Menschen für Menschen

Agro Technical and Technology College

General Physics (Phys 1011)

By: Tadesse L.
Haramaya University, Department of Physics

Objective

The main objective of the course is

- To refresh the previous preparatory physics concepts before tackling the advanced physics and other related courses.
- To provide the student with a broad understanding of the physical principles.
- To help them develop critical thinking and quantitative reasoning skills,
- To empower them to think creatively and critically about scientific problems and experiments.

Expected outcomes: At the end of the course students should be able to:

- Discuss about the physical quantities and measurement.
- Explain the kinematics and dynamics of particles in one and two dimensions.
- State principles of fluids in equilibrium and solve problems applying Pascal’s principle, Archimedes’s, principles and Bernoulli’s equation in various situations.
- Explain the first law of thermodynamics for a closed system and apply it to solve problems.
- Discuss systems that oscillate with simple harmonic motion.

- Explain the basic concepts of charges, fields and potentials.
- Analyze direct and alternating current circuits containing different electric elements and solve circuit problems.
- Demonstrate the use and the working system of cells (batteries), resistors, generators, motors and transformers.
- Distinguish differences between conductors, insulators and semiconductors .
- Explain different elements of electronic circuits.
- Explain the application of physics in different sciences and technology fields.

UNIT 1
PRELIMINARIES

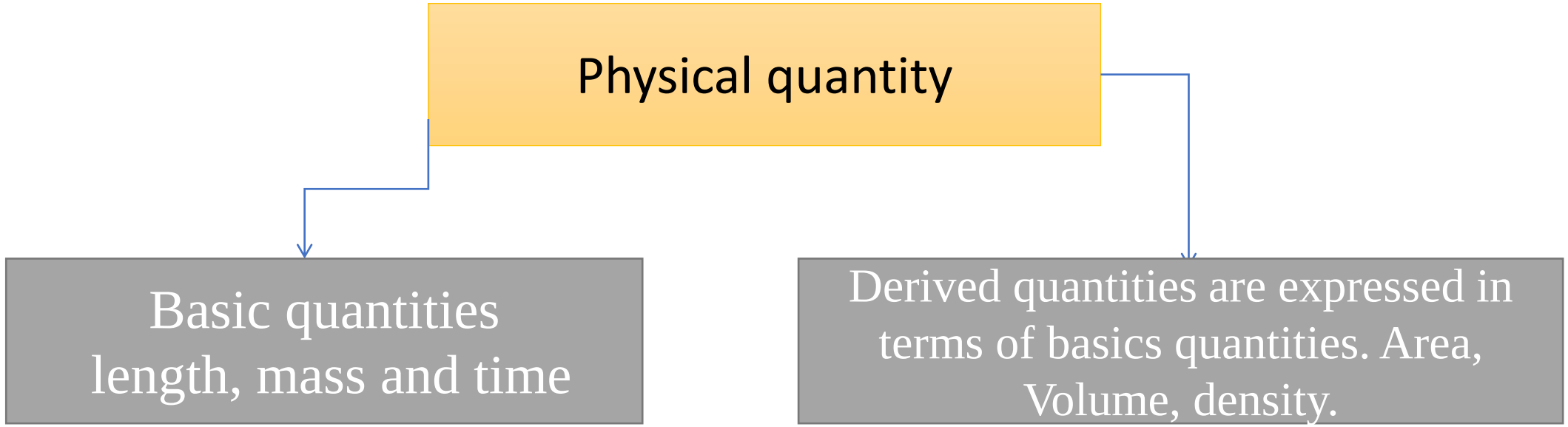
- **What is physics?**
- **Physics**, science that deals with the structure of matter and the interactions between the fundamental constituents of the observable universe.
- In the broadest sense, physics is concerned with all aspects of nature on both the macroscopic and sub microscopic levels.
- Its scope of study encompasses not only the behaviour of objects under the action of given forces but also the nature and origin of gravitational, electromagnetic, and nuclear force fields.

- Physics plays an important role in all the natural sciences.
- Hence, all such fields have branches in which physical laws and measurements receive special emphasis, bearing such names as astrophysics, geophysics, biophysics, medical physics, psychophysics, Agro physics, soil physics etc.

Physical Quantities and Measurement

- Why do we need measurement in physics and our day-to-day lives?

- **Physical quantity** is a quantifiable or assignable property ascribed to a particular phenomenon or body, for instance the length of a rod or the mass of a body.
- **Measurement** is the act of comparing a physical quantity with a certain standard. Measurements of physical quantities are expressed in terms of units, which are standardized values.
- Measurements of physical quantities are expressed in terms of *units*, which are standardized values or SI Units.



SI Units: Basic and Derived Units

- SI unit is the abbreviation for International System of Units and is the modern form of metric system finally agreed up on at the eleventh International conference of weights and measures, 1960.
- This system of units is now being adopted throughout the world and will remain the primary system of units of measurement.

Basic quantities and their SI units

Property	Symbol	Unit	Dimension
Length	L	Meter (m)	L
Mass	m	Kilogram (Kg)	M
Time	t	second (s)	T
Temperature	T	Kelvin (K)	θ
Electric Current	I	Ampere (A)	I
Amount of Substance	N	Mole (N)	l
Luminous Intensity	F	Candela (cd)	J

Derived quantities, their SI units and dimensions

Property	Symbol	Definition	Unit	Dimension
Area	A	length x length	m^2	$[L^2]$
Volume	*V*	$(length)^3$	m^3	$[L^3]$
Speed, velocity	*v*	length/time	m/s	$[LT^{-1}]$
Acceleration	a	velocity/time	m/s^2	$[LT^{-2}]$
Momentum	p	mass x velocity	kgm/s	$[MLT^{-1}]$
Force	*F*	momentum/time	newton (N)	$[MLT^{-2}]$
Pressure	*P*	force/area	pascal (Pa)	$[ML^{-1}T^{-2}]$
Density	ρ	mass/volume	kg/m^3	$[ML^{-3}]$
Energy	*E*	force x distance	joule (J)	$[ML^2T^{-2}]$
Power	*W*	work/time	watt (W)	$[ML^2T^{-3}]$

Unit conversion of basic quantities

Quantity	Form	To	Operation
Length	inch (in)	m	(inch) x 0.0254
	foot (ft)	m	(foot) x 0.3048
	mile (mi)	m	(mile) x 1609.34
Mass	pound (lb)	kg	(pound) x 0.4536
	metric ton (t)	kg	(ton) x 1000
	ounce	kg	(ounce) x 0.02835
Volume	liter (l)	m^3	(liter) x 0.001
	gallon (ga)	m^3	(gallon) x 0.00379
Temperature	fahrenheit (F)	K	{(farhrenheit) – 32} x 5/9 + 273.15
	celsius (C)	K	(celcius) + 273.15

Uncertainty in Measurement and Significant Digits

- All measurements have some degree of uncertainty that may come from a variety of sources such as limitations of the measuring device, the skill of the person making the measurement, Irregularities in the object being measured etc.
- The **uncertainty** in a measurement is the interval of confidence around the measured value such that the measured value is certain not to lie outside this stated interval.

- The following general rules of thumb are often used to determine the uncertainty in a single measurement.

1. Uncertainty in a scale measuring device is equal to the smallest increment divided by 2.

$$\sigma_x = \frac{Smallest\ increament}{2}$$

Example: Meter Stick (scale device)

$$\sigma_x = \frac{1mm}{2} = 0.5mm = 0.05cm$$

2. Uncertainty in a digital measuring device is equal to the smallest increment.

$$\sigma_x = smallest\ increament$$

Example: A reading from digital Balance is 5.7513 kg, therefore

$$\sigma_x = 0.0001$$

- The most common way of writing any measurement is

$$Measurement = x \pm \sigma_x$$

- Where, x is the measured value, σ_x is uncertainty in measurement.
- For example, if we measure a length of 5.7cm with a meter stick, this implies that the length can be anywhere in the range 5.65 cm ≤ L ≤ 5.75 cm.
- **An error in a Measurement** is defined as the difference between the actual or true value and the measured value. An error should not be confused with a mistake, the mistake can be avoided, while the error cannot be avoided but they can be reduced (minimized).

- Errors in measurement can be grouped into two categories:
- **Systematic Error -** errors resulting from measuring devices being out of calibration. Such measurements will be consistently too small or too large. These errors can be eliminated by pre-calibrating against a known, trusted standard.

- **Random Errors -** errors resulting in the fluctuation of measurements of the same quantity about the average. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations.

Precision, Accuracy and Significance

- **Accuracy** the closeness of a measurement to its true value.
- **Precision** the quality of being exact and the degree to which repeated measurements under the same conditions give the same value**.**
- **Significance** the number of significant figures used in a reading, which should be appropriate to the precision of the measuring instrument.

