

NUCLEUS

ENTHUSE

IIT PHYSICS

GRAVITATION



NUCLEUS
Centre of Excellence



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GRAVITATION

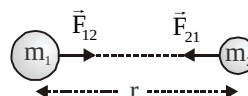
KEY CONCEPTS

NEWTON'S LAW OF GRAVITATION

It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

$$F \propto m_1 m_2 \text{ and } F \propto \frac{1}{r^2} \text{ so } F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore F = -\frac{Gm_1 m_2}{r^2} \hat{r} \quad [G = \text{Universal gravitational constant}]$$



Note : This formula is only applicable for spherical symmetric masses or point masses.

Vector form of Newton's law of Gravitation :

Let $\vec{r}_{12}$ = Displacement vector from m_1 to m_2

$\vec{r}_{21}$ = Displacement vector from m_2 to m_1

$\vec{r}_{21}$ = Gravitational force exerted on m_2 by m_1

$\vec{r}_{12}$ = Gravitational force exerted on m_1 by m_2

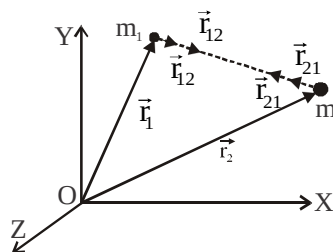
$$\vec{r}_{12} = -\frac{Gm_1 m_2}{r_{21}^2} \vec{r}_{21} = -\frac{Gm_1 m_2}{r_{21}^3} \vec{r}_{21}$$

Negative sign shows that :

- (i) The direction of $\vec{F}_{12}$ is opposite to that $\hat{r}_{21}$
- (ii) The gravitational force is attractive in nature

$$\text{Similarly } \vec{F}_{21} = -\frac{Gm_1 m_2}{r_{12}^2} \hat{r}_{12} \text{ or } \vec{F}_{21} = -\frac{Gm_1 m_2}{r_{12}^3} \vec{r}_{12} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$$

The gravitational force between two bodies are equal in magnitude and opposite in direction.



Gravitational constant "G"

- Gravitational constant is a scalar quantity.
- **Unit :**

SI : $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$

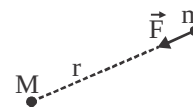
CGS : $6.67 \times 10^{-8} \text{ dyne-cm}^2/\text{g}^2$

Dimensions : $M^{-1}L^3T^{-2}$
- The total gravitational force on one particle due to number of particles is the resultant of forces of attraction exerted on the given particle due to individual particles i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$ it means the principle of superposition is valid.

Gravitational Field Intensity [g or E_g]

Gravitational force acting per unit mass at any point in the gravitational field is

$$g = \frac{GMm}{r^2} / m = \frac{GM}{r^2} \quad \text{Vector form : } \vec{g} = \frac{\vec{F}}{m} \text{ or } \vec{g} = -\frac{GM}{r^2} \hat{r}$$



Gravitational field intensity is a vector quantity having dimension $[LT^{-2}]$ and unit N/kg .

- Since the force between two point masses is having the similar expression as that of force between two point charges, we can write the gravitational field & gravitational potential in the same manner as the electric field & electric potential.
- Notice gravitational force is always attractive and hence gravitational potential is always –ve. (for a repulsive force potential is positive). This can be explained from the sign of W_{ext} in moving the test charge from ∞ to the point under consideration.
- Since $\vec{g}$ points from B towards A potential increases as we move from A to B. Just like electric potential gravitational potential also increases opposite to field direction.

ANALOGY BETWEEN ELECTROSTATICS & GRAVITATION

(1) Point Charge

(a) $E = \frac{kQ}{r^2}$

(b) $v = \frac{kQ}{r}$

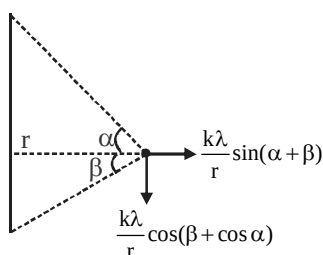
(2) Uniform charged ring

(a) $E = \frac{kQx}{(r^2 + x^2)^{3/2}}$ on axis

E is max. when $x = \frac{r}{\sqrt{2}}$

(b) $v = \frac{kQ}{\sqrt{r^2 + x^2}}$ on axis, $\frac{kQ}{r}$ at center

(3) Uniform linear charge



Point Mass

$$g = \frac{GM}{r^2}$$

$$v_G = \frac{-GM}{r}$$

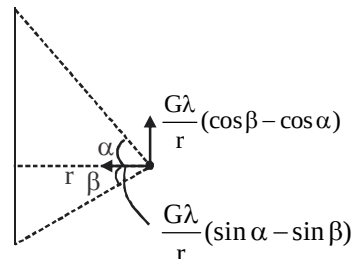
Ring of uniform mass distribution

$$g = \frac{GMx}{(r^2 + x^2)^{3/2}} \text{ on axis}$$

$$g \text{ is max. when } x = \frac{r}{\sqrt{2}}$$

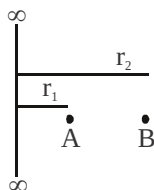
$$v_G = \frac{-GM}{\sqrt{r^2 + x^2}} \text{ on axis, } \frac{-GM}{r} \text{ at center}$$

Uniform linear mass



(λ = mass per unit mass)

(4) Infinite Linear charge



(a) $E = \frac{2K\lambda}{r}$

(b) $V_B - V_A = -2K\lambda \ln \left(\frac{r_2}{r_1} \right)$

(5) Infinite Sheet of charge

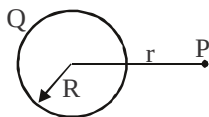
$$E = \frac{\sigma}{2\epsilon_0}$$

(6) Uniformly charged hollow sphere

Charge Q, radius R

distance of field point from center r

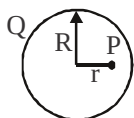
Case I $r > R$



$$E = \frac{kQ}{r^2}$$

$$V = \frac{kQ}{r}$$

Case II $r < R$



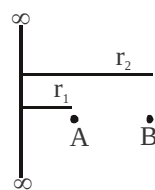
$$E = 0$$

$$V = \frac{kQ}{R}$$

(7) Electrostatics self energy of uniformly charged thin spherical shell.

$$U = \frac{KQ^2}{2R}$$

Infinite linear mass



$$g = \frac{2G\lambda}{r}$$

$$V_B - V_A = 2G\lambda \ln \left(\frac{r_2}{r_1} \right)$$

Infinite Sheet of mass

$$g = \frac{\sigma}{2} \times 4\pi G = 2\pi G\sigma$$

(σ = mass per unit area)

Hollow sphere of uniform mass

Mass M, radius R

distance of field point from center r

$$g = \frac{GM}{r^2}$$

$$V_G = -\frac{GM}{r}$$

$$g = 0$$

$$V_G = \frac{GM}{R}$$

Gravitational self energy of uniform thin spherical shell.

$$U = \frac{GM^2}{2R}$$

(8) **Uniformly charged solid sphere**
mass M, radius R

$$E = \frac{kQ^2}{r^2}, r > R$$

$$\frac{kQr}{R^3}, r < R$$

$$v = \frac{kQ}{r}, r > R$$

$$\frac{kQ}{2R^3}(3R^3 - r^2), r > R$$

Uniformly solid sphere
mass M, radius R

$$g = \frac{GM}{r^2}, r > R$$

$$\frac{GM}{R^3}r, r < R$$

$$v_a = \frac{GM}{r}, r > R$$

$$\frac{-GM}{2R^3}(3R^3 - r^2), r > R$$

(9) **Electrostatics self energy of uniformly charged solid sphere**

$$U = \frac{3}{5} \frac{KQ^2}{R}$$

Gravitational self energy of uniform solid sphere.

$$U = \frac{3}{5} \frac{GM^2}{R}$$

ACCELERATION DUE TO GRAVITY (g)

Gravitational Force $F_g = ma$ if R_e = Radius of Earth, M_e = Mass of Earth.

$$\text{then } g = \frac{GM_e}{R_e^2} \dots(i)$$

Variation in Acceleration due gravity

(a) **Due to Altitude (height)**

$$[\text{If } h \ll R_e, \therefore g_h = g \left[1 - \frac{2h}{R_e} \right]]$$

(b) **Due to depth :**

Assuming density of Earth remains same throughout. At depth d inside the Earth :

$$g_d = g \left[1 - \frac{d}{R_e} \right] \text{ valid for any depth}$$

(c) **Due to Rotation of the Earth**

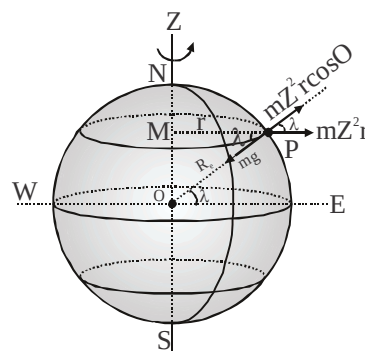
$$g' = g - R_e \omega^2 \cos^2 \lambda$$

If latitude angle $\lambda = 0$.

