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MATHEMATICS GRADE -9

PART -TWO

PROBABILITY

ADDIS ABABA EDUCATION BUREAU

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Probability

- Probability is a numerical value that describes the likelihood of the occurrence of an event in an experiment
- An experiment is a trial by which an observation is obtained but whose outcome cannot be predicted

❖ Outcomes, Events and sample spaces

In describing an experiment, we use the following terminology.

1. An experiment

An experiment is any process of measurement or observation that generates results (outcomes).tossing a coin or rolling a fair die is an example of experiment

2.Sample space

The set of possible outcomes or the set of all elementary events is called or possible set of an experiment, and denoted by S . Any subset of (E) of the possibility (S) is called an event

3. Sample point

A sample point (outcome) is any result obtained from the experiment

4. An event

An event is any subset of a sample space. That is, an event is a collection of a sample point

- Note
- Note
- If a sample space consists of equally likely outcomes which can be counted, then the probability that an event E will occur, $P(E)$, is the ratio of the number of outcomes (number of events) in the event to the number of outcomes in the sample space (sample space of events).
- $P(E) =$
- That is, $P(E) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$
- That is, $P(E) = \frac{n(E)}{n(S)}$

Example: A bag contains 12 identical balls. 5 of them are black, 4 blue, and 3 red one ball drawn at random .what is the probability of getting

a) Red ball?

Number of outcomes in the event is equal to 3 and 12 of them have the same chance, therefore the number of sample space is equal to 12

$$P(E) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

That is, $P(E) = \frac{n(E)}{n(S)}$

$$P(\text{red}) = P(E) = \frac{n(E)}{n(S)}$$

=0.25 or it is equal to 25 % (meaning the probability of getting red is 25%)

or it is equal to 25 % (meaning the probability of getting red is 25%)

b) Not blue?

Getting not blue has the same meaning with getting black or red ball

So the number of event is equal to $7 = 4 + 3$

$$P(E) = \frac{7}{12}$$

c) Black or blue ball?

The number of event is equal to $8 = 5 + 3$

$$P(E) = \frac{8}{12}$$

Or equivalently probability of this event can be expressed as sum of probability of getting black plus probability of getting blue ball

$$\text{That is } P(\text{blue}) = \frac{5}{12} \text{ and } P(\text{black}) = \frac{3}{12}$$

$$\begin{aligned} \text{Therefore } P(\text{blue or black}) &= P(\text{blue}) + P(\text{black}) \\ &= \frac{5}{12} + \frac{3}{12} \\ &= \frac{8}{12} \end{aligned}$$

d) White ball?

There is no white ball

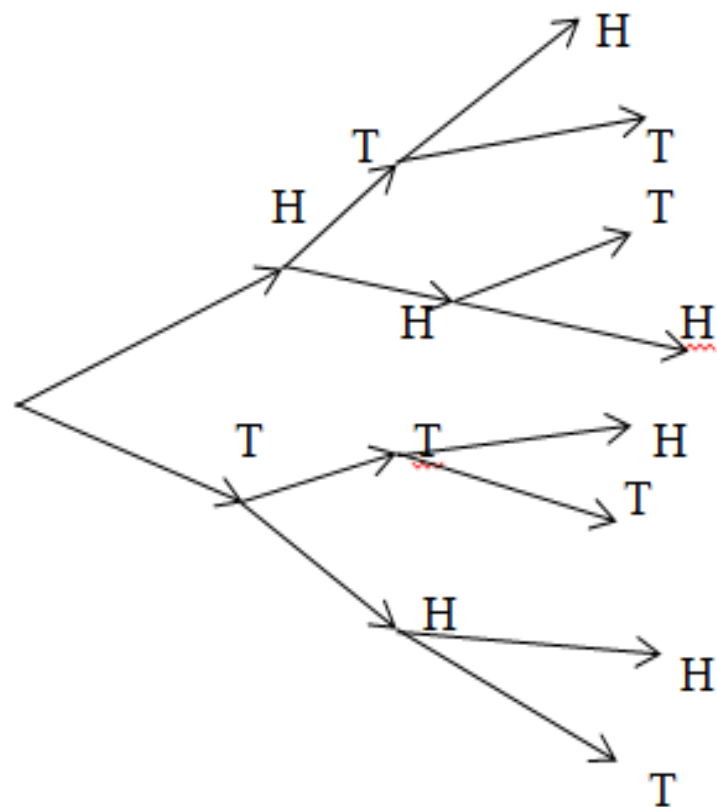
$$n(E)=0 \quad p(E)=\frac{n(E)}{n(S)}=\frac{0}{n(S)}=0$$

