MA : MATHEMATICS

Duration: Three Hours Maximum Marks: 100

Read the following instructions carefully.

1. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
2. Take out the **O**ptical **R**esponse **S**heet (**ORS)** from this Question Booklet **without breaking the seal** and read the instructions printed on the **ORS** carefully.
3. On the right half of the **ORS,** using ONLY a **black ink ball point pen**, (i) darken the bubble corresponding to your test paper code and the appropriate bubble under each digit of your registration number and (ii) write your registration number, your name and name of the examination centre and put your signature at the specified location.
4. This Question Booklet contains **20** pages including blank pages for rough work. After you are permitted to open the seal, please check all pages and report discrepancies, if any, to the invigilator.
5. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Each question has only **one** correct answer. Questions must be answered on the left hand side of the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using ONLY a **black ink ball point pen** against the question number. **For each question darken the bubble of the correct answer**. More than one answer bubbled against a question will be treated as an incorrect response.
6. Since bubbles darkened by the black ink ball point pen **cannot** be erased, candidates should darken the bubbles in the **ORS very carefully**.
7. Questions Q.1 – Q.25 carry 1 mark each. Questions Q.26 – Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
8. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1 mark each, and questions Q.61 – Q.65 carry 2 marks each.
9. Unattempted questions will result in zero mark and wrong answers will result in **NEGATIVE** marks. For all 1 mark questions, ⅓ mark will be deducted for each wrong answer. For all 2 marks questions, ⅔ mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question.
10. Calculator is allowed whereas charts, graph sheets or tables are **NOT** allowed in the examination hall.
11. Rough work can be done on the question paper itself. Blank pages are provided at the end of the question paper for rough work.
12. Before the start of the examination, write your name and registration number in the space provided below using a black ink ball point pen.

Name								
Registration Number	**MA**							

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Notations and Symbols used

$\mathbb{R}$: Set of all real numbers

$\mathbb{C}$: Set of all complex numbers

$\mathbb{Z}$: Set of all integers

F : A field

$\mathbb{C}^n$: The set of all n-tuples of complex numbers

F^n : The set of all n-tuples over F

$R_1 \times R_2 \times ... \times R_n$: Cartesian product of rings $R_1, R_2, ..., R_n$

$D_x f(x, y)$: Partial derivative with respect to x.

$N(\mu, \sigma^2)$: Normal distribution with mean μ and variance σ^2

$E(X)$: Expectation of X

$Cov(X, Y)$: Covariance between X and Y

S_n : The group of all permutations on n symbols

P_n : The set of all polynomials of degree at most n

C_n : Cyclic Group of Order n

Z(G) : Centre of the Group G

$i = \sqrt{-1}$

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Q. 1 – Q. 25 carry one mark each.

Q.1 The straight lines $L_1 : x = 0,\ L_2 : y = 0$ and $L_3 : x + y = 1$ are mapped by the transformation $w = z^2$ into the curves C_1, C_2 and C_3 respectively. The angle of intersection between the curves at $w = 0$ is

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Q.2 In a topological space, which of the following statements is **NOT** always true :

(A) Union of any finite family of compact sets is compact.
(B) Union of any family of closed sets is closed.
(C) Union of any family of connected sets having a non empty intersection is connected.
(D) Union of any family of dense subsets is dense.

Q.3 Consider the following statements:

P: The family of subsets $\left\{A_n = \left(-\frac{1}{n}, \frac{1}{n}\right), n = 1, 2, \ldots\right\}$ satisfies the finite intersection property.

Q: On an infinite set X, a metric $d : X \times X \to R$ is defined as $d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$.

The metric space (X, d) is compact.

R: In a Frechet (T_1) topological space, every finite set is closed.

S: If $f : R \to X$ is continuous, where R is given the usual topology and (X, τ) is a Hausdorff (T_2) space, then f is a one-one function.

Which of the above statements are correct?

(A) P and R (B) P and S (C) R and S (D) Q and S

Q.4 Let H be a Hilbert space and $S^{\perp}$ denote the orthogonal complement of a set $S \subseteq H$. Which of the following is **INCORRECT**?

