SHORT NOTE

GENERAL PHYSICS



edited by MUJA

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SHORT NOTE

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Rectilinear Kinematics: Continuous Motion

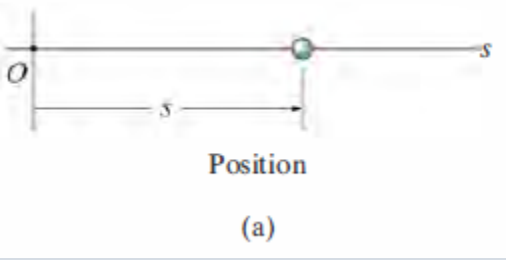
We will begin our study of dynamics by discussing the kinematics of a particle that moves along a rectilinear or straight line path. Recall that a *particle* has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion. In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position. The straight-line path of a particle will be defined using a single coordinate axis s, Fig. 12–1*a*. The origin O on the path is a fixed point, and from this point the *position coordinate* s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s. Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O. Realize that position is a vector quantity since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s since the direction always remains along the coordinate axis.

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Position

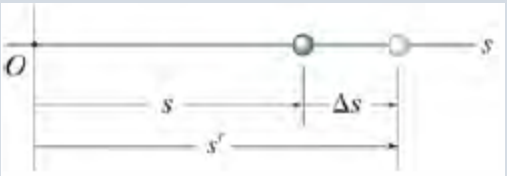
(a)

Displacement. The *displacement* of the particle is defined as the *change* in its *position*. For example, if the particle moves from one point to another, Fig. 12–1*b*, the displacement is

$$\Delta s = s' - s$$

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.



Displacement

(b)

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Velocity. If the particle moves through a displacement Δs during the time interval Δt, the *average velocity* of the particle during this time interval is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the *instantaneous velocity* is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

$$(\stackrel{+}{\rightarrow}) \qquad \boxed{v = \frac{ds}{dt}} \qquad (12\text{–}1)$$

Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$, Fig. 12–1*e*.

The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v/\Delta t)$, or

$$(\stackrel{+}{\rightarrow}) \qquad \boxed{a = \frac{dv}{dt}} \qquad (12\text{–}2)$$

$$\boxed{a\,ds = v\,dv}$$

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Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$$(\stackrel{+}{\rightarrow}) \qquad \boxed{v = v_0 + a_c t \quad \text{Constant Acceleration}} \qquad (12\text{–}4)$$

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\,dt$$

$$(\stackrel{+}{\rightarrow}) \qquad \boxed{s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2 \quad \text{Constant Acceleration}} \qquad (12\text{–}5)$$

Velocity as a Function of Position. Either solve for t in Eq. 12–4 and substitute into Eq. 12–5, or integrate $v\,dv = a_c\,ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$(\stackrel{+}{\rightarrow}) \qquad \boxed{v^2 = v_0^2 + 2a_c(s - s_0) \quad \text{Constant Acceleration}} \qquad (12\text{–}6)$$

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Procedure for Analysis

Coordinate System.

- Establish a position coordinate s along the path and specify its *fixed origin* and positive direction.
- Since motion is along a straight line, the vector quantities position, velocity, and acceleration can be represented as algebraic scalars. For analytical work the sense of s, v, and a is then defined by their *algebraic signs*.
- The positive sense for each of these scalars can be indicated by an arrow shown alongside each kinematic equation as it is applied.

Kinematic Equations.

- If a relation is known between any *two* of the four variables a, v, s and t, then a third variable can be obtained by using one of the kinematic equations, $a = dv/dt$, $v = ds/dt$ or $a\,ds = v\,dv$, since each equation relates all three variables.*
- Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.
- Remember that Eqs. 12–4 through 12–6 have only limited use. These equations apply *only* when the *acceleration is constant* and the initial conditions are $s = s_0$ and $v = v_0$ when $t = 0$.

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EXAMPLE 12.1

The car in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



Fig. 12–2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\stackrel{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\stackrel{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$

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EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.



Fig. 12–4

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(s_B - 40\text{ m})$$

$$s_B = 327\text{ m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81\text{ m/s}^2)(0 - 327\text{ m})$$

$$v_C = -80.1\text{ m/s} = 80.1\text{ m/s}\downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward.

Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(0 - 40\text{ m})$$

$$v_C = -80.1\text{ m/s} = 80.1\text{ m/s}\downarrow \qquad \textit{Ans.}$$

F12–2. A ball is thrown vertically upward with a speed of 15 m/s. Determine the time of flight when it returns to its original position.



F12–5. The position of the particle is given by $s = (2t^2 - 8t + 6)$ m, where t is in seconds. Determine the time when the velocity of the particle is zero, and the total distance traveled by the particle when $t = 3$ s.



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•**12–1.** A car starts from rest and with constant acceleration achieves a velocity of 15 m/s when it travels a distance of 200 m. Determine the acceleration of the car and the time required.

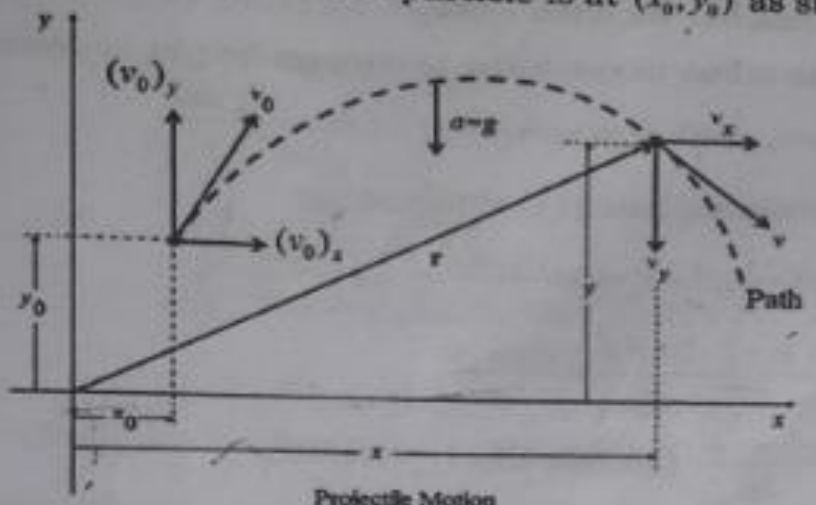
12–2. A train starts from rest at a station and travels with a constant acceleration of 1 m/s^2. Determine the velocity of the train when $t = 30$ s and the distance traveled during this time.

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Analysis of Projectile Motions

Projectile motions are the most common type of motion that are convenient to be analyzed using rectangular coordinates. Suppose the initial position of a particle is at (x_0, y_0) as shown.



Projectile Motion

I) Horizontal Motion: The motion of the particle in the direction of the x-axis. In this motion, the particle is assumed to move with zero acceleration. Since $a_x = 0$ (the x-component of the acceleration is zero), the particle moves with constant speed.

Then, the motion is described by the following equations.

$$\begin{cases} v_x = v_{0x} + a_x t \Rightarrow v_x = v_{0x} = v_0 \cos\theta \\ x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \Rightarrow x = x_0 + v_{0x} t = x_0 + v_0 \cos\theta\, t \\ v^2 = v_{0x}^2 + 2a_x (x - x_0) \Rightarrow v = v_{0x} = v_0 \cos\theta \end{cases}$$

II) Vertical Motion: The motion of the particle in the direction of the y-axis. In this motion, the particle moves with a constant acceleration of $a_y = -g$ (the y-component of the acceleration always points downward and its magnitude is the same as that of the gravitational acceleration).

Then, the motion of the particle is described as follow.

$$\begin{cases} v_y = v_{0y} + a_y t \\ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\ v_y^2 = v_{0y}^2 + 2a_y (y - y_0) \end{cases} \Rightarrow \begin{cases} v_y = v_0 \sin\theta - gt \\ y = y_0 + v_0 \sin\theta\, t - \frac{1}{2} g t^2 \\ v_y^2 = v_{0y}^2 - 2g(y - y_0) \end{cases}$$

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Maximum height and Range of a projectile:

The maximum height of a projectile motion is attained when the particle takes a momentary rest and starts to fall down ward. At this momentary rest (that means when the particle attains its maximum height), we have $v_y = 0$.

So, the time taken to reach the maximum height is determined as follow: $v_y = v_0 \sin\theta - gt \Rightarrow v_0 \sin\theta - gt = 0 \Rightarrow t = \frac{v_0 \sin\theta}{g}$.

And the maximum height is obtained as;

$$h_{max} = v_0 \sin\theta\, t - \frac{1}{2} g t^2 = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{1}{2}\frac{v_0^2 \sin^2\theta}{g} = \frac{v_0^2 \sin^2\theta}{2g}$$

$$\Rightarrow h_{max} = \frac{v_0^2 \sin^2\theta}{2g} \quad \left(\text{Attained when } t_{max} = \frac{v_0 \sin\theta}{g}\right)$$

Range: In a projectile motion, a range means the horizontal distance that the particle moves in its total time of flight.

First, recall the basic assumption that the time to move upward is equal to the time to fall down.

That means the time to reach the maximum height and the time to return to the ground are the same. That means the total time of flight is $t_f = t_{up} + t_{down} = 2.t_{up} = 2.t_{max} = \frac{2v_0 \sin\theta}{g}$

Now, the range of the projectile is calculated as follow:

$$\text{Range} = R = v_{0x}.t_f = v_0 \cos\theta \left(\frac{2v_0 \sin\theta}{g}\right) = \frac{v_0^2(2\cos\theta \sin\theta)}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

Under what condition does the range will be maximum?

As we see from the formula $R = \frac{v_0^2 \sin 2\theta}{g}$, since v_0 and g are constant the range depends on the firing angle θ.

So, using derivative, the range is maximum when

$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{2v_0^2 \cos 2\theta}{g} = 0 \Rightarrow 2v_0^2 \cos 2\theta = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} = 45^0$$

That means the maximum range is $R = \frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 \sin 2(45^0)}{g} = \frac{v_0^2}{g}$

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A T C

Motion of a Projectile

The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (x_0, y_0), with an initial velocity of $\mathbf{v}_0$, having components $(\mathbf{v}_0)_x$ and $(\mathbf{v}_0)_y$, Fig. 12–20. When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a *constant downward acceleration* of approximately $a_c = g = 9.81\ \text{m/s}^2$ or $g = 32.2\ \text{ft/s}^2$.*

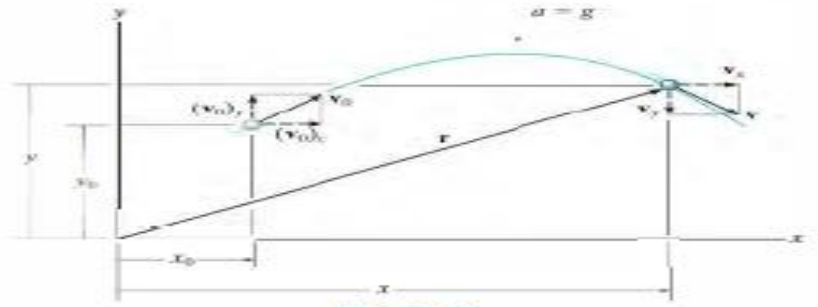


Fig. 12–20

Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\rightarrow) \quad v = v_0 + a_c t; \qquad v_x = (v_0)_x$$
$$(\rightarrow) \quad x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad x = x_0 + (v_0)_x t$$
$$(\rightarrow) \quad v^2 = v_0^2 + 2a_c(x - x_0); \qquad v_x = (v_0)_x$$

The first and last equations indicate that *the horizontal component of velocity always remains constant during the motion.*

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$(+\uparrow) \quad v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$
$$(+\uparrow) \quad y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(y - y_0); \qquad v^2 = (v_0)_y^2 - 2g(y - y_0)$$

Recall that the last equation can be formulated on the basis of eliminating the time t from the first two equations, and therefore *only two of the above three equations are independent of one another.*

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Procedure for Analysis

Coordinate System.

