

# GRADE 10

MATHEMATICS

*ESSENTIAL HANDBOOK BASED  
ON THE NEW NATIONAL AND  
INTERNATIONAL  
CURRICULUM*



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FRAMED ON THE BASIS OF G10 CHAPTER SIX

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# UNIT 6

## SOLID FIGURES

Keep in mind that

- ✍ A **solid figure** is a three-dimensional figure because it has three dimensions length, width and height.
- ✍ **Solid figures** can be identified by the *shape and number of their bases*, and also the *shape of their lateral faces*.

In this section, you will revise the most common solid figures you have learnt in grade 8 namely prism, cylinder, pyramid and cone, additionally you will learn about spheres, frustum of pyramids, frustum of cones and composite solids.

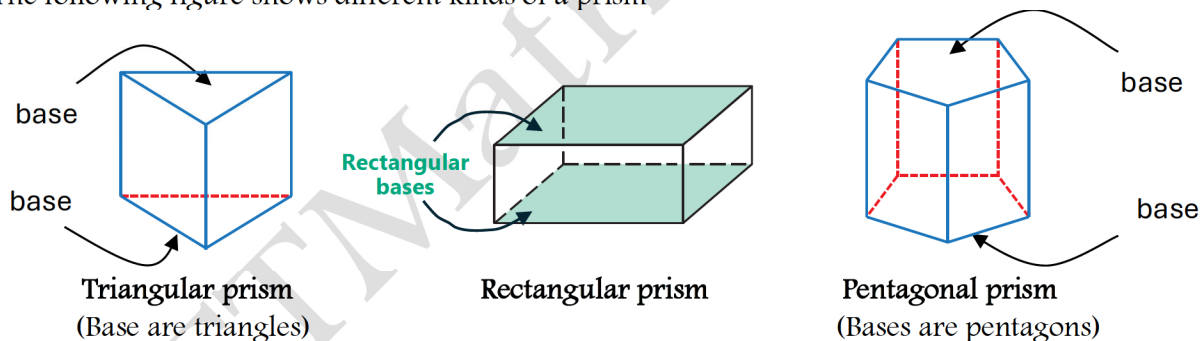
### 6.1 Revision of Prisms, Cylinders, Pyramids and Cones

#### A. Prism

##### Definition 6.1

- **Prism** is a solid figure, with two parallel bases, which are polygons and lateral faces are parallelograms.
- The prism takes its name from the name of the base.  
**Example:** triangular prism, square prism, rectangular prism, pentagonal prism and so on.

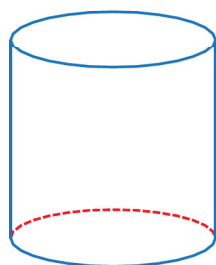
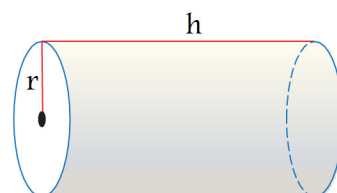
The following figure shows different kinds of a prism



#### B. Cylinder

##### Definition 6.2

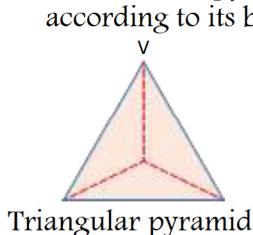
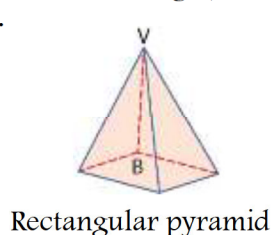
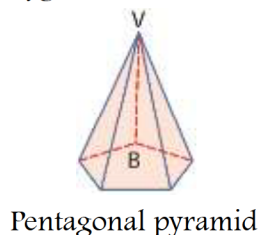
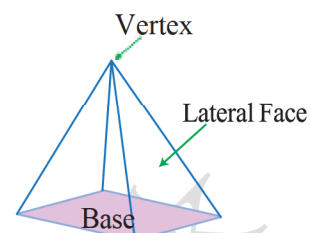
- **Cylinder** is a solid figure whose upper and lower bases are congruent circles joined by curved surface on parallel planes.
- The distance between the two bases of the cylinder is called height of the cylinder



### C. Pyramid

#### Definition 6.3

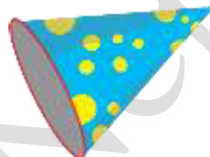
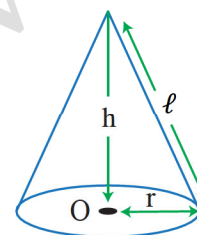
- **Pyramid** is a solid figure with a polygonal base and triangular lateral faces meeting at a vertex.
- The base of a pyramid can be triangle, or any polygon and the corresponding pyramid is named according to its base.



### D. Cone

#### Definition 6.4

- **Cone** is a solid figure with a circular base and a curved lateral surface meeting at a vertex.
- A pyramid whose base is a circular region is called a circular cone.



#### 6.1.1 Surface Area and Volume of Prisms, Cylinders, Pyramids and Cones

##### A. Surface Area

- The surface area of a solid figure is the sum of the area of each face of the solid figure.

There are two types of surface areas:

1. **Lateral surface area**
2. **Total surface area**

1. The Lateral Surface Area (LSA) is the sum of the areas of all lateral faces of the solid figure.
2. The Total Surface Area (TSA) is the sum of the areas of all the faces.