(A) For $S_1, S_2 \subseteq H; S_1 \subseteq S_2 \Rightarrow S_1^{\perp} \subseteq S_2^{\perp}$ (B) $S \subseteq (S^{\perp})^{\perp}$
(C) $\{0\}^{\perp} = H$ (D) $S^{\perp}$ is always closed.

Q.5 Let H be a complex Hilbert space, $T : H \to H$ be a bounded linear operator and let T^* denote the adjoint of T. Which of the following statements are always **TRUE**?

P: $\forall x, y \in H, \langle Tx, y\rangle = \langle x, T^* y\rangle$ Q: $\forall x, y \in H, \langle x, Ty\rangle = \langle T^* x, y\rangle$

R: $\forall x, y \in H, \langle x, Ty\rangle = \langle x, T^* y\rangle$ S: $\forall x, y \in H, \langle Tx, Ty\rangle = \langle T^* x, T^* y\rangle$

(A) P and Q (B) P and R (C) Q and S (D) P and S

Q.6 Let $X = \{a, b, c\}$ and let $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology defined on X. Then which of the following statements are **TRUE**?

P: $(X, \Im)$ is a Hausdorff space. Q: $(X, \Im)$ is a regular space.

R: $(X, \Im)$ is a normal space. S: $(X, \Im)$ is a connected space.

(A) P and Q (B) Q and R (C) R and S (D) P and S

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Q.7 Consider the statements

P: If X is a normed linear space and $M \subseteq X$ is a subspace, then the closure $\bar{M}$ is also a subspace of X.

Q: If X is a Banach space and $\sum x_n$ is an absolutely convergent series in X, then $\sum x_n$ is convergent.

R: Let M_1 and M_2 be subspaces of an inner product space such that $M_1 \cap M_2 = \{0\}$. Then $\forall m_1 \in M_1,\ m_2 \in M_2;\ \|m_1 + m_2\|^2 = \|m_1\|^2 + \|m_2\|^2$.

S: Let $f : X \to Y$ be a linear transformation from the Banach Space X into the Banach space Y. If f is continuous, then the graph of f is always compact.

The correct statements amongst the above are:

(A) P and R only (B) Q and R only (C) P and Q only (D) R and S only

Q.8 A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{5} e^{-\frac{3}{5}x}, & x > 0 \\ 0, & x \le 0. \end{cases}$$

The probability density function of $Y = 3X + 2$ is

(A) $f(y) = \begin{cases} \frac{1}{5} e^{-\frac{1}{5}(y-2)}, & y > 2 \\ 0, & y \le 2 \end{cases}$ (B) $f(y) = \begin{cases} \frac{2}{5} e^{-\frac{2}{5}(y-2)}, & y > 2 \\ 0, & y \le 2 \end{cases}$

(C) $f(y) = \begin{cases} \frac{3}{5} e^{-\frac{3}{5}(y-2)}, & y > 2 \\ 0, & y \le 2 \end{cases}$ (D) $f(y) = \begin{cases} \frac{4}{5} e^{-\frac{4}{5}(y-2)}, & y > 2 \\ 0, & y \le 2 \end{cases}$

Q.9 A simple random sample of size 10 from $N(\mu, \sigma^2)$ gives 98% confidence interval (20.49, 23.51). Then the null hypothesis $H_0 : \mu = 20.5$ against $H_A : \mu \neq 20.5$

(A) can be rejected at 2% level of significance
(B) cannot be rejected at 5% level of significance
(C) can be rejected at 10% level of significance
(D) cannot be rejected at any level of significance

Q.10 For the linear programming problem

Maximize $z = x_1 + 2x_2 + 3x_3 - 4x_4$

Subject to
$$2x_1 + 3x_2 - x_3 - x_4 = 15$$
$$6x_1 + x_2 + x_3 - 3x_4 = 21$$
$$8x_1 + 2x_2 + 3x_3 - 4x_4 = 30$$
$$x_1,\ x_2,\ x_3,\ x_4 \ge 0,$$

$x_1 = 4,\ x_2 = 3,\ x_3 = 0,\ x_4 = 2$ is

(A) an optimal solution
(B) a degenerate basic feasible solution
(C) a non-degenerate basic feasible solution
(D) a non-basic feasible solution

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Q.11 Which one of the following statements is **TRUE**?