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns*. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2 or 32.2 ft/s^2. The particle's initial and final velocities should be represented in terms of their x and y components.
- Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

Kinematic Equations.

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \tfrac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

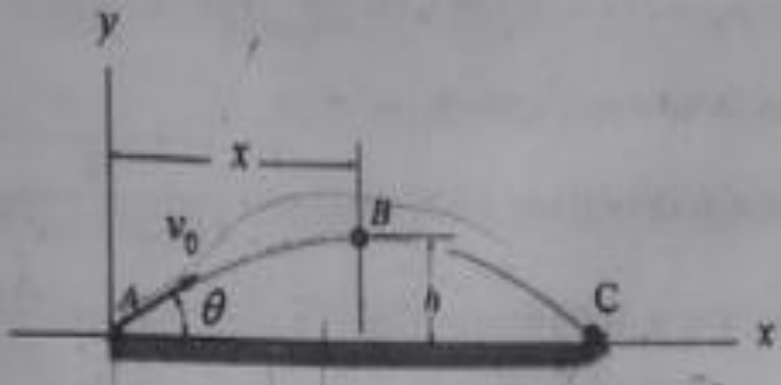
For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.

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Examples:

1. The ball is kicked from point A with initial velocity of $v_0 = 40\text{m/s}$ at an angle of $\theta = 30^0$ as shown below. Determine the range R and the speed of the ball when it strikes the ground. What are the maximum range and height it reaches?



Solution: At point C: When the ball hits the ground at C, the height becomes $y_C = 0$. So, using vertical motion equations,

$$y_C = v_0 \sin\theta t - \frac{1}{2}gt^2 \Rightarrow v_0 \sin 30^0 t - \frac{1}{2}(10)t^2 = 0 \Rightarrow 20t - 5t^2 = 0 \Rightarrow t = 0, t = 4$$

Besides, $v_x = v_0 \cos\theta = 20\sqrt{3}\text{m/s}$, $v_y = v_0 \sin\theta - gt = 20 - 40 = -20\text{m/s}$.

So, when the ball hits the ground, the velocity is $v = 20\sqrt{3}\mathbf{i} - 20\mathbf{j}$.

Then, the speed is simply the magnitude of the velocity.

That is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{1200 + 400} = \sqrt{1600} = 40\text{m/s}$.

The range is $x = x_0 + v_0 \cos\theta t = (40\cos 30^0)(4) = 80\sqrt{3}m$.

Besides, the maximum height is $h_{max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{1600(0.25)}{20} = 20m$.

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EXAMPLE

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.

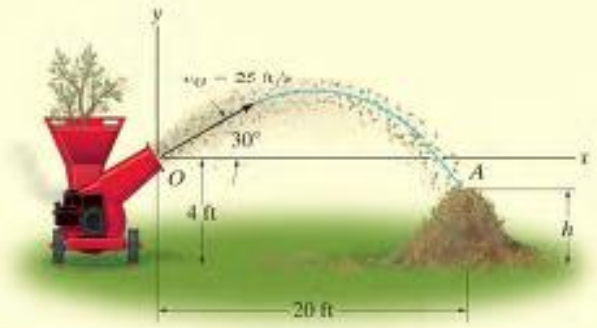


Fig. 12–22

SOLUTION

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°)\ \text{ft/s} = 21.65\ \text{ft/s} \rightarrow$$
$$(v_O)_y = (25 \sin 30°)\ \text{ft/s} = 12.5\ \text{ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2\ \text{ft/s}^2$. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$$(\pm\rightarrow) \qquad x_A = x_O + (v_O)_x t_{OA}$$
$$20\ \text{ft} = 0 + (21.65\ \text{ft/s}) t_{OA}$$
$$t_{OA} = 0.9238\ \text{s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \qquad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$
$$(h - 4\ \text{ft}) = 0 + (12.5\ \text{ft/s})(0.9238\ \text{s}) + \tfrac{1}{2}(-32.2\ \text{ft/s}^2)(0.9238\ \text{s})^2$$
$$h = 1.81\ \text{ft} \qquad \textit{Ans.}$$

NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_c t_{OA}$.

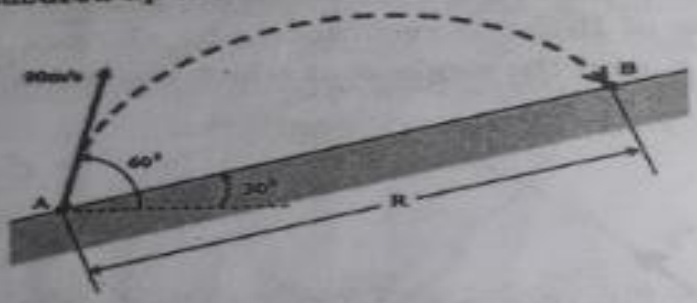
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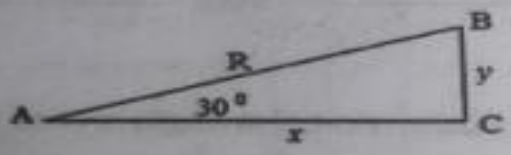
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5. A projectile is launched with an initial velocity of 90m/s at an angle of 60° with respect to the horizontal as shown. Compute the range R as measured up the inclined plane.



Solution: Consider the following diagram.



Horizontal Motion: Using geometry, the horizontal distance is expressed in terms of the range R as $x = R\cos 30^\circ = \frac{\sqrt{3}}{2}R$.

Using projectile, $x = v_0 \cos 60^\circ t = 45t$.

Equating the two gives $45t = \frac{\sqrt{3}}{2}R \Rightarrow t = \frac{\sqrt{3}}{90}R$.

Vertical Motion: Using geometry, the vertical height at B is expressed in terms of R as $y = R\sin 30^\circ = \frac{1}{2}R$.

Using projectile, $y = (v_0)_y t - \frac{1}{2}gt^2 = v_0 \sin 60^\circ t - 5t^2 = 45\sqrt{3}t - 5t^2$.

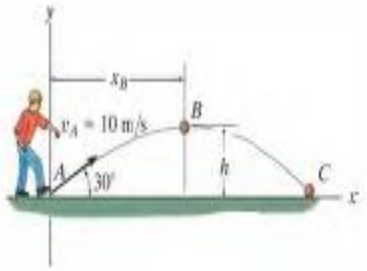
Equating the two gives $45\sqrt{3}t - 5t^2 = \frac{1}{2}R \Rightarrow 10t^2 - 90\sqrt{3}t + R = 0$.

Using the result $t = \frac{\sqrt{3}}{90}R$ from the horizontal motion, we have

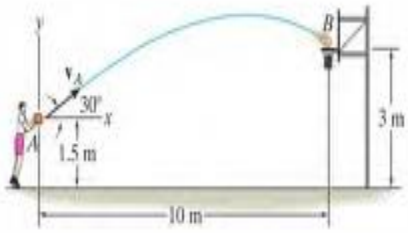
$$10t^2 - 90\sqrt{3}t + R = 0 \Rightarrow 10\left(\frac{\sqrt{3}}{90}R\right)^2 - 90\sqrt{3}\left(\frac{\sqrt{3}}{90}R\right) + R = 0 \Rightarrow \frac{30}{8100}R^2 - 3R + R = 0$$

$$\Rightarrow R^2 - 540R = 0 \Rightarrow R(R - 540) = 0 \Rightarrow R = 540\text{m}$$

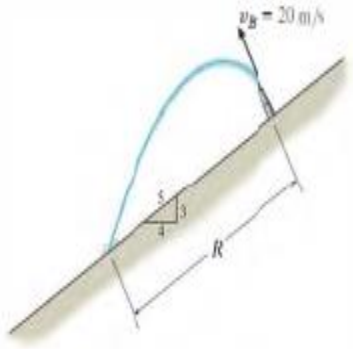
The ball is kicked from point A with the initial velocity $v_A = 10$ m/s. Determine the range R, and the speed when the ball strikes the ground.



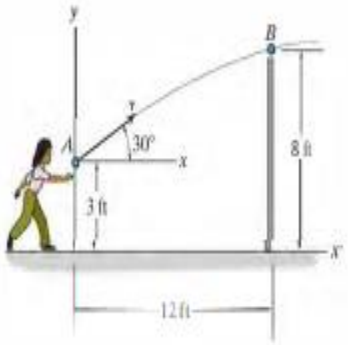
Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B.



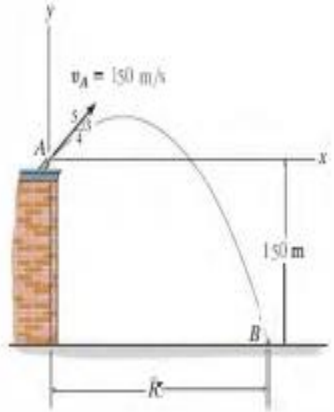
Water is sprayed at an angle of 90° from the slope at 20 m/s. Determine the range R.



A ball is thrown from A. If it is required to clear the wall at B, determine the minimum magnitude of its initial velocity v_A.



A projectile is fired with an initial velocity of $v_A = 150$ m/s off the roof of the building. Determine the range R where it strikes the ground at B.



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6. A projectile is launched from A with initial conditions as shown in the figure. Determine the slant distance D which locates point B and the total time of flight from A to B.



Solution: Set up the x-y coordinate axis with origin at A.

Horizontal Motion: From A to B, $x = v_0 \cos\theta t = (40\cos 45°)t$.

But from geometry, the horizontal distance from A to B is given by $x = 80 + D\cos\alpha$. Equating the two distances, we get

$$(40\cos 45°)t = 80 + D\cos\alpha \Rightarrow 20\sqrt{2}t = 80 + \frac{4}{5}D \Rightarrow t = 2\sqrt{2} + \frac{\sqrt{2}}{50}D.$$

Vertical Motion: $y = v_0 \sin\theta t - \frac{1}{2}gt^2 = (40\sin 45°)t - 5t^2$

$$40\sin 45°t - 5t^2 = -\frac{3}{5}D \Rightarrow (40\sin 45°)(2\sqrt{2} + \frac{\sqrt{2}}{50}D) - 5(2\sqrt{2} + \frac{\sqrt{2}}{50}D)^2 = -\frac{3}{5}D$$

$$\Rightarrow (20\sqrt{2})(2\sqrt{2} + \frac{\sqrt{2}}{50}D) - 5(8 + \frac{4}{25}D + \frac{2}{2500}D^2) = -\frac{3}{5}D$$

$$\Rightarrow 80 + \frac{4}{5}D - 40 - \frac{4}{5}D - \frac{1}{250}D^2 = -\frac{3}{5}D \Rightarrow D^2 - 150D - 10000 = 0 \Rightarrow D = 250\text{m}$$

Besides, the time is $t = 2\sqrt{2} + \frac{\sqrt{2}}{50}D = 2\sqrt{2} + \frac{\sqrt{2}}{50}(250) = 7\sqrt{2}\text{s}$.