It means at equator. $g'_{\min} = g_e = g - R_e \omega^2$

If latitude angle $\lambda = 90^\circ$.

It means at poles. $g'_{\max} = g_p = g \Rightarrow g_p > g_e$

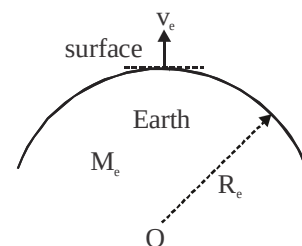


ESCAPE ENERGY

Minimum energy given to a particle in form of kinetic energy so that it can just escape from Earth's gravitational field.

$$\text{Escape energy} = \frac{GM_e m}{R_e} \quad (\text{-ve of P.E. at Earth's surface})$$

$$\text{Escape energy} = \text{Kinetic Energy} \Rightarrow \frac{GM_e m}{R_e} = \frac{1}{2}mv_e^2 \Rightarrow v_e = \sqrt{\frac{2GM_e}{R_e}}$$



$$\bullet \quad v_e = \sqrt{\frac{2GM_e}{R_e}} \quad (\text{In form of mass}) \quad \text{If } M = \text{constant} \quad v_e \propto \frac{1}{\sqrt{R_e}}$$

$$\bullet \quad v_e = \sqrt{2gR_e} \quad (\text{In form of } g) \quad \text{If } g = \text{constant} \quad v_e \propto \sqrt{R_e}$$

$$\bullet \quad v_e = R_e \sqrt{\frac{8\pi G \rho}{3}} \quad (\text{In form of density}) \quad \text{If } \rho = \text{constant} \quad v_e \propto R_e$$

- Escape velocity does not depend on mass of body, angle of projection or direction of projection.

$$v_e \propto m^0 \quad \text{and} \quad v_e \propto \theta^0$$

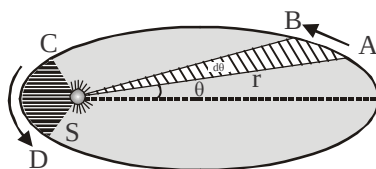
- Escape velocity at : Earth surface $v_e = 11.2 \text{ km/s}$ Moon surface $v_e = 2.31 \text{ km/s}$

- Atmosphere on Moon is absent because root mean square velocity of gas particle is greater than escape velocity. $v_{\text{rms}} > v_e$

KEPLER'S LAWS

Kepler found important regularities in the motion of the planets. These regularities are known as “Kepler’s three laws of planetary motion”.

- First Law (Law of Orbits) :** All planets move around the Sun in elliptical orbits, having the Sun at one focus of the orbit.
- Second Law (Law of Areas) :** A line joining any planet to the Sun sweeps out equal areas in equal times, that is, the areal speed of the planet remains constant. According to the second law, when the planet is nearest the Sun, then its speed is maximum and when it is farthest from the Sun, then its speed is minimum. In figure if a planet moves from A to B in a given time-interval, and from C to D in the same time-interval, then the areas ASB and CSD will be equal.



$$\frac{dA}{dt} = \frac{J}{2m} \quad \dots(iii)$$

Now, the areal speed dA/dt of the planet is constant, according to Kepler’s second law. Therefore, according to eq. (iii), the angular momentum J of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler’s second law is equivalent to conservation of angular momentum.

- (c) **Third Law : (Law of Periods) :** The square of the period of revolution (time of one complete revolution) of any planet around the Sun is directly proportional to the cube of the semi-major axis of its elliptical orbit.

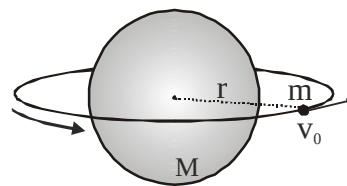
$$T^2 \propto a^3$$

So it is clear through this rule that the farthest planet from the Sun has largest period of revolution. The period of revolution of the closest planet Mercury is 88 days, while that of the farthest dwarf planet Pluto is 248 years.

SATELLITE MOTION

Orbital velocity (v_0) : A satellite of mass m moving in an orbit of radius r with speed v_0 then required centripetal force is provided by gravitation.

$$F_{cp} = F_g \Rightarrow \frac{mv_0^2}{r} = \frac{GMm}{r^2} \Rightarrow v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R_e + h)}} \quad (r = R_e + h)$$



For a satellite very close to the Earth surface $h \ll R_e \therefore r = R_e$

$$v_0 = \sqrt{\frac{GM}{R_e}} = \sqrt{gR_e} = 8 \text{ km/s}$$

- If a body is taken at some height from Earth and given horizontal velocity of magnitude 8 km/sec then the body becomes satellite of Earth.
- v_0 depends upon : Mass of planet, Radius of circular orbit of satellite, g (at planet), Density of planet
- If orbital velocity of a nearby satellite becomes $\sqrt{2} v_0$ (or increased by 41.4%, or K.E. is doubled) then the satellite escapes from gravitational field of Earth.

Time Period of a Satellite $T = \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{R\sqrt{g}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3 \Rightarrow T^2 \propto r^3 (r = R + h)$

For Geostationary Satellite $T = 24 \text{ hr}$, $h = 36,000 \text{ km} \approx 6R_e$ ($r \approx 7R_e$), $v_0 = 3.1 \text{ km/s}$

For near by satellite $v_0 = \sqrt{\frac{GM_e}{R_e}} \approx 8 \text{ km/s}$

$$T_{Ns} = 2\pi\sqrt{\frac{R_e}{g}} = 84 \text{ minute} = 1 \text{ hour } 24 \text{ minute} = 1.4 \text{ hr} = 5063 \text{ s}$$

In terms of density $T_{Ns} = \frac{2\pi(R_e)^{1/2}}{(G \times 4/3\pi R_e \times \rho)^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$

Time period of near by satellite only depends upon density of planet.

For Moon

$$h_m = 380,000 \text{ km and } T_m = 27 \text{ days}$$

$$v_{om} = \frac{2\pi(R_e + h)}{T_m} = \frac{2\pi(386400 \times 10^3)}{27 \times 64 \times 60 \times 60} \approx 1.04 \text{ km/sec}$$

Energies of a Satellite:

$$\text{Kinetic energy} \quad K.E. = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$$

$$\text{Potential energy} \quad P.E. = -\frac{GMm}{r} = -mv_0^2 = -\frac{L^2}{mr^2}$$

$$\text{Total mechanical energy T.E.} = P.E. + K.E. = -\frac{mv_0^2}{2} = -\frac{GMm}{2r} = -\frac{L^2}{2mr^2}$$

Essential Condition's for Satellite Motion

- Centre of satellite's orbit coincides with centre of Earth.
- Plane of orbit of satellite is passing through centre of Earth.

Special Points about Geo-Stationary Satellite

- All three essential conditions for satellite motion should be followed.
- It rotates in equatorial plane.
- Its height from Earth surface is 36000 km. ($\sim 6R_e$)
- Its angular velocity and time period should be same as that of Earth.
- Its rotating direction should be same as that of Earth (West to East).
- Its orbit is called parking orbit and its orbital velocity is 3.1 km./sec.
- Maximum latitude at which message can be received by geostationary satellite is

$$\theta = \cos^{-1}\left(\frac{R_e}{R_e + h}\right)$$

- The area of earth's surface covered by geostationary satellite is $S = \Omega R_e^2 = \frac{2\pi h R_e^2}{R_e + h}$

BINDING ENERGY

Total mechanical energy (potential + kinetic) of a closed system is negative. The modulus of this total mechanical energy is known as the binding energy of the system. This is the energy due to which system is closed or different parts of the system are bound to each other.

Binding energy of satellite (system)

$$B.E. = -T.E., \quad B.E. = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}, \quad \text{Hence } B.E. = K.E. = -T.E. = \frac{-P.E.}{2}$$

Work done in changing the orbit of Satellite

$$W = \text{Change in mechanical energy of system but } E = \frac{-GMm}{2r} \text{ so } W = E_2 - E_1 = \frac{GMm}{2r} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Typical Example of Effect of Air Resistance on satellite

An artificial satellite (mass m) of a planet (mass M) revolves in a circular orbit whose radius is n times the radius R of the planet. In the process of motion, the satellite experiences a slight resistance due to cosmic dust. Assuming the force of resistance on satellite to depend on velocity as $F = av^2$ where ' a ' is a constant, calculate how long the satellite will stay in the space before it falls onto the planet's surface.