##### B. Volume

- Volume: the product of the area of the base and height

$$V = A_b h$$

#### Summary of LSA, SA and Volume of Prisms, Cylinders, Pyramids and Cones

| Solid figures   | Lateral surface area                       | Total surface area                      | Volume                                             |
|-----------------|--------------------------------------------|-----------------------------------------|----------------------------------------------------|
| <b>Prism</b>    | $LSA = ph$                                 | $TSA = LSA + 2A_b$                      | $V = A_b h$                                        |
| <b>Cylinder</b> | $LSA = ph$<br>$= 2\pi r h$                 | $TSA = LSA + 2A_b$<br>$= 2\pi r(h + r)$ | $V = A_b h$<br>$= \pi r^2 h$                       |
| <b>Pyramid</b>  | $LSA = \frac{1}{2}p\ell$                   | $TSA = LSA + A_b$                       | $V = \frac{1}{3}A_b h$                             |
| <b>Cone</b>     | $LSA = \frac{1}{2}p\ell$<br>$= \pi r \ell$ | $TSA = LSA + A_b$                       | $V = \frac{1}{3}A_b h$<br>$= \frac{1}{3}\pi r^2 h$ |

Where:  $p$  = perimeter of the base,

$h$  = height (the distance between the two bases)

$A_b$  = Area of the base,

$\ell$  = Slant height

### 6.1.2 Worked examples on LSA, TSA and Volume of Prisms, Cylinders, Pyramids and Cones

#### Example 1

1. Find the lateral surface Area and the Total surface Area of the following right rectangular prism shown in figure 6.1

**Solution**

$$LSA = ph = 2(6\text{cm} + 3\text{cm})5\text{cm} = 90\text{cm}^2$$

$$TSA = AL + 2A_B = 90\text{cm}^2 + 2(6\text{cm} \times 3\text{cm}) = 126\text{cm}^2$$

$$V = A_B h = (6\text{cm} \times 3\text{cm}) 5\text{cm} = 90\text{cm}^3$$

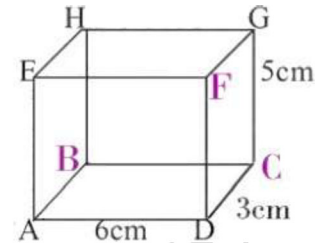


Figure 6.1

#### Example 2

2. Find the total surface area and the volume of the right triangular prism shown in figure 6.2

**Solution**

$$\text{hypotenuse} = \sqrt{8^2 + 6^2} = 10$$

$$LSA = ph = (8\text{cm} + 6\text{cm} + 10\text{cm}) 6\text{cm} = 240\text{cm}^2$$

$$TSA = LSA + 2A_B = 240\text{cm}^2 + 2 \times \frac{1}{2}(6\text{cm} \times 8\text{cm}) = 288\text{cm}^2$$

$$V = A_B h = \frac{1}{2}(6\text{cm} \times 8\text{cm}) 8\text{cm} = 192\text{cm}^3$$

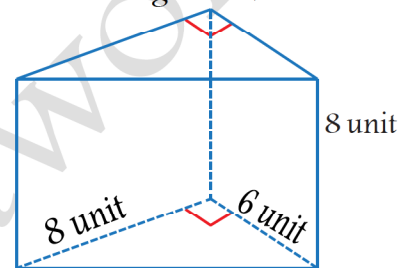


Figure 6.2

#### Example 3

3. Find the total surface area and volume of the rectangular prism shown in figure 6.3.

**Solution**

$$LSA = ph = 2(3\text{cm} + 2\text{cm})5\text{cm} = 50\text{cm}^2$$

$$TSA = LSA + 2A_B = 50\text{cm}^2 + 2(3\text{cm} \times 2\text{cm}) = 62\text{cm}^2$$

$$V = A_B h = (3\text{cm} \times 2\text{cm}) 4\text{cm} = 24\text{cm}^3$$

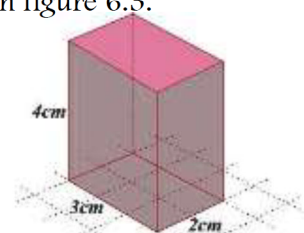


Figure 6.3

#### Example 4

4. Find the total surface area and volume of the prism shown in figure 6.4.

**Solution**

$$A_B = \text{area of the rectangle} + \text{area of the trapezium} = (6 \times 7) + \frac{1}{2}(7+1)8 = 74$$

$$LSA = ph = (7 + 14 + 1 + 10 + 6)6 = 228$$

$$TSA = AL + 2A_B = 228 + 2(74) = 376$$

$$V = A_B h = (74) 6 = 444$$

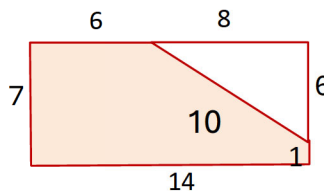
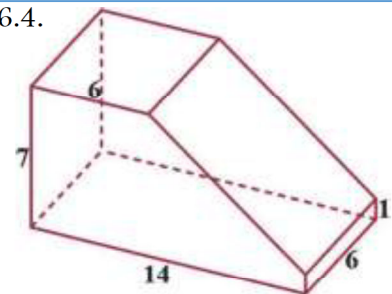