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Force

Before dealing with a group or *system* of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in Chapter 1 as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

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Newton's Laws

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.* Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

The correctness of these laws has been verified by innumerable accurate physical measurements. Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m, it may be stated as

$$\mathbf{F} = m\mathbf{a}$$

where **F** is the vector sum of forces acting on the particle and **a** is the resulting acceleration. This equation is a *vector* equation because the direction of **F** must agree with the direction of **a**, and the magnitudes of **F** and $m\mathbf{a}$ must be equal.

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at rest or is moving with a uniform velocity. The first law adds nothing new to the description of motion but is included here because it was part of Newton's classical statements.

The third law is basic to our understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on the desk by the pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source, and holds at every instant of time during which the forces are applied. Lack of careful attention to this basic law is the cause of frequent error by the beginner.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as

$$F = G\frac{m_1 m_2}{r^2}$$

where

F = force of attraction between the two particles
G = universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \mathrm{m^3/(kg \cdot s^2)}$
m_1, m_2 = mass of each of the two particles
r = distance between the centers of the two particles

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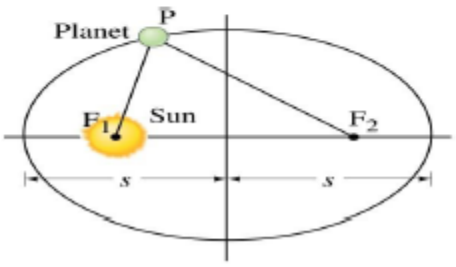
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Kepler's Laws, Satellites Motion and Weightlessness

The basic laws of planetary motion were established by Johannes Kepler (1571-1630) based on the analysis of astronomical observations of Tycho Brahe (1546–1601). In 1609, Kepler formulated the first two laws. The third law was discovered in 1619. Later, in the late 17th century, Isaac Newton proved mathematically that all three laws of Kepler are a consequence of the law of universal gravitation.

Kepler's First Law (Law of Orbits)

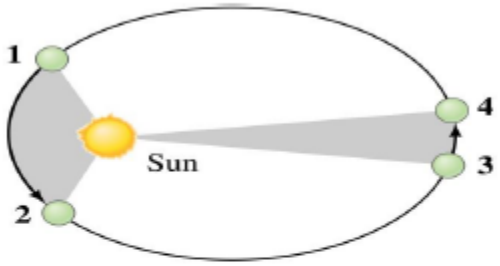
States that the orbit of each planet in the solar system is an ellipse, the Sun will be on one focus. The points F_1 and F_2represented in figure are known as the **foci** of the ellipse. The closer together that these points are, the more closely that the ellipse resembles the shape of a circle. In fact, a circle is the special case of an ellipse in which the two foci are at the same location. Kepler's first law is rather simple - all planets orbit the sun in a path that resembles an ellipse, with the sun being located at one of the foci of that ellipse.



Kepler's Second Law (The Law of Areas)

States that "the radius vector connecting the centers of the Sun and the Planet sweepsout equal areas in equal intervals of time."The Figure below shows the two sectors of the ellipse having equal areas corresponding to the same time intervals. The second

law describes the speed (which is constantly changing) at which any given planet will move while orbiting the sun. A planet moves fastest when it is closest to the sun and slowest when it is furthest from the sun. Yet, if an imaginary line were drawn from the center of the planet to the center of the sun, that line would sweep out the same area in equal periods of time.



Kepler's Third Law (The Law of Harmony)

States that *"the square of the orbital period of a planet is proportional to the cube of the average distance between the centers of the planet and the sun."*

Unlike Kepler's first and second laws that describe the motion characteristics of a single planet, the third law makes a comparison between the motion characteristics of different planets. The comparison being made is that the ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets. As an illustration, consider the orbital period and average distance from sun (orbital radius) for Earth and mars as given in the table below.

Planet	Period (s)	Average Distance (m)	T^2/R^3 (s^2/m^3)
Earth	3.156×10^7 s	1.4957×10^{11}	2.977×10^{-19}
Mars	5.93×10^7 s	2.278×10^{11}	2.975×10^{-19}

Observe that the T^2/R^3 ratio is the same for Earth as it is for mars. In fact, if the same T^2/R^3 ratio is computed for the other planets, it can be found that this ratio is nearly the same value for all the planets (see table below). Amazingly, every planet has the same T^2/R^3 ratio.

Planet	Period	Average	T^2/R^3

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	(yr)	Distance (au)	(yr^2/au^3)
Mercury	0.241	0.39	0.98
Venus	.615	0.72	1.01
Earth	1.00	1.00	1.00
Mars	1.88	1.52	1.01
Jupiter	11.8	5.20	0.99
Saturn	29.5	9.54	1.00
Uranus	84.0	19.18	1.00
Neptune	165	30.06	1.00
Pluto	248	39.44	1.00

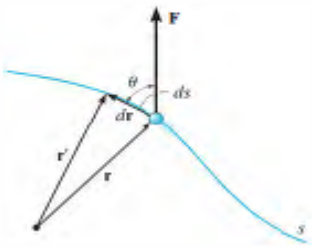
(**NOTE**: The average distance value is given in astronomical units where 1 a.u. is equal to the distance from the earth to the sun - 1.4957×10^{11} m. The orbital period is given in units of earth-years where 1 earth year is the time required for the earth to orbit the sun - 3.156×10^{7} seconds.)

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The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force **F** will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*. For example, if the force **F** in Fig. 14–1 causes the particle to move along the path s from position **r** to a new position **r′**, the displacement is then $d\mathbf{r} = \mathbf{r}' - \mathbf{r}$. The magnitude of $d\mathbf{r}$ is ds, the length of the differential segment along the path. If the angle between the tails of $d\mathbf{r}$ and **F** is θ, Fig. 14–1, then the work done by **F** is a *scalar quantity*, defined by

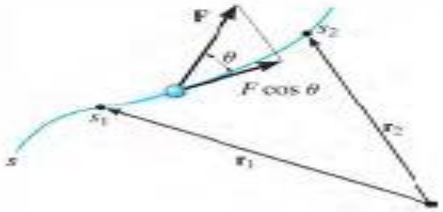


$$dU = F\,ds\cos\theta$$

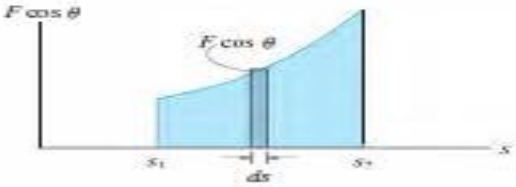
Work of a Variable Force. If the particle acted upon by the force **F** undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2, Fig. 14–2*a*, the work of force **F** is determined by integration. Provided **F** and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r} = \int_{s_1}^{s_2} F\cos\theta\,ds$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of $F\cos\theta$ vs. s. Then the *area* under this graph bounded by s_1 and s_2 represents the total work, Fig. 14–2*b*.



(a)



(b)

$$W = mg$$

By comparison with $F = ma$, we term g the acceleration due to gravity. For most engineering calculations g is a point on the surface of the earth at sea level, and at a latitude of 45°, which is considered the "standard location." Here the values $g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$ will be used for calculations.

In the SI system the mass of the body is specified in kilograms, and the weight must be calculated using the above equation, Fig. 13–1*a*. Thus,

$$W = mg\ (\text{N}) \qquad (g = 9.81\ \text{m/s}^2)$$

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Work of a Constant Force Moving Along a Straight Line.

If the force $\mathbf{F}_c$ has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 14–3*a*, then the component of $\mathbf{F}_c$ in the direction of displacement is always $F_c \cos\theta$. The work done by $\mathbf{F}_c$ when the particle is displaced from s_1 to s_2 is determined from Eq. 14–1, in which case

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

or

$$U_{1-2} = F_c \cos\theta (s_2 - s_1) \qquad (14\text{–}2)$$

Here the work of $\mathbf{F}_c$ represents the *area of the rectangle* in Fig. 14–3*b*.

Work of a Weight.

Consider a particle of weight $\mathbf{W}$, which moves up along the path s shown in Fig. 14–4 from position s_1 to position s_2. At an intermediate point, the displacement $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$ Since $\mathbf{W} = -W\mathbf{j}$, applying Eq. 14–1 we have

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\, dy = -W(y_2 - y_1)$$

or

$$U_{1-2} = -W\, \Delta y \qquad (14\text{–}3)$$

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 14–4 the work is *negative*, since W is downward and Δy is upward. Note, however, that if the particle is displaced *downward* $(-\Delta y)$, the work of the weight is *positive*. Why?

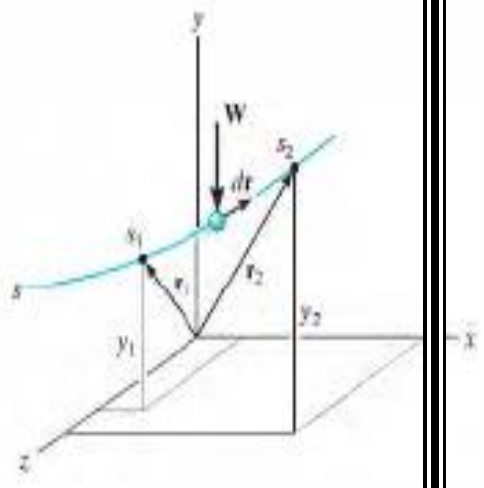


Fig. 14–4

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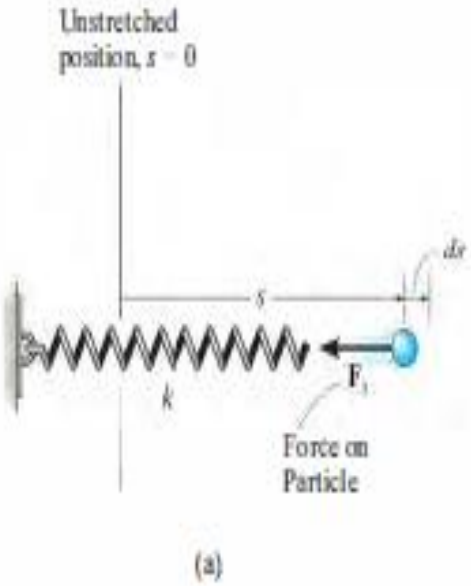
Work of a Spring Force. If an elastic spring is elongated a distance ds, Fig. 14–5*a*, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\,ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to ds. If the particle displaces from s_1 to s_2, the work of $\mathbf{F}_s$ is then

$$U_{1-2} = \int_{s_1}^{s_2} F_s\,ds = \int_{s_1}^{s_2} -ks\,ds$$

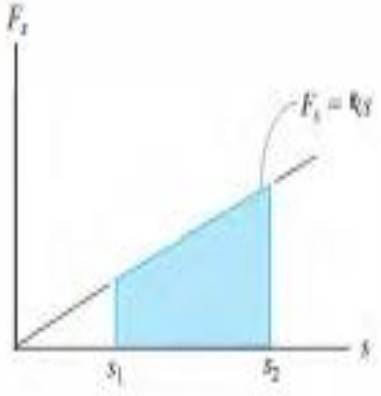
$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$

This work represents the trapezoidal area under the line $F_s = ks$, Fig. 14–5*b*.

