

GRADE 9; MATHEMATICS

SIMPLIFIED NOTES

CBC RATIONALIZED WORK

Well-organized, simplified learning notes that provide clear, concise information in a logical sequence, making key concepts easily accessible and memorable, enhancing understanding and retention while fostering engagement and facilitating effective study.

By providing a clear focus on key ideas, these Mwalimu Consultancy Notes foster better retention and comprehension, making learning more accessible and enjoyable.

UPDATED VERSION

This is a Free Sample/Overview of the Original Notes

CONTACT US FOR COMPLETE VERSION OF THE NOTES

Mr Isaboke 0746 222 000 / 0742 999 000

MWALIMU CONSULTANCY

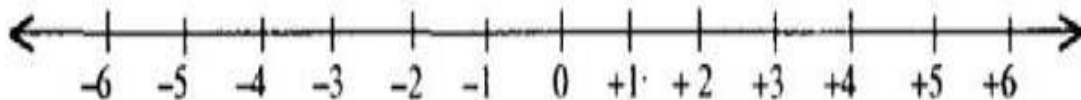
STRAND 1: NUMBERS

SUBSTRAND: Integers Introduction to Integers

- Meaning of integers: An integer is a positive whole number, negative whole numbers and including zero.
- Examples of integers include 2, -3, 5, 0, 4, 7, etc.

The number line

- A number line is a pictorial representation of numbers on a straight line. It's a reference for comparing and ordering numbers. It can be used to represent any integer.
- Integers can be illustrated on a number line as shown below.



Any integer is less than all other integers to the right of it. Thus, -2 is less than -1 but greater than -3.

- The symbol $<$ and $>$ are used to denote 'less than' and 'greater than' respectively.

- Thus, $-2 < -1$ and $-2 > -3$

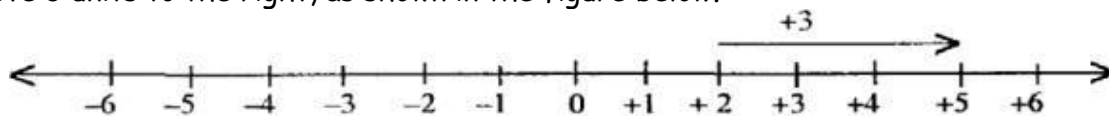
ASSIGNMENT

Use $<$ or $>$ to compare the following pairs of numbers;

1. -5 and +1
2. -3 and +4
3. -5 and +5
4. -10 and +1
5. -7 and -9
6. -20 and -36
7. 1 and -25
8. 15 and -30
9. 25 and -38

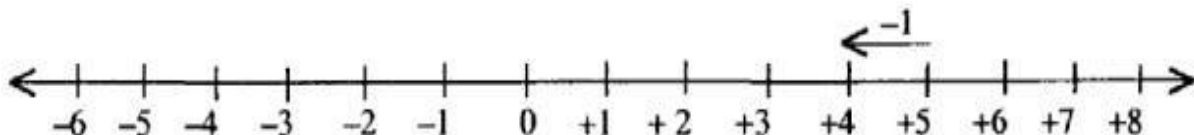
Addition of integers

- Addition of integers can be represented on a number line. For example, to add +3 to +2, we begin at +2 and move 3 units to the right, as shown in the figure below.



If instead we begin at +3 and move 2 units to the right, what would be the result?

Similarly, to add -1 and +5, we begin at +5 and move 1 unit to the left, as shown in the figure below.



EXERCISE

Show how the following additions can be done using a number line and give the results.

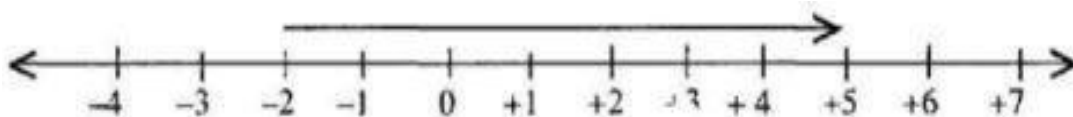
Subtraction of Integers

Steps for Subtracting Integers

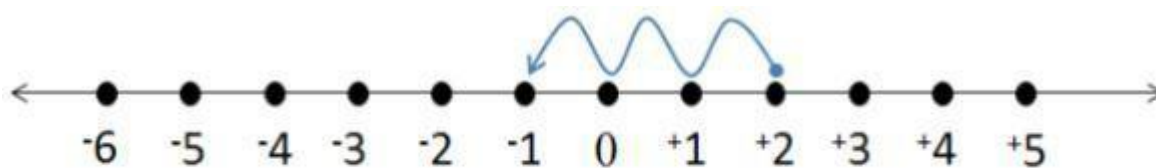
- To subtract integers on a number line, we need to move towards the left side when subtracting a positive number from a given number. On the other hand, when we subtract a negative integer from a given number, we move towards the right side of the number line.

Examples

(1). $-2 - (-7)$



(2).



$$2 - 3 = -1$$

Multiplication of Integers Steps for Multiplying Integers

Multiplication of integers follows the following rules.

- A positive number times a positive number equals a positive number.
- A positive number times a negative number equals a negative number. A negative number times a positive number equals a negative number. A negative number times a negative number equals a positive number.

EXAMPLES

$$5 \times -6 = -30 \quad -2 \times 5 = -10$$

$$4 \times -6 = -24 \quad -2 \times 4 = -8$$

$$3 \times -6 = -18 \quad -2 \times 3 = -6$$

$$2 \times -6 = -12 \quad -2 \times 2 = -4$$

$$1 \times -6 = -6 \quad -2 \times 1 = -2$$

$$0 \times -6 = 0 \quad -2 \times 0 = 0$$

$$-1 \times -6 = 6 \quad -2 \times -1 = 2$$

$$-2 \times -6 = 12 \quad -2 \times -2 = 4$$

$$-3 \times -6 = 18 \quad -2 \times -3 = 6$$

$$-4 \times -6 = 24 \quad -2 \times -4 = 8$$

$$-5 \times -6 = 30 \quad -2 \times -5 = 10$$

DIVISION OF INTEGERS

Steps for Dividing Integers

Division of integers follows the following rules.

