



UNIT 6

ELECTROSTATICS AND ELECTRIC CIRCUIT

At the end of this unit, you should be able to:

- ♣ Calculate the electric field resulting from a point charge.
- ♣ Determine the magnitude and direction of the electric force among any point charges.
- ♣ Acquire knowledge and understanding in electrostatic phenomenon demonstrate an understanding of the components and functions of electrical circuits that are commonly found at home and in the workplace
- ♣ Construct, analyze, simple electrical circuits, using schematic diagrams, working with electrical tools and components, and examining small everyday electrical devices and appliances
- ♣ Define a capacitor and explain some of its applications in life.
- ♣ Apply Kirchhoff's rules to solve circuit problems.
- Matter is made of certain elementary particles.
- With the advancement in technology, we have discovered hundreds of elementary particles.
- The three most common elementary particles are electrons, protons and neutrons having masses

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67262 \times 10^{-27} \text{ kg}$$

$$m_n = 1.67493 \times 10^{-27} \text{ kg}$$

- Because of their mass these particles attract each other by **gravitational forces**.
- Thus, an electron attracts another electron, placed 1 cm away, with a gravitational force

$$F = G \frac{m_1 m_2}{r^2}$$

$$F = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \frac{(9.1 \times 10^{-31} \text{ kg})^2}{(10^{-2})^2} = 5.5 \times 10^{-67} \text{ N}$$

- However, an electron is found to repel another electron at 1 cm with a force of $2.3 \times 10^{-24} \text{ N}$.
- This extra force is called the electric force.
- The electric force is very large as compared to the gravitational force.
- The electrons must have some additional property, apart from their mass, which is responsible for the electric force. This is called property of charge
- Just as masses are responsible for the gravitational force, charges are responsible for the electric force.

6.1. Coulomb's Law

By the end of this section, you will be able to:

- ♣ State Coulomb's law.
- ♣ Calculate the magnitude and direction of electric force between any two charges.
- ♣ Solve problems involving Coulomb's law.
- ♣ Explain Coulomb's law using the idea of vectors.
- ♣ Explain the meaning of a coulomb.

Brainstorming Questions:

1. What are the compositions of an atom?
2. What are the two types of charges and what are their origins?

3. How can a body be charged positively and negatively?

4. What does the law of electrostatics say?

Properties of Electric Charges

- Charge is the fundamental property of matter associated with electrons and protons which are found in any atom.
- There are only two types of charges: negative and positive charges.
- By convention, negative charges are carried by electrons and positive charges are carried by protons.
- When the number of electrons and protons in an object are equal that object is said to be electrically neutral.
- However, if this electrical balance is disturbed, then the object is said to be electrically charged.
- A negatively charged object contains more electrons than protons, but a positively charged body contains more protons than electrons.
- The properties of charges are:
 1. **There are two kinds of electric charges: Positive and Negative.**
 2. **Two similarly charged bodies repel each other and two oppositely charged bodies attract each other.**
 3. **Electric charge is always conserved.**
 - Charge cannot be created or destroyed.

Note: The charge of an isolated system is conserved.

- ▶ It is possible to create or destroy charged particles but it is not possible to create or destroy *net charge*.
- ▶ In a beta decay process, a neutron converts itself into a proton and a fresh electron is created. The charge however, remains zero before and after the event.

- In any charging process, the number of electrons gained by one object is equal to the number of electrons lost by the other.

Note

- ▶ By convention, charge on glass rod and fur is positive and charge on silk cloth and plastic rod is negative.

4. Electric charge is quantized.

- Charge comes in discrete packets, it means that charge (q) always occurs as some integral multiple of the charge of an electron, e .

$$Q = \pm ne$$

$$n = 1, 2, 3, \dots$$

- The smallest unit of charge that is carried by one electron or proton is called the elementary charge, e . The magnitude of the charge of an electron is:

$$1e = -1.6 \times 10^{-19} C$$

- The charge carried by a proton is also the same as that of the charge carried by an electron except it is positive.

$$1p = +1.6 \times 10^{-19} C$$

- If protons and electrons are the only charge carriers in the universe, all observable charges must be integral multiples of e .
- If an object contains n_1 protons and n_2 electrons, the net charge on the object is

$$n_1 e + n_2(-e) = (n_1 - n_2)e$$

Example

1. How many electrons are there in 1 coulomb of charge?

Solution

$$n = \frac{Q}{e} = \frac{1C}{1.6 \times 10^{-19} C} = 6.25 \times 10^{18} \text{ electrons}$$

2. If 1.875×10^{19} electrons are transferred from a cloth to a plastic rod, how much charge did the rod acquire?

Solution

$$Q = (1.875 \times 10^{19})(1.6 \times 10^{-19} C)$$

$$Q = 3C$$

3. Two spheres carried charges of $-11 \times 10^{-11} C$ and $5 \times 10^{-11} C$. If they made contact and then separated from each other. Calculate the final charge on each sphere and how many excess electrons are there on each sphere?

Solution

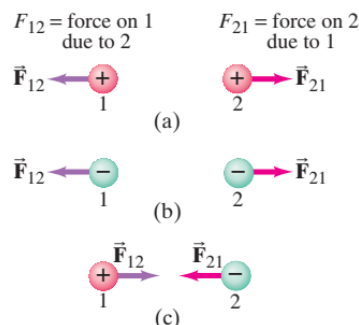
$$Q_{net} = \frac{Q_1 + Q_2}{2}$$

$$Q_{net} = \frac{-11 \times 10^{-11} C + 5 \times 10^{-11} C}{2} = -3 \times 10^{-11} C$$

$$n = \frac{Q}{e} = \frac{-3 \times 10^{-11} C}{-1.6 \times 10^{-19} C} = 1.875 \times 10^8 \text{ electrons}$$

Electrostatic Force

- **Electrostatic** is a branch of physics that studies slow moving or stationary electric charges.
- The force between charges that are at rest is called **electrostatic force**.
- An electrostatic phenomenon arises from the forces that electric charges exert on each other and are described by Coulomb's law.
- A force of attraction or repulsion between two charged bodies is known as **the electrostatic force**.
- Electrostatic force is either attractive or repulsive, unlike the gravitational force which is only attractive.
- If the objects have opposite charges they attract each other, while if their charges are similar, they repel each other.



- The direction of the static electric force one point charge exerts on another is always along the line joining the two charges, and depends on whether the charges have the same sign or not.
 - ▶ The amount of charge with which two charges are repelling or attracting each other is determined by **coulombs law**

Coulomb's law state that:

"The force between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them"

If two charges Q_1 and Q_2 are at a distance r apart, then the electrostatic force between them is given by:

Where K is called the electrostatic constant.

$$F = K \frac{Q_1 Q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

ϵ_0 is epsilon which is permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$$

$$k = 9 \times 10^9 \frac{Nm^2}{C^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$F = \text{Positive}$ for **Repulsive**

$F = \text{Negative}$ for **Attractive**

Example

- Two point charges $Q_1 = 3 \times 10^{-9} C$ and $Q_2 = 5 \times 10^{-9} C$ are 2m apart. Determine the magnitude of force between them.

Given

$$Q_1 = 3 \times 10^{-9} C$$

$$Q_2 = 5 \times 10^{-9} C$$

$$r = 2m$$

Required

$$F = ?$$

Solution

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$$F = 9 \times 10^9 \frac{Nm^2}{C^2} \left(\frac{3 \times 10^{-9} C \times 5 \times 10^{-9} C}{2^2} \right)$$

$$F = 9 \times 10^9 \frac{Nm^2}{C^2} \left(\frac{15 \times 10^{-18} C}{4} \right) = \frac{135}{4} \times 10^{-9} C$$

$$F = 33.75 \times 10^{-9} N$$

$$F = 3.375 \times 10^{-8} N$$

2. Compare the magnitude of the electric force between two charges of each charge 1C separated by a distance of 1m with the magnitude of the gravitational force if each have a mass of 1kg and placed 1m apart

Solution

$$F_e = k \frac{Q_1 Q_2}{r^2} = 9 \times 10^9 \frac{Nm^2}{C^2} \frac{(1C)(1C)}{(1m)^2} = 9 \times 10^9 N$$

$$F_g = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \frac{(1kg)(1kg)}{(1)^2} = 6.67 \times 10^{-11} N$$

Taking the ratio: $\frac{F_e}{F_g} = \frac{9 \times 10^9 N}{6.67 \times 10^{-11} N} = 1.3 \times 10^{20} \Rightarrow F_e = 1.3 \times 10^{20} F_g$

The electrostatic force is about 10^{20} times the gravitational force

EXERCISE:

1. Calculate the electrostatic force between two charges of $3 \times 10^{-7} C$ each placed 10mm apart
2. Two charges 6nC and 3nC are placed 30mm apart. What is the magnitude and nature of the force acting on the charges?
3. When charges $Q_1 = 6\mu C$ and $Q_2 = 15\mu C$ are separated at some distance, they experience an electric 900N. What is the separation between the charges?

Similarities between Newton's law of universal gravitational and Coulomb's law

1. Both obeys inverse square law
2. Both forces operate in vacuum
3. Both forces are conservative force
4. Both forces having central forces- always directed along the line connecting the centers of the objects.

Difference between Newton's law of universal gravitational and Coulomb's law

The following table compares electric force vs. gravitational force

Electric force	Gravitational force
Force occurs between charged objects	Force occurs between any object with mass
Can involve attraction (pull towards) or repulsion (push away)	Only involves attraction
Force originates from the object's charge and can vary depending on the medium	Force originates from the object's mass and does not vary depending on the medium
Stronger than gravitational force	Weaker than electric force

Force on a Charge due to multiple electric charges

- The electric force between charged particles at rest (sometimes referred to as the electrostatic force or as the Coulomb force) is, like all forces, a vector: it has both magnitude and direction.
- When several forces act on an object $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots$ the net force ($\vec{F}_{net}$) on the object is the vector sum of all the forces acting on it:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots \Rightarrow \text{This is the **principle of superposition** for forces.}$$

- Consider a system of n stationary charges $Q_1, Q_2, Q_3, \dots, Q_n$ in vacuum.
- The force on Q_1 due to $Q_2, Q_3, \dots, Q_n$ is obtained by taking the vector sum of the individual forces.

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1n}$$

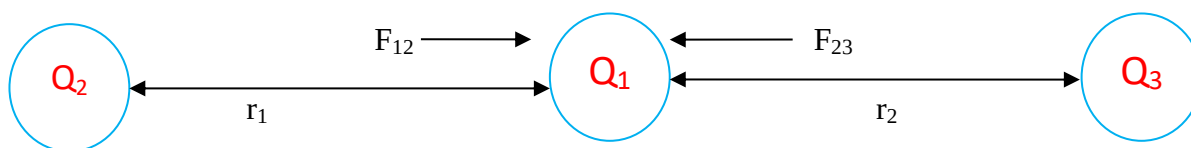
Where, $\vec{F}_{12}$ is the force that charge Q_2 exert on Q_1

$\vec{F}_{13}$ is the force that Q_3 exert on Q_1

$\vec{F}_{14}$ is the force that Q_4 exert on Q_1

$\vec{F}_{1n}$ is the force that Q_n exert on Q_1

When more than one charge exerts a force on another charge, the total force on that charge is the sum of the individual forces, taking into account both their sizes and direction.



The force exerted by Q_2 on Q_1 is given by:

$$F_{12} = k \frac{Q_1 Q_2}{r_1^2}$$

The force exerted by Q_3 on Q_1 is given by:

$$F_{13} = k \frac{Q_1 Q_3}{r_2^2}$$

The net force on Q_1 is:

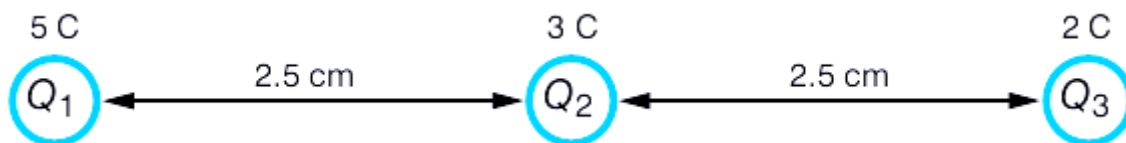
$$F_{net} = F_{12} + F_{13}$$

$$F_{net} = k \frac{Q_1 Q_2}{r_1^2} + \left(-k \frac{Q_1 Q_3}{r_2^2} \right)$$

$$F_{net} = k \frac{Q_1 Q_2}{r_1^2} - k \frac{Q_1 Q_3}{r_2^2}$$

Example

1. Three charges are arranged in a line with 2.5 cm between them, as shown in Figure below.



What is the force exerted on Q_2 by the other two charges?

Solution

Consider the forces exerted on Q_2 by the other two:

From Q_1 : $F_{12} = k \frac{Q_1 Q_2}{r_1^2}$

$$F_{12} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{5\text{C} \times 3\text{C}}{(2.5 \times 10^{-2} \text{m})^2} \right)$$

$$F_{12} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{15\text{C}^2}{6.25 \times 10^{-4} \text{m}^2} \right)$$

$$F_{12} = \frac{135 \times 10^9}{6.25 \times 10^{-4}} \text{N}$$

$$F_{12} = 21.6 \times 10^{13} \text{N} = 2.16 \times 10^{14} \text{N}$$

From Q_2 : $F_{12} = k \frac{Q_2 Q_3}{r_2^2}$

$$F_{23} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{3\text{C} \times 2\text{C}}{(2.5 \times 10^{-2} \text{m})^2} \right)$$

$$F_{23} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{6\text{C}^2}{6.25 \times 10^{-4} \text{m}^2} \right)$$

$$F_{23} = \frac{64 \times 10^9}{6.25 \times 10^{-4}} \text{N}$$

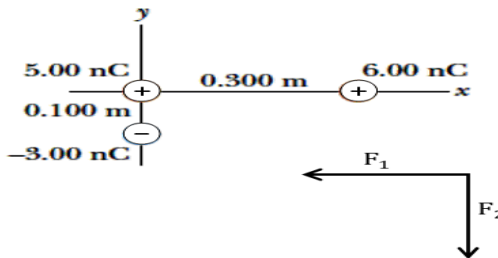
$$F_{23} = 8.64 \times 10^{13} \text{N} = 0.864 \times 10^{14} \text{N}$$

$$F_{\text{net}} = F_{12} - F_{23}$$

$$F_{\text{net}} = 2.16 \times 10^{14} \text{N} - 0.864 \times 10^{14} \text{N}$$

$$F_{\text{net}} = 1.296 \times 10^{14} \text{N}$$

2. Three point charges are arranged as shown in figure below. Find **the magnitude** of the net electrostatic force on the point charge at the origin.



Given

$$Q_1 = 6\text{nC} = 6 \times 10^{-9} \text{C}$$

$$Q_2 = 5\text{nC} = 5 \times 10^{-9} \text{C}$$

$$Q_3 = -3\text{nC} = -3 \times 10^{-9} \text{C}$$

$$r_1 = 0.3\text{m}$$

$$r_2 = 0.1\text{m}$$

Required

$$F_{\text{net}} = ?$$

Solution

$$F_1 = 9 \times 10^9 \times \frac{5 \times 10^{-9} \times 6 \times 10^{-9}}{(0.3)^2}$$

$$= \frac{270 \times 10^{-9}}{0.09} = 3 \times 10^{-6} \text{N}$$

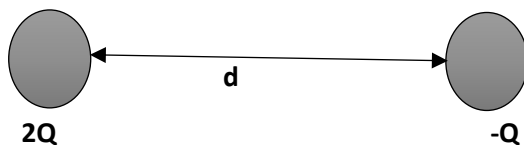
$$F_2 = 9 \times 10^9 \times \frac{5 \times 10^{-9} \times 3 \times 10^{-9}}{(0.1)^2} = \frac{1.35 \times 10^{-9}}{(0.1)^2} = 1.35 \times 10^{-5} \text{N}$$

$$F_{\text{net}} = \sqrt{F_1^2 + F_2^2} = \sqrt{(3 \times 10^{-6})^2 + (1.35 \times 10^{-5})^2} = \sqrt{1.912 \times 10^{-10}}$$

$$F_{\text{net}} = 1.38 \times 10^{-5} \text{N}$$

Exercise

1. Two identical conducting spheres are charged to $+2Q$ and $-Q$, respectively, and are separated by a distance d (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is F_1 . After the two spheres are made to touch and then are re-separated by distance d , the magnitude of the force on the left sphere is F_2 . Which of the following relationships is correct?



A. $2F_1 = F_2$

B. $F_1 = F_2$

C. $F_1 = 2F_2$

D. $F_1 = 8F_2$

6.2. Electric Fields

By the end of this section, you will be able to:

- ♣ Define the terms, electric field, and electric flux.
- ♣ Sketch electric field lines.
- ♣ Solve problems involving electric field.
- ♣ Map an electric field lines pattern using electric lines of force.
- ♣ Calculate the magnitude and direction of electric field due to a point charge and two point charges.

Brainstorming Questions

1. What is a gravitational field?
2. What causes gravitational field?
3. Discuss the concepts and recall the equations of:
 - A. Gravitational field lines, and
 - B. Gravitational field strength.

Answers

1. The region of space around every body of matter where the gravitational force could be experienced by other body is called **a gravitational field (gravitational acceleration field)**.
2. Gravitational fields are caused by the presence of mass or energy in space.
 - According to Einstein's theory of general relativity, mass and energy curve the fabric of space-time, creating what we perceive as a gravitational field. This curvature of space-time causes objects with mass or energy to follow curved paths, which we interpret as the force of gravity.
 - The more mass or energy an object has, the stronger its gravitational field. This is why larger objects, such as planets or stars, have stronger gravitational fields than smaller objects like asteroids or moons. Additionally, the distance between objects also affects the strength of the gravitational field. The closer two objects are to each other, the stronger the gravitational attractions between them.
 - It's important to note that the concept of gravitational fields is a way of describing the effects of gravity mathematically.
 - In reality, gravity is a fundamental force of nature, and the exact nature of how mass and energy interact to create gravity is still an area of active research in physics.
3. Gravitational field lines are a way to visualize the gravitational field surrounding an object.
 - They are imaginary lines that represent the direction and strength of the gravitational force at different points in space.