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle—if both are in the *same sense*, *positive work* results; if they are *opposite* to one another, the *work is negative*.



(a)



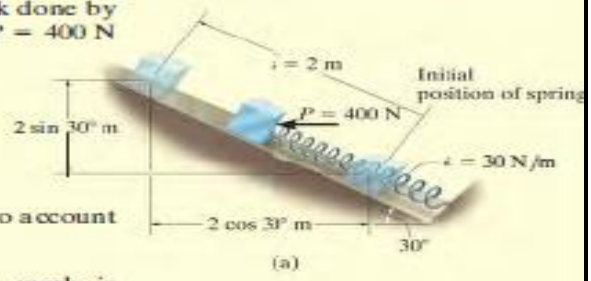
(b)

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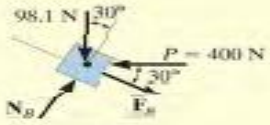
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EXAMPLE

The 10-kg block shown in Fig. 14–6*a* rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.



(a)



(b)

Fig. 14–6

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14–6*b*.

Horizontal Force P. Since this force is *constant*, the work is determined using Eq. 14–2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_P = 400\text{ N}\ (2\text{ m}\cos 30^\circ) = 692.8\text{ J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400\text{ N}\cos 30^\circ(2\text{ m}) = 692.8\text{ J}$$

Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5\text{ m} + 2\text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30\text{ N/m})(2.5\text{ m})^2 - \tfrac{1}{2}(30\text{ N/m})(0.5\text{ m})^2\right] = -90\text{ J}$$

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1\text{ N})\ (2\text{ m}\sin 30^\circ) = -98.1\text{ J}$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1\sin 30^\circ\text{ N})\ (2\text{ m}) = -98.1\text{ J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

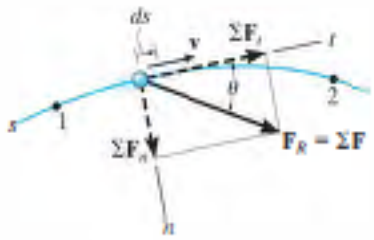
Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8\text{ J} - 90\text{ J} - 98.1\text{ J} = 505\text{ J} \qquad \textit{Ans.}$$

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Principle of Work and Energy



Consider the particle in Fig. 14–7, which is located on the path defined relative to an inertial coordinate system. If the particle has a mass m and is subjected to a system of external forces represented by the resultant $\mathbf{F}_R = \Sigma\mathbf{F}$, then the equation of motion for the particle in the tangential direction is $\Sigma F_t = ma_t$. Applying the kinematic equation $a_t = v\,dv/ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have

$$\Sigma \int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\Sigma \int_{s_1}^{s_2} F_t\,ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

From Fig. 14–7, note that $\Sigma F_t = \Sigma F \cos\theta$, and since work is defined from Eq. 14–1, the final result can be written as

$$\Sigma U_{1\text{-}2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

This equation represents the *principle of work and energy* for the particle. The term on the left is the sum of the work done by *all* the forces acting on the particle as the particle moves from point 1 to point 2. The two terms on the right side, which are of the form $T = \frac{1}{2}mv^2$, define the particle's final and initial *kinetic energy*, respectively. Like work, kinetic energy is a *scalar* and has units of joules (J) and ft · lb. However, unlike work, which can be either positive or negative, the kinetic energy is *always positive*, regardless of the direction of motion of the particle.

When Eq. 14–6 is applied, it is often expressed in the form

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

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Procedure for Analysis

Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy.

- Apply the principle of work and energy, $T_1 + \Sigma U_{1-2} = T_2$.
- The kinetic energy at the initial and final points is *always positive* since it involves the speed squared $\left(T = \frac{1}{2}mv^2\right)$.
- A force does work when it moves through a displacement in the direction of the force.
- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force–displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_w = \pm Wy$. It is positive when the weight moves downwards.
- The work of a spring is of the form $U_s = \frac{1}{2}ks^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

Principle of Work and Energy for a System of Particles

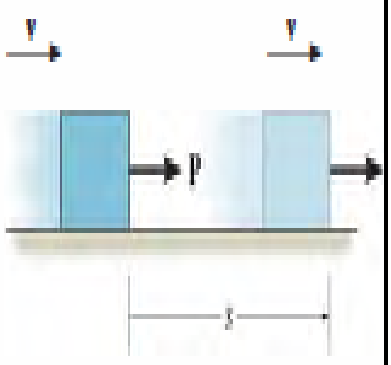
The principle of work and energy can be extended to include a system of particles isolated within an enclosed region of space as shown in Fig. 14–8. Here the arbitrary ith particle, having a mass m_i, is subjected to a resultant external force $\mathbf{F}_i$ and a resultant internal force $\mathbf{f}_i$ which all the other particles exert on the ith particle. If we apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically, which gives

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

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Work of Friction Caused by Sliding. A special class of problems will now be investigated which requires a careful application of Eq. 14–8. These problems involve cases where a body slides over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance s over a rough surface as shown in Fig. 14–9*a*. If the applied force **P** just balances the *resultant* frictional force $\mu_k N$, Fig. 14–9*b*, then due to equilibrium a constant velocity $\mathbf{v}$ is maintained, and one would expect Eq. 14–8 to be applied as follows:



(a)



$$\tfrac{1}{2}mv^2 + Ps - \mu_k Ns = \tfrac{1}{2}mv^2$$

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3.3 The concepts of Work and Energy

Here, first we will discuss the concept of work, next the concept of energy and finally the relation of work and energy.

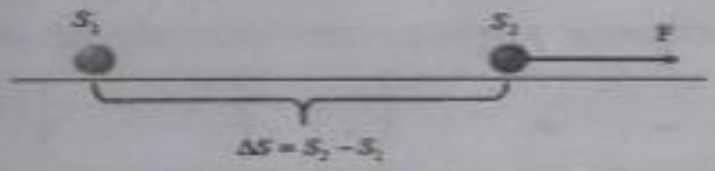
Work: Generally, work is a scalar quantity which is defined as the product of a force with a distance moved in the direction of the force. Work is commonly expressed using a unit of joules(J).

Energy: It is defined as the capacity for doing work. Like a work, energy is also a scalar quantity and expressed with the same unit of joule (J). But unlike work, energy is associated with the status of an object where as work is a parameter associated with a process not associated with the the status of an object.

3.3.1 Work Done by Different Forces

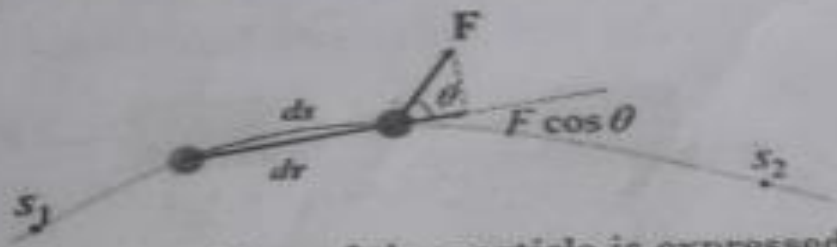
I) Work done by a constant force:

Suppose a constant force F displaces a particle from position S_1 to position S_2 along the direction of the force as shown.



Then, the work done by this force which is denoted by U is defined as $U = F\Delta S = F.(S_2 - S_1) = FS$.

II) Work done by general force: Suppose a force F displaces a particle from the position S_1 to S_2 as shown.



Then, the change in position of the particle is expressed by dr and the length of moved is ds. But for sufficiently small change of time $dr = ds$

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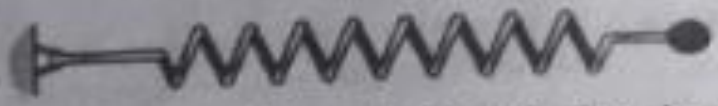
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Then, the work done by the force during ... of time is $dU = F\cos\theta\,ds = F\,dr$. Therefore, by taking integration both sides, the total work done $U_{1\to 2}$ in displacing from position S_1 to S_2 is derived as follow: $dU = F\,dr \Rightarrow \int_0^U dU = \int_{S_1}^{S_2} F\,dr \Rightarrow U_{1\to 2} = \int_{S_1}^{S_2} F\,dr$

III) Work done by Weight force:

Suppose a particle of weight W is moved from a height h_1 to h_2. Then, the work done by the weight force which is directed down ward is $U_{w_{1\to 2}} = -W(h_2 - h_1) = -W\Delta h$. If the particle is moving upward, the work done is negative and if it is moving downward, the work done is positive.

IV) Work done by Spring Force: Consider a spring with a spring constant k in its neutral position as shown.

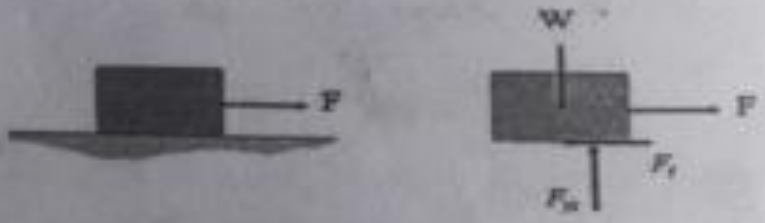


If a particle is attached to this spring such that it compresses the spring in to position S_1 and then it is stretched to position S_2 so as to return to its neutral position. First of all recall that a spring force with spring constant k is given by $F_s = -kS$. Then, the work done by this spring force in the process of compressing and stretching the spring is $U_{s_{1\to 2}} = \int_{S_1}^{S_2} F_s\,ds = \int_{S_1}^{S_2} -kS\,ds = -\frac{1}{2}k(S_2^2 - S_1^2)$

Notice that S_1 and S_2 are displacements measured from the neutral position of the spring.

V) Work done by Frictional Force:

Suppose a force F displaces a particle to a distance of S on a surface with coefficient of kinetic friction μ_k as shown.



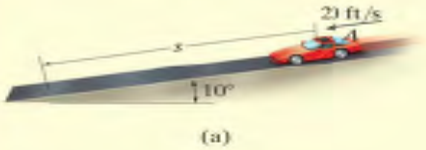
Then, the work done by this frictional force is defined as follow.

$$U_{f_{1\to 2}} = -F_f S = -\mu_k F_N S$$

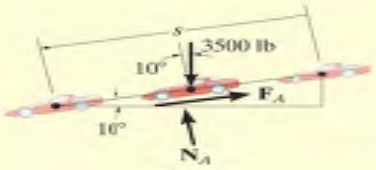
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EXAMPLE



(a)



(b)

The 3500-lb automobile shown in Fig. 14–10*a* travels down the inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far *s* the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.