Figure 6.4

#### Example 5

5. Find the SA and volume of a right circular cylinder whose radius is 8cm and height is 12cm.

**Solution**

$$LSA = ph = 2\pi rh = 2\pi(8\text{cm} \times 12\text{cm}) = 192\pi\text{cm}^2$$

$$TSA = LSA + 2A_B = 192\pi + 2\pi(8\text{cm})^2 = 320\pi\text{cm}^2$$

$$V = A_B h = \pi r^2 h = \pi(8\text{cm})^2 \times 12 = 768\pi\text{cm}^3$$

#### Example 6

6. A regular square pyramid has a base side of 8cm and a slant height of 10cm. Find surface area.

**Solution:**

$$\text{Base Area} = (8\text{cm})^2 = 64\text{cm}^2$$

$$LSA = \frac{1}{2} \times (32\text{cm}) \times (10\text{cm}) = 160\text{cm}^2$$

$$\text{Perimeter of Base} = 4 \times 8\text{cm} = 32\text{cm}$$

$$TSA = 160\text{cm}^2 + 64\text{cm}^2 = 224\text{cm}^2$$

### Example 7

7. A circular hole of radius 5cm is drilled through the center of a right circular cylindrical shape object whose base has radius 8 cm and whose altitude is 10cm find
- The total surface area of the object (annulus).
  - Volume of the resulting object

#### Solution

- TSA of the object = TSA of larger - TSA of smaller  
 $LSA(\text{inner}) = ph = 2\pi(5)10\text{cm} = 100\pi\text{cm}^2$   
 $TSA(\text{inner}) = A_L + 2A_B = 100\pi\text{cm}^2 + 2\pi(5^2) = 150\pi\text{cm}^2$   
 $LSA(\text{outer}) = ph = 2\pi(8)10\text{cm} = 160\pi\text{cm}^2$   
 $TSA(\text{outer}) = A_L + 2A_B = 160\pi\text{cm}^2 + 2\pi(8^2) = 288\pi\text{cm}^2$   
 $\therefore \text{TSA of the object} = 288\pi\text{cm}^2 - 150\pi\text{cm}^2 = 138\pi\text{cm}^2$
- V of the object = Volume of larger - Volume of smaller  
 $\therefore \text{V of the object} = [\pi(8\text{cm})^2 \times 10] - [\pi(5\text{cm})^2 \times 10] = 390\pi\text{cm}^3$

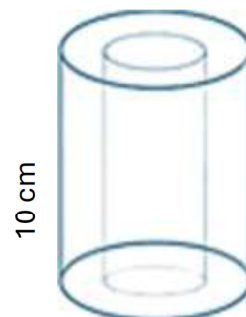


Figure 6.6

### Example 8

8. A rectangular prism has dimensions length 8 cm, width 5 cm, and height 3 cm. Find the TSA and volume.

#### Solution

$$TSA = LSA + 2A_B = 2(8\text{cm} + 5\text{cm})3\text{cm} + 2(8\text{cm} \times 5\text{cm}) = 158\text{cm}^2$$

$$\text{Volume} = A_B h = 8\text{cm} \times 5\text{cm} \times 3\text{cm} = 120\text{cm}^3$$

### Example 9

9. A cylinder has radius 4 cm and height 7 cm. Find the TSA and volume.

**Solution:**  $TSA = 2\pi(4\text{cm}) \times (7\text{cm}) + 2\pi(4\text{cm})^2 = 88\pi\text{cm}^2$

$$\text{Volume} = \pi(4\text{cm})^2 \times (7\text{cm}) = 112\pi\text{cm}^3$$

### Example 10

10. A right cone has a radius of 6cm and a slant height of 12cm. Find surface area.

#### Solution:

$$LSA = \pi(6\text{cm}) \times (12\text{cm}) = 72\pi\text{cm}^2$$

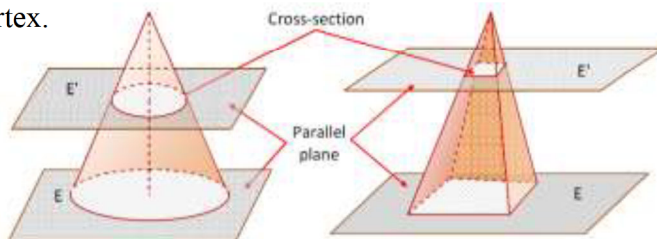
$$TSA = 72\pi\text{cm}^2 + \pi(6\text{cm})^2 = 72\pi\text{cm}^2 + 36\pi\text{cm}^2 = 108\pi\text{cm}^2$$

### 6.1.3 Horizontal Cross-Sections of Pyramids and Cones

If a pyramid or a cone is cut by a plane parallel to the base, the intersection of the plane and the pyramid (or the cone) is called a horizontal cross section of the pyramid (or the cone).

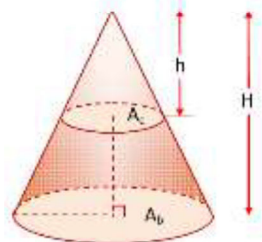
#### • Principle:

- ✍ A horizontal cross-section of a pyramid or cone creates a smaller, similar figure to the base.
- ✍ The ratio of corresponding lengths in the cross-section and the base is equal to the ratio of their distances from the vertex.



- If the height of the original pyramid/cone is H, and the distance from the apex to the cross-section is h, then:

- $\frac{\text{Side of cross-section}}{\text{Side of Base}} = \frac{h}{H} = k \text{ (constant)}$
- $\frac{\text{Radius of cross-section}}{\text{Radius of Base}} = \frac{h}{H} = k \text{ (constant)}$
- $\frac{\text{Area of the cross-section}}{\text{Area of base}} = k^2$



## 6.1.2 Worked examples on Horizontal Cross-Sections of Pyramids and Cones

**Example 11**

11. A cone has a height of 15cm and base radius 8cm. A horizontal cross-section is made 5cm from the vertex. Find the radius of the cross-section.