$P(E)=0$, which is impossible event

Note: probability of an event that is certain to occur is 1, the probability of an event that cannot occur is 0. Thus, the probability of an event E , $P(E)$, is a number such that $0 \leq P(E) \leq 1$

- Example 2: Three coins are tossed .specify the following
 - a) The sample space ,S
 - b)The probability of an event getting exactly two heads.
 - c)Probability of getting at least one tail.
 - d)Probability of getting at most one head
- Solution
 - If we toss a coin, there are two possible outcomes: head (H) or tail (T)
- In tossing three coins, there are three experiments and each experiment has two possible outcomes head (H) and tail (T).

The possible outcomes for the two experiments can be described using a tree diagram



Hence there are 8 possible outcomes

a) HTH, HTT, HHT, TTH, TTT, THT, TTH, THT

b) Probability of getting two heads

$$E = \{HTH, HHT, THH\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

c) Probability of getting at least one tail

$E = \{HTH, HTT, HHT, TTH, TTT, THT\}$

$$E = \{HTH, HTT, HHT, TTH, TTT, THT\}$$

d) Probability of getting at most one head

$$E = \{HTT, TTH, TTT, THT\}$$

d) Probability of getting at most one head

$$E = \{HTT, TTH, TTT, THT\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

- Example 3: probability of a sum of 7 when two dice are rolled. Here the number of sample space is equal to 36 and the event $E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

- Exercise

1If a coin and a die is rolled at the same time a)
what is the number of sample space ?

b)What is the probability of getting tail and even number?

Solution

Solution

a) A coin has two outcomes H and T

a) A coin has two outcomes H and T

a die has six outcomes $\{1, 2, 3, 4, 5, 6\}$

a die has six outcomes $\{1, 2, 3, 4, 5, 6\}$

Since the two events are independent the sample space is the product of the two which is equal to 12

b) The event $E = \{2T, 4T, 6T\}$

b) The event $E = \{2T, 4T, 6T\}$

$$= 3$$

$$= 3$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

2. A quiz contains a multiple choice question with five possible answers, and only one is correct. A student plans to guess the answer

- a) what is the number of sample space ?
- b) probability of a student guesses wrong answer ?
- c) probability of a student guesses the correct answer ?

Solution

Solution

a) Number of sample space is 5

a) Number of sample space is 5

b) From five choices the correct answer is one and the remaining

b) From five choices the correct answer is one and the remaining

four are wrong answers
 $n(E) = 4$

~~$P(E) = \frac{4}{5}$~~

~~$P(E) = 1 - \frac{n(E)}{n(S)} = \frac{4}{5}$~~

~~$P(E) = 1$~~

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{5}$$

Mathematics Grade -9
Unit 7
vector in two dimension

Main contents

- Vector quantities
- Magnitude and direction of vectors
- Addition, subtraction and scalar multiplication of vectors
- Coordinate representation of vectors