(A) A convex set cannot have infinite many extreme points.
(B) A linear programming problem can have infinite many extreme points.
(C) A linear programming problem can have exactly two different optimal solutions.
(D) A linear programming problem can have a non-basic optimal solution.

Q.12 Let $\alpha = e^{2\pi i/5}$ and the matrix

$$M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}.$$

Then the trace of the matrix $I + M + M^2$ is

(A) -5 (B) 0 (C) 3 (D) 5

Q.13 Let $V = \mathbb{C}^2$ be the vector space over the field of complex numbers and $B = \{(1,i),(i,1)\}$ be a given ordered basis of V. Then for which of the following, $B^* = \{f_1, f_2\}$ is a dual basis of B over $\mathbb{C}$?

(A) $f_1(z_1, z_2) = \frac{1}{2}(z_1 - iz_2)$, $f_2(z_1, z_2) = \frac{1}{2}(z_1 + iz_2)$

(B) $f_1(z_1, z_2) = \frac{1}{2}(z_1 + iz_2)$, $f_2(z_1, z_2) = \frac{1}{2}(iz_1 + z_2)$

(C) $f_1(z_1, z_2) = \frac{1}{2}(z_1 - iz_2)$, $f_2(z_1, z_2) = \frac{1}{2}(-iz_1 + z_2)$

(D) $f_1(z_1, z_2) = \frac{1}{2}(z_1 + iz_2)$, $f_2(z_1, z_2) = \frac{1}{2}(-iz_1 - z_2)$

Q.14 Let $R = \mathbb{Z}\times\mathbb{Z}\times\mathbb{Z}$ and $I = \mathbb{Z}\times\mathbb{Z}\times\{0\}$. Then which of the following statement is correct?

(A) I is a maximal ideal but not a prime ideal of R.
(B) I is a prime ideal but not a maximal ideal of R.
(C) I is both maximal ideal as well as a prime ideal of R.
(D) I is neither a maximal ideal nor a prime ideal of R.

Q.15 The function $u(r, \theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2$$

subject to the conditions $u(e, \theta) = 1$, $u(e^2, \theta) = 0$ is

(A) $\ln(e/r)$ (B) $\ln(e/r^2)$ (C) $\ln(e^2/r)$ (D) $\sum_{n=1}^{\infty}\left(\frac{r - e^2}{e - e^2}\right)\sin n\theta$

Q.16 The functional

$$\int_0^1 \left(y'^2 + (y + 2y')y'' + kxyy' + y^2\right)dx, \; y(0) = 0, y(1) = 1, y'(0) = 2, y'(1) = 3$$

is path independent if k equals

(A) 1 (B) 2 (C) 3 (D) 4

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Q.17 If a transformation $y = uv$ transforms the given differential equation $f(x)y'' - 4f'(x)y' + g(x)y = 0$ into the equation of the form $v'' + h(x)v = 0$, then u must be

(A) $1/f^2$ (B) xf (C) $1/2f$ (D) f^2

Q.18 The expression $\frac{1}{D_x^2 - D_y^2}\sin(x-y)$ is equal to

(A) $-\frac{x}{2}\cos(x-y)$ (B) $-\frac{x}{2}\sin(x-y)+\cos(x-y)$

(C) $-\frac{x}{2}\cos(x-y)+\sin(x-y)$ (D) $\frac{3x}{2}\sin(x-y)$

Q.19 The function $\phi(x)$ satisfying the integral equation

$$\int_0^x e^{x-\xi}\phi(\xi)\,d\xi = \frac{x^2}{2}$$

is

(A) $\frac{x^2}{2}$ (B) $x+\frac{x^2}{2}$ (C) $x-\frac{x^2}{2}$ (D) $1+\frac{x^2}{2}$

Q.20 Given the data:

x	1	2	3	4	5
y	–1	2	–3	4	–5

If the derivative of $y(x)$ is approximated as: $y'(x_k) \approx \frac{1}{h}(\Delta y_k + \frac{1}{2}\Delta^2 y_k - \frac{1}{4}\Delta^3 y_k)$, then the value of $y'(2)$ is