- Gravitational field strength can be calculated as:

$$g = G \frac{M_E}{r^2}$$

G = Gravitational constant

M_E = Mass of the earth

r = The distance between two mass

- An electric field is the physical field that surrounds electrically charged particles.
- **An electric field** is a region in space where one charge experiences a force from another charge.
- The electric field produced by a charged particle can be represented by lines of force called electric field lines.
- Electric field lines are helpful to visualize the vector nature of an electric field.

Properties of electric Field lines

Electric field lines have the following properties

1. Electric field lines never cross each other.
2. Electric field lines start from the positive charge and end at a negative charge.
- In case of single charge electric field lines end at infinity.
3. Electric field lines are close together where the electric force is stronger.
4. Electric field lines never form a closed loop because electric field is conservative in nature of electric field (i.e. work done by electric field depends on final and initial position and not the path followed).
5. Electric field is uniform inside a charged parallel plate.
6. Electric field due to a point charge is not uniform. It varies from point to point.
7. Electric field lines always start at right angle (perpendicular) to the charged object (surface) causing the field.
8. The direction of the electric field is the direction of the force on a **positive test charge**.

Types of an Electric Field

- The electric field is mainly classified into two types.

1. uniform electric field and

2. No uniform electric field.

1. Uniform Electric Field

- ◆ When the electric field is constant at every point, then the field is called **the uniform electric field**.
- ◆ The constant field is obtained by placing the two conductors parallel to each other, and the potential difference between them remains same at every point.

2. Non-Uniform Electric Field

- The field which is irregular at every point is called the non-uniform electric field.
- The non-uniform field has a different magnitude and directions.

The Electric field strength

- The electric field ($\vec{E}$), at any point in space is defined as the force ($\vec{F}$) exerted on a tiny positive test charge placed at that point divided by the magnitude of the test charge q .

$$\vec{E} = \frac{\vec{F}}{q}$$

$\vec{E}$ = electric field strength in $\frac{N}{C}$

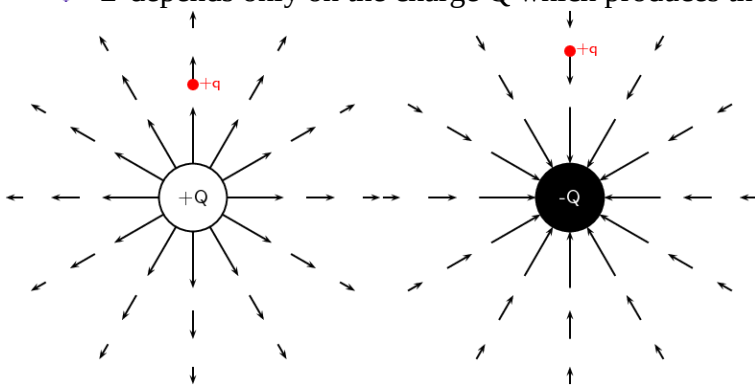
$\vec{F}$ = force acting the positive test charge in N

q = charge of the positive test charge in C

$$\vec{E} = k \frac{Q}{r^2} \Rightarrow \text{Single point charge}$$

Notice:

- ◆ $\vec{E}$ is independent of the test charge q
- ◆ $\vec{E}$ depends only on the charge Q which produces the field.



- The direction of electric field is in the direction of electric force.
- The electric field due to a positive charge points **away from the charge**, whereas due to a **negative charge** points toward that charge.

Example

1. An electron experiences a force of $6\mu\text{N}$ when passing through an electric field. Calculate the electric field strength

$$\vec{E} = \frac{\vec{F}}{q} = \frac{6 \times 10^{-6} \text{ N}}{1.6 \times 10^{-19} \text{ C}} = 0.375 \times 10^{13} \frac{\text{N}}{\text{C}}$$

2. A uniform charged sphere exerts a force $\vec{F} = 12i\text{N} + 6j\text{N} - 4k\text{N}$ on a test charge of $2\mu\text{C}$. Find the electric field produced by the charged body at the position of the test particle.

Solution

$$\vec{E} = \frac{\vec{F}}{q} = \frac{12i\text{N} + 6j\text{N} - 4k\text{N}}{2 \times 10^{-6} \text{ C}} = 6 \times 10^6 i\text{N} + 3 \times 10^6 j\text{N} - 2 \times 10^6 k\text{N}$$

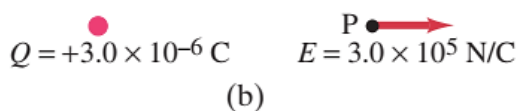
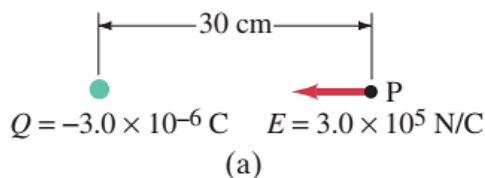
3. Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge $-3 \times 10^{-6} \text{ C}$.

Solution

$$\vec{E} = k \frac{Q}{r^2}$$

$$\vec{E} = \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{-3 \times 10^{-6} \text{ C}}{(30 \times 10^{-2})^2} = -3 \times 10^5 \frac{\text{N}}{\text{C}}$$

- The direction of the electric field is toward the charge Q .



4. A positive test charge of 30C produces an electric field of 5 N/C, what is the magnitude and direction of the charge?

Solution

$$\vec{F} = \vec{E}q = 150N$$

- Since the charge is positive, the direction of the force is in the direction of the electric field.
5. An electron initially at rest accelerated through 1cm by an electric field of 30kN/C. what speed will it acquire in the region? How long does it take to move this distance?

Solution

$$\vec{F} = \vec{E}q$$

From Newton's second law $F = ma$

$$ma = \vec{E}q$$

$$ma = \vec{E}e$$

$$a = \frac{\vec{E}e}{m}$$

$$a = \frac{\left(30 \times 10^3 \frac{N}{C}\right) (1.6 \times 10^{-19} C)}{9.1 \times 10^{-31} kg} = 5.27 \times 10^{15} \frac{m}{s^2}$$

$$v^2 = u^2 + 2as$$

$$v^2 = 2as$$

$$v^2 = 2 \left(5.27 \times 10^{15} \frac{m}{s^2} \right) (10^{-2} m) = 1.054 \times 10^{14} \frac{m^2}{s^2}$$

$$v = \sqrt{1.054 \times 10^{14} \frac{m^2}{s^2}} = 1.026 \times 10^7 \frac{m}{s}$$

$$v = u + at$$

$$v = at$$

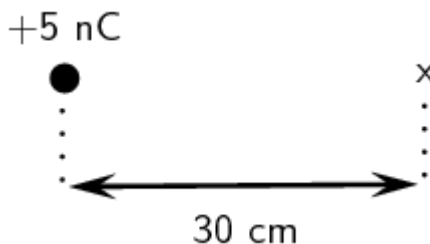
$$t = \frac{v}{a}$$

$$t = \frac{1.026 \times 10^7 \frac{m}{s}}{5.27 \times 10^{15} \frac{m}{s^2}}$$

$$t = 1.953 \times 10^{-9} sec$$

$$t = 1.953 \text{ nanosec}$$

6. Determine the magnitude and direction of an electric field 30cm from +5nC charge.



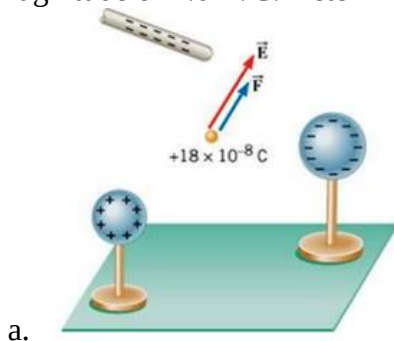
Solution

$$\vec{E} = k \frac{Q}{r^2}$$

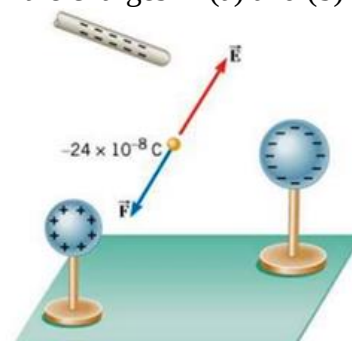
$$\vec{E} = \left(9 \times 10^9 \frac{Nm^2}{C^2} \right) \frac{5 \times 10^{-9} C}{(30 \times 10^{-2})^2} = 500 \frac{N}{C} \text{ to the right}$$

Exercise

- The positive test charge has a magnitude of $3.0 \times 10^{-8} C$ and experiences a force of $6.0 \times 10^{-8} N$. Calculate the electric field that the test charge experiences.
- The charges on the two metal spheres and the ebonite rod create an electric field at the spot indicated. The field has a magnitude of 2.0 N/C. Determine the force on the charges in (a) and (b)



a.



b.

Electric field for multiple point charges

- If the electric field at a given point in space is due to more than one charge, the individual fields ($\vec{E}_1, \vec{E}_2, \vec{E}_3, \dots$) due to each charge are added vectorially to get the total field at that point:

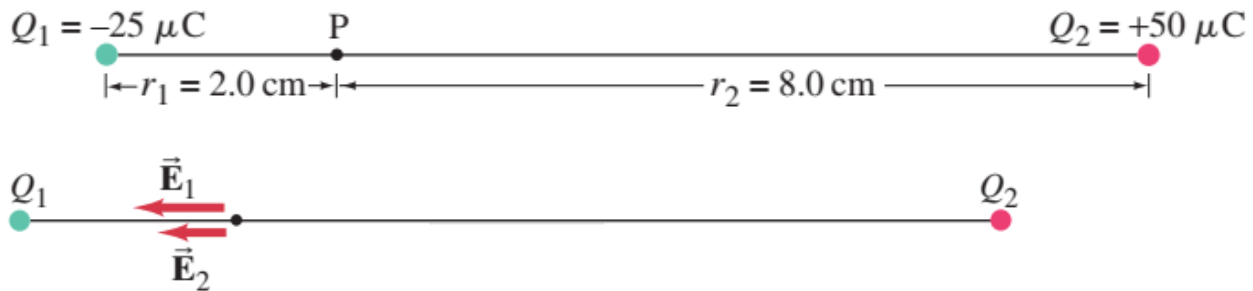
$$\vec{E}_{net} = |\vec{E}_1| + |\vec{E}_2| + |\vec{E}_3| + \dots + |\vec{E}_n| \Rightarrow \text{This superposition principle for electric fields}$$
- The validity of this superposition principle for electric fields is fully confirmed by experiment.

Example

- Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25\mu\text{C}$ and the other $+50\mu\text{C}$. Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge.

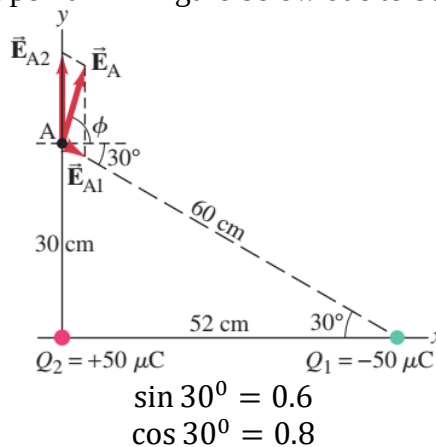
Solution

$$\vec{E}_{net} = |\vec{E}_1| + |\vec{E}_2|$$



$$\begin{aligned} \vec{E}_{net} &= |\vec{E}_1| + |\vec{E}_2| \\ \vec{E}_{net} &= k \frac{Q_1}{r_1^2} + k \frac{Q_2}{r_2^2} \\ \vec{E}_{net} &= \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{25 \times 10^{-6} \text{C}}{(2 \times 10^{-2})^2} + \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{50 \times 10^{-6} \text{C}}{(8 \times 10^{-2})^2} \\ \vec{E}_{net} &= 56.25 \times 10^7 \frac{\text{N}}{\text{C}} + 7.3125 \times 10^7 \frac{\text{N}}{\text{C}} \\ \vec{E}_{net} &= 63.5625 \times 10^7 \frac{\text{N}}{\text{C}} \end{aligned}$$

- Calculate the total electric field at point A in Figure below due to both charges Q_1 and Q_2 .



Solution

- The magnitude of the electric field produced at point A by each of the charges Q_1 and Q_2 is given by:

$$\vec{E} = k \frac{Q}{r^2}$$

$$\vec{E}_1 = \left(9 \times 10^9 \frac{Nm^2}{C^2}\right) \frac{50 \times 10^{-6} C}{(60 \times 10^{-2})^2} = 1.25 \times 10^6 \frac{N}{C}$$

$$\vec{E}_2 = \left(9 \times 10^9 \frac{Nm^2}{C^2}\right) \frac{50 \times 10^{-6} C}{(30 \times 10^{-2})^2} = 5 \times 10^6 \frac{N}{C}$$

➤ The direction of $\vec{E}_1$ points from A toward Q_1 (negative charge), whereas $\vec{E}_2$ points from A away from Q_2 as shown.

➤ The component of $\vec{E}_1$ can be calculated as:

$$\begin{aligned}\vec{E}_{1x} &= \vec{E}_1 \cos \theta i \\ \vec{E}_{1x} &= \left(1.25 \times 10^6 \frac{N}{C}\right) \cos 30^\circ i \\ &= 1 \times 10^6 \frac{N}{C} \hat{i}\end{aligned}$$

$$\begin{aligned}\vec{E}_{1y} &= \vec{E}_1 \sin \theta i \\ \vec{E}_{1y} &= \left(1.25 \times 10^6 \frac{N}{C}\right) \sin 30^\circ i \\ &= 0.75 \times 10^6 \frac{N}{C} \hat{j}\end{aligned}$$

➤ so the total electric field at A $\vec{E}$

$$\begin{aligned}\vec{E}_x &= 1 \times 10^6 \frac{N}{C} \hat{i} \\ \vec{E}_y &= \vec{E}_{2y} - \vec{E}_{1y} \hat{j}\end{aligned}$$

$$\begin{aligned}\vec{E}_y &= 5 \times 10^6 \frac{N}{C} \hat{j} - 0.75 \times 10^6 \frac{N}{C} \hat{j} \\ &= 4.25 \times 10^6 \frac{N}{C} \hat{j}\end{aligned}$$

$$\vec{E} = 1 \times 10^6 \frac{N}{C} \hat{i} + 4.25 \times 10^6 \frac{N}{C} \hat{j}$$

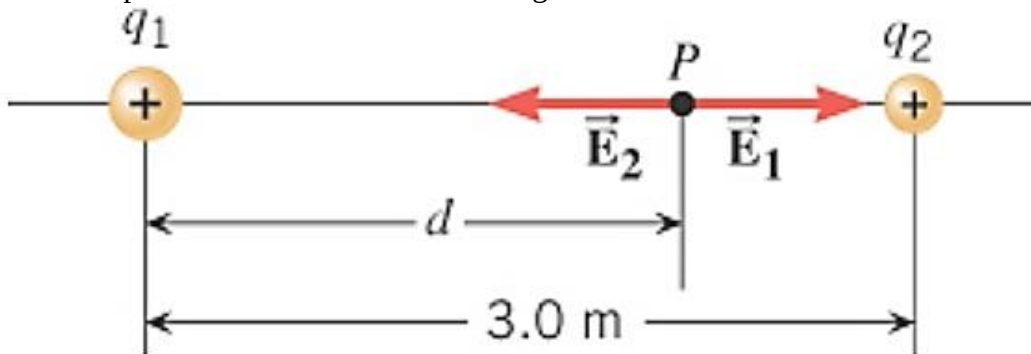
$$|\vec{E}| = \sqrt{\left(1 \times 10^6 \frac{N}{C}\right)^2 + \left(4.25 \times 10^6 \frac{N}{C}\right)^2}$$

$$|\vec{E}| = \sqrt{1 \times 10^{12} \frac{N^2}{C^2} + 18.0625 \times 10^{12} \frac{N^2}{C^2}}$$

$$|\vec{E}| = \sqrt{19.0625 \times 10^{12} \frac{N^2}{C^2}}$$

$$|\vec{E}| = 4.36 \times 10^6 \frac{N}{C}$$

3. Two positive point charges, $q_1 = +16\mu C$ and $q_2 = +4.0\mu C$ are separated in a vacuum by a 3.0m. Find the spot on the line between the charges where the net electric field is zero.



Solution

$$\begin{aligned}\vec{E} &= k \frac{Q}{r^2} \\ \vec{E}_1 &= k \frac{16 \times 10^{-6} C}{d^2} \\ \vec{E}_2 &= k \frac{4 \times 10^{-6} C}{(3-d)^2} \\ \vec{E}_{net} &= \vec{E}_1 - \vec{E}_2 = 0 \\ \vec{E}_1 &= \vec{E}_2\end{aligned}$$

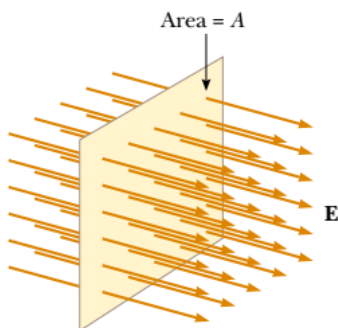
$$\begin{aligned}k \frac{16 \times 10^{-6} C}{d^2} &= k \frac{4 \times 10^{-6} C}{(3-d)^2} \\ \frac{16}{d^2} &= \frac{4}{(3-d)^2} \\ 16(3-d)^2 &= 4d^2 \\ 16(9-6d+d^2) &= 4d^2 \\ 4(9-6d+d^2) &= d^2 \\ 36-24d+4d^2 &= d^2 \\ 36-24d+4d^2-d^2 &= 0 \\ 36-24d+3d^2 &= 0\end{aligned}$$

$$\begin{aligned}
 12 - 8d + d^2 &= 0 \\
 d^2 - 8d + 12 &= 0 \\
 d &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 d &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} \\
 d &= \frac{8 \pm \sqrt{64 - 48}}{2} \\
 d &= \frac{8 \pm \sqrt{16}}{2}
 \end{aligned}$$

$$\begin{aligned}
 d &= \frac{8 \pm 4}{2} \\
 d &= \frac{8+4}{2} \text{ or } \frac{8-4}{2} \\
 d &= 6m \text{ or } 2m \\
 \text{Answer is } d &= 2m \\
 d = 6m &\text{ is to the right of } q_2. \\
 \text{So it is extraneous solution}
 \end{aligned}$$

Electric flux

- Electric flux is a measure of the number of electric field lines that crosses a given area.