SOLUTION

This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

Work (Free-Body Diagram). As shown in Fig. 14–10*b*, the normal force $\mathbf{N}_A$ does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced *s* sin 10° and does positive work. Why? The frictional force $\mathbf{F}_A$ does both external and internal work when it undergoes a displacement *s*. This work is negative since it is in the opposite sense of direction to the displacement. Applying the equation of equilibrium normal to the road, we have

$$+\nwarrow \Sigma F_n = 0; \quad N_A - 3500 \cos 10^\circ \text{ lb} = 0 \quad N_A = 3446.8 \text{ lb}$$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10^\circ) - (1723.4 \text{ lb})s = 0$$

Solving for *s* yields

$$s = 19.5 \text{ ft} \qquad \text{Ans.}$$

NOTE: If this problem is solved by using the equation of motion, two steps are involved. First, from the free-body diagram, Fig. 14–10*b*, the equation of motion is applied along the incline. This yields

$$+\swarrow \Sigma F_s = ma_s; \quad 3500 \sin 10^\circ \text{ lb} - 1723.4 \text{ lb} = \frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2} a$$

$$a = -10.3 \text{ ft/s}^2$$

Then, since *a* is constant, we have

$$(+\swarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0);$$

$$(0)^2 = (20 \text{ ft/s})^2 + 2(-10.3 \text{ ft/s}^2)(s - 0)$$

$$s = 19.5 \text{ ft} \qquad \text{Ans.}$$

•14–1. A 1500-lb crate is pulled along the ground with a constant speed for a distance of 25 ft, using a cable that makes an angle of 15° with the horizontal. Determine the tension in the cable and the work done by this force. The coefficient of kinetic friction between the ground and the crate is $\mu_k = 0.55$.

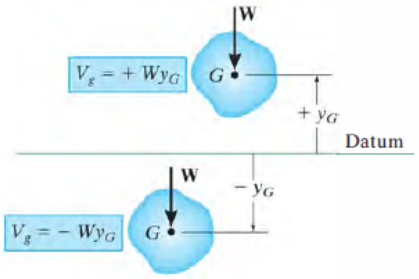
***14–4.** When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.

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Conservation of Energy

When a force system acting on a rigid body consists only of *conservative forces*, the conservation of energy theorem can be used to solve a problem that otherwise would be solved using the principle of work and energy. This theorem is often easier to apply since the work of a conservative force is *independent of the path* and depends only on the initial and final positions of the body. It was shown in Sec. 14.5 that the work of a conservative force can be expressed as the difference in the body's potential energy measured from an arbitrarily selected reference or datum.



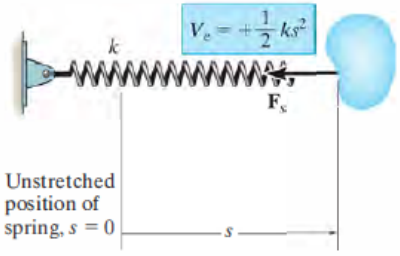
Gravitational potential energy

Gravitational Potential Energy. Since the total weight of a body can be considered concentrated at its center of gravity, the *gravitational potential energy* of the body is determined by knowing the height of the body's center of gravity above or below a horizontal datum.

$$V_g = W y_G$$

Elastic Potential Energy. The force developed by an elastic spring is also a conservative force. The *elastic potential energy* which a spring imparts to an attached body when the spring is stretched or compressed from an initial undeformed position $(s = 0)$ to a final position s,

$$V_e = +\tfrac{1}{2} k s^2$$



Elastic potential energy

In the deformed position, the spring force acting *on the body* always has the ability for doing positive work when the spring returns back to its original undeformed position

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The Principle (Equation) of Work and Energy

Kinetic energy: It is the measure of the particle's capacity to do work associated with the motion. In other words, kinetic energy is the amount of work that can be done to bring an object from the state of rest to a state of motion with speed v.

Equation of Kinetic Energy: For any particle of mass m and moving with a speed of v, its kinetic energy is defined as

$$T = \frac{1}{2}mv^2$$

The difference between kinetic energy and work is that unlike work, kinetic energy is always non-negative and it is associated with the status of an object where as work can be negative and it is a parameter associated with a process.

Principle of work and energy: This principle states that the total work done in moving a particle from position-1 to position-2 is equal to the change in kinetic energy of the particle.

That is $\sum U_{1\text{-}2} = T_2 - T_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ (Work-Energy Equation).

Alternatively, principle of work and energy is stated as the initial kinetic energy of the particle plus the work done by all external forces acting on the particle while it is moving from the its initial position to its final position is equal to the final kinetic energy of the particle. That is

$$\left\{\begin{matrix}\text{The initial}\\ \text{kinetic energy of}\\ \text{the particle}\end{matrix}\right\} + \left\{\begin{matrix}\text{The sum of the work done}\\ \text{by all external forces}\\ \text{during the process}\end{matrix}\right\} = \left\{\begin{matrix}\text{The final}\\ \text{kinetic energy}\\ \text{of the particle}\end{matrix}\right\}$$

Mathematically, $T_1 + \sum U_{1\text{-}2} = T_2$ where T_1 is the initial kinetic energy, $\sum U_{1\text{-}2}$ is the total work done by all forces and T_2 is the final kinetic energy of the particle.

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Conservation of Energy In general, if a body is subjected to both gravitational and elastic forces, the total *potential energy* can be expressed as a potential function represented as the algebraic sum

$$V = V_g + V_e$$

Here measurement of V depends upon the location of the body with respect to the selected datum.

Realizing that the work of conservative forces can be written as a difference in their potential energies, i.e., $(\Sigma U_{1-2})_{\text{cons}} = V_1 - V_2$, , we can rewrite the principle of work and energy for a rigid body as

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons}} = T_2 + V_2$$

Here $(\Sigma U_{1-2})_{\text{noncons}}$ represents the work of the nonconservative forces such as friction. If this term is zero, then

$$T_1 + V_1 = T_2 + V_2$$

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Procedure for Analysis

The conservation of energy equation is used to solve problems involving *velocity*, *displacement*, and *conservative force systems*. For application it is suggested that the following procedure be used.

Potential Energy.

- Draw two diagrams showing the body located at its initial and final positions along the path.
- If the center of gravity, G, is subjected to a *vertical displacement*, establish a fixed horizontal datum from which to measure the body's gravitational potential energy V_g.
- Data pertaining to the elevation y_G of the body's center of gravity from the datum and the extension or compression of any connecting springs can be determined from the problem geometry and listed on the two diagrams.
- The potential energy is determined from $V = V_g + V_e$. Here $V_g = W y_G$, which can be positive or negative, and $V_e = \frac{1}{2} k s^2$, which is always positive.

Kinetic Energy.

- The kinetic energy of the body consists of two parts, namely translational kinetic energy, $T = \frac{1}{2} m v_G^2$, and rotational kinetic energy, $T = \frac{1}{2} I_G \omega^2$.
- Kinematic diagrams for velocity may be useful for establishing a *relationship* between v_G and ω.

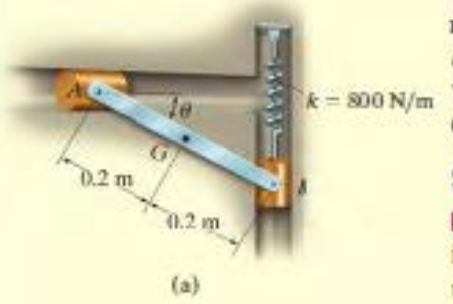
Conservation of Energy.

- Apply the conservation of energy equation $T_1 + V_1 = T_2 + V_2$.

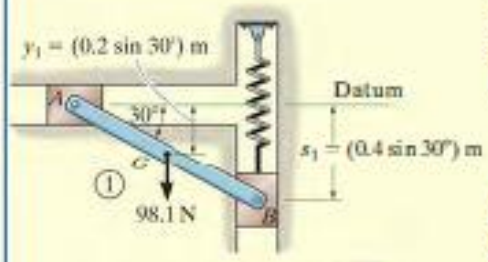
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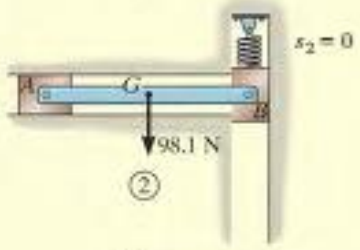
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EXAMPLE

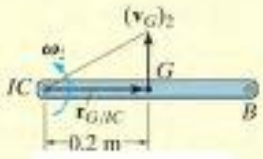


(a)





(b)



(c)

Fig. 18–18

The 10-kg rod AB shown in Fig. 18–18*a* is confined so that its ends move in the horizontal and vertical slots. The spring has a stiffness of $k = 800$ N/m and is unstretched when $\theta = 0°$. Determine the angular velocity of AB when $\theta = 0°$, if the rod is released from rest when $\theta = 30°$. Neglect the mass of the slider blocks.

SOLUTION

Potential Energy. The two diagrams of the rod, when it is located at its initial and final positions, are shown in Fig. 18–18*b*. The datum, used to measure the gravitational potential energy, is placed in line with the rod when $\theta = 0°$.

When the rod is in position 1, the center of gravity G is located *below the datum* so its gravitational potential energy is *negative*. Furthermore, (positive) elastic potential energy is stored in the spring, since it is stretched a distance of $s_1 = (0.4 \sin 30°)$ m. Thus,

$$V_1 = -Wy_1 + \tfrac{1}{2}ks_1^2$$
$$= -(98.1\text{ N})(0.2 \sin 30°\text{ m}) + \tfrac{1}{2}(800\text{ N/m})(0.4 \sin 30°\text{ m})^2 = 6.19\text{ J}$$

When the rod is in position 2, the potential energy of the rod is zero, since the center of gravity G is located at the datum, and the spring is unstretched, $s_2 = 0$. Thus,

$$V_2 = 0$$

Kinetic Energy. The rod is released from rest from position 1, thus $(\mathbf{v}_G)_1 = \boldsymbol{\omega}_1 = \mathbf{0}$, and so

$$T_1 = 0$$

In position 2, the angular velocity is ω_2 and the rod's mass center has a velocity of $(\mathbf{v}_G)_2$. Thus,

$$T_2 = \tfrac{1}{2}m(v_G)_2^2 + \tfrac{1}{2}I_G\omega_2^2$$
$$= \tfrac{1}{2}(10\text{ kg})(v_G)_2^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(10\text{ kg})(0.4\text{ m})^2\right]\omega_2^2$$

Using *kinematics*, $(\mathbf{v}_G)_2$ can be related to $\boldsymbol{\omega}_2$ as shown in Fig. 18–18*c*. At the instant considered, the instantaneous center of zero velocity (IC) for the rod is at point A; hence, $(v_G)_2 = (r_{G/IC})\omega_2 = (0.2\text{ m})\omega_2$. Substituting into the above expression and simplifying (or using $\tfrac{1}{2}I_{IC}\omega_2^2$), we get

$$T_2 = 0.2667\omega_2^2$$

Conservation of Energy.