Representation of vectors

Vector can be represented using rays as

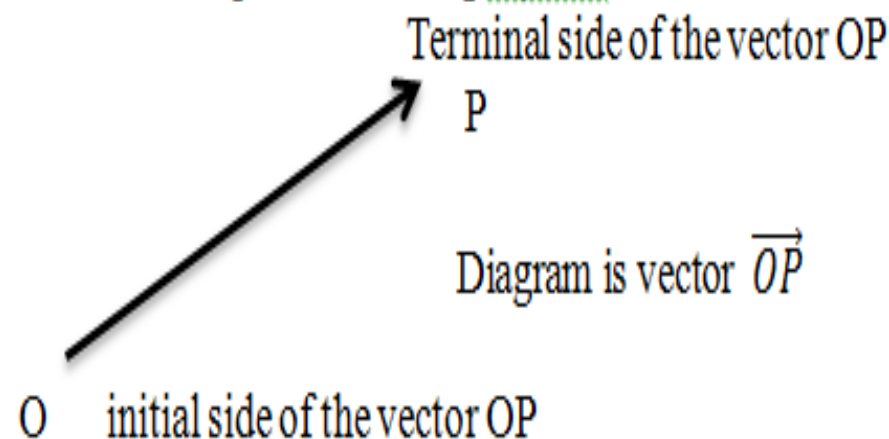


Diagram is vector $\overrightarrow{OP}$

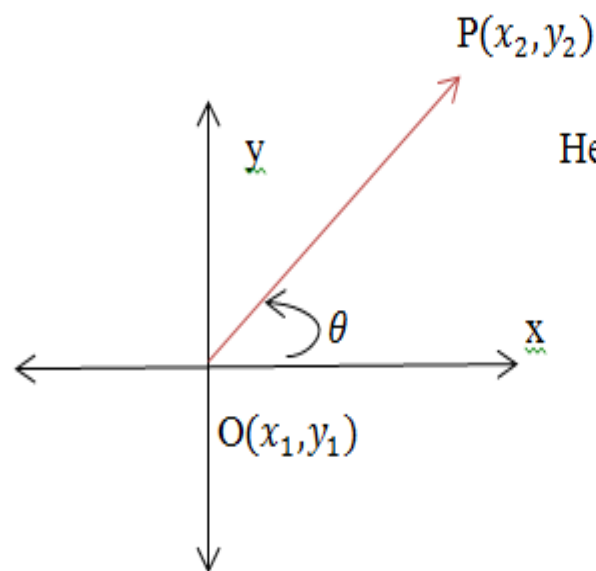
Magnitude and direction of vectors

Magnitude of a vector $\overrightarrow{OP}$ is denoted by $|\overrightarrow{OP}|$ and it represents the length of the vector.

If the initial side and terminal side is given as $O(x_1, y_1)$ and $P(x_2, y_2)$, then the magnitude (length) of

$$\overrightarrow{OP} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{and}$$

The direction of vector $|\overrightarrow{OP}|$ is an angle



Here direction of $|\overrightarrow{OP}|$ is $E\theta N$ or $N(90-\theta)E$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$\theta = \tan^{-1} \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

θ is the angle formed between the positive x-axis and the vector in counter clockwise direction

The direction of $\overrightarrow{OP}$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$$

$$\theta = \tan^{-1} \left(\frac{3 - (-3)}{5 - (-3)} \right)$$

$$= \tan^{-1} \left(\frac{6}{8} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} \right)$$

$$\theta = 37^\circ$$

- Example: find the magnitude and direction of vector with initial position $O(-3, -3)$ and terminal side $P(5, 3)$

Then magnitude of vector $|\overrightarrow{OP}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