- One of the properties of electric field lines is that, the number of field lines crossing a unit area, placed normal to the field at a point, is a measure of the strength of electric field at that point.
- This means that the number of field lines per unit area, is proportional to the magnitude of the electric field.

$$E \propto \frac{N}{A}$$

- The number of field lines is proportional to the product of E and A.

$$N \propto EA$$

- The product of the electric field and the surface area A, perpendicular to the field is called **the electric flux**.

$$\phi = EA$$

- In a vector form, the above equation can be written as the dot product of the electric field vector and area vector.

$$\phi = \vec{E} \cdot \vec{A}$$

- The area vector of a flat surface of area A has the following magnitude and direction:

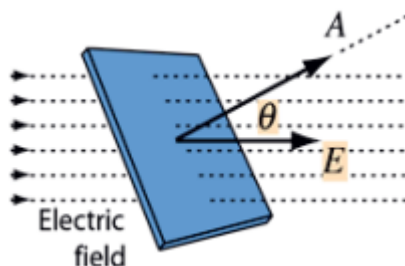
- ◆ Magnitude is equal to area of the surface (A)
- ◆ Direction along normal or perpendicular to the surface.

- If the surface under consideration is not perpendicular to the field the expression for the electric flux became

$$\phi = EA \cos \theta$$

$E \cos \theta$ is the projection of area normal to E.

θ is the angle between the electric field lines and the normal (perpendicular) to the surface.



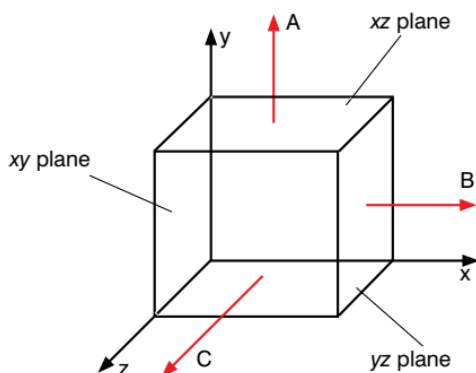
- This means, if the electric field vectors are tangent to the surface at all points, that is if they don't penetrate the surface, the electric flux through the surface is zero.
- Electric flux ϕ is a scalar quantity, whose SI unit is $\frac{Nm^2}{C}$.

Example

1. A $2cm \times 2cm$ square lies in the xy -plane. Find the electric flux through the square for the electric field vectors $(\vec{E} = 50\hat{k} + 20\hat{j}) \frac{N}{C}$

Solution

- In this case direction of the area vector, which is perpendicular to the surface, is in the z -axis. Because the square lies in the xy -plane.

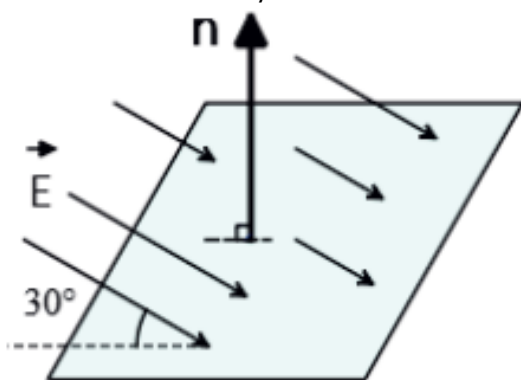


$$\begin{aligned}\phi &= \vec{E} \cdot \vec{A} \\ \phi &= (50\hat{k} + 20\hat{j}) \frac{N}{C} \cdot (4 \times 10^{-4} m^2 \hat{k}) \\ \phi &= (50\hat{k} + 20\hat{j}) \frac{N}{C} \cdot (4 \times 10^{-4} m^2 \hat{k}) \\ \phi &= 200 \times 10^{-4} \frac{Nm^2}{C} \\ \phi &= 0.02 \frac{Nm^2}{C}\end{aligned}$$

$$\vec{A} = (2cm \times 2cm) \hat{k}$$

$$\vec{A} = 4cm^2 \hat{k} = 4 \times 10^{-4} m^2 \hat{k}$$

2. Find the electric flux through the surface with sides of $15cm \times 15cm$ positioned in a uniform electric field of $E = 150N/C$ as shown in the figure below.



Solution

$$\begin{aligned}\phi &= EA \cos \theta \\ A &= 15cm \times 15cm = 225cm^2 \\ \phi &= \left(150 \frac{N}{C}\right) (225 \times 10^{-4} m^2) \cos 60^\circ \\ \phi &= 1.6875 \frac{Nm^2}{C}\end{aligned}$$

3. A rectangular surface of sides 10 cm and 15 cm is placed in a uniform electric field of $120 \frac{N}{C}$ such that the surface makes an angle of 30° with the direction of electric field. Find the flux through the rectangular surface.

Solution

$$\phi = EA \cos \theta$$

$$A = 10 \text{ cm} \times 15 \text{ cm} = 150 \text{ cm}^2$$

$$\phi = \left(120 \frac{N}{C}\right) (150 \times 10^{-4} \text{ m}^2) \cos 60^\circ$$

$$\phi = 0.9 \frac{Nm^2}{C}$$

6.3. Electric Potential

By the end of this section, you will be able to:

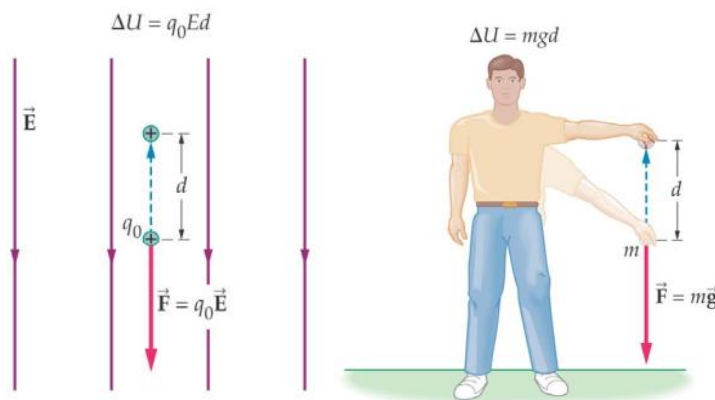
- ♣ Define the terms electric potential, equipotential surface.
- ♣ Solve problems involving electric potential.
- ♣ Explain the meaning of a volt, potential difference, and emf.

Brainstorming Questions

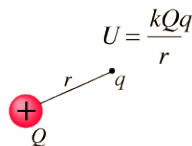
1. Discuss the concepts and recall the equations of (a) gravitational potential, and (b) gravitational potential energy.
 - **Gravitational potential** is defined as the amount of work done by the gravitational field per unit mass in bringing a small test mass from infinity to that point, without any change in its kinetic energy.
 - **Gravitational potential energy** is the energy possessed by an object due to its position in a gravitational field.
2. What is the relationship between the following pair of quantities?
 - A. Electric potential and electrical potential energy?
 - B. Electrical potential energy and electric work?
 - C. Electric potential and potential difference?
 - D. Potential difference and electric field?
 - E. Electrical potential and gravitational potential?

Electrical potential energy

- Electric charges have fields around them and thus a charge has to do work if it intends to change its position.
- This work is the electric energy or the electric potential energy of the charge.
- Just like mass in a gravitational field has gravitational potential energy, charges in an electric field have an electric potential energy.
- Electric potential energy is the energy that is needed to move a charge against an electric field.



- A positive charge q_0 is placed in the field. If the charge moves upward a distance d , the work done by the electric force is negative.



- The change of the potential energy is

$$W = -Fd$$

$$\Delta U = -W$$

$$\Delta U = -(-q_0Ed)$$

$$\Delta U = q_0Ed$$

$$E = k \frac{Q}{d^2}$$

$$\Delta U = q_0 \left(k \frac{Q}{d^2} \right) d$$

$$\Delta U = k \frac{Qq_0}{d}$$

$$\Delta U = k \frac{Q_1Q_2}{r}$$

- As the positive charge moves in the direction of the field, its electric potential energy decreases and its kinetic energy increase. So, the change in potential energy is negative.

Example

1. The electric field in a particle-accelerator machine is $4.5 \times 10^5 \frac{N}{C}$. How much work is done to move a proton 25 cm through that field?

Solution

$$W = q_0Ed$$

$$W = (1.6 \times 10^{-19} C) \left(4.5 \times 10^5 \frac{N}{C} \right) (25 \times 10^{-2} m) = 1.8 \times 10^{-14} J$$

2. A uniform electric field of magnitude $4 \times 10^5 \frac{N}{C}$ points in the positive x direction. Find the change in electric potential energy of a $45 \mu C$ charge as it moves from the origin to $x = 3m$.

Solution

$$\Delta U = (45 \times 10^{-6} C) \left(4 \times 10^5 \frac{N}{C} \right) (3m) = 5.4 J$$

Electric potential

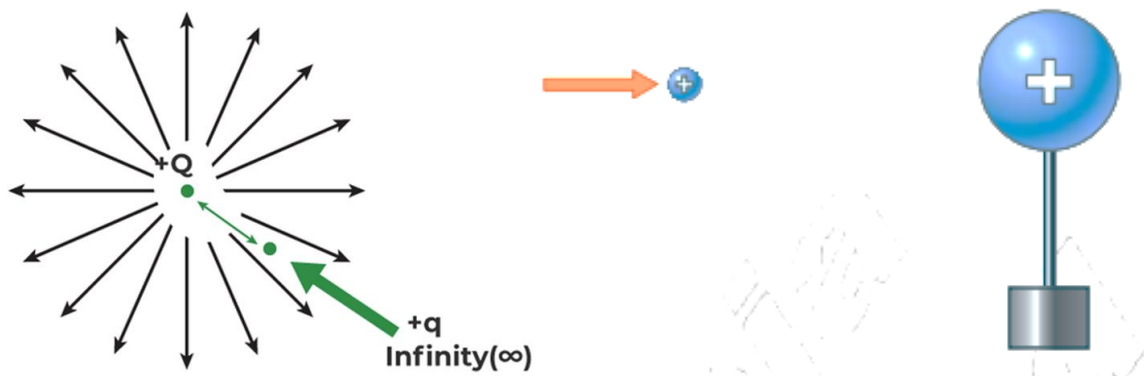
- The electric potential (absolute potential) at a point is the electric potential energy divided by charge.
- Electric potential, also known as voltage, is a scalar quantity that describes the electric potential energy per unit charge at a given point in an electric field.
- It represents the amount of work required to move a positive test charge from a reference point (usually infinity) to the given point, divided by the charge of the test particle.
- Electric potential is measured in volts (V).

$$V = \frac{\Delta U}{q_0} = \frac{W}{q_0}$$

$$\Delta U = k \frac{Qq_0}{d}$$

$$V = k \frac{Qq_0}{d} \times \frac{1}{q_0}$$

$$V = \frac{kQ}{d} \Rightarrow V = \frac{kQ}{r}$$



Example

1. What is the potential at in space 1mm from a 1nc point charge?

Solution

$$V = \frac{kQ}{r}$$

$$V = \frac{\left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right) (1 \times 10^{-9} \text{C})}{1 \times 10^{-3} \text{m}}$$

$$V = 9 \times 10^3 \frac{\text{Nm}}{\text{C}} = 9000 \text{V}$$

2. What is the electric potential between two terminals of a cell if it requires 7J of work to transfer 3C between the terminals?

Solution

$$V = \frac{W}{q}$$

$$V = \frac{7\text{J}}{3\text{C}} = 3.5\text{V}$$

3. What is the potential $2 \times 10^{-14} \text{m}$ from a fragment that has 46 protons in it?

Solution

$$Q = 46 \times 1.6 \times 10^{-19} \text{C} = 73.6 \times 10^{-19} \text{C}$$

$$V = \frac{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} (73.6 \times 10^{-19} \text{C})}{2 \times 10^{-14} \text{m}} = 3.312 \times 10^6 \text{V}$$

Exercise

1. Determine the electric potential at a point located at 1 cm from a charge $5.0 \mu\text{C}$.

Electrical Potential Difference

- Electric potential difference is the work done per unit charge when moving a charge between two points in an electric field.
- **Potential difference** is the potential created (the work done per unit charge) when moving a charge from one point in the field to another whereas **electric potential** is the work done per unit charge to get a charge from infinity to a point in an electric field
- The potential difference between two points A and B in an electric field is defined as **the work required to moving a unit positive test charge from the point of lower potential to that of higher potential.**

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}$$

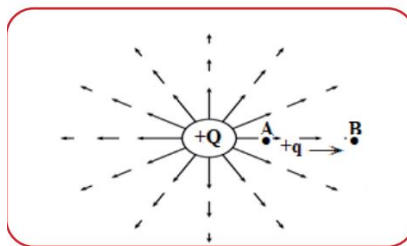
- If an amount of work W is required to move a charge Q from one point to another, then the potential difference between the two points is given by

$$\Delta V = V_B - V_A = \frac{W}{q}$$

$$\Rightarrow W = q\Delta V$$

$$\Rightarrow W = q(V_B - V_A)$$

$$\Rightarrow W = q\left(\frac{kQ}{r_B} - \frac{kQ}{r_A}\right)$$



- Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

Example

1. What potential difference is required to do 100 J of work to carry a charge of 10 C between two points?

Solution

$$\Delta V = V_B - V_A = \frac{W}{q}$$

$$\Delta V = \frac{100J}{10C} = 10V$$

2. What is the voltage difference between the positions $x_f = 11m$ and $x_i = 5m$ in an electric field of $6 \frac{N}{C}$?

Solution

$$\Delta V = Ed$$

$$\Delta V = \left(6 \frac{N}{C}\right)(6m) = 12V$$

3. The work needed to move a 10C charge between two charged plates is 500 J. What is the voltage between the plates?

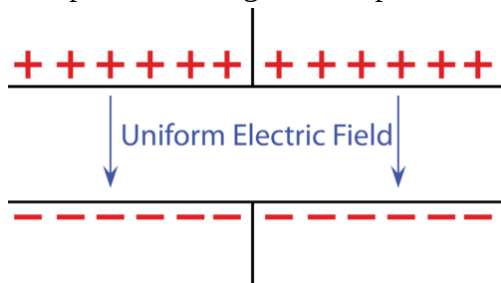
Solution

$$\Delta V = V_B - V_A = \frac{W}{q}$$

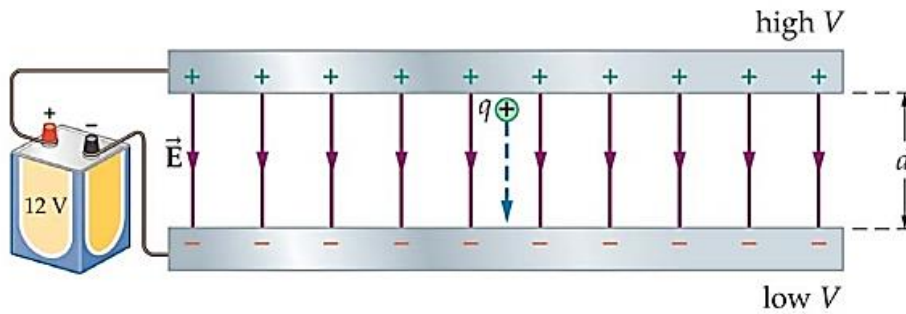
$$\Delta V = \frac{500J}{10C} = 50V$$

Potential difference in a uniform electric field

- If one of the plates is positively charged and the other negatively charged, the space between the plates will have a constant electric field except near the edges of the plates.



- A uniform electric field E is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B.



- The work done by the electric field in to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = \Delta PE = q\Delta V$$

$$W = q\Delta V$$

$$Fd = q\Delta V$$

$$qEd = q\Delta V$$

$$Ed = \Delta V$$

$$\Delta V = \mathbf{Ed}$$

$$E = \frac{\Delta V}{d}$$

$$E = \frac{F}{q}$$

$$\Rightarrow \frac{F}{q} = \frac{\Delta V}{d}$$

- The motion of a charged particle in an electric field depends on the direction of the electric field.
- With the force on a charged particle in a uniform field established, the acceleration of a particle can be easily established from Newton's second law.

$$F = ma$$

$$\frac{F}{q} = \frac{\Delta V}{d}$$

$$\frac{ma}{q} = \frac{\Delta V}{d}$$

$$a = \frac{\Delta V q}{dm}$$

- From work energy theorem

$$W = \Delta KE$$

$$W = q\Delta V$$

$$W = \Delta KE = q\Delta V$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = q\Delta V$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = qEd$$

Example

1. If a potential difference of 20 V applied across two parallel plates produces an electric field of 500 N/C (or V/m) what distance separates these two plates?

Solution

$$d = \frac{\Delta V}{E} = \frac{20V}{500 \frac{V}{m}} = 0.04m$$

2. An electric field of 4200 N/C exists between two parallel charged plates separated by 15 cm. What is the potential difference that is causing this?

Solution

$$\Delta V = Ed$$

$$\Delta V = 4200 \frac{N}{C} \times 15 \times 10^{-2} m = 630V$$

3. What is the electric field that could be created by two parallel charged plates separated by 20 cm that have a potential difference of 140 V?