- For example, when you measure the length of standard A4 paper three times and obtain the 11.1 in., 11.2 in., and 10.9 in. your measurement is accurate when we compare to its true length is 11.0 inches long. It is also precise.
- Whenever you make a measurement, the number of meaningful digits that you write down implies the error in the measurement.
- **For example** if you say that the length of an object is 0.428 m, you imply an uncertainty of about 0.001m. You should only report as many significant figures as are consistent with the estimated error. The quantity 0.428m is said to have three significant digits, that is, three digits that make sense in terms of the measurement.

Rules for Determining a Significant Figures

- **Rule 1:** All non zero numbers are significant. 1-9.
- **Rule 2:** Zeros located between non-zero digits are significant. Eg. 5004 has four sig figs and 602 has three sig figs.
- **Rule 3:** Trailing zeros (those at the end) are significant only if the number contains a decimal point; otherwise they are insignificant. Eg. 5.640 has four sig figs, 120000. has six sig figs and 120000 has two sig figs

- **Rule 4:** Zeros to left of the first nonzero digit are *insignificant. Eg.* 0.000456 has three sig figs and 0.052 has two sig figs.
- ***Rule 5:*** Zeros that follow a number may be significant.
- **Rule 6:** *Numbers with no decimal points are ambiguous. Does 5000 ml mean exactly 5000? May be or May not be! So 5000, 500, 50, and 5 are all assumed to have 1 significant figure*

Rules for addition/subtraction

- When decimal numbers are added or subtracted, the number of significant digits should equal the smallest number of decimal places of any term in the sum or difference. Add/subtract and put the answer with least significant digits after decimal point.

Example: 7.939 + 6.26 + 11.1 = 25.299

- The final answer is 25.3

Rules for multiplication/division

- When decimal numbers are multiplied or divided, the number of significant digits in the final answer is the same as the number of significant digits in the least accurate of the factors. Thus put the final result with least significant figures.
- Example. 27.2 x 15.63 ÷ 1.846 = 230.3011918 =230. (3 S.f.)

Vectors: Composition and Resolution

A. Algebraic Method: Vectors are represented algebraically by a letter (or symbol) with an arrow over its head (Eg.$\vec{v}$, $\vec{p}$) and the magnitude of a vector is a positive scalar and is written as either by |**A**| or *A*.

B. **Geometric Method.**

• Vectors are nothing but straight line arrows drawn from one point. Zero vector is just a vector of zero length **- a point**. Length of vectors is the magnitude of vectors.

Vectors Algebra

- 1. **Addition of vectors:** A single vector that is obtained by adding two or more vectors is called **resultant vector** and it is obtained using the following two methods.
- A. **Graphical method of vector addition**
- Graphically vectors can be added by joining their head to tail and in any order their resultant vector is the vector drawn from the tail of the first vector to the head of the last vector.

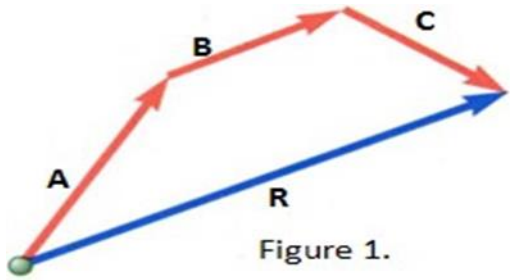


Figure 1.

- The **parallelogram law of Vector addition** States that the resultant R of two vectors A and B is the diagonal of the parallelogram for which the two vectors A and B becomes adjacent sides.

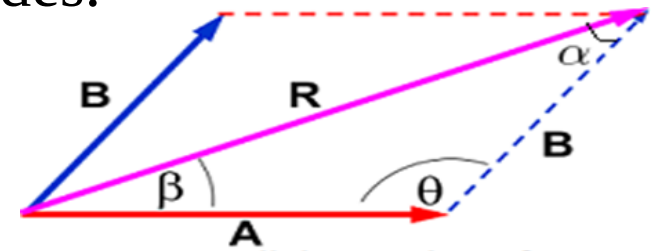


Figure 2. Parallelogram law of vector addition

- The magnitude of the diagonal (resultant vector) is obtained using cosine law.

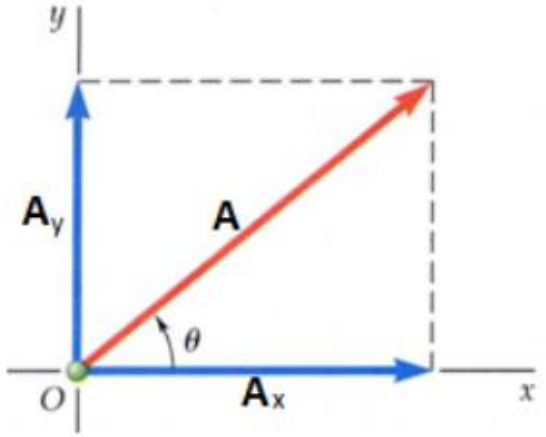
$$\boldsymbol{R = \sqrt{A^2 + B^2 - 2ABcos\theta}}$$

- The direction (i.e. the angle that the diagonal vector makes with the sides) is obtained using the sine law.

$$\frac{\boldsymbol{sin\theta}}{\boldsymbol{R}} = \frac{\boldsymbol{sin\alpha}}{\boldsymbol{A}} = \frac{\boldsymbol{sin\beta}}{\boldsymbol{B}}$$

Components of Vector

- The components of the given vector **A** are obtained by applying the trigonometric functions of sine and cosine.
- $A_x = Acos\theta$
- $A_y = Asin\theta$



Unit Vector

- A unit vector is a vector that has magnitude of one and it is dimensionless and its sole purpose is to point a given vector in specified direction. It is usually denoted with a “hat”.

$$\boldsymbol{A} = A\hat{u}$$

- Any vector can be expressed in terms of unit vectors as follows.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

Where $i, j\ and\ k$ are unit vector in cartesian coordinate system and A_x, A_y and A_z are components of vector $\boldsymbol{A}$.

Vector addition in Unit Vector Notation(Algebraic method)

- Let's consider two vectors $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$.

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$R = R_x i + R_y j + R_z k$$

Finding a Unit Vector

- Consider the vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
- The unit vector r^ in the same direction as the vector $\vec{r}$is simply the vector $\vec{r}$divided by its magnitude r.

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{r} = \frac{x}{r}\hat{\imath} + \frac{y}{r}\hat{\jmath} + \frac{z}{r}\hat{k}$$

Exercise 1. Vector $\vec{A}$ has magnitude of 8units and makes an angle of 45^0 with the positive x-axis. Vector $\vec{B}$ also has the same magnitude of 8units and directed along the negative x-axis. Find

a. The magnitude and direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$

2. Given the displacement vectors $\vec{A}=3\hat{\imath}-4\hat{\jmath}+4\hat{k}$, $\vec{B}=2\hat{\imath}+3\hat{\jmath}-7\hat{k}$. Find the magnitudes of the vectors a) $\vec{A}$+$\vec{B}$ b) 2$\vec{A}$-$\vec{B}$

3. If $\vec{A}=6\hat{\imath}-8\hat{\jmath}$, $\vec{B}=-8\hat{\imath}+3\hat{\jmath}$ and $\vec{C}=26\hat{\imath}+19\hat{\jmath}$. Find a and b Such that $a\vec{A}$ +b$\vec{B}+\vec{C}=0$

4. Find a unit vector in the direction of the resultant of vectors $\vec{A}=2\hat{\imath}-3\hat{\jmath}+\hat{k}$, $\vec{B}=\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{C}=3\hat{\imath}-2\hat{\jmath}+4\hat{k}$

CHAPTER TWO
KINEMATICS AND DYNAMICS OF PARTICLES

- Mechanics is the study of the motion of objects under the action of given forces.

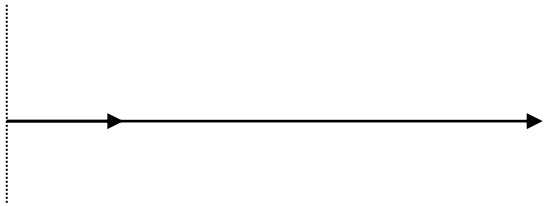
Branches of Mechanics

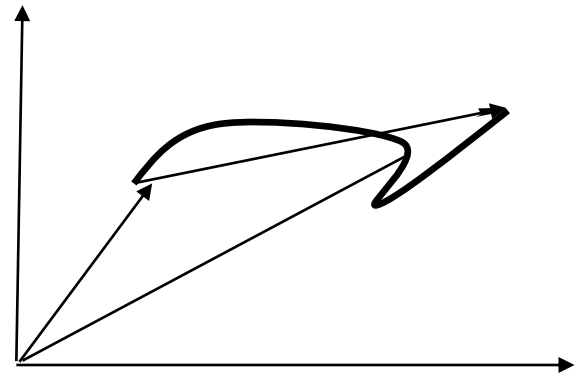
Kinematics: The Study the motions of an objects without the cause.

