General Physics (Phys1011)

Winter 2019

Semester I, 2019/20 Academic Year

Chapter 5
Oscillations, Waves and Optics

- Simple Harmonic Motion
- The Simple Pendulum
- Wave and Its Characteristics
- Resonance
- Doppler Effect
- Image formation by thin lenses and mirrors

Oscillations and Mechanical Waves

- *Periodic motion is the repeating motion of an object in which it continues to return to a given position after a fixed time interval. The repetitive movements are called oscillations. A special case of periodic motion called simple harmonic motion will be the focus.*

- Simple harmonic motion also forms the basis for understanding mechanical waves. Oscillations and waves also explain many other phenomena quantity.
- Oscillations of bridges and skyscrapers
- Radio and television
- Understanding atomic theory

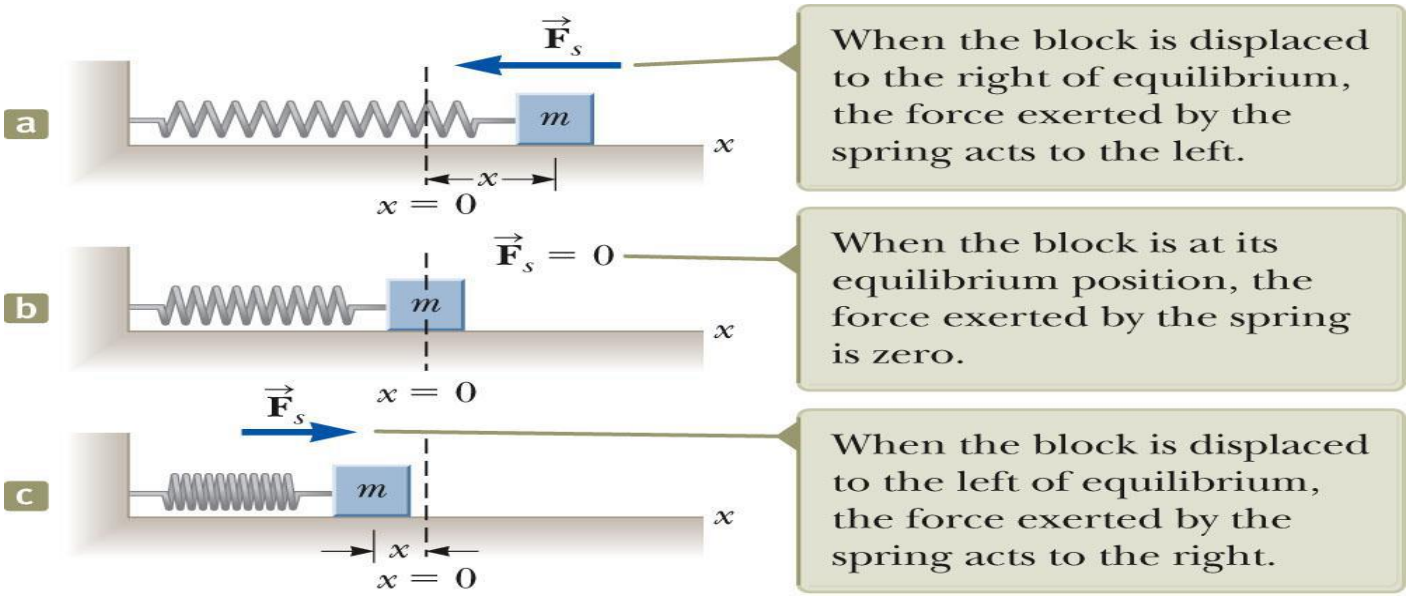
Periodic Motion

- ***Periodic motion is motion of an object that regularly returns to a given position after a fixed time interval. A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position.***
- If the force is always directed toward the equilibrium position, the motion is called ***simple harmonic motion.***

Motion of a Spring-Mass System

- A block of mass *m is attached to a spring, the block is free to move on a frictionless horizontal surface.When the spring is neither stretched nor compressed, the block is at the* ***equilibrium position.***

  $x = 0$ *; Such a system will oscillate back and forth if disturbed from its equilibrium position.*



Hooke's Law

Hooke's Law states $F_s = -kx$

- *Fs is the restoring force.*
  - It is always directed toward the equilibrium position. Therefore, it is always opposite the displacement from equilibrium.
- *k is the force (spring) constant.*
- *x is the displacement.*

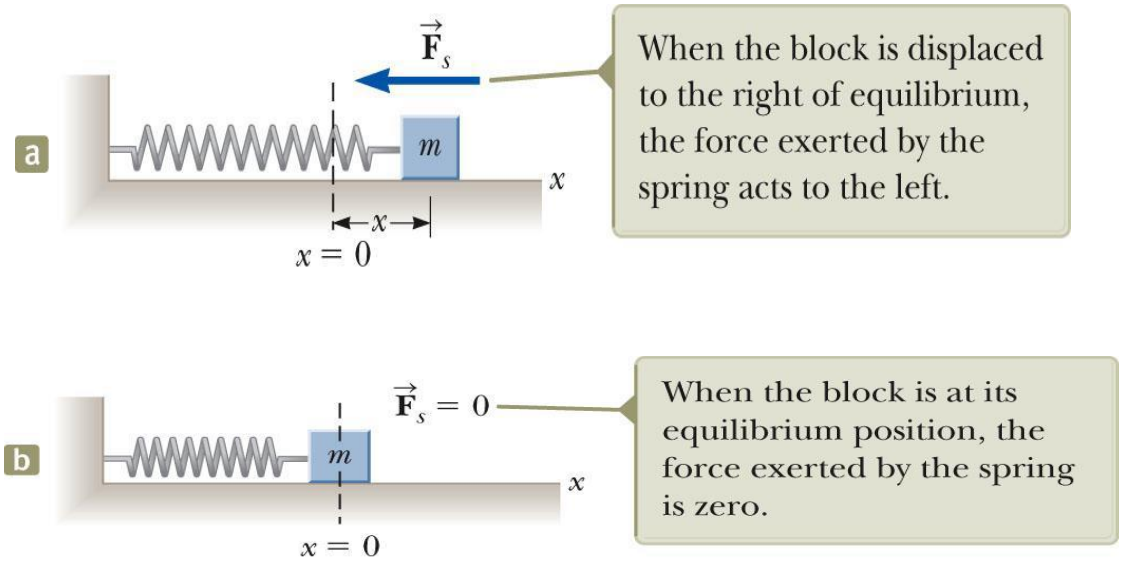
Restoring Force and the Spring Mass System

In a, the block is displaced to the right of $x = 0$.

- The position is positive and the restoring force is directed to the left.

In b, the block is at the equilibrium position. $x = 0$

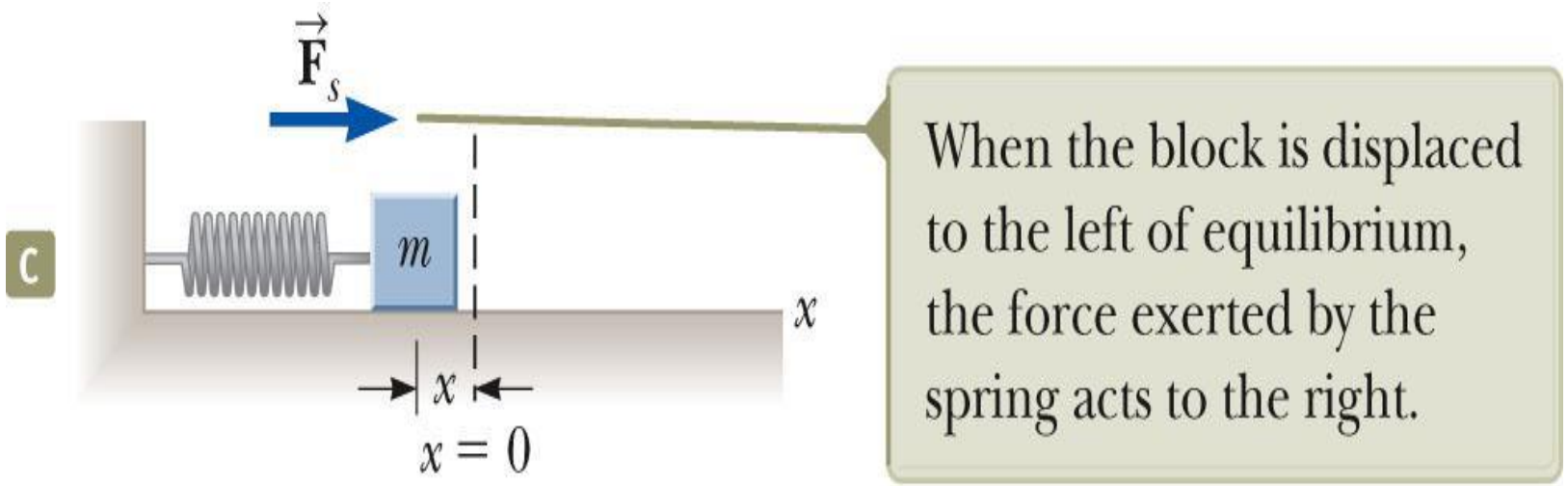
- The spring is neither stretched nor compressed and the force is 0.



Restoring Force, cont.

The block is displaced to the left of $x = 0$.

- The position is negative.
- The restoring force is directed to the right.



Acceleration

- When the block is displaced from the equilibrium point and released, it is a particle under a net force and therefore has an acceleration. The force described by Hooke's Law is the net force in Newton's Second Law.

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

- The acceleration is proportional to the displacement of the block. The direction of the acceleration is opposite the direction of the displacement from equilibrium.
- An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

Acceleration, cont'd...

The acceleration is ***not constant.***

- Therefore, the kinematic equations cannot be applied.
- If the block is released from some position $x = A$, *then the initial acceleration is* $-kA/m$.
- When the block passes through the equilibrium position, $a = 0$.
- The block continues to $x = -A$ *where its acceleration is* $+kA/m$.

Motion of the Block

The block continues to oscillate between –A and +A.

- These are turning points of the motion. The force is conservative. In the absence of friction, the motion will continue forever.
- Real systems are generally subject to friction, so they do not actually oscillate forever.

Analysis Model:
A Particle in Simple Harmonic Motion

Model the block as a particle.

- The representation will be **particle in simple harmonic motion model. Choose *x as the axis along which the oscillation occurs*.**

Acceleration

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

We let $\omega^2 = \frac{k}{m}$

Then $a = -\omega x$

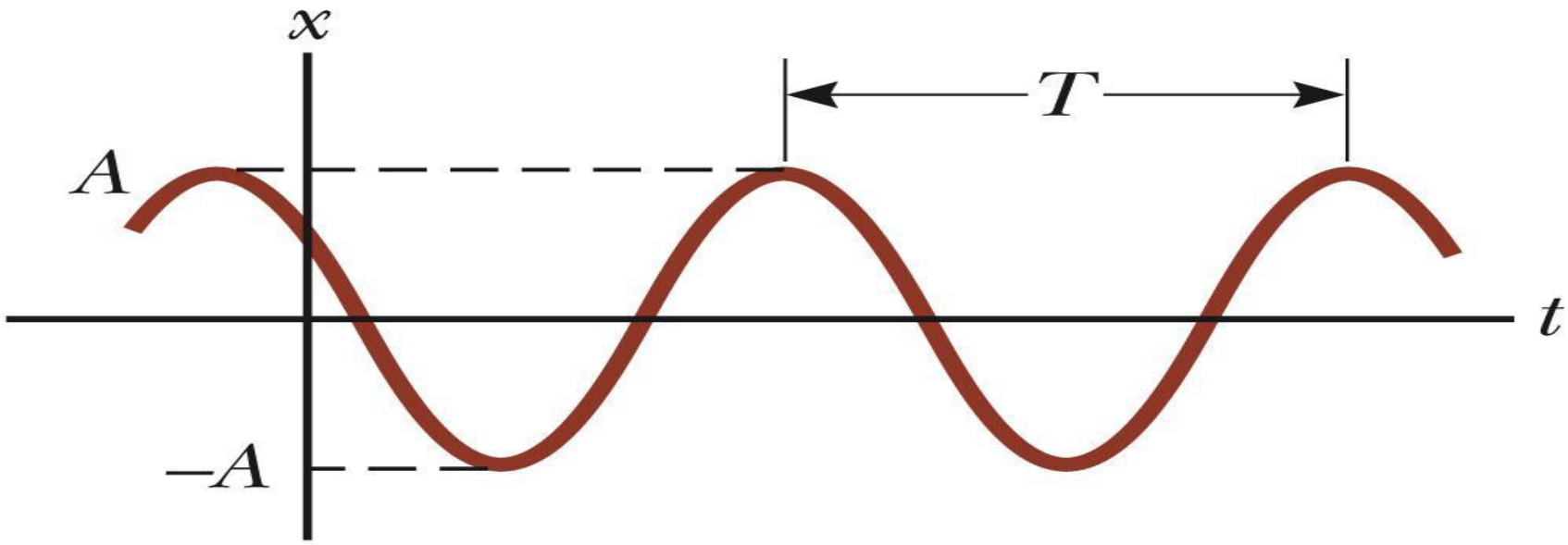
A Particle in Simple Harmonic Motion

- A function that satisfies the equation is needed.
- Need a function *x(t) whose second derivative is the same as the original function with a negative sign and multiplied by* $\omega 2$.
- The sine and cosine functions meet these requirements.

Simple Harmonic Motion – Graphical Representation

- A solution is $x(t) = A \cos(\omega t + \phi)$

A, ω, ϕ are all constants A cosine curve can be used to give physical significance to these constants.



a

Simple Harmonic Motion – Definitions

A is the amplitude of the motion.