- (i) (a positive number) $\div$ (a positive number) = (a positive number)
 - (ii) (a positive number) $\div$ (a negative number) = (a negative number)
 - (iii) (a negative number) $\div$ (a positive number) = (a negative number)
 - (iv) (a negative number) $\div$ (a negative number) = (a positive number)
- In general, for multiplication and division of integers:
- Two like signs give positive sign,
 - Two unlike signs give negative sign

EXAMPLES

1. $-4 \div -2 = 2$
2. $-2 \div -2 = 1$
3. $-4 \div -4 = 1$
4. $-8 \div 4 = -2$
5. $-6 \div 3 = -2$
6. $4 \div -2 = -2$

EXERCISE

Evaluate

- 1.
2. $10 \div 2$
3. $50 \div -25$
4. $98 \div -14$
5. $126 \div 9$
6. $288 \div -24$
8. $42 \div 6$
9. $-90 \div 10$
10. $-125 \div 5$
11. $-615 \div 15$
12. $-1080 \div 90$
13. $-140 \div -20$
14. $-256 \div 16$
15. $-289 \div 17$
16. $-560 \div 16$
17. $-912 \div 19$
18. $-570 \div 19$

COMBINED OPERATIONS ON INTEGERS

Steps for Combined Operations on Integers

The order in which operations are performed can be shown by use of brackets.

(1). 'Subtract 8 from 18 and then subtract 5 from the result' can be written as $(18 - 8) - 5 = 10 - 5 = 5$

(2). 'Subtract 5 from 8 and then subtract the result from 18' can be written as $18 - (8 - 5) = 18 - 3 = 15$

Note that;

• $(18-8)-5 \neq 18-(8-5)$

At times, more than two operations may occur in one expression, e.g, $6 \times 3 - 4 \div 2 + 5$. In such a case, we begin by brackets, then division, followed by multiplication, addition and finally subtraction, in that order. This can be shown by the use of the brackets, as below:

$$(6 \times 3) - (4 \div 2) + 5 = 18 - 2 + 5 = 21$$

The BODMAS rule is applicable when more than operation appears in the same question.

EXERCISE

(1).

a. $(18 - 24) + 30 =$ b. $32 + (17 + 30) =$

c. $13 - (18 + 7) =$ d. $(62 - 94) + 20 =$

e. $84 - (100 + 2) =$ f. $(74 - 24) + 30 =$

g. $(77 + 54) - 110 =$ h. $(100 - 150) + 180 =$

i. $222 - (158 + 90) =$ j. $1120 - (1450 + 120) =$

(2).

a. $72 - 30 + 25 =$ b. $86 - 109 + 4 =$

c. $209 + 43 - 300 =$ d. $348 + 60 - 510 =$

e. $890 - 100 + 23 =$ f. $989 + 100 - 1470 =$

g. $763 - 26 + 471 =$ h. $1190 + 340 + 670 =$

i. $666 - 892 + 238 =$ j. $3004 - 563 + 1044 =$

(3).

a. $2 \times (10 \div 5) =$ b. $(6 \times 18) \div 9 =$

c. $90 \div (10 \times 3) =$ d. $-84 \div (7 \times 4) =$

e. $(-39 \div 13) - 8 =$ f. $21 \times (14 \div 7) =$

g. $1320 \div (11 \times 5) =$ h. $(-420 \div 28) \times 5 =$

i. $20 \times (525 \div 21) =$ j. $(1125 \div 15) \times 19 =$

k. $11 \times 12 \div 4 =$ l. $19 \times 8 \div 2 =$

m. $64 \div 16 \times 9 =$ n. $-256 \div 64 \times 10 =$

o. $3 \times 68 \div 17 =$ p. $91 \div 13 \times 5 =$

q. $-11 \times 125 \div 5 =$ r. $-369 \div 123 \times 8 =$

WORD PROBLEMS ON INTEGERS

1. If $x = -2$, $y = -6$ and $z = 4$, find the values of each of the following:
2. $4z + 2y - x$
3. $2y - 3x + z$
4. $4xy/z$
5. $3yz/x$
6. On a certain day, a student measured the temperature inside a deep freezer and found that it was -3°C while the room temperature was 24°C . What was the temperature difference between the room and the deep freezer?
7. Rhoda walked four floors down from the tenth floor and then took a lift to the eighteenth floor. How many floors did she go through while in the lift?
8. Kericho is a town on Kisumu-Nakuru road. The distance between Kisumu and Kericho is 85km, while that between Kericho and Nakuru is 105km. What is the distance between Nakuru and Kisumu?
9. A man was born in 1966. His father was born in 1928 and the mother three years later. If the man's daughter was born in 1992 and the son 5 years earlier, find the difference between the age of the man's mother and that of his son.
10. The temperature of a patient admitted to a hospital with fever was 42°C . After treatment, his temperature settled at 36.8°C . Find the change in temperature.