Solution

$$E = \frac{\Delta V}{d} = \frac{140V}{20 \times 10^{-2} m} = 700 \frac{V}{m}$$

4. An electron accelerates from rest in a uniform electric field $6.0 \times 10^3 \frac{N}{C}$ to a speed of $2.0 \times 10^4 \frac{m}{s}$. How far did it travel and through what potential difference was the electron accelerated?

Solution

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = qEd$$

$$\frac{1}{2}(9.1 \times 10^{-31} kg) \left(2.0 \times 10^4 \frac{m}{s}\right)^2 - \frac{1}{2}(9.1 \times 10^{-31} kg)(0)^2 = (1.6 \times 10^{-19} C) \left(6.0 \times 10^3 \frac{N}{C}\right) d$$

$$d = \frac{\frac{1}{2}(9.1 \times 10^{-31} kg) \left(2.0 \times 10^4 \frac{m}{s}\right)^2}{(1.6 \times 10^{-19} C) \left(6.0 \times 10^3 \frac{N}{C}\right)} = 1.9 \times 10^{-7} m$$

$$\Delta V = Ed$$

$$\Delta V = \left(6.0 \times 10^3 \frac{N}{C}\right) (1.9 \times 10^{-7} m) = 1.1 \times 10^{-3} V$$

Exercise

1. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of $7.37 \times 10^{-17} J$. What is the charge on the ion? (EUEE:2004)

A. $6.41 \times 10^{-19} C$

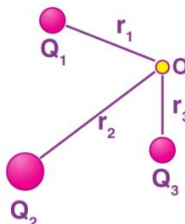
C. $4.51 \times 10^{-15} C$

B. $2.32 \times 10^{-15} C$

D. $2 \times 10^{-21} C$

Potential due to a system of point charges

- The electric potential at a point due to a group of point charges is the algebraic sum of all the potentials due to individual charges.
- Consider a system of charges $Q_1, Q_2, \dots, Q_n$ with position vectors $r_1, r_2, \dots, r_n$ relative to some origin.



- By the superposition principle, the potential V at O due to the total charge configuration is the algebraic sum of the potentials due to the individual charges.

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} + \dots + \frac{kQ_n}{r_n}$$

$$V = k \sum_{i=1}^n \frac{Q_i}{r_i}$$

Note

- ▶ The sum in this equation is an algebraic sum.
- ▶ It is not a vector sum. It means the signs of the charges are used in calculating the potential.
- ▶ As a result, the potential of a positive charge is positive and the potential of a negative charge is negative.

Example

1. Two point charges $3.0 \mu\text{C}$ and $-6.0 \mu\text{C}$ are separated by 1.20 m . What is the electric potential midway between them?

Solution

$$V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2}$$

$$V = \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{3 \times 10^{-6} \text{C}}{(0.6 \text{m})} + \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{-6 \times 10^{-6} \text{C}}{(0.6 \text{m})}$$

$$V = \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{3 \times 10^{-6} \text{C}}{(0.6 \text{m})} - \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{6 \times 10^{-6} \text{C}}{(0.6 \text{m})}$$

$$V = 45000 \text{V} - 90000 \text{V}$$

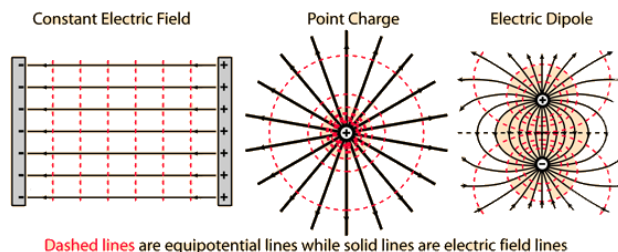
$$V = -45000 \text{V}$$

Exercise

1. Two-point charges are located on the corners of a rectangle with a height of 0.05 m and a width of 0.15 m . The first charge ($q_1 = -5 \mu\text{C}$) is located at the upper left-hand corner, while the second charge ($q_2 = +2 \mu\text{C}$) is at the lower right-hand corner. (a) Determine the electric potential at the upper right-hand corner of the rectangle. (b) What is the potential difference for a point at the right-hand corner of the rectangle relative to the lower left-hand corner?

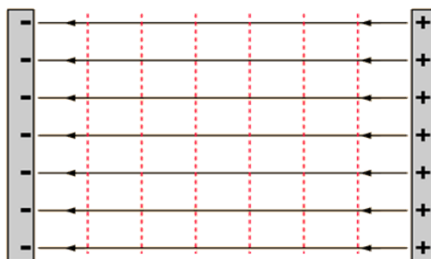
Equipotential

- An equipotential is a line joining points within an electric field with the same potential.
- **Equipotential Points:** If the points in an electric field are all at the same electric potential, they are known as **the equipotential points**.
- If these points are connected by a line or a curve, it is known as an equipotential line.
- If such points lie on a surface, it is called an equipotential surface.
- Further, if these points are distributed throughout a space or a volume, it is known as an equipotential volume.
- **Equipotential lines** are always perpendicular to the electric field.
- In three dimensions, the lines form equipotential surfaces.
- Movement along an equipotential surface requires no work because such movement is always perpendicular to the electric field.
- If a charge moves along a line of equipotential then its potential remains constant and there is no change in its electrical potential energy.
- This means all lines of equipotential are perpendicular to the electric field lines.



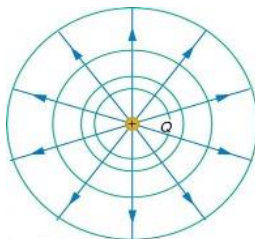
Equipotential Lines: Constant Field (Uniform Field)

In a uniform field the lines of equipotential are equidistant parallel lines.



Equipotential Lines: Radial field

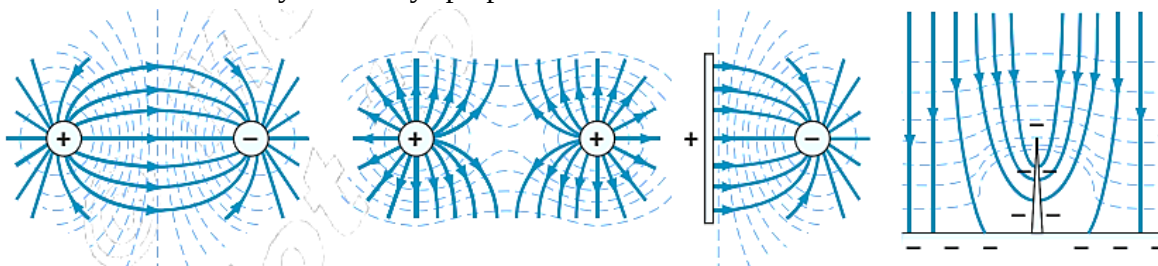
In a radial field such as the field around a point charge the line of equipotential are concentric circles centred on the single charge.



These lines move further apart as the distance from the charge increases. This is due to the fact that the field gets weaker as the distance increases.

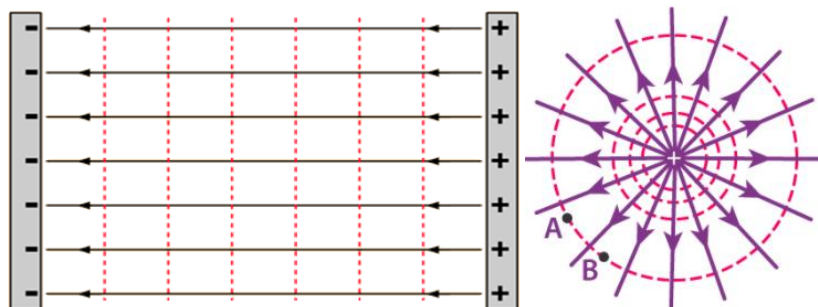
Equipotential Lines: More complex fields

If multiple charges or different shapes are involved the lines of equipotential can get much more complex. However in each case they are always perpendicular to the electric field lines.



Characteristics of equipotential lines/surfaces

1. Two equipotential surfaces never intersect each other.
2. The equipotential lines of a uniform field are parallel to each other and the equipotential lines of a point charge are concentric spheres.



3. Since the electric field (E) is stronger near the charge, so equipotential surfaces of point charge are crowded near the charge.
4. No work is required to move a charge along an equipotential line because there is no change in potential.

$$W = q\Delta V$$

$$W = q(V_A - V_B) = 0$$

5. Equipotential lines are always perpendicular to the electric field lines.

$$W = Fd\cos\theta$$

$$W = Fd\cos 90^\circ$$

$$W = 0$$

6.4. Electric Current, Resistance and ohm's law

By the end of this section, you will be able to:

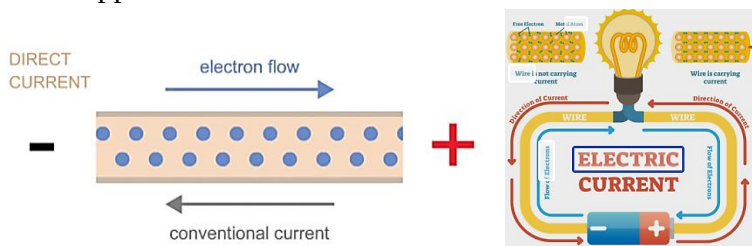
- ♣ Define the terms electric current, current density, resistance, conductivity, resistivity and drift velocity.
- ♣ Explain the effect on the current (brightness) and potential difference of connecting light bulbs in series.
- ♣ Explain the effect on the current (brightness) and potential difference of connecting light bulbs in parallel.
- ♣ Draw electric circuits consisting of three resistors connected in series and parallel.
- ♣ Determine equivalent resistance of resistors connected in series and parallel.
- ♣ Describe the principles of potentiometer and Wheatstone bridge.
- ♣ Describe how to convert galvanometer to a voltmeter and ammeter.
- ♣ Explain the meaning of an ohm, resistance.
- ♣ Explain the SI units of electric current, current density, resistance, resistivity, conductivity.
- ♣ State and apply Kirchhoff's laws.
- ♣ Solve problems involving network of resistors.

Electric Current, Resistance and Ohm's law

Electric current

- An electric current is a flow of charged particles, such as electrons or ions, moving through an electrical conductor or space.
- It is defined as the net rate of flow of electric charge through a surface.
- The moving particles are called **charge carriers** (In conductors charge carriers are free electrons and in electrolytes charge carriers are ions), which may be one of several types of particles, depending on the conductor.
- In electric circuits the charge carriers are often electrons moving through a wire.
- In semiconductors they can be **electrons** or **holes**.
- In an electrolyte the charge carriers are ions, while in plasma, an ionized gas, they are ions and electrons.
- Electric current is a flow of charge.

- In metals, current is due to a flow of electrons.
- When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field.
- The electric field in turn exerts force on the free or conduction charges of the conductor, causing current.
- Electric currents create magnetic forces, which are used in motors, generators, inductors, and transformers.
- In ordinary conductors, they cause Joule heating, which creates light in incandescent light bulbs.
- Time-varying currents emit electromagnetic waves, which are used in telecommunications to broadcast information.
- **The conventional direction of current**, also known as conventional current is arbitrarily defined as the direction in which positive charges flow.
- Current flows from positive to negative.
- The **flow of electrons** is termed electron current.
- Electrons flow from the negative terminal to the positive.
- Current and electron flow in the opposite direction.



- **Current** is determined by the number of electrons passing through a cross-section of a conductor in one second.

$$I = \frac{Q}{t}$$

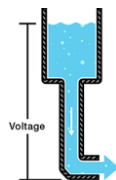
- One ampere is a flow of one **coulomb** (1 C) of charge in 1 s.
- In the International System of Units (SI), electric current is expressed in units of ampere (sometimes called an "amp", symbol A), which is equivalent to one coulomb per second.
- The ampere is an SI base unit and electric current is a base quantity in the International System of Quantities
- Electric current is also known as amperage and is measured using a device called an ammeter.
- Small current are measured in milliampere (mA) and microampere (μA).
- When describing voltage, current, and resistance, a common analogy is a water tank.
- In this analogy, charge is represented by the water amount, voltage is represented by the water pressure, and current is represented by the water flow.

Water = Charge

Pressure = Voltage

Flow = Current

- Consider a water tank at a certain height above the ground. At the bottom of this tank there is a hose.



➤ Voltage is like the pressure created by the water.

DID YOU KNOW?

If two wires are placed 1 m apart and carry the same current, if the force between them is $2 \times 10^{-7} \text{ N}$ then the current flowing in each wire is 1 A. How??

Example:

1. 300 C of charge passes in 1 minute. What is the current?

Solution

$$I = \frac{Q}{t} = \frac{300 \text{ C}}{60 \text{ s}} = 5 \text{ A}$$

2. How long will a current of 5 A take to pass 100 C of charge?

Solution

$$t = \frac{Q}{I} = \frac{100 \text{ C}}{5 \text{ A}} = 20 \text{ s}$$

Exercise:

1. A steady current of 20 A is maintained in a metal conductor for 1/3 minute. What charge is transferred through it in that time?
2. A charge of 600 C flows through a resistor in which a steady current is maintained for 120 s. what is the current?
3. There is current of 10mA in a wire. How much charge passes any cross-section of the wire in 30 s? If this current is caused by the flow of electrons, then how many electrons pass through the wire during this time?

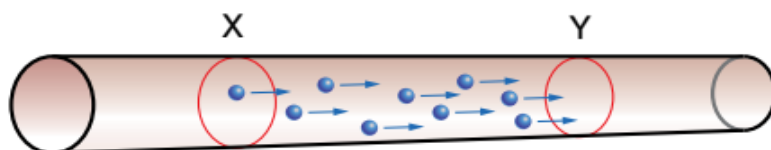
Drift velocity

Suppose current flow through the conductor of length l between two ends X and Y due to difference in electric potential (voltage), then there are basic terms in microscopic model of current and these are:

1. Net charges (ΔQ)

Suppose N number of charges flow through the conductor between X and Y, then the net charge is given as

$$\Delta Q = Nq$$



2. Charge density (n)

It is the measure of number of charges (electrons) (N) per unit volume (V).

$$n = \frac{N}{V} \Rightarrow N = nV$$

But volume (V) can be given as: $V = A\Delta l$

$$\Rightarrow N = nA\Delta l$$

Therefore the net charge (ΔQ) can be given as

$$\Delta Q = Nq$$

$$\Delta Q = nqA\Delta l$$

Where

n is the number of charge carriers per unit volume

A is the cross sectional area

Δl a small length along the wire

q is the charge of the charge carriers

3. Drift velocity (v_d)

It is the measure of average velocity of charges in the conductors for the length l .

$$v_d = \frac{\Delta l}{\Delta t}$$

$$\Delta l = v_d \Delta t$$

Since net charge

$$\Delta Q = nqA\Delta l$$

$$\Delta Q = nqA(v_d \Delta t)$$

$$\Delta Q = nqAv_d \Delta t$$

$$v_d = \frac{\Delta Q}{nqA\Delta t}$$

4. Electric current (I)

It is the rate flow of charges.

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{nqAv_d \Delta t}{\Delta t}$$

$$I = nqAv_d$$

$$v_d = \frac{I}{nAq}$$

$$I = nqAv_d$$

5. Current density (J)

It is the measure of amount of current to cross sectional area.

$$J = \frac{I}{A}$$

$$J = \frac{nqAv_d}{A}$$

$$J = nqv_d$$

$$v_d = \frac{J}{nq}$$

$$J = nqv_d$$

Where v_d is the drift velocity

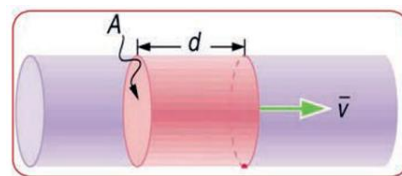
J is the current density

n is the number of charge carriers per unit volume

q the elementary charge on the charge carriers

There is a common approximation to the current density which assumes that the current is proportional to the electric field E

$$J = \sigma E$$



Example

- Find the approximate current density when an electric field of 5 V/m is applied to a copper conductor. The conductivity of copper is $59.6 \times 10^6 \frac{S}{m}$.

Solution

$$J = \sigma E = 59.6 \times 10^6 \frac{S}{m} \left(5 \frac{V}{m} \right) = 298 \times 10^6 \frac{A}{m^2} = 2.98 \times 10^8 \frac{A}{m^2}$$

2. Find the number of charge carriers per unit volume in a copper wire of cross sectional area 1mm^2 carrying a current of 3 A if the drift velocity of the charge carriers is 0.00028 m/s. (The elementary charge is $1.6 \times 10^{-19} \text{C}$)

Solution

$$J = nqv_d$$

$$n = \frac{J}{v_d q}$$

$$J = \frac{I}{A} = \frac{3A}{1\text{mm}^2} = \frac{3A}{1 \times 10^{-6} \text{m}^2} = 3 \times 10^6 \frac{A}{\text{m}^2}$$

$$n = \frac{3 \times 10^6 \frac{A}{\text{m}^2}}{0.00028 \text{ m/s} (1.6 \times 10^{-19} \text{C})} = \frac{3 \times 10^6 \frac{A}{\text{m}^2}}{0.000448 \times 10^{-19} \text{m/sC}} = 6696.42 \times 10^{25} = 6.7 \times 10^{28}$$

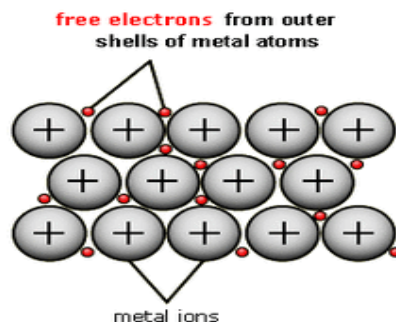
Exercise:

- Find the approximate current density when an electric field of 12 V/m is applied to a copper conductor. The conductivity of copper is $63.0 \times 10^6 \frac{S}{m}$.
- Find the number of charge carriers per unit volume in a copper wire of cross sectional area 2mm^2 carrying a current of 6 A if the drift velocity of the charge carriers is 0.00028 m/s. (The elementary charge is $1.6 \times 10^{-19} \text{C}$)
- A wire with a circular cross-sectional area 10^{-6}m^2 carries current of 1A. Find the drift speed of electrons if the number of mobile charge carriers is 10^{28} electrons per cubic meter.