**Solution:**

$$k = \frac{\text{height of the cross-section}}{\text{height of original cone}} = \frac{5\text{cm}}{15\text{cm}} = \frac{1}{3}$$

$$\text{radius of horizontal cross-section} = k \times \text{radius of base} = (1/3) \times 8\text{cm} = 8/3\text{cm}$$

**Example 12**

12. A cone is 18 cm tall with a base radius of 9 cm. A horizontal cut is made 6 cm from the apex. Find the radius and area of the cross-section.

**Solution**

The ratio of cross section is, height of small cone/height of large cone =  $k = 6/18 = 1/3$

radius of the cross-section =  $k \times \text{radius of base} = 1/3 \times 9\text{ cm} = 3\text{ cm}$

Area of cross section =  $\pi \times 3\text{cm} \times 3\text{cm} = 9\pi\text{ cm}^2$

**Example 13**

13. If the area of horizontal cross section of a pyramid is  $25\text{cm}^2$  and the height of the pyramid at that point is 10cm from the top, what is the area of the horizontal cross section, if the height is doubled.

**Solution:**

Since, it is similar the scale factor is,  $20/10=2$

Area =  $(2)^2 \times 25\text{cm}^2 = 100\text{cm}^2$

**Cavalieri's Principle:**

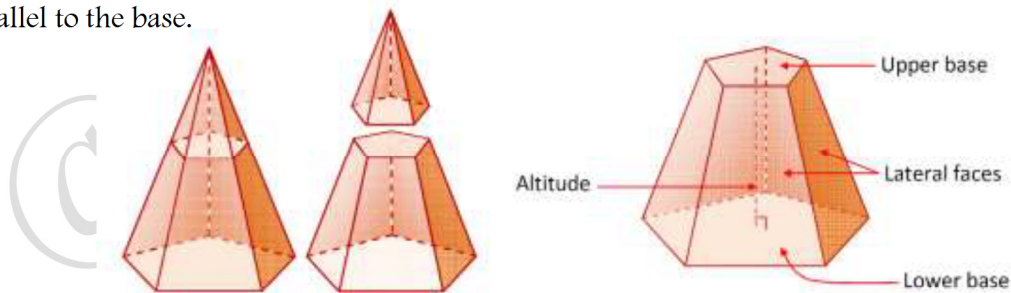
✍ If two solids have the same height and if, at every level, the cross-sectional areas are equal, then the solids have the same volume.

**6.3 Frustum of Pyramids and Cones**

**Frustum** is a solid obtained when a pyramid or cone is cut by a plane parallel to its base, removing the smaller pyramid/cone that contains the vertex.

**Frustum of a pyramid**

A **frustum** of a pyramid is the portion of a pyramid remaining after the top portion is cut off by a plane parallel to the base.



✍ The lateral surface area (LSA) of a frustum of a regular pyramid is equal to half the product of the slant height  $\ell$  and the sum of the perimeter  $p$  of the lower base and perimeter  $p'$  of the upper base. That is,

- $LSA = \frac{1}{2} \ell (p + p')$
- $TSA = LSA + A' + A$
- $V = \frac{1}{3} h (A + A' + \sqrt{AA'})$  where  $A$  and  $A'$  are base areas and  $h$  is the altitude of the pyramid

## Frustum of a cone

A **frustum** of a cone is the portion of a cone remaining after the top portion is cut off by a plane parallel to the base

Surface Area of Frustum

For frustum of cone with radii  $r$  (bottom) and  $r'$  (top), slant height  $\ell$ :

- Lateral Surface Area:  $LSA = \pi(r + r') \ell$

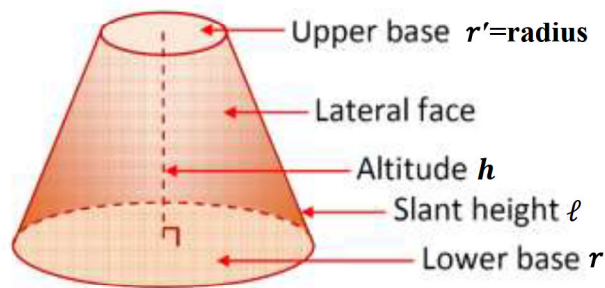
$A_b$  = the area of the two bases =  $\pi(r^2 + r'^2)$

- Total Surface Area:  $SA = \pi(r + r') \ell + \pi r^2$

Volume of Frustum

$$V = \frac{1}{3} \pi h (r^2 + r'^2 + r r')$$

$$V = \frac{1}{3} h (A + A' + \sqrt{AA'}), \quad \text{where } h \text{ is the height (altitude) of the frustum}$$



Alternatively,

$$\Rightarrow V(\text{frustum}) = V(\text{original}) - V(\text{removed cone/pyramid})$$

$$\Rightarrow \text{Lateral Area}(\text{frustum}) = \text{Total Area} - \text{Top Area} - \text{Lower Area}$$

### Example 14

14. Find the lateral surface area of a frustum of a pyramid with slant height 5 cm, and base perimeters 24 cm and 16 cm.

**Solution:**

$$LSA = \frac{1}{2} (P_1 + P_2) \ell = \frac{1}{2} (24 + 16) \times 5 = 20 \times 5 = 100 \text{ cm}^2$$