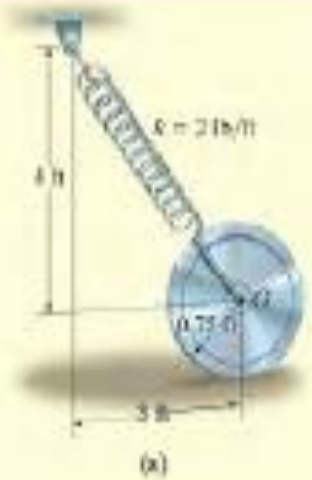
$$\{T_1\} + \{V_1\} = \{T_2\} + \{V_2\}$$
$$\{0\} + \{6.19\text{ J}\} = \{0.2667\omega_2^2\} + \{0\}$$
$$\omega_2 = 4.82\text{ rad/s}\circlearrowleft \qquad \textit{Ans.}$$

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EXAMPLE

The wheel shown in Fig. 18–19a has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. It is attached to a spring which has a stiffness $k = 2$ lb/ft and an unstretched length of 1 ft. If the disk is released from rest in the position shown and rolls without slipping, determine its angular velocity at the instant G moves 3 ft to the left.



(a)

SOLUTION

Potential Energy. Two diagrams of the wheel, when it is at the initial and final positions, are shown in Fig. 18–19b. A gravitational datum is not needed here since the weight is not displaced vertically. From the problem geometry the spring is stretched $s_1 = \left(\sqrt{3^2 + 4^2} - 1\right) = 4$ ft in the initial position, and $s_2 = (4 - 1) = 3$ ft in the final position. Hence,

$$V_1 = \tfrac{1}{2}ks_1^2 = \tfrac{1}{2}(2\text{ lb/ft})(4\text{ ft})^2 = 16\text{ J}$$
$$V_2 = \tfrac{1}{2}ks_2^2 = \tfrac{1}{2}(2\text{ lb/ft})(3\text{ ft})^2 = 9\text{ J}$$



(b)

Kinetic Energy. The disk is released from rest and so $(\mathbf{v}_G)_1 = 0$, $\omega_1 = 0$. Therefore,

$$T_1 = 0$$

Since the instantaneous center of zero velocity is at the ground, Fig. 18–19c, we have

$$T_2 = \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}\left[\left(\frac{30\text{ lb}}{32.2\text{ ft/s}^2}\right)(0.6\text{ ft})^2 + \left(\frac{30\text{ lb}}{32.2\text{ ft/s}^2}\right)(0.75\text{ ft})^2\right]\omega_2^2$$
$$= 0.4297\omega_2^2$$

Conservation of Energy.

$$\{T_1\} + \{V_1\} = \{T_2\} + \{V_2\}$$
$$\{0\} + \{16\text{ J}\} = \{0.4297\omega_2^2\} + \{9\text{ J}\}$$
$$\omega_2 = 4.04\text{ rad/s}) \qquad \textit{Ans.}$$

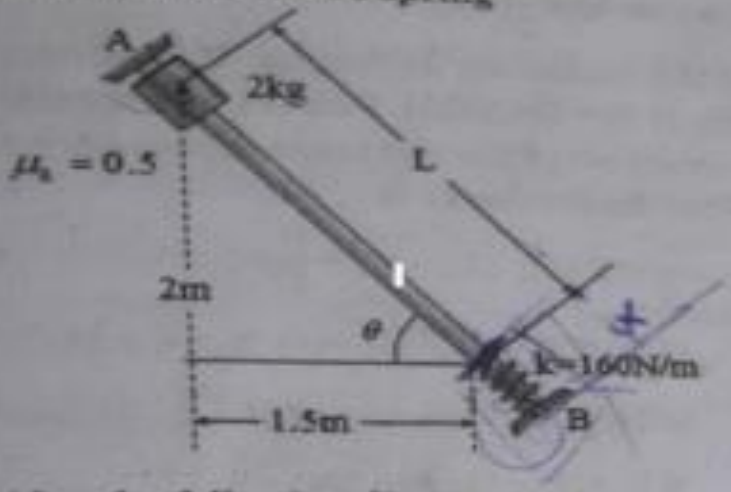


(c)

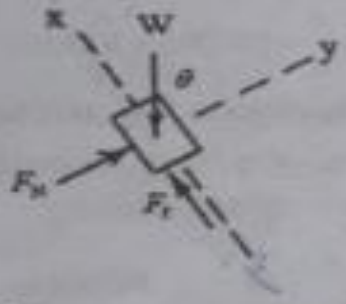
NOTE: If the principle of work and energy were used to solve this problem, then the work of the spring would have to be determined by considering both the change in magnitude and direction of the spring force.

The 2kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. If the coefficient of kinetic friction is $\mu_k = 0.5$, calculate

a) The velocity of the collar as it strikes the spring
b) The maximum deflection x of the spring



Solution: Consider the following diagram.



First identify necessary data:

Length of the rod: By Pythagoras Theorem, the length of the ro found to be $L^2 = (1.5)^2 + (2)^2 = 6.25 \Rightarrow L = 2.5\text{m}$.

Using this, we have $\sin\theta = \frac{2}{2.5} = \frac{4}{5}, \cos\theta = \frac{1.5}{2.5} = \frac{3}{5}$.

Now, calculate the total work done and the kinetic energy at initial point and at the final point.

Work done by frictional force:

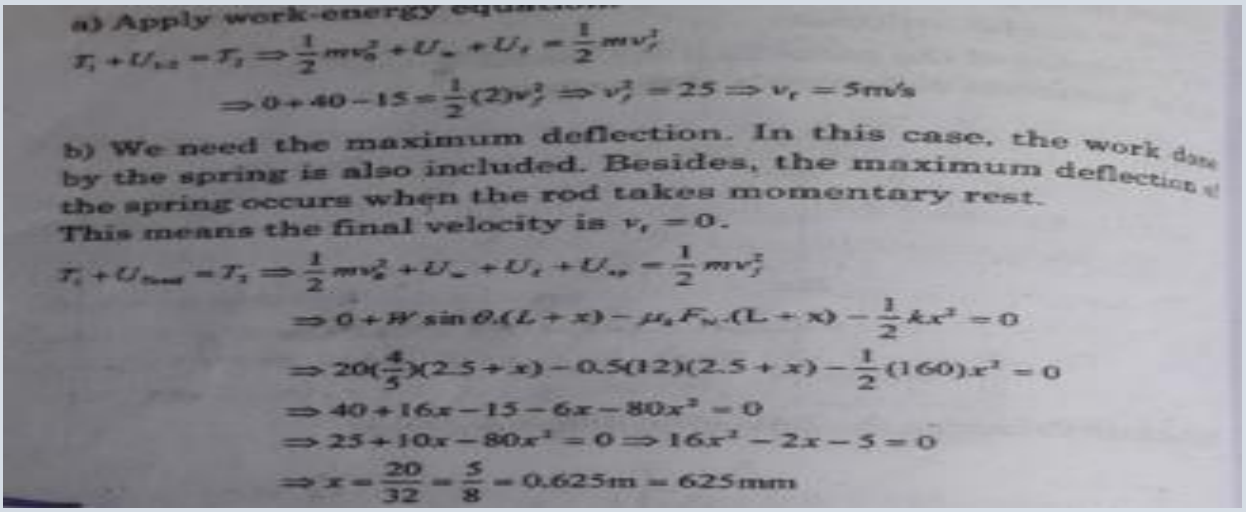
$\sum F_y = 0 \Rightarrow F_N - W_y = 0 \Rightarrow F_N - 20\cos\theta = 0 \Rightarrow F_N - 12 = 0 \Rightarrow F_N = 12\text{N}$

So, the work done is $U_f = -F_f L = -\mu_k F_N L = -0.5(12)(2.5) = -15\text{J}$.

Work done by the weight: $U_w = W\sin\theta L = 20(\frac{4}{5})(2.5) = 4$

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a) Apply work-energy equation:

$$T_1 + U_{1-2} = T_2 \Rightarrow \frac{1}{2}mv_0^2 + U_w + U_s = \frac{1}{2}mv_f^2$$

$$\Rightarrow 0 + 40 - 15 = \frac{1}{2}(2)v_f^2 \Rightarrow v_f^2 = 25 \Rightarrow v_f = 5\text{m/s}$$

b) We need the maximum deflection. In this case, the work done by the spring is also included. Besides, the maximum deflection of the spring occurs when the rod takes momentary rest. This means the final velocity is $v_f = 0$.

$$T_1 + U_{1-max} = T_2 \Rightarrow \frac{1}{2}mv_0^2 + U_w + U_s + U_{sp} = \frac{1}{2}mv_f^2$$

$$\Rightarrow 0 + W\sin\theta(L + x) - \mu_k F_N(L + x) - \frac{1}{2}kx^2 = 0$$

$$\Rightarrow 20\left(\frac{4}{5}\right)(2.5 + x) - 0.5(12)(2.5 + x) - \frac{1}{2}(160)x^2 = 0$$

$$\Rightarrow 40 + 16x - 15 - 6x - 80x^2 = 0$$

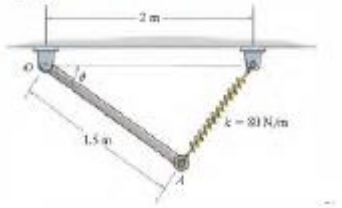
$$\Rightarrow 25 + 10x - 80x^2 = 0 \Rightarrow 16x^2 - 2x - 5 = 0$$

$$\Rightarrow x = \frac{20}{32} = \frac{5}{8} = 0.625\text{m} = 625\text{mm}$$

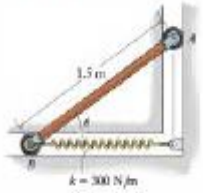
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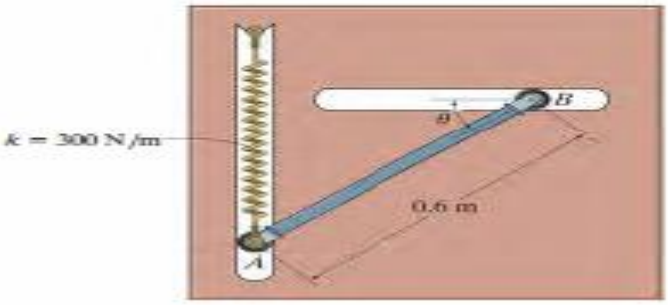
The 30-kg rod is released from rest when $\theta = 0°$. Determine the angular velocity of the rod when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.



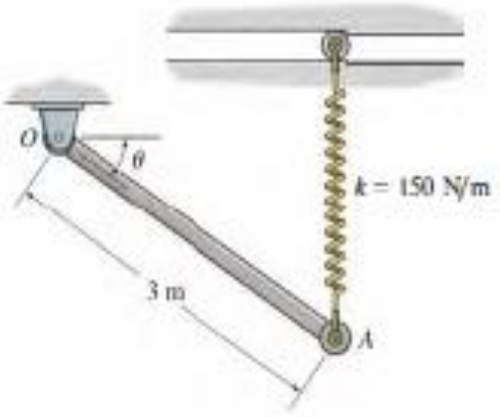
The 30-kg rod is released from rest when $\theta = 45°$. Determine the angular velocity of the rod when $\theta = 0°$. The spring is unstretched when $\theta = 45°$.