Dynamics: The study of the action of forces in producing either motion or static equilibrium.

- In physics motion is defined as the change in position with respect to time. It is described as: Change in speed and Change in direction.

- To describe the motion of an object by a **physical quantities**, their position must known in the space.





$\Delta x = x_f - x_i$ $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$

- **Average Velocity ($\vec{v}_{av}$):-**is the total displacement divided by the total time.

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

- **Average acceleration:** is the change in velocity ($\Delta\vec{v}$) of an object divided by the time interval during which that change occurs.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

- **Instantaneous acceleration**: -The limit of average acceleration as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

- **Average Speed**: - is the total distance traveled by the object divided by the total elapsed time.

$$v_{av} = \frac{total\ distance(S)}{total\ time\ interval(\Delta t)}$$

- Average speed and average velocity of an object do not provide the detail information of the entire motion.

- **Instantaneous Velocity ($\vec{v}$):** - is the limiting value of the ratio $\frac{\Delta \vec{r}}{\Delta t}$ as Δt approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

Examples: A car moving along the x-axis starts from the position $x_i = 2\,m$ when $t_i = 0$ and stops at $x_f = -3\,m$ when $t_f = 2\,s$.

A. Find the displacement, the average velocity, and the average speed during this interval of time.

B. If the car goes backward and takes 3 s to reach the starting point, then repeat part (a) for the whole time interval.

Solution:

- The car's displacement is given by:

$$\Delta x = x_f - x_i = -3\,m - 2\,m = -5\,m$$

- The average velocity is given by:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-5\ m}{2s} = -2.5\ m/s$$

- Since Δx and $\bar{v}$ are negative for this time interval, then the car has moved to the left. The total distance covered is d = 5m and the average speed is thus:

$$\bar{s} = \frac{\text{total distance}}{\Delta t} = \frac{5m}{2s} = 2.5\ m/s$$

B. After the backward movement, the final position and final time of the car are $x_f = 2\ m$ and $t_f = 2\ s + 3\ s = 5\ s$, respectively, while the total distance covered by the car is d = 5 m + 5 m = 10 m.

- Therefore, the displacement will be,

$$\Delta x = x_f - x_i = 2\,m - 2\,m = 0\,m$$

- Then, the average velocity will be:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0m}{5s} = 0\ m/s$$

- Finally, the average speed for the whole movement of the car will be:

$$\bar{s} = \frac{\text{total distance}}{\Delta t} = \frac{10m}{5s} = 2\ m/s$$

- Therefore, as you can expect, the average velocity is zero, while the average speed is 2 m/s since the speed depends only on the total covered distance.

Motion with Constant Acceleration

• For motion with constant acceleration, the velocity changes at the same rate throughout the motion and an average acceleration over any time interval is equal to the instantaneous acceleration at any instant of time.

$$a = \bar{a} = \text{Constant}$$

$$\overrightarrow{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t},$$

Assuming $t_i = 0$ and Rearranging this equation gives

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

Similarly we can drive other kinematic equation

Kinematics Equations in 1D and 2D motion

1D equations	2D equations	Descriptions
$\boldsymbol{v_f = v_i + at}$	$v_f = v_i + at$	Enables us to determine an object's velocity at *any* time *t*
$\boldsymbol{x_f = x_i + v_i t + \frac{1}{2} a t^2}$	$r_f = r_i + v_i t + \frac{1}{2} a t^2$	Provides the final position of the particle at time *t* in terms of the initial velocity and the acceleration.
$\boldsymbol{{v_f}^2 = {v_i}^2 + 2a(x_f - x_i)}$	${v_f}^2 = {v_i}^2 + 2a(r_f - r_i)$	Provides the final velocity in terms of the acceleration and the displacement of the particle

• Example:1. A track covers 40m in 8.5s while smoothly slowing down to a final speed of 2.8m/s. Find its original speed and acceleration.

Soln.

$$i. \quad S = \left(\frac{v_f + v_i}{2}\right) t \text{ or } v_i = \frac{2S}{t} - v_f = 6.6 m/s$$

$$ii. \quad \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t} \frac{6.6 m/s - 2.8 m/s}{8.5 s} = -0.447 m/s^2$$

Free Fall Motion

- **Why an object fall?**
- Consider you have feather and apple. When you release both at the same time in vacuum which will land first? Why?
- **Free fall motion** is a uniformly accelerated motion of an object near the Earth surface under the influence of the force of gravity only.
- In the absence of air resistance, all objects fall with constant acceleration, toward the surface of the Earth surface (g = 9.8 m/s^2). i.e during each second of fall the speed of an object changes by 9.8m/s due to the force of gravity.
- On the Moon, for example, the acceleration due to gravity is only 1.67m/s2.

- A freely falling motion is uniformly accelerated motion in a vertical direction, the kinematics equation of motion with constant acceleration are used.
- However, we can make them simpler to use with the following minor changes:

❑ The acceleration due to gravity is negative if the y-axis is chosen to be upward and positive if the y-axis is chosen to be downward.

- Therefore,

$$v_f = v_i \pm gt$$

$$y_f = y_i + v_i t \pm \frac{1}{2} g t^2$$

$$v_f{}^2 = v_i{}^2 \pm 2g(y_f - y_i)$$

- Example: A ball is dropped from a tall building. Choose the positive y to be downward with its origin at the top of the building, i.e. $y_i = 0$. Find the following for the ball's motion:

A. Its acceleration

B. The distance it falls in 2s

C. Its velocity after falling 15 m, and

D.The time it takes to fall 25 m.

- **Solution:**

A. Since the positive y is downward, then the ball's acceleration is positive (downward) and will be given by $a = g = 9.8\, m/s^2$. Also, the ball's velocity will be always positive.

B. To find y_f, we use

$$y_f = v_i\, t + \frac{1}{2} g\, t^2 = 0 + \frac{1}{2}\left(9.8 \frac{m}{s^2}\right) 2s^2 = 19.6m$$

C. To find v_f, we use

$$v_f{}^2 = v_i{}^2 + 2g\left(y_f - y_i\right) = 0 + 2 \times \left(9.8 \frac{m}{s^2}\right) \times 15\, m = 294 \frac{m^2}{s^2}$$

$$v_f = \sqrt{294}\ \frac{m}{s} = 17.2\, m/s$$

D. To find t, we use,

$$y_f = y_i + v_i t + \frac{1}{2} g t^2$$

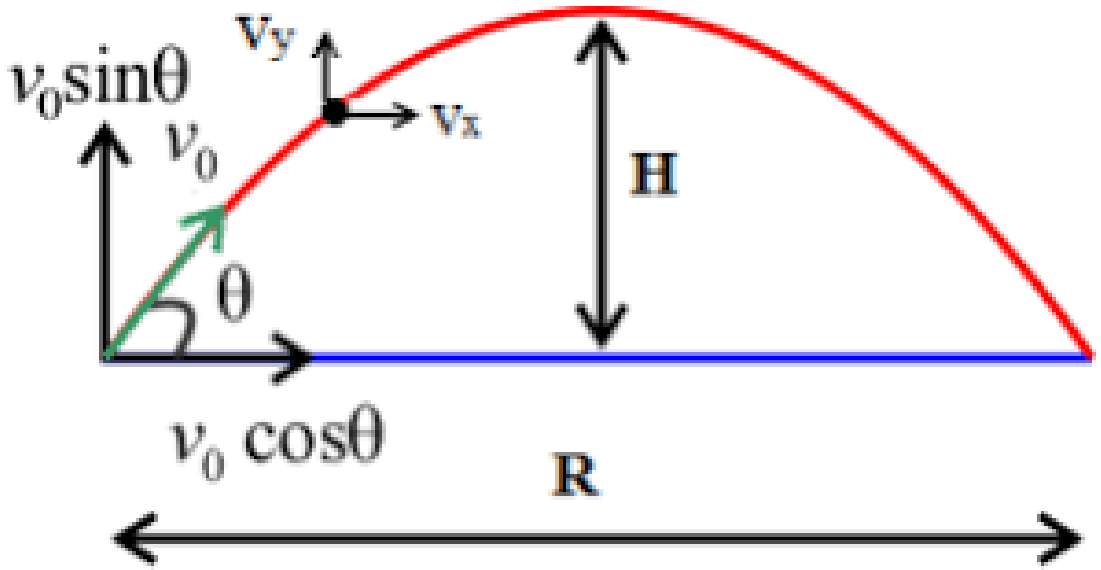
So solving for t, we have,

$$25m = 0 + 0 + \frac{1}{2}(9.8\ m/s^2)t^2$$

$$t = \sqrt{\frac{50}{9.8} s^2} = 2.3s$$

Projectile Motion

- When an object is projected obliquely near the earth's surface, it moves simultaneously in the direction of horizontal and vertical. The motion of such object is called Projectile Motion.