CUBES AND CUBE ROOT

Cubes from the Tables

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	3	7	10	13	16	20	23	26	30
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685	4	8	12	16	20	24	28	31	35
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147	5	9	14	19	23	28	33	37	42
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686	5	11	16	22	27	33	38	43	49
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308	6	13	19	25	31	38	44	50	56
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020	7	14	21	29	36	43	50	57	64
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827	8	16	24	32	41	49	57	65	73
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735	9	18	27	37	46	55	64	73	82
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751	10	20	31	41	51	61	71	82	92
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881	11	23	34	45	57	68	79	91	102
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129	13	25	38	50	63	75	88	100	113
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.078	10.218	10.360	10.503	14	28	41	55	69	83	97	110	124
2.2	10.648	10.794	10.941	11.090	11.239	11.391	11.543	11.697	11.852	12.009	15	30	45	60	76	91	106	121	136
2.3	12.167	12.326	12.487	12.649	12.813	12.978	13.144	13.312	13.481	13.652	16	33	49	66	82	99	115	132	148
2.4	13.824	13.998	14.172	14.349	14.527	14.706	14.887	15.069	15.253	15.438	18	36	54	72	90	108	126	143	161
2.5	15.625	15.813	16.003	16.194	16.387	16.581	16.777	16.975	17.174	17.374	19	39	58	78	97	117	136	155	175
2.6	17.576	17.780	17.985	18.191	18.400	18.610	18.821	19.034	19.249	19.465	21	42	63	84	105	126	147	168	189
2.7	19.683	19.903	20.124	20.346	20.571	20.797	21.025	21.254	21.485	21.718	23	45	68	90	113	136	158	181	203
2.8	21.952	22.188	22.426	22.665	22.906	23.149	23.394	23.640	23.888	24.138	24	49	73	97	121	146	170	194	219
2.9	24.389	24.642	24.897	25.154	25.412	25.672	25.934	26.198	26.464	26.731	26	52	78	104	130	156	182	208	234
3.0	27.000	27.271	27.544	27.818	28.094	28.373	28.653	28.934	29.218	29.504	28	56	83	111	139	167	195	223	250
3.1	29.791	30.080	30.371	30.664	30.959	31.256	31.554	31.855	32.157	32.462	30	59	89	119	148	178	208	237	267
3.2	32.768	33.076	33.386	33.698	34.012	34.328	34.646	34.966	35.288	35.611	32	63	95	126	158	190	221	253	284
3.3	35.937	36.265	36.594	36.926	37.260	37.595	37.933	38.273	38.614	38.958	34	67	101	134	163	201	235	269	302
3.4	39.304	39.652	40.002	40.354	40.708	41.064	41.422	41.782	42.144	42.509	36	71	107	142	178	214	249	285	320
3.5	42.87	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	4	8	11	15	19	23	26	30	34
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	4	8	12	16	20	24	28	32	36
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	4	8	13	17	21	25	29	34	38
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	4	9	13	18	22	27	31	35	40
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	5	9	14	19	23	28	33	37	42
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	5	10	15	20	25	29	34	39	44
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	5	10	15	21	26	31	36	41	46
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	5	11	16	22	27	32	38	43	49
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	6	11	17	23	28	34	40	45	51
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	6	12	18	24	30	36	41	47	53
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	6	12	19	25	31	37	43	50	56
4.6	97.34	97.97	98.61	99.25	99.90	100.54	101.19	101.85	102.50	103.16	6	13	19	26	32	39	45	52	58
4.7	103.82	104.49	105.15	105.82	106.50	107.17	107.85	108.53	109.22	109.90	7	14	20	27	34	41	47	54	61
4.8	110.59	111.28	111.98	112.68	113.38	114.08	114.79	115.50	116.21	116.93	7	14	21	28	35	42	49	56	63
4.9	117.65	118.37	119.10	119.82	120.55	121.29	122.02	122.76	123.51	124.25	7	15	22	29	37	44	51	59	66
5.0	125.000	125.75	126.51	127.26	128.02	128.79	129.55	130.32	131.10	131.87	8	15	23	31	38	46	53	61	69
5.1	132.65	133.43	134.22	135.01	135.80	136.59	137.39	138.19	138.99	139.80	8	16	24	32	40	48	56	64	71
5.2	140.61	141.42	142.24	143.06	143.88	144.70	145.53	146.36	147.20	148.04	8	17	25	33	41	50	58	66	74
5.3	148.88	149.72	150.57	151.42	152.27	153.13	153.99	154.85	155.72	156.59	9	17	26	34	43	51	60	69	77
5.4	157.46	158.34	159.22	160.10	160.99	161.88	162.77	163.67	164.57	165.47	9	18	27	36	44	53	62	71	80

Cubes

- The cube of a number is simply a number multiplied by itself three times e.g.

$$a \times a \times a = a^3 \quad 1 \times 1 \times 1 = 1$$

$$3^3 = 2 \times 2 \times 2 = 23; 27 = 3 \times 3 \times 3 = 33;$$

Example 1

What is the value of 6^3 ?

Solution

$$6^3 = 6 \times 6 \times 6$$

$$10. \quad 36 \times 6$$

$$11. \quad 216$$

Example 2

Find the cube of 1.4

Solution

10. $1.4 \times 1.4 \times 1.4$

11. 1.96×1.4

12. 2.744

USE OF TABLES TO FIND ROOTS

- The cubes can be read directly from the tables just like squares and square root.

a. $(1.8)^3 = 5.832$

b. $(2.12)^3 = 9.528$

c. $(3.254)^3 = 34.454$
 $= 34.45$ (4 s.f)

QUESTIONS

(1). Use mathematical tables to find the cube of each of the following:

(a) 8.3 (b) 1.01 (c) 2.504 (d) 0.87 (e) 15.45

(2). Use tables to find:

31. (4.06)

32. (6.312)

33. (0.0912)

34. (381.7)

35. (2.1534)

36. (5.3679)

(3). A cubic building block measures 21 cm. Find its volume.

(4). A cubic water tank has sides of length 2.143 m. What is the capacity of the tank in litres?

Cube Root by Factor Method

Cube Roots Using Factor Methods

- Cubes and cubes roots are opposite. The cube root of a number is the number that is multiplied by itself three times to get the given number

Example

The cube root of 64 is written as;

$= 4$ Because $4 \times 4 \times 4 = 64$

$\sqrt[3]{27}$

$= 3$ Because $3 \times 3 \times 3 = 27$

Example

Evaluate: $\sqrt[3]{216}$

Solution

10. $\sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)}$

11. 2×3

12. 6

The volume of a cube is 1000 cm What is the length of the cube?

