

**ADDIS ABABA CITY ADMINISTRATION EDUCATION
BUREAU
GRADE 12 MATHEMATICS MODEL EXAMINATIONS FOR
NATURAL SCIENCE
MESKEREM 2015/ SEPTEMBER 2022**

NUMBER OF QUESTIONS: 65

TIME ALLOWED: 3 HOURS

GENERAL DIRECTIONS

THIS BOOKLET CONTAINS MATHEMATICS EXAMINATION FOR THE NATURAL SCIENCE CANDIDATES ONLY. IN THIS EXAMINATION, THERE ARE A TOTAL OF 65 MULTIPLE CHOICE QUESTIONS.

THERE IS ONLY ONE BEST ANSWER FOR EACH QUESTION. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET. USE ONLY PENCIL TO MARK YOUR ANSWERS.

YOU WILL BE ALLOWED TO WORK ON THE EXAM FOR 3 HOURS. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, PUT YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

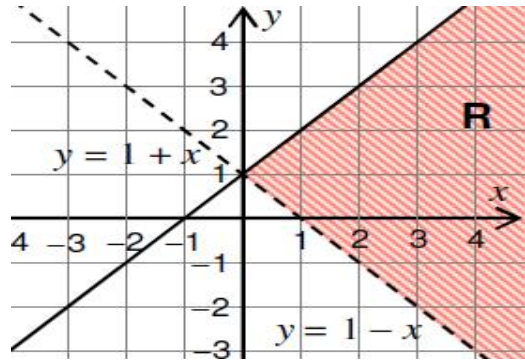
ANY FORM OF CHEATING OR AN ATTEMPT TO CHEAT IN THE EXAMINATION WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE(S).

PLEASE MAKE SURE THAT YOU HAVE WRITTEN ALL THE REQUIRED INFORMATION ON THE ANSWER SHEET BEFORE YOU START TO WORK ON THE EXAMINATION.

DO NOT TURN THIS PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following **is true** about the relation R given by the graph below?

- A. Domain of $R = (1, \infty)$.
 B. The range of $R^{-1} = (0, \infty)$.
 C. $(1, 0)$ is belongs to R^{-1} .
 D. The relation $R = \{(x, y) : y - x \geq 1 \text{ and } y + x > 1, x, y \in \mathbb{R}\}$.



2. Which one of the following functions is a power function?

- A. $f(x) = e^x$
 B. $f(x) = x^x$
 C. $f(x) = x^2 + 1$
 D. $f(x) = x^{-1}$

3. Which one of the following functions is a one-to-one correspondence?

- A. $f : [1, \infty) \rightarrow (0, \infty)$, defined by $f(x) = \sqrt{x-1}$.
 B. $f : [1, \infty) \rightarrow [0, \infty)$, defined by $f(x) = (x-1)^2 + 1$.
 C. $f : [0, \infty) \rightarrow [0, \infty)$, defined by $f(x) = x^4$.
 D. $f : (0, \infty) \rightarrow (0, \infty)$, defined by $f(x) = \log_3 x$.

4. If $f(x) = \frac{x-2}{x-1}$ and $h(x) = f\left(\frac{1}{x}\right)$, then which of the following **is not** true?

- A. Domain of h is $\mathbb{R} \setminus \{0, 1\}$.
 B. $h(x) = \frac{1-2x}{1-x}$, for $x \neq 0$ and $x \neq 1$.
 C. The line $x = 0$ is a vertical asymptote to the graph of h .
 D. $h\left(\frac{1}{x}\right) = f(x)$, for $x \neq 0$ and $x \neq 1$.