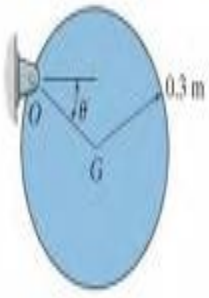
A spring having a stiffness of $k = 300$ N/m is attached to the end of the 15-kg rod, and it is unstretched when $\theta = 0°$. If the rod is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 30°$. The motion is in the vertical plane.



The 60-kg rod OA is released from rest when $\theta = 0°$. Determine its angular velocity when $\theta = 45°$. The spring remains vertical during the motion and is unstretched when $\theta = 0°$.



If the 30-kg disk is released from rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$.



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Linear and Angular Momentum

In this chapter we will use the principles of linear and angular impulse and momentum to solve problems involving force, velocity, and time as related to the planar motion of a rigid body. Before doing this, we will first formalize the methods for obtaining a body's linear and angular momentum, assuming the body is symmetric with respect to an inertial x–y reference plane.

Linear Momentum. The linear momentum of a rigid body is determined by summing vectorially the linear momenta of all the particles of the body, i.e., $\mathbf{L} = \Sigma m_i \mathbf{v}_i$. Since $\Sigma m_i \mathbf{v}_i = m\mathbf{v}_G$ (see Sec. 15.2) we can also write

$$\mathbf{L} = m\mathbf{v}_G$$

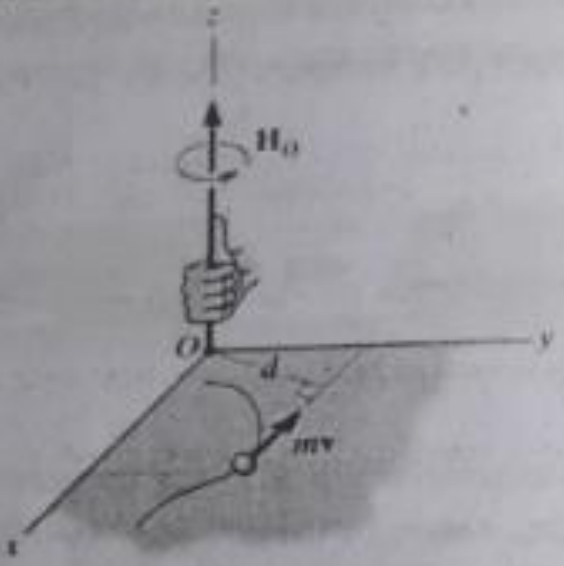
This equation states that the body's linear momentum is a vector quantity having a *magnitude* mv_G, which is commonly measured in units of kg · m/s or slug · ft/s and a *direction* defined by $\mathbf{v}_G$ the velocity of the body's mass center.

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Moment of a force and Angular Momentum

Consider a particle having a linear velocity v as shown.



Angular momentum:

The angular momentum of a particle about point O is defined as the moment of the particle's linear momentum about O.

In short, angular momentum H_o is the moment of momentum.

That is $H_o = Ld = dmv$ where L is the linear momentum and d is the perpendicular distance from O to the line of action of the linear momentum vector $L = mv$.

Vector Equation: In vector form, $H_o = r \times mv$ where r is the position vector from O to any point on the line of action of L.

If $r = r_x i + r_y j + r_z k$, $L = mv_x i + mv_y j + mv_z k$, then the angular momentum becomes

$$H_o = r \times L = r \times mv = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}.$$

Angular impulse: It is defined as the integral of the moment of a force acting on the particle in the time interval from t_1 to t_2.

That is angular impulse $K = \int_{t_1}^{t_2} M_o \, dt$.

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How does the moment caused by a force that we know in statics is related to the angular momentum? In statics we have seen the moment of a force is $\mathbf{M}_o = \mathbf{r} \times \mathbf{F}$. Thus, the angular impulse can be expressed in vector form as $\mathbf{K} = \int_{t_1}^{t_2} \mathbf{M}_o \, dt = \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F}) dt$.

Again, using the definition of the moment about a reference point O, we have $\sum \mathbf{M}_o = \mathbf{r} \times \sum \mathbf{F} = \mathbf{r} \times m\mathbf{v}'$.

Recall derivative of cross product $\frac{d}{dt}(\mathbf{r} \times m\mathbf{v}) = \mathbf{r}' \times m\mathbf{v} + \mathbf{r} \times m\mathbf{v}'$.

Now, take the derivative of angular momentum $\mathbf{H}_o = \mathbf{r} \times m\mathbf{v}$.

That is $\frac{d\mathbf{H}_o}{dt} = \frac{d}{dt}(\mathbf{r} \times m\mathbf{v}) \Rightarrow \mathbf{H}'_o = \mathbf{r}' \times m\mathbf{v} + \mathbf{r} \times m\mathbf{v}'$.

But observe that $\mathbf{r}' \times m\mathbf{v} = 0$ (Do you see why it is zero?)

Therefore, we have the relation, $\sum \mathbf{M}_o = \mathbf{r} \times m\mathbf{v}' \Rightarrow \sum \mathbf{M}_o = \mathbf{H}'_o$.

This means the resultant moment of all forces acting on a particle about point O is equal to the time rate of change of the particle's angular momentum $\mathbf{H}_o$ about the same point.

Angular impulse:

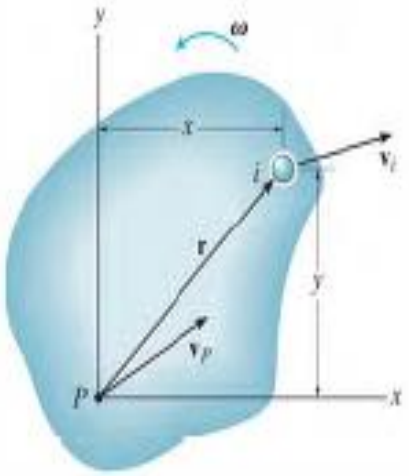
As we have derived above, the relation between the moment $\sum \mathbf{M}_o$ of a force about point O and angular momentum is given by $\sum \mathbf{M}_o = \mathbf{H}'_o$. Then,

$$\sum \mathbf{M}_o = \mathbf{H}'_o \Rightarrow \sum \mathbf{M}_o = \frac{d\mathbf{H}_o}{dt} \Rightarrow \sum \mathbf{M}_o \, dt = d\mathbf{H}_o$$

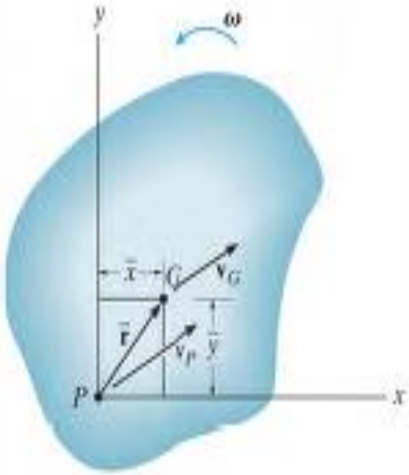
$$\Rightarrow \sum \int_{t_1}^{t_2} \mathbf{M}_o \, dt = \int_{(\mathbf{H}_o)_1}^{(\mathbf{H}_o)_2} d\mathbf{H}_o \Rightarrow \sum \int_{t_1}^{t_2} \mathbf{M}_o \, dt = (\mathbf{H}_o)_2 - (\mathbf{H}_o)_1$$

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(a)



(b)

Angular Momentum. Consider the body in Fig. 19–1*a*, which is subjected to general plane motion. At the instant shown, the arbitrary point P has a known velocity $\mathbf{v}_P$, and the body has an angular velocity $\boldsymbol{\omega}$. Therefore the velocity of the ith particle of the body is

$$\mathbf{v}_i = \mathbf{v}_P + \mathbf{v}_{i/P} = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}$$

The angular momentum of this particle about point P is equal to the "moment" of the particle's linear momentum about P, Fig. 19–1*a*. Thus,

$$(\mathbf{H}_P)_i = \mathbf{r} \times m_i \mathbf{v}_i$$

Expressing $\mathbf{v}_i$ in terms of $\mathbf{v}_P$ and using Cartesian vectors, we have

$$(H_P)_i \mathbf{k} = m_i(x\mathbf{i} + y\mathbf{j}) \times [(v_P)_x \mathbf{i} + (v_P)_y \mathbf{j} + \omega \mathbf{k} \times (x\mathbf{i} + y\mathbf{j})]$$

$$(H_P)_i = -m_i y (v_P)_x + m_i x (v_P)_y + m_i \omega r^2$$

Letting $m_i \rightarrow dm$ and integrating over the entire mass m of the body, we obtain

$$H_P = -\left(\int_m y\, dm\right)(v_P)_x + \left(\int_m x\, dm\right)(v_P)_y + \left(\int_m r^2\, dm\right)\omega$$

Here H_P represents the angular momentum of the body about an axis (the z axis) perpendicular to the plane of motion that passes through point P. Since $\bar{y}m = \int y\, dm$ and $\bar{x}m = \int x\, dm$ the integrals for the first and second terms on the right are used to locate the body's center of mass G with respect to P, Fig. 19–1*b*. Also, the last integral represents the body's moment of inertia about point P. Thus,

$$H_P = -\bar{y}m(v_P)_x + \bar{x}m(v_P)_y + I_P\omega$$

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Principle of Impulse and Momentum

Like the case for particle motion, the principle of impulse and momentum for a rigid body can be developed by *combining* the equation of motion with kinematics. The resulting equation will yield a *direct solution to problems involving force, velocity, and time.*

Principle of Linear Impulse and Momentum. The equation of translational motion for a rigid body can be written as $\Sigma \mathbf{F} = m\mathbf{a}_G = m(d\mathbf{v}_G/dt)$. Since the mass of the body is constant,

$$\Sigma \mathbf{F} = \frac{d}{dt}(m\mathbf{v}_G)$$

Multiplying both sides by dt and integrating from $t = t_1$, $\mathbf{v}_G = (\mathbf{v}_G)_1$ to $t = t_2$, $\mathbf{v}_G = (\mathbf{v}_G)_2$ yields

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m(\mathbf{v}_G)_2 - m(\mathbf{v}_G)_1$$

This equation is referred to as the *principle of linear impulse and momentum.* It states that the sum of all the impulses created by the *external force system* which acts on the body during the time interval t_1 to t_2 is equal to the change in the linear momentum of the body during this time interval, Fig. 19–4.