(A) 4 (B) 8 (C) 12 (D) 16

Q.21 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then A^{50} is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

Q.22 If $y = \sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be a solution of the differential equation

$$x^2y'' - xy' - 3(1+x^2)y = 0,$$

then the values of r are

(A) 1 and 3 (B) –1 and 3 (C) 1 and –3 (D) –1 and –3

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Q.23 Let the linear transformation $T : F^2 \to F^3$ be defined by $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$. Then the nullity of T is

(A) 0 (B) 1 (C) 2 (D) 3

Q.24 The approximate eigenvalue of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

obtained after two iterations of Power method, with the initial vector $[1\ 1\ 1]^T$, is

(A) 7.768 (B) 9.468 (C) 10.548 (D) 19.468

Q.25 The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of bisection method, is

(A) 0.25 (B) 0.50 (C) 0.75 (D) 0.65

Q. 26 to Q. 55 carry two marks each.

Q.26 Let $\int_C \left[\frac{1}{(z-2)^4} - \frac{(a-2)^2}{z} + 4\right] dz = 4\pi$, where the close curve C is the triangle having vertices at i, $\left(\frac{-1-i}{\sqrt{2}}\right)$ and $\left(\frac{1-i}{\sqrt{2}}\right)$, the integral being taken in anti-clockwise direction. Then one value of a is

(A) $1+i$ (B) $2+i$ (C) $3+i$ (D) $4+i$

Q.27 The Lebesgue measure of the set $A = \left\{0 < x \le 1 : x \sin\left(\frac{\pi}{2x}\right) \ge 0\right\}$ is

(A) 0 (B) 1 (C) ln2 (D) $1 - \ln\sqrt{2}$

Q.28 Which of the following statements are **TRUE**?

P : The set $\{x \in R : |\cos x| \le \frac{1}{2}\}$ is compact.

Q : The set $\{x \in R : \tan x \text{ is not differentiable}\}$ is complete.

R : The set $\{x \in R : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \text{ is convergent}\}$ is bounded.

S : The set $\{x \in R : f(x) = \cos x \text{ has a local maxima}\}$ is closed.

(A) P and Q (B) R and S (C) Q and S (D) P and S

Q.29 If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, \quad x = 1, 2, 3, \ldots,$$

then $E(X)$ is

(A) 2/9 (B) 2/3 (C) 1 (D) 3/2

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Q.30 The probability density function of the random variable X is

$$f(x)=\begin{cases}\frac{1}{\lambda}e^{-x/\lambda}, & x>0\\ 0, & x\le 0,\end{cases}$$

where $\lambda>0$. For testing the hypothesis $H_0:\lambda=3$ against $H_A:\lambda=5$, a test is given as "Reject H_0 if $X\ge 4.5$". The probability of type I error and power of this test are, respectively,

(A) 0.1353 and 0.4966 (B) 0.1827 and 0.379
(C) 0.2021 and 0.4493 (D) 0.2231 and 0.4066

Q.31 The order of the smallest possible non trivial group containing elements x and y such that $x^7=y^2=e$ and $yx=x^4y$ is

(A) 1 (B) 2 (C) 7 (D) 14

Q.32 The number of 5-Sylow subgroup(s) in a group of order 45 is

(A) 1 (B) 2 (C) 3 (D) 4

Q.33 The solution of the initial value problem

$$y''+2y'+10y=6\,\delta(t),\qquad y(0)=0,\quad y'(0)=0,$$

where $\delta(t)$ denotes the Dirac-delta function, is

(A) $2e^t\sin 3t$ (B) $6e^t\sin 3t$ (C) $2e^{-t}\sin 3t$ (D) $6e^{-t}\sin 3t$

Q.34 Let $\omega=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$, $M=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}$, $N=\begin{pmatrix}\omega & 0\\ 0 & \omega^2\end{pmatrix}$ and $G=\langle M,N\rangle$ be the group generated by the matrices M and N under matrix multiplication. Then