Solution

Volume of the cube, $V = P^3 = 1000$

$P = \sqrt[3]{1000}$

20. $\sqrt[3]{10 \times 10 \times 10}$

21. 10

∴ The length of the cube is 10 cm.

QUESTIONS

1. The volume of a sphere is given by $\frac{4}{3} \pi r^3$ Find the radius of a sphere whose volume is 104.816 cm^3 . (Take π to be $\frac{22}{7}$)
2. The volume of material used to make a cube is 1728 cm^3 . What is the length of the side of the cube.
3. The volume of water in a measuring cylinder reads 200 cm^3 . When a cube is immersed into the water, the cylinder reads 543 cm^3 . Find:
 - a. The volume of the cube.
 - b. The length of the side of the cube.
4. A metallic cuboid measuring 16 cm by 8 cm by 4 cm was melted. The material was then used to make a cube. What was the length of the cube?

INDICES AND LOGARITHMS

Law of indices

Indices

Index and Base Form

The power to which a number is raised is called index or indices in plural. $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

5 is called the power or index while 2 two is the base. $100 = 10^2$

2 is called the index and 10 is the base.

Laws of Indices

- For the laws to hold the base must be the same.

Rule 1

Any number, except zero whose index is 0 is always equal to 1

Example

$$5^0=1$$

$$1000000000000000000=1$$

Rule 2

To multiply an expression with the same base, copy the base and add the indices. $a^m \times a^n = a^{m+n}$

Example

$$5^2 \times 5^3 = 5^5$$

$$= 3125$$

Rule 3

To divide an expression with the same base, copy the base and subtract the powers. $a^m \div a^n = a^{m-n}$

Example

$$9^5 \div 9^2 = 9^3$$

Rule 4

To raise an expression to the nth index, copy the base and multiply the indices a mx^n

$$= a^m n$$

Example

$$(5^3)^2$$

$$= 5^3 \times 2 = 5^6$$

QUESTIONS

Solve for x in $5^{2x-1} = 12^x$ (3mks)

Solve for x in $4^{x+1} = 32$ (3mks)

Find the value of x which satisfies the equation $16^{x^2} = 8^{4x-3}$ Solve for x in the equation $32^{(x-3)} \div 8^{(x+4)} = 64 \div 2^x$

$$\frac{81^{2x} \times 27^x}{9^x} = 729$$

Solve each of the following equations:

a. $(3^{2 \times})^3 = 3^4 \times 3^8$

b. $(7^5) = (7^4) \times 7^2$

c. $(3^{2 \times})^4 = 81$

d. $4^{5 \times} \div (2^{3 \times})^2 = 256$

e. $9^{2 \times} = 729$

f. $2^{8 \times} = 512$

g. $(7^4)^{2 \times} = (7^4)^3$

h. $(5^3)^a (5^8)^a = 5^{7^2}$

i. $94^x \div 3^{2 \times} = 2187$

Negative Indices

Rule 5

• When dealing with a negative power, you simply change the power to positive by changing it into a fraction with 1 as the numerator. $a^{-m} = \frac{1}{a^m}$

a^m

Example

$$2^{-2} = \frac{1}{2^2}$$

$$2^2$$

$$= \frac{1}{4} \text{ Example Evaluate:}$$

a. $2^3 \times 2^{-3} = 2^{(3 + -3)}$

$$= 2^0 = 1$$

b. $(\frac{2}{3})^{-2} = (1)^2 (\frac{2}{3})^2$

$$1$$

$$\frac{4}{9}$$

$$= 1 \div \frac{4}{9}$$

$$= 1 \times \frac{9}{4} = 2 \frac{1}{4} \text{ or } (\frac{2}{3})^{-2} = (\frac{3}{2})^2 = \frac{9}{4}$$

QUESTIONS

$$6^2 \times 7^{-4} \times (8^{-2})^2 \times 6^3 \times 7^2 \times 8^4$$

$$7^{-3} \times 8^4 \times 7^2 \times 8^{-3}$$

$$20^{-3} \times 25^2 \times 20^3 \times 25^{-4}$$

$$\frac{2^{-2} \times (2^2)^{-6}}{2^{-4} \times 2^{-6}}$$

$$\frac{20^{-2} \times 20^{-2} \times 20^{-2}}{20}$$

$$\frac{a^{-a} b^2 \times b^{-a} \times c^{-a}}{a^{-2a} \times b^{-2a} \times c^{-2a}}$$

FRACTIONAL INDICES

Fractional Indices

- Fractional indices are written in fraction form. In summary if $a^n = b$, a is called the n th root of b written as $n\sqrt{b}$.

Example

$$27^{1/3} = \sqrt[3]{27} = 3$$

$$16^{3/4}$$

$$= (4\sqrt[4]{16})^3 = 2^3 = 8$$

$$4^{-1/2} = 1$$

$$4^{1/2}$$

$$= 1$$

$$\sqrt{4}$$

$$= \frac{1}{2}$$

QUESTIONS

$$\frac{27^{\frac{2}{3}} \div 2^4}{32^{-\frac{3}{5}}}$$

$$27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{-\frac{1}{4}}$$

a.

b.

$$64^{-\frac{1}{2}} \times 27000^{\frac{2}{3}}$$

$$\frac{2^{-4} \times 3^0 \times 5^2}{243^{\frac{-2}{5}} \times 125^{\frac{2}{3}}}$$

d.

$$\frac{(9a^2)^{\frac{1}{3}}}{6^4} \times \frac{(4b^2)^{\frac{1}{3}}}{5a}$$

INTRODUCTION TO LOGARITHMS

- The indices 0, 1, 2, 3, 4 ... are called the **logarithms** of the corresponding numbers to **base 3**.
- For example; logarithm of 9 to **base 3** is 2.
- Logarithm of 81 to **base 3** is 4.