Resistance and Ohm's Law

Resistance

- An electric wire is made of metal.
- Inside a metal, there are negatively charged electrons that are free to move. This free electron is called **conduction electrons (free electrons)**.
- **Conduction electrons** are the particles that allow a metal to conduct an electric current.
- Conduction electrons are called **valence electrons**.
- The atoms of a metal bind tightly together; they usually form a regular array.

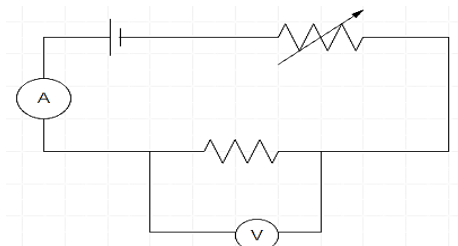


- In a typical metal, such as copper or silver, one or more electrons from each atom break free to become conduction electrons.
- Since there are equal numbers of free electrons (negative) and ions (positive), the metal has no overall charge - it is neutral.

- When electrons are moving through a conductor, they collide with each other and with the fixed positive ions of the material.
- This electric property that impedes current is called **resistance**.
- **Resistance** is a measure of the opposition to current flow in an electrical circuit.

Ohm's Law

- According to Ohm's law, the current flowing through a conductor at constant temperature is precisely proportional to the potential difference across the conductor's ends.



- Voltage is directly proportional to current.

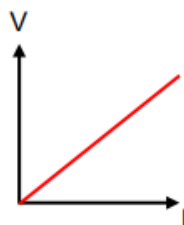
$$V \propto I$$

$$V = RI$$

R is the proportionality constant, often known as **conductor resistance**.

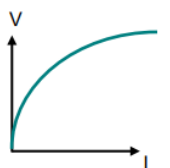
$$R = \frac{V}{I}$$

- **Ohmic conductors** are those electrical conductors that follow Ohm's law.
- In other words there is a linear relationship between voltage and current for all values.
- Devices that have a constant resistance, as described above, are known as Ohmic conductors.
- The graph of V versus I is a straight line that must go through the origin.
- If the graph is not a straight line then the device is called a non-Ohmic conductor.

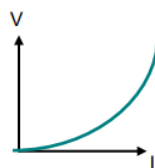


Ohmic Conductor
eg, resistor

- **Non-Ohmic conductors** are those electrical conductors that do not follow Ohm's law.
- In other words the relationship between voltage and current is not linear for all values.
- Light bulbs such as car headlamps are common examples of non-Ohmic conductors.
- As the voltage increases, the current will not increase in proportion, as shown in the graph below.
- Other non-Ohmic conductors include devices whose resistance changes with light or temperature.
- These are particularly useful as detectors in sensors which need to respond to changes in light (see diode graph below) or temperature levels.



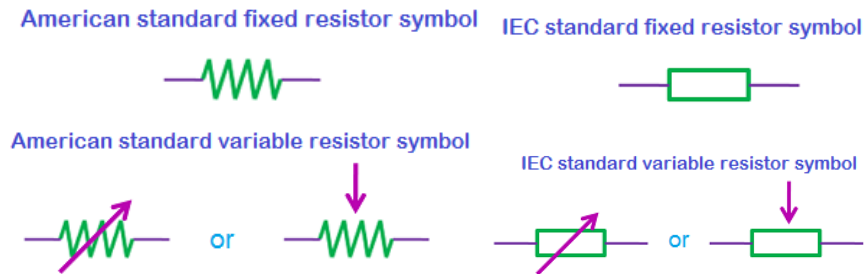
Non-Ohmic Conductor
eg, car headlamp bulb



Non-Ohmic Conductor
eg, diode

Resistors

- A resistor is an electrical component that limits or regulates the flow of electrical current in an electronic circuit.
- The IEC (International Electro-technical Commission) standard and American standard fixed and variable resistor symbol is shown in the below figure.



Combination of Resistors

- There are two basic ways in which to connect more than two resistors. They are series and parallel.

1. Resistors in Series

In series connection:

- ✓ Resistors are connected end-to-end in a line to form a single path for electrons to flow.
- ✓ There is only one path for electrons to flow.
- ✓ Current is constant at every point.

$$I_1 = I_2 = I_3 = I_n = I_T = I$$

- ✓ The total potential difference in the system is the sum of individual p.d across each resistor.

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$V = IR$$

$$I_T R_T = I_1 R_1 + I_2 R_2 + I_3 R_3 + \dots + I_n R_n$$

$$I_1 = I_2 = I_3 = I_n = I_T = I$$

$$I R_T = I R_1 + I R_2 + I R_3 + \dots + I R_n$$

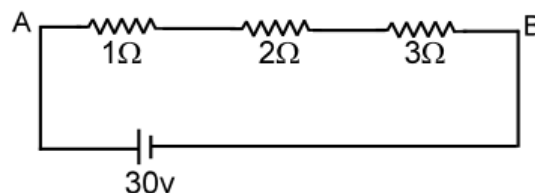
$$I R_T = I (R_1 + R_2 + R_3 + \dots + R_n)$$

$$R_T = R_1 + R_2 + R_3 + \dots + R_n$$

$$R_T = R_1 + R_2 + R_3 + \dots + R_n$$

Example

- Find the current in the circuit



Solution

$$R_T = R_1 + R_2 + R_3$$

$$R_T = 1\Omega + 2\Omega + 3\Omega = 6\Omega$$

$$I = \frac{V}{R} = \frac{30V}{6\Omega} = 5A$$

2. Three resistors 20Ω , 30Ω , and 50Ω are connected in series to a battery which provides $240V$. calculate
- The total resistance
 - The current in the circuit
 - The voltage drop across each resistor
 - Compare the sum of the voltage drops with voltage provided by the battery.

Solution

A. For resistors in series:

$$R = R_1 + R_2 + R_3 = 20\Omega + 30\Omega + 50\Omega = 100\Omega$$

B. $I = \frac{V}{R} = \frac{240V}{100\Omega} = 2.4A$

C. The same current $I = 2.4A$ flows through each resistor.

$$V_1 = IR_1 = (2.4A)(20\Omega) = 48V$$

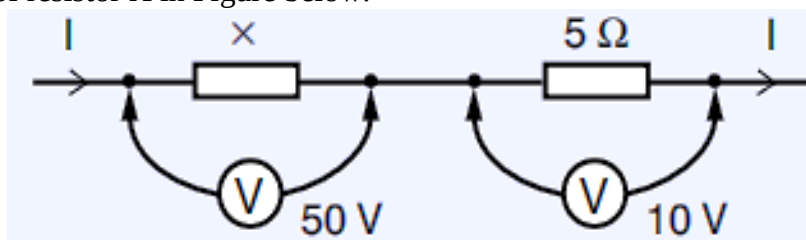
$$V_2 = IR_2 = (2.4A)(30\Omega) = 72V$$

$$V_3 = IR_3 = (2.4A)(50\Omega) = 120V$$

D. $V_1 + V_2 + V_3 = 48V + 72V + 120V = 240V$

Hence, the sum of the voltage drop across each resistor is equal to the voltage of the source.

3. Calculate the size of resistor X in Figure below.



Solution

First look at the 5Ω resistor. From $I = \frac{V}{R}$ a p.d. of $10V$ across it must mean that a current I is flowing through it, given by:

$$I = \frac{V}{R} = \frac{10V}{5\Omega} = 2A$$

Now look at resistor 'X'. Since they are in series the same $2A$ is flowing through it. To achieve this there is a p.d. of $50V$ across it.

$$R = \frac{V}{I} = \frac{50V}{2A} = 25\Omega$$

2. Resistors in parallel

In parallel connection:

- ✓ Resistors are connected face to face.
- ✓ The current has many paths.
- ✓ Since the polarity of each p.d on each resistor is the same, the potential drop in the system is equal to the potential drop across each resistor.

$$V_1 = V_2 = V_3 = V_n = V_T = V$$

- ✓ The total current flowing in the circuit is the sum of individual current flowing.

$$I_T = I_1 + I_2 + I_3 + \dots + I_n$$

$$V = IR \rightarrow I = \frac{V}{R}$$

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots + \frac{V_n}{R_n}$$

$$\frac{V_1}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots + \frac{V_n}{R_n}$$

$$V\left(\frac{1}{R_T}\right) = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}\right)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

Example

1. Three resistors of 8Ω , 12Ω , and 24Ω are connected in parallel, and a current of $20A$. determine

- A. The potential difference across the combination.
- B. The current through each resistor.

Solution

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{8\Omega} + \frac{1}{12\Omega} + \frac{1}{24\Omega}$$

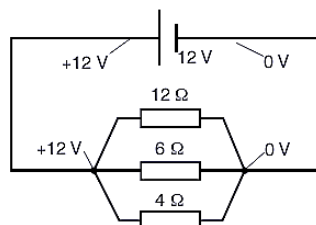
$$\frac{1}{R_T} = \frac{3 + 2 + 1}{24\Omega} = \frac{6}{24\Omega}$$

$$\frac{1}{R_T} = \frac{6}{24\Omega}$$

$$\frac{R_T}{1} = \frac{24\Omega}{6}$$

$$R_T = 4\Omega$$

2. For the circuit shown in figure below.
 - A. Calculate the voltage drop across each resistor
 - B. Calculate the current through each resistor
 - C. Calculate the effective resistance of the circuit.



Solution

A. $V_1 = V_2 = V_3 = 12V$

B. $I_1 = \frac{V_1}{R_1} = \frac{12V}{12\Omega} = 1A$

$I_2 = \frac{V_2}{R_2} = \frac{12V}{6\Omega} = 2A$

$I_3 = \frac{V_3}{R_3} = \frac{12V}{4\Omega} = 3A$

C. $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\frac{1}{R_T} = \frac{1}{12\Omega} + \frac{1}{6\Omega} + \frac{1}{4\Omega}$$

$$\frac{1}{R_T} = \frac{1+2+3}{12\Omega}$$

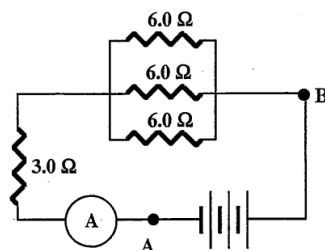
$$\frac{1}{R_T} = \frac{6}{12\Omega}$$

$$\frac{R_T}{1} = \frac{12\Omega}{6}$$

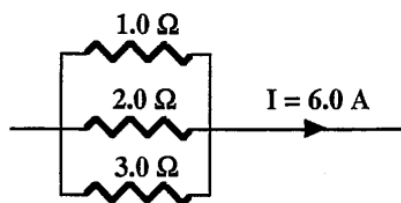
$$R_T = 2\Omega$$

Exercise

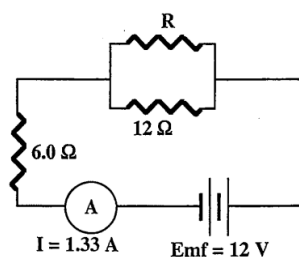
1. In the circuit shown below, what is the potential difference between points A and B if the reading on the ammeter is 1.2 A? Assume that the ammeter has zero resistance.



- A. 0.24 V B. 4.4 V C. 6 V D. 10.8 V
2. What is the potential difference across the section of the circuit shown above if the input current is 6.0 A?



- A. 3 V B. 3.3 V C. 12 V D. 36 V
3. What is the value of the resistor marked 'R' shown above?

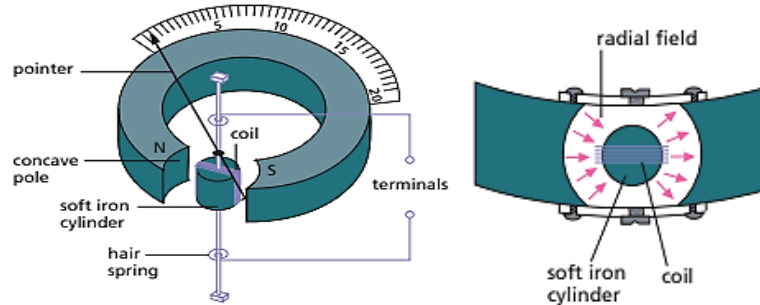


- A. 1Ω B. 2Ω C. 3Ω D. 4Ω

Measuring Instruments

1. Moving-coil galvanometer

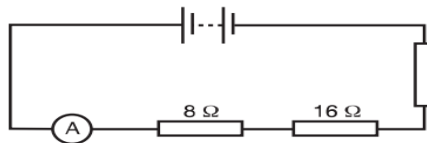
- A galvanometer is an instrument used for measuring small electric current in electric circuit.
- A galvanometer is very sensitive instrument to detect the current.
- It can be easily converted into ammeter and voltmeter.
- In the moving-coil **pointer-type** meter, a coil is pivoted between the poles of a permanent magnet.
- Current enters and leaves the coil by hair springs above and below it.
- The greater the current, the greater the deflection which is shown by a pointer attached to the coil.



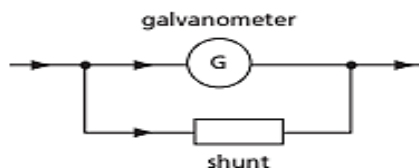
- The soft iron cylinder at the centre of the coil is fixed and along with the concave poles of the magnet it produces a **radial** field i.e. the field lines are directed to and from the centre of the cylinder.

2. Ammeters and shunts

- An ammeter is a device used to measure current flowing in a circuit.
- An ammeter is a galvanometer that has a known low resistance, called **a shunt**, in parallel with it to take most of the current.
- An ammeter is placed in series in a circuit and must have a low resistance.

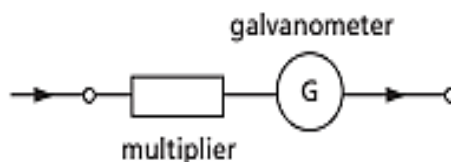


- The basic Galvanometer can be converted into an ammeter by adding a low resistance (shunt).



3. Voltmeters and multipliers

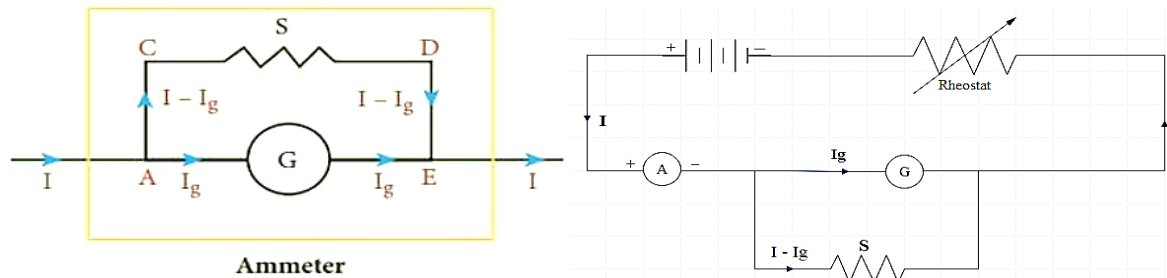
- Voltmeter is a device used to measure the potential difference between two points in the circuit.
- A voltmeter is a galvanometer having a known **high resistance**, called **a multiplier**, in series with it.



- A voltmeter is placed in parallel with the part of the circuit across which the p.d. is to be measured and must have a high resistance otherwise the total resistance of the whole circuit is reduced so changing the current and the p.d. measured.
- You convert a galvanometer to be a voltmeter by adding a large resistance in series with the meter.

Converting a galvanometer into an ammeter

- A galvanometer can detect only small currents.
- Thus, to measure large currents it is converted into an ammeter.
- It can be converted into an ammeter by connecting a low resistance called **shunt resistance** in parallel to the galvanometer.
- This low resistance is called shunt resistance S .
- The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.



I = The current passing through the circuit.

$$I = I_g + I_{sh} \Rightarrow I_{sh} = I - I_g$$

- The potential difference galvanometer is same as the potential difference across shunt resistance.

$$V_{\text{galvanometer}} = V_{\text{shunt}}$$

$$I_g R_g = I_{sh} S$$

$$I_g R_g = (I - I_g) S$$

$$I_g R_g = (I - I_g) R_{sh}$$

$$R_{sh} = \left(\frac{I_g}{I - I_g} \right) R_g$$

Example

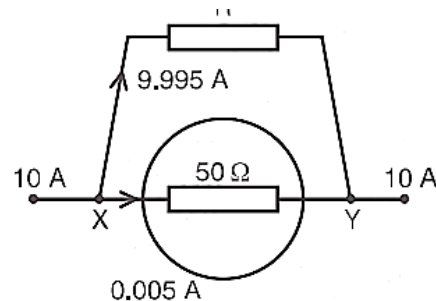
1. A galvanometer of full-scale deflection 5mA is to be converted into a 0-10A ammeter. If its coil has a resistance of 50Ω, what value shunt must be fitted?

Solution

$$R_{sh} = \left(\frac{I_g}{I - I_g} \right) R_g$$

$$R_{sh} = \left(\frac{0.005A}{10A - 0.005A} \right) 50\Omega$$

$$R_{sh} = \left(\frac{0.005A}{9.995A} \right) 50\Omega = 0.025\Omega$$



2. A moving coil galvanometer has a coil of resistance 59 Ω. It shows a full-scale deflection for a current of 50 mA. How will you convert it to an ammeter having a range of 0 to 3A?