(A) $G/Z(G)\cong C_6$ (B) $G/Z(G)\cong S_3$ (C) $G/Z(G)\cong C_2$ (D) $G/Z(G)\cong C_4$

Q.35 The flux of the vector field $\vec{u}=x\hat{i}+y\hat{j}+z\hat{k}$ flowing out through the surface of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ a>b>c>0,$$

is

(A) πabc (B) $2\pi abc$ (C) $3\pi abc$ (D) $4\pi abc$

Q.36 The integral surface satisfying the partial differential equation $\frac{\partial z}{\partial x}+z^2\frac{\partial z}{\partial y}=0$ and passing through the straight line $x=1, y=z$ is

(A) $(x-1)z+z^2=y^2$ (B) $x^2+y^2-z^2=1$
(C) $(y-z)x+x^2=1$ (D) $(x-1)z^2+z=y$

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Q.37 The diffusion equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\ u=u(x,t),\quad u(0,t)=0=u(\pi,t),\quad u(x,0)=\cos x\sin 5x$$

admits the solution

(A) $\frac{e^{-36t}}{2}\left[\sin 6x+e^{20t}\sin 4x\right]$ (B) $\frac{e^{-36t}}{2}\left[\sin 4x+e^{20t}\sin 6x\right]$

(C) $\frac{e^{-20t}}{2}\left[\sin 3x+e^{15t}\sin 5x\right]$ (D) $\frac{e^{-36t}}{2}\left[\sin 5x+e^{20t}\sin x\right]$

Q.38 Let $f(x)$ and $xf(x)$ be the particular solutions of a differential equation

$$y''+R(x)y'+S(x)y=0.$$

Then the solution of the differential equation $y''+R(x)y'+S(x)y=f(x)$ is

(A) $y=\left(-\frac{x^2}{2}+\alpha x+\beta\right)f(x)$ (B) $y=\left(\frac{x^2}{2}+\alpha x+\beta\right)f(x)$

(C) $y=(-x^2+\alpha x+\beta)f(x)$ (D) $y=(x^3+\alpha x+\beta)f(x)$

Q.39 Let the Legendre equation $(1-x^2)y''-2xy'+n(n+1)y=0$ have n^{th} degree polynomial solution $y_n(x)$ such that $y_n(1)=3$. If $\int_{-1}^{1}\left(y_n^2(x)+y_{n-1}^2(x)\right)dx=\frac{144}{15}$, then n is

(A) 1 (B) 2 (C) 3 (D) 4

Q.40 The maximum value of the function $f(x,y,z)=xyz$ subject to the constraint $xy+yz+zx-a=0, a>0$ is

(A) $a^{\frac{3}{2}}$ (B) $(a/3)^{3/2}$ (C) $(3/a)^{3/2}$ (D) $(3a/2)^{3/2}$

Q.41 The functional $\int_0^1\left(y'^2+4y^2+8ye^x\right)dx,\quad y(0)=-\frac{4}{3},\quad y(1)=-\frac{4e}{3}$ possesses :

(A) strong minima on $y=-\frac{1}{3}e^x$ (B) strong minima on $y=-\frac{4}{3}e^x$

(C) weak maxima on $y=-\frac{1}{3}e^x$ (D) strong maxima on $y=-\frac{4}{3}e^x$

Q.42 A particle of mass m is constrained to move on a circle with radius a which itself is rotating about its vertical diameter with a constant angular velocity ω. Assume that the initial angular velocity is zero and g is the acceleration due to gravity. If θ be the inclination of the radius vector of the particle with the axis of rotation and $\dot{\theta}$ denotes the derivative of θ with respect to t, then the Lagrangian of this system is

(A) $\frac{1}{2}ma^2(\dot{\theta}^2+\omega^2\sin^2\theta)+mga\cos\theta$ (B) $\frac{1}{2}ma^2(\dot{\theta}^2+2\omega\sin\theta)-mga\sin\theta$

(C) $\frac{1}{2}ma^2(\dot{\theta}^2+2\omega^2\cos\theta)-mga\sin\theta$ (D) $\frac{1}{2}ma^2(\dot{\theta}^2+\omega\sin 2\theta)+mga\sin\theta$

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Q.43 For the matrix

$$M = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix},$$

which of the following statements are correct?