- Basic assumptions in projectile motion
- The free fall acceleration (g) is constant over the range of motion and it is directed downward.
- The effect of air resistance is negligible.
- The path described by the projectile from the point of projection to the point where the projectile reaches the horizontal plane passing through the point of projection is called **trajectory**. The trajectory of the projectile is a **parabola**.

- The horizontal position of the projectile after some time t is:
  - $x = ucos\theta t$ *

- The vertical position of the projectile after some time t

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

$$y = usin\theta t - \frac{1}{2} g t^2 \quad **$$

- Solving for t from eqn * and substituting into eqn **, the trajectory of the projectile becomes,

$$y = tan\theta x - \frac{g x^2}{2(vcos)^2}$$

Which exactly the equation of parabola.

Horizontal Range and Maximum Height

- When the projectile reaches the maximum height (the peak), $v_y = 0$

$$0 = usin\theta - gt$$

$$t = \frac{usin\theta}{g}$$

$$h = usin\theta(\frac{usin\theta}{g}) - \frac{1}{2}g(\frac{usin\theta}{g})^2$$

$$\boldsymbol{h = \frac{u^2 sin^2\theta}{2g}}$$

- The Range(R) is the horizontal displacement of the projectile covered in a total time of flight.

$$\bullet\ t_{tot} = 2t = \frac{2usin\theta}{g}$$

- When $x = R = ucos\theta t_{tot}$

$$R = (ucos\theta)(\frac{2usin\theta}{g}) \qquad \text{But,} \quad 2sin\theta cos\theta = sin2\theta$$

$$\boldsymbol{R = \frac{u^2 sin2\theta}{g}}$$

- The range (R) is maximum, when $sin2\theta = 1 \rightarrow 2\theta = 90^0 \rightarrow \theta = 45^0$

$$R_{max} = \frac{u^2}{g}$$

Particle Dynamics and Planetary Motion

- In this section, we investigate what causes changes in the state of motion. i.e What cause particles to remain at rest or accelerate?
- It is because of the **mass of the object** and **forces acting on it**. Force is an external agent capable of changing the state of rest or motion of a particular body.
- Knowledge of Newton's laws and the ability to apply them to various situations will allow us to explain much of the motion we observe in the world around us.

- Based on how particles interact with each other, there are four basic forces in nature.
- Gravitational, Electromagnetic, Strong nuclear and weak nuclear forces.
- There are two types of forces based on their applications: Contact and non-contact force.
- **Contact force** is the forces which act on a body either directly or through a medium.
- Eg. Muscular Forces, frictional force, normal force, spring force, tension force etc.

Non-contact forces: is the forces which act through spaces without making direct contact with the body. Eg. gravitational force, electrostatic force and magnetic force.

Newton's Laws of Motion and Applications

- Laws of motions are formulated for the first time by English physicist Sir Isaac Newton in 1687.
- He developed the three laws of motion in order to explain why the orbits of the planets are ellipses rather than circles.
- Newton's laws continue to give an accurate account of nature, except for very small bodies such as electrons or for bodies moving close to the speed of light.

Newton’s First Law of Motion

- States that **“In a given reference frame, an object at rest remains at rest, and an object in motion continues in motion with constant velocity unless it experiences a net external force.”**
- It’s also known as the law of inertia because it defines the special set of the inertia reference frame in which the acceleration of an object zero.
- The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. It is the tendency of an object to resist any attempt to change its velocity.
- The inertia of an object is measured by its mass. As we know from experience, some objects have more inertia than others.

Newton's Second law of Motion

- The acceleration acquired by a point particle is directly proportional to the net force acting on the particle and inversely proportional to its mass and the acceleration is always in the direction of the net force.

$$\sum \mathrm{F} = \mathrm{ma}$$

Example. A 3kg object undergoes an acceleration given by $\vec{a} = (2\hat{\imath} + 5\hat{\jmath})m/s^2$. Find the magnitude of the resultant force. $\vec{F} = m\vec{a}$

$$m = 3kg, \overrightarrow{a} = (2\hat{\imath} + 5\hat{\jmath})\frac{m}{s^2}$$

$$F = (3kg)(2\hat{\imath} + 5\hat{\jmath})m/s^2 = (6\hat{\imath} + 15\hat{\jmath})N$$

- *The magnitude of the force is*

$$\left|\vec{F}\right| = \sqrt{6^2 + 15^2} = 16.15N$$

1. **Exercise:** Find the force needed to accelerate a mass of 40kg from velocity $\vec{v}_i = (4\hat{\imath} - 5\hat{\jmath} + 3\hat{k})m/s$ to $\vec{v}_f = (8\hat{\imath} + 3\hat{\jmath} - 5\hat{k})m/s$ in 10s.

Newton's Third law of Motion

- States that “For every action there is always an equal and opposite reaction.”
- Let’s consider two bodies A and B that are interacting. The forces they exert on each other is equal in magnitude and opposite in direction.

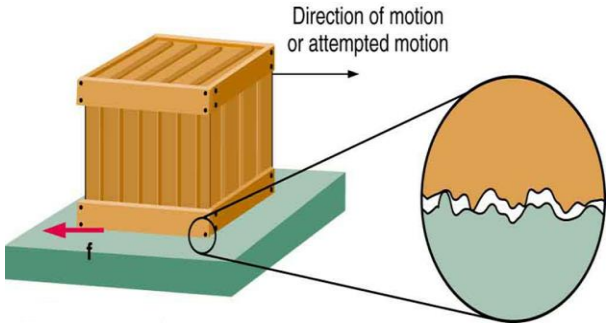
$$\boldsymbol{F_{BA} = -F_{AB}\ or}$$

$$\boldsymbol{F_{AB} + F_{BA} = 0}$$

- Note that: Action and reaction forces are always exist in pair, A single isolated force cannot exist and Action and reaction forces act on different objects.

Frictional Force

- Friction is a force that opposes relative motion between surfaces in contact. It is not a fundamental force, like gravity or electromagnetism.



- Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view.
- In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion.

- Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free.
- Generally friction force is always proportional to the normal force between the two interacting surfaces. Mathematically.

$$\boldsymbol{F_{frict}} \propto \boldsymbol{F_{norm}} \text{ or } \boldsymbol{F_f} = \mu F_N$$

- Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.
- There are two types of Friction force. Static friction operates between two surfaces that aren't moving relative to each other, while kinetic friction acts between objects in motion.

$$f_k = \mu_k N \text{ and } f_s = \mu_s N$$

- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s (which implies that $f_k < f_s$).

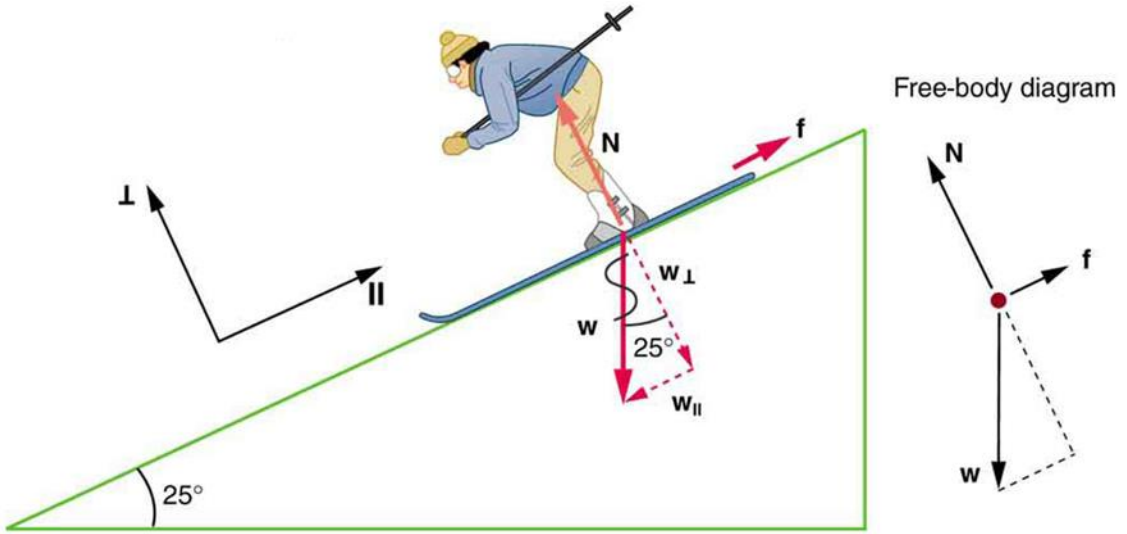
- Example: A 25.0-kg block is initially at rest on a horizontal surface. A. horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.
- *Soln.* $f_s = \mu_s N$ *but* $N = mg$