In this case,

$$\text{Air resistance } F = -av^2, \text{ where orbital velocity } v = \sqrt{\frac{GM}{r}}$$

$$r = \text{the distance of the satellite from planet's centre} \Rightarrow F = -\frac{GMa}{r}$$

$$\text{The work done by the resistance force } dW = Fdx = Fvdt = \frac{GMa}{r} \sqrt{\frac{GM}{r}} dt = \frac{(GM)^{3/2}}{r^{3/2}} a dt \dots (i)$$

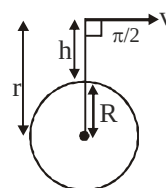
$$\text{The loss of energy of the satellite} = dE \therefore \frac{dE}{dr} = \frac{d}{dr} \left[-\frac{GMm}{2r} \right] = \frac{GMm}{2r^2} \Rightarrow dE = \frac{GMm}{2r^2} dr \dots (ii)$$

$$\text{Since } dE = -dW \text{ (work energy theorem)} - \frac{GMm}{2r^2} dr = \frac{(GM)^{3/2}}{r^{3/2}} a dt$$

$$\Rightarrow t = -\frac{m}{2a\sqrt{GM}} \int_{nR}^R \frac{dr}{\sqrt{r}} = \frac{m\sqrt{R}(\sqrt{n}-1)}{a\sqrt{GM}} = (\sqrt{n}-1) \frac{m}{a\sqrt{gR}}$$

CONDITIONS FOR DIFFERENT TRAJECTORY

For a body being projected tangentially from above earth's surface, say at a distance r from earth's center, the trajectory would depend on the velocity of projection v .



Velocity

$$1. \text{ velocity, } v < \sqrt{\frac{GM}{r} \left(\frac{2R}{r+R} \right)}$$

$$2. \sqrt{\frac{GM}{r}} > v > \sqrt{\frac{GM}{r} \left(\frac{2R}{r+R} \right)}$$

3. Velocity is equal to the critical velocity

Orbit

Body returns to earth

Body acquires an elliptical orbit with earth as

Circular orbit with radius r of the orbit, $v = \sqrt{\frac{GM_e}{r}}$

4. Velocity is between the critical and escape velocity of the orbit

Body acquires an elliptical orbit with earth as the near focus w.r.t. the point of projection.

$$\sqrt{\frac{2GM_e}{r}} > v > \sqrt{\frac{GM_e}{r}}$$

5. $v = v_{esc} = \sqrt{\frac{2GM_e}{r}}$

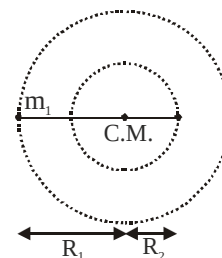
Body just escapes earth's gravity, along a parabolic path.

6. $v > v_{esc} = \sqrt{\frac{2GM_e}{r}}$

Body escape earth's gravity along a hyperbolic path.

BINARY STAR SYSTEM

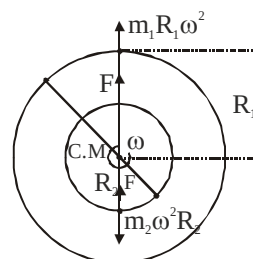
Figure shows two particles moving due to mutually attractive gravitational force about center of mass. Since there is no external force CM of system remains fixed and time period of revolution must be same. Both bodies have comparable mass and both are moving in circular orbit centre of mass as shown in diagram



$$\omega = \sqrt{\frac{G(m_1 + m_2)}{R^3}}$$

Angular momentum of the system about centre of mass.

$$L = \left(\frac{m_1 m_2}{m_1 + m_2} \right) R^2 \omega$$



THE TIME PERIOD OF A SIMPLE PENDULUM OF INFINITE LENGTH.

In deriving the formula $T_0 = 2\pi\sqrt{\frac{L}{g}}$ we have assumed that length of the pendulum L is much less than

the radius of the earth R so that 'g' always remains vertical. However, if length of pendulum is comparable of the radius of earth, 'g' will not remain vertical but will be directed towards the centre of the earth.

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{1}{g \left[\frac{1}{L} + \frac{1}{R} \right]}} \quad (< T_0)$$

From this expression it is clear that:

(a) If $L \ll R$, $(1/L) \gg (1/R)$ so $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ which is expected.

(b) If $L \gg R \rightarrow \infty$ $(1/L) \ll (1/R)$ so

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}}$$

$$= 800 \times 2\pi \text{ sec} \approx 83.8 \text{ minute}$$

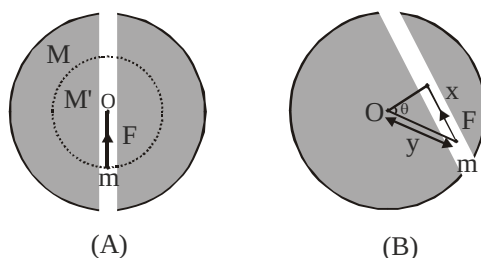
And it is also the maximum time period which an oscillating simple pendulum can have.

(c) If L is comparable to R (say $L = R$),

$$T = 2\pi\sqrt{\frac{R}{2g}} \approx 1 \text{ hour.}$$

Motion of a ball in a tunnel through the earth :

Case I : If the tunnel is along a diameter and the ball is released from the surface. The ball executes SHM.



$$\text{so that } T = 2\pi\sqrt{\frac{R}{GM}} ; T = 2\pi\sqrt{\left(\frac{R}{g}\right)}$$

Which is same as that of a simple pendulum of infinite length and is equal to 84.6 minute.

Case II : If the tunnel is along a chord and ball is released from the surface. The motion is SHM with the same time period.

GRAVITATIONAL PRESSURE :

A uniform sphere has a mass M and radius R . The pressure p inside the sphere, caused by gravitational compression, as a function of the distance r from its centre can be found to be

$$p = \frac{3}{8} \left(1 - \frac{r^2}{R^2}\right) \gamma M^2 / \pi R^4$$

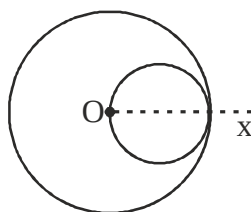
EXERCISE

EXERCISE (S-1)

FORCE/FIELD

HCV Worked out Examples (Chapter No. 11 - Q.No. 1 to 8, 10)

1. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius $r = R/2$ whose centre is at $x = R/2$ as in fig. (a) Find gravitational field at points on the axis for $|x| > R$ (b) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.



2. A small body of mass m is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius $2R$ and mass $3M$) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planets is $6R$. The distance of the body from the center of bigger planet is ' x ' at any moment. During the journey, find the distance x where the speed of the body is (a) maximum (b) minimum. Assume motion of body along the line joining centres of planets.

HCV Exercises (Chapter No. 11 - Q.No. 1 to 15)

POTENTIAL/ENERGY

HCV Worked out Examples (Chapter No. 11 - Q.No. 9)

3. Two uniform spherical stars made of same material have radii R and $2R$. Mass of the smaller planet is m . They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution $1/2$.
(a) Find the kinetic energy of the system just after the collision.
(b) Find the maximum separation between their centres after their first collision.
4. A body is launched from the earth's surface at an angle $\alpha = 30^\circ$ to the horizontal at a speed $v_0 = \sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (b) the radius of curvature of trajectory at its top point.

HCV Exercises (Chapter No. 11 - Q.No. 16 to 21, 37,38)

VARIATION IN ACCELERATION DUE TO GRAVITY 'g'

HCV Worked out Examples (Chapter No. 11 - Q.No. 11)

5. Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

6. The Earth may be regarded as a spherically shaped uniform core of density ρ_1 and radius $R/2$ surrounded by a uniform shell of thickness $R/2$ and density ρ_2 . Find the ratio of $\frac{\rho_1}{\rho_2}$ if the value of acceleration due to gravity is the same at surface as at depth $R/2$ from the surface.

HCV Exercises (Chapter No. 11 - Q.No. 6, 22 to 27, 34)

ESCAPE VELOCITY

7. A particle is fired vertically from the surface of the earth with a velocity $k v_e$, where v_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e . Calculate the height to which it will rise from the surface of the earth.

HCV Exercises (Chapter No. 11 - Q.No. 39)

CIRCULAR ORBIT

HCV Worked out Examples (Chapter No. 11 - Q.No. 12)

8. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5 \text{ J}$. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5 \text{ J}$ is equal to _____.
9. A satellite of mass m is orbiting the earth in a circular orbit of radius r . It starts losing energy due to small air resistance at the rate of $C \text{ J/s}$. Then the time taken for the satellite to reach the earth is _____.
10. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time _____.
11. A binary star has a period (T) of 2 earth years while distance L between its components having masses M_1 and M_2 is four astronomical units. If $M_1 = M_s$ where M_s is the mass of sun, find the ratio $M_2/5M_s$.
12. A pair of stars rotates about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are at a distance d apart, d being large compared to the size of either star. (a) Derive an expression for the period of rotation of the stars about their common centre of mass in terms of d, m, G . (b) Compare the angular momentum of the two stars about their common centre of mass by calculating the ratio L_m/L_M . (c) Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M .
13. A hypothetical spherical planet of radius R and its density varies as $\rho = Kr$, where K is constant and r is the distance from the center. Determine the pressure caused by gravitational pull inside ($r < R$) the planet at a distance r measured from its center.

HCV Exercises (Chapter No. - Q.No. 30 to 33, 36)

KEPLER'S LAWS

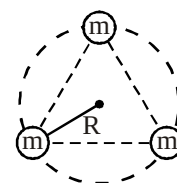
HCV Worked out Examples (Chapter No. 11 - Q.No. 13)

14. The minimum and maximum distances of a satellite from the centre of the Earth are $2R$ and $4R$ respectively, where R is the radius of Earth and M is the mass of the Earth. Find radius of curvature at the point of minimum distance.