P : M is skew-Hermitian and iM is Hermitian
Q : M is Hermitian and iM is skew Hermitian
R : eigenvalues of M are real
S : eigenvalues of iM are real

(A) P and R only (B) Q and R only (C) P and S only (D) Q and S only

Q.44 Let $T : P_3 \to P_3$ be the map given by $T(p(x)) = \int_1^x p'(t)\,dt$. If the matrix of T relative to the standard bases $B_1 = B_2 = \{1, x, x^2, x^3\}$ is M and M' denotes the transpose of the matrix M, then $M + M'$ is

(A) $\begin{bmatrix} 0 & -1 & -1 & -1 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 0 & 2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ -1 & 0 & -1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \end{bmatrix}$

Q.45 Using Euler's method taking step size = 0.1, the approximate value of y obtained corresponding to $x = 0.2$ for the initial value problem $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$, is

(A) 1.322 (B) 1.122 (C) 1.222 (D) 1.110

Q.46 The following table gives the unit transportation costs, the supply at each origin and the demand of each destination for a transportation problem.

		Destination				
		D_1	D_2	D_3	D_4	Supply
Origin	O_1	3	4	8	7	60
	O_2	7	3	7	6	80
	O_3	3	9	3	4	100
Demand		40	70	50	80	

Let x_{ij} denote the number of units to be transported from origin i to destination j. If the u-v method is applied to improve the basic feasible solution given by $x_{12} = 60$, $x_{22} = 10$, $x_{23} = 50$, $x_{24} = 20$, $x_{31} = 40$ and $x_{34} = 60$, then the variables entering and leaving the basis, respectively, are

(A) x_{11} and x_{24} (B) x_{13} and x_{23} (C) x_{14} and x_{24} (D) x_{33} and x_{24}

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Q.47 Consider the system of equations

$$\begin{bmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ -1 \end{bmatrix}.$$

Using Jacobi's method with the initial guess $\begin{bmatrix} x^{(0)} & y^{(0)} & z^{(0)} \end{bmatrix}^T = \begin{bmatrix} 2.0 & 3.0 & 0.0 \end{bmatrix}^T$, the approximate solution $\begin{bmatrix} x^{(2)} & y^{(2)} & z^{(2)} \end{bmatrix}^T$ after two iterations, is

(A) $\begin{bmatrix} 2.64 & -1.70 & -1.12 \end{bmatrix}^T$

(B) $\begin{bmatrix} 2.64 & -1.70 & 1.12 \end{bmatrix}^T$

(C) $\begin{bmatrix} 2.64 & 1.70 & -1.12 \end{bmatrix}^T$

(D) $\begin{bmatrix} 2.64 & 1.70 & 1.12 \end{bmatrix}^T$

Common Data Questions

Common Data for Questions 48 and 49:

The optimal table for the primal linear programming problem:

Maximize $z = 6x_1 + 12x_2 + 12x_3 - 6x_4$

Subject to $x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0,$

is

Basic variables (x_B)	x_1	x_2	x_3	x_4	RHS Constants (b)
x_3	3/4	0	1	-1/4	2
x_2	1/4	1	0	1/4	2
$z_j - c_j$	6	0	0	6	$z = 48$

Q.48 If y_1 and y_2 are the dual variables corresponding to the first and second primal constraints, then their values in the optimal solution of the dual problem are, respectively,

(A) 0 and 6 (B) 12 and 0 (C) 6 and 3 (D) 4 and 4

Q.49 If the right hand side of the second constraint is changed from 8 to 20, then in the optimal solution of the primal problem, the basic variables will be

(A) x_1 and x_2 (B) x_1 and x_3 (C) x_2 and x_3 (D) x_2 and x_4

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Common Data for Questions 50 and 51:

Consider the Fredholm integral equation $u(x) = x + \lambda \int_0^1 x e^t u(t)\, dt$.