- This is the maximum position of the particle in either the positive or negative *x direction*. ω *is called the angular frequency.*
- Units are rad/s

$$\omega = \sqrt{\frac{k}{m}}$$

ϕ is the phase constant or the initial phase angle.

Simple Harmonic Motion, cont.

A and **ϕ** are determined uniquely by the position and velocity of the particle at t = 0.

- If the particle is at x = A at t = 0, then **ϕ**= 0 The **phase of the motion is the quantity (ωt + ϕ).**
- **x (t) is periodic and its value is the same each time ωt increases by 2π radians.**

Period

The ***period, T, of the motion is the time interval required for the particle to go through one full cycle of its motion.***

- The values of *x and v for the particle at time t equal the values of x and v at t + T.*

$$T = \frac{2\pi}{\omega}$$

Frequency : The inverse of the period is called the ***frequency. The frequency represents the number of oscillations that the particle undergoes per unit time interval.***

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Units are cycles per second = hertz (Hz).

Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

Simple harmonic motion is one-dimensional and so directions can be denoted by + or - sign. Remember, simple harmonic motion is **not uniformly accelerated motion.**

Maximum Values of v and a

Because the sine and cosine functions oscillate between ± 1, we can easily find the maximum values of velocity and acceleration for an object in SHM.

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}\,A$$

$$a_{max} = \omega^2 A = \frac{k}{m}A$$

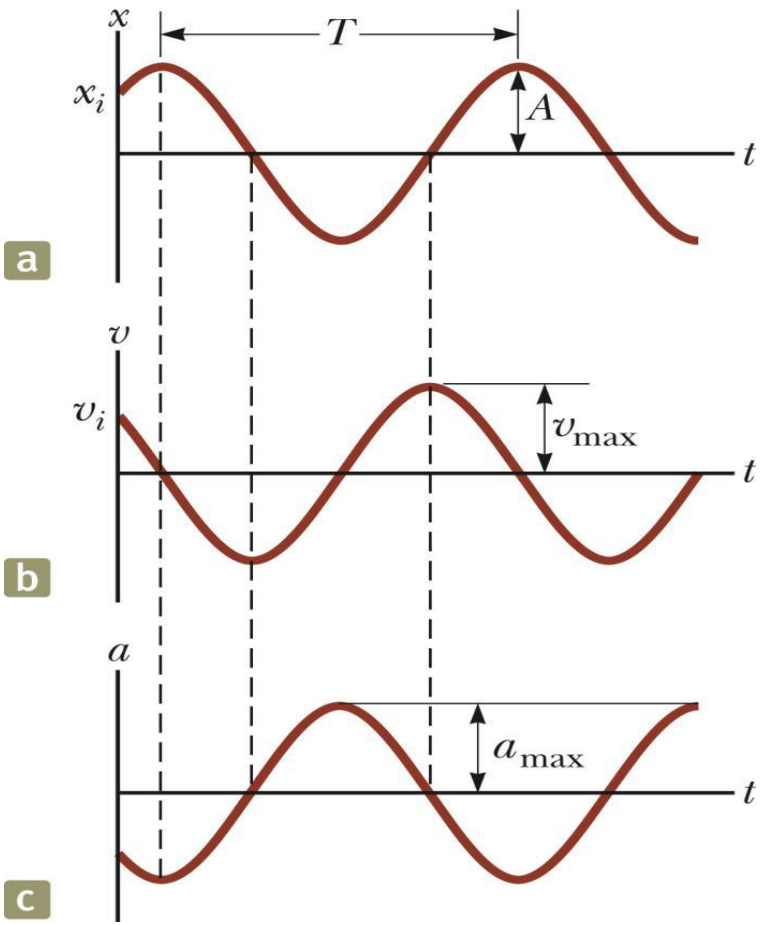
Graphs

The graphs show:

(a) displacement as a function of time

(b) velocity as a function of time

(c) acceleration as a function of time The velocity is 90 degree out of phase with the displacement and the acceleration is 180 degree out of phase with the displacement.



Energy of the SHM Oscillator

Mechanical energy is associated with a system in which a particle undergoes simple harmonic motion.

- For example, assume a spring-mass system is moving on a frictionless surface. Because the surface is frictionless, the system is isolated.
- This tells us the total energy is constant. The kinetic energy can be found by
  - $K = \frac{1}{2} mv = \frac{1}{2} m\omega A\sin(\omega t + \phi)$

Assume a massless spring, so the mass is the mass of the block. The elastic potential energy can be found by

- $U = \frac{1}{2} kx = \frac{1}{2} kA \cos(\omega t + \phi)$

The total energy is $E = K + U = \frac{1}{2} kA$

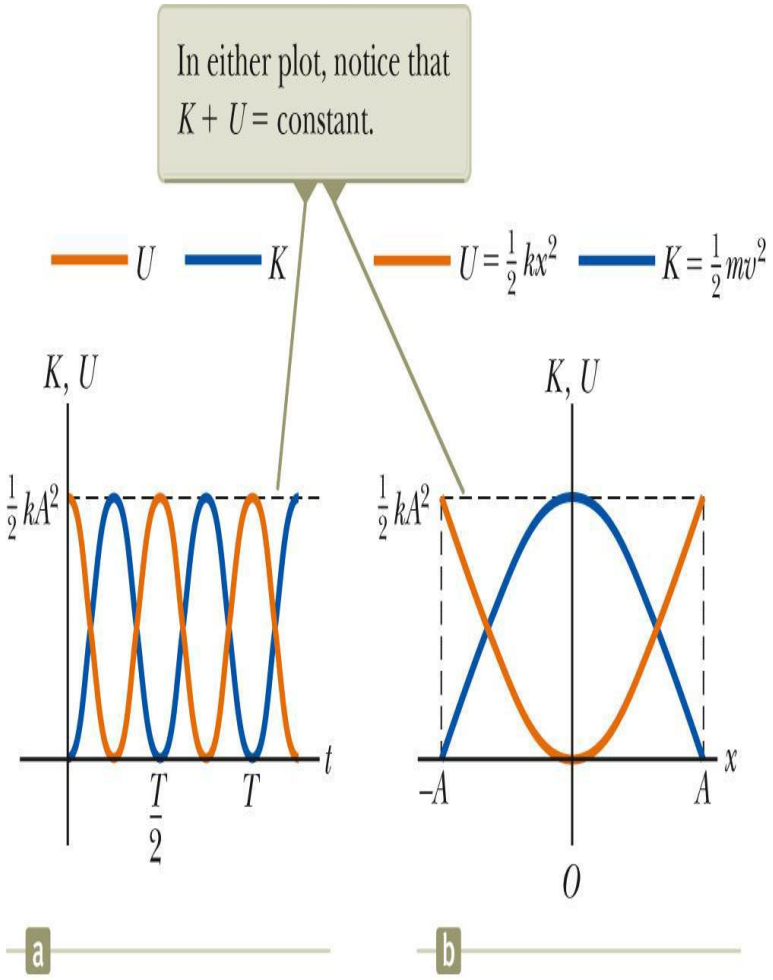
Energy of the SHM Oscillator, cont'd..

The total mechanical energy is constant. ½ k A 2

- At all times, the total energy is

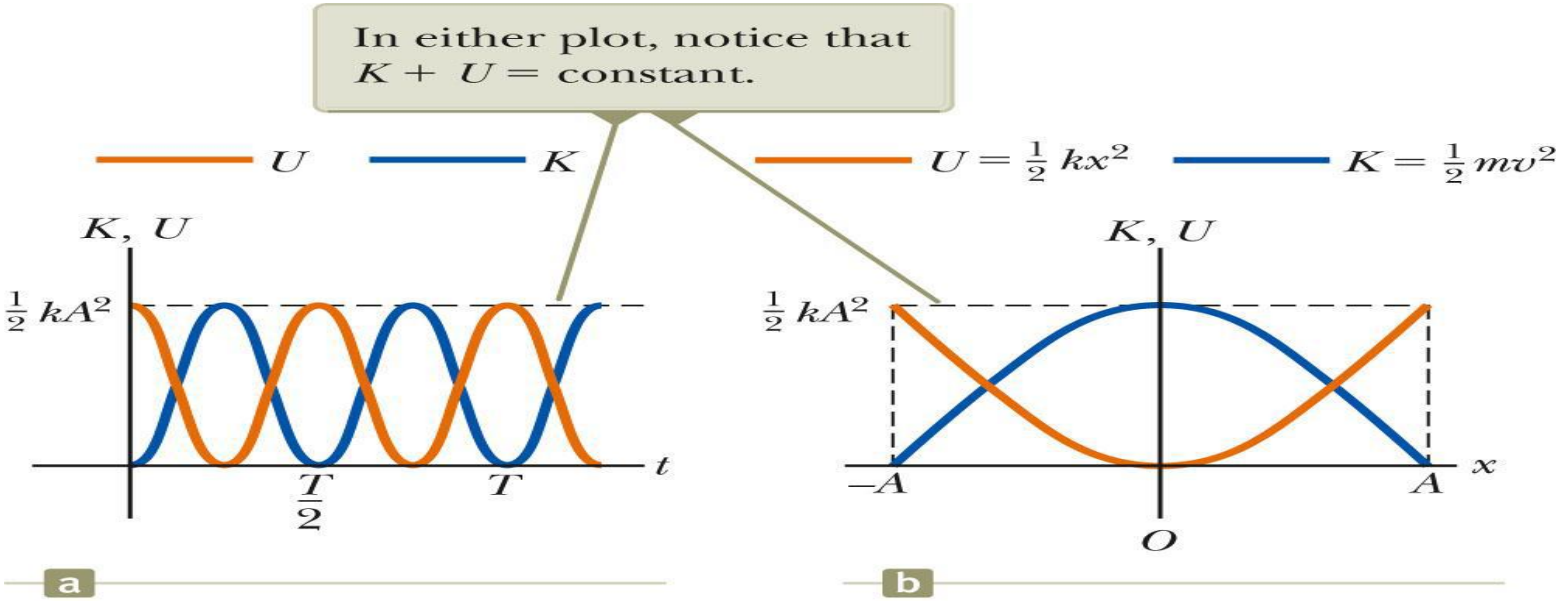
The total mechanical energy is proportional to the square of the amplitude. Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block.

- In the diagram, $\Phi = 0$

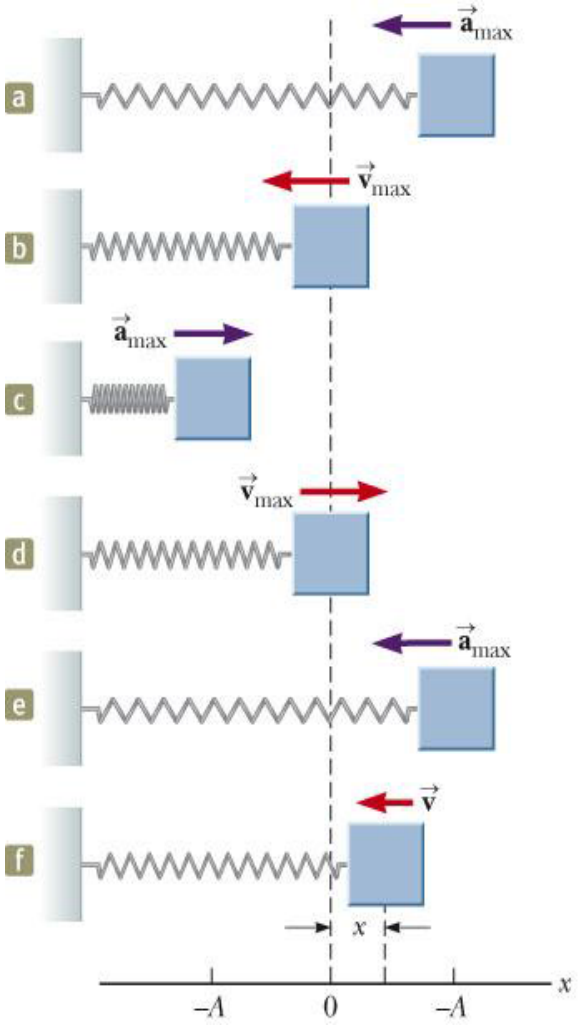


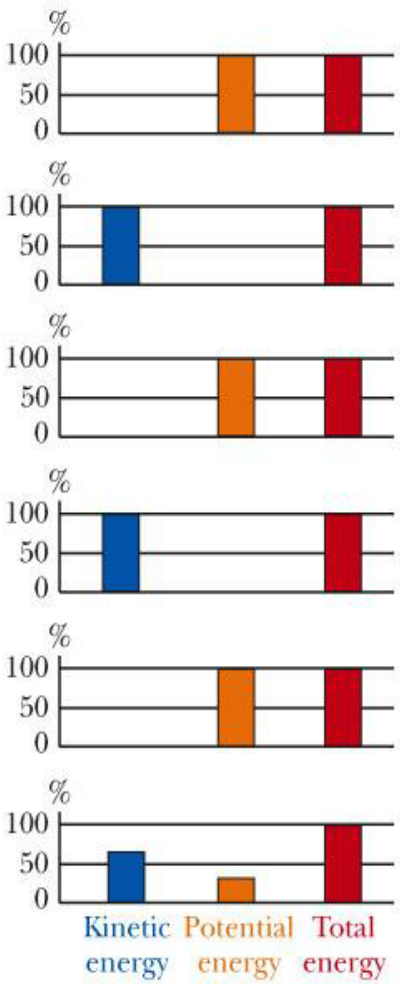
Energy of the SHM Oscillator, cont'd..