These are usually written in short form as; $\text{Log}_3 9 = 2$

$\text{Log}_3 81 = 4$

Examples

Index notation	Logarithm form
$2^2 = 4$	$\log_2 4 = 2$
$9^{\frac{1}{2}} = 3$	$\log_9 3 = \frac{1}{2}$
$b^n = m$	$\log_b m = n$

Generally,

$a^m = n$ is the index notation while $\log_a n = m$ is the logarithmic notation.

Write in logarithmic form:

a. $2^4 = 16$

b. $9^{\frac{1}{2}} = 3$

c. $b^n = m$

Solution

a. If $2^4 = 16$, then $\log_2 16 = 4$

b. If $9^{\frac{1}{2}} = 3$, then $\log_9 3 = \frac{1}{2}$

c. If $b^n = m$, then $\log_b m = n$

QUESTIONS

(1). write in logarithmic form

a. $3^2 = 9$

b. $2^4 = 16$

c. $3^3 = 27$

d. $2^5 = 32$

e. $3^4 = 81$

f. $5^3 = 125$

g. $10^0 = 1$

h. $2^{10} = 1024$

i. $a^n = b$

(2). Write each of the following in index form.

a. $\log_2 8 = 3$

b. $\log_4 16 = 2$

c. $\log_5 125 = 3$

d. $\log_{10} 8 = x$

e. $\log_3 27 = 3$

f. $\log_6 216 = 3$

g. $\log_x 40 = y$

h. $\log_4 6 = y$

j

k. $\log_{10} 10\,000 = 4$

l. $\log_2 16 = 4$

STANDARD FORM OF LOGARITHMS

Standard Form Consider the following:

$$12 = 1.2 \times 10^1$$

$$120 = 1.2 \times 10^2$$

$$1200 = 1.2 \times 10^3$$

$$0.12 = 1.2 \times 10^{-1}$$

$$= 1.2 \times 10^{-1}$$

$$0.286 = 2.86 \times 10^{-1}$$

$$= 2.86 \times 10^{-1}$$

$$0.0074 = 7.4 \times 10^{-3}$$

$$= 7.4 \times 10^{-3}$$

Any number can be written in the form $A \times 10^n$, Where A is a number between 1 and 10 (10 not included) or $1 \leq A < 10$, and n is an integer. When written in this way, a number is said to be in **standard form**.

Write each of the following numbers in standard form.

12. 26

13. 357

14. 4068 iv.15

000 000

13. 0.031

14. 0.00215

15. 0.005012

16. 0.000000152

17. 100 000

18. $46/100000$

COMMON LOGARITHMS

Reading logarithms from the tables is the same as reading squares square roots and reciprocals.

We can read logarithms of numbers between 1 and 10 directly from the table. For numbers greater than 10 we proceed as follows:

- Express the number in standard form, $A \times 10^n$. Then n will be the whole number part of the logarithms.
- Read the logarithm of A from the tables, which gives the decimal part of the logarithm.
 - Then add it to n which is the power of 10 to form the positive part of the logarithm.

Logarithms to Base 10

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
1.1	.0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	27	30	34
1.2	.0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
1.3	.1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
1.4	.1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
1.5	.1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
1.6	.2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
1.7	.2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
1.8	.2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
1.9	.2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
2.0	.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
2.1	.3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
2.2	.3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
2.3	.3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
2.4	.3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
2.5	.3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
2.6	.4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
2.7	.4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
2.8	.4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
2.9	.4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
3.0	.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13

Example

Find the logarithm of: 379

Solution

In standard form

$$= 3.79 \times 10^2$$

$$\log 3.79 = 0.5786$$

Therefore the logarithm of 379 is $2 + 0.5786 = 2.5786$

Note: The whole number part of the logarithm is called **the characteristic** and the decimal part is the **mantissa**.

QUESTIONS

Use logarithm tables to express each of the following numbers in the form 10^x .

37. 8.2

38. 1.47

39. 4.73

40. 7.25

41. 9.89

42. 5.672

43. 8.137

h. 3.142

Logarithms of Positive Numbers Less Than 1

Example

Log to base 10 of 0.034

Solution

We proceed as follows:

Express 0.034 in standard form, i.e., $A \times 10^n$. Read the logarithm of A and add to n

Thus $0.034 = 3.4 \times 10^{-2}$

Log 3.4 from the tables is 0.531 5

Hence $3.4 \times 10^{-2} = 100.5315 \times 10^{-2}$

Using laws of indices add $0.531\ 5 + -2$ which is written as $\bar{2}.5315$.

It reads bar two point five three one five. The negative sign is written directly above two to show that it's only the characteristic is negative.

Example

Find the logarithm of: 0.00063

Solution

6.3×10^{-3} (Find the logarithm of 6.3)

$10^{0.6993} \times 10^{-3}$

$-3 + 0.7993$

3.7993

QUESTIONS

Find the algorithm of;

1. 0.57
2. 0.00063
3. 0.0029
4. 0.0765
5. 0.82

Antilogarithms

Antilogarithms

• Finding antilogarithm is the reverse of finding the logarithms of a number.

For example;

The logarithm of 1000 to base 10 is 3. So the antilogarithm of 3 is 1000. In algebraic notation, if

$\log x = y$ then antilog of $y = x$.