Principle of Angular Impulse and Momentum. If the body has *general plane motion* then $\Sigma M_G = I_G\alpha = I_G(d\omega/dt)$. Since the moment of inertia is constant,

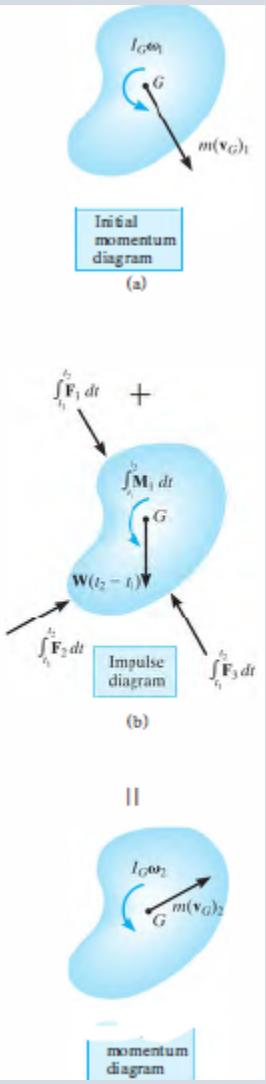
$$\Sigma M_G = \frac{d}{dt}(I_G\omega)$$

Multiplying both sides by dt and integrating from $t = t_1$, $\omega = \omega_1$ to $t = t_2$, $\omega = \omega_2$ gives

$$\Sigma \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2 - I_G\omega_1$$

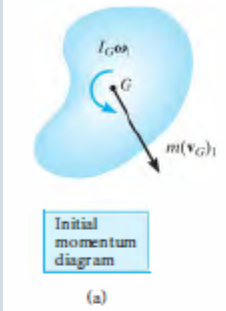
In a similar manner, for *rotation about a fixed axis* passing through point O, Eq. 17–16 ($\Sigma M_O = I_O\alpha$) when integrated becomes

$$\Sigma \int_{t_1}^{t_2} M_O\, dt = I_O\omega_2 - I_O\omega_1$$



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Initial momentum diagram

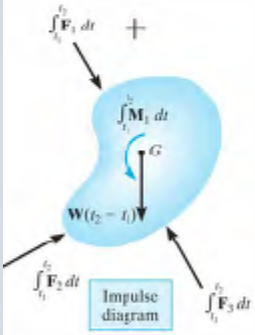
(a)

To summariz. these concepts, if motion occurs in the x–y plane, the following *three s alar equations* can be written to describe the *planar motion* of the boc

$$(v_{Gx})_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_{Gy})_2$$

$$I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2$$



(b)

The terms in these equations can be shown graphically by drawing a set of impulse and momentum diagrams for the body, Fig. 19–4. Note that the linear momentum $m\mathbf{v}_G$ is applied at the body's mass center, Figs. 19–4*a* and 19–4*c*; whereas the angular momentum $I_G\boldsymbol{\omega}$ is a free vector, and therefore, like a couple moment, it can be applied at any point on the body. When the impulse diagram is constructed, Fig. 19–4*b*, the forces $\mathbf{F}$ and moment $\mathbf{M}$ vary with time, and are indicated by the integrals. However, if $\mathbf{F}$ and $\mathbf{M}$ are *constant* integration of the impulses yields $\mathbf{F}(t_2 - t_1)$ and $\mathbf{M}(t_2 - t_1)$, respectively. Such is the case for the body's weight $\mathbf{W}$, Fig. 19–4*b*.

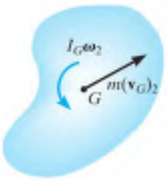
Equations 19–14 can also be applied to an entire system of connected bodies rather than to each body separately. This eliminates the need to include interaction impulses which occur at the connections since they are *internal* to the system. The resultant equations may be written in symbolic form as

$$\left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{x1} + \left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{impulse}\end{matrix}\right)_{x(1-2)} = \left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{x2}$$

$$\left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{y1} + \left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{impulse}\end{matrix}\right)_{y(1-2)} = \left(\Sigma \begin{matrix}\text{syst. linear}\\ \text{momentum}\end{matrix}\right)_{y2}$$

$$\left(\Sigma \begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{O1} + \left(\Sigma \begin{matrix}\text{syst. angular}\\ \text{impulse}\end{matrix}\right)_{O(1-2)} = \left(\Sigma \begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{O2}$$

=



Final momentum diagram

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Procedure For Analysis

Impulse and momentum principles are used to solve kinetic problems that involve *velocity*, *force*, and *time* since these terms are involved in the formulation.

Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the free-body diagram in order to account for all the forces and couple moments that produce impulses on the body.
- The direction and sense of the initial and final velocity of the body's mass center, $\mathbf{v}_G$, and the body's angular velocity $\boldsymbol{\omega}$ should be established. If any of these motions is unknown, assume that the sense of its components is in the direction of the positive inertial coordinates.
- Compute the moment of inertia I_G or I_O.
- As an alternative procedure, draw the impulse and momentum diagrams for the body or system of bodies. Each of these diagrams represents an outlined shape of the body which graphically accounts for the data required for each of the three terms in Eqs. 19–14 or 19–15, Fig. 19–4. These diagrams are particularly helpful in order to visualize the "moment" terms used in the principle of angular impulse and momentum, if application is about the IC or another point other than the body's mass center G or a fixed point O.

Principle of Impulse and Momentum.

- Apply the three scalar equations of impulse and momentum.
- The angular momentum of a rigid body rotating about a fixed axis is the moment of $m\mathbf{v}_G$ plus $I_G\boldsymbol{\omega}$ about the axis. This is equal to $H_O = I_O\omega$, where I_O is the moment of inertia of the body about the axis.
- All the forces acting on the body's free-body diagram will create an impulse; however, some of these forces will do no work.
- Forces that are functions of time must be integrated to obtain the impulse.
- The principle of angular impulse and momentum is often used to eliminate unknown impulsive forces that are parallel or pass through a common axis, since the moment of these forces is zero about this axis.

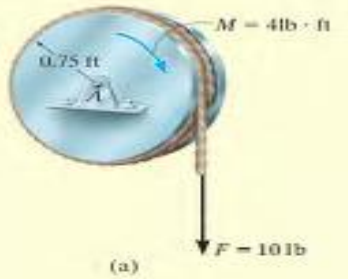
Kinematics.

- If more than three equations are needed for a complete solution, it may be possible to relate the velocity of the body's mass center to the body's angular velocity using *kinematics*. If the motion appears to be complicated, kinematic (velocity) diagrams may be helpful in obtaining the necessary relation.

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EXAMPLE



(a)

The 20-lb disk shown in Fig. 19–5*a* is acted upon by a constant couple moment of 4 lb · ft and a force of 10 lb which is applied to a cord wrapped around its periphery. Determine the angular velocity of the disk two seconds after starting from rest. Also, what are the force components of reaction at the pin?

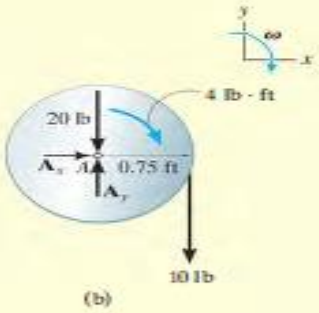
SOLUTION

Since angular velocity, force, and time are involved in the problems, we will apply the principles of impulse and momentum to the solution.

Free-Body Diagram. Fig. 19–5*b*. The disk's mass center does not move; however, the loading causes the disk to rotate clockwise.

The moment of inertia of the disk about its fixed axis of rotation is

$$I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{20\text{ lb}}{32.2\text{ ft/s}^2}\right)(0.75\text{ ft})^2 = 0.1747\text{ slug}\cdot\text{ft}^2$$



(b)

Principle of Impulse and Momentum.

$$(\stackrel{+}{\rightarrow}) \qquad m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$

$$0 + A_x(2\text{ s}) = 0$$

$$(+\uparrow) \qquad m(v_{Ay})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Ay})_2$$

$$0 + A_y(2\text{ s}) - 20\text{ lb}(2\text{ s}) - 10\text{ lb}(2\text{ s}) = 0$$

$$(\circlearrowleft +) \qquad I_A\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + 4\text{ lb}\cdot\text{ft}(2\text{ s}) + [10\text{ lb}(2\text{ s})](0.75\text{ ft}) = 0.1747\omega_2$$

Solving these equations yields

$$A_x = 0 \qquad \textit{Ans}$$

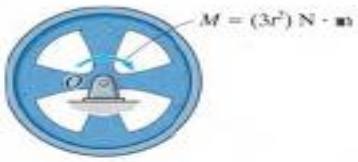
$$A_y = 30\text{ lb} \qquad \textit{Ans}$$

$$\omega_2 = 132\text{ rad/s}\circlearrowright \qquad \textit{Ans}$$

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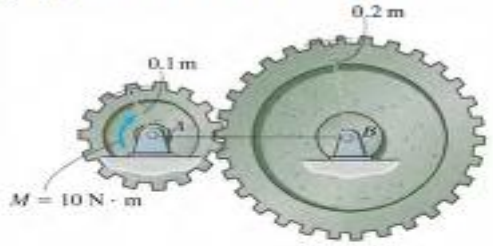
The 60-kg wheel has a radius of gyration about its center O of $k_O = 300$ mm. If it is subjected to a couple moment of $M = (3t^2)$ N·m, where t is in seconds, determine the angular velocity of the wheel when $t = 4$ s, starting from rest.



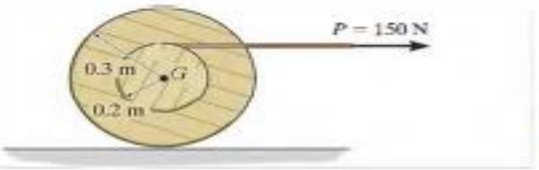
The 300-kg wheel has a radius of gyration about its mass center O of $k_O = 400$ mm. If the wheel is subjected to a couple moment of $M = 300$ N·m, determine its angular velocity 6 s after it starts from rest and no slipping occurs. Also, determine the friction force that develops between the wheel and the ground.



Gears A and B of mass 10 kg and 50 kg have radii of gyration about their respective mass centers of $k_A = 80$ mm and $k_B = 150$ mm. If gear A is subjected to the couple moment $M = 10$ N·m, determine the angular velocity of gear B 5 s after it starts from rest.



The 50-kg spool is subjected to a horizontal force of $P = 150$ N. If the spool rolls without slipping, determine its angular velocity 3 s after it starts from rest. The radius of gyration of the spool about its center of mass is $k_G = 175$ mm.



Power

Power is defined as the time rate of energy transfer. If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the average power during this interval is defined as

$$P_{av} = \frac{W}{\Delta t}$$

The instantaneous power P is defined as the limiting value of the average power as Δt approaches zero:

$$P = \lim_{\Delta t \to o} \frac{W}{\Delta t} = \frac{W}{t}$$

But, $W = \boldsymbol{F}.\triangle \boldsymbol{r}$

$$P = \frac{W}{\Delta t} = \vec{F}.\frac{\Delta \vec{r}}{\Delta t} = \vec{F}.\vec{v}$$

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is

$$P = \frac{E}{t}$$

The SI unit of power is joules per second (J/s), also called the watt (W) (after James Watt):

$$1W = \frac{1J}{s} = 1kg.\frac{m^2}{s^3}$$

A unit of power in the U.S. customary system is the horsepower (hp):

$$1hp = 746W$$

A unit of energy (or work) can now be defined in terms of the unit of power. One kilowatt-hour (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s. The amount of energy represented by 1 kWh is:

$$1kWh = (10^3W)(3600s) = 3.6 \times 10^6J$$