Variations of K and U can also be observed with respect to position. The energy is continually being transformed between potential energy stored in the spring and the kinetic energy of the block. The total energy remains the same



Energy in SHM, Summary





t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{T}{4}$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$\frac{T}{2}$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{3T}{4}$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
t	x	v	$-\omega^2 x$	$\frac{1}{2}mv^2$	$\frac{1}{2}kx^2$

Velocity at a Given Position

- Energy can be used to find the velocity:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$= \pm\omega^2\sqrt{A^2 - x^2}$$

Example

An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation

$$x = (4.00\ \text{m}) \cos\left(\pi t + \frac{\pi}{4}\right)$$

where t is in seconds and the angles in the parentheses are in radians.

(A) Determine the amplitude, frequency, and period of the motion.

$x = A\cos(\omega t + \phi)$, we see that $A =$ 4.00 m and

$\omega = \pi$ rad/s. Therefore, $f = \omega/2\pi = \pi/2\pi =$ 0.500 Hz

and $T = 1/f =$ 2.00 s.

(B) Calculate the velocity and acceleration of the object at any time t.

Solution Differentiating x to find v, and v to find a, we obtain

$$v = \frac{dx}{dt} = -(4.00\ \text{m/s}) \sin\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t)$$

$$= -(4.00\pi\ \text{m/s}) \sin\left(\pi t + \frac{\pi}{4}\right)$$

$$a = \frac{dv}{dt} = -(4.00\pi\ \text{m/s}) \cos\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t)$$

$$= -(4.00\pi^2\ \text{m/s}^2) \cos\left(\pi t + \frac{\pi}{4}\right)$$

(C) Using the results of part (B), determine the position, velocity, and acceleration of the object at $t = 1.00$ s.

Solution Noting that the angles in the trigonometric functions are in radians, we obtain, at $t = 1.00$ s,

$$x = (4.00\ \text{m}) \cos\left(\pi + \frac{\pi}{4}\right) = (4.00\ \text{m}) \cos\left(\frac{5\pi}{4}\right)$$

$$= (4.00\ \text{m})(-0.707) = \boxed{-2.83\ \text{m}}$$

$$v = -(4.00\pi\ \text{m/s}) \sin\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi\ \text{m/s})(-0.707) = \boxed{8.89\ \text{m/s}}$$

$$a = -(4.00\pi^2\ \text{m/s}^2) \cos\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi^2\ \text{m/s}^2)(-0.707) = \boxed{27.9\ \text{m/s}^2}$$

(D) Determine the maximum speed and maximum acceleration of the object.

Solution In the general expressions for v and a found in part (B), we use the fact that the maximum values of the sine and cosine functions are unity. Therefore, v varies between $\pm 4.00\pi$ m/s, and a varies between $\pm 4.00\,\pi^2$ m/s^2. Thus,

$$v_{\max} = 4.00\pi \text{ m/s} = 12.6 \text{ m/s}$$

$$a_{\max} = 4.00\pi^2 \text{ m/s}^2 = 39.5 \text{ m/s}^2$$

We obtain the same results using the relations $v_{\max} = \omega A$ and $a_{\max} = \omega^2 A$, where $A = 4.00$ m and $\omega = \pi$ rad/s.

(E) Find the displacement of the object between $t = 0$ and $t = 1.00$ s.

Solution The position at $t = 0$ is

$$x_i = (4.00\text{ m})\cos\left(0 + \frac{\pi}{4}\right) = (4.00\text{ m})(0.707) = 2.83\text{ m}$$

In part (C), we found that the position at $t = 1.00$ s is -2.83 m; therefore, the displacement between $t = 0$ and $t = 1.00$ s is

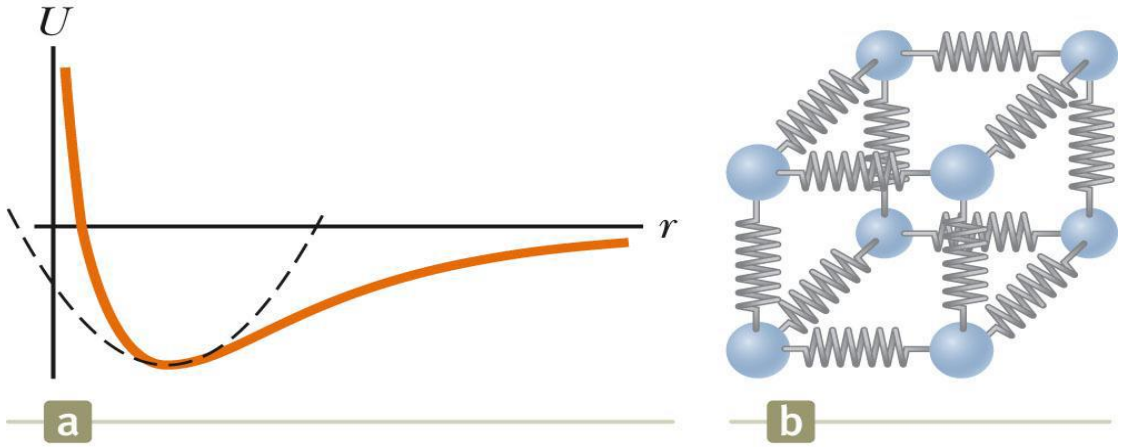
$$\Delta x = x_f - x_i = -2.83\text{ m} - 2.83\text{ m} = -5.66\text{ m}$$

Because the object's velocity changes sign during the first second, the magnitude of Δx is not the same as the distance traveled in the first second. (By the time the first second is over, the object has been through the point $x = -2.83$ m once, traveled to $x = -4.00$ m, and come back to $x = -2.83$ m.)

Importance of Simple Harmonic Oscillators

Simple harmonic oscillators are good models of a wide variety of physical phenomena. Molecular example

- If the atoms in the molecule do not move too far, the forces between them can be modeled as if there were springs between the atoms.
- The potential energy acts similar to that of the SHM oscillator



Simple Pendulum

A simple pendulum also exhibits periodic motion. It consists of a particle-like bob of mass *m suspended by a light string of length L. The motion occurs in the vertical plane and is driven by gravitational force. The motion is very close to that of the SHM oscillator.* If the angle is <10 degree

The forces acting on the bob are the tension and the weight.

- **T** is the force exerted on the bob by the string.
- m**g** is the gravitational force. The tangential component of the gravitational force is a restoring force.

Simple Pendulum cont'd...

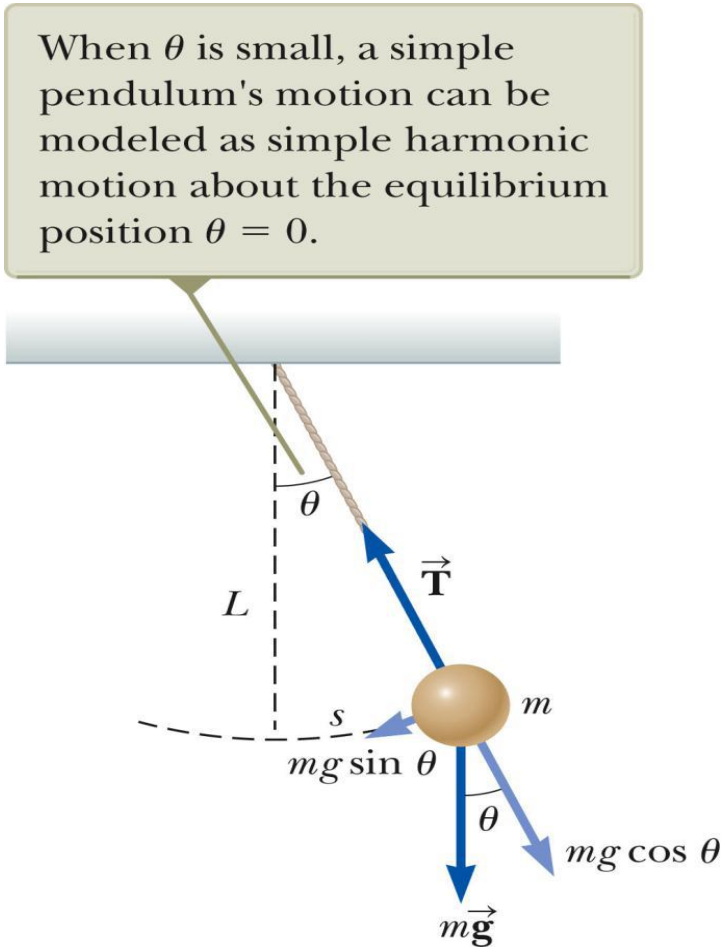
In the tangential direction,

$$F_t = ma_t \rightarrow -mg\sin\theta = m\frac{d^2s}{dt^2}$$

The length, *L, of the pendulum is constant, and for small values of* $\boldsymbol{\theta}$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$

This confirms the mathematical form of the motion is the same as for SHM.



Simple Pendulum, cont'd..

The function θ can be written as $\theta = \theta_{max} \cos(\omega t + \phi)$.

The angular frequency is

$$\omega = \sqrt{\frac{g}{L}}$$

The period is

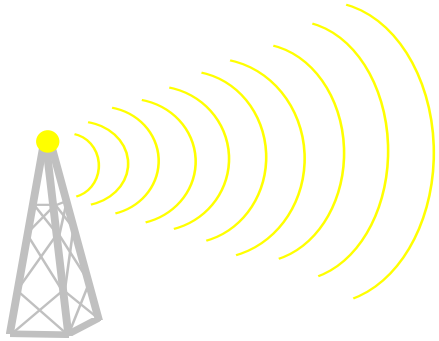
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

The period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity. The period is independent of the mass. All simple pendula that are of equal length and are at the same location oscillate with the same period.