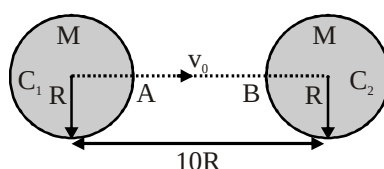
HCV Exercises (Chapter No. 11 - Q. No. 29)

EXERCISE (S-2)

- Mean solar day is the time interval between two successive noon when sun passes through zenith point (meridian). Sidereal day is the time interval between two successive transit of a distant star through the zenith point (meridian). If mean solar day is X minutes longer than the sidereal day. In other words, distant stars would rise X minutes early every successive day. Find the nearest integer of X. [You may assume circular orbit for the earth]
- Assume that a geosynchronous communications satellite is in orbit at the longitude of Kota. You are in Kota and want to pick up its signals. In what direction should you point the axis of your parabolic antenna? The latitude of Kota is 30°N .
- The fastest possible rate of rotation of a planet such that for which the gravitational force on material at the equator barely provides the centripetal force needed for the rotation.. Show then that the corresponding shortest period of rotation is given by $T = \sqrt{\frac{3\pi}{G\rho}}$, where ρ is the density of the planet, assumed to be homogeneous.
- A satellite is orbiting the Earth of mass M in equatorial plane in a circular orbit having radius 2R and same sense of rotation as that of the Earth. Find duration of time for which a man standing on the equator will be able to see the satellite continuously. Assume that the man can see the satellite when it is above horizon. Take Earth's angular velocity = ω .
- A launching pad with a spaceship is moving along a circular orbit of the moon, whose radius R is triple that of moon's radius R_m . The space ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity $\vec{v}_0$ and the launching pad then falls to the moon. Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.
- A body is launched from the earth's surface at an angle $\alpha=30^\circ$ to the horizontal at a speed $v_0 = \sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (b) the radius of curvature of trajectory at its top point.
- A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find:
 - the total P.E. of the system
 - the orbital speed of each moon such that they maintain this configuration.
- A body moving radially away from a planet of mass M, when at distance r from the centre of planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be
 - then velocity in circular orbits.
 - maximum distance between the two fragments before collision and
 - magnitude of their relative velocity just before they collide.



9. A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth surface. Assume that the spaceship and the astronaut fall on a straight line from the earth centre. The radius of the earth is 6400 km.
10. Imagine a planet of mass M with a small moon of mass m and radius a orbiting it and keeping the same face toward it. If the moon now approaches the planet, there will be a critical distance from the planet's center at which loose material lying on the moon's surface will be lifted off. Show that this distance is given by $|r_c = a(3M/m)^{1/3}$. This critical distance is called Roche's limit.
11. An astronaut in a circular orbit around earth observes a celestial body moving in a lower circular orbit around earth in same plane as his orbit and in the same sense. He observes that the body moves at a speed of 5 m/s relative to himself when it is closest to him. The minimum distance between him and the body if he is moving at a speed of 5000 m/s is α km. Fill the value of $\frac{\alpha}{4}$ in OMR sheet.
(Mass of earth = 6×10^{24} kg & round off the value of α to the nearest integer).
12. Two large spherical object of mass M each (uniformly distributed) are fixed as shown in figure. A small point mass m is projected from point A heading towards center C_2 of second sphere. The minimum velocity of point mass so that it can reach upto second object at point B is $\frac{n}{3} \sqrt{\frac{GM}{5R}}$ then calculate n.
[Neglect other gravitational forces]

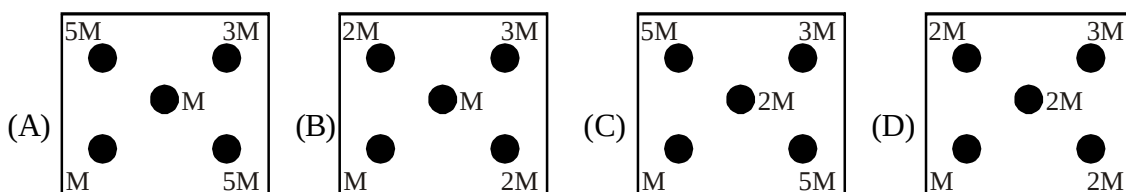


13. Two point-like objects, each with mass m , are connected by a massless rope of length ℓ . The objects are suspended vertically near the surface of Earth, so that one object is hanging below the other. Then the objects are released. Show that the tension in the rope is $T = \frac{GMm\ell}{R^3}$ where M is the mass of the Earth and R is its radius [$\ell \ll R$].
14. There is a crater of depth $\frac{R}{100}$ on the surface of the Moon (radius R). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the Moon. Find the maximum height attained by the projectile. [IIT-JEE 2003]

EXERCISE (O-1)

FORCE/FIELD

- Choose the **INCORRECT** option.
 (A) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.
 (B) That the gravitational mass and inertia mass are equal is an experimental result
 (C) That the acceleration due to gravity on earth is the same for all bodies is due to the equality of gravitational mass and inertia mass.
 (D) Gravitational mass of a particle like proton can depend on the present of neighbouring heavy objects but the inertia mass cannot.
- If the distance between the centres of Earth and Moon is D and mass of Earth is 81 times that of Moon. At what distance from the centre of Earth gravitational field will be zero ?
 (A) $\frac{D}{2}$ (B) $\frac{2D}{3}$ (C) $\frac{4D}{5}$ (D) $\frac{9D}{10}$
- A mass is at the center of a square, with four masses at the corners as shown.

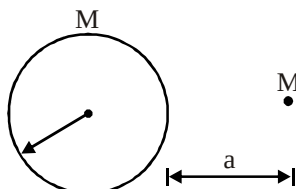


Rank the choices according to the magnitude of the gravitational force on the center mass.

- (A) $F_A = F_B < F_C = F_D$ (B) $F_A > F_B < F_D < F_C$ (C) $F_A = F_B > F_C = F_D$ (D) None

POTENTIAL/ENERGY

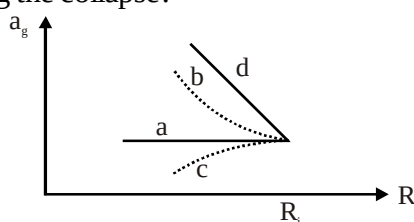
- A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre
 (A) increases
 (B) decreases
 (C) remains same
 (D) during the compression increases then returns at the previous value.
- A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a .



- (A) Gravitational field and potential both are zero at centre of the shell.
 (B) Gravitational field is zero not only inside the shell but at a point outside the shell also.
 (C) Inside the shell, gravitational field alone is zero.
 (D) Neither gravitational field nor gravitational potential is zero inside the shell.

VARIATION IN ACCELERATION DUE TO GRAVITY 'g'

6. At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R)?
(A) $R/4$ (B) R (C) $3R/8$ (D) $R/2$
7. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles in absence of rotation of earth. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole ($d < R$). The value of d is
(A) $\frac{\omega^2 R^2}{g}$ (B) $\frac{\omega^2 R^2}{2g}$ (C) $\frac{2\omega^2 R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$
8. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?
(A) $1/25$ (B) $1/5$ (C) $1/\sqrt{5}$ (D) 5
9. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth?
(A) $\sqrt{2}$ second (B) $2\sqrt{2}$ seconds (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second
10. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V . Due to the rotation of planet about its axis the acceleration due to gravity g at equator is $1/2$ of g at poles. The escape velocity of a particle on the pole of planet in terms of V is
(A) $V_e = 2V$ (B) $V_e = V$ (C) $V_e = \sqrt{2} V$ (D) $V_e = \sqrt{3} V$
11. A (non-rotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration ' a_g ' on the surface of the star as a function of the radius of the star during the collapse?



