^2 = \sin^2\theta$, $(\cos\theta)^2 = \cos^2\theta$, $(\tan\theta)^2 = \tan^2\theta$
 $(\csc\theta)^2 = \csc^2\theta$, $(\sec\theta)^2 = \sec^2\theta$, $(\cot\theta)^2 = \cot^2\theta$

$$\text{2. Reciprocal Identities} \quad \text{i. } \csc\theta = \frac{1}{\sin\theta} \quad \text{ii. } \sec\theta = \frac{1}{\cos\theta} \quad \text{iii. } \cot\theta = \frac{1}{\tan\theta}$$

$$\text{3. Quotient Identities} \quad \text{i. } \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{ii. } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

❖ An identity is an equation that is true for all values of the variable for which both sides of the equation are defined.

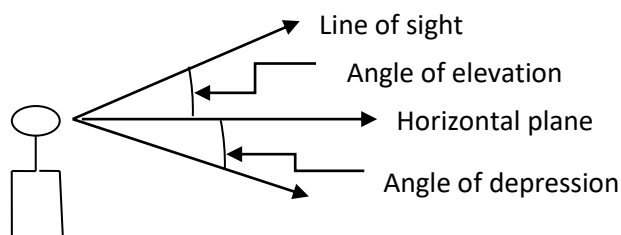
Note: 1. Sine, Cosine and tangent functions are the basic trigonometric functions.

2. Cosecant is the reciprocal of sine, secant is the reciprocal cosine and cotangent is the reciprocal of tangent.

3, Sine and cosine, Secant and Cosecant and tangent and cotangent are co-functions.

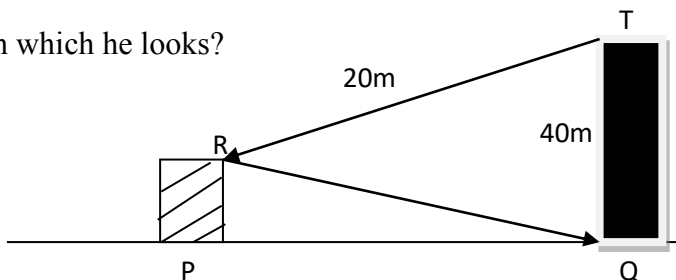
Angle of Elevation and Angle of Depression

- Angle of elevation is an angle between the horizontal line and line of sight
- The line of sight of an object is the line joining the eye of an observer and the object.
- If the object is above the horizontal line (plane) through the eye of the observer the angle between the line of sight and the horizontal plane is called *angle of elevation* .
- If the object is below the horizontal line (plane) through the eye of the observer the angle between the line of sight and the horizontal plane is called *angle of depression* .



Example 1: A 20m cable from T, on top of building is joined to R, the top of another building across the street PQ, making an angle of depression of 30° . The building TQ is 40m high. An observer at R looks at Q, the base of building across the street.

- What is the distance RQ through which he looks?
- How high is the building?



Solution: The problem can be represented as follows.

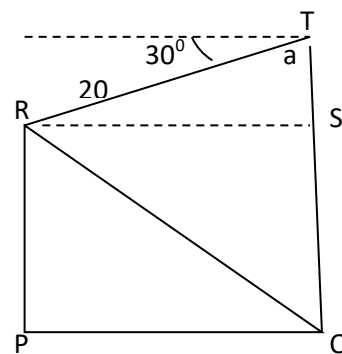
In $\triangle TRQ$, $a = 90^\circ - 30^\circ = 60^\circ$

$$\begin{aligned} RQ^2 &= 20^2 + 40^2 - 2(20)(40\cos 60^\circ) \\ &= 400 + 1600 - 1600\cos 60^\circ \\ &= 2000 - 1600(0.5) \\ &= 2000 - 800 = 1200 \end{aligned}$$

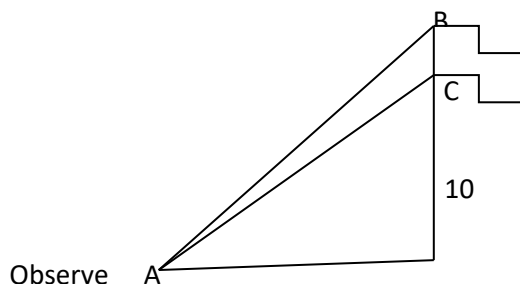
$$RQ = \sqrt{1200} = 20\sqrt{3}m$$

$$\begin{aligned} \text{In } \triangle RST, \cos 60^\circ &= \frac{TS}{20} \Rightarrow TS = 20\cos 60^\circ \\ &= 20(0.5) = 10m \end{aligned}$$

$$\text{Thus } PR = TQ - TS = 40 - 10 = 30m$$



Exercise 1: An observer looks through an angle of elevation of 60° at the top B of a flag on a flagpole BD. He then looks through an angle of 45° at the bottom C of flag on the pole. The bottom of the flag is 10m above the ground. Find the width BC of the flag and the distance of the observer from the pole.



Exercise 2: An observer walks towards a church tower. At B, the angle of elevation of the top of the tower is 20° . He walks 100m closer to A and notes that the angle of elevation is now 50° . Determine

- The height of the tower CD
- The distance from the tower at A.

UNIT-6 PLANE GEOMETRY

6.1. THEOREMS ON TRIANGLES

- A triangle is a polygon with three sides and the simplest type of polygon
- A line that divides an angle into two congruent angles is called angle bisector.
- A line that divides a line segment into two congruent line segments is called a bisector of the line segment.
- When a bisector of a line segment forms a right angle with line segment, then it is called the perpendicular bisector of the line segment.

Median of a triangle

- ✓ A median of a triangle is a line segment drawn from any vertex to the mid point of the opposite side.
- ✓ The medians of a triangle are concurrent at a point $\frac{2}{3}$ of the distance from each vertex to the mid point of the opposite side.
- ✓ The point of intersection of the medians of a triangle is called the centroid of the triangle.
- ✓ The medians of an equilateral triangle are equal in length.
- ✓ In a right angled triangle the length of the median to the hypotenuse is equals one half the length of the hypotenuse.

Altitude of a triangle

- ❖ The altitude of a triangle is a line segment drawn from a vertex perpendicular to the opposite side, or to the opposite side produced.
- ❖ The altitudes of a triangle are concurrent at a point called Orthocenter of the triangle.
- ❖ For acute angled triangle Orthocenter lies inside the triangle.
- ❖ For right angled triangle Orthocenter is the vertex of a right angle.
- ❖ For obtuse triangle the Orthocenter lies outside the triangle.

Bisector of a triangle

- A line segment that passes through the mid-point of a side of a triangle and perpendicular to the side is called perpendicular bisector of a triangle.
- The perpendicular bisectors of the sides of any triangle are concurrent at a point which is equidistant from the vertices of the triangle.
- The point of intersection of the perpendicular bisectors of a triangle is called Circumcenter of the triangle.
- The three perpendicular side bisector of a triangle for,
 - i. An acute triangle lies inside the triangle.
 - ii. An obtuse triangle lies outside the triangle.
 - iii. A right triangle lies on hypotenuse of the triangle.
- If a circle passes through the three vertices of a given triangle, then the center of the circle lies at Circumcenter.

Angle bisector of a triangle

- ✚ The angle bisectors of any triangle are concurrent at a point which is equidistant from the sides of the triangle.
- ✚ The point of intersection of the angle bisectors of a triangle is called the incentre of the triangle
- ✚ For all types of triangles incentre is inside the triangle.
- ✚ Incentre is the center of an inscribed circle of the triangle.