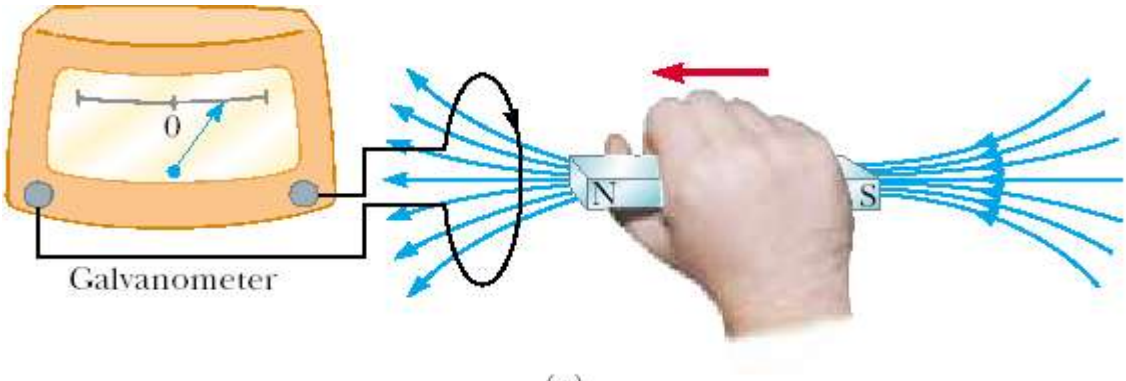
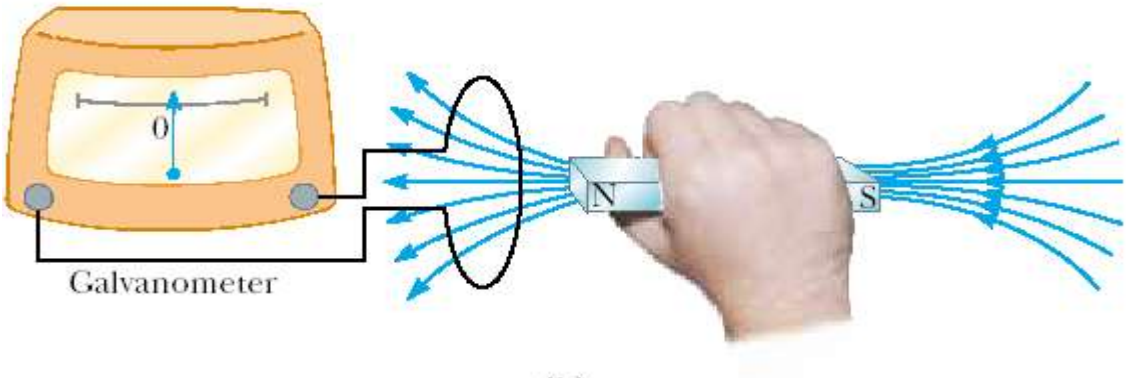
UNIT 7: ELECTROMAGNETIC INDUCTION AND AC CIRCUIT

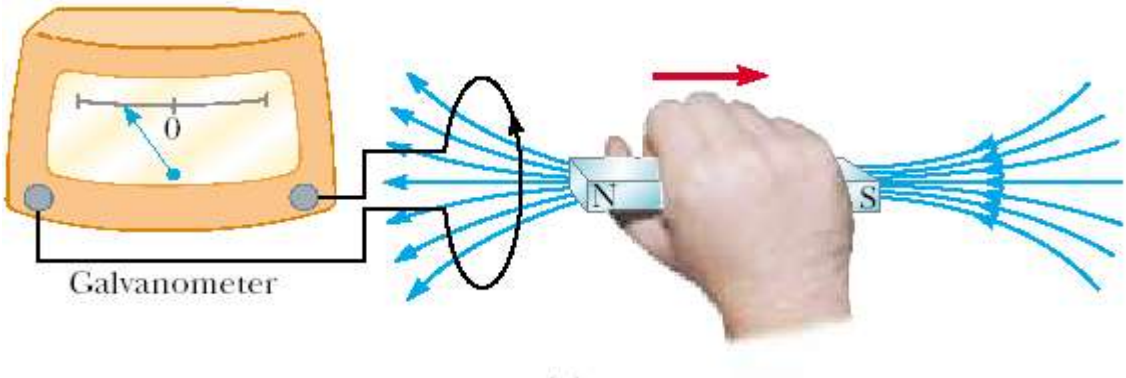
In 1819 Hans Christian Oersted discovered that electric current produces magnetic field. This finding was the first evidence of a link between electricity and magnetism. The discovery that electric current produces magnetic field led scientists to suspect that magnetic field could produce electric current.
Experiments conducted by Michael Faraday in England and independently by Joseph Henry in the United States in 1831 showed that a changing magnetic field could induce an electric current in a circuit. The results of these experiments led to a basic and important law known as *Faraday's law of induction.*
Electromagnetic induction:- is the production of Ɛmf in a conductor when there is a change of magnetic flux linkage with the conductor or when there is relative motion of the conductor across a magnetic field.
To see how an Ɛmf can be induced by a changing magnetic field, consider a loop of wire connected to a sensitive galvanometer, as illustrated in figure below. When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the rght in figure (a). When the magnet is brought to rest and held stationary relative to the loop figure (b), no deflection is observed. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in figure (c). Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects.
The following figures show the production of Ɛmf in a conductor.



(a)



(b)



(c)

Faraday's Law:- the *Ɛmf induced in a circuit is directly proportional to the time rate of change of the magnetic flux* through the circuit.

The induced Ɛmf is given by

$$\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$$ Where N is number of turns

The –ve sign indicates that the induced Ɛmf is in such a direction to oppose the change of magnetic flux.

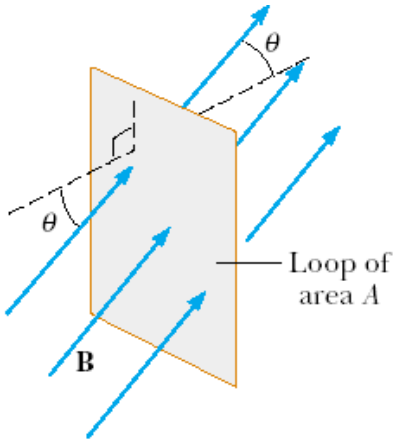
Suppose that a loop enclosing an area *A* lies in a uniform magnetic field B, as in figure below.

The magnetic flux through the loop is equal to *BA* cos θ; hence, the induced Ɛmf can be expressed as

$$\varepsilon = -\frac{\Delta N(BA\cos\theta)}{\Delta t}$$

$$OR\ \ \varepsilon = -\frac{d(NBA\cos\theta)}{dt}$$

ε = NBA sin θ



From this expression, we see that an emf can be induced in the circuit if:

- the magnitude of **B** changes with time.
- the area enclosed by the loop changes with time.
- the angle θ between **B** and the normal to the loop changes with time.
- any combination of the above occurs.

*The magnitude of the induced Ɛmf increases if

- the relative speed of the magnet and coil increases.
- the number of turns of the coil increases.
- a strong magnet is used.