Waves

- Sound
- Light
- Water
- Radio
- Seismic



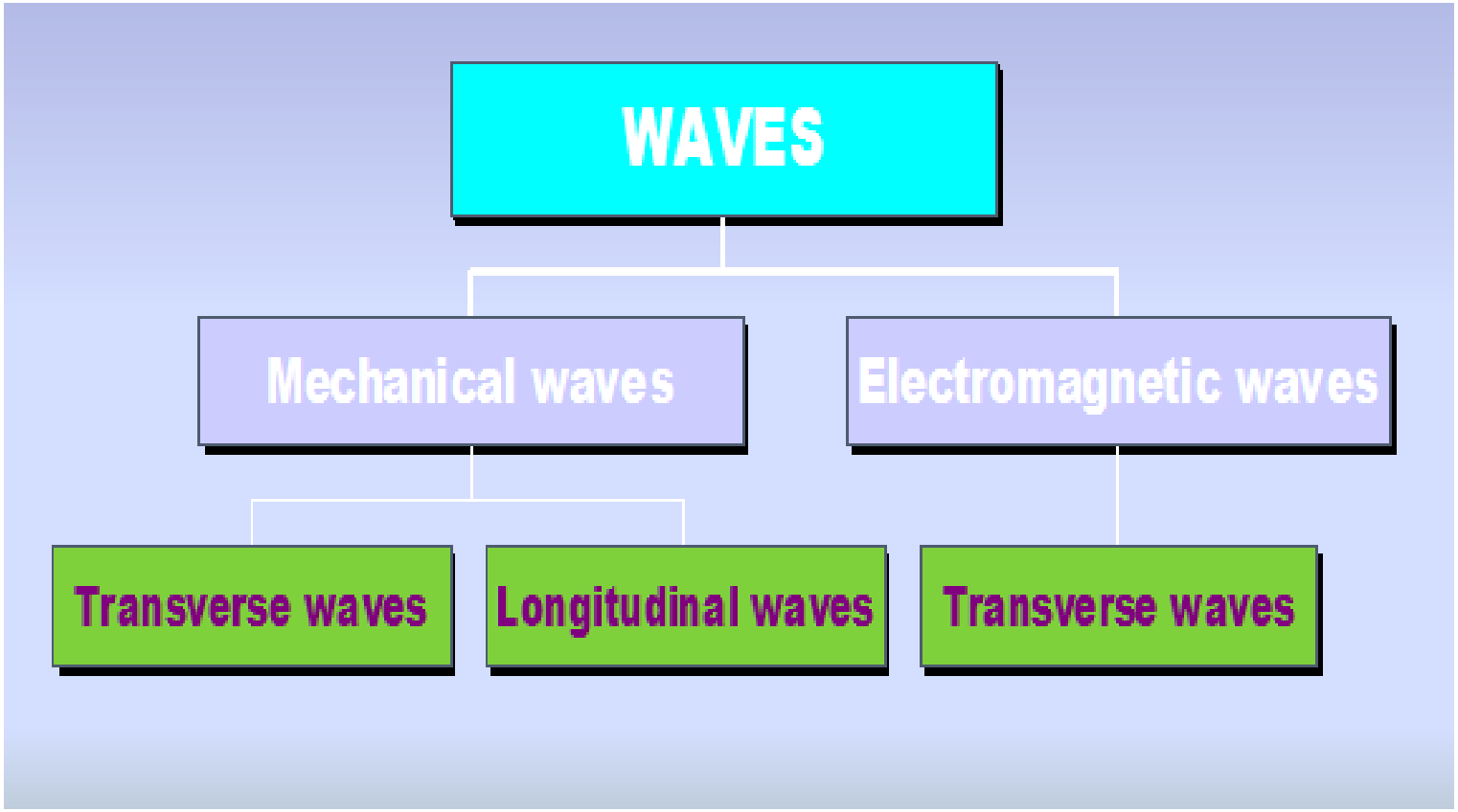




A **wave** is a means of transferring **energy** from one place to another

TYPES OF WAVES

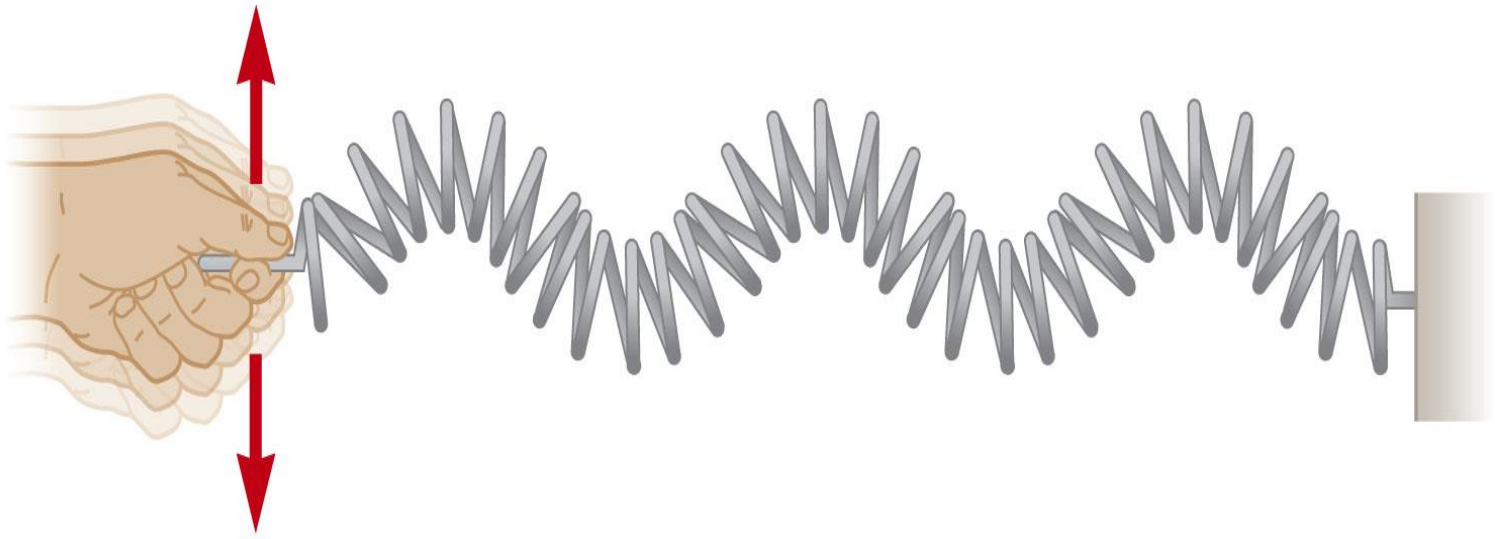
- **Waves** are classified into different types according to their natures :



Mechanical	Electromagnetic
e.g. Water waves, waves on a rope, **sound**	e.g. Radio, X-Rays, **Light**
Must have a substance to travel through Cannot travel in a vacuum	Does not need a substance to travel through but can travel through various substances Can travel in a vacuum
Vibrations passed on from molecule to molecule	Travel at very fast speed in a vacuum: **3×10^8 m/s**

Types of Waves – Transverse

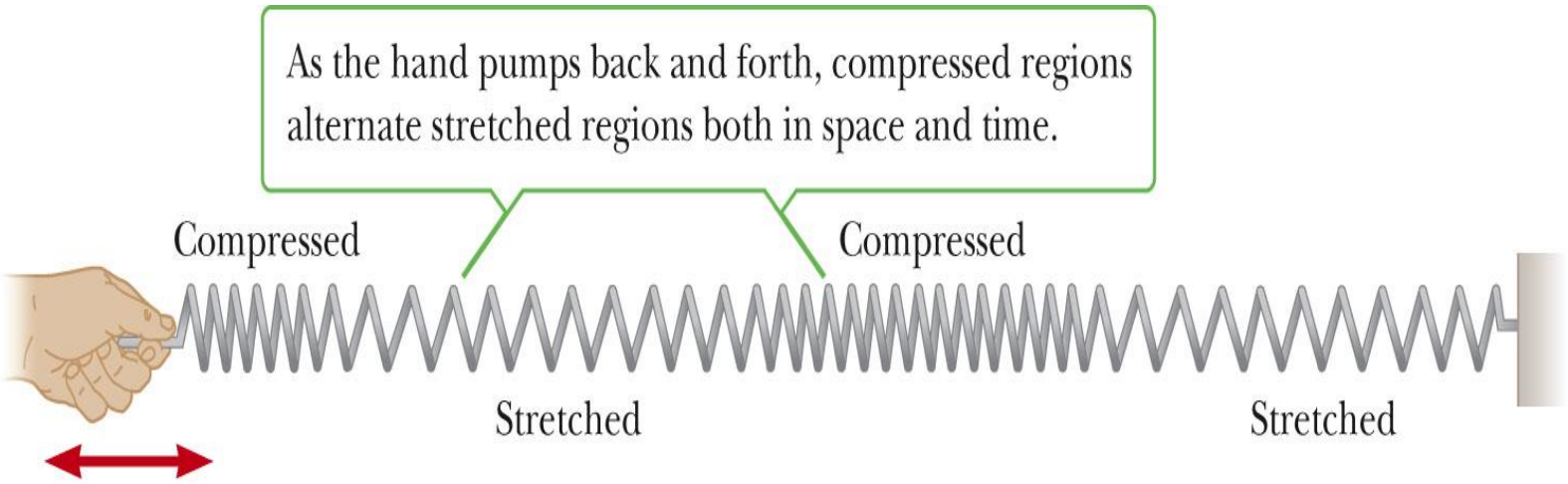
- In a transverse wave, each element that is disturbed moves in a direction perpendicular to the wave motion



Transverse wave

Types of Waves – Longitudinal

- In a longitudinal wave, the elements of the medium undergo displacements parallel to the motion of the wave
- A longitudinal wave is also called a compression wave

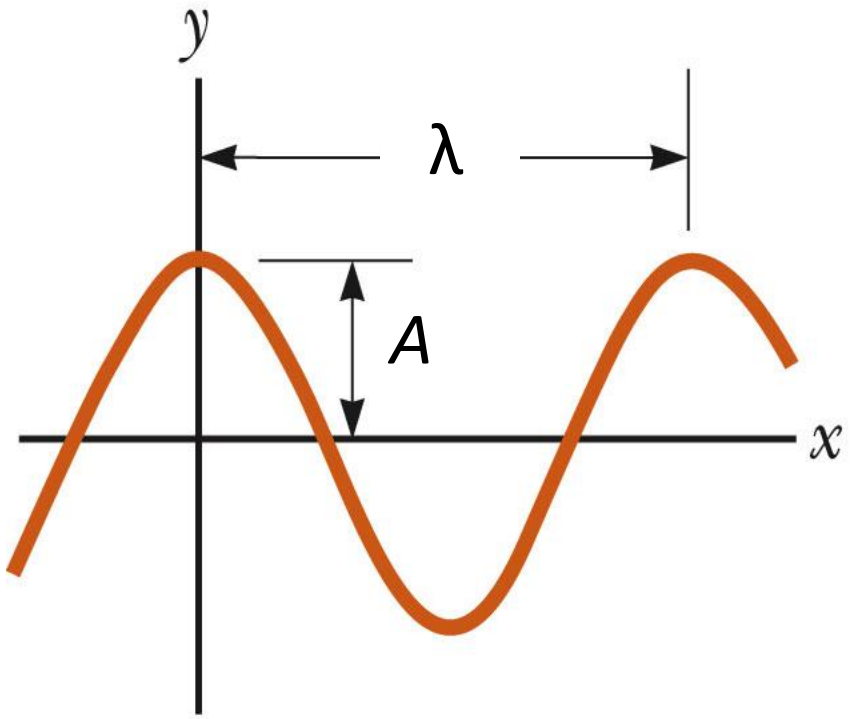


Longitudinal wave

Amplitude and Wavelength

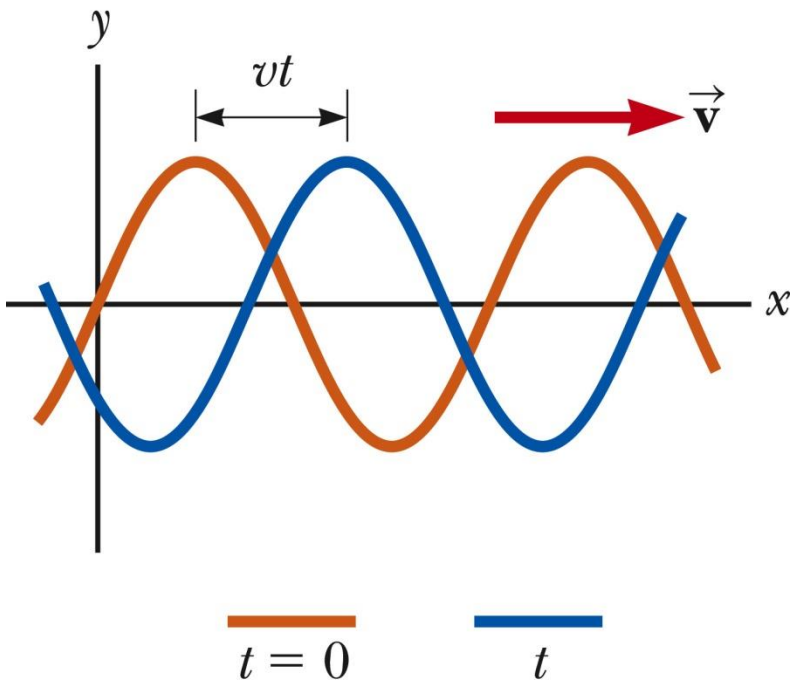
- Amplitude A is the maximum displacement of string above the equilibrium position

- Wavelength, λ, is the distance between two successive points that behave identically



Waveform – A Picture of a Wave

- The ***brown curve*** is a "snapshot" of the wave at $t = 0$ instant in time
- The ***blue curve*** is that *at a later in time, t*
- The high points are *crests* of the wave
- The low points are *troughs* of the wave



Wavelength λ = vT

Speed of a Wave

- $v = f\lambda$

  Is derived from the basic speed equation of

  Distance = Speed x Time

  $\lambda = vT = v/f \qquad T = 1/f$

- This is a general equation that can be applied to many types of waves

Interference of Waves;

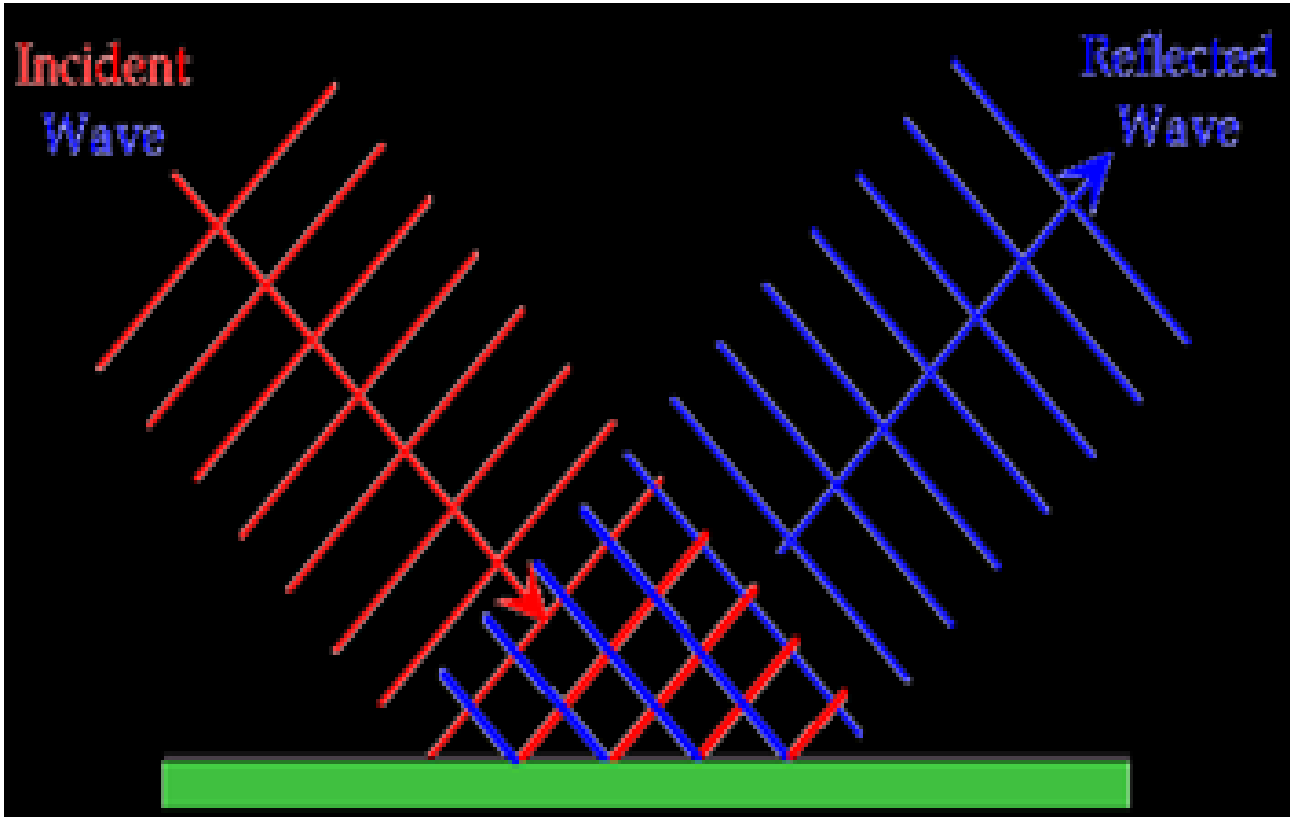
- Two traveling waves can meet and pass through each other without being destroyed or even altered
- Waves obey the *Superposition Principle*
  - When two or more traveling waves encounter each other while moving through a medium, the resulting wave is found by adding together the displacements of the individual waves point by point