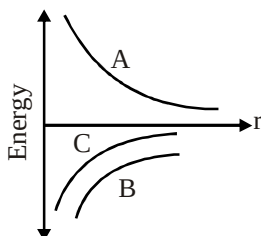
- (A) a (B) b (C) c (D) d

ESCAPE VELOCITY

12. The escape velocity for a planet is v_e . A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be
(A) v_e (B) $\frac{v_e}{\sqrt{2}}$ (C) $\frac{v_e}{2}$ (D) zero
13. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is
(A) 2 (B) $\sqrt{2}$ (C) $1/\sqrt{2}$ (D) $1/2$

CIRCULAR ORBIT

14. Two identical satellites are at the heights R and $7R$ from the Earth's surface. Then which of the following statement is incorrect. (R = radius of the Earth)
- (A) Ratio of total energy of both is 5
(B) Ratio of kinetic energy of both is 4
(C) Ratio of potential energy of both 4
(D) Ratio of total energy of both is 4 and ratio of magnitude of potential to kinetic energy is 2
15. A satellite of the earth is revolving in circular orbit with a uniform velocity V . If the gravitational force suddenly disappears, the satellite will
- (A) continue to move with the same velocity in the same orbit.
(B) move tangentially to the original orbit with velocity V .
(C) fall down with increasing velocity.
(D) come to a stop somewhere in its original orbit.
16. A satellite of mass m , initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is
- (A) $\frac{\sqrt{3}}{4} mgR$ (B) $\frac{1}{2} mgR$ (C) $\frac{1}{4} mgR$ (D) $\frac{3}{4} mgR$
17. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



- (A) A shows the kinetic energy, B the total energy and C the potential energy of the system.
(B) C shows the total energy, B the kinetic energy and A the potential energy of the system.
(C) C and A are kinetic and potential energies respectively and B is the total energy of the system.
(D) A and B are kinetic and potential energies and C is the total energy of the system.
18. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is
- (A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$
19. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change
- (A) gravitational potential energy (B) angular velocity
(C) linear orbital velocity (D) centripetal acceleration

20. A double star system consists of two stars A and B which have time period T_A and T_B . Radius R_A and R_B and mass M_A and M_B . Choose the correct option : **[IIT-JEE 2006]**
- (A) If $T_A > T_B$ then $R_A > R_B$ (B) If $T_A > T_B$ then $M_A > M_B$
- (C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (D) $T_A = T_B$

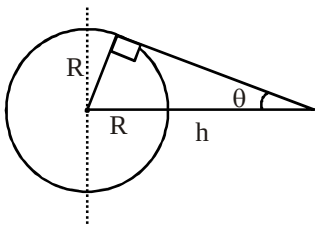
KEPLER'S LAWS

21. A planet of mass m is in an elliptical orbit about the sun ($m \ll M_{\text{sun}}$) with an orbital period T . If A be the area of orbit, then its angular momentum would be:
- (A) $\frac{2mA}{T}$ (B) mAT (C) $\frac{mA}{2T}$ (D) $2mAT$
22. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth
- (A) the acceleration of S is always directed towards the centre of the earth
- (B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
- (C) the total mechanical energy of S varies periodically with time
- (D) the linear momentum of S remains constant in magnitude
23. Two satellites A and B of same mass are orbiting around the earth in orbits of ratio R and $4R$ respectively. The ratio of their areal velocities is:
- (A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16
24. A planet revolves about the sun in elliptical orbit. The areal velocity $\left(\frac{dA}{dt}\right)$ of the planet is $4.0 \times 10^{16} \text{ m}^2/\text{s}$. The least distance between planet and the sun is $2 \times 10^{12} \text{ m}$. Then the maximum speed of the planet in km/s is :
- (A) 10 (B) 20 (C) 40 (D) None of these
25. A satellite of mass $5M$ orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass $4M$. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass $4M$ is
- (A) In a circular orbit
- (B) unbound
- (C) elliptical orbit
- (D) data is insufficient to determine the nature of the orbit.

EXERCISE (O-2)

- Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start moving under their mutual gravitational attraction
 - acceleration of m_1 is more than that of m_2
 - acceleration of m_2 is more than that of m_1
 - centre of mass of system will remain at rest in all the references frame
 - total energy of system remains constant

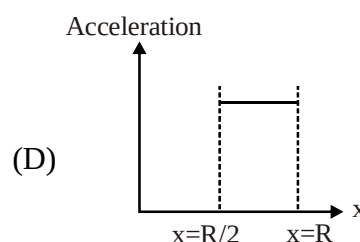
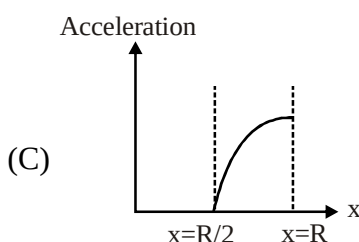
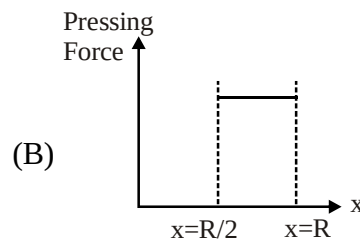
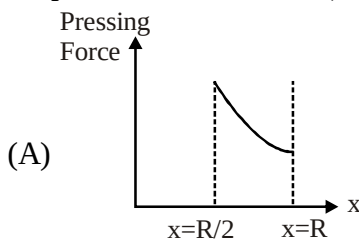
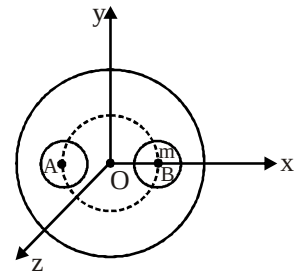
- A geostationary satellite is at a height h above the surface of earth. If earth radius is R



- The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1} (R/R+h)$
 - The maximum latitudes on earth upto which the satellite can be used for communication is $\cos^{-1} (R/R+h)$
 - The area on earth escaped from this satellite is given as $2\pi R^2 (1 + \sin\theta)$
 - The area on earth escaped from this satellite is given as $2\pi R^2 (1 + \cos\theta)$
- When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then
 - its kinetic energy increases
 - its kinetic energy decreases
 - its angular momentum about the earth decreases
 - its period of revolution around the earth increases
 - A communications Earth satellite

(A) goes round the earth from east to west	(B) can be in the equatorial plane only
(C) can be vertically above any place on the earth	(D) goes round the earth from west to east
 - If a satellite orbits as close to the earth's surface as possible,
 - its speed is maximum
 - time period of its rotation is minimum
 - the total energy of the 'earth plus satellite' system is minimum
 - the total energy of the 'earth plus satellite' system is maximum
 - For a satellite to orbit around the earth, which of the following must be true?
 - It must be above the equator at some time
 - It cannot pass over the poles at any time
 - Its height above the surface cannot exceed 36,000 km
 - Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth

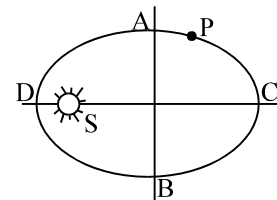
7. Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & $4R$
- (A) the ratio of period of revolution of s_1 & s_2 is $1 : 8$.
 (B) their velocities are in the ratio $2 : 1$
 (C) their angular momentum about the planet are in the ratio $2 : 1$
 (D) the ratio of angular velocities of s_2 w.r.t. s_1 when all three are in the same line is $9 : 5$.
8. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then
- (A) The gravitational field due to this object at the origin is zero
 (B) The gravitational field at the point $B(2, 0, 0)$ is zero
 (C) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$
 (D) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$
9. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform, sphere of radius R and mass M are F_1 and F_2 respectively. then
- (A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$ (B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$
 (C) $\frac{F_1}{F_2} = \frac{r_1^3}{r_2^3}$ if $r_1 < R$ and $r_2 < R$ (D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$
10. For a satellite to be geo-stationary, which of the following is/are essential conditions?
- (A) It must always be stationed above the equator
 (B) It must be rotated from west to east
 (C) It must be about 36000 km above the Earth
 (D) Its orbit must be circular, and not elliptical
11. A tunnel is dug along a chord of the Earth at a perpendicular distance $R/2$ from the Earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall, and the acceleration of the particle varies with x (distance of the particle from the centre) according to :



12. A double star is a system of two stars of masses m and $2m$, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to :
- (A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

Paragraph for question nos. 13 to 14

Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.



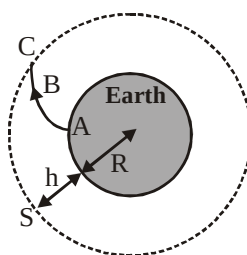
13. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then
 (A) $t_1 = t_2$ (B) $t_1 > t_2$ (C) $t_1 < t_2$ (D) nothing can be concluded
14. If U is the potential energy and K kinetic energy then $|U| > |K|$ at
 (A) Only D (B) Only C (C) both D & C (D) neither D nor C

Paragraph for question nos. 15 to 17

An Earth satellite is a body placed in a stable orbit about the Earth. These satellites are used for communications, meteorology, Earth measurements (gravitation and magnetic fields), resource evaluation (water, minerals), transmission of radio and TV signals, and as reference points for navigation.

Although most satellites are launched from ground based stations, more recently some have been placed in orbit from one of NASA's space shuttles.

There are two requirements needed to place a satellite in a stable orbit at an insertion point C. (Figure). It is first necessary to bring the satellite to that altitude and then the satellite must be given the necessary orbiting velocity. The orbiting velocity for a circular orbit, also called the insertion velocity.