Solution

$$R_{sh} = \left(\frac{I_g}{I - I_g} \right) R_g$$

$$R_{sh} = \left(\frac{0.05A}{3A - 0.05A} \right) 59\Omega$$

$$R_{sh} = \left(\frac{0.05A}{2.95A} \right) 59\Omega = 1\Omega$$

3. A galvanometer of resistance 100Ω gives a full-scale deflection for a current of $10^{-5}A$. To convert it into an ammeter capable of measuring up to $1A$ we should connect a resistance of_____.

A. 1Ω in parallel B. $10^{-3}\Omega$ in parallel C. $10^5\Omega$ in series D. $10^3\Omega$ in series

Solution

$$R_{sh} = \left(\frac{I_g}{I - I_g} \right) R_g$$

$$R_{sh} = \left(\frac{10^{-5}A}{1A - 0.00001A} \right) 100\Omega$$

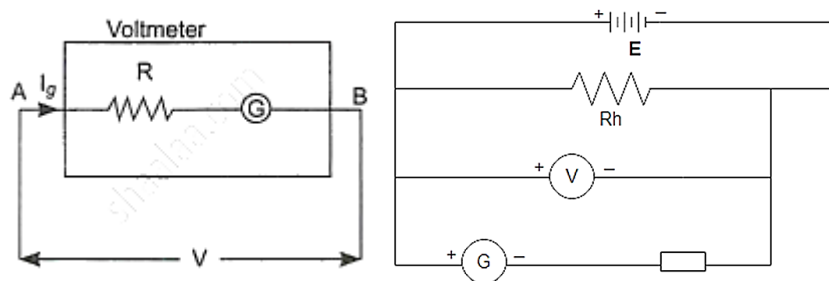
$$R_{sh} = \left(\frac{10^{-5}A}{0.99999A} \right) 100\Omega = 10^{-3}\Omega$$

Exercise

1. A moving coil galvanometer of resistance 55Ω produces a full scale deflection for a current of 250 mA . How will you convert it into an ammeter with a range of $0\text{-}3A$? (Answer: 5Ω)
2. A galvanometer coil has a resistance of 15Ω and the metre shows full scale deflection for a current of 4 mA . How will you convert the metre into an ammeter of range $0\text{ to }6\text{ A}$? (Answer: 0.01Ω)
3. A galvanometer has a resistance of 16Ω . It shows full scale deflection, when a current of 20 mA is passed through it. The only shunt resistance available is 0.06 which is not appropriate to convert a galvanometer into an ammeter. How much resistance should be connected in series with the coil of galvanometer, so that the range of ammeter is 8 A ? (Answer: 7.94Ω)

Converting a galvanometer into voltmeter

- A galvanometer can be converted into voltmeter by connecting with it with a very high resistance (R) called multiplier in series with it.



- Potential difference across the given load resistance is the sum of potential difference across galvanometer and potential difference across the high resistance.

$$V = I_g R_g + I_g R$$

$$V = I_g (R_g + R)$$

$$\frac{V}{I_g} = R_g + R$$

$$R = \frac{V}{I_g} - R_g$$

Example

1. A moving coil galvanometer has a resistance of 25Ω and gives a full scale deflection for a current of 10mA . How will you convert it into a voltmeter having range $0 - 100\text{V}$?

Solution

$$R = \frac{V}{I_g} - R_g$$

$$R = \frac{100\text{V}}{10 \times 10^{-3}\text{A}} - 25\Omega$$

$$R = 10000\Omega - 25\Omega = 9975\Omega$$

2. Find the resistance that must be placed in series with a 150Ω galvanometer having a $5000\mu\text{A}$ sensitivity to allow it to be used as a voltmeter with a $0 - 100\text{V}$ full-scale reading.

Solution

$$R = \frac{V}{I_g} - R_g$$

$$R = \frac{100\text{V}}{5000 \times 10^{-6}\text{A}} - 150\Omega$$

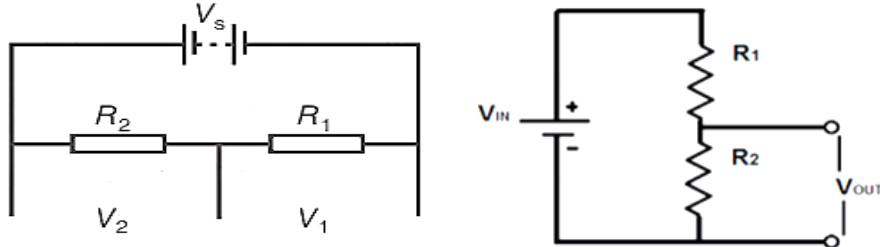
$$R = 20000\Omega - 150\Omega = 19850\Omega$$

Exercise

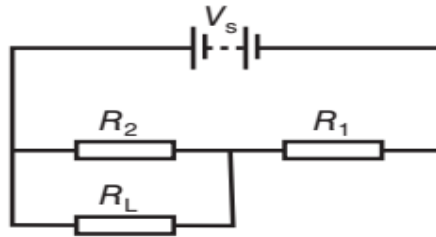
1. 100 mA current gives a full scale deflection in a galvanometer of 2Ω resistance. The resistance connected with the galvanometer to convert it into a voltmeter to measure 5 V is
- A. 40Ω B. 36Ω C. 48Ω D. 28Ω
2. An ammeter of 5Ω resistance can read 5 mA . If it is to be used to read 100 volts , how much resistance is to be connected in series?
- A. 19.9995Ω B. 199.995Ω C. 1999.95Ω D. 19995Ω

The Potential divider

- A potential divider is a circuit arrangement (configuration) used in an electronics to obtain a desired voltage output from a voltage source.
- A potential divider consists of two or more resistors in series with each other and with a source of fixed potential difference.
- The potential difference of the source is divided between the components in the circuit.



- If we put a load resistance R_L in parallel with R_2 as shown in figure below, then the potential difference V_L across R_L can be calculated as follows.



$$\text{Since } R_2 \text{ \& } R_L \text{ are parallel together, } \frac{1}{R_{eff}} = \frac{1}{R_2} + \frac{1}{R_L}$$

$$\frac{1}{R_L} = \frac{R_2 + R_L}{R_2 R_L}$$

$$R_2 R_L = \frac{R_2 R_L}{R_L + R_2}$$

$$R_{eff} = \frac{R_2 R_L}{R_L + R_2} + R_1$$

$$R_{eff} = \frac{R_2 R_L + R_1 (R_L + R_2)}{R_L + R_2}$$

$$R_{eff} = \frac{R_2 R_L + R_1 R_L + R_1 R_2}{R_L + R_2}$$

$$\frac{1}{R_{eff}} = \frac{R_L + R_2}{R_2 R_L + R_1 R_L + R_1 R_2}$$

$$I_{2L} = I = I_2 + I_L, V_2 = V_L \text{ since they are parallel.}$$

$$\text{From Ohm's law: } I = \frac{V}{R}$$

$$I_{2L} = \frac{V_s}{R_{eff}}, I_2 = \frac{V_2}{R_2}, I_L = \frac{V_L}{R_L}$$

$$\frac{V_s}{R_{eff}} = \frac{V_2}{R_2} + \frac{V_L}{R_L}$$

$$\frac{V_s}{R_{eff}} = \frac{V_L}{R_2} + \frac{V_L}{R_L} = V_L \left(\frac{1}{R_2} + \frac{1}{R_L} \right)$$

$$V_s \left(\frac{1}{R_{eff}} \right) = V_L \left(\frac{1}{R_2} + \frac{1}{R_L} \right)$$

$$V_s \left(\frac{1}{R_{eff}} \right) = V_L \left(\frac{R_L + R_2}{R_2 R_L} \right)$$

$$V_s \left(\frac{R_L + R_2}{R_2 R_L + R_1 R_L + R_1 R_2} \right) = V_L \left(\frac{R_L + R_2}{R_2 R_L} \right)$$

$$V_s \left(\frac{1}{R_2 R_L + R_1 R_L + R_1 R_2} \right) = V_L \left(\frac{1}{R_2 R_L} \right)$$

$$V_L = \frac{R_2 R_L}{R_1 R_L + R_2 R_L + R_1 R_2} V_s$$

$$V_L = \frac{R_L}{R_L} \left(\left(\frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} \right) V_s \right)$$

$$V_L = \left(\frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} \right) V_s$$

- If R_L is large in comparison with R_1 and R_2 [$R_L \gg R_1 \& R_2$] (as it would be in a practical application such as the input to an operational amplifier) then this equation can be:

$$V_L = \frac{R_2}{R_1 + R_2} V_s$$

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

- If R_L is in parallel with R_1 , the potential difference V_L is given by:

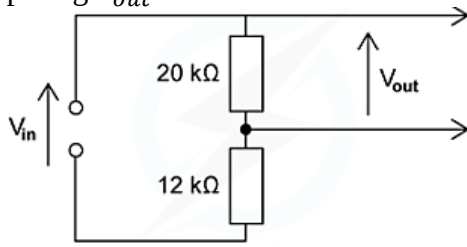
$$V_L = \frac{R_1 R_L}{R_1 R_L + R_2 R_L + R_1 R_2} V_s$$

- If R_L is large in comparison with R_1 and R_2 [$R_L \gg R_1 \& R_2$]

$$V_L = \frac{R_1}{R_1 + R_2} V_s$$

Examples

1. The circuit is designed to light up a lamp when the input voltage exceeds a preset value. It does this by comparing V_{out} with a fixed reference voltage of 5.3 V.



$$V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$$

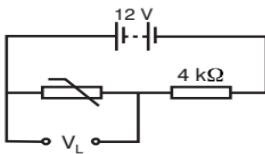
$$5.3V = \frac{12k\Omega}{12k\Omega + 20k\Omega} V_{in}$$

$$5.3V = \frac{12}{32} V_{in}$$

$$V_{in} = 8.48V$$

Solution

2. A thermistor is connected in a circuit shown in figure below. The resistance of the thermistor varies with temperature. It has a resistance of 12Ω at a temperature of 0°C and a resistance of $0.25k\Omega$ at the temperature of 25°C . Find the output p.d (V_L) at a) 0°C and b) 25°C



Solution

$$V_L = \frac{R_2}{R_1 + R_2} V_s$$

R_2 = resistance of thermistor = 12Ω

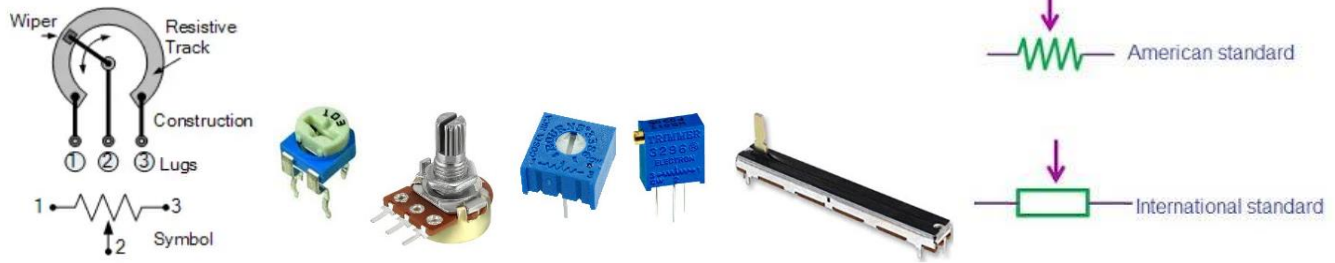
$R_1 = 4\Omega$

$$\text{a) } V_L = \frac{R_2}{R_1 + R_2} V_s = \frac{12}{4 + 12} \times 12 = 9V$$

$$\text{b) } V_L = \frac{R_2}{R_1 + R_2} V_s = \frac{0.25}{4 + 0.25} \times 12 = 0.706V$$

Potentiometer

- A potentiometer (also known as a pot or potmeter) is defined as a 3 terminal variable resistor in which the resistance is manually varied to control the flow of electric current.



- A potentiometer acts as an adjustable voltage divider.
- A potentiometer is a three-terminal resistor with a sliding or rotating contact that forms an adjustable voltage divider.
- It works by varying the position of a sliding contact across a uniform resistance wire.
- In a potentiometer, the entire input voltage is applied across the whole length of the resistor, and the output voltage is the voltage drop between the fixed and sliding contact.
- Thus, a potentiometer is essentially a type of potential divider circuit.
- There are two main types of potentiometers:

1. Rotary potentiometer

2. Linear potentiometer

Rotary potentiometer

- Rotary potentiometer is used mainly for obtaining adjustable supply voltage to a part of electronic circuits and electrical circuits.
- The volume controller of a radio transistor is an example of a rotary potentiometer where the rotary knob of the potentiometer controls the supply to the amplifier.



Linear potentiometer

- A linear potentiometer, also known as a **slide potentiometer** or a **slider potentiometer**, is a type of potentiometer that has a linear resistive element. It is commonly used to control or vary electrical signals such as voltage or audio levels.



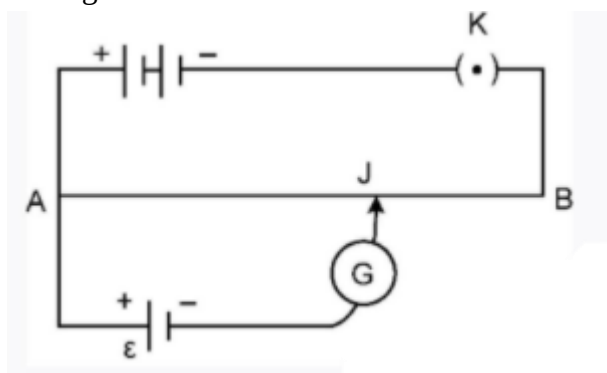
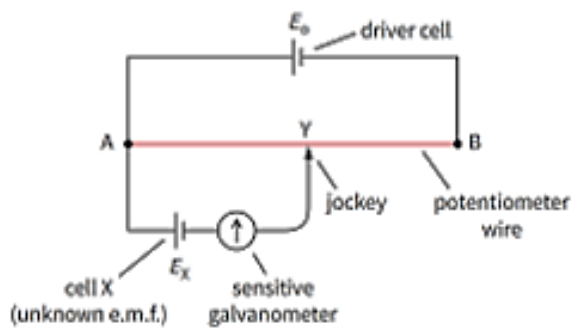
- The Potentiometer can also be used for the following purposes.
 - ▶ To measure the emf of a given cell,
 - ▶ To measure the internal resistance of a cell,
 - ▶ To compare the emf of different cells, and used as a variable resistor.

Exercise: A potentiometer can be used to measure

- A. Emf of newly designed cell
- B. Internal resistance of a cell
- C. Unknown resistance
- D. Potential difference across a resistor
- E. All

Potentiometer to measure the emf of a cell

- The principle of a potentiometer is that the potential dropped across a segment of a wire of uniform cross-section carrying a constant current is directly proportional to its length.
- The potentiometer is a simple device used to measure the electrical potentials (or compare the e.m.f of a cell).
- One form of potentiometer is a uniform high-resistance wire attached to an insulating support, marked with a linear measuring scale.
- In use, an adjustable regulated voltage source V_s , of greater magnitude than the potential to be measured, is connected across the wire so as to pass a steady current through it.



- Now, suppose we wish to measure the emf of cell X (E_x) (this must have a value less than that of the driver cell). The positive terminal of cell X is connected to point A.
- Both cells have their positive terminals connected to A.
- A lead from the negative terminal is connected to a sensitive galvanometer, and a lead from the other terminal of the galvanometer ends with a metal jockey.
- This is a simple connecting device with a very sharp edge that allows very precise positioning on the wire.
- If the jockey is touched onto the wire close to point A, the galvanometer needle will deflect in one direction.
- If the jockey is touched close to B, the galvanometer needle will deflect in the opposite direction.
- Clearly, there must be some point Y along the wire that, when touched by the jockey, gives zero deflection.
- In finding this position, the jockey must be touched gently and briefly onto the wire; the deflection of the galvanometer shows whether the jockey is too far to the left or right.
- It is important not to slide the jockey along the potentiometer wire as this may scrape its surface, making it non-uniform so that the voltage does not vary uniformly along its length.
- When the jockey is positioned at Y, the galvanometer gives zero deflection, showing that there is no current through it.
- This can only happen if the potential difference across the length of wire AY is equal to the emf of cell X.
- We can say that the potentiometer is balanced.
- If the balance point was exactly halfway along the wire, we would be able to say that the emf of X was half that of the driver cell.
- This technique finding a point where there is a reading of zero is known as **a null method**.

$$\frac{AY}{AB} = \frac{AJ}{AB} = \frac{\text{cell voltage}}{E_0}$$

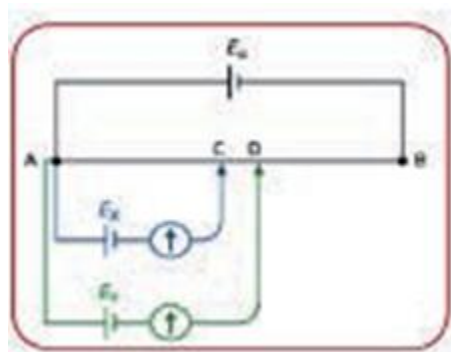
$$\frac{AY}{AB} = \frac{E_x}{E_0}$$

- Now we can calculate cell voltage as

$$E_x = \frac{AY}{AB} \times E_0$$

Potentiometer to compare the emf of two cells

- Suppose we have two cells whose emfs E_x and E_y we want to compare.
- Each is connected in turn to the potentiometer, giving balance points at C and D.



- In the diagram, we can see immediately that E_y must be greater than E_x because D is further to the right than C.
- The ratio of the emfs of the two cells will be equal to the ratio of the two lengths AC and AD:

$$\frac{E_x}{E_y} = \frac{AC}{AD}$$

$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

Example

1. In a potentiometer arrangement, a cell of emf 1.25 volts gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and balance point shifts to 63 cm, what is the emf of the second cell?