    only true for waves with small amplitudes

Properties of waves

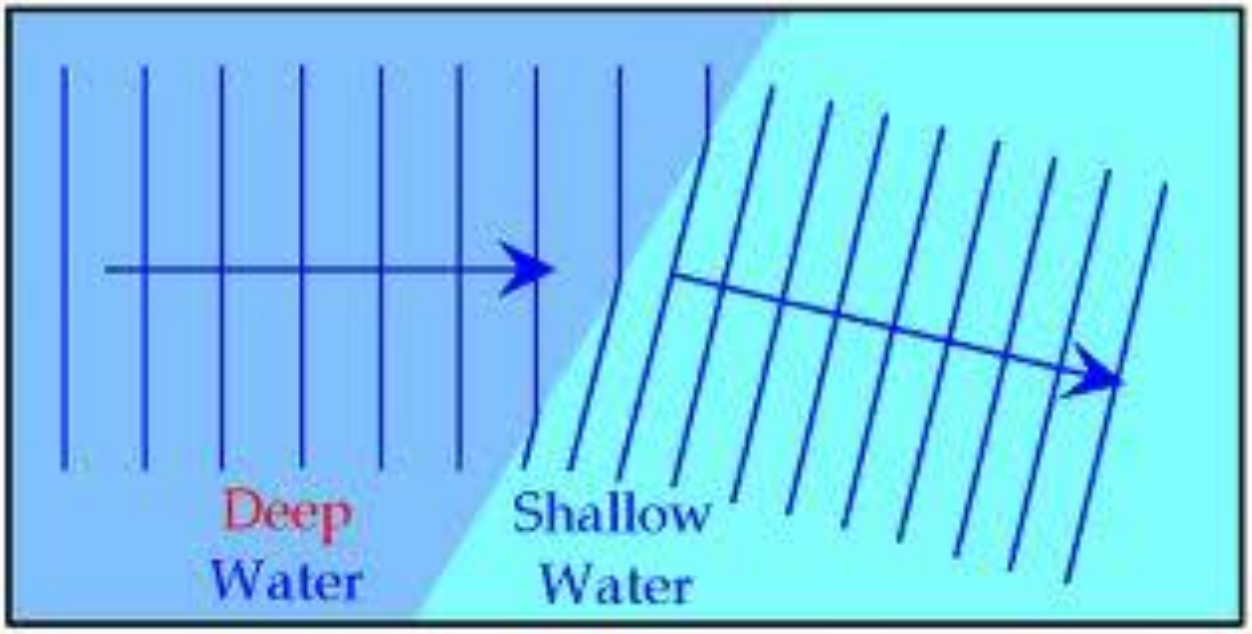
1. **Reflection:**

Reflection is the bouncing of a wave off an object.



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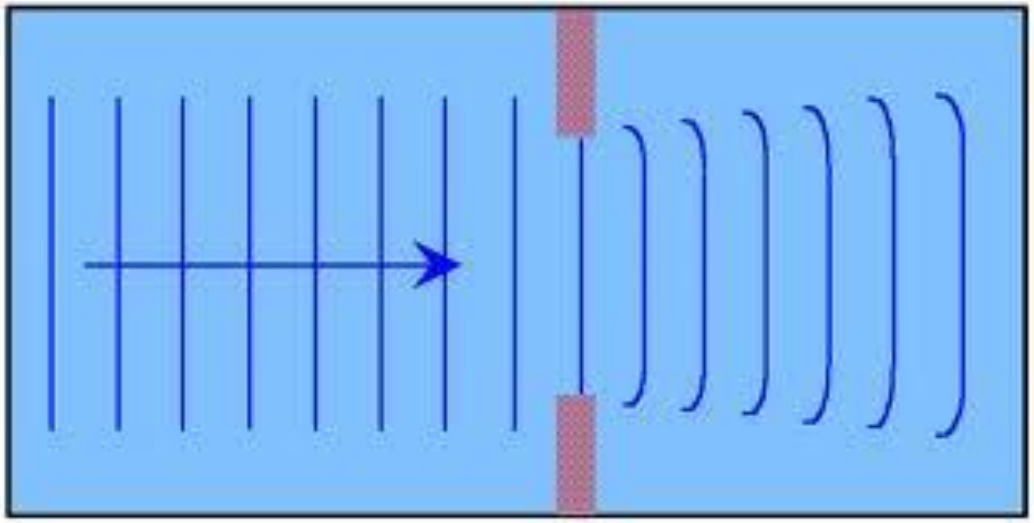
2 Refraction is the bending of a wave as it travels from one medium to another. Note that when a wave travels from one medium to another its frequency does not change.



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3. **Diffraction** is the spreading of waves around a slit or an obstacle.

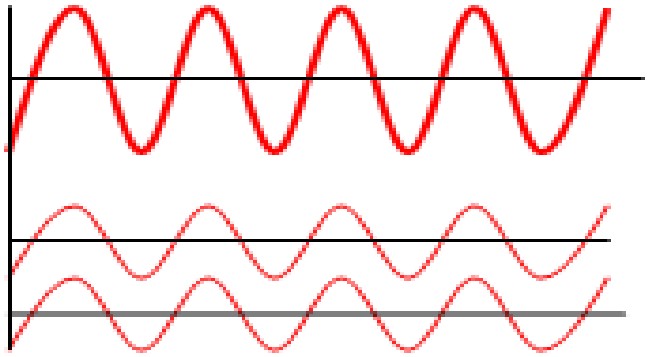
- This effect is only significantly noticeable if the slit width is approximately the same size as the wavelength of the waves.



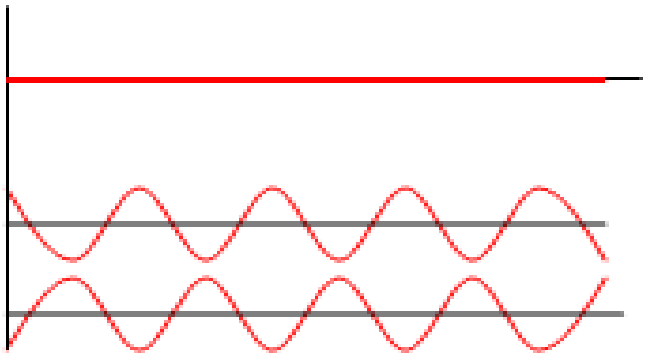
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4 **Interference** occurs when waves from two sources meet to produce a wave of different amplitude.

Two types of interference;



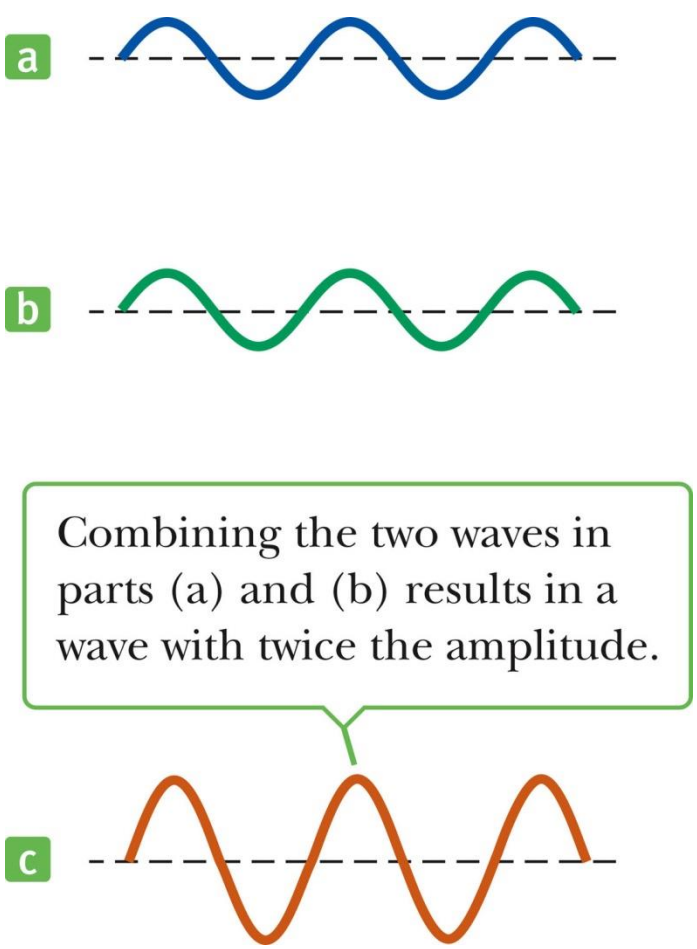
Constructive Interference



Destructive Interference

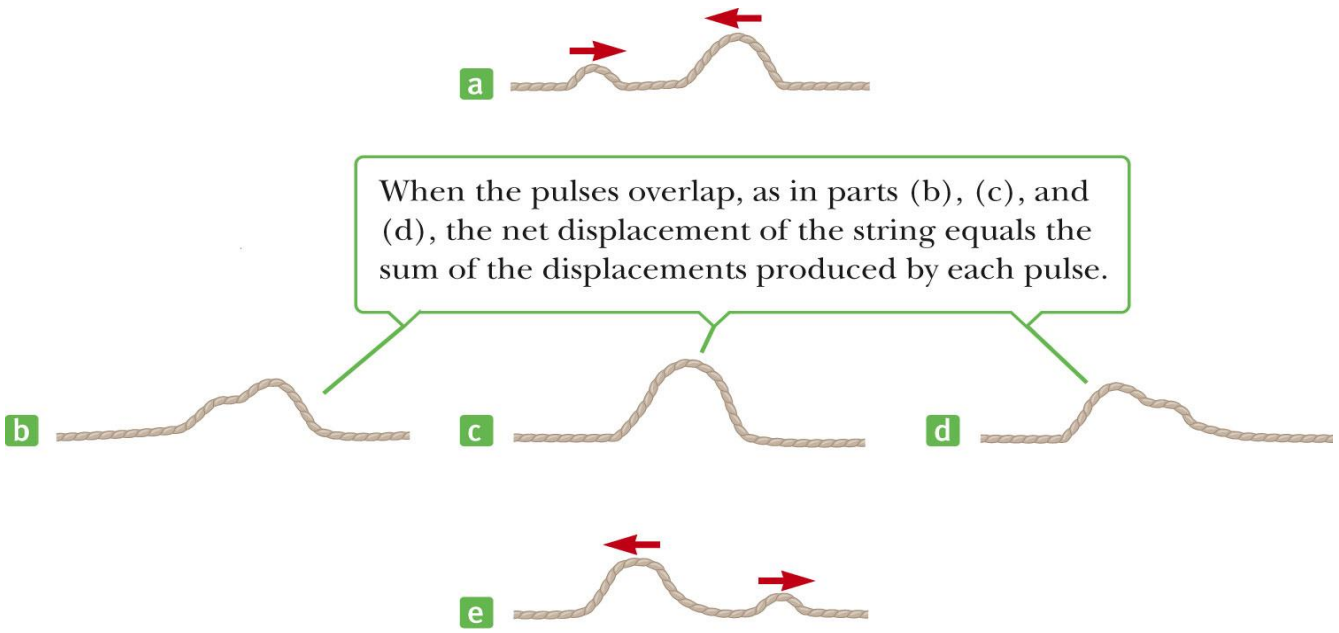
Constructive Interference

- Two waves, *a* and *b*, have the same frequency and amplitude
  - Are *in phase*
- The combined wave, *c*, has the same frequency and a greater amplitude



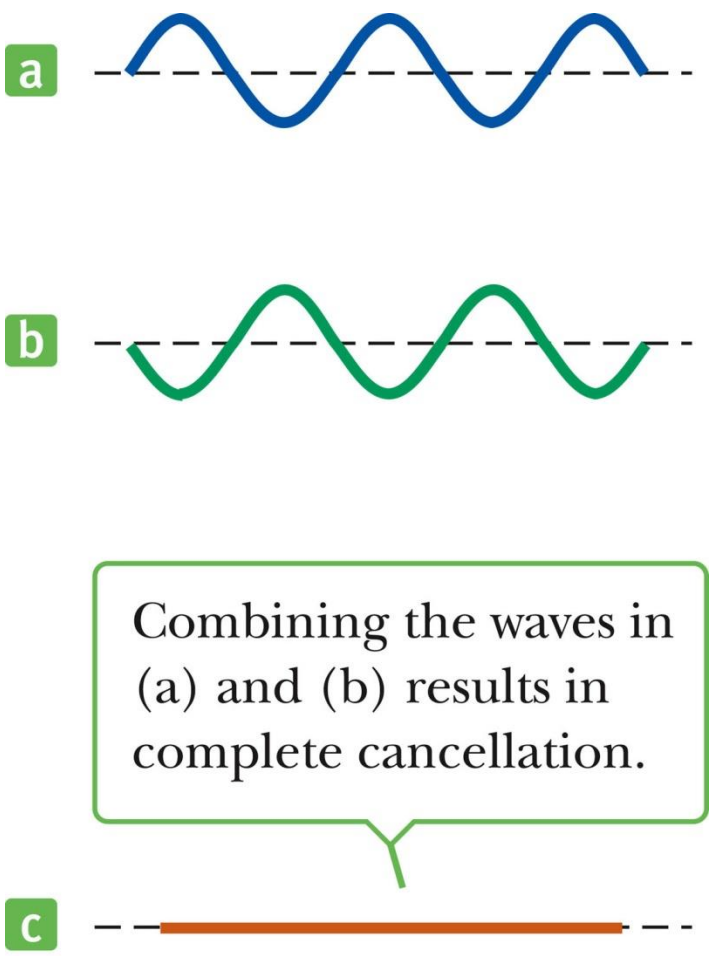
Constructive Interference in a String

- Two pulses are traveling in opposite directions
- The net displacement when they overlap is the sum of the displacements of the pulses
- Note that the pulses are unchanged after the interference