15. What is insertion velocity for very small h ($\ll R$)
- (A) $\left(\frac{GM}{R}\right)^{1/2} \left[1 - \frac{h}{2R}\right]$ (B) $\left(\frac{2GM}{R}\right)^{1/2} \left[1 - \frac{h}{2R}\right]$ (C) $\left(\frac{GM}{R}\right)^{1/2} \left[1 - \frac{2h}{R}\right]$ (D) $\left(\frac{GM}{2R}\right)^{1/2} \left[1 - \frac{h}{2R}\right]$
16. If velocity at insertion point is k times escape velocity of the satellite at the insertion height h , then mark incorrect statement.
- (A) For $k = 1$ satellite moves in a circular orbit
 (B) For $k \geq 1$ satellite moves in an unbound orbit
 (C) For $k < 1$ satellite may move in an elliptical orbit
 (D) For $k < 1$ satellite can move in circular orbit

17. A particle is projected from surface of Earth with velocity v , such that it's total energy at height h becomes same as the total energy of a satellite of same mass, moving in circular orbit of same height. Value of v is

(A) $\sqrt{\frac{2GM}{R}} \left(1 + \frac{h}{R+h}\right)^{\frac{1}{2}}$ (B) $\sqrt{\frac{GM}{R}} \left(1 + \frac{h}{R+h}\right)^{\frac{1}{2}}$ (C) $\sqrt{\frac{2GM}{R}} \left(1 - \frac{h}{R+h}\right)^{\frac{1}{2}}$ (D) $\sqrt{\frac{GM}{R}} \left(1 - \frac{h}{R+h}\right)^{\frac{1}{2}}$

Paragraph for Q. 18 to 20

A problem that has fascinated philosophers & scientists is how the universe began, if it had a beginning, and how long ago it happened. Much has been learned in the last few decades about the conditions of the early universe. Current thinking assumes that the universe began as a big-bang, at that time everything was condensed into a very small space, where there were both extremely high densities & exceptionally high temperatures (actually, it was beginning of space and time as we know them). Since that time the universe has been in continuous expansion, so that the average density & temperature have been decreasing continuously. The rate of expansion of the universe (that is, the rate at which galaxies are receding from each other) is given by Hubble's Law, discovered in 1929 by the astronomer Edwin Hubble, it states that the rate of separation of any two galaxies in the universe is directly proportional to their separation. Thus, if we have two galaxies a distance R apart, their receding relative speed is given by $v = HR$ where, H is a proportionality factor called as Hubble's parameter. The currently accepted value for H is : $22 \text{ kms}^{-1} \text{ MLyr}^{-1} = 2.32 \times 10^{-18} \text{ s}^{-1}$ where one ML_{yr} (mega light - year) is $9.46 \times 10^{21} \text{ m}$.

18. If we define a time t_H , which corresponds to the time when two galaxies have reached a separation of $2R$ from R , using Hubble's Law, then t_H is

(A) $t_H = \frac{1}{H}$ (B) $t_H = \frac{\ln(2)}{H}$ (C) $t_H = \frac{R}{H}$ (D) $t_H = \frac{H}{R} \ln(2)$

19. Some recent observations by Hubble Space Telescope has fascinated astronomers and baffled the scientists that even with the presence of gravitational attraction to retard the separation between the galaxies, it seems that they are accelerating which according to some scientists proves the existence of dark energy which is supplying the increasing kinetic energy to accelerate the expansion. To measure the amount of dark energy supply we need to calculate the rate at which separation velocity is increasing at different levels of separation, find the ratio of relative acceleration at separation of R and $2R$ respectively.

(A) 2 : 1 (B) 1 : 1 (C) 1 : 2 (D) 1 : 4

20. There is a distance called as the horizon of the universe, which is equal to the farthest distance that can be observed from earth, it corresponds to the time taken by light to reach earth almost very close to the age of universe. The oldest & farthest object observed from earth is near the constellation of ursa major. It is a star like object called a quasar and is estimated to be at a distance of $1.3 \times 10^{26} \text{ m}$, therefore we see it as it was about how much years ago :

(A) $1.36 \times 10^{10} \text{ years}$ (B) $4.3 \times 10^{17} \text{ years}$ (C) $2.32 \times 10^{18} \text{ years}$ (D) $9.46 \times 10^{21} \text{ years}$

21. A satellite is in a circular equatorial orbit of radius 7000 km around the Earth. If it is transferred to a circular orbit of double the radius

Column I

- (A) Angular momentum
(B) Area of Earth covered by satellite signal
(C) Potential energy
(D) Kinetic energy

Column II

- (P) Increases
(Q) Decreases
(R) Becomes double
(S) Becomes half

22. In elliptical orbit of a planet, as the planet moves from aphelion position to perihelion position,

Column I

- (A) Speed of planet
(B) Distance of planet from centre of Sun
(C) Potential energy
(D) Angular momentum about centre of Sun

Column II

- (P) Remains same
(Q) Decreases
(R) Increases
(S) Cannot say

- 23.

Column I

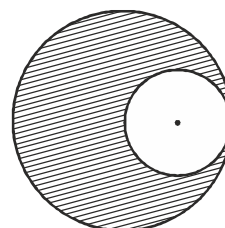
- (A) Kinetic energy of a particle in gravitational field is increasing
(B) Potential energy of a particle in gravitational field is increasing
(C) Mechanical energy of a particle in gravitational field is increasing

Column II

- (P) work done by gravitational force should be positive
(Q) work done by external force should be non zero
(R) work done by gravitational force in should be negative
(S) cannot say anything

EXERCISE (JM)

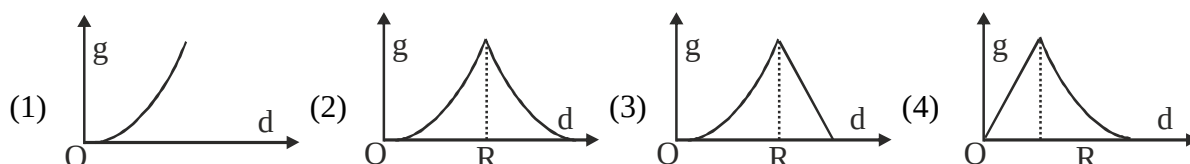
1. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1} , the escape velocity from the surface of the planet would be [AIEEE-2008]
 (1) 11 km s^{-1} (2) 110 km s^{-1} (3) 0.11 km s^{-1} (4) 1.1 km s^{-1}
2. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R , the radius of the earth, is [AIEEE-2009]
 (1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$ (3) $\sqrt{2} R$ (4) $2R$
3. Two bodies of masses m and $4m$ are placed at a distance r . The gravitational potential at a point on the line joining them where the gravitational field is zero is : [AIEEE-2011]
 (1) zero (2) $-\frac{4Gm}{r}$ (3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$
4. Two particles of equal mass ' m ' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is : [AIEEE 2011]
 (1) $\sqrt{\frac{Gm}{4R}}$ (2) $\sqrt{\frac{Gm}{3R}}$ (3) $\sqrt{\frac{Gm}{2R}}$ (4) $\sqrt{\frac{Gm}{R}}$
5. The mass of a spaceship is 1000 kg . It is to be launched from the earth's surface out into free space. The value of ' g ' and ' R ' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be : [AIEEE 2012]
 (1) $6.4 \times 10^{11} \text{ Joules}$ (2) $6.4 \times 10^8 \text{ Joules}$
 (3) $6.4 \times 10^9 \text{ Joules}$ (4) $6.4 \times 10^{10} \text{ Joules}$
6. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? [JEE Main 2013]
 (1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$ (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$
7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. the speed of each particle is : [JEE Main 2014]
 (1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2} \frac{GM}{R}}$ (3) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$
8. From a solid sphere of mass M and radius R , a spherical portion of radius $R/2$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant) [JEE Main 2015]
 (1) $-\frac{GM}{R}$ (2) $-\frac{2GM}{3R}$
 (3) $-\frac{2GM}{R}$ (4) $-\frac{GM}{2R}$



9. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; $h \ll R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) [JEE Main 2016]

(1) $\sqrt{gR}$ (2) $\sqrt{gR/2}$ (3) $\sqrt{gR}(\sqrt{2}-1)$ (4) $\sqrt{2gR}$

10. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) : [JEE Main 2017]



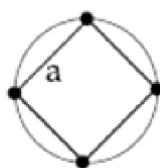
11. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2 . The value of h for which E_1 and E_2 are equal, is: [JEE Main-2019]

(1) 1.28×10^4 km (2) 1.6×10^3 km (3) 6.4×10^3 km (4) 3.2×10^3 km

12. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be : [JEE Main-2019]

- (1) in a circular orbit of a different radius (2) in an elliptical orbit
(3) in the same circular orbit of radius R (4) such that it escapes to infinity

13. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others gravitational field in a circular orbit circumscribing the square? [JEE Main-2019]



(1) $1.41\sqrt{\frac{GM}{a}}$ (2) $1.21\sqrt{\frac{GM}{a}}$ (3) $1.35\sqrt{\frac{GM}{a}}$ (4) $1.16\sqrt{\frac{GM}{a}}$

14. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :

[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg Radius of Earth : 6.4×10^6 m Gravitational acceleration on Earth = 10 ms^{-2}] [JEE Main-2019]

(1) 800 K (2) 10^4 K (3) 650 K (4) 3×10^5 K

15. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

[JEE Main-2019]

- (1) $\frac{E}{4}$ (2) $\frac{E}{16}$ (3) $\frac{E}{64}$ (4) $\frac{E}{32}$

16. A solid sphere of mass ' M ' and radius ' a ' is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance ' $3a$ ' from the centre will be:

[JEE Main-2019]

- (1) $\frac{GM}{3a^2}$ (2) $\frac{2GM}{9a^2}$ (3) $\frac{GM}{9a^2}$ (4) $\frac{2GM}{3a^2}$

17. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$.