Solution

$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

$$E_2 = \frac{l_2}{l_1} \times E_1$$

$$E_2 = \frac{63\text{cm}}{35\text{cm}} \times 1.25\text{V}$$

$$E_2 = 2.25\text{V}$$

2. When the emf of two cells is compared in a potentiometer experiment, the ratio of balancing length is obtained as 2: 5. If one of the cells has an emf of 1.2 V, calculate the other cell's emf.

Solution

$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

$$\frac{E_2}{1.2\text{V}} = \frac{2}{5}$$

$$E_2 = 0.48\text{V}$$

$$\frac{1.2\text{V}}{E_1} = \frac{2}{5}$$

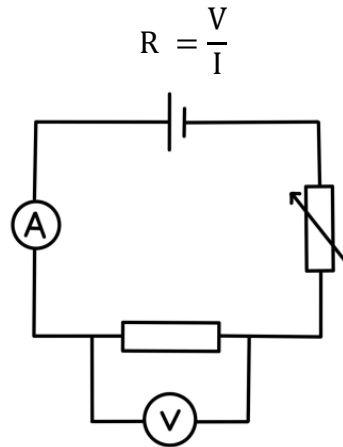
$$E_1 = 3\text{V}$$

Exercise

1. Consider a potentiometer set-up where a cell of emf 2.25 V gives a balance point at 63.0 cm length of the wire. Now the cell is replaced by another cell and the balance point shifts to 21.0 cm. Then the emf of the second cell is:
 A. 1.25V B. 0.75V C. 1.75V D. 0.35V

Measurement of Resistance

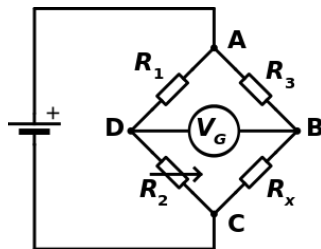
- Resistance can be measured directly by using ohmmeter.
- Experimentally resistance is measured using the following two methods.
 1. The ammeter-voltmeter method.
 2. Using Wheatstone bridge
 1. The ammeter-voltmeter method
- The ammeter-voltmeter method is a general method for measuring electrical resistances.
- The readings of current (I) and voltage (V) allow the calculation of electrical resistance (R) using Ohm's law



- The variable resistor is used to adjust the current and voltage as needed.

2. Using Wheatstone bridge

- Wheatstone bridge is an instrument designed to measure unknown resistance in electrical circuits.
- It calculates the unknown resistance by balancing the two legs of the bridge circuit where one leg contains both known resistors and the other leg contains one known (variable) and one unknown resistor.
- Since it estimates unknown resistance in an electric circuit, it is also known as a resistance bridge.
- Wheatstone bridge is a very reliable instrument as it measures the resistance very precisely.
- Wheatstone bridge works on the **principle of null deflection** i.e., there is no current flowing through the galvanometer, and its needle shows no deflection.

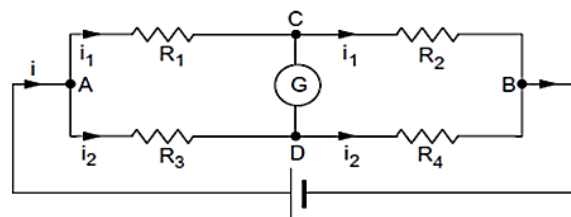


$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

$$R_1 R_x = R_2 R_3$$

$$R_x = \frac{R_2 R_3}{R_1}$$

- If four resistors with resistances R_1 , R_2 , R_3 and R_4 are connected to form a loop there are four joints A, B, C and D.

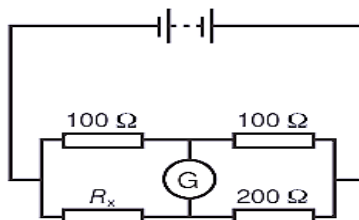


- A battery is connected between two opposite joints A and B and a galvanometer is connected between the other two opposite joints C and D.
- The current in R_2 is the same as the current in R_1 and the current in R_4 is the same as that in R_3 .
- When no current is flowing through, the bridge, it is said to be balanced.
- As there is no current through the galvanometer, the potential difference across its terminals is zero.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Examples

1. A Wheatstone bridge circuit is set up as shown below. It is balanced. What is resistance R_x ?



Solution

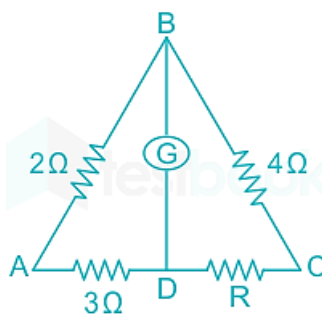
$$R_x = R_2 \times \frac{R_3}{R_1} = \frac{100\Omega \times 200\Omega}{100\Omega} = 200\Omega$$

2. Resistances of the ratio arms of a Wheatstone bridge are 300Ω and 30Ω . The fourth arm is connected to an unknown resistor. Find the value of the unknown resistance if the third arm has a resistance of 250Ω in a balanced condition?

Solution

$$\begin{aligned} \frac{R_1}{R_2} &= \frac{R_3}{R_x} \\ \frac{300\Omega}{30\Omega} &= \frac{250\Omega}{R_x} \\ R_x &= 25\Omega \end{aligned}$$

3. In the following circuit if deflection in the galvanometer is zero then calculate the value of resistance R.



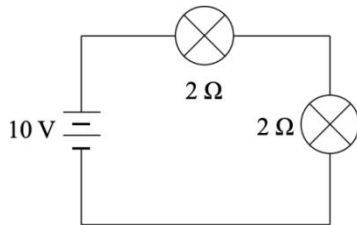
Solution

$$\begin{aligned} \frac{R_1}{R_2} &= \frac{R_3}{R_x} \\ \frac{2\Omega}{4\Omega} &= \frac{3\Omega}{R_x} \\ R_x &= 6\Omega \end{aligned}$$

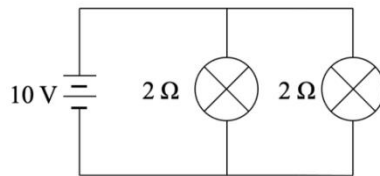
Comparing the Brightness of Bulbs

- Watts are not measure brightness; they're a measure of energy consumption per unit time.
- It measures, how much electricity a bulb uses.
- The amount of light emitted by a bulb (brightness) is measured in **Lumens**.
- A lumen is a measure of the amount of brightness of a lightbulb.
- The higher the lumens, the more light the bulb emits.
- We can use power (watts) to compare the brightness of bulbs.
- The higher the wattage rating, the more energy consumed.
- From the equation for electric power, we can see that brightness depends on current and resistance.

$$P = I^2 R$$



- Higher total resistance
- Lower total current
- Lower total power
- Brightness is reduced

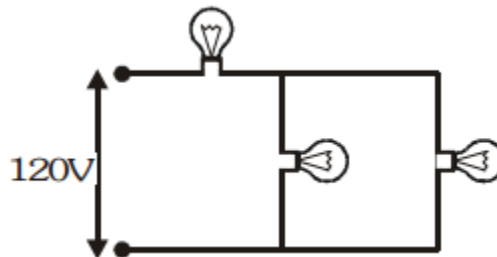


- Lower total resistance
- Higher total current
- Higher total power
- Brightness is unchanged

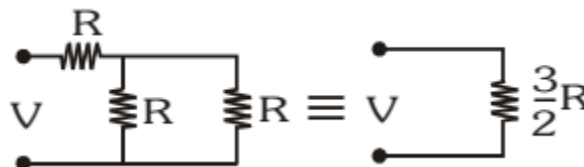
EFFICIENCY	LEAST			MOST
BULB TYPE	STANDARD	HALOGEN	CFL	LED
450 LUMENS	40W	29W	9W	6W
800 LUMENS	60W	42W	14W	9W
1100 LUMENS	75W	53W	18W	11W
1600 LUMENS	100W	72W	23W	14W
RATED LIFE	1 YEAR	1-3 YEARS	6-10 YEARS	12-25 YEARS
SAVINGS	X	UP TO 30%	UP TO 75%	UP TO 90%

Example

- Three 60W, 120V light bulbs are connected across a 120 V power source. If resistance of each bulb does not change with current then find out total power delivered to the three bulbs.



Solution



$$P = \frac{V^2}{R}$$

$$P_{total} = \frac{V^2}{\frac{3}{2}R} = \frac{2V^2}{3R}$$

$$P_{total} = \frac{2}{3} \left(\frac{V^2}{R} \right)$$

$$P_{total} = \frac{2}{3} (P) = \frac{2}{3} (60W) = 40W$$

Kirchhoff's Rules

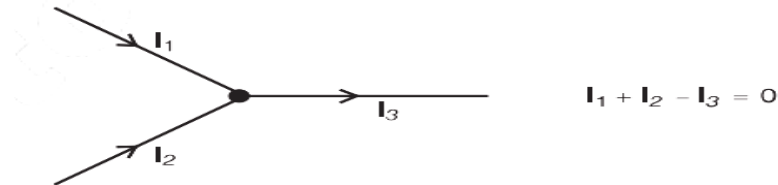
- Kirchhoff's rules can be used to analyze any DC circuit, regardless of how many loops and cells are in the circuit.
- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules.

1. Kirchhoff's first rule (point or junction rule).

- The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

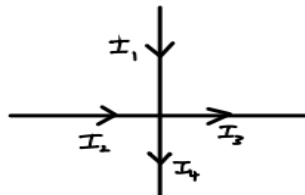
$$\sum I_{in} = \sum I_{out}$$

- ✚ It is a **statement of conservation of charge**
- ✚ A large number of electrons enter the battery at the positive terminal every second is equal to the number of electrons leaves the battery at the negative terminal every second.
- ✚ Similarly the rate at which electrons arrive at one end of a wire is exactly the same as the rate at which they leave the other. This is **Kirchhoff's point (junction) rule**.
- ✚ **The total current flowing into a point is equal to the total current flowing out of that point.**

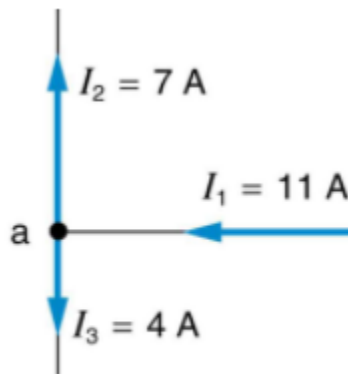


- By convention, currents going into a junction are **positive** but currents leaving a junction are **negative**.
- The sum of currents at any junction is zero.

$$I_3 = I_1 + I_2$$

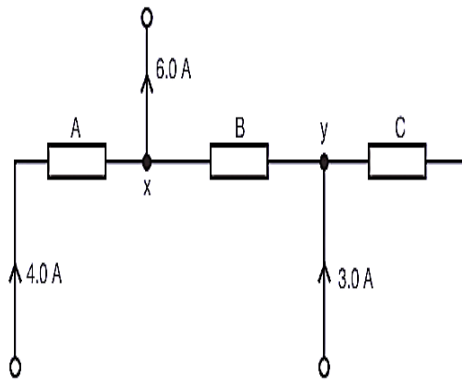


$$I_1 + I_2 = I_3 + I_4$$



$$I_1 = I_2 + I_3$$

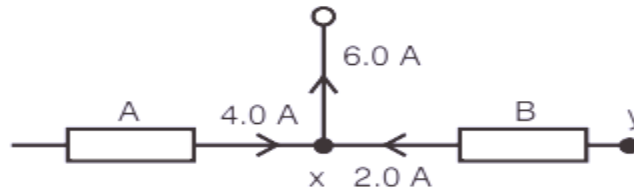
1. Look at the figure below, state the value of current in each of the resistors A, B, C



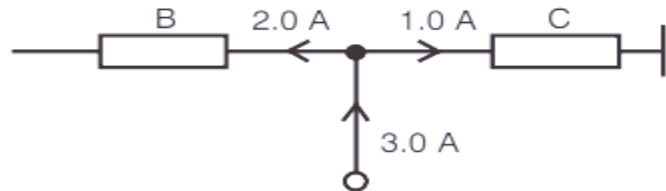
Solution

Resistor A has the full 4.0 A flowing through it.

For resistor B, we need to consider junction x. there are 6.0 A leaving x and which means a total of 6.0 A must enter x. since 4.0 A enter from the left, Kirchhoff's junction rule states that 2.0 A must enter from the right, as shown below.

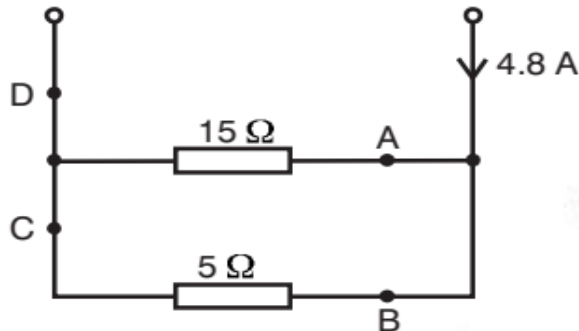


For resistor C we need to consider junction y.



There are 3.0 A flowing into the junction, so a total 3.0 A must leave the junction (2.0 A flow through B and 1.0 A

2. In the following circuits find the current at points A, B, C, and D.



Solution

From Kirchhoff's junction rule

$$4.8A = I_1 + I_2 \dots \dots \dots (1)$$

$$R_1 = 3R_2$$

Since R_1 and R_2 are parallel, the p.d. (V) across them will be the same.

$$V = IR$$

$$V_1 = I_1 R_1$$

$$\begin{aligned}
 V_2 &= I_2 R_2 \\
 V_1 &= V_2 \\
 I_1 R_1 &= I_2 R_2 \\
 I_1 (3R_2) &= I_2 R_2 \\
 3I_1 &= I_2 \dots \dots \dots (2) \\
 \text{Use equation 1 into 2} \\
 4.8\text{A} &= I_1 + 3I_1 \\
 4.8\text{A} &= 4I_1 \\
 \frac{4.8\text{A}}{4} &= I_1 \\
 I_1 &= 1.2\text{A}
 \end{aligned}$$

$$4.8\text{A} = 1.2\text{A} + I_2$$

$$I_2 = 3.6\text{A}$$

Now use Kirchhoff's junction rule at junction.

$$\text{Current in} = I_1 + I_2 = 4.8\text{A}$$

$$\text{Current out} = I_D = 4.8\text{A}$$

So at A, current is 1.2 A

at B, current is 3.6 A

at C, current is 3.6 A

at D, current is 4.8 A

Kirchhoff's second rule (loop rule)

- The sum of the potential differences across all elements around any closed circuit loop must be zero:

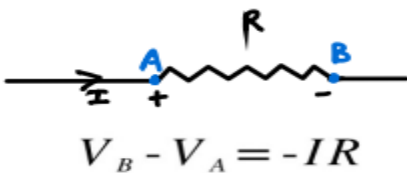
$$\sum_{\text{closed loop}} \Delta V = 0$$

- ✚ Kirchhoff's second rule is based on conservation of energy.

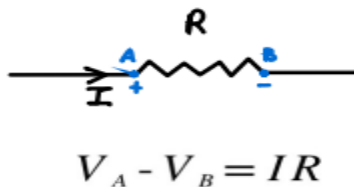
- ✚ Kirchhoff's loop rule state that: **In any closed loop in a circuit the sum of the emf is equal to the sum of the p.ds.**

Some conventions:

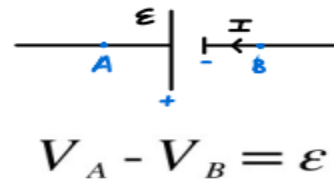
- Potential across a resistor is negative in the direction of current



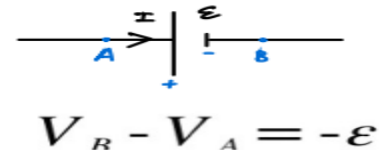
- Potential across a resistor is positive in the direction opposite to current



- Potential is gained on crossing a battery from negative to positive terminal

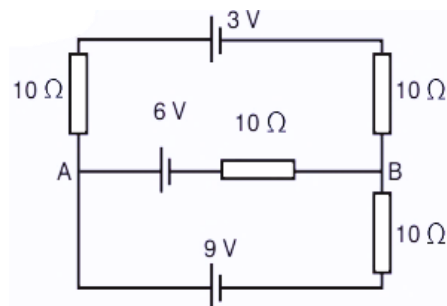


- Potential drops on crossing a battery from positive to negative terminal



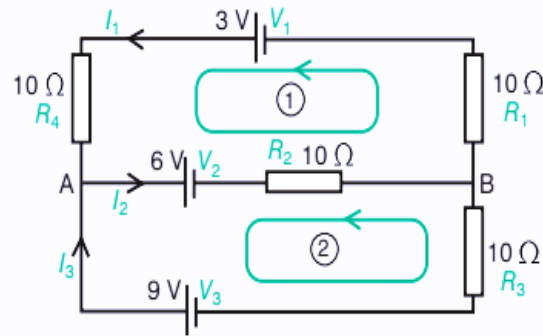
Examples

- Find the current flowing in each of the resistors in this circuit.



Solution

First label the diagram as shown below.