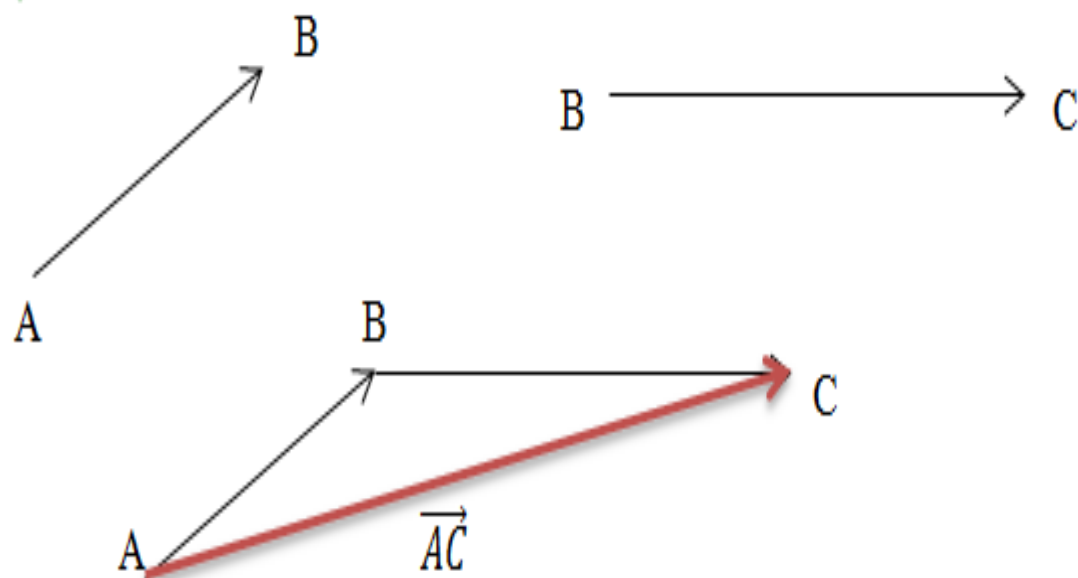
$$\begin{aligned}
 |\overrightarrow{OP}| &= \sqrt{(5 - (-3))^2 + (3 - (-3))^2} \\
 &= \sqrt{(8)^2 + (6)^2} \\
 &= \sqrt{64 + 36} \\
 &= 10 \text{ units and} \\
 &= \sqrt{100} \\
 &= 10 \text{ units and}
 \end{aligned}$$

- Parallel vectors are vectors that have the same or opposite direction
- Equal vectors are vectors that have the same direction and the same magnitude
- That is, if two vectors $\vec{a}$ and $\vec{b}$ are equal
- Then, if two vectors $\vec{a}$ and $\vec{b}$ are equal vectors then, $|\vec{a}| = |\vec{b}|$ and the direction of both vector is the same

Addition, subtraction and multiplication of vectors by a scalar

Addition of vectors

- Triangular law of vectors
- Geometrical addition of vectors is described by placing the initial point of one vector onto the terminal side of other without changing the magnitude and direction
- without changing the magnitude and direction
- Let $\vec{a}$ and $\vec{b}$ two vectors in a coordinate system .if
 $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{BC}$
, then their sum, $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC}$ is the vector represented by the directed line segment $\overrightarrow{AC}$
That is, $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

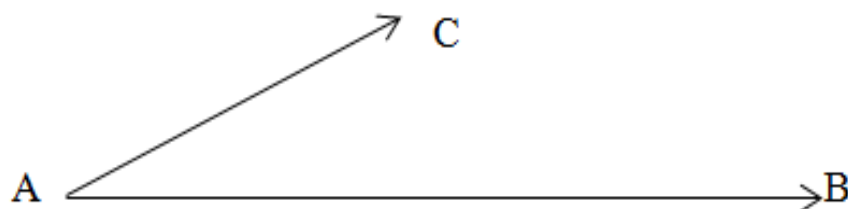


Triangular law of addition of vectors is applicable if the initial side of one vector is terminal side of the other

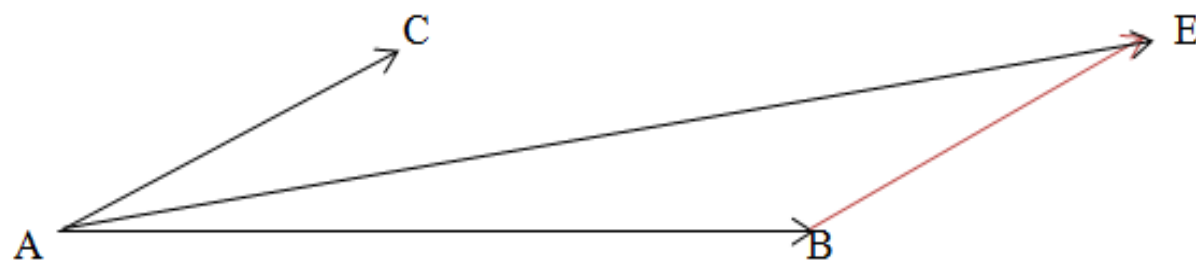
❖ Parallelogram law of addition of vectors