Identify the correct relation between the radius R of the particle's orbit and its period T :

- (1) T/R^2 is a constant (2) T^2/R^3 is a constant (3) TR is a constant (4) T/R is a constant

[JEE Main-2019]

18. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth.

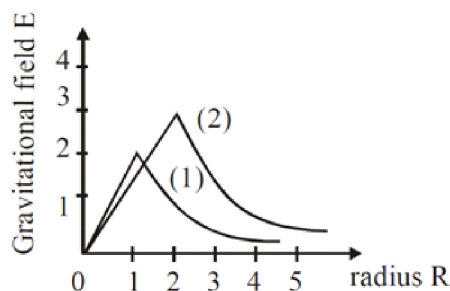
After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

[JEE Main-2020]

- (1) $\frac{m}{10} \left(u^2 + \frac{113}{200} \frac{GM}{R} \right)$ (2) $\frac{3m}{8} \left(u + \sqrt{\frac{5GM}{6R}} \right)^2$
(3) $5m \left(u^2 - \frac{119}{200} \frac{GM}{R} \right)$ (4) $\frac{m}{20} \left(u - \sqrt{\frac{2GM}{3R}} \right)^2$

19. Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2 , respectively. The

gravitational field due to sphere ① and ② are shown. The value of $\frac{M_1}{M_2}$ is: [JEE Main-2020]



- (1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

20. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :
[JEE Main-2020]
- (1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$ (3) $T^2 \propto R^3$ (4) $T^2 \propto R$
21. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) :
[JEE Main-2020]
- (1) $\frac{\sqrt{5}R - R}{2}$ (2) $\frac{R}{2}$ (3) $\frac{\sqrt{3}R - R}{2}$ (4) $\frac{\sqrt{5}}{2}R - R$
22. A satellite L_s moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e . By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is:
[JEE Main-2020]
- (1) $3R_e$ (2) $4R_e$ (3) $2.5R_e$ (4) $2R_e$
23. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at :
[JEE Main-2020]
- (1) $r = \sqrt{\frac{3}{4}}R$ (2) $r = \frac{1}{\sqrt{3}}R$ (3) $r = \sqrt{\frac{5}{9}}R$ (4) $r = R$
24. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2 + a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :
[JEE Main-2020]
- (1) $A(x^2 + a^2)^{3/2}$ (2) $A(x^2 + a^2)^{1/2}$ (3) $\frac{A}{(x^2 + a^2)^{1/2}}$ (4) $\frac{A}{(x^2 + a^2)^{3/2}}$
25. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is:
[JEE Main-2020]
- (1) $\frac{1}{\sqrt{2}}$ (2) 1 (3) 2 (4) $\sqrt{2}$

26. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals:
[JEE Main-2020]
- (1) $\frac{1}{3}$ (2) $\frac{5}{9}$ (3) $\frac{4}{9}$ (4) $\frac{7}{9}$
27. A physical quantity z depends on four observables a, b, c and d , as $z = \frac{a^2 b^{\frac{2}{3}}}{\sqrt{c} d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is:
[JEE Main-2020]
- (1) 13.5% (2) 16.5% (3) 12.25% (4) 14.5%
28. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω . An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : ($h \ll R$, where R is the radius of the earth)
[JEE Main-2020]
- (1) $\frac{R^2 \omega^2}{g}$ (2) $\frac{R^2 \omega^2}{4g}$ (3) $\frac{R^2 \omega^2}{2g}$ (4) $\frac{R^2 \omega^2}{8g}$
29. A satellite is in an elliptical orbit around a planet P . It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :
[JEE Main-2020]
- (1) 1 : 6 (2) 3 : 4 (3) 1 : 3 (4) 1 : 2
30. Two planets have masses M and $16M$ and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :
[JEE Main-2020]
- (1) $\sqrt{\frac{GM^2}{ma}}$ (2) $\frac{3}{2} \sqrt{\frac{5GM}{a}}$ (3) $4 \sqrt{\frac{GM}{a}}$ (4) $2 \sqrt{\frac{GM}{a}}$
31. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is :
[JEE Main-2021]
- (1) $\frac{1}{2\pi} \sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi \sqrt{\frac{d^3}{3Gm}}$ (3) $\frac{1}{2\pi} \sqrt{\frac{3Gm}{d^3}}$ (4) $2\pi \sqrt{\frac{3Gm}{d^3}}$

32. Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

[JEE Main-2021]

(1) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$ (2) $\sqrt{G(1+2\sqrt{2})}$ (3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$

33. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is :

[JEE Main-2021]

(1) 8 : 1 (2) 1 : 4 (3) 2 : 1 (4) 1 : 8

34. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ?

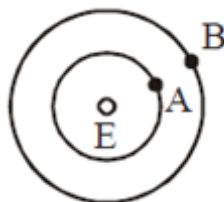
[JEE Main-2021]

(Use $g = \frac{GM}{R^2} = 9.8 \text{ ms}^{-2}$ and radius of earth, $R = 6400 \text{ km}$)

(1) 49 N (2) 48.83 N (3) 49.83 N (4) 49.17 N

35. Two satellites A and B of masses 200kg and 400kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

[JEE Main-2021]



[Given : radius of earth = 6400km, mass of earth = $6 \times 10^{24} \text{ kg}$]

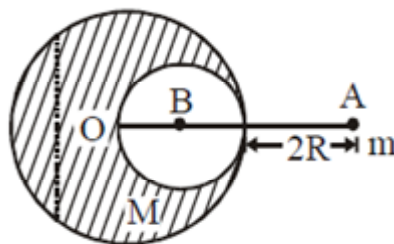
(1) $1.33 \times 10^3 \text{ s}$ (2) $3.33 \times 10^2 \text{ s}$
(3) $4.24 \times 10^3 \text{ s}$ (4) $4.24 \times 10^2 \text{ s}$

36. A solid sphere of radius R gravitationally attracts a particle placed at $3R$ from its centre with a force F_1 .

Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes

F_2 . The value of $F_1 : F_2$ is :

[JEE Main-2021]



(1) 25 : 36 (2) 36 : 25 (3) 50 : 41 (4) 41 : 50

37. Given below are two statements : one is labeled as Assertion A and the other is labelled Reason R.

[JEE Main-2021]

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same, $M_1 R_1 = M_2 R_2$

In the light of the above statements, choose the most appropriate answer from the options given below :

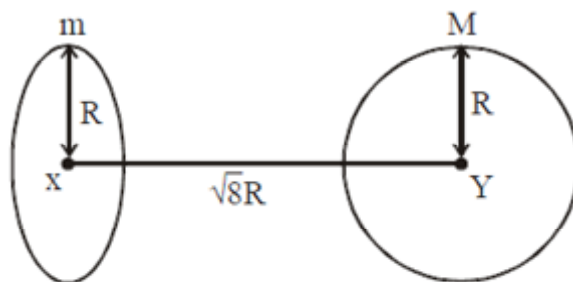
- (1) Both A and R are correct but R is NOT the correct explanation of A
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) A is not correct but R is correct
38. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of $10R$, where R is the radius of the earth, may be described in terms of escape velocity v_e such

that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be _____.

[JEE Main-2021]

39. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8} R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.

[JEE Main-2021]



- (1) $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$ (2) $\frac{2\sqrt{2}}{3} \cdot \frac{GmM}{R^2}$ (3) $\frac{1}{3\sqrt{8}} \cdot \frac{GmM}{R^2}$ (4) $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$
40. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance ($R/2$) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

[JEE Main-2021]

- (1) $\frac{2\pi R}{g}$ (2) $\frac{g}{2\pi R}$ (3) $\frac{1}{2\pi} \sqrt{\frac{g}{R}}$ (4) $2\pi \sqrt{\frac{R}{g}}$

41. A planet revolving in elliptical orbit has :

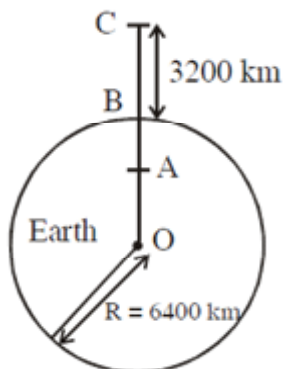
[JEE Main-2021]

- (A) a constant velocity of revolution.
(B) has the least velocity when it is nearest to the sun.
(C) its areal velocity is directly proportional to its velocity.
(D) areal velocity is inversely proportional to its velocity.
(E) to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

- (1) A only (2) D only (3) C only (4) E only

42. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is
[JEE Main-2021]