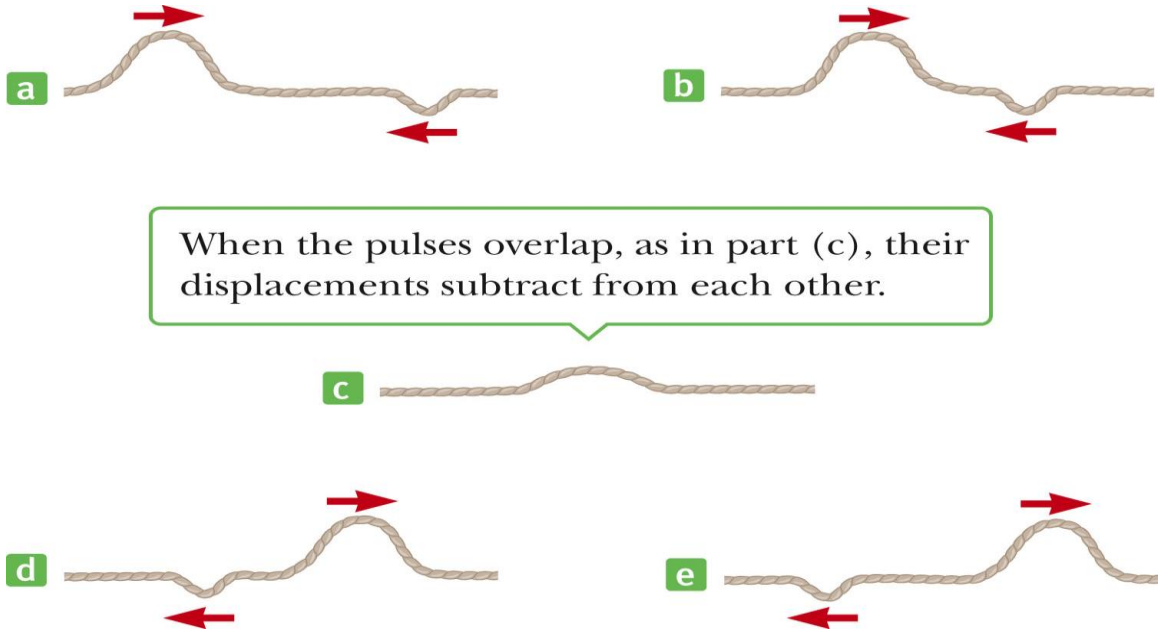
Destructive Interference

- Two waves, *a* and *b*, have the same amplitude and frequency
- One wave is *inverted* relative to the other
- They are 180° out of phase
- When they combine, the waveforms cancel



Destructive Interference in a String

- Two pulses are traveling in opposite directions
- The net displacement when they overlap is decreased since the displacements of the pulses subtract
- Note that the pulses are unchanged after the interference



Resonance

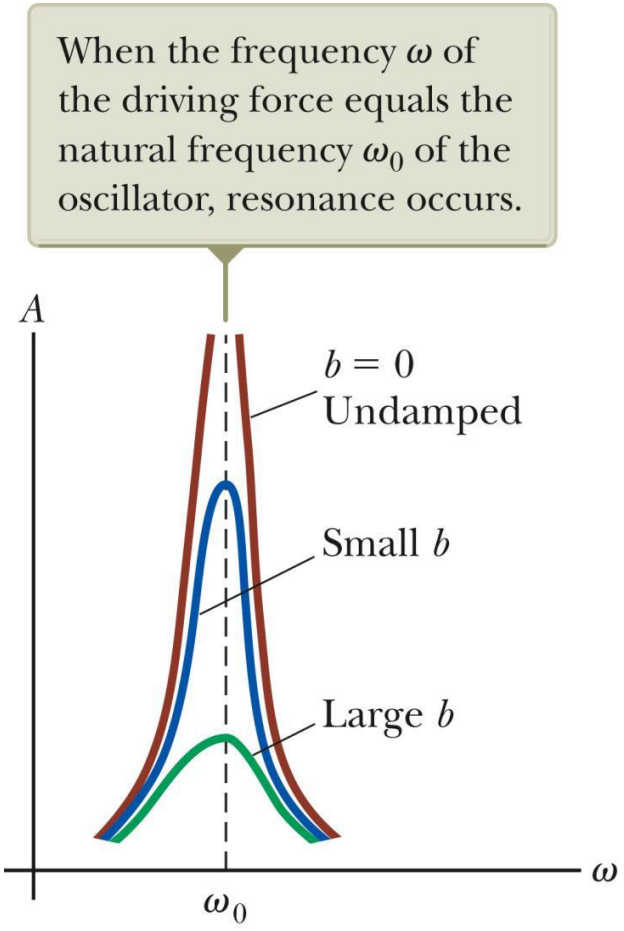
- When the frequency of the driving force is near the natural frequency ($\omega \approx \omega_0$) an increase in amplitude occurs. This dramatic increase in the amplitude is called ***resonance. The natural frequency*** ω_0
- ***is also called the resonance frequency of the system. At resonance, the applied force is in phase with the velocity and the power transferred to the oscillator is a maximum.***
- The applied force and v are both proportional to sin ($\omega t + \phi$).

Resonance, cont.

The power delivered is **F.V**

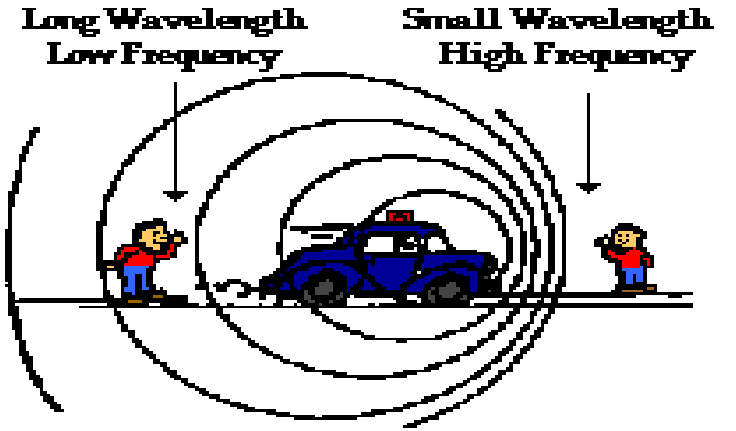
- This is a maximum when the force and velocity are in phase.
- The power transferred to the oscillator is a maximum.

Resonance (maximum peak) occurs when driving frequency equals the natural frequency. The amplitude increases with decreased damping. The curve broadens as the damping increases. The shape of the resonance curve depends on *b*.



The Doppler Effect

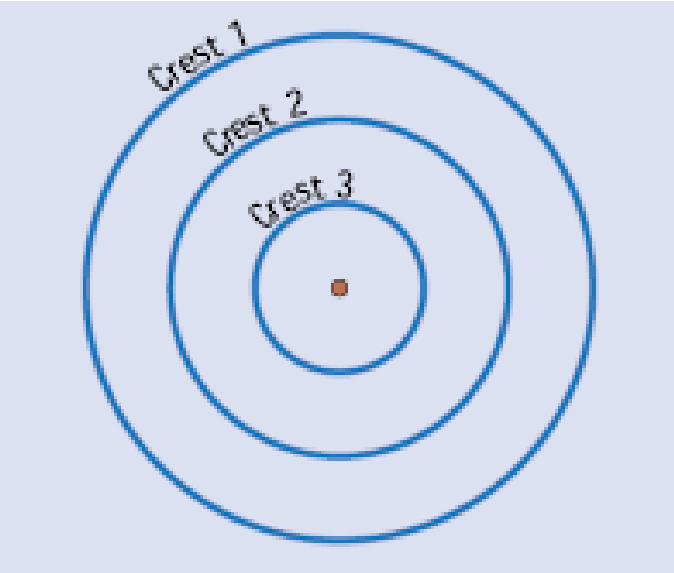
- The Doppler effect is the apparent change in frequency of a wave due to the motion of the source or the observer.
- The observed frequency is higher when the source and observer are getting closer.
- The observed frequency is lower when the source and observer are getting farther away.



The Doppler Effect for a moving sound source

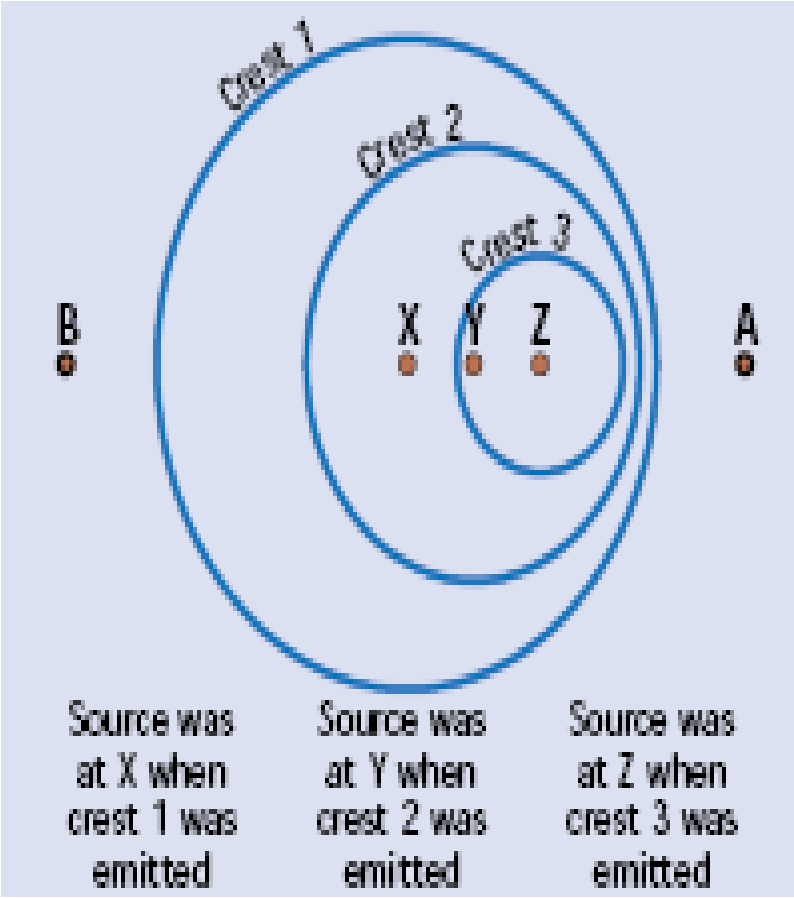
Con'd..

- **Consider a source S emitting a wave with crests 1, 2, 3 as shown.**



- The distance between successive crests is the constant and so the number of crests passing a point in one second is **the frequency of the wave**.
- These waves will pass an observer in equal intervals of time.
- This means that the wavelength, and the frequency, will be constant.

In this case the source is moving to the right while emitting the waves.



The result is that:

- **Ahead** of the moving source the crests are **closer** together than crests from the stationary source would be. This means that the **wavelength is smaller** and the **frequency is greater**.
- **Behind** the moving source, the crests are **further apart** than crests from the stationery source would be. This means the **wavelength is greater** and so the **frequency is less**.

Cont'd

$$f' = \frac{fc}{c \pm u}$$

Formula:

f' = apparent frequency

f = actual frequency

c = speed of the wave

u = speed of the moving source

Example of the Doppler Effect

- The noise from a racing car as it approaches
- and then moves away from an observer is
- *an example* of the Doppler effect.

This is **NOT** an APPLICATION of the Doppler effect.

Applications of the Doppler Effect

- Police speed traps
- Measuring the red shift of galaxies in astronomy

Image Formation by Thin Lenses and Mirrors

Mirrors and Lenses;

The development of mirrors and lenses led to the *micro*scopes and *tele*scopes

Study of objects from microbes to distance planets

Images can be formed by:

reflection from mirrors



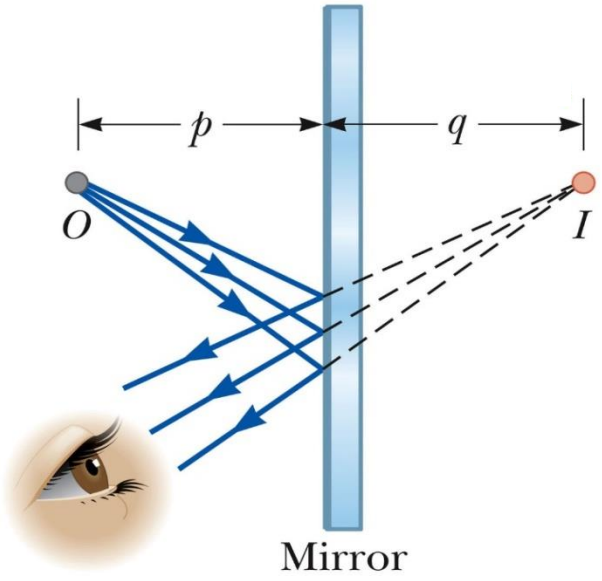


refraction through lenses

combinations of these



Notation for mirrors and lenses

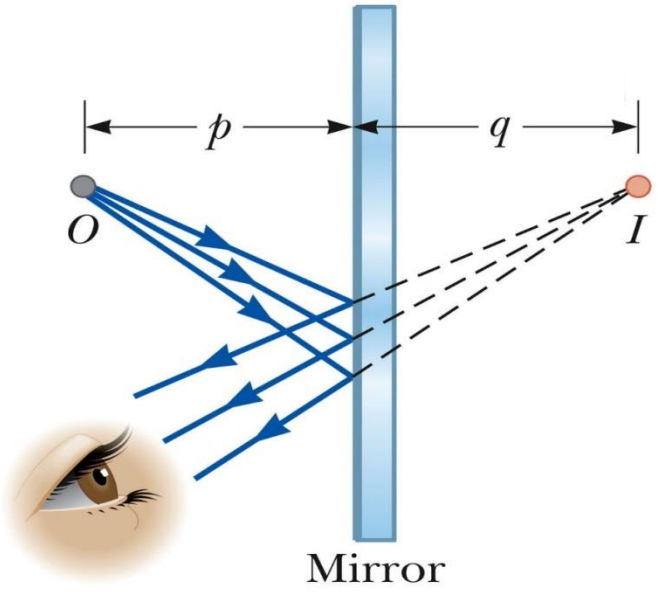


- The *object distance* p: from the object O to the mirror or lens
- The *image distance* q: from the image I to the mirror or lens