At junction A, from Kirchhoff's junction rule,

$$I_1 + I_3 = I_2 \quad (1)$$

In loop 1, from Kirchhoff's loop rule, $V_1 - I_1 R_4 -$

$$V_2 - I_2 R_2 - I_1 R_1 = 0$$

$$3 - 10I_1 - 6 - 10I_2 - 10I_1 = 0$$

$$-3 - 20I_1 - 10I_2 = 0 \quad (2)$$

In loop 2, from Kirchhoff's loop rule, $V_3 - V_2 -$

$$I_2 R_2 - I_3 R_3 = 0$$

$$9 - 6 - 10I_2 - 10I_3 = 0$$

$$3 - 10I_2 - 10I_3 = 0 \quad (3)$$

From equation (1) we can derive I_1

$$I_1 = I_2 - I_3 \quad (4)$$

Now we can use equation (4) in equation (2)

$$-3 - 20(I_2 - I_3) - 10I_2 = 0$$

$$-3 - 20I_2 + 20I_3 - 10I_2 = 0$$

$$-3 - 30I_2 + 20I_3 = 0 \quad (5)$$

Multiply equation (3) by -3 and add with equation

(5)

$$-3(3 - 10I_2 - 10I_3 = 0)$$

$$-9 + 30I_2 + 30I_3 = 0$$

$$+ \begin{cases} -9 + 30I_2 + 30I_3 = 0 \\ -3 - 30I_2 + 20I_3 = 0 \end{cases}$$

$$-12 + 50I_3 = 0$$

$$12 = 50I_3$$

$$I_3 = \frac{12}{50}$$

$$I_3 = 0.24A$$

Put I_3 in equation (3)

$$3 - 10I_2 - 10(0.24A) = 0$$

$$3 - 10I_2 - 2.4 = 0$$

$$0.6 - 10I_2 = 0$$

$$0.6 = 10I_2$$

$$I_2 = \frac{0.6}{10}$$

$$I_2 = 0.06A$$

$$I_1 = I_2 - I_3$$

$$I_1 = 0.06A - 0.24A$$

$$I_1 = -0.18A$$

6.6. Capacitors and Capacitance

By the end of this section, you will be able to:

- ♣ Define the terms capacitors, capacitances, dielectric.
- ♣ Explain the effect of inserting dielectric in the gap between the plates of a capacitor.
- ♣ Solve problems related to capacitance of parallel plate capacitor.
- ♣ Calculate the equivalent capacitance of capacitors connected in series and parallel.

Capacitor and Capacitance

- A capacitor is a two-terminal electrical device that can store energy in the form of an electric charge.
- It consists of two electrical conductors that are separated by a distance.
- The space between the conductors may be filled by vacuum or with an insulating material known as a **dielectric**.
- The ability of the capacitor to store charges is known as **capacitance**.



- A combination of two conductors placed close to each other but not touching form a capacitor.
- In charging a capacitor one of the plates is connected to the positive terminal and the other to the negative terminal of a DC source.
- The electric field created forces electrons to flow from the positive plate to the negative plate until the potential difference between the plates of the capacitor becomes equal to the emf of the battery.
- For a given capacitor, the charge Q stored on the capacitor is proportional to the potential difference V between the plates.

$$Q = CV$$

$$C = \frac{Q}{V}$$

- The proportionality constant C is called the capacitance of the capacitor.
- It is independent of Q and V but depends on geometrical factors.
- The SI unit of capacitance is farad (F). Small values of capacitances are measured in micro, nano and pico farads.

Example

1. What is the capacitance of a capacitor which can store 18mC of charge when the pd across it is 6V ?

Solution

$$C = \frac{Q}{V}$$

$$C = \frac{18 \times 10^{-3} \text{C}}{6\text{V}}$$

$$C = 3 \times 10^{-3} \text{F}$$

$$C = 3\text{mF}$$

2. How much charge will be stored on this capacitor if the voltage is increased to 20V ?

$$Q = CV$$

$$Q = 3 \times 10^{-3} \text{F} \times 20\text{V}$$

$$Q = 60 \times 10^{-3} \text{C}$$

Exercise

1. A conductor gets a charge of $50\text{ }\mu\text{C}$ when it is connected to a battery of e.m.f. 5V . Calculate capacity of the conductor.

2. A 10-farad capacitor connected to a 12-volt battery what is amount of charge can be stored in the capacitor?
3. A capacitor having a capacitance of $6\mu F$ is connected to a 100 V battery. What charge will be stored in the capacitor?
4. What amount of charge accumulates on the plates of a $30\mu F$ capacitor when it is connected to a 300 V power supply?
5. What is the capacitance of a capacitor which acquires a charge of $600\mu C$ when connected to a 1200 V power supply?

Parallel-plate Capacitor

- A parallel-plate capacitor consists of two large plane plates placed parallel to each other with a small separation between them.
- Suppose, the area of each of the facing surfaces is A and the separation between the two plates is d . Also, assume that the space between the plates is filled with vacuum.
- If a charge Q is stored on the positive plate, a charge $-Q$ will be stored on the negative plate. The charges will appear on the facing surfaces.
- **Charge density** is the charge per unit area on a charged surface.
- For a given plate with a charge Q and an area A the charge density σ is given by:

$$\sigma = \frac{Q}{A}$$

- The charge density on each of these surfaces has a magnitude

$$\sigma = \frac{Q}{A}$$

- Using **Gauss's law** we can derive an expression for the electric field strength between the plates.
- **Gauss's Law** states that the net electric flux through any surface is equal to the total charge enclosed within that surface divided by the permittivity of free space.

$$\Phi_E = \frac{Q}{\epsilon_0}$$

- This statement basically implies that any particular surface will only have a net flux as long as a net charge is found inside that surface.
- Using Gauss's law the electric field through each plate is

$$E = \frac{\sigma}{2\epsilon_0}$$

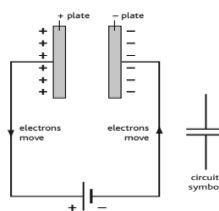
- As the plates have an equal but opposite charge they mutually attract and hold each other together. Therefore in between plates the electric fields are in the same direction.

- The total electric field between the plates is the sum of the two electric fields.

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = 2\left(\frac{\sigma}{2\epsilon_0}\right)$$

$$E = \frac{\sigma}{\epsilon_0}$$

- An electric field can cause charged particles to move.



- The electric field between the plates is given by the equation

$$E = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{Q}{A\epsilon_0}$$

- The potential difference between each plate is given by:

$$V = Ed$$

$$V = \left(\frac{Q}{A\epsilon_0} \right) d$$

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\left(\frac{Q}{A\epsilon_0} \right) d}$$

$$C = \epsilon_0 \frac{A}{d}$$

- This equation gives the capacitance of two parallel plates in a vacuum.
 ➤ If a dielectric material of dielectric constant k is inserted between the plates, the equation for capacitance became:

$$C = k\epsilon_0 \frac{A}{d}$$

- The capacitance depends on:

- ✚ The size of the plates
- ✚ Their separation
- ✚ The nature of the insulator between them

- Two parallel plates are arranged as a capacitor having an air as the dielectric and being 1 cm apart. If the plates is 200cm^2 what is the capacitance of the capacitor?

Solution

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} (200 \times 10^{-2} \text{m})}{1 \times 10^{-2} \text{m}} = 1770 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} = 1.77 \times 10^{-9} \text{F} = 1.77 \text{nF}$$

Exercise

- An air-filled parallel-plate capacitor has plates of area 2.30 cm^2 separated by 1.50 mm.
 - Find the value of its capacitance. The capacitor is connected to a 12.0V battery.
 - What is the charge on the capacitor?
 - What is the magnitude of the uniform electric field between the plates?
- Calculate the area of a parallel plate's air capacitor having a capacitance of $8.85 \mu\text{F}$ and separated by 1mm.

Dielectrics

- A dielectric is an insulating or non-conducting material such as glass, paraffin, wood or plastic that is inserted in the gap between the plates of a capacitor.
- When a dielectric is inserted in the electric field between the plates, there will be some charge distribution in it.
- The positive charges within the dielectric are displaced lightly towards the negative plate, and the negative charges are displaced slightly towards the positive plate.
- This slight separation of charge is known as **polarization**.
- Due to polarization the potential difference and hence the electric field between the plates will decrease.
- Since, the dielectric is not disconnected from the source; the potential difference between the plates finally will be that of the source.
- As a result, the capacitor can hold more charge, and thus, its capacitance increased,

$$C = kC_0$$

- A dielectric constant (k) is also called **relative permittivity** or **specific inductive capacity**.

- It measures the ability of a capacitor to store charge.
- It is the property of the dielectric.
- It is equal to the ratio of the capacitance of a capacitor filled with the given material to the capacitance of an identical capacitor in a vacuum without the dielectric material.

$$k = \frac{C}{C_0}$$

- The dielectric constant is a number without dimensions.
- The value of the dielectric constant for vacuum is one and for any other material is it is greater than one.
- **Inserting a dielectric always increases the capacitance of the capacitor.**

Dielectric strength

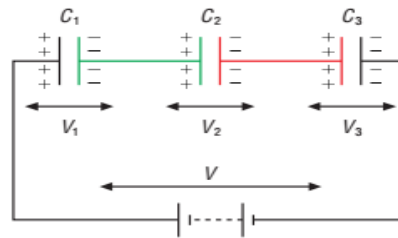
- For any insulator, there is a maximum electric field that can be maintained without ionizing the molecules.
- This means that there is a maximum allowable voltage that that can be placed across the conductors.
- This maximum voltage depends on the dielectric in the capacitor.
- The corresponding maximum field E_b is called the dielectric strength of the material.
- It is the maximum electric field that the material can withstand without undergoing electrical breakdown and becoming electrically conductive (i.e. without failure of its insulating properties).

Dielectric strength is the maximum electric field strength that a material can withstand before break down.

Combination of Capacitors

Capacitors in series

✚ When capacitors are connected in series, they are connected one after the other.



- For a series connection, the charge stored in each capacitor is the same as the charge stored in the connection.

$$Q_1 = Q_2 = Q_3 = Q_T = Q \dots \dots \dots (1)$$

- The total potential is equal to the sum of the individual potential drops across each capacitor

$$V_T = V_1 + V_2 + V_3 + \dots + V_n \dots \dots \dots (2)$$

$$C = \frac{Q}{V} \text{ And } V = \frac{Q}{C} \quad V_T = \frac{Q_T}{C_T}$$

Now, equation (2) became

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \dots + \frac{Q_n}{C_n}$$

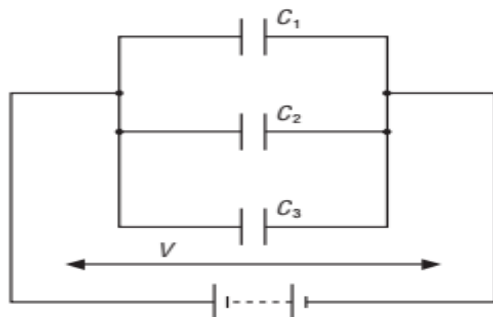
$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots + \frac{Q}{C_n}$$

$$Q \left(\frac{1}{C_T} \right) = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n} \right)$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n} \dots \dots \dots (3)$$

Capacitors in parallel

When capacitors are connected in parallel, they are connected face to face.



For a parallel connection:

🌸 The total potential difference is the same as the potential drop across each capacitor

$$V_1 = V_2 = V_3 = V_T = V \dots \dots \dots (1)$$

➤ The total charge stored in the system is equal to the sum of charges stored in each capacitor

$$Q_T = Q_1 + Q_2 + Q_3 + \dots + Q_n \dots \dots \dots (2)$$

$$Q = CV$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 + \dots + C_n V_n$$

$$C_T V = C_1 V + C_2 V + C_3 V + \dots + C_n V$$

$$C_T V = V(C_1 + C_2 + C_3 + \dots + C_n)$$

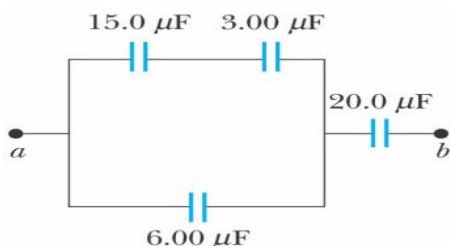
$$C_T = C_1 + C_2 + C_3 + \dots + C_n$$

$$C_T = C_1 + C_2 + C_3 + \dots + C_n$$

1. Four capacitors are connected as shown in figure below.

A. Find the equivalent capacitance between points a and b.

B. Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0 \text{ V}$.



Solution

$$\text{A. } \frac{1}{C_s} = \frac{1}{15\mu F} + \frac{1}{3\mu F} = \frac{6}{15\mu F}$$

$$C_s = \frac{15\mu F}{6} = 2.5\mu F$$

$$C_p = 2.5\mu F + 6\mu F = 8.5\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{8.5\mu F} + \frac{1}{20\mu F} = \frac{285}{1700\mu F}$$

$$C_{eq} = \frac{1700\mu F}{285} = 5.96\mu F$$

B. $Q = C\Delta V = 5.96\mu F(15V) = 89.5\mu C$
on $20\mu F$

$$\Delta V = \frac{Q}{C} = \frac{89.5\mu F}{20\mu F} = 4.47 \text{ V}$$

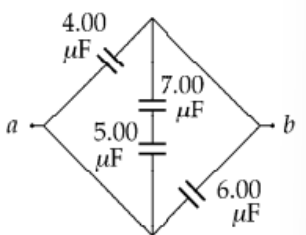
$$\Delta V = 15V - 4.47 \text{ V} = 10.53V$$

$Q = C\Delta V = 6\mu F(10.53V) = 63.2\mu C$
on $6\mu F$

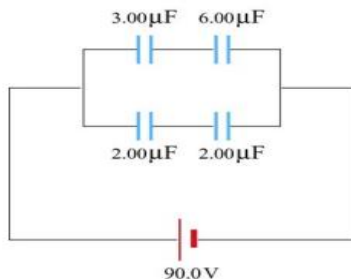
$89.5\mu C - 63.2\mu C = 26.3\mu C$ on
 $15\mu F$ and $3\mu F$

Exercise

- Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure below



- For the system of capacitors shown in Figure below find (a) the equivalent capacitance of the system, (b) the charge on each capacitor



Energy stored in a capacitor

The energy stored by a capacitor is given by:

$$E = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

Example: Two capacitors, of $6.0\mu F$ and $12.0\mu F$ capacitance, are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

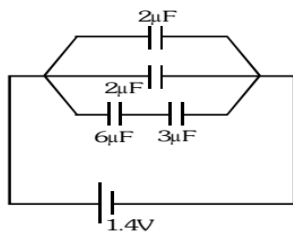
Solution

$$C_T = C_1 + C_2 = 6.0\mu + 12.0\mu F = 18\mu F = 18 \times 10^{-6} F$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} (18 \times 10^{-6} F)(300 V)^2 = 2700 \times 10^{-6} FV = 2.7 \times 10^{-3} J = 2.7 mJ$$

EXERCISE

- A capacitor of capacitance $10\mu F$ is fully charged from 20V power supply. Calculate the energy stored in the capacitor.
- Find the energy held in capacitors in the given circuit, given in mJ.



- How much energy can be stored if 10V is applied to a 5F capacitor?
- It takes 20V to store 80C of charge on a certain capacitor. (a) How much potential energy is stored in this capacitor (b) Calculate the capacitance of this capacitor.
- How much energy is stored if 500C of charge rests on a 10F capacitor? (b) What is the voltage across this capacitor?
- A certain capacitor has 25J of stored potential energy when connected across a 10V battery. How much energy can the capacitor store when connected across a 20 V battery?
- How much energy is stored in a $150\mu F$ capacitor if a 12V battery is connected across it? (b) How much charge is stored in this capacitor?

6.7. Electric Circuits in Our Surroundings

By the end of this section, you will be able to:

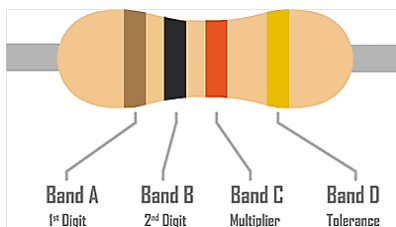
- ♣ Apply the knowledge about electric circuits to explain household electric installation.

Household Electric Installations

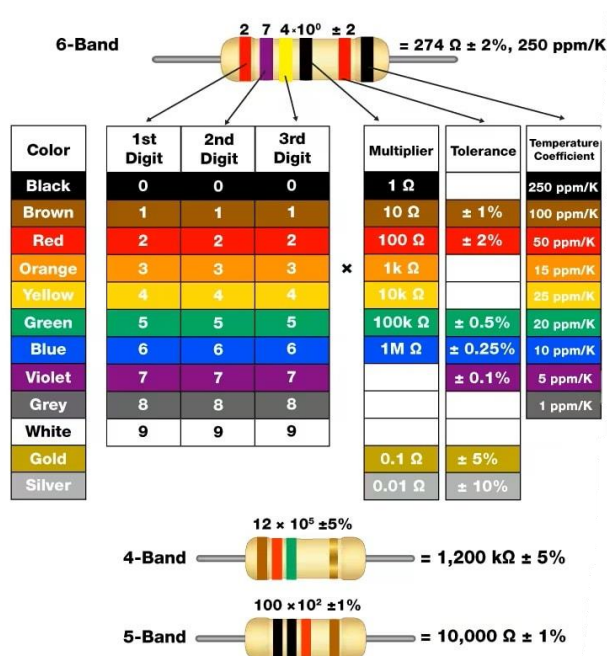
- In our country Ethiopia, the voltages of the mains are 220 V - 240 V.
- Heating appliances include **electric kettle**, **immersion heater**, **electric irons**, **hair driers** and **electric stoves**.
- All heating appliances contain a heating element that converts electrical energy into heat energy.

Color Code of Resistors

- The value of the resistance of resistors in electronic devices is labeled by a color code.
- There are colored rings in any resistor.
- According to the international standard organization ISO, the orders of the rings are arranged and each color is represented by a number.



- The first band represents the first digit number, the second band the second digit number and the third band represents multiplier to the base 10.
- The fourth band is the tolerance (the permissible percentage error $\pm$ of the value).
- The trick in reading color code is noticing that the first three digits (numerical values) in a four band resistor are closer in equal gap, whereas the fourth band (the tolerance) on the other end is in a bit wider gap
- In a five band resistor, the first four digits are closer in equal gap and the fifth band (the tolerance) is a bit further
- The first three closer bands are: **brown**, **black** and **red**, which represent first digit number, second digit number and the multiplier respectively.
- Referring the color chart, the first band is brown = 1 the second band is black = 0 , the third band is red= 2.



- 5ppm/K means that the resistance will change by 5 parts per million per 1 Kelvin change in temperature.