5. Which of the following is the universal set of the expression $\frac{x^4 - 8x}{3x^3 - 2x^2 - 8x}$?

- A. $\mathbb{R} / \{0, 2\}$
 B. $\mathbb{R} / \left\{-\frac{4}{3}, 0, 2\right\}$
 C. $\mathbb{R} / \{-2, \frac{4}{3}, 0\}$
 D. $\mathbb{R} / \left\{-\frac{4}{3}\right\}$

6. Suppose that ℓ_1 and ℓ_2 represent two perpendicular lines in a plane. If ℓ_1 passes through the point $(0, 4)$ and the equation of ℓ_2 is $2x - y - 3 = 0$, then what is the equation of ℓ_1 ?
- A. $2y + x = 8$ C. $y - 4x + 3 = 0$
 B. $x - 2y - 4 = 0$ D. $2x + y - 8 = 0$
7. What is the equation of the circle whose center is at $(2, -3)$ and which passes through the intersection of the lines $3x + 2y = 11$ and $2x + 3y = 4$?
- A. $x^2 + y^2 + 4x + 6y + 3 = 0$ C. $x^2 + y^2 - 4x + 6y - 3 = 0$
 B. $x^2 + y^2 + 4x + 6y - 3 = 0$ D. $x^2 + y^2 - 4x + 6y + 3 = 0$
8. What is the focus of the parabola $y^2 - 8x + 6y + 1 = 0$?
- A. $(2, 0)$ C. $(8, 0)$
 B. $(1, -3)$ D. $(-8, 6)$
9. If base of a triangle is the major axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and third vertex moves on the ellipse, then what is the maximum area of the triangle?
- A. 24 C. 6
 B. 72 D. 12
10. Suppose a proposition $p \Rightarrow \neg q$ is false (F), then which of the following is true?
- A. $\neg q \wedge (p \Rightarrow q)$ C. $(\neg q \vee p) \Leftrightarrow q$
 B. $\neg p \vee (q \Rightarrow \neg p)$ D. $(p \vee q) \Leftrightarrow \neg p$
11. Which of the following is a valid argument?
- A. $\neg p \Rightarrow \neg q, q \mid \neg p$ C. $\neg p \vee q, r \Rightarrow p, r \mid \neg q$
 B. $p \Rightarrow \neg q, p, r \Rightarrow q \mid \neg r$ D. $p \Rightarrow q, \neg q \Rightarrow r \mid p$
12. What is the negation of the statement “If it is raining, then you take your umbrella.”?
- A. If it is raining, then you do not take your umbrella.
 B. It is not raining and you do not take your umbrella.
 C. It is raining and you do not take your umbrella.
 D. It is not raining and you take your umbrella.

13. The following table gives the age distribution in a certain class. Which of the following **is not true** about the data? (to the nearest whole number)

Age (in year)	12-14	15 - 17	18 - 20	21 - 23	24 - 26
Frequency(<i>f</i>)	6	4	7	10	3

- A. The fifth deciles (D_5) is 20
 B. The mean is 19
 C. The mode is 21
 D. The standard deviation is 15
14. If Q_i , D_i and P_i are respectively the i^{th} quartile, decile and percentile of data arranged in an increasing order, then which one of the following is **necessarily true**?
- A. $D_3 > P_{25}$
 B. $Q_2 = \frac{Q_1 + Q_3}{2}$
 C. $P_{25} > Q_1$
 D. $Q_2 = \text{mean of the data}$
15. Among 6000 students who took a regional exam of Addis Ababa city Administration, the percentile of certain student's score is 90. Which of the following is correct about the student's score?
- A. The student's score is greater than or equal to that of 5400 students.
 B. The students have answered 90% of the questions correctly.
 C. The student's score is the same as the top 10% of the scores.
 D. The score of the student is as good as that of a 90% of the students.
16. A bag contains 3 red, 4 blue and 3 white balls. Three balls are drawn one after the other. What is the probability of getting a red ball on the first draw, a blue ball on the second draw and a white ball on the third draw if the balls are drawn without replacement?
- A. $\frac{9}{250}$
 B. $\frac{1}{20}$
 C. $\frac{3}{80}$
 D. $\frac{4}{5}$
17. What is the constant term in the expansion of $\left(\frac{2}{x} + 3x\right)^6$?
- A. 160
 B. 216
 C. 540
 D. 4320

18. A student needs to select 3 books from 3 different mathematics, 3 different physics and 1 history books. What is the probability that one of them is a mathematics and the other two are either physics or history books?