Given, $m = 25kg,\ f_s = 75N,\ f_k = 60N$ then req: $\mu_s \& \mu_k = ?$

$$\mu_s = \frac{f_s}{mg} = \frac{75N}{(25kg)(9.8m/s^2)} \mu_s = 0.31$$

$$\mu_k = \frac{f_k}{mg} = \frac{60N}{(25kg)(9.8m/s^2)} = 0.245$$

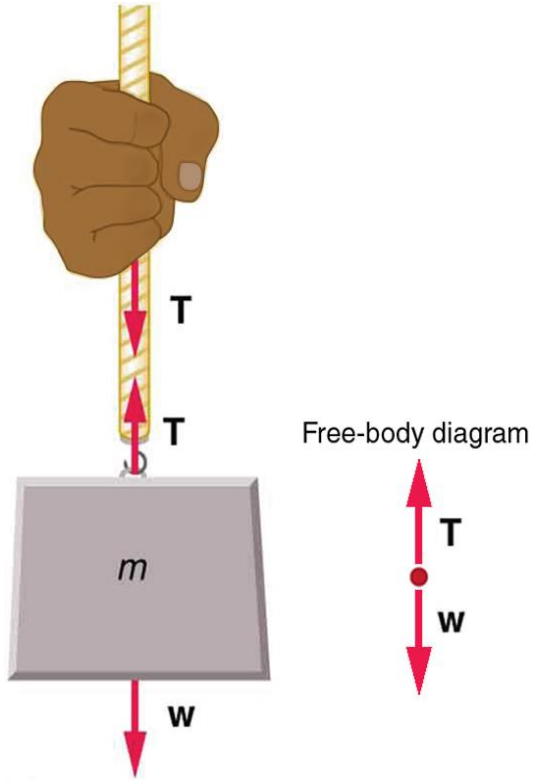
- Example. A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if frictional force is 45.0 N.



- Ans $\mu_k = 0.082$ check it.

- A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable.

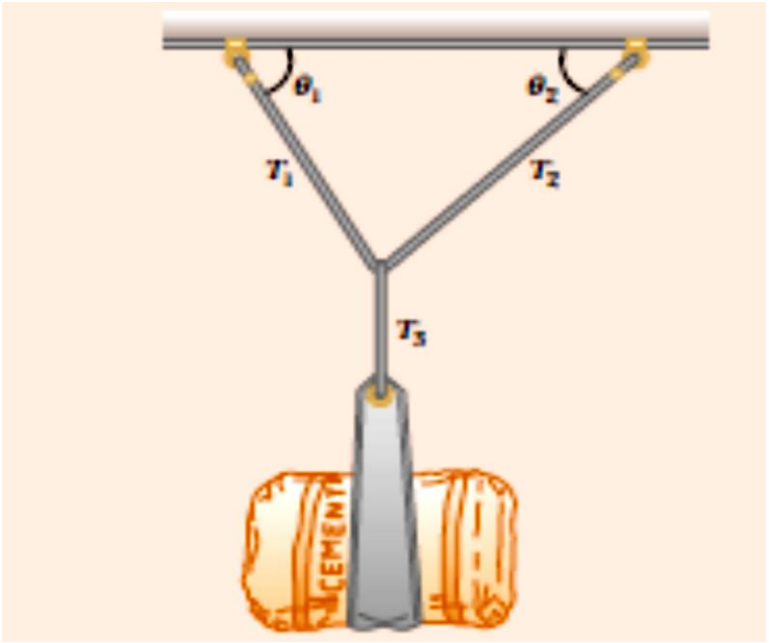
$$F_{net} = T - W = 0$$



Applications of Newtons Laws

- We apply Newton's laws to objects that are either in equilibrium ($\vec{a}= 0$) or accelerating along a straight line under the action of constant external forces.
- Procedure, 1st , draw a free-body diagram for each body separately, showing all the forces acting on that body.
- 2nd apply Newton's second law to X and Y components of force.
- 3rd Solve the equation for the unknown(s).

• A bag of cement of weight 300 N hangs from three ropes as shown in the figure below. Two of the ropes make angles of $\theta_1 = 53.0°$ and$\theta_2 = 37.0°$ with the horizontal. If the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the ropes.



- **Soln.** 1st draw the free body diagram.

Apply newtons law of motion, $\sum \vec{F} = 0 \rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$

- From free body diagram (a) $\sum F_x = T_2 cos37^0 - T_1 cos53^0 = 0$

$$0.8T_2 = 0.6\,T_1 \qquad (1)$$

- $\sum F_y = T_1 sin53^0 + T_2 sin37^0 - T_3 = 0$

$$0.8\,T_1 + 0.6\,T_2 = T_3 \qquad (2)$$

$$\sum F_y = T_3 - W = 0 \rightarrow \boldsymbol{T_3 = W = 300N} \qquad (3)$$

- Substituting(3) in (2) $\quad 0.8\,T_1 + 0.6\,T_2 = 300N \qquad (4)$

$$\text{But } 0.8T_2 = 0.6\,T_1 \rightarrow T_2 = \frac{0.6\,T_1}{0.8} = 0.75T_1$$

- Substituting for T_2 in eqn(4) Gives

$$0.8\,T_1 + 0.6(0.75T_1) = 300N$$

$$\boldsymbol{T_1 = 240N}$$

$$T_2 = 0.75T_1 = 0.75(240N)$$

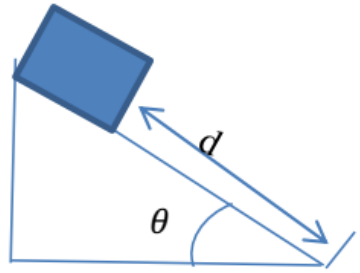
$$\boldsymbol{T_2 = 180N}$$

Example 2. A block of mass m = 2 kg is placed on an inclined plane of angle θ = 30◦, as shown in Figure. The block is released from rest at the top of the plane, where the distance from the bottom is d = 10 m.

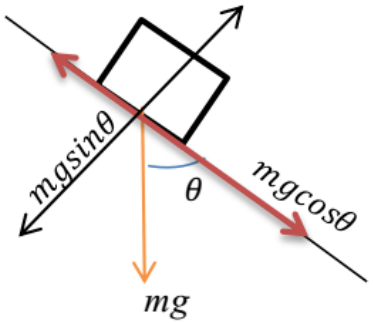
A. Find the magnitude of the acceleration of the block and the normal force exerted on the block.

B. How long does it take the block to reach the bottom, and what is its speed just as it gets there?

Solution:



- First construct a free-body diagram. The only forces on the block are the weight W (acting downward) and the normal force N.



- Applying Newton's second law to the x and a y component gives:

$$\sum F_x = mg\cos\theta = m\,ax \qquad (1)$$

$$\sum F_y = N - mg\sin\theta = 0 \qquad (2)$$

- From Eqn. 1,

$$a_x = g\cos\theta = 10\frac{m}{s^2} \times \cos 30° = ???\ m/s^2$$

- From Eqn.2, we find N to be:

$$N = mg\sin\theta = (2\,kg)\left(10\frac{m}{s^2}\right)(\sin 30°) = ??\ N$$

- Since a_x = constant, we apply the equation $x = v_{xi}\, t + \frac{1}{2} a_x\, t^2$ to the block with $v_{xi} = 0$ and $x = d = 10\, m$ to get the following relation:

$$d = \frac{1}{2} a_x\, t^2$$

- Solving for t, we have,

$$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2 \times 10m}{5\ m/s^2}} = 2s$$

- Also, we can apply the kinematics equation ${v_x}^2 = {v_{xi}}^2 + 2a_x x$ with $v_{xi} = 0$ and

$x = d = 10\, m$ to get the following relation:

$${v_x}^2 = 2a_x d$$

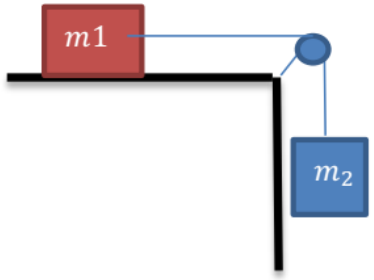
- Then, solving for v_x and taking the positive root yields:

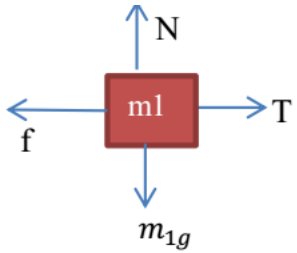
$$v_x = \sqrt{2a_x d} = \sqrt{2 \times 5\ m/s^2 \times 10m} = 10\ m/s$$