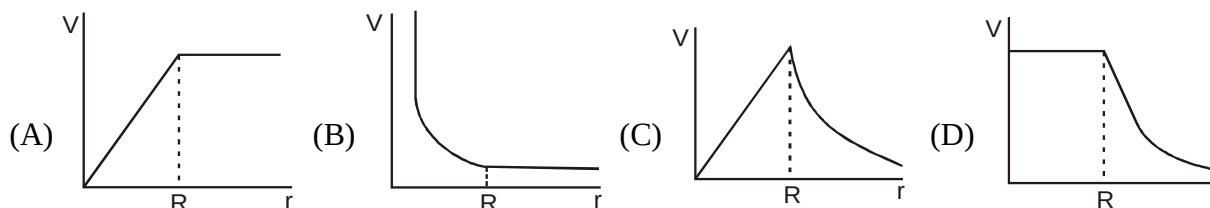


EXERCISE (JA)

1. A spherically symmetric gravitational system of particles has a mass density [IIT-JEE 2008]

$$\rho = \begin{cases} \rho_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



2. Statement – 1 : An astronaut in an orbiting space station above the Earth experiences weightlessness.
and

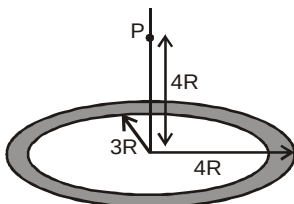
Statement – 2 : An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'. [IIT-JEE 2008]

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is Not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km s^{-1} , the escape speed on the surface of the planet in km s^{-1} will be : [IIT-JEE 2010]

4. A binary star consists of two stars A (mass $2.2 M_{\odot}$) and B (mass $11 M_{\odot}$) where $M_{\odot}$ is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is : [IIT-JEE 2010]

5. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is : [IIT-JEE 2010]

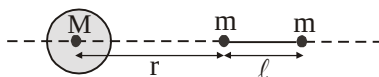


- (A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

6. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is [IIT-JEE 2010]
- (A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$
7. Two spherical planets P and Q have the same uniform density ρ , masses M_p and M_Q , with surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_p + M_Q)$. The escape velocities from the planets P, Q and R, are V_p , V_Q and V respectively. Then [IIT-JEE 2012]
- (A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = \frac{1}{2}$
8. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) :- [IIT-JEE 2013]
- (A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$
- (B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$
- (C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$
- (D) The energy of the mass m remains constant
9. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3}kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2}) [IIT-JEE 2015]
- (A) 96 N (B) 108 N (C) 120 N (D) 150 N
10. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planets gravity is $1/4^{\text{th}}$ of its value at the surface of the planet. If the escape velocity from the planet is $V_{\text{esc}} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) [IIT-JEE 2015]

11. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $= 3\ell$

from M , the tension in the rod is zero for $m = k \left(\frac{M}{288} \right)$. The value of k is [IIT-JEE 2015]



12. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2 \text{ km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

[IIT-JEE 2017]

- (A) $v_s = 22 \text{ km s}^{-1}$ (B) $v_s = 42 \text{ km s}^{-1}$
(C) $v_s = 62 \text{ km s}^{-1}$ (D) $v_s = 72 \text{ km s}^{-1}$

13. A planet of mass M , has two natural satellites with masses m_1 and m_2 . The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. [JEE-ADVANCE-2018]

List - I

List - II

- | | | | |
|----|-------------------|-----|---------------|
| P. | $\frac{v_1}{v_2}$ | (1) | $\frac{1}{8}$ |
| Q. | $\frac{L_1}{L_2}$ | (2) | 1 |
| R. | $\frac{K_1}{K_2}$ | (3) | 2 |
| S. | $\frac{T_1}{T_2}$ | (4) | 8 |

Codes :

- | | P | Q | R | S | | P | Q | R | S |
|-----|---|---|---|---|-----|---|---|---|---|
| (A) | 4 | 2 | 1 | 3 | (B) | 3 | 2 | 4 | 1 |
| (C) | 2 | 3 | 1 | 4 | (D) | 2 | 3 | 4 | 1 |

14. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K . The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is [G is universal gravitational constant] [IIT-JEE 2019]

- (A) $\frac{K}{6\pi r^2 m^2 G}$ (B) $\frac{K}{2\pi r^2 m^2 G}$ (C) $\frac{3K}{\pi r^2 m^2 G}$ (D) $\frac{K}{\pi r^2 m^2 G}$

ANSWER KEY

EXERCISE (S-1)

- $\vec{g} = \frac{\pi G \rho_0 R^3}{6} \left[\frac{1}{(x - (R/2))^2} - \frac{8}{x^2} \right] \hat{i}, \vec{g} = \frac{2\pi G \rho_0 R}{3} \hat{i}$
- $2R, 3R[3 - \sqrt{3}]$
- (a) $\frac{2Gm^2}{3R}$ (b) $4R$
- (a) $h = \left(\frac{\sqrt{7}}{2} + 1 \right) R$, (b) $1.125 R$
- $\frac{\sqrt{5}-1}{2} R$
- $7/3$
- $\frac{R_e k^2}{1-k^2}$
- $1 \times 10^5 J$
- $t = \frac{GMm}{2C} \left(\frac{1}{R_e} - \frac{1}{r} \right)$
- 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west
- 3
- (a) $T - \frac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2
- $\frac{1}{4} K^2 G \pi (R^4 - r^4)$
- $8R/3$

EXERCISE (S-2)

- 4
- $\cot^{-1} \left(\sqrt{3} - \frac{2}{7} \right) \approx 35^\circ$ to vertical
- $T = \sqrt{\frac{3\pi}{G\rho}}$
- $3 \left[\sqrt{\frac{GM}{8R^3}} - \omega_e \right]$
- $\cos \theta = \frac{3}{\sqrt{10}}$
- (a) $h = \left(\frac{\sqrt{7}}{2} + 1 \right) R$, (b) $1.13R$
- (i) $-\frac{3Gm}{R} \left(\frac{m}{\sqrt{3}} + m \right)$, (ii) $\sqrt{\frac{G}{R} \left(\frac{m}{\sqrt{3}} + M \right)}$
- (a) $\sqrt{\frac{GM}{r}}$; (b) $r\sqrt{2}$; (c) $\sqrt{\frac{2GM}{r}}$
- $T = 3 \times 10^{-2} N$
- $r_c = a (3M/m)^{1/3}$
- 8
- 8
- $h = 99.5 R$

EXERCISE (O-1)

- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1. D | 2. D | 3. A | 4. B | 5. D | 6. B | 7. A |
| 8. B | 9. B | 10. A | 11. B | 12. B | 13. A | 14. A |
| 15. B | 16. D | 17. D | 18. C | 19. A | 20. D | 21. A |
| 22. A | 23. A | 24. C | 25. B | | | |

EXERCISE (O-2)

- | | | | | | | | | | | | | | |
|-----|---|-----|-----|-----|------|-----|----|-----|-----|-----|----|-----|-----|
| 1. | AD | 2. | ABC | 3. | AC | 4. | BD | 5. | ABC | 6. | AD | 7. | ABD |
| 8. | ACD | 9. | AB | 10. | ABCD | 11. | BC | 12. | AD | 13. | B | 14. | C |
| 15. | A | 16. | A | 17. | B | 18. | B | 19. | C | 20. | A | | |
| 21. | (A) $\rightarrow$ P; (B) $\rightarrow$ P; (C) $\rightarrow$ PS; (D) $\rightarrow$ QS; | | | | | | | | | | | | |
| 22. | (A) $\rightarrow$ R; (B) $\rightarrow$ Q; (C) $\rightarrow$ Q; (D) $\rightarrow$ P; | | | | | | | | | | | | |
| 23. | (A) $\rightarrow$ S; (B) $\rightarrow$ R; (C) $\rightarrow$ Q; | | | | | | | | | | | | |

EXERCISE (JM)

- | | | | | | | | | | | | | | |
|-----|---|-----|---|-----|----|-----|---|-----|---|-----|---|-----|---|
| 1. | 2 | 2. | 4 | 3. | 4 | 4. | 1 | 5. | 4 | 6. | 1 | 7. | 4 |
| 8. | 1 | 9. | 3 | 10. | 4 | 11. | 4 | 12. | 2 | 13. | 4 | 14. | 2 |
| 15. | 2 | 16. | 1 | 17. | 4 | 18. | 3 | 19. | 1 | 20. | 4 | 21. | 1 |
| 22. | 1 | 23. | 3 | 24. | 3 | 25. | 1 | 26. | 2 | 27. | 4 | 28. | 3 |
| 29. | 1 | 30. | 2 | 31. | 2 | 32. | 4 | 33. | 1 | 34. | 2 | 35. | 1 |
| 36. | 3 | 37. | 2 | 38. | 10 | 39. | 4 | 40. | 4 | 41. | 4 | 42. | 4 |

EXERCISE (JA)

- | | | | | | | | | | | | | | |
|----|----|----|---|-----|---|-----|---|-----|---|-----|---|-----|----|
| 1. | C | 2. | A | 3. | 3 | 4. | 6 | 5. | A | 6. | B | 7. | BD |
| 8. | BD | 9. | B | 10. | 2 | 11. | 7 | 12. | B | 13. | B | 14. | B |

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