A. $\frac{3}{35}$

C. $\frac{15}{35}$

B. $\frac{9}{25}$

D. $\frac{18}{35}$

19. What is the value of x such that the matrix $A = \begin{pmatrix} x & -3 & x \\ -1 & 0 & 2 \\ 2 & 1 & -5 \end{pmatrix}$ is singular (non-invertible)?

A. -2

C. 1

B. -1

D. 2

20. What should be the values of a and b so that the system $\begin{cases} x + y + z = 0 \\ 3x - ay - z = 4 \\ x + 5y + 5z = b \end{cases}$

will have **no solution**?

A. $a = 1, b \neq -4$

C. $a \neq -1, b = -1$

B. $a = -1, b \in \mathbb{R}$

D. $a \neq 1, b = 4$

21. Suppose A and B be 3×3 matrices such that $A = \begin{pmatrix} 2 & 0 & 0 \\ -4 & 6 & 0 \\ 0 & -1 & \frac{2}{3} \end{pmatrix}$ and $|B| = \frac{1}{4}$, then

which of the following is equal to $|3A^T B|$?

A. 6

C. 54

B. 18

D. $\frac{27}{32}$

22. Let $A = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 5 & 7 \\ 8 & -3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 3 & -5 \\ 2 & 1 & 7 \\ 6 & 5 & 0 \end{pmatrix}$. If $C = AB + A^T$, then which of the

following is the entry of C_{23} ?

A. 15

C. 33

B. 12

D. 28

23. Which one of the following is the conjugate of $|3-4i| - \frac{(2+i)(3-i)}{3+4i}$?
- A. $4-i$ C. $-4-i$
 B. $4+i$ D. $-1-i$
24. In the set of complex numbers, what is the solution set of $x^2 + 2x + 5 = 0$?
- A. $\emptyset$ C. $\{1-2i, 1+2i\}$
 B. $\{-2-4i, -2+4i\}$ D. $\{-1-2i, -1+2i\}$
25. If $z = 2 + 2\sqrt{3}i$, then which one of the following is the polar representation of z ?
- A. $16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ C. $16\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
 B. $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ D. $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
26. A contest offers 10 prizes with a total value of Birr 13,250. If the difference in value between consecutive prizes is Birr 250, what is the value of the **first** prizes?
- A. Birr 2,400 C. Birr 2,450
 B. Birr 200 D. Birr 2,700
27. Suppose $f_1 = 3$, and for $n = 2, 3, 4, \dots$, $f_n = 1 + f_{n-1}$. Which of the following is **true**?
- A. f_{37} is a prime number.
 B. For any natural number n , $f(f_n) = n + 6$
 C. If f_n is an odd composite number, then n is even.
 D. $\sum_{i=1}^n f_i = \frac{n(n+5)}{2}$
28. Which of the following is the sum of the series $\frac{1}{2} - \frac{1}{3} + \frac{2}{9} - \frac{4}{27} + \dots$?
- A. $\frac{3}{10}$ C. $\frac{5}{6}$
 B. ∞ D. 3

29. Which one of the following **is not true** about the arithmetic progression whose sixth term and tenth terms are 18 and 30 respectively?

- A. The common difference is 3.
- B. The first term is 3.
- C. The sum of the first one hundred terms is 15,150.
- D. The arithmetic mean of the first 10 terms is 30.

30. Which of the following **is true** about the sequence $\left\{2 - \frac{(-1)^n}{n+1}\right\}_{n=1}^{\infty}$?

- A. $\frac{8}{5}$ is its greatest lower bound.
- B. The sequence converges to the least upper bound.
- C. The sequence converges to 2.
- D. The sequence is divergent.

31. Which of the following is equal to $\lim_{n \rightarrow \infty} \frac{(3-4n)^2}{2-\sqrt{n}-2n^2}$?

- A. $\frac{3}{2}$
- B. -8
- C. 8
- D. 2

32. Which of the following is equal to $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$?

- A. $\frac{1}{4}$
- B. 0
- C. does not exist
- D. 4

33. Which one of the following **is true** about the function $f(x) = \begin{cases} \frac{x^2-9}{|x-3|}, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$?

- A. f is continuous from the right at $x = 3$.
- B. f is continuous from the left at $x = 3$.
- C. f is continuous everywhere.
- D. f has $x = 3$ as a vertical asymptote.