Example

Find the antilogarithm of $\bar{2}.303$

Solution

Let the number be x $x = 10^{\bar{2}.3031}$

$10^{-2+0.3031}$

$10^{-2} \times 100.3031$ (Find the antilog, press shift and log then key in the number)

$10^{-2} \times 2.01$

$1/100 \times 2.01$

$2.01/100$

0.0201

QUESTIONS

(1). Read from the table

18. 0.1461

19. 0.248

7

20. 1.4900

21. 2.4835

(2). Find the number whose logarithm is;

(a) $\bar{2}.3031$ (b) $\bar{4}.5441$

Applications of Logarithms

Example

Use logarithm tables to evaluate:

logarithm tables to evaluate:

$\frac{456 \times 398}{271}$			
Number	Standard form	logarithm	
456	4.56×10^2	2.6590	
398	3.98×10^2	2.5999	
<u>5.2589</u>			
		+	
271	2.71×10^2	2.4330 -	
<u>6.697 $\times 10^2$ ←</u>		2.8259	

Evaluate $\frac{415.2 \times 0.0761}{135}$

Solution

Number	logarithm
415.2	2.6182
0.0761	<u>2.8814 +</u>
	1.4996
135	2.1303 -
2.341×10^{-1}	1.3693
0.2341	

$$h \ 3.14^2 = 3.14 \times 3.14$$

No.	log
3.14^2	0.4969×2
9.858×10^0	0.9938

Therefore, $3.14^2 = 9.858 \times 10^0$

No.	log
8.36^3	0.9222×3
5.842×10^2	2.7666

(v)9

.858 c.

8.36^3

Therefore, $8.36^3 = 5.842 \times 10^2$

= 584.2

Questions

Use algorithms to evaluate.

a.

$$\frac{1.34}{(5.24)^{0.8} \times 0.0029}$$

b. 21.47×362.1

c.

$$\frac{18^2 \times 391^4}{15^3 \times 56^4}$$

$$\frac{94.7 \times 16.45}{12.5 \times 8.93}$$

d.

e.

$$\frac{48.35 \times 125.3}{39.3 \times 50.4}$$

Roots

$$\begin{aligned}\sqrt{0.945} &= (9.45 \times 10^{-1})^{\frac{1}{2}} \\ &= (10^{1.9754 \pm \frac{1}{2}})\end{aligned}$$

Example

Note;

- In order to divide 1.9754 by 2, We write the logarithm in such a way that the characteristic is exactly divisible by 2. if we are looking for the n^{th} root, we arrange the characteristic to be exactly divisible by n)

$$1.9754 = -1 + 0.9754$$

$$= -2 + 1.9754$$

$$\text{Therefore, } \frac{1}{2}(1.9754) = \frac{1}{2}(-2 + 1.9754)$$

$$= -1 + 0.9877$$

$$= 1.9877$$

Find the antilog of 1.9877 by writing the mantissa as power of 10 and then find the antilog
characteristic = $9.720 \times 10^{-1} = 0.9720$

$$\sqrt[3]{0.0618}$$

Solution

Number	Logarithm
$\sqrt[3]{0.0618} = 0.0618^{1/3}$	$2.7910 \times \frac{1}{3}$
$3.954 \times 10^{-1} \leftarrow \text{antilog}$	$(3 + 1.7910) \times \frac{1}{3}$
	$= 1.5970$

Questions

Use logarithm tables to evaluate:

(1).

$$\sqrt[3]{(35.6 \times 0.0613^2)}$$

(2).

$$\sqrt{\left[\frac{2.935 \times 0.0765}{32.74} \right]}$$

(3).

$$\sqrt[4]{\frac{4.562 \times 0.038}{0.82}}$$

(4).

$$\frac{(0.07284)^2}{\sqrt[3]{0.06195}}$$

(5).

$$\sqrt[3]{\frac{36.15 \times 0.02573}{1.938}}$$

Compound Proportions and Rates of Work Compound Proportions

- The proportion involving two or more quantities is called compound proportion. Any four quantities a, b, c and d are in proportion if;

$$\frac{a}{b} = \frac{c}{d}$$

Example

Find the value of a that makes 2, 5, a and 25 to be in proportion;

Solution

Since 2, 5, a, and 25 are in proportion

$$\frac{2}{5} = \frac{a}{25}$$

$$5a = 2 \times 25$$

$$a = \frac{2 \times 25}{5}$$

$$a = 10$$

Continued Proportions

- In continued proportion, all the ratios between different quantities are the same; but always remember that the relationship exists between two quantities for example:

P:Q	Q:R	R:S
10:5	16:8	4:2

- Note that in the example, the ratio between different quantities i.e. P:Q, Q:R and R:S are the same i.e. 2:1 when simplified.
- Continued proportion is very important when determining the net worth of individuals who own the same business or even calculating the amounts of profit that different individual owners of a company or business should take home.

Proportional Parts

In general, if n is to be divided in the ratio a: b: c, then the parts of n proportional to a, b, c are

$$\frac{a}{a+b+c} \times n, \frac{b}{a+b+c} \times n \text{ and } \frac{c}{a+b+c} \times n \text{ respectively}$$

Example

Omondi, Joel, cheroot shared sh 27,000 in the ratio 2:3:4 respectively. How much did each get?

Solution

The parts of sh 27,000 proportional to 2, 3, 4 are

$$\frac{2}{9} \times 27,000 = \text{sh } 6000 \rightarrow \text{Omondi}$$

$$\frac{3}{9} \times 27,000 = \text{sh } 6000 \rightarrow \text{Joel}$$

$$\frac{4}{9} \times 27,000 = \text{sh } 6000 \rightarrow \text{Cheroot}$$

Example

Three people – John, Debby and Dave contributed ksh 119, 000 to start a company. If the ratio of the contribution of John to Debby was 12:6 and the contribution of Debby to Dave was 8:4, determine the amount in dollars that every partner contributed.

Solution

Ratio of John to Debby's contribution = 12:6 = 2:1 Ratio of Debby to Dave's contribution = 8:4 = 2:1
As you can see, the ratio of the contribution of John to Debby and that of Debby to Dave is in continued proportion.