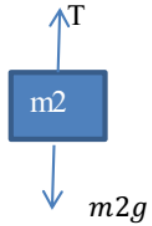
- A block of mass $m_1 = 4\,kg$ lying on a rough horizontal surface is connected to the second block of mass $m_2 = 6\,kg$ by a light non-stretchable cord over a massless, frictionless pulley as shown below. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

A. Find the magnitudes of the acceleration of the system and the tension in the cord.

B. Find the relation between m_1 and m_2 in the case when the system is on the verge of slipping.







- When Newton's second law is applied to m_2 in part (b) of the figure, we find:

$$\sum F_x = 0 \text{ and } \sum F_y = m_2 g - T = m_2 a$$

$$T = m_2(g - a) \qquad (1)$$

- When Newton's second law is applied to m_1,

$$\sum F_x = T - f_k = m_1 a \qquad (2)$$

$$\sum F_y = m_1 g - T = 0 \qquad (3)$$

- But, $f_k = N\mu_k = m_1 g \mu_k$
- Then using this value into eqn. 2, we have,

$$T = m_1(a + g\mu_k) \qquad (4)$$

- Equating eqn. 1 and 4, we have,

$$m_2(g - a) = m_1(a + g\mu_k)$$

- Solving for $\boldsymbol{a}$ we get:

$$\vec{a} = \frac{m_2 - m_1\mu_k}{m_1 + m_2} g = 3.92\ m/s^2$$

- We can find T by substituting the expression of $\boldsymbol{a}$ into eqn. 4, we get

$$T = \frac{(\mu_k+1)m_1 m_2}{m_1+m_2} g = 35.28\ \text{N}$$

B. When the system is on the verge of slipping, the magnitude of the force T that acts on mass m1 must equal the maximum static friction

$$fs, max = \mu_s N, \quad \text{i.e. } T = \mu_s N = \mu_s m_1 g.$$

- Also, the weight of the mass m_2 must equal the magnitude of the tension, i.e. T = m_2g.

Thus:

$$m_2 = \mu_s m_1$$

Newton’s Law of Universal Gravitation

- According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun.
- The inspiration of Newton’s apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton’s universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature.

- The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them.
- Newton’s universal law of gravitation states that every particle in the universe attracts every other particle with a force along a line joining them.
- The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton’s universal law of gravitation is

$$F_g = G\frac{mM}{r^2}$$

- Where G is the Universal gravitational constant ($G = 6.674 \times 10^{-11}\ Nm^2/kg^2$).
- Weight is the *gravitational force* exerted on an object of a certain mass.
- Why the acceleration due to gravity is 9.8m/s2? Now, let's combine the gravitational force and the weight of an object to answer the questions.

$$mg = \frac{GmM.}{r^2}$$

$$g = \frac{GM.}{r^2}$$

Substituting known values for Earth's mass and radius, we have,

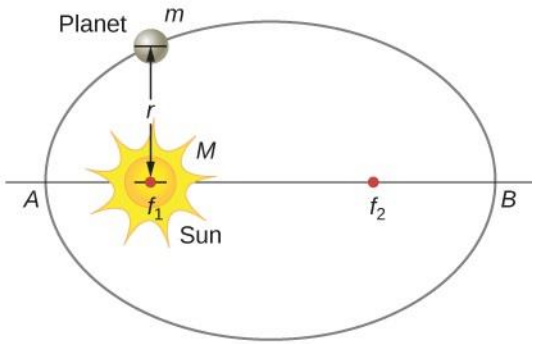
$$g = (6.67 \times 10^{-11}\ \frac{\mathrm{Nm}^2}{\mathrm{kg}^2})(\frac{5.98x10^{24}kg}{6.38x10^{6}m^2}) = 9.8\frac{m}{s^2}$$

- This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration.

Keplers law of planetary motion

- The German astronomer and mathematician Johannes Kepler, worked briefly with the Danish astronomer Tycho Brahe, who gathered some of the most accurate observational data on planetary motion.
- After the death of Brahe died in 1601, Kepler inherited his data books and devoted many years of intensive effort to find a mathematical description for the planetary motion described by the data.
- From his analysis, he formulated three laws and he announce the first two laws in the year 1609 and a third law nearly a decade later, in 1618.

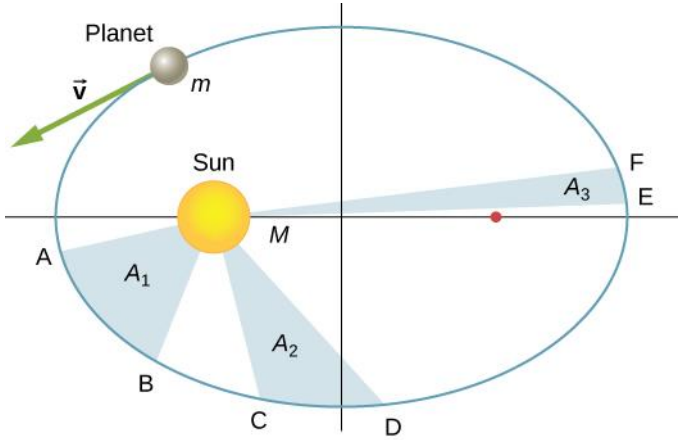
- **Keplers first law** states that “All planets move around the Sun in elliptical orbits, with the Sun as one focus of the ellipse”.



- For elliptical orbits, the point of closest approach of a planet to the Sun is called the perihelion (piont A) and the farthest point is the aphelion (point B).

- **Keplers second law** states that a planet sweeps out equal areas in equal times, that is, the area divided by time, called the areal velocity, is constant.
- Comparing the areas and the distance traveled

along the ellipse in each case, we can see that in order for the areas to be equal, the planet must speed up as it gets closer to the Sun and slow down as it moves away.



- Keplers second law is actually a consequence of the conservation of angular momentum, which holds for any system with only radial forces.

- **Keplers third law** states that the square of the period is proportional to the cube of the semi-major axis of the orbit. For an ellipse, the semi-major axis is one-half the sum of the perihelion and the aphelion.
- In Satellite Orbits, the Keplers third law can expressed as,

$$T^2 = \frac{4\pi^2}{GM} a^3$$

- Where T is the period of satellite, G is Universal gravitation Constant, M mass of satellite, and a is semi-major axis of ellipse.

- Unlike Keplers 1st and 2nd laws that describe the motion characteristics of a single planet, the third law makes a comparison between the motion characteristics of different planets.
- The comparison being made is that the ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets.

Planet	**Period (s)**	**Average Distance (m)**	**T^2/R^3 (s^2/m^3)**
Earth	3.156×10^7 s	1.4957×10^{11}	2.977×10^{-19}
Mars	5.93×10^7 s	2.278×10^{11}	2.975×10^{-19}

Apparent weight and Weightlessness

- The true weight of an object is defined as the weight due to force of gravity that the object feels.

$$W = mg$$

- An apparent weight defined as the weight due to the force of gravity and other external force. It depends on the states of motion.

$$\boldsymbol{w}_{ap} = \boldsymbol{w} - m\boldsymbol{a}$$

- For example, consider a body in an elevator. When an elevator is at rest or moving with constant velocity,

$$\boldsymbol{w}_{ap} = mg$$

- If a body in elevator is accelerating upward,

$$\boldsymbol{w}_{ap} = mg + ma$$

- Hence, an apparent weight is larger than the actual weight of a body.
- When the elevator is moving downward with constant acceleration,

$$\boldsymbol{W_{ap}} = mg - ma$$

- The apparent weight is smaller than the actual weight.
- When the elevator falls freely,

$$\boldsymbol{W_{ap}} - mg = -mg$$

$$\boldsymbol{W_{ap}} = 0$$

- That is the apparent weight of a body is Zero and a body is in state of weightlessness.

Work, Energy and Linear Momentum

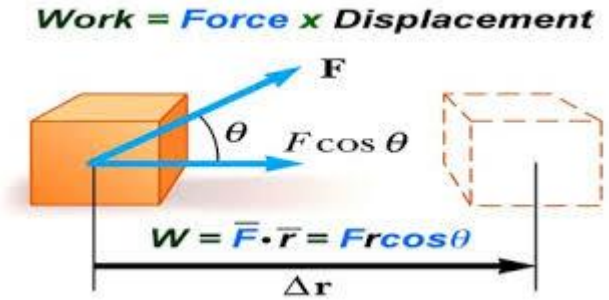
- In our day to day activity work can be just any activity we perform for the purpose of getting food. For example, construction, selling something, driving a bus, teaching a class, cooking food, treating patients, playing a music and so much more.
- However, in physics, work is more specific. Work can be defined as transfer of energy due to an applied force. The applied force must cause the motion. If no displacement takes place, no work is said to be done.