34. Which of the following is equal to $\lim_{x \rightarrow 0} \frac{\sin(2x^3)}{\sin^3(4x)}$?

A. $\frac{1}{8}$

C. $\frac{1}{32}$

B. $\frac{1}{2}$

D. 2

35. Which of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{2x}{2x+4} \right)^{-5x}$?

A. e^{-10}

C. $e^{\frac{-5}{2}}$

B. e^{20}

D. e^{10}

36. If $f(x) = \ln x$, then what is the value of $f^{(n)}(x)$?

A. $f^{(n)}(x) = \frac{(-1)^{n+1} n!}{x^n}$

C. $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$

B. $f^{(n)}(x) = \frac{(-1)^{n+1} (n-1)!}{x^n}$

D. $f^{(n)}(x) = \frac{(-1)^{n+1} (n+1)!}{x^{n+1}}$

37. If $f(x) = 2x^3 - 6x + 2 \ln x$, then which of the following is equal to

$\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$?

A. 0

C. 2

B. -4

D. 1

38. Which one of the following is **not true** about the derivative of $f(x) = x|x|$?

A. $f'(-1) = -2$

B. $f'(x) = 2|x|$, for every $x \in (-\infty, \infty)$.

C. $f'(x) = 2$, at $x = 1$.

D. f is differentiable at $x = 0$.

39. If $F(x) = f(x^2 + 1) + \frac{f(1-x)}{\cos x}$, $f'(2) = 10$, $f'(0) = f(2) = -4$ and $f(0) = 6$, then

$F'(1)$ is equal to

A. -12

C. 24

B. 16

D. -4

40. Which one of the following is the equation of the tangent line to the graph of

$f(x) = \begin{cases} x - 2, & \text{if } x \leq 0 \\ -2 \sin x, & \text{if } x > 0 \end{cases}$ at the point $(\theta, f(\theta))$?

A. $y = x - 2$

C. $y = -2x - 3$

B. $y = 2x - 2$

D. $y = 2x - 3$

41. Which one of the following **is not true** about the function $f(x) = 5x^4 - x^5$?
- f has a local maximum value at $x = 4$ and local minimum value at $x = 0$.
 - f is concave down ward on $(3, \infty)$.
 - $(0, 0)$ and $(3, 162)$ are inflection points of f .
 - f is decreasing on $(-\infty, 0]$ and $[4, \infty)$.
42. A box is to have a square base, an open top and volume of $32m^3$. What is the dimension (base (x) and height (h)) of the box that use the least amount of material?
- $x = 8\sqrt[3]{2}m$ and $h = 2\sqrt[3]{4}m$
 - $x = 4m$ and $h = 2m$
 - $x = 2m$ and $h = 4m$
 - $x = 2\sqrt[3]{4}m$ and $h = 8\sqrt[3]{2}m$
43. The height of rectangular box is $10m$. Its length increases at the rate of $2m/sec$, its width decreases at the rate of $4m/sec$. When the length is $8m$ and the width is $6m$, then at what rate the volume of the box is changing?
- $200m^3/sec$
 - $80m^3/sec$
 - $-200m^3/sec$
 - $-20m^3/sec$
44. For what value of c the conclusion of mean value theorem is satisfied for the function $f(x) = x^3 + x - 1$ on $[0, 2]$?
- $\pm \frac{2\sqrt{3}}{3}$
 - $-\frac{2\sqrt{3}}{3}$
 - $2\sqrt{3}$
 - $\frac{2\sqrt{3}}{3}$
45. Which one of the following is the set of all critical numbers of $f(x) = 2\sqrt{x}(6-x)$?
- $\{0, 2\}$
 - $\{2\}$
 - $\{0\}$
 - $\{-2, 0\}$
46. Which of the following is equals to $\int_{\frac{1}{2}}^0 e^{-2x} dx$?
- $2(e-1)$
 - $\frac{1}{2}(e-1)$
 - 1
 - $-\frac{1}{2}(e-1)$
47. Which of the following is equals to $\int \frac{\sin(2x)}{2\sin x} dx$?
- $\sin x + c$
 - $\frac{1}{2}\sin x + c$
 - $-\sin x + c$
 - $\frac{1}{2}\cos x + c$