We may sometimes have vectors whose initial point is the same such vectors sum is obtained using parallelogram law

Consider the vector $\vec{AC}$ and $\vec{AB}$ as given below in figure



Recall that equal vectors have the same magnitude and the same direction

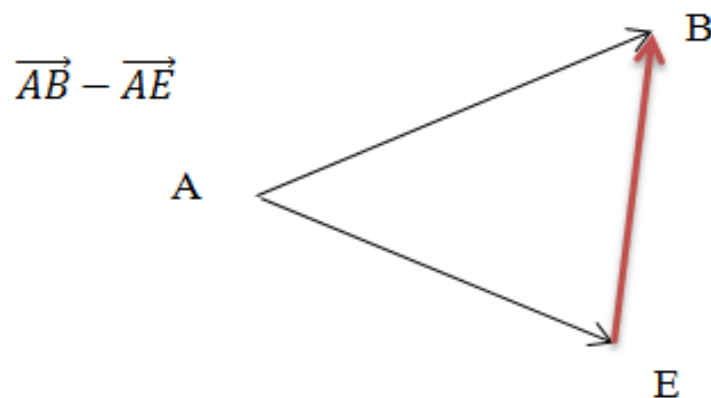
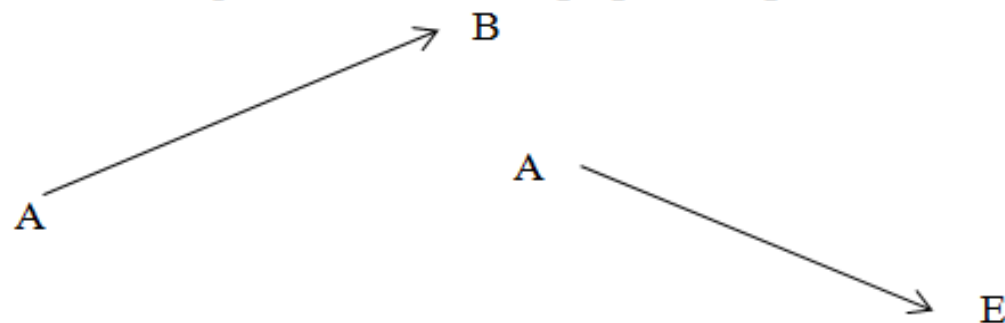


$$\vec{AC} = \vec{BE}$$

$\vec{AB} + \vec{AC}$ and $\vec{AB} + \vec{BE}$ represents the same vector. But from triangular law and $\vec{AB} + \vec{BE} = \vec{AE}$

Subtraction of vectors

For two vectors and $\vec{AB}$ and $\vec{AE}$ geometrically and $\vec{AB} - \vec{AE}$ is obtained by putting the initial side at a common point without changing the magnitude and direction of both vectors



from addition of vectors $\vec{AE} + \vec{EB} = \vec{AB}$
 $\vec{AB} - \vec{AE} = \vec{BE}$

Multiplication of a vector by a scalar

Let $\vec{a}$ be any vector and K be any scalar. Then the vector $K\vec{a}$ is defined as follows

- i. A vector whose magnitude is $|\vec{ka}|$
- ii. If $\vec{a} \neq \vec{0}$, then
for $k > 0$, $k\vec{a}$ has the same direction as $\vec{a}$
for $k < 0$, $k\vec{a}$ has opposite direction to $\vec{a}$
- iii. $k\vec{a} = \vec{0}$, if $\vec{a} = \vec{0}$ or $k=0$

Note

- if $\vec{a}$ and $\vec{b}$ are non-zero vectors which are parallel, then one can be expressed as a scalar multiple of an other