• Therefore for work to be done on an object, three essential conditions should be satisfied:

- Force must be exerted on the object
- The force must cause a motion or displacement
- The force should have a component along the line of displacement

• If a particle subjected to a constant force $\vec{F}$ undergoes a certain displacement, $\Delta\vec{r}$, the work done W by the force is given by:

$$W = \vec{F}.\Delta\vec{r} = \vec{F}||\Delta\vec{r}|\cos\theta$$

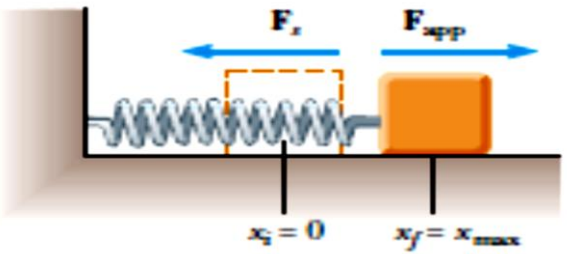


• The work done by the applied force is positive when the projection of $\overrightarrow{F}$ onto $\Delta\vec{r}$ is in the same direction as $\Delta\vec{r}$.

• Consider the Mass-spring system where the force applied varies with position constantly. If the spring is either stretched or compressed a small distance from its un-stretched (equilibrium) configuration, it exerts on the block a force that can be expressed as:

$$F_s = -kx \text{ (Hook's Law)}$$

• The negative sign in the equation signifies the force exerted by the spring is always directed *opposite* to the displacement from equilibrium.



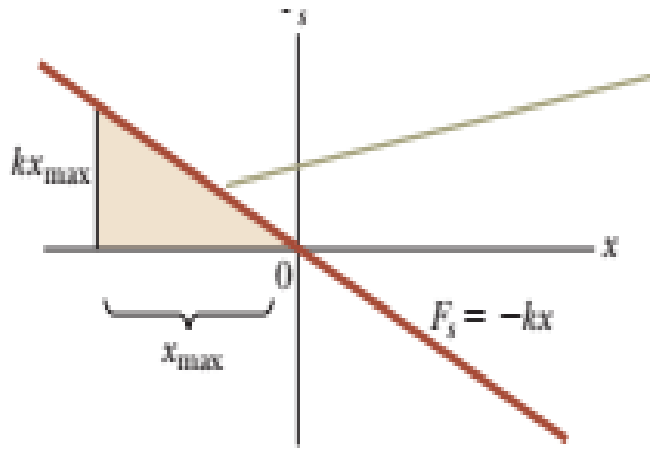
- Work done by the restoring force if the block undergoes an arbitrary displacement from x_i to x_f is the area enclosed by the above graph.

$$Area = W = \frac{1}{2}(F_s \, x \, \Delta X), \qquad but \qquad F_s = -k\Delta X$$

$$W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- If the work is done by the applied force

$$W_{F_{app}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$



- **Example:** An applied force of -5 N is exerted on the block that is attached to the free end of the spring. As a result of this force, the spring is compressed by 1 cm from its relaxed length.

A. What is the spring constant of the spring?

B. What force does the spring exert on the block if the spring is compressed by 2.5 cm?

C. How much work does the spring force do on the block as the spring is compressed from the relaxed state by 2.5 cm?

D. How much work does the spring force do on the block during a total displacement starting from compression of 2.5 cm, passing through the equilibrium, and then to a stretch of 2.0 cm?

- **Solution:**

A. The compressed spring pushes the block with a force $f_s = -F_a = +5\ N$. From $f_s = -k\ x$, with $x = -1$ cm, we have:

$$k = -\frac{f_s}{x} = -\frac{5N}{-0.01m} = 500N/m$$

B. Using Hooke's law, with $x = -2.5\ cm = -0.025\ m$, we have:

$$f_s = -k\ x = -500N/m \times -0.025\ m = 12.5N$$

C. Since the spring is initially at its relaxed state, the work done by the spring force on the block from $xi = 0\ to\ xf = -0.025\ m$ will be:

$$W_s = \frac{1}{2}k\left({x_i}^2 - {x_f}^2\right) = -\frac{1}{2} \times 500N/m \times (-0.025m)^2 = -0.156\ J$$

D. The work is negative because the spring force and the displacement are in opposite directions.

Here, we have $x_i = -0.025m$ and $x_f = 0.02\ m$, then,

$$W_s = \frac{1}{2}k\left({x_i}^2 - {x_f}^2\right) = \frac{1}{2} \times 500N/m\left((-0.025m)^2 - (0.02m)^2\right) = 0.06J$$

ENERGY and ITS TYPES

- It is defined as the capacity of a physical system to perform work. It exists in several forms such as thermal, chemical, mechanical, sound, light etc. which all fall into two forms **kinetic and potential** energy.
- **Kinetic energy (KE)** is the capacity of an object to do work by virtue of its motion.

For an object of mass *m* and moving with speed *v*, the kinetic energy is calculated as:

$$KE = \frac{1}{2}mv^2$$

Work-Energy Theorem

- When a complex force is exerted on an object, solving such a problem using Newton's second law can be difficult. Hence, we have an alternate methods. That is to relate the speed of an object to the net work done by such a force.
- Consider a force F is applied on an object initially moving with velocity v_i. When the object is displaced x distance from its initial position, the final velocity of an object can be expressed using the kinematics equation as

$$v_f{}^2 - v_i{}^2 = 2ax$$

- Multiply both sides above eqn. by m, and rearranging the equation, one can obtain,

$$\frac{m}{2}\left({v_f}^2 - {v_i}^2\right) = max$$

- This equation tells us that, the work done by the net force on a particle of mass m is equal to the difference between the initial and final values of kinetic energy.
- In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

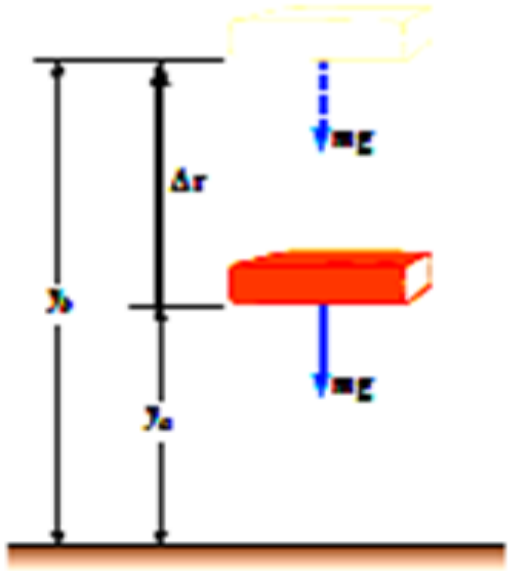
$$W_{net} = \Delta K$$ Work-Energy Theorem

Conservative Forces and Potential Energy

• An object possesses gravitational potential energy if it is positioned at a height above (or below) the zero height. The potential energy concept can be used only when dealing with a special class of forces called *conservative forces*.

• The potential energy the near the earth surface is given by

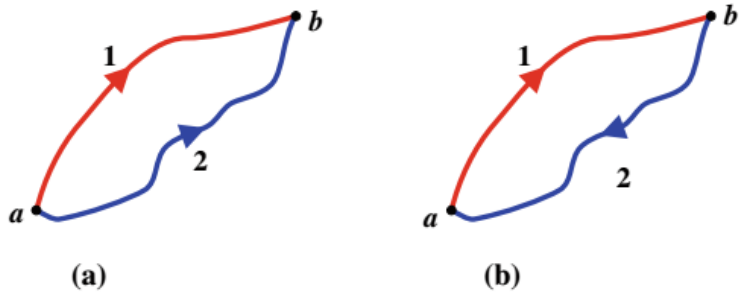
$$\Delta U_g = mg\Delta y$$



- **A conservative force** is a force that conserves the mechanical energy of the system.
- A conservative force has two important properties:

✓The work done by a conservative force is independent of the path.

✓The work done by a conservative force on an object moving through any closed path is zero.



For example, the force of gravity, the electromagnetic force and spring forces are the best examples of a conservative force.