48. Which of the following is equal to $\int \frac{x^3}{(x+1)(x+2)} dx$?

A. $8\ln|x+2| - \ln|x+1| + c$

C. $\frac{x^2}{2} - 3x + 8\ln|x+2| - \ln|x+1| + c$

B. $\frac{x^2}{2} + 3x + \ln|x+2| - 8\ln|x+1| + c$

D. $\ln|x+2| - 8\ln|x+1| + c$

49. If $F(x)$ is an anti-derivative of $f(x) = 2x + \sqrt{x}e^{\sqrt{x}}$ and $F(0) = 6$, then $F(4)$ is equal to

A. $20e^2 + 18$

C. $-20e^2 + 18$

B. $-12e^4 + 32e^2 + 2$

D. $4e^2 + 18$

50. What is the area of the region bounded by the lines $y = e$, $y = e^2$, $x = -1$ and the curve $f(x) = \frac{1}{x}$?

A. 1

C. $e^2 - e + 1$

B. $-e^2 + e + 1$

D. $e^2 - e - \frac{1}{e} + 1$

51. What is the volume of the solid of revolution about the x -axis generated by revolving the region enclosed by the curve $y = x^2$ and the line $y = 4$?

A. $\frac{64}{5}$ ®

C. $\frac{256}{5}$ ®

B. 64 ®

D. $\frac{384}{5}$ ®

52. Suppose a and b are vectors with $|a| = 4$, $|b| = 3$ and the angle between a and b is $\frac{\pi}{3}$, then what is the cosine of the angle between $a - b$ and a ?

A. 10

C. $\sqrt{13}$

B. $\frac{5}{2\sqrt{13}}$

D. $\frac{5}{\sqrt{13}}$

53. What is the image of the line given by $(x, y) = (-1, 0) + t(3, 6)$ under the translation that takes $(1, 0)$ to $(0, 1)$ followed by the reflection about the line $y = 2x$?

A. $y = 2x + 3$

C. $y = 2x + 6$

B. $y = 2x - 3$

D. $y = 2x - 5$

54. What is the image of the circle $(x-3)^2 + (y+5)^2 = 1$ when it is rotated through $\frac{5\pi}{3}$ about the point $(4, -3)$?

A. $\left(x - \frac{7}{2} + \sqrt{3}\right)^2 + \left(y + 4 - \frac{\sqrt{3}}{2}\right)^2 = 1$

C. $\left(x + \frac{7}{2} + \sqrt{3}\right)^2 + \left(y - 4 - \frac{\sqrt{3}}{2}\right)^2 = 1$

B. $\left(x - \frac{7}{2} - \sqrt{3}\right)^2 + \left(y + 4 + \frac{\sqrt{3}}{2}\right)^2 = 1$

D. $\left(x + \frac{7}{2} - \sqrt{3}\right)^2 + \left(y - 4 + \frac{\sqrt{3}}{2}\right)^2 = 1$

55. Which of the following is the vector equation of the line passing through the points $(3, 5)$ and $(-2, 3)$?

A. $(x, y) = (3, 5) + t(-5, 2)$

C. $(x, y) = (3, -2) + t(-5, -2)$

B. $(x, y) = (3, 5) + t(-5, -2)$

D. $(x, y) = (2, -3) + t(-5, 2)$

56. Which of the following is **true** about the graph $f(x) = -3\sin\left(\frac{1}{2}x - \frac{\pi}{3}\right) + 1$?

A. The period of f is 4π .

B. The amplitude of f is -3 .

C. The graph completes one cycle on $\left[\frac{2\pi}{3}, \frac{13\pi}{3}\right]$.

D. The maximum value of f is 6 .

57. Which of the following is **true** about the function $f(x) = \csc x$?

A. The domain is the set of real numbers.

B. The period is π .

C. The domain is $\left\{x \in \mathbb{R} : x \neq \frac{(2k+1)\pi}{2}; k \in \mathbb{Z}\right\}$.

D. The range of f is $(-\infty, -1] \cup [1, \infty)$.

58. As shown in the figure below, the angle of elevation of the top of a cliff when seen from the foot and the top of a building are 45° and 30° respectively.