Hence $\frac{\text{John}}{\text{Debby}} = \frac{\text{Debby}}{\text{Dave}}$

To determine the ratio of the contribution between the three members, we do the calculation as follows: John : Debby : Dave

12 : 6

8 : 4

We multiply the upper ratio by 8 and the lower ratio by 6, thus the resulting ratio will be: John : Debby : Dave

96 : 48 : 24

= 4:2:1

The total ratio = 7

The contribution of the different members can then be found as follows:

John $\frac{4}{7} \times \text{ksh } 119,000 = \text{ksh } 68,000$

Debby $\frac{2}{7} \times \text{ksh } 119,000 = \text{ksh } 34,000$

Dave $\frac{1}{7} \times \text{ksh } 119,000 = \text{ksh } 17,000$

John contributed ksh 68, 000 to the company while Debby contributed ksh 34, 000 and Dave contributed ksh 17, 000

Example 2

You are presented with three numbers which are in continued proportion. If the sum of the three numbers is 38 and the product of the first number and the third number is 144, find the three numbers.

Solution

Let us assume that the three numbers in continued proportion or Geometric Proportion are a , ar and ar^2 where a is the first number and r is the rate.

$$a + ar + ar^2 = 38 \dots (1)$$

The product of the 1st and 3rd is $a \times ar^2 = 144$

Or

$$(ar)^2 = 144 \dots (2)$$

If we find the square root of $(ar)^2$, then we will have found the second number:

$$\sqrt{(ar)^2} = \sqrt{144}$$

$$ar = 12$$

Since the value of the second number is 12, it then implies that the sum of the first and the third number is 26.

We now proceed and look for two numbers whose sum is 26 and product is 144. Clearly, the numbers are 8 and 18.

Thus, the three numbers that we were looking for are 8, 12 and 18.

Let us work backwards and try to prove whether this is actually true: $8+12+18=30$

What about the product of the first and the third number? $8 \times 18=144$

What about the continued proportion $\underline{a} = \underline{ar} = \underline{ar^2}$

or $ar^2 = 3$

The numbers are in continued proportion

Example

Given that $x: y = 2:3$, Find the ratio $(5x - 4y): (x + y)$.

Solution

Since $x: y = 2:3$

$$x/2 = y/3 = k,$$

$$x = 2k \text{ and } y = 3k$$

$$(5x - 4y): (x + y) = (10k - 12k): (2k + 3k)$$

$$-2k: 5k$$

$$-2:5$$

Example

If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a - 3b}{b - 3a} = \frac{c - 3d}{d - 3c}$.

$$b - 3a$$

$$d - 3c$$

Solution

$$\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{c} = \frac{b}{d} \quad \frac{a}{b} = \frac{b}{d} = k$$

$$a = kc \text{ and } b = kd$$

Substituting kc for a and kd for b in the expression $\frac{a - 3b}{b - 3a}$

$$\frac{kc - 3kd}{kd - 3kc} = \frac{k(c - 3d)}{k(d - 3c)}$$

Therefore expression $\frac{a - 3b}{b - 3a} = \frac{c - 3d}{d - 3c}$

$$b - 3a \quad d - 3c$$

RATES OF WORK AND MIXTURES

Examples

195 men working 10 hours a day can finish a job in 20 days. How many men employed to finish the job in 15 days if they work 13 hours a day.

Solution:

Let x be the no. of men required

Days	hours	Men
20	10	195
15	13	x

$$20 \times 10 \times 195 = 15 \times 13 \times x$$

$$x = \frac{20 \times 10 \times 195}{15 \times 13} = 200 \text{ men}$$

Example

Tap P can fill a tank in 2 hrs, and tap Q can fill the same tank in 4 hrs. Tap R can empty the tank in 3 hrs.

If tap R is closed, how long would it take taps P and Q to fill the tank?

Calculate how long it would take to fill the tank when the three taps P, Q and R. are left running?

Solution

Tap P fills $\frac{1}{2}$ of the tank in 1 h. Tap Q fills $\frac{1}{4}$ of the tank in 1 h. Tap R empties $\frac{1}{3}$ of the tank in 1 h.

In one hour, P and Q fill $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ of the tank

Therefore $\frac{3}{4}$ of the tank is filled in 1 h.

Time taken to fill the tank $(\frac{4}{4}) = (\frac{4}{4} \div \frac{3}{4})\text{h}$

$$\frac{4}{3} \text{ h}$$

In 1h, P and Q fill $\frac{3}{4}$ of tank while R empties $\frac{1}{3}$ of the tank.

When all taps are open, $(\frac{1}{2} + \frac{1}{4} - \frac{1}{3} = \frac{5}{12})$ of the tank is filled in 1 hour.

$\frac{5}{12}$ of tank is filled in 1 hour.

Therefore time required to fill the tank $\frac{12}{12} = (\frac{12}{12} \div \frac{5}{12}) \times 1 \text{ h}$

$$2\frac{2}{5} \text{ h}$$

Example

In what proportion should grades of sugars costing sh. 45 and sh. 50 per kilogram be mixed in order to produce a blend worth sh. 48 per kilogram?

Solution Method 1

Let n kilograms of the grade costing sh. 45 per kg be mixed with 1 kilogram of grade costing sh. 50 per kg.

Total cost of the two blends is sh. $b(45n+50)$ The mass of the mixture is $(n+1)$ kg

Therefore total cost of the mixture is $(n+1)48$ $45n + 50 = 48(n+1)$

$$45n + 50 = 48n + 48$$

$$50 = 3n + 48$$

$$2 = 3n$$

$$n = \frac{2}{3}$$

The two grades are mixed in the proportion $\frac{2}{3} : 1 = 2 : 3$

Method 2

Let x kg of grade costing sh 45 per kg be mixed with y kg of grade costing sh.50 per kg. The total cost will be sh. $(45x + 50y)$