Mechanical Energy and Its Conservations

- **It** is defined as the sum of kinetic energy and potential energy. Consider the book-earth system shown in the figure. As the book falls from y_b to y_a, the work done by the gravitational force on the book is

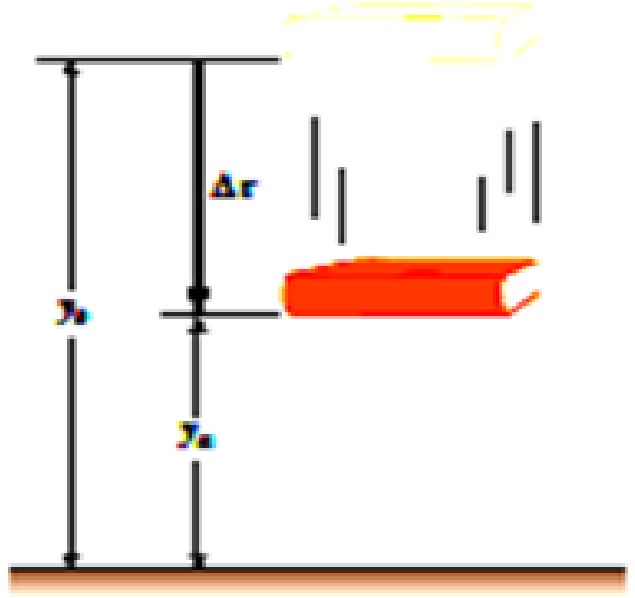
$$W_{onbook} = (-mg).(y_a - y_b)$$

$$= mgy_b - mgy_a$$

$$= -(mgy_a - mgy_b) = -(U_f - U_i)$$

But $-\Delta U_g = \Delta K_{book}$ (Work-energy)

$\boldsymbol{K_f + U_f = K_i + U_i}$ (*Conservation of mechanical energy*)

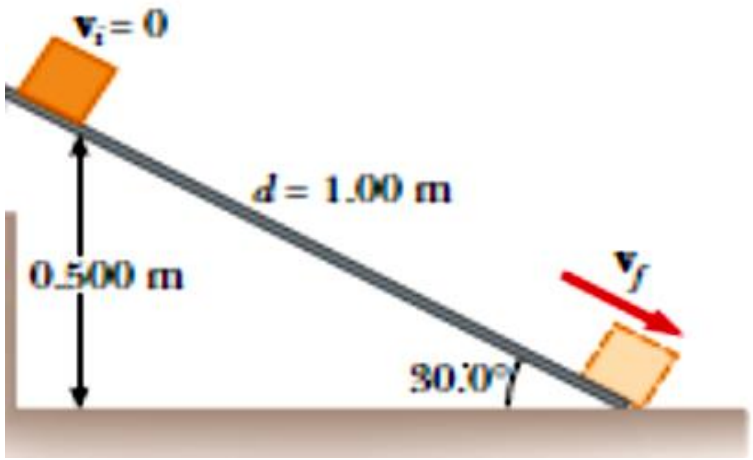


- *The mechanical energy of an isolated system is conserved.*

$$\frac{1}{2}mv_f{}^2 + mgy_f = \frac{1}{2}mv_i{}^2 + mgy_i$$

- Example:- A 3kg crate slides down a ramp. The ramp is 1.00m in length and inclined at an angle of 30.0°, as shown in the figure below. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

- **Solution:**
- *Since the non-conservative force exists in the system,*



$$\Delta K + \Delta U = -f_k d$$

$$K_f - K_i + U_f - U_i = -f_k d$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = -(5N)(1m)$$

$$\frac{1}{2}mv_i^2 = mgh_f = 0 \; because \; both \;\; v_i = h_f = 0$$

$$\frac{1}{2}(3kg)v_f^2 - (3kg)(9.8m/s^2)(0.5m) = -5J$$

$$v_f^2 = 6.47 \;\; \rightarrow v_f = 2.54m/s$$

Power

- Power is defined as the time rate of energy transfer.
- If an external force is applied to an object the work done by this force in the time interval Δt, then the average power during this interval is defined as

$$P_{av} = \frac{W}{\Delta t}$$

- The instantaneous power P is the limiting value of the average power as Δt approaches zero:

$$P = \lim_{\Delta t \to o} \frac{W}{\Delta t} = \frac{W}{t} \qquad \text{But, } W = \boldsymbol{F}.\triangle \boldsymbol{r}$$

$$P = \frac{W}{\triangle t} = \vec{F}.\frac{\triangle \vec{r}}{\triangle t} = \vec{F}.\vec{v}$$

Linear Momentum

- What happens when two automobiles collides? Anything that has a lot of momentum is going to be hard to stop. Physically, the more momentum an object has, the more force has to be applied to stop it in a given time.
- The linear momentum $\vec{P}$ of an object of mass m moving with velocity $\vec{v}$ is the product of its mass and velocity.

$$\boldsymbol{P} = m\boldsymbol{v}$$

- Changing the momentum of an object requires the application of a force. This is, in fact, how Newton originally stated his second law of motion. Starting from the more common version of the second law, we have

$$\vec{F} = m\frac{\triangle \vec{v}}{\triangle t} \Longrightarrow \vec{F} = \frac{\triangle (m\vec{v})}{\triangle t} = \frac{\triangle \vec{P}}{\triangle t}$$

- Therefore, the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

$$\triangle \vec{P} = \vec{F} \triangle t$$

$$\vec{I}(Impulse) = \Delta\vec{P} = \vec{P}_f - \vec{p}_i = \overrightarrow{F} \triangle t$$

Conservation of Linear Momentum

• When two objects of masses m_1 and m_2 interact, the force that object 2 applies to object 1 is equal in magnitude and opposite in direction to the force that object 1 applies on object 2.

$$\vec{F}_{12} + \vec{F}_{21} = 0 \; or \; m_1 \frac{\triangle \vec{v}_1}{\triangle t} + m_2 \frac{\triangle \vec{v}_2}{\triangle t} = 0$$

$$\Longrightarrow m_1 \Delta\vec{v}_1 + m_2 \Delta\vec{v}_2 = 0$$

$$\Longrightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = \; m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 = constant$$

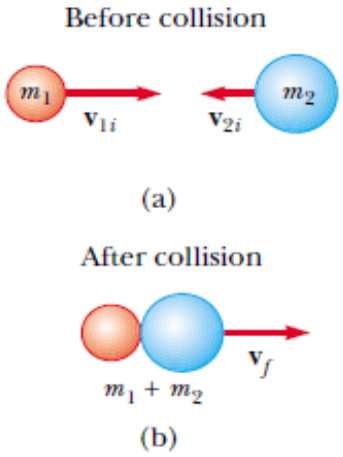
- **When no net external force acts on a system, the total momentum of the system remains constant in time. Or** “*Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant”*.

Collisions

- A **collision** is the event in which two or more bodies exert forces on each other during short period of time.
- There are two types of collision: An **elastic collision**, both momentum and kinetic energy are conserved.
- An **inelastic collision**, only momentum is conserved but kinetic energy is not.
- In a **perfectly inelastic collision**, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same..

- The total kinetic energy during an inelastic collision is converted to internal energy, sound energy, and the work needed to permanently deform the objects involved.

- **Inelastic Collision**



When the two particles collide head-on, stick together, and then move with some common velocity v_f after the collision.

Using conservation of linear momentum,

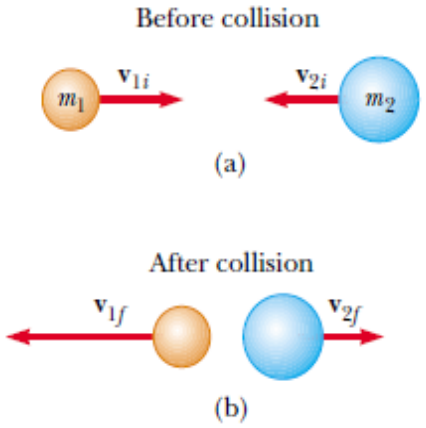
$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Solving for the final velocity gives,

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Elastic Collisions

- Consider



A one-dimensional elastic collision between masses m_1 and m_2 we can relate the final velocities to the initial velocities.

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Special cases:

1. If $m1 = m2$,

Then $v1f = v2i$ and $v2f = v1i$. That is, the particle exchange velocities if they have equal masses.

2. If particle 2 is initially at rest, then $v2i = 0$,

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right) v_{1i}$$

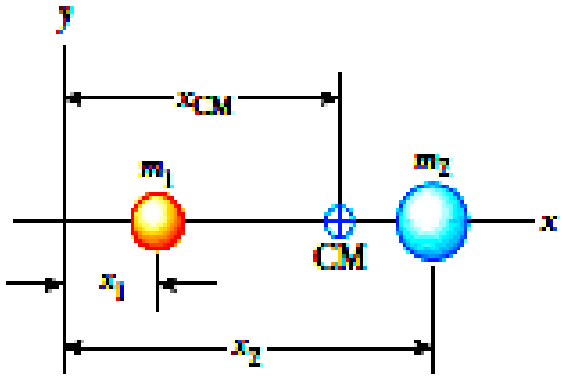
3. If m_2 is much greater than m_1 and m_2 is at rest, then

$$v_{1f} \approx v_{1i} \text{and } v_{2f} \approx 2v_{1i}$$

- That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.
- Example:

Center of Mass

- The center of mass is the point at which all the mass can be considered to be "concentrated". The center of mass of the system is located somewhere on the line joining the particles and is closer to the particle having the larger mass.
- The center of mass of the pair of particles located on the *x* axis as shown in the figure below lies somewhere between the particles, it is given by:



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$