### Example 15

15. A frustum of a regular square pyramid has height 5 cm. The upper base is of side 2 cm and the lower base is of side 6 cm. Find the volume of the frustum

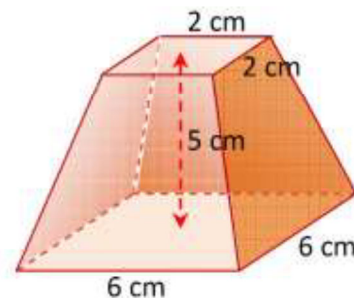
**Solution:**

Since the upper base and lower base are squares,

$$A = (6 \text{ cm})^2 = 36 \text{ cm}^2$$

$$A' = (2 \text{ cm})^2 = 4 \text{ cm}^2$$

$$\begin{aligned} V_f &= \frac{1}{3} h (A + A' + \sqrt{AA'}) = \frac{1}{3} \times 5 (36 + 4 + \sqrt{36 \times 4}) \\ &= \frac{1}{3} \times 5 (40 + 12) = \frac{260}{3} \text{ cm}^3 \end{aligned}$$



### Example 16

16. Original cone: radius 8cm, height 12cm. Removed cone: height 4cm. Find the volume of the frustum.

**Solution:**

Since the removed cone's height is  $\frac{1}{3}$  of original cone's height.

$$\text{Radius of Removed Cone} = \left(\frac{1}{3}\right) \times \text{radius of original cone} = \left(\frac{1}{3}\right) \times 8 \text{ cm} = \frac{8}{3} \text{ cm}$$

$$\text{Original Volume} = \left(\frac{1}{3}\right) \times \pi \times (8 \text{ cm})^2 \times (12 \text{ cm}) = 256\pi \text{ cm}^3$$

$$\text{Volume of Removed Cone} = \left(\frac{1}{3}\right) \times \pi \times \left(\frac{8}{3} \text{ cm}\right)^2 \times (4 \text{ cm}) = \left(\frac{256}{27}\right)\pi \text{ cm}^3$$

$$\text{Volume of Frustum} = 256\pi \text{ cm}^3 - \left(\frac{256}{27}\right)\pi \text{ cm}^3 = \left(\frac{6656}{27}\right)\pi \text{ cm}^3 \approx 775.7 \text{ cm}^3$$

## 6.2.4 Surface Area and Volume of a Sphere

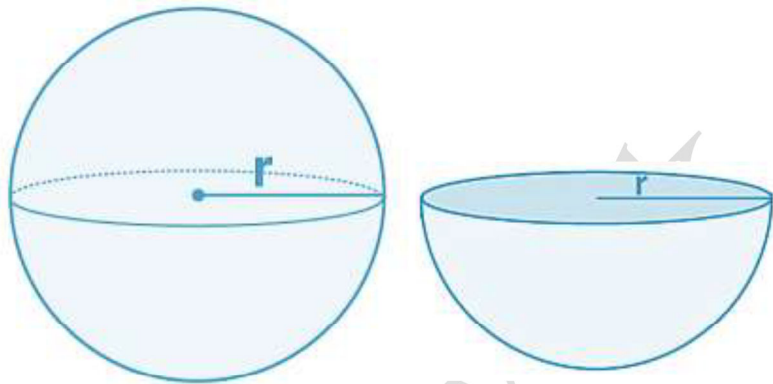
## • Sphere

Sphere is a perfectly round 3D object where every point on the surface is equidistant from the center.

- The distance from the center to the surface is the radius  $r$ .
- Has only a curved surface

$$\text{Surface Area: } TSA = 4\pi r^2$$

$$\text{Volume: } V = \frac{4}{3}\pi r^3$$



## • Hemisphere

- A hemisphere is half of a sphere, divided by a plane passing through its center.
- It has a curved surface and a flat circular base.

$$\text{Curved surface area: } LSA = \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

$$\text{Surface Area: } TSA = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$\text{Volume: } V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$$

← curved surface and flat surface

← half of the volume of a sphere

**Example 17**

17. A sphere has a radius of 7cm. Find its surface area and volume.

**Solution:**

$$\text{Surface Area} = 4\pi \times (7\text{cm})^2 = 196\pi \text{ cm}^2$$

$$\text{Volume} = (4/3)\pi \times (7\text{cm})^3 = (1372/3)\pi \text{ cm}^3$$

**Example 18**

18. Find the total surface area of a hemisphere with radius 10 cm.

**Solution:**

$$TSA = 3\pi r^2 = 3\pi (10)^2 = 3\pi \times 100 = 300\pi$$

**Example 19**

19. Find the Curved Surface Area, Total Surface Area, and Volume a hemisphere with radius 4 cm.

**Solution:**

$$\text{Curved Surface Area } LSA = 2\pi r^2 = 2\pi(4^2) = 32\pi \text{ cm}^2$$

$$\text{Total Surface Area } TSA = 3\pi r^2 = 3\pi(4^2) = 48\pi \text{ cm}^2$$

$$\text{Volume } V = \frac{2}{3}\pi r^3 = (2/3)\pi(4^3) = (128/3)\pi \text{ cm}^3$$

**Example 20**

20. Find the total surface area of a sphere with a diameter of 12 cm.

**Solution:**

$$\text{First, find the radius: } r = d/2 = 12/2 = 6 \text{ cm}$$

$$\text{Now calculate TSA: } TSA = 4\pi r^2 = 4\pi (6)^2 = 4\pi (36) = 144\pi \text{ cm}^2$$