Formation of Images

Images are formed at the point where rays *actually* *intersect*



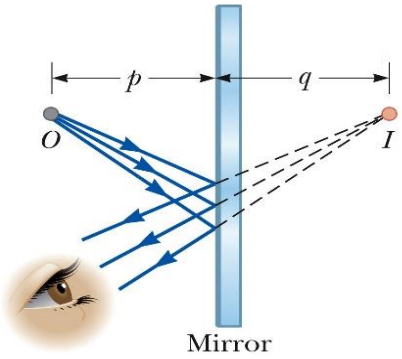


Types of Images

- ***Real images*** are formed at the point the rays of light actually intersect
  can be displayed on screens

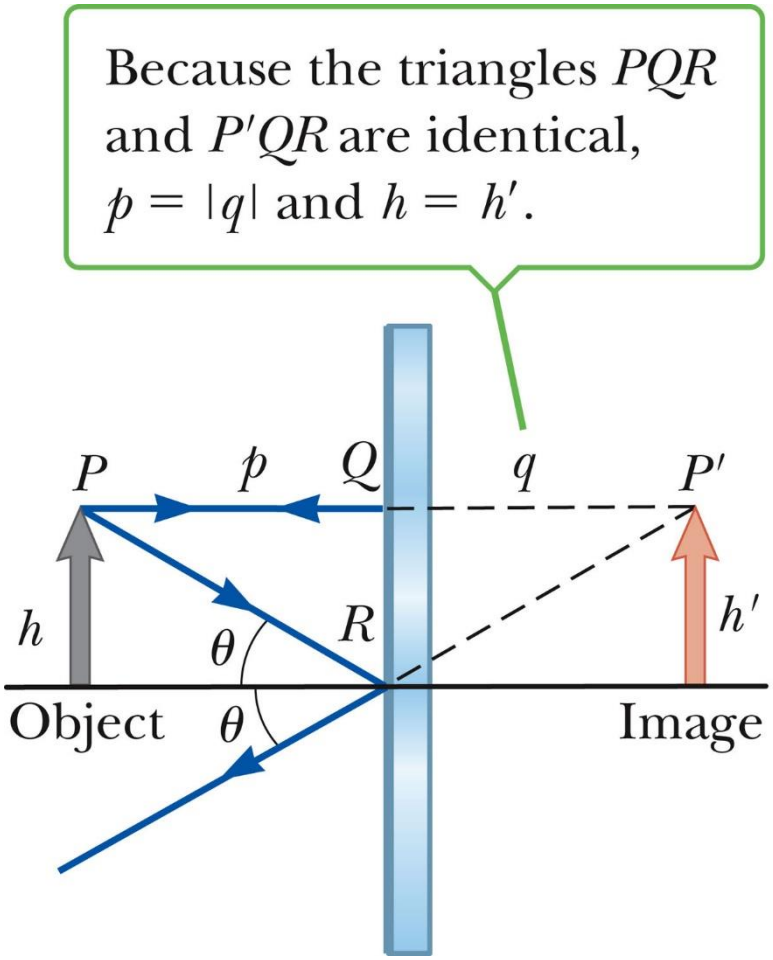


- ***Virtual images*** are formed at the point the rays of light *appear* to originate
  The light appears to *diverge* from that point
  Virtual images cannot be displayed on screens



Flat Mirror

- Properties of the image can be determined by geometry
- One ray starts at *P*, follows path *PQ* and reflects back on itself
- A second ray follows path *PR* and reflects according to the Law of Reflection



Properties of the Image Formed by a Flat Mirror

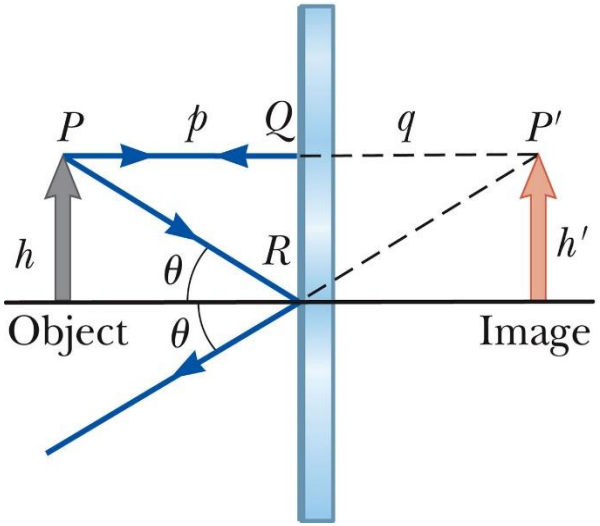
- The image is as far behind the mirror as the object is in front

$$p = |q|$$

- The image is unmagnified:

  The image height is the same as the object height,

$$h' = h \text{ and } M = 1$$

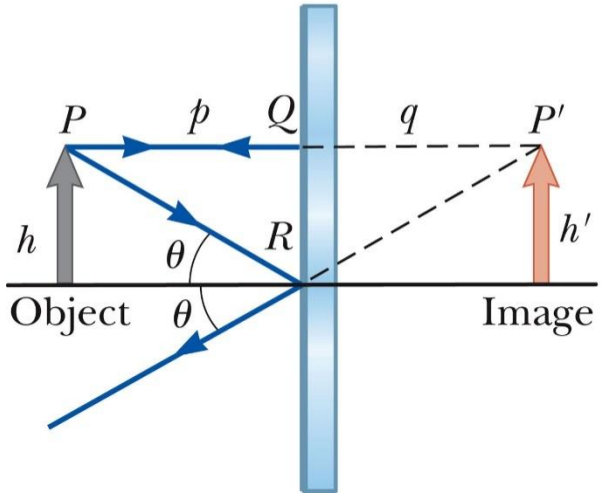


More Image Properties – Flat Mirror

- The image is *virtual*
- The image is upright

It has the same orientation as the object

- There is an apparent left-right reversal in the image



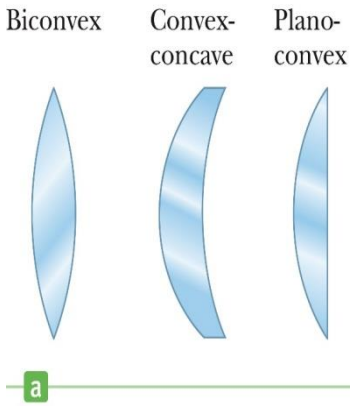
Thin Lenses

- A thin lens consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane

- Lenses are commonly used to form images by refraction in optical instruments

Thin Lens Shapes

These are examples of *converging* lenses
They have positive focal lengths
They are thickest in the middle



More Thin Lens Shapes



These are examples of *diverging* lenses
They have negative focal lengths
They are thickest at the edges

Focal Length of Lenses

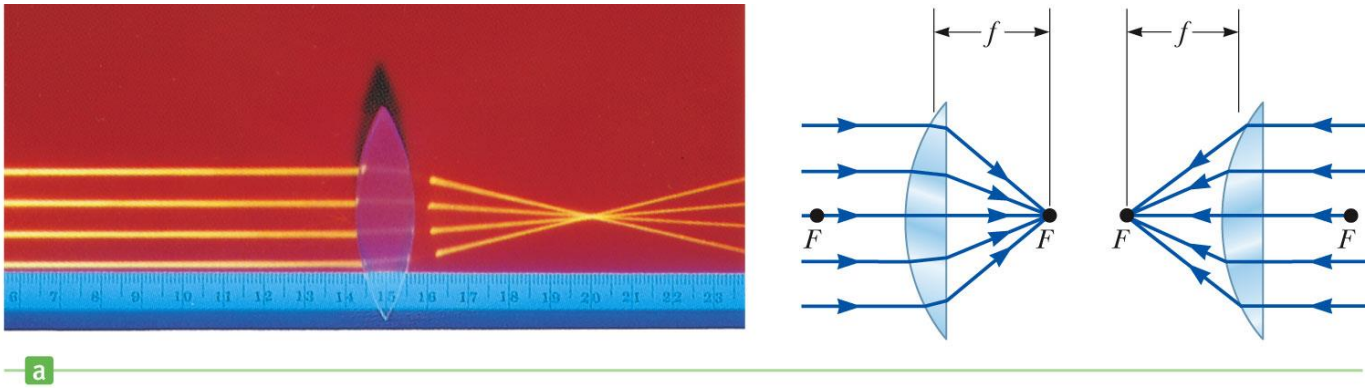
- The focal length, f, is the image distance that corresponds to an infinite object distance

  This is the same as for mirrors

- A thin lens has *two focal points*, corresponding to parallel rays from the left and from the right

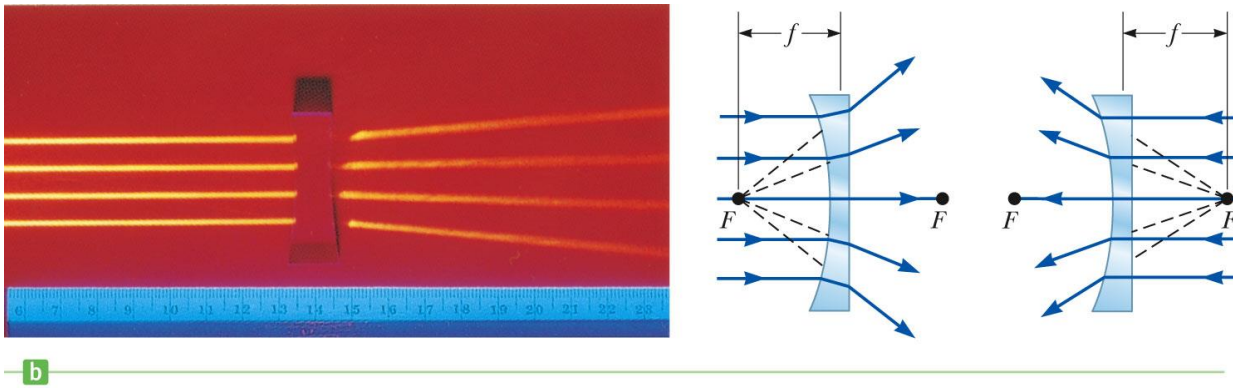
  A thin lens is one in which the distance between the surface of the lens and the center of the lens is negligible

Focal Length of a Converging Lens



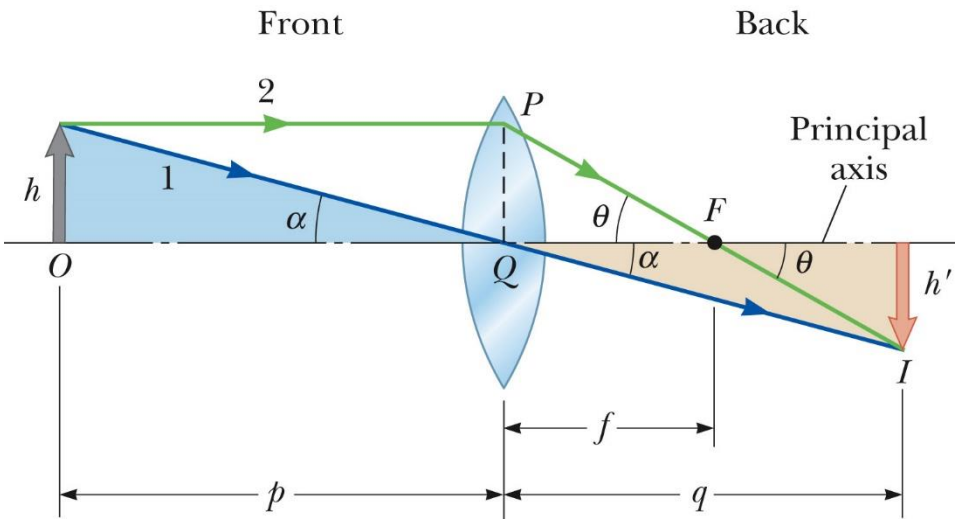
- The parallel rays pass through the lens and converge at the focal point
- The parallel rays can come from the left or right of the lens

Focal Length of a Diverging Lens



- The parallel rays diverge after passing through the diverging lens
- The focal point is the point where the rays appear to have originated