Cost per kg of the mixture is sh. $\frac{45x+50y}{x+y}$

$$\frac{45x+50y}{x+y} = 48$$

$$45x + 50y = 48(x + y)$$

$$45x + 50y = 48x + 48y$$

$$45x + 50y = 48x + 48y$$

$$2y = 3x$$

$$\frac{x}{y} = \frac{2}{3}$$

The proportion is $x : y = 2:3$

Questions

1. Akinyi bought maize and beans from a wholesaler. She then mixed the maize and beans the ratio 4:3 she brought the maize at Kshs. 12 per kg and the beans 4 per kg. If she was to make a profit of 30% what should be the selling price of 1 kg of the mixture?
2. A rectangular tank of base 2.4 m by 2.8 m and a height of 3m contains 3,600 liters of water initially. Water flows into the tank at the rate of 0.5 litres per second. Calculate the time in hours and minutes, required to fill the tank.
3. A company is to construct a parking bay whose area is 135m^2 . It is to be covered with concrete slab of uniform thickness of 0.15. To make the slab cement, ballast and sand are to be mixed so that their masses are in the ratio 1 : 4 : 4. The mass of m^3 of dry slab is 2,500kg. Calculate
 - a.
 - i. The volume of the slab
 - ii. The mass of the dry slab
 - iii. The mass of cement to be used
 - b. If one bag of the cement is 50 kg, find the number of bags to be purchased
 - c. If a lorry carries 7 tonnes of sand, calculate the number of lorries of sand to be purchased.
4. The mass of a mixture A of beans and maize is 72 kg. The ratio of beans to maize is 3:5 respectively
 - a. Find the mass of maize in the mixture
 - b. A second mixture of B of beans and maize of mass 98 kg is mixed with A. The final ratio of beans to maize is 8:9 respectively. Find the ratio of beans to maize in B
5. A retailer bought 49 kg of grade 1 rice at Kshs. 65 per kilogram and 60 kg of grade II rice at Kshs

Contact Mwalimu Consultancy 0746 222 000 / 0742 999 000 for Complete Notes.

27.50 per kilogram. He mixed the two types of rice.

- a. Find the buying price of one kilogram of the mixture
- b. He packed the mixture into 2 kg packets
 - i. If he intends to make a 20% profit find the selling price per packet
 - ii. He sold 8 packets and then reduced the price by 10% in order to attract customers. Find the new selling price per packet.
 - iii. After selling $\frac{1}{3}$ of the remainder at reduced price, he raised the price so as to realize the original goal of 20% profit overall. Find the selling price per packet of the remaining rice.
6. A trader sells a bag of beans for Kshs 1, 200. He mixed beans and maize in the ratio 3 : 2. Find how much the trader should he sell a bag of the mixture to realize the same profit?
7. Pipe A can fill an empty water tank in 3 hours while, pipe B can fill the same tank in 6 hours, when the tank is full it can be emptied by pipe C in 8 hours. Pipes A and B are opened at the same time when the tank is empty.
If one hour later, pipe C is also opened, find the total time taken to fill the tank
8. A solution whose volume is 80 litres is made 40% of water and 60% of alcohol. When litres of water are added, the percentage of alcohol drops to 40%
 - a. Find the value of x
 - b. Thirty litres of water is added to the new solution. Calculate the percentage
 - c. If 5 litres of the solution in (b) is added to 2 litres of the original solution, calculate in the simplest form, the ratio of water to that of alcohol in the resulting solution

STRAND 2; ALGEBRA

Matrices

Introduction

- A matrix is a rectangular arrangement of numbers in rows and columns.
- For instance, matrix A below has two rows and three columns. The dimensions of this matrix are 2 x 3 (read 2 by 3).
- The numbers in a matrix are its entries. In matrix A, the entry in the second row and third column is 5.
- Some matrices (the plural of matrix) have special names because of their dimensions or entries.

Order of Matrix

Note;

Matrix consist of rows and columns. Rows are the horizontal arrangement while columns are the vertical arrangement.

Order of matrix is being determined by the number of rows and columns. The order is given by statin the number of rows followed by columns.

- If the number of rows is m and the number of columns n, the matrix is of order $m \times n$.

E.g. If a matrix has m rows and n columns, it is said to be order $m \times n$.

e.g. $\begin{bmatrix} 2 & 0 & 3 & 6 \\ 3 & 4 & 7 & 0 \\ 1 & 9 & 2 & 5 \end{bmatrix}$ is a matrix of order 3×4 .

e.g. $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$ is a matrix of order 3.

e.g. $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -8 & 5 \end{bmatrix}$ is a 2×3 matrix.

e.g. $\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ is a 3×1 matrix.

Elements of Matrix

POLITE NOTE!

*Take note that this is a Sample of the Well Organized Detailed
Simplified Notes.*

*Call/Text/WhatsApp 0746 222 000 / 0742 999 000 for the
Complete Notes.*

**FOR MORE EDUCATIONAL
RESOURCES,**

CONTACT MR ISABOKE

0746 222 000 / 0742 999 000

FOR THE FOLLOWING;

- ✓ **ONLINE TUITION**
- ✓ **REVISION NOTES**
- ✓ **SCHEMES OF WORK**
- ✓ **SETBOOKS VIDEOS**
- ✓ **TERMLY EXAMS**
- ✓ **QUICK REVISION KITS**
- ✓ **KCSE PREMOCKS**
- ✓ **TOP SCHOOLS PREMOCKS**
- ✓ **KPLEA JOINT PREMOCKS**
- ✓ **TOP SCHOOLS MOCKS**
- ✓ **KPLEA JOINT MOCKS**
- ✓ **KJSEA POSTMOCKS**
- ✓ **TOP SCHOOLS PREDICTIONS**
- ✓ **FINAL EXAM PREDICTIONS**
- ✓ **KPLEA REVEALED SETS**
- ✓ **KPSEA EXAMS**

CALL/TEXT 0746 222 000

mwalimuconsultancy@gmail.com

THIS IS A PROPERTY OF MWALIMU CONSULTANCY LTD.

**POWERED BY MR
ISABOKE**

SUCCESS

Contact Mwalimu Consultancy 0746 222 000 | 0742 999 000 for Complete Notes.