**Example 21**

21. A sphere has a volume of  $36\pi \text{ cm}^3$ . Find its radius and surface area.

**Solution:**

$$\text{Volume} = (4/3)\pi r^3 = 36\pi$$

$$r^3 = (36\pi \times 3) / (4\pi) = 27$$

$$r = \sqrt[3]{27} = 3 \text{ cm}$$

$$\text{Surface Area} = 4\pi(3^2) = 36\pi \text{ cm}^2$$

## 6.4 Surface Area and Volume of Composed Solids

## Strategy:

- ✍ Identify the individual solids, calculate their surface areas and volumes **separately**, and then **subtract** the smaller if it is removed or **add** if it is combined.
- ✍ Pay careful attention to overlapping surfaces for surface area if it is hidden; these must be subtracted to avoid double-counting. (These step works only when we calculate surface areas not volume)

Consider the following examples.

## Example 22

22. A solid figure is made of a circular cylinder of radius 4 cm at bottom and a right circular cone of altitude 3 cm, as shown in below. If the overall height is 10 cm, find

- total surface area of the solid
- the volume of the solid

## Solution:

The total surface area of the solid is the sum of the area of the three surfaces. These are the lateral surface of the cone, the lateral surface of the cylinder and the lower base of the cylinder.

- TSA of the solid = LSA of the cone + (LSA + Lower  $A_B$ ) of the cylinder

✍ Slant height of the cone is  $l = \sqrt{3^2 + 4^2} = 5$  cm

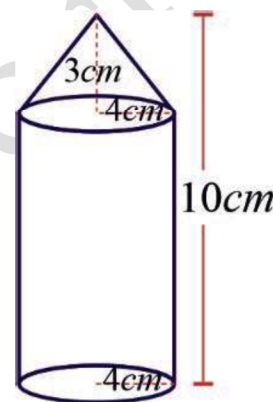
- LSA of the cone =  $\pi r l = 4 \times 5 \times \pi = 20\pi$  cm<sup>2</sup>,
- LSA of the cylinder =  $2\pi r h = 2\pi \times 4 \times 7 = 56\pi$  cm<sup>2</sup>, since  $h = 10 - 3 = 7$  cm
- Lower base area of the cylinder =  $\pi r^2 = 16\pi$  cm<sup>2</sup>,

$\therefore$  Total surface area of the solid =  $20\pi + 56\pi + 16\pi = 92\pi$  cm<sup>2</sup>.

- Total volume of the solid = Volume of the cylinder + Volume of the cone

$$= \pi r^2 h + \frac{1}{3} \pi r^2 h = (\pi \times 4^2 \times 7) + \left(\frac{1}{3} \pi \times 4^2 \times 3\right)$$

$$= 128 \pi \text{ cm}^3$$



## Example 23

23. Through a right circular cylinder whose base radius is 10 cm and whose height is 12 cm is drilled a triangular prism hole whose base has edges 3 cm, 4 cm and 5 cm as shown in the Figure. Find

- the total surface area of the remaining solid
- the volume of the remaining solid

## Solution:

TSA of the object = TSA of larger + LSA of smaller -  $2A_B$  of smaller

LSA(inner) =  $ph = (3+4+5)12\text{cm} = 144\text{cm}^2$

$2A_B = 2 \times \frac{1}{2}(3 \times 4) = 12 \text{ cm}^2$

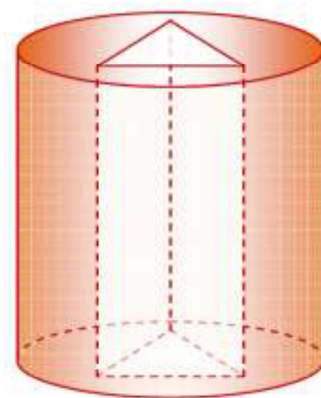
LSA(outer) =  $ph = 2\pi(10)12\text{cm} = 240\pi\text{cm}^2$

TSA(outer) = LSA +  $2A_B = 240\pi\text{cm}^2 + 2\pi(10^2) = 440\pi \text{ cm}^2$

$\therefore$  TSA of the object =  $440\pi \text{ cm}^2 + 144 - 12 \text{ cm}^2 = 572\pi \text{ cm}^2$

V of the object = Volume of larger - Volume of smaller

$\therefore$  V of the object =  $[\pi (10\text{cm})^2 \times 12] - \left[\frac{1}{2} (3\text{cm} \times 4\text{cm}) \times 12\right] = 1200\pi - 72 \text{ cm}^3$



## Example 24

24. Find the volume of a solid composed of a cylinder of radius 3 cm and height 10 cm with a hemisphere of radius 3 cm on top.

## Solution:

$V_{\text{cylinder}} = \pi r^2 h = \pi \times 9 \times 10 = 90\pi$

$V_{\text{hemisphere}} = \frac{2}{3} \pi r^3 = \frac{2}{3} \pi \times 27 = 18\pi$

$V_{\text{total}} = 90\pi + 18\pi = 108\pi \approx 339.29 \text{ cm}^3$

## 6.5 REAL LIFE APPLICATIONS

**Example 25**

25. An ice-cream consists of a hemisphere and a cone. Calculate its

- volume
- total surface area.