Q.50 The resolvent kernel $R(x,t;\lambda)$ for this integral equation is

(A) $\frac{xe^t}{1-\lambda}$ (B) $\frac{\lambda xe^t}{1+\lambda}$ (C) $\frac{xe^t}{1+\lambda^2}$ (D) $\frac{xe^t}{1-\lambda^2}$

Q.51 The solution of this integral equation is

(A) $\frac{x+1}{1-\lambda}$ (B) $\frac{x^2}{1-\lambda^2}$ (C) $\frac{x}{1+\lambda^2}$ (D) $\frac{x}{1-\lambda}$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

The joint probability density function of two random variables X and Y is given as

$$f(x,y)=\begin{cases}\frac{6}{5}(x+y^2), & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{elsewhere}\end{cases}$$

Q.52 $E(X)$ and $E(Y)$ are, respectively,

(A) $\frac{2}{5}$ and $\frac{3}{5}$ (B) $\frac{3}{5}$ and $\frac{3}{5}$ (C) $\frac{3}{5}$ and $\frac{6}{5}$ (D) $\frac{4}{5}$ and $\frac{6}{5}$

Q.53 $Cov(X,Y)$ is

(A) -0.01 (B) 0 (C) 0.01 (D) 0.02

Statement for Linked Answer Questions 54 and 55:

Consider the functions $f(z)=\frac{z^2+\alpha z}{(z+1)^2}$ and $g(z) = \sinh(z-\frac{\pi}{2\alpha})$, $\alpha \neq 0$.

Q.54 The residue of $f(z)$ at its pole is equal to 1. Then the value of α is

(A) -1 (B) 1 (C) 2 (D) 3

Q.55 For the value of α obtained in Q.54, the function $g(z)$ is not conformal at a point

(A) $\frac{\pi(1+3i)}{6}$ (B) $\frac{\pi(3+i)}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{i\pi}{2}$

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General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

Q.56 Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his ___ was impressive.

(A) beggary (B) nomenclature (C) jealousy (D) nonchalance

Q.57 Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he ___ the mattress out on the balcony.

(A) should take
(B) shall take
(C) should have taken
(D) will have taken

Q.58 If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23 (B) 4.33 (C) 11.37 (D) 27.64

Q.59 One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is **INCORRECT**?

I requested that he should be given the driving test today instead of tomorrow.

(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

Q.60 Which one of the following options is the closest in meaning to the word given below?

Latitude

(A) Eligibility (B) Freedom (C) Coercion (D) Meticulousness

Q. 61 - Q. 65 carry two marks each.

Q.61 There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(A) 2 (B) 3 (C) 4 (D) 8

Q.62 Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5 (B) 6 (C) 9 (D) 10

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Q.63 **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

(A) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals.
(C) Discipline was the armies' inheritance from their seniors.
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

Q.64 A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4 (B) 1/16 (C) 7/16 (D) 9/16

Q.65 The data given in the following table summarizes the monthly budget of an average household.

Category	**Amount (Rs.)**
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is

(A) 10% (B) 14% (C) 81% (D) 86%

END OF THE QUESTION PAPER

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GATE 2012 - Answer Key - Paper : MA

Paper	Question no.	Key
MA	1	D
MA	2	B
MA	3	A
MA	4	A
MA	5	A
MA	6	C
MA	7	C
MA	8	A
MA	9	C
MA	10	D
MA	11	D
MA	12	D
MA	13	C
MA	14	B
MA	15	C
MA	16	B
MA	17	D
MA	18	A
MA	19	C
MA	20	B
MA	21	C
MA	22	B
MA	23	A
MA	24	C
MA	25	C
MA	26	C
MA	27	D
MA	28	C
MA	29	D
MA	30	D
MA	31	B
MA	32	A
MA	33	Marks to All
MA	34	B
MA	35	D

Paper	Question no.	Key
MA	36	D
MA	37	A
MA	38	B
MA	39	B
MA	40	B
MA	41	B
MA	42	A
MA	43	B
MA	44	Marks to All
MA	45	C
MA	46	A
MA	47	C
MA	48	B
MA	49	D
MA	50	A
MA	51	D
MA	52	B
MA	53	A
MA	54	D
MA	55	A
MA	56	D
MA	57	A
MA	58	D
MA	59	B
MA	60	B
MA	61	A
MA	62	A
MA	63	A
MA	64	C
MA	65	D

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