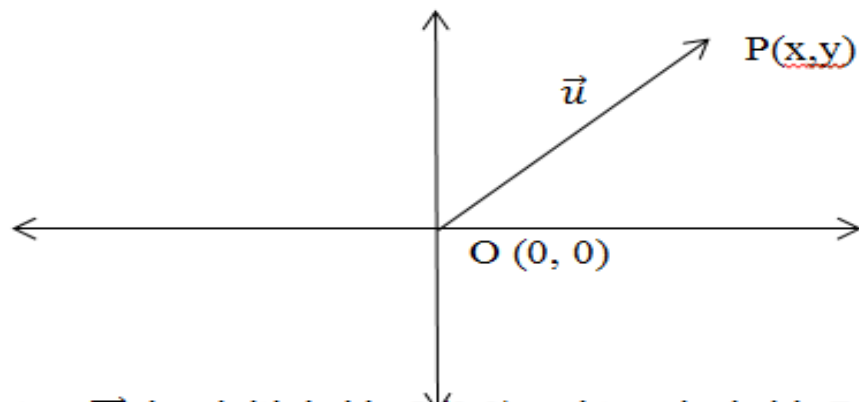
That is $\vec{a} = k\vec{b}$

- if $\vec{a}$ is non-zero vector then the vector $\vec{u} = \frac{1}{|\vec{a}|} \vec{a}$ has a magnitude 1 and called unit vector

Coordinate representation of vectors

Position vector of a point

Position vector is a vector whose initial position is at the origin of the coordinate plane



Vector $\vec{u}$ has initial side $O(0,0)$ and terminal side $P(x,y)$ is magnitude is given by

$$\begin{aligned} |\vec{u}| &= \sqrt{x^2 + y^2} \quad \text{Which is the length of the directed line segment } \overrightarrow{op} \quad \text{and the direction of the} \\ \text{vector is } \tan \theta &= \left(\frac{\text{opposite}}{\text{adjacent}} \right) \\ &= \frac{y}{x} \\ \theta &= \tan^{-1} \left(\frac{y}{x} \right) \end{aligned}$$

• And vector $\vec{u}$ can be represented in different ways

- $\vec{u} = (x, y)$

- $\vec{u} = \begin{pmatrix} x \\ y \end{pmatrix}$

- $\vec{u} = x\vec{i} + y\vec{j}$ where $\vec{i} = (1, 0)$ and $\vec{j} = (0, 1)$ and x and y in vector $\vec{u}$ are called components of vectors.

- If a vector is given as initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$, then the position vector

- $\vec{AB} = (x_2 - x_1, y_2 - y_1)$

- $= (x_2 - x_1, y_2 - y_1)$

Example 1: which one of the following is not parallel to vector $\vec{u} = 2i - 4j$? For a parallel one identify the direction as opposite or have the same direction

- a) $\vec{u} = 2i + 8j$
- b) $\vec{u} = i - 2j$
- c) $\vec{u} = -4i + 8j$
- d) $\vec{u} = 5i - 10j$

- Example 2: find a position vector, magnitude and direction of a vector with initial position A(2,7) and terminal position B(-2,8)

Solution

$$\begin{aligned} \text{Position vector } \overrightarrow{AB} &= (-2-2, 8-7) \\ &= (-4, 1) \end{aligned}$$

$$\begin{aligned} &= -4i + 1j \\ &= -4 + 1j \\ \text{Magnitude of } \overrightarrow{AB} &= \sqrt{(-4)^2 + (1)^2} \\ |AB| &= \sqrt{17} \text{ unit} \end{aligned}$$

=

= unit

$$\text{Direction of vector is } \theta = \tan^{-1}\left(\frac{1}{-4}\right)$$

Example 3: for a vector $\vec{u} = 2\vec{i} + \frac{1}{3}\vec{j}$ and

$$\vec{v} = -4\vec{i} + \vec{j}$$

a) $2\vec{u}$

b) $\vec{v} + 2\vec{u}$

c) $3\vec{v} - 2\vec{u}$