**Solution:**

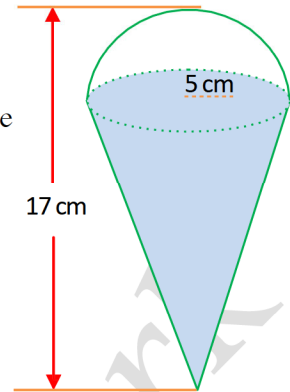
- volume of the ice-cream = Volume of the cone + Volume of the hemisphere

$$V \text{ of the object} = \frac{1}{3} [\pi (5\text{ cm})^2 \times 12] + \left[ \frac{2}{3} \pi (5\text{ cm})^3 \right] = \frac{550}{3} \pi \text{ cm}^3$$

- TSA of the solid = LSA of the cone + LSA of the hemisphere

$$= \frac{1}{2} (2\pi \times 5) \times 13 + (2\pi \times 5^2)$$

$$= 125\pi \text{ cm}^2$$

**Example 26**

26. A candle is made in the form of a circular cylinder of radius 4 cm at the bottom and a right circular cone of altitude 3 cm, as shown in Figure 7.47. If the overall height is 12 cm, Find

- total surface area of the candle
- the volume of the candle

**Solution:**

The total surface area of the solid is the sum of the area of three surfaces.

These are the lateral surface of the cone, the lateral surface of the cylinder and the lower base of the cylinder.

- TSA of the solid = LSA of the cone + (LSA + Lower  $A_b$ ) of the cylinder

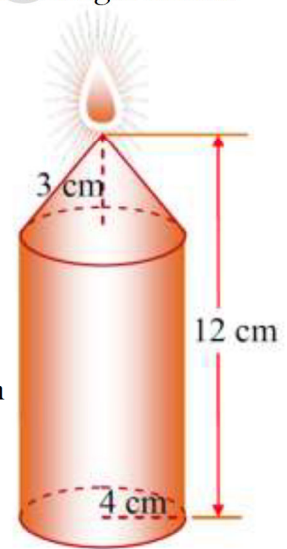
$$\text{Slant height of the cone is } l = \sqrt{3^2 + 4^2} = 5 \text{ cm}$$

- LSA of the cone =  $\pi r l = 4 \times 5 \times \pi = 20\pi \text{ cm}^2$ ,
- LSA of the cylinder =  $2\pi r h = 2\pi \times 4 \times 9 = 72\pi \text{ cm}^2$ , since  $h = 12 - 3 = 9 \text{ cm}$
- Lower base area of the cylinder =  $\pi r^2 = 16\pi \text{ cm}^2$ ,

$$\therefore \text{Total surface area of the solid} = 20\pi + 72\pi + 16\pi = 108\pi \text{ cm}^2.$$

- Total volume of the solid = Volume of the cylinder + Volume of the cone

$$\therefore V \text{ of the object} = [\pi (4\text{ cm})^2 \times 9] + \left[ \frac{1}{3} \pi (4\text{ cm})^2 \times 3 \right] = 160\pi \text{ cm}^3$$

**Example 27**

27. A cylindrical tank has a radius of 2 m and a height of 5 m. How much water can it hold in liters? ( $1 \text{ m}^3 = 1000 \text{ liters}$ )

**Solution:**

$$\text{Volume} = \pi r^2 h = \pi (2^2) (5) = 20\pi \text{ m}^3$$

$$\text{Water capacity} = 20\pi \times 1000 = 20,000\pi \text{ liters}$$

**Example 28**

28. A spherical balloon has a radius of 15 cm. How much air is needed to inflate it?

$$\text{Solution: Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (15^3) = 4500\pi \text{ cm}^3$$

**Example 29**

29. A conical tent has a radius of 4 m and a height of 3 m. How much canvas is needed to make it?

**Solution:**

$$\text{Slant Height (l)} = \sqrt{r^2 + h^2} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$\text{Lateral Surface Area} = \pi r l = \pi (4) (5) = 20\pi \text{ m}^2 \approx 62.83 \text{ m}^2$$

**Example 30**

30. A company manufactures cylindrical cans that are 10 cm high and have a radius of 4 cm. How much material is needed to make one can (including top and bottom)?

$$\text{Solution: Total Surface Area} = \pi r^2 h + 2\pi r^2 = \pi (4)^2 (10) + 2\pi (4)^2 = 502.65 \text{ cm}^2$$