If the height of the building is $10m$

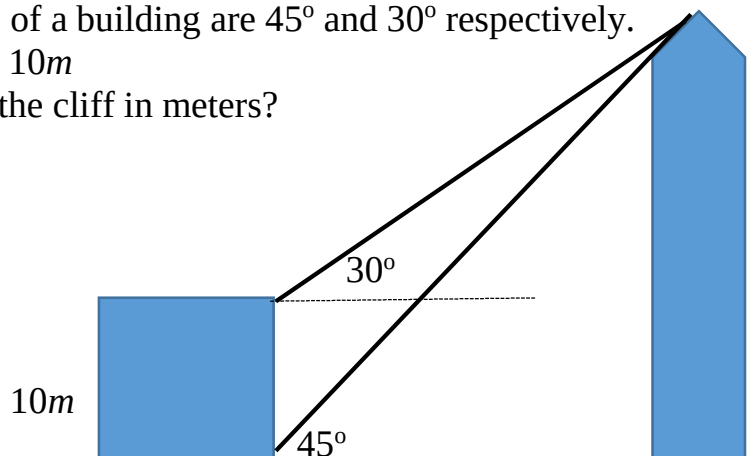
, then how high is the top of the cliff in meters?

A. $15 + \sqrt{3}$

B. $5(3 + \sqrt{3})$

C. $\frac{10}{\sqrt{3} - 1}$

D. $\frac{10\sqrt{3}}{\sqrt{3} - 2}$



59. Which of the following is equal to the value of $\sin^{-1}\left(\sin \frac{5\pi}{4}\right)$?

A. $-\frac{\pi}{4}$

C. $\frac{5\pi}{4}$

B. $\frac{\pi}{4}$

D. $-\frac{\sqrt{2}}{2}$

60. Suppose $A = 2i - j - 3k$, $B = i - 2j + ak$ and $D = ai - 4k$ be vectors in space and i, j and k are standard unit vectors. Then for what value of a is the vector from A to B perpendicular to the vector from A to D ?

A. -1

C. 3

B. 1

D. 5

61. Suppose that the equation $x^2 + y^2 + z^2 + 2x - 6y + 8z = 7$ represents a sphere. Where is the point $(3, -1, -4)$ located relative to the sphere?

A. Outside the sphere.

C. Inside the sphere.

B. On the sphere.

D. At the center of the sphere.

62. If one of the end point of the line segment is $(-3, 2, 4)$ and the midpoint is $(2, -5, -3)$, then the coordinates of the other end point in space is

A. $(-7, 12, 10)$

C. $(-8, -9, 11)$

B. $(-1, -3, 1)$

D. $(7, -12, -10)$

63. Consider the following assertion of a person and his proof.

“If n is odd, then n^2 is odd”

Proof: Suppose that n is odd.

Then $n = 2k + 1$ for some integer k . Then it follows that

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1.$$

(where $m = 2k^2 + 2k$ which is an integer).

Therefore, n^2 is odd.

This can be proved or disproved by which one of the following mathematical proofs?

A. Direct proof

C. Indirect proof

B. Proof by exhaustion

D. proof by contradiction

64. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

- A. $n < 2^n$, for every integer $n \leq 100$.
- B. $6^r - 1$ is a multiple of 5, for every real number $r \geq 1$.
- C. $2^{n-1} \leq n!$, for every integer $n \geq 1$.
- D. $n^2 - 11n + 121$ is prime, for every natural number $n \geq 1$.

65. Let $p(x) \equiv x^2 + x$ is positive. Which one of the following is **equivalent** to $\neg(\exists x)p(x)$?

- | | |
|-----------------------------------|----------------------------------|
| A. $(\exists x)\neg(x^2 + x > 0)$ | C. $(\forall x)(x^2 + x < 0)$ |
| B. $(\exists x)(x^2 + x \leq 0)$ | D. $(\forall x)(x^2 + x \leq 0)$ |

THE END