Lens Equations

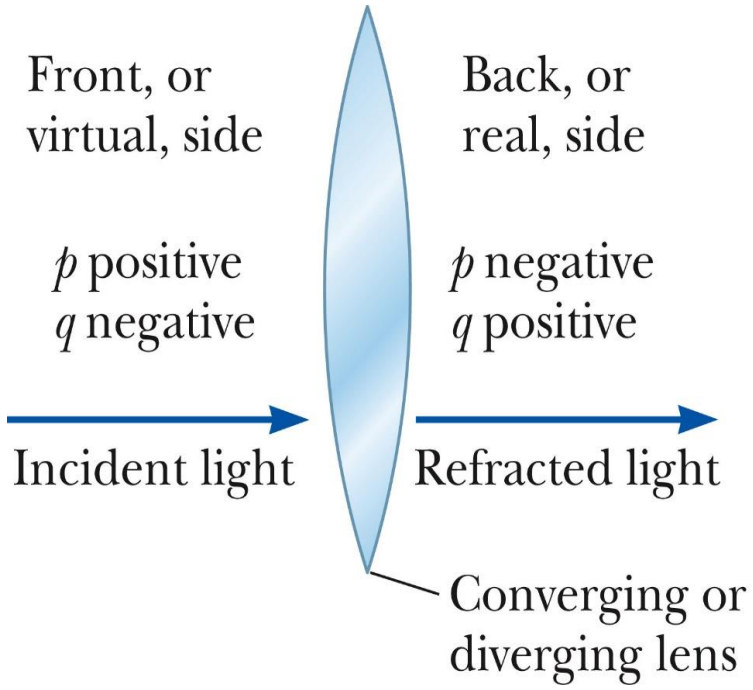


- The geometric derivation of the equations is very similar to that of mirrors

$$M = \frac{h'}{h} = -\frac{q}{p} \quad \text{and} \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Lens Equations and Signs

- The equations can be used for both converging and diverging lenses
  - A converging lens has a positive focal length.
  - A diverging lens has a negative focal length.
- See other sign conventions in the diagram



Sign Conventions, Table

Quantity	Symbol	In Front	In Back	Convergent	Divergent
Object location	p	+	−		
Image location	q	−	+		
Lens radii	R_1, R_2	−	+		
Focal length	f			+	−

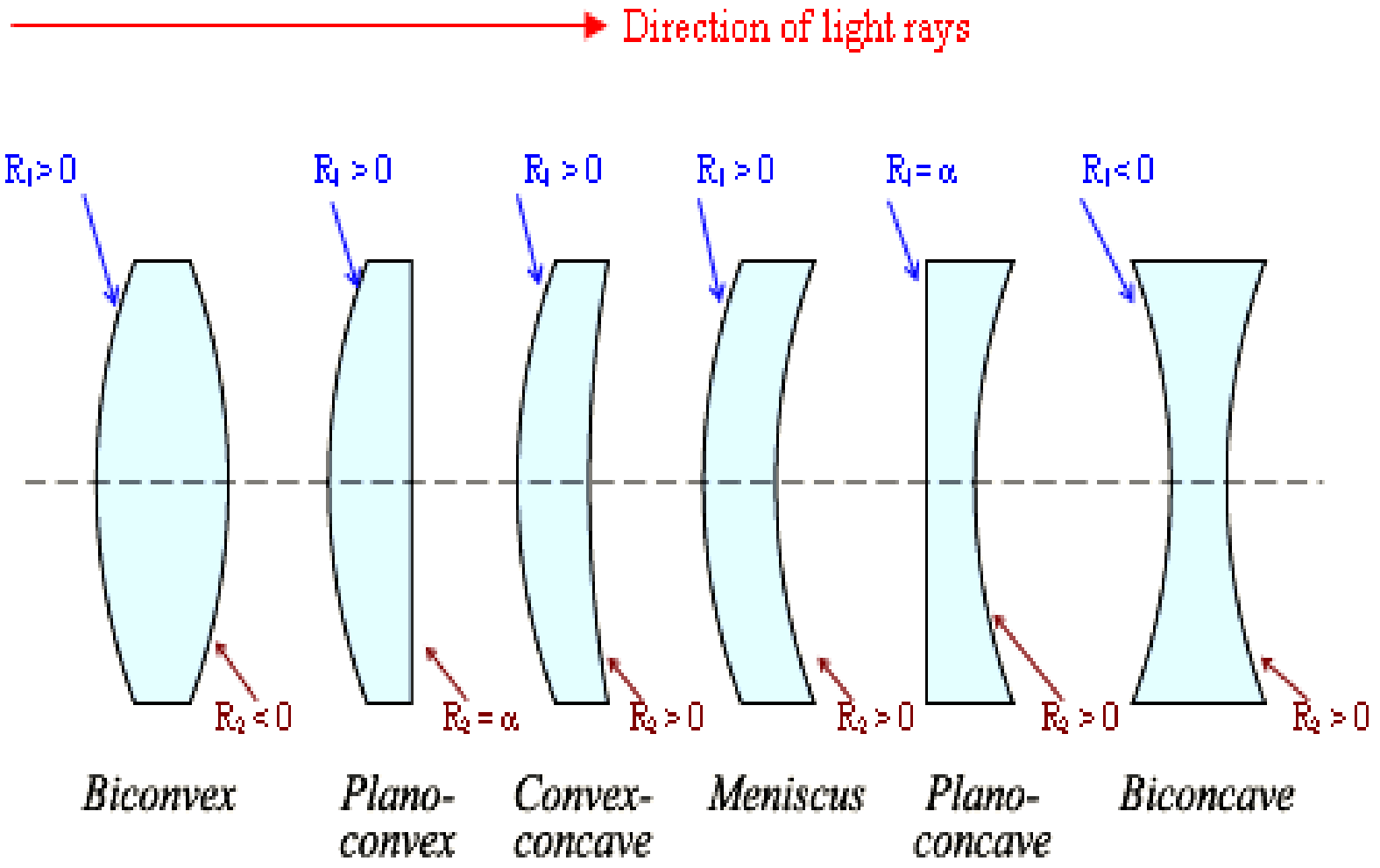
Focal Length for a Lens

- The focal length of a lens is related to the curvature of its front and back surfaces and the index of refraction of the material:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This is called the *lens-maker's equation*

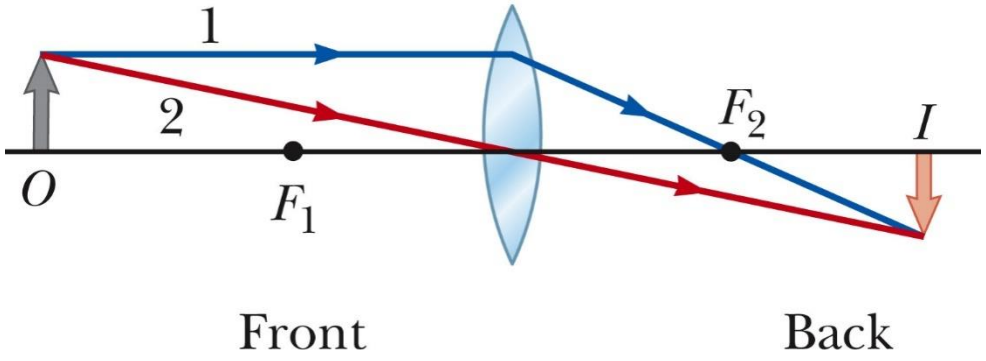
Signs of R_1 and R_2 in $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



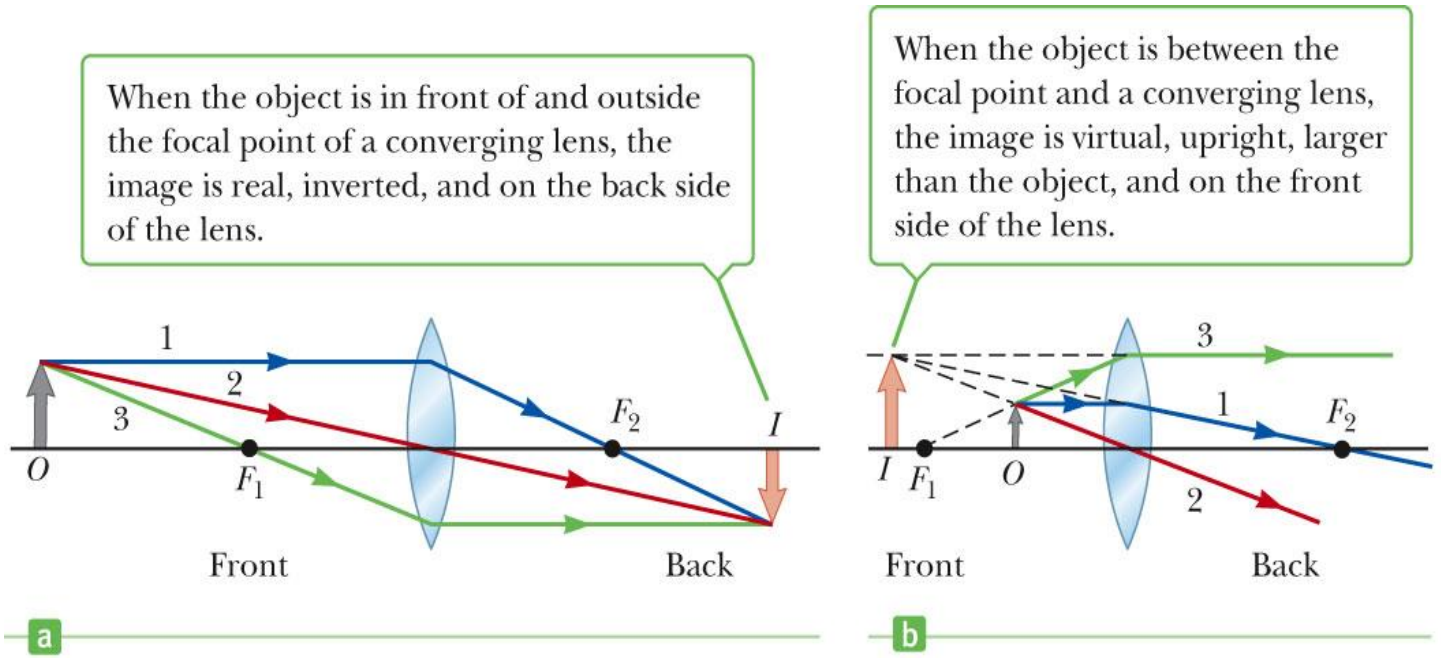
Ray Diagrams for Thin Lenses

- Three rays are drawn
  - 1st ray: parallel to the principle axis and then passes through (or appears to come from) one of the focal points *F*
  - 2nd ray is drawn through the center of the lens *C* and continues in a straight line.
  - 3rd ray is drawn through the other focal point *F* and emerges from the lens parallel to the principle axis **(not shown)**

There are an infinite number of rays, these are convenient



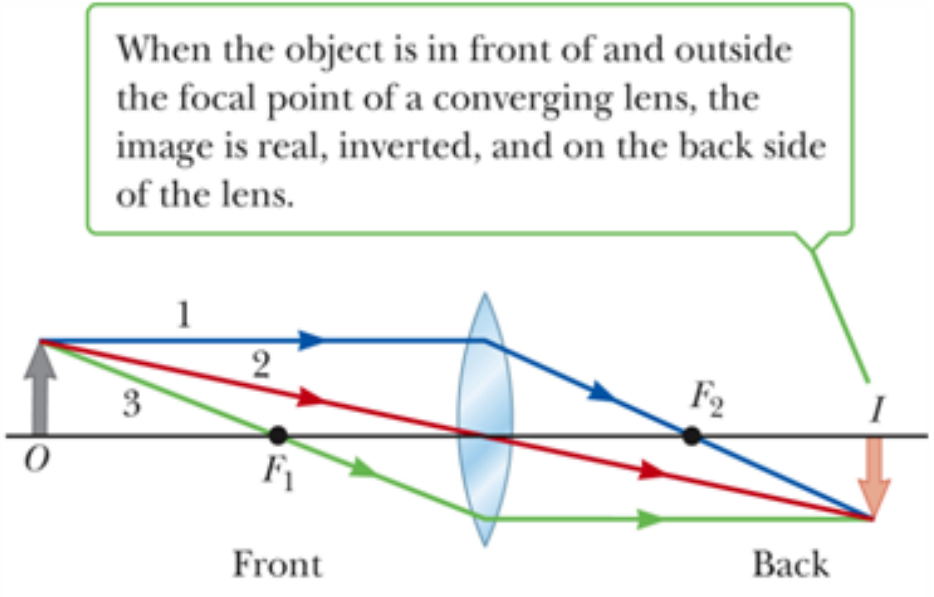
Ray Diagram Examples



- Note the changes in the image as the object moves through the focal point

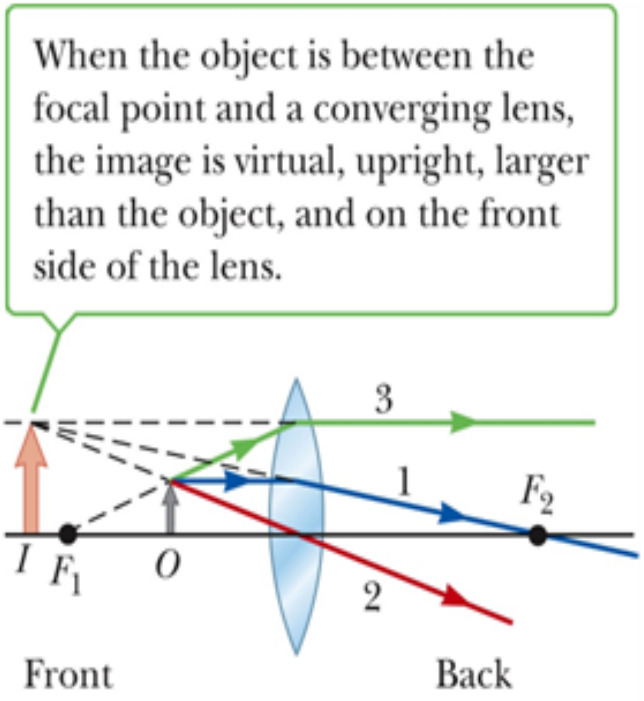
Ray Diagram for Converging Lens, p > f

- The image is real
- The image is inverted
- The image is on the back side of the lens



Ray Diagram for Converging Lens, p < f

- The image is virtual
- The image is upright
- The image is on the front side of the lens



Ray Diagram for Diverging Lens

- The image is virtual
- The image is upright
- The image is on the front side of the lens