### Miscellaneous Questions on Unit 6

1. Which of the following is NOT an example of solid figure?  
 A. Circle                      B. Cube                      C. Cone                      D. Sphere
2. The lateral surface area of a cylinder is given by:  
 A.  $\pi r^2 h$                       B.  $2\pi r h$                       C.  $\pi r^2 + 2\pi r h$                       D.  $2\pi r^2 + 2\pi r h$
3. A pyramid is named based on the shape of its:  
 A. Apex                      B. Lateral faces                      C. Base                      D. Height
4. The slant height of a pyramid is:  
 a) The height of the pyramid.                      b) The length of the base.  
 c) The height of a lateral face.                      d) The distance from the center of the base to a vertex.
5. The volume of a cylinder is given by:  
 A.  $\pi r h$                       B.  $2\pi r h$                       C.  $\pi r^2 h$                       D.  $(1/3)\pi r^2 h$
6. What is the volume of a cylinder with a radius of 3 cm and a height of 7 cm?  
 A.  $21\pi \text{ cm}^3$                       B.  $12\pi \text{ cm}^3$                       C.  $63\pi \text{ cm}^3$                       D.  $189\pi \text{ cm}^3$
7. What is the surface area of a sphere with a radius of 4 cm?  
 A.  $16\pi \text{ cm}^2$                       B.  $32\pi \text{ cm}^2$                       C.  $64\pi \text{ cm}^2$                       D.  $128\pi \text{ cm}^2$
8. What is a pyramid?  
 A. A solid with two congruent and parallel faces.  
 B. A solid with a circular base and a curved surface.  
 C. A solid with a polygonal base and triangular faces meeting at an apex.  
 D. A solid with a circular base and curved surface meeting at an apex.
9. The volume of a pyramid is given by:  
 A. (Area of Base) x Height                      C. (Perimeter of Base) x Height  
 B.  $(1/3) \times (\text{Area of Base}) \times \text{Height}$                       D.  $2 (\text{Area of Base}) + (\text{Perimeter of Base}) \times \text{Height}$
10. What is the volume of a cone with a radius of 4 cm and a height of 6 cm?  
 A.  $8\pi \text{ cm}^3$                       B.  $16\pi \text{ cm}^3$                       C.  $24\pi \text{ cm}^3$                       D.  $32\pi \text{ cm}^3$
11. What is the curved surface area of a cone with a radius of 5 cm and a slant height of 8 cm?  
 A.  $20\pi \text{ cm}^2$                       B.  $40\pi \text{ cm}^2$                       C.  $65\pi \text{ cm}^2$                       D.  $80\pi \text{ cm}^2$
12. A cube has sides of length 5 cm. Its total surface area is:  
 A.  $25 \text{ cm}^2$                       B.  $100 \text{ cm}^2$                       C.  $125 \text{ cm}^2$                       D.  $150 \text{ cm}^2$
13. What is the volume of a sphere with a radius of 3 cm?  
 A.  $12\pi \text{ cm}^3$                       B.  $36\pi \text{ cm}^3$                       C.  $4\pi \text{ cm}^3$                       D.  $110 \text{ cm}^3$
14. What is the slant height of a cone with radius 3 and height 4?  
 A. 7                      B. 25                      C. 5                      D. 1
15. A horizontal cross-section of a cone is always a:  
 A. Triangle                      B. Square                      C. Circle                      D. Rectangle
16. A horizontal cross-section of a pyramid is always similar to its:  
 A. Lateral face                      B. Apex                      C. Base                      D. Slant height
17. According to Cavalieri's Principle, two solids with the same height have equal volumes if:  
 A. They have the same base area.                      C. They are both oblique solids.  
 B. They are both right solids.                      D. Their cross-sections at every level have the same area.
18. The formula for the volume of a sphere is:  
 A.  $\pi r^2 h$                       B.  $4\pi r^2$                       C.  $(4/3)\pi r^3$                       D.  $2\pi r^3$
19. A swimming pool is in the shape of a rectangular prism. To calculate how much water it can hold, you need to find its:  
 A. Surface area                      B. Volume                      C. Perimeter                      D. Diagonal
20. A frustum is formed when:  
 A. Two cones are joined together.  
 B. A pyramid is folded.  
 C. The top portion of a cone or pyramid is cut off by a plane parallel to the base.  
 D. A cylinder is tilted.

21. To find the surface area of a composed solid, you generally need to:
- Add the volumes of all the solids.
  - Add the surface areas of all the solids.
  - Add the surface areas and subtract any areas that are hidden.
  - Multiply the volumes of all the solids.
22. What solid can be generated by rotating a rectangle about one of its sides?
- Sphere
  - Cone
  - Cylinder
  - Pyramid
23. What solid has only one curved surface?
- Cone
  - Cube
  - Prism
  - Sphere
24. Two spheres have radii 3 cm and 6 cm. What is the ratio of their volumes?
- 1:2
  - 1:4
  - 1:8
  - 1:16
25. A composite solid consists of a cube with a hemisphere placed on top. The total volume is found by adding:
- The surface area of the cube and the surface area of the hemisphere.
  - The volume of the cube and half the surface area of a sphere.
  - The volume of the cube and half the volume of a sphere.
  - The surface area of the cube and the volume of the hemisphere.
26. The amount of air in a ball is related to which measure?
- Surface Area
  - Volume
  - Perimeter
  - Diameter
27. Which of these does a Sphere not have?
- Diameter
  - Centre
  - Vertex
  - Radius
28. If the height of a pyramid is doubled, the volume will be:
- Halved
  - Doubled
  - Quadrupled
  - Unchanged
29. Cavalieri's Principle states that if two solids have the same height and the same cross-sectional area at every level, then they have the same:
- Surface area
  - Weight
  - Volume
  - Density
30. A hemisphere is:
- A quarter of a sphere.
  - Half of a sphere.
  - A flattened sphere.
  - A hollow sphere.
31. The total surface area of a hemisphere is:
- $2\pi r^2$
  - $3\pi r^2$
  - $4\pi r^2$
  - $\pi r^2$
32. A grain storage container is shaped like a cylinder with a cone on top. If the radius of the cylinder and cone is 5 meters, the height of the cylinder is 10 meters, and the height of the cone is 3 meters, what is the total volume of the container?
- $250\pi \text{ m}^3$
  - $275\pi \text{ m}^3$
  - $83.33\pi \text{ m}^3$
  - $325\pi \text{ m}^3$
33. An ice cream cone has a diameter of 6 cm and a height of 10 cm. If it's filled perfectly with ice cream (including a hemispherical scoop on top with the same diameter) what is the total volume of ice cream?
- $126\pi \text{ cm}^3$
  - $360\pi \text{ cm}^3$
  - $30\pi \text{ cm}^3$
  - $96\pi \text{ cm}^3$



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