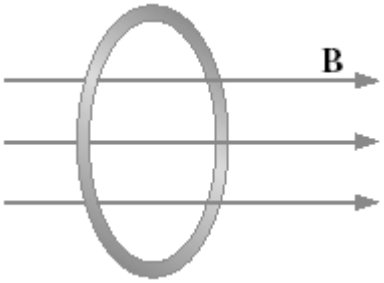
Example

1. The magnitude of the field changes from 0.1 T to 0.2 T in 0.1 sec. If the plane of the coil with area 10 cm^2 is perpendicular to the field and has 10 turns, what is the magnitude of the induced emf? **Ans**. $\mathcal{E}_{ind}$ = 0.01V
2. A magnetic field that increases at a rate of 4×10^{-3} T/ sec is at right angles to a plane of closed loop of wire having a cross-sectional area of 7.5 cm^2. What emf is induced in the loop? **Ans**. Ɛ = 3×10^{-6}V
3. A magnetic field is perpendicular to the plane of a rectangular coil of cross-sectional area 8 cm^2. If the coil has 50 turns and the magnitude of the field varies with time according to B = (4*t* + 5) T, what is the induced Ɛmf in the coil? **Ans**. Ɛ = 0.16 V
4. A single loop of wire with an area of 0.8 m^2 is placed in a region where there is a varying magnetic field at a rate of 0.3 T/sec. If the resistance of the loop is 4Ω, what is the current induced in the loop? **Ans**. I = 0.06 A
5. How long will it take for a magnetic flux to increase from 3×10^{-3} Wb to 5×10^{-3} Wb there by inducing an Ɛmf of magnitude 0.02V in a single loop coil? **Ans**. Δ*t* = 0.1 sec

Exercise

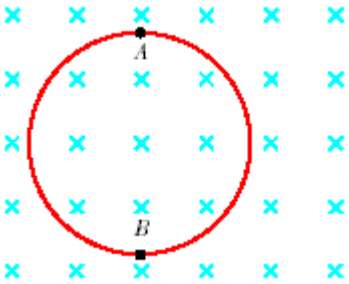
1. A single loop coil with an area 4×10^{-2} m^2 is placed in a uniform magnetic field perpendicular to the plane of the loop. If the coil has a resistance of 8 Ω, at what rate must the magnetic field change in order to induce a current of 5 A in the coil? **Ans**. $\Delta B/\Delta t = 10^3$ T/sec
2. A circular coil of area 10 cm^2 is placed perpendicular to a magnetic field that varies with time as B = (10t + 2) T. If the coil has 100turns, what is the magnitude of the induced emf? **Ans**. Ɛ = 1V
3. A wire loop of radius 0.3m lies so that an external magnetic field of magnitude 0.3T is perpendicular to the loop. The field reverses its direction, and its magnitude changes to 0.2T in 1.5 sec. Find the magnitude of the average induced emf in the loop during this time. **Ans**. ε = 0.0942V
4. A circular loop of wire of radius r is in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field as in the figure below. The magnetic field varies with time according to $B(t)$ = a + bt, where a and b are constants. Calculate

   a) the magnetic flux through the loop at t = 0. **Ans**. $\phi(0) = a\,\pi\,r^2$
   b) the emf induced in the loop. **Ans**. Ɛ = $b\,\pi\,r^2$
   c) the induced current if the resistance of the loop is R.
   Ans. I = $b\,\pi\,r^2$ /R
   d) the rate at which energy is being delivered to the resistance of the loop. **Ans**. P = $b^2\,\pi^2\,r^4$ /R



5. The magnetic flux through a metal ring varies with time t according to $\Phi_B = 3(a\,t^3 - bt^2)$ T.m^2, with a = 2 s^{-3} and b = 6 s^{-2}. The resistance of the ring is 3 Ω. Determine the maximum current induced in the ring during the interval from t= 0 to t = 2 s. **Ans**. I_{max} = 6 A
6. The flexible loop in figure below has a radius of 12 cm and is in a magnetic field of strength 0.15 T. The loop is grasped at points A and B and stretched until its area is nearly zero. If it takes 0.20 s to close the loop, find the magnitude of the average induced emf in it during this time.

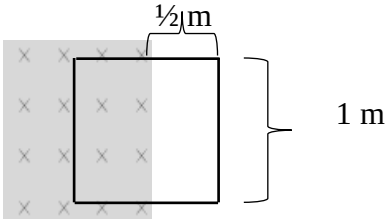
   Ans. $\mathcal{E}_{av} = 34\,m$ V



Questions from national exam

1. A square loop of wire, 1 m on a side, is placed halfway into a uniform magnetic field of strength 2 T as shown in the figure. If the magnetic field decreases to zero at a constant rate over 2 sec, then what is the magnitude of the induced Ɛmf on the loop?

   A) 0.2 V
   B**)** 0.5 V
   C) 1.0 V
   D) 1.5 V



2. A rectangular loop of wire of area 200 cm^2 is placed perpendicular to the magnetic field given by B(t) = (0.4T) cos(100t). What is the induced Ɛmf in the circuit as a function of time?

   A) 0.8 cos(100t)
   B**)** 0.8 sin(100t)
   C) 0.008 sin(100t)
   D) 800 sin(100t)

3. A circular loop of wire expands due to rise in temperature so that area enclosed by the loop increases slowly with time according to $A = A_0 + bt$, where b is a constant given by 1×10^{-6} m^2/sec. What is the induced Ɛmf if the circuit is placed in a uniform magnetic field of magnitude B = 0.4 T and is perpendicular to the plane formed by the circuit?

   A) 3.0×10^{-5}V B) 4.0×10^{-6}V C) 3.0×10^{-6}V D**)** 4.0×10^{-7}V

4. A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 37^0 with the direction of the field. When the magnetic field is increased uniformly from 200 *m*T to 600 *m*T in 4.00 s, an emf of magnitude 160.0 *m*V is induced in the coil. What is the total length of the wire in the coil? **Ans**. $L_{Tot} = 40$ *m*

   A) 100 *m* B) 200 *m* C) 300 *m* D) 400 *m*

5. A rectangular loop of areab 0.4 m^2 is placed in a magnetic field that is changing at a rate of 100 T/s. If the normal to the loop makes angle 60^o with the magnetic field, what is the magnitude of inducedbemf?

   A) 20 V B) 40 V C) 80 V D) 34.64 V

Motional EMF

In this section we describe the emf induced in a conductor moving through a constant magnetic field. The straight conductor of length ℓ shown in the figure below is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. Because the area enclosed by the circuit at any instant is ℓx, where x is the position of the bar, the magnetic flux through that area is

$$\Phi_B = B\ell x$$

Using Faraday's law, and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(B\ell x)}{dt} = -B\ell\frac{dx}{dt}$$

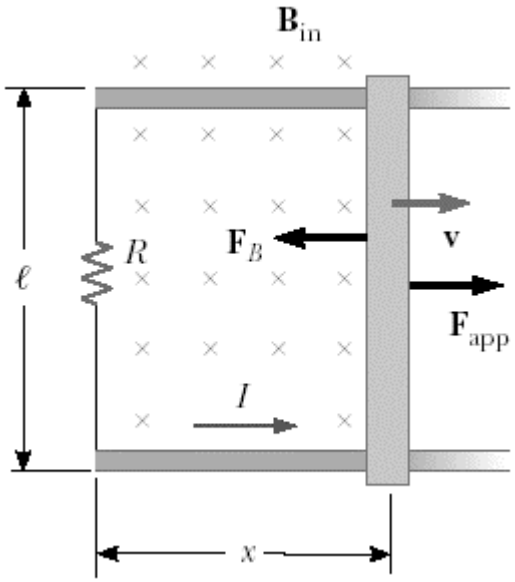
$$Ɛ = -B\ell v$$

If the velocity of the conductor makes an angle θ with the magnetic field, the induced Ɛmf is given by

$$Ɛ = B\ell v \sin\theta$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$



The power delivered by the applied force is

$$P = F_{app}\, v$$

$$= (I\ \ell\ B)\, v$$

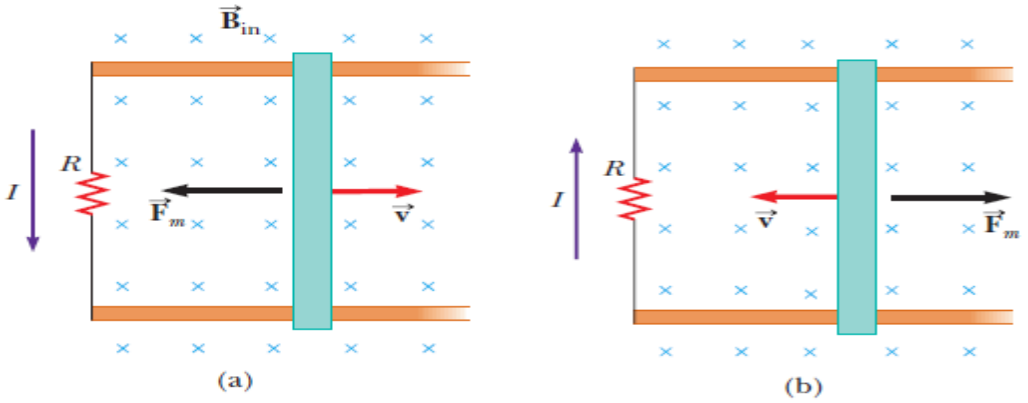
$$P = \left(\frac{\varepsilon}{R}\ell B\right)v$$

$$P = \frac{\varepsilon^2}{R}$$

Lenz's Law

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop. **That is, the induced current tends to keep the original magnetic flux through the circuit from changing. As we shall see, this law is a consequence of the law of conservation of energy.**

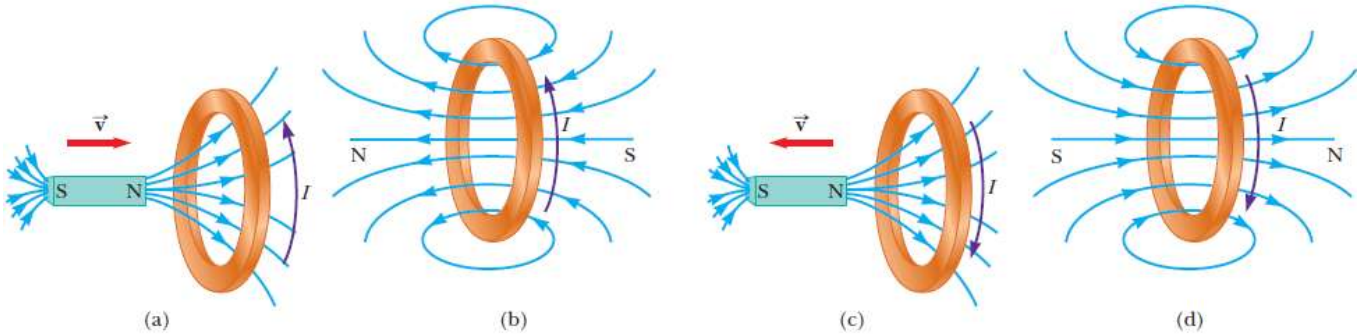
Consider a bar moving to the right on two parallel rails in the presence of a uniform magnetic field directed into the paper as in figure (a). As the bar moves to the right, the magnetic flux through the circuit increases with time because the area of the loop increases. Lenz's law says that the induced current must be in a direction such that the flux *it* produces opposes the change in the external magnetic flux.



Because the flux due to the external field is increasing *into* the paper, the induced current, to oppose the change, must produce a flux *out* of the paper. Hence, the induced current must be counterclockwise when the bar moves to the right. On the other hand, if the bar is moving to the left, as in figure (b), the magnetic flux through the loop decreases with time. Because the flux is into the paper, the induced current has to be clockwise to produce its own flux into the paper (which opposes the decrease in the external flux).
In either case, the induced current tends to maintain the original flux through the circuit.

Now we examine this situation from the viewpoint of energy conservation. Suppose that the bar is given a slight push to the right. In the preceding analysis, we found that this motion led to a counter-clockwise current in the loop. Let's see what would happen if we assume that the current is clockwise, opposite the direction required by Lenz's law. For a clockwise current I, the direction of the magnetic force $BI\ell$ on the sliding bar is to the right. This force accelerates the rod and increases its velocity. This, in turn, causes the area of the loop to increase more rapidly, thereby increasing the induced current, which increases the force, which increases the current, and so forth. In effect, the system acquires energy with zero input energy. **This is inconsistent with all experience and with the law of conservation of energy, so we're forced to conclude .**

Consider another situation. A bar magnet is moved to the right toward a stationary loop of wire, as in figure (a). As the magnet moves, the magnetic flux through the loop increases with time. To counteract this rise in flux, the induced current produces a flux to the left, as in figure (b); hence, the induced current is in the direction shown. Note that the magnetic field lines associated with the induced current oppose the motion of the magnet. The left face of the current loop is therefore a north pole and the right face is a south pole. On the other hand, if the magnet were moving to the left, as in figure (c), its flux through the loop, which is toward the right, would decrease with time. Under these circumstances, the induced current in the loop would be in a direction to set up a field directed from left to right through the loop, in an effort to maintain a constant number of flux lines. Hence, the induced current in the loop would be as shown in figure (d). In this case, the left face of the loop would be a south pole and the right face would be a north pole.

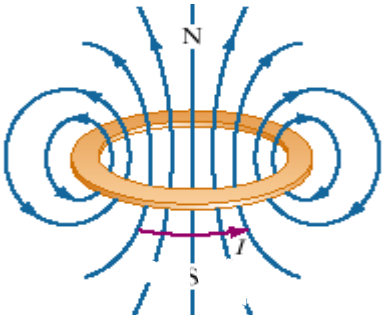


(a) When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. (b) This induced current produces its own flux to the left to counteract the increasing external flux to the right. (c) When the magnet is moved away from the stationary conducting loop, a current is induced in the direction shown. (d) This induced current produces its own flux to the right to counteract the decreasing external flux to the right.

Example

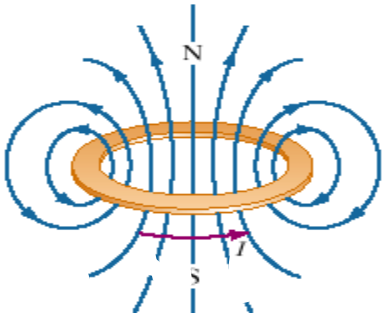
1. Use Lenz's law to find the direction of the induced current.

a) **B** increasing



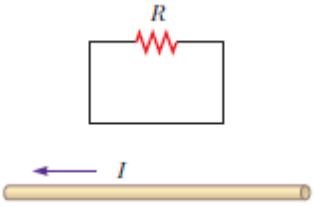
Ans. I_{indu} is in the clockwise direction.

b) **B** decreasing

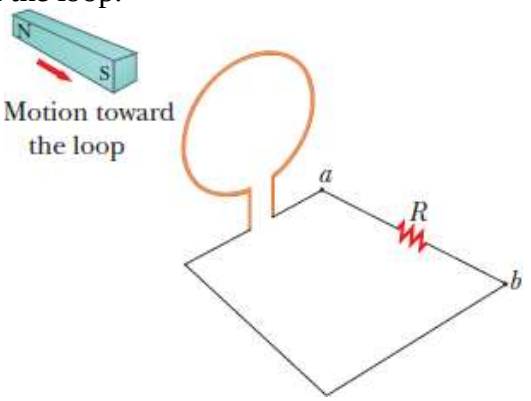


Ans. I_{indu} is in the counterclockwise direction.

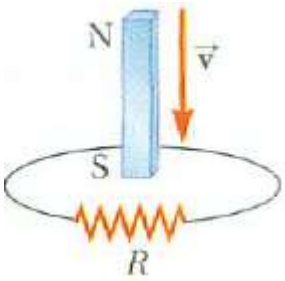
2. What is the direction of the current induced in the resistor when the current in the long, straight wire in figure bellow:
   a) increases from zero to constant current I?
   b) decreases rapidly to zero?



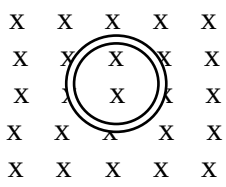
3. In the following figure, the bar magnet is being moved toward the loop.
   a) will the induced current flow from a to b or b to a across the resistor?
      Ans. using Lenz's law, the induced current flows from b to a.
   b) Is (V_a - V_b) positive, *negative*, or zero during this motion? Explain.



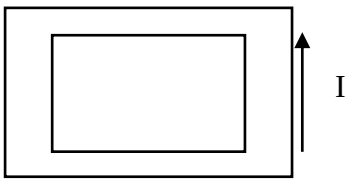
4. A bar magnet is held above the center of a wire loop lying in the horizontal plane, as shown in figure below. After the magnet is dropped, what is true of the current in the resistor as viewed from above?
   a) It is counterclockwise as the magnet falls toward the loop.
   b) It is always counterclockwise.
   c) It is clockwise after the magnet has moved through the loop and moves away from it.
   d) It is always clockwise.
   e) It is first clockwise as the magnet approaches the loop and then counterclockwise after it has passed through the loop.



5. A copper ring is in a uniformly increasing magnetic field directed into the page of this paper as shown in the fig. The induced current in the ring is
   A) clockwise and constant.
   B) counterclockwise and constant.
   C) clockwise and changing.
   D) counterclockwise and changing.



6. Two rectangular loops of wire lie in the plane of this paper as shown in figure below. If the current I in the outer loop is counterclockwise and increases with time, one of the following statements is correct about which the current induced in the inner loop?
   A) A current is induced in the inner loop in the clockwise direction.
   B) A current is induced in the inner loop in the counterclockwise direction.
   C) The induced current in the inner loop creates a magnetic field inside the area bounded by the inner loop in the same direction of the magnetic field that the outer loop produces in this same area.
   D) It is zero.



Motional emf Induced in a Rotating Bar

A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. If a uniform magnetic field **B** is directed perpendicular to the plane of rotation as shown in figure below, an expression for the *motional emf induced between the ends of the bar* can be derived as follows.

Consider a segment of the bar of length *dr* having a velocity v.
The magnitude of the emf induced in this segment is

$$d\mathcal{E} = Bv\,dr$$

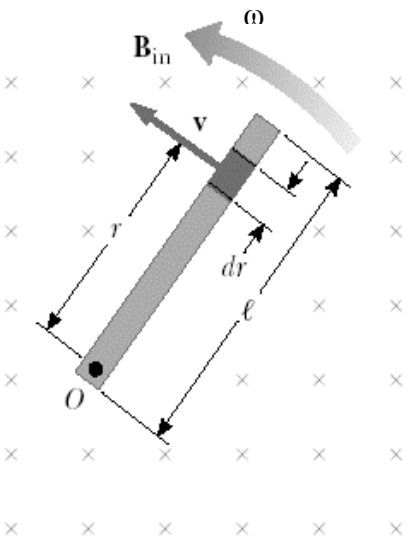
Because every segment of the bar is moving perpendicular to B, an emf $d\mathcal{E}$ of the same form is generated across each segment. Summing the emfs induced across all segments, which are in series, gives the total emf between the ends of the bar:

$$\mathcal{E} = \int Bv\,dr$$

$$= B\int v\,dr$$

$$= B\omega \int_0^{\ell} r\,dr \quad \text{because } v = \omega r$$

$$\mathcal{E} = \tfrac{1}{2} B\,\omega\ell^2$$

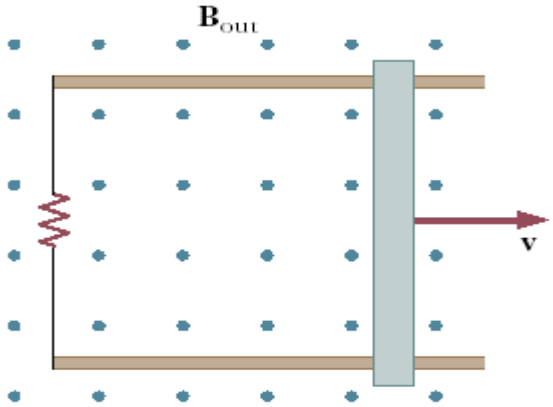


Example

1. The bar in the following figure moves on rails to the right with a velocity **v**, and the uniform, constant magnetic field is directed out of the page.

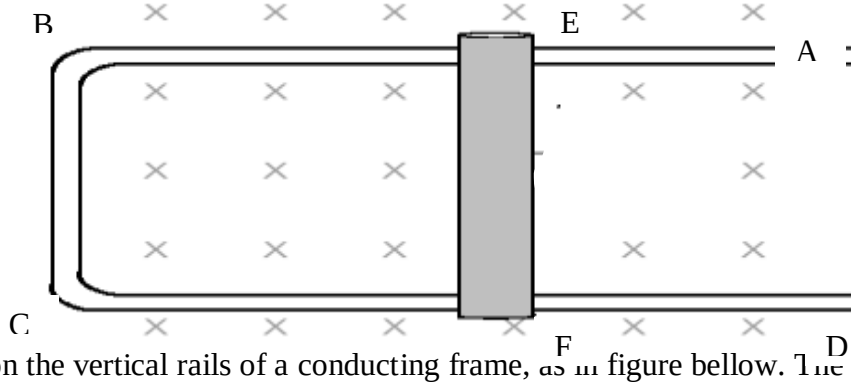
   a) What is the direction of induced current?
   Ans. clockwise.

   b) If the bar were moving to the left, what would be the direction of the induced current?



2. A copper bar 30 cm long is perpendicular to a field of B = 0.8T and moves at right angles to the field with a speed of 0.5 m/s. What is the induced emf in the bar? **Ans**. $\mathcal{E} = 0.12V$
3. Calculate the emf induced across a 2m long bumper of a car which is moving with a velocity of 30 m/s in the magnetic field of the earth, which has a vertical component equal to 5×10^{-5}T. **Ans**. $\mathcal{E} = 3\times10^{-3}$ V
4. A 40 cm long conductor moves at right angles to a uniform field of 1.2 T. If the emf induced between the ends of the conductor is 2.4V, what is the speed of the conductor? **Ans**. v = 5m/s
5. A portion of a conducting wire of length 2m moves with a constant speed of 10m/s at right angles across a uniform magnetic field of 0.6T. If the resistance of the conductor is 6Ω, calculate the induced current in the conductor. **Ans**. I = 2A
6. On a metal wire ABCD, another wire EF is placed in contact and free to slide. If a magnetic field directed into the plane of the wires is decreased, wire EF will

   A) move up.
   B) remain stationary.
   C) slid to the left.
   D) slid to the right.

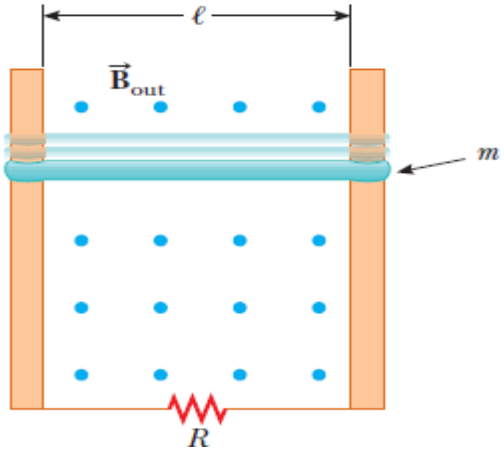


7. A horizontal wire is free to slide on the vertical rails of a conducting frame, as in figure bellow. The wire has mass *m* and length ℓ, and the resistance of the circuit is *R*. If a uniform magnetic field is directed perpendicular to the frame, show that the terminal speed of the wire as it falls under the force of gravity, neglecting friction, is given by

$$V_{ter} = \frac{mgR}{B^2\ell^2}$$

Hint:- Terminal speed is a maximum constant speed of the wire when the downward force on the wire $F_{down} = mg$ is balanced by upward electric force of $F_{up} = I_{ind}\,\ell B$

Where I_{ind} is induced current in the wire.



Questions from national exam

1. A conducting bar of length 1*m* placed in a uniform magnetic field of 2.5T, moves with a velocity of 2m/s perpendicular to both the bar and magnetic field. What is the induced emf?

   A) 2V B) 2.5V C**)** 5V D) 10V

2. An induced emf is obtained between the ends of a horizontal steel axle X of a train moving due east. This is because

   A) X points due east. C) the earth's magnetic field has a horizontal component.

   B) X moves parallel to the earth's magnetic field. D) the earth has a vertical magnetic field component.

3. Which of the following statements is NOT true?

   A) An electric current can be induced in a circuit with a change in the cross-sectional area of the wire.

   B) The induced emf in a circuit is directly proportional to the time rate of change of magnetic flux.

   C) Ɛmf can be induced in a circuit by changing in the area enclosed by the loop.

   D) An electric current can be induced in a circuit with a changing magnetic field.

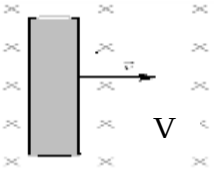
4. A conducting rod of length ℓ moves with a constant velocity V perpendicular to a long, straight wire carrying current I. What is the magnitude of the Ɛmf generated between the ends of the rod?

   A) $\frac{\mu_0 IV\ell}{2\pi r}$ B) $\frac{\mu_0 IV\, r}{2\pi \ell}$ C) $\frac{\mu_0 IV\, r}{2\ell}$ D) $\frac{\mu_0 IV\, \ell}{2r}$

5. The diagram shows a wire moving to the right at a speed of V through a uniform magnetic field that is directed into the page. If the strength of the magnetic field is increased, the induced emf will

   A) decrease. C) remain the same.

   B) increase. D) become zero.



6. On which of the following does the induced emf in a coil NOT depend when a magnet is inserted into it?

   A) Resistance of the coil. C) Speed of approach of the magnet.

   B) Number of turns in the coil. D) Magnetic moment of the magnet.

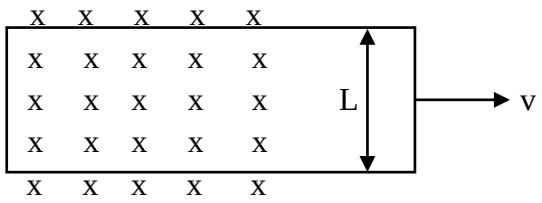
7. The figure below shows a rectangular loop of wire of width L and resistance R. One end of the loop is in a uniform magnetic field of strength B directed perpendicular into the plane of the page. The loop is pulled to the right at a constant speed v. What are the magnitude and direction of the induced current in the loop?

   A) I = BLvR and clockwise.

   B) I = BLv/R and clockwise.

   C) I = BLvR and counterclockwise.

   D) I = BLv/R and counterclockwise.



8. The induced *electromotive force* in a coil placed in a magnetic field does **NOT** depend on which of the following?

   A) The speed at which the coil moves.

   B) The change in magnetic flux.

   C) The length of the coil that is moving in a direction perpendicular to that the magnetic field.

   D**)** The area of the coil outside the field.

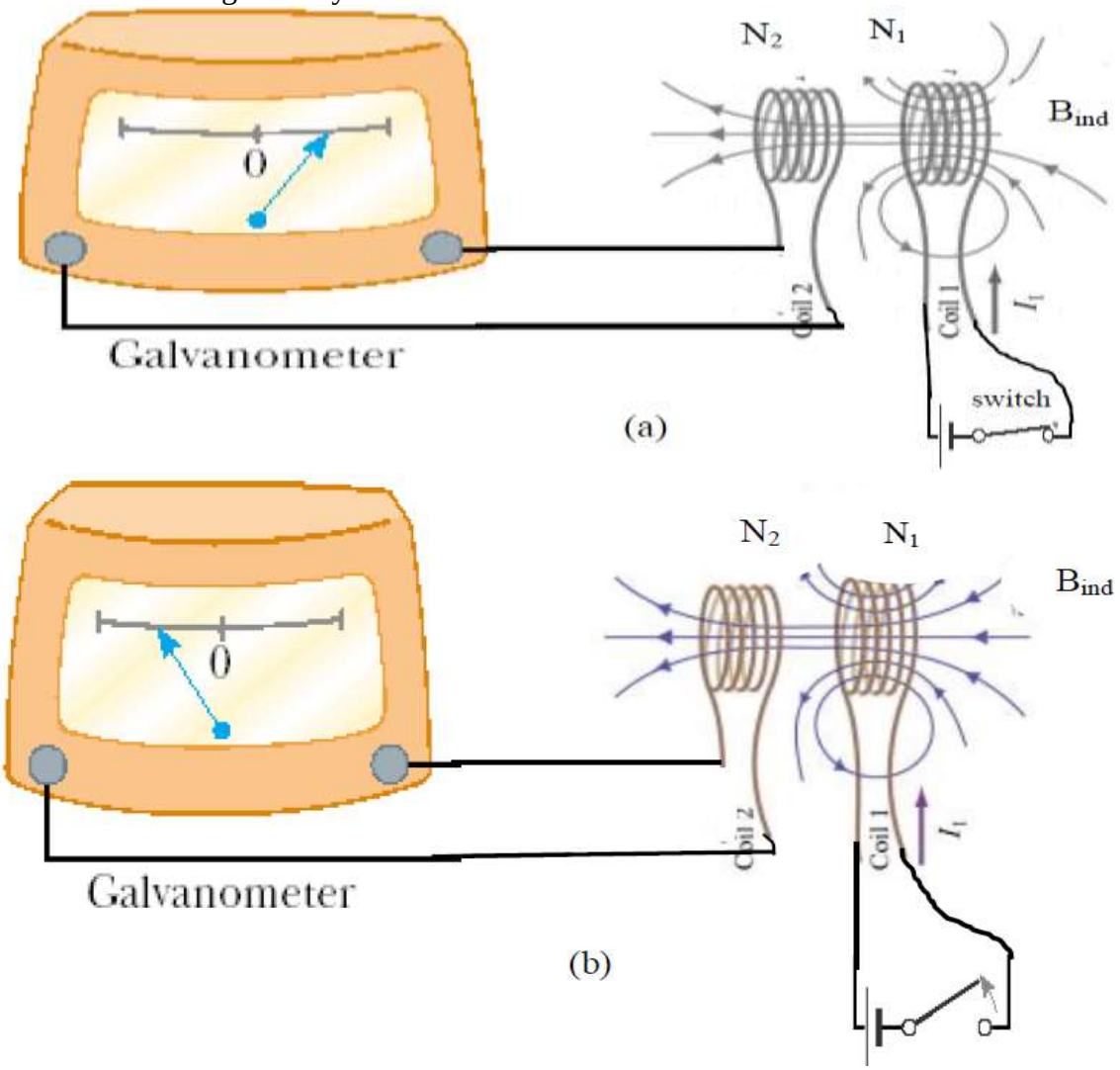
Inductance

Inductance:- is the property of an electric circuit that causes an emf to be generated as result of a change in the current in the neighboring circuit with which it is magnetically inked (*mutual inductance*), or through the same circuit (*self-inductance*). A coil in a circuit is known as *inductor* and represented by a symbol —ᔓᔓᔓ—.

Mutual inductance

An airport metal detector contains a large coil of wire around the frame. This coil has a property called inductance. When a passenger carries metal through the detector, the inductance of the coil changes, and the change in inductance signals an alarm to sound.

Mutual inductance:- is the induction of an Ɛmf in a circuit due to a changing current in a neighboring circuit with which it is magnetically linked.



At the instant the switch is closed as in fig (a), the galvanometer needle deflects in one direction and then returns to zero. *This is similar to moving the north pole of a magnet toward the loop.*

At the instant the switch is opened as in fig (b), the needle deflects in the opposite direction and again returns to zero. *Which is similar to moving the north pole of a magnet away from the loop.*

Finally, the galvanometer reads zero when there is either a steady current or no current in circuit 1.

The same effect is observed when a magnet is brought to rest and held stationary relative to the loop.

As a result of these observations, Faraday concluded that **an electric current can be induced in coil 2 by a changing magnetic field**.

The induced current exists for only a short time while the magnetic field through coil 2 is changing. Once the magnetic field reaches a steady value, the current in the coil 2 disappears. In effect, the coil 2 behaves as though a source of emf were connected to it for a short time.

The current I_1 in coil 1 produces a flux $\Phi_2 = B_1A_2$ in coil 2 and then there will be $\mathcal{E}_2$ in coil 2.

$$\mathcal{E}_2 = -\frac{d(N_2\Phi_2)}{dt} = -\frac{d(N_2B_1A_2)}{dt}$$

$$= -\frac{d\left[N_2\left(\frac{\mu_0 N_1 I_1}{\ell}\right)A_2\right]}{dt} = -\left(\frac{\mu_0 N_1 N_2 A}{\ell}\right)\frac{dI_1}{dt} = -M\frac{dI_1}{dt}$$

Where $M = \frac{\mu_0 N_1 N_2 A}{\ell}$ is called mutual inductance.

The mutual inductance is purely a geometric factor, and depends on the number of turns in each coil, their shapes and their arrangement.

A mutually induced emf is proportional to the time rate of change of the current.
For any coil, we find that

$$\varepsilon_2 = -M\frac{dI_1}{dt} \qquad \ldots\ldots\ldots \quad (1)$$

According to Faraday's Law the induced Ɛmf is

$$\varepsilon_2 = -N_2\frac{d\Phi_2}{dt} \qquad \ldots\ldots\ldots (2)$$

Equate (1) and (2)

$$M\frac{dI_1}{dt} = N_2\frac{d\Phi_2}{dt}$$

$$M = \frac{N_2\Phi_2}{I_1}$$

In the same manner

$$M = \frac{N_1\Phi_1}{I_2}$$

$$\text{M} = \frac{N_1\Phi_1}{I_2} = \frac{N_2\Phi_2}{I_1}$$

The SI unit of inductance is the henry (H). $1\text{H} = \frac{1Wb}{A} = \frac{1Vs}{A}$

Example

1. $N_1 = 1000$, $N_2 = 50$, A = 0.5 cm^2 and ℓ = 10 cm. What is M? **Ans**. M = 3.14 ×10^{-5}H
2. How large an average Ɛmf is generated in a second coil if the current in the primary is increased from 2A to 4A in 0.01s? M= 0.5H **Ans**. $\mathcal{E}_2$ = 100V
3. Two coils have mutual inductance of 0.01 H. The current in the first coil increases at a rate of 0.05 A/s. What is the induced current in the second coil? **Ans**. $\mathcal{E}_2 = 5\times10^{-4}$V
4. When the current in a coil changes from 5A to 25A in 0.005 s, and emf induced in a nearby coil is 60 V. What is M? **Ans**. M = 0.015H
5. A long solenoid of length ℓ carrying current I has N_1 turns. A coil of small area A, containing N_2 turns is placed at the center of the solenoid with the plane of the coil making angle θ to the axis of the solenoid. What is the mutual inductance?

   A) $\frac{\mu_0 N_1 N_2 A}{\ell}$ B) $\frac{\mu_0 N_1 N_2 A\cos\theta}{\ell}$ C) $\frac{\mu_0 N_1 N_2 A\sin\theta}{\ell}$ D) $\frac{\mu_0 N_1 N_2 A\tan\theta}{\ell}$

6. Two coils, held in fixed positions, have a mutual inductance of 100μ H. What is the peak voltage in one when the sinusoidal current given by I(t) = (10A) sin (1000t) flows in the other?

   A) 1.0 μ V B) 100.0 V C) 10.0 V D**)** 1.0 V

Self-Induction

The production of an emf in a circuit by a varying current in the same circuit is known as *self-induction*.

Because of the direction of the induced emf, it is also known as a *back emf*.

A self-induced emf is proportional to the time rate of change of the current. For any coil, we find that

$$\mathcal{E}_L = -L\frac{dI}{dt} \qquad (1)$$

According to Faraday's Law the induced Ɛmf is

$$\mathcal{E}_L = -N\frac{d\Phi}{dt} \qquad (2)$$

Equate (1) and (2)

$$L\frac{dI}{dt} = N\frac{d\Phi}{dt}$$

$$L = \frac{N\Phi}{I}$$

$$L = \frac{N(BA)}{I} = \frac{NA}{I}\frac{(\mu_0 NI)}{\ell}$$

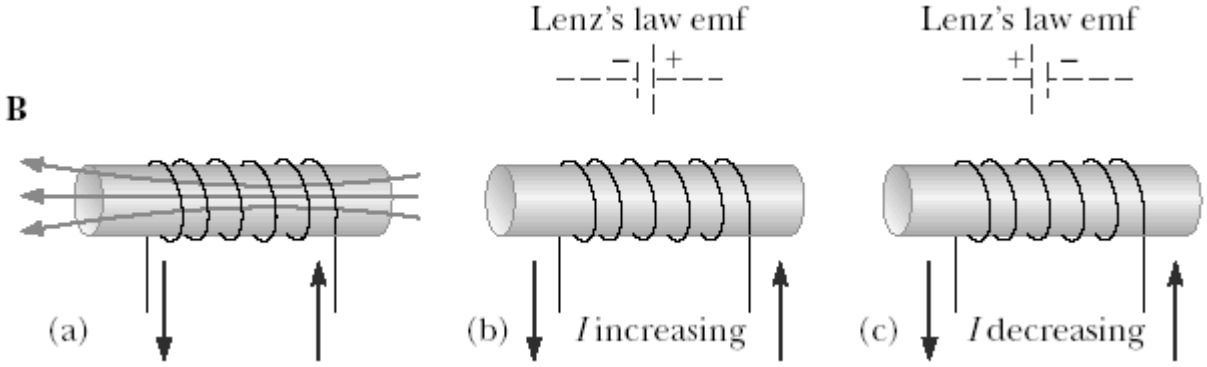
$$L = \frac{\mu_0 N^2 A}{\ell} \quad \text{Or} \quad L = \mu_0 n^2 \ell A \quad \text{Where } n = N/\ell$$

The self-inductance is also purely a geometric factor.

Recall that **resistance is a measure of the opposition to current** $R = \frac{V}{I}$; in comparison,

inductance is a measure of the opposition to a *change in current*. $L = -\frac{\varepsilon_L}{dI/dt}$

Consider the following figure which shows a coil wound on a cylindrical core. Assume that the current in the coil either increases or decreases with time.



When the current is in the direction shown, a magnetic field directed from right to left is set up inside the coil, as seen in figure (a). As the current changes with time, the magnetic flux through the coil also changes and induces an emf in the coil. From Lenz's law, the polarity of this induced emf must be such that it opposes the change in the magnetic field from the current. If the current is increasing, the polarity of the induced emf is as pictured in figure (b), and if the current is decreasing, the polarity of the induced emf is as shown in figure (c).

Example

1. A solenoid of length 25 cm and cross-sectional area 4 cm^2 contains 300 turns. If the current it carries is decreasing at the rate of 50 A/s, calculate
   a) the inductance of the solenoid. **Ans**. L = 1.81×10^{-4} H
   b) the self-induced emf in the solenoid . **Ans**. $\mathcal{E}_L$ = 9.05 *m*V

2. The current in a coil increases uniformly from 9 mA to 15mA in 1.5 s. If the induced emf is 32mV, what is the self- inductance of the coil? **Ans**. L = 8H
3. As current in a 100 turn coil increases from zero to 10A, it produces a magnetic flux of 0.09 Wb. What is the self-inductance? **Ans**. L = 9H
4. By what factor does the inductance of a solenoid change when the number of turns of the solenoid is doubled? **Ans**. 4
5. What is the self-inductance of a coil which experiences a 1.6V induced emf when the current is changing at the rate of 80A/s?
   a) 10mH B) 15mH C**)** 20mH D) 25mH
6. The current flowing through the inductor L = 60mH, in an ac circuit is given by $i(t) = 12 \sin (100\, t + \pi/6)$ A. What is the voltage across the inductor?
   A) $2 \cos (100\, t + \pi/6)$ V
   B) $72 \sin (100\, t + \pi/6)$ V
   C) $2 \sin (100\, t + \pi/6)$ V
   D**)** $72 \cos (100\, t + \pi/6)$ V
7. Self-inductance is a quantity that describes:
   A) the number of turns in a circuit.
   B) the induction of an electric field by an AC current in a lone coil.
   C**)** the induction of a magnetic field by an AC current in lone coil.
   D) the steady state current in a coil.
8. Which of the following statements is correct?
   A) The direction of induced e.m.f is in such a direction as to reinforce the change produces it.
   B**)** Self-inductance of a coil is e.m.f divided by magnitude of rate of change of current with time.
   C) The induced e.m.f in a coil is inversely proportional to its number of turns.
   D) Self-inductance of a coil is inversely proportional to the cross-sectional of the coil.
9. An inductor has a self inductance of 12 *m*H. The current in the circuit decreases at a rate of 2.0 A/s. What is the magnitude of the induced electromotive force?
   (A) 2.4π V (B) 1.2π V (C) 1.2×10^{-2}V (D**)** 2.4×10^{-2}V

Energy stored in an inductor

When an electric current passes through an inductor, the induced emf is given by

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

The power supplied to the coil by the battery is

$$P = I\mathcal{E}$$

$$P = I\left(L\frac{dI}{dt}\right)$$

The energy stored by an inductor is

$$U = \int P dt = \int I\left(L\frac{dI}{dt}\right)dt$$

$$= \int LI dI$$

$$U = \tfrac{1}{2} L I^2$$

Since $L = \mu_0 n^2 \ell A$ and $B = \mu_0 n I$, $I = \dfrac{B}{\mu_0 n}$

$$U = \frac{1}{2}\left(\mu_0 n^2 \ell A\right)\left(\frac{B}{\mu_0 n}\right)^2$$

$$U = \frac{1}{2}\frac{B^2}{\mu_0} A\ell$$

The energy density (U_B), or the energy stored per unit volume in the inductor is

$$U_B = \frac{U}{A\ell}$$

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$ Because volume of the solenoid = $A\ell$

Although this expression was derived for the special case of a solenoid, it is valid for any region of space in which a magnetic field exists.

Note that equation $U_B = \frac{1}{2}\frac{B^2}{\mu_0}$ is similar in form to equation $U_e = \frac{1}{2}\varepsilon_0 E^2$

In both cases, the energy density is proportional to the square of the field magnitude.

Example

1. L= 0.5 H, I = 1A. What is the energy stored in the inductor? **Ans**. U = 0.25J
2. An inductor, 10 cm long and cross-sectional radius 2 cm, has a self-inductance of 0.4H. Calculate
   a) the amount of energy stored in the coil. **Ans**. U = 3.2J
   b) the energy density. **Ans**. $U_B = 2.55\times10^{-4}$J/m^3
3. Which one of the following actions increases the magnetic energy density in the interior of a very long current-carrying solenoid?
   A) Increasing the number of turns per unit length of the solenoid.
   B) Increasing the cross sectional area of the solenoid.
   C) Increasing the length of the solenoid keeping the number of turns per unit length fixed.
   D) Decreasing the cross sectional area of the solenoid.

Alternating current generator

A **generator** is a device that converts *mechanical energy* to *electrical energy*. In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field as in figure below. In commercial power plants, the energy required to rotate the loop can be derived from a variety of sources. For example, in a hydroelectric plant, falling water directed against the blades of a turbine produces the rotary motion; in a coal-fired plant, the energy released by burning coal is used to convert water to steam, and this steam is directed against the turbine blades. As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time; this induces an emf and a current in the loop according to Faraday's law. The ends of the loop are connected to slip rings that rotate with the loop. Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary brushes in contact with the slip rings.

The current from this generator like the emf is varying sinusoidally in the resistor R. Because of this the generator is said to be *alternating current (ac) generator*.

If the coil has *N* turns, the total emf is

$$\varepsilon = -\frac{Nd(\Phi)}{dt}$$

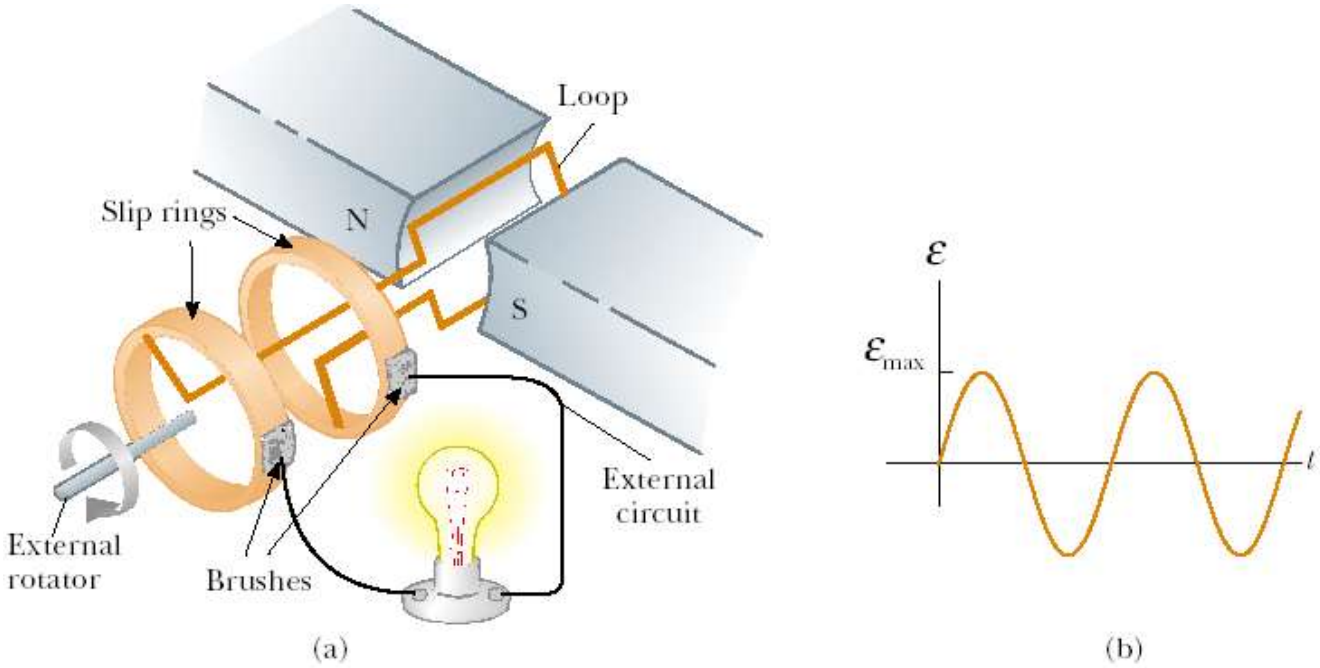
$$\varepsilon = -\frac{Nd(BA\cos\omega t)}{dt}$$

$$\varepsilon = NBA\,\omega \sin \omega t$$

This result shows that the emf varies sinusoidally with time, as plotted in figure below (b).

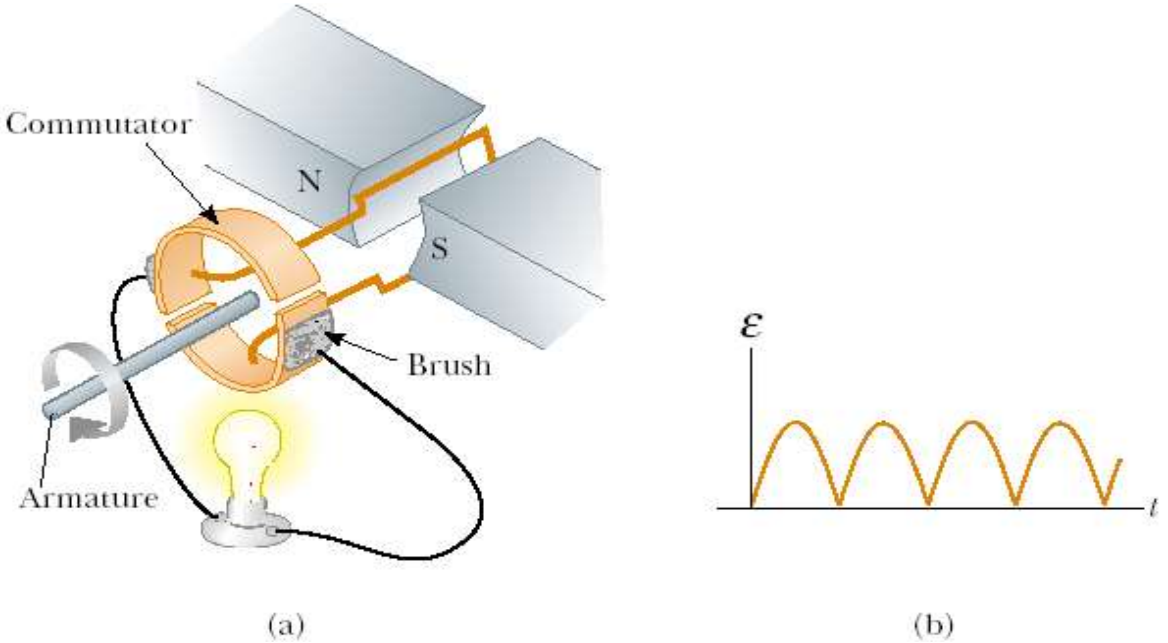
Note that the maximum emf has the value

$$\varepsilon_{max} = NBA\omega$$



(a) Schematic diagram of an AC generator. An emf is induced in a loop that rotates in a magnetic field. (b) The alternating emf induced in the loop plotted as a function of time.

An output voltage that always has the same polarity, although fluctuates in magnitude, can be obtained by connecting the loop to a split ring (commutator) as shown in figure below. Such generators are used in older cars to charge the storage batteries. The components are essentially the same as those of the AC generator except that the contacts to the rotating loop are made using a split ring called a *commutator*. Note that the contacts to the split ring reverse their roles every half cycle. At the same time, the polarity of the induced emf reverses; hence, the polarity of the split ring (which is the same as the polarity of the output voltage) remains the same.



(a) Schematic diagram of a DC generator. (b) The magnitude of the emf varies in time but the polarity never changes.

1. An electric generator has a coil of 400 turns and cross-sectional area 5 m^2 rotating at an angular frequency 0.524 rad/s in a uniform magnetic field of 0.2 T. What is the induced emf produced from the electric generator in each second?

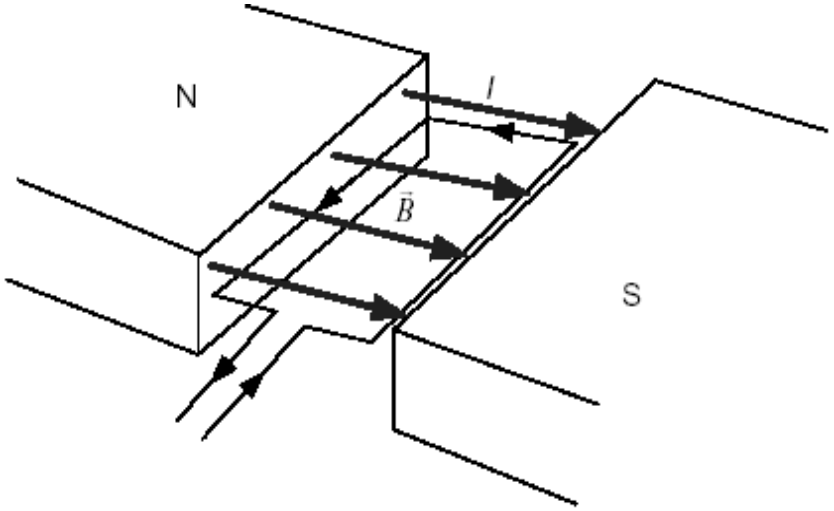
A**)** 209.6 V B) 104.8 V C) 200 V D) 182.4 V

Electric motor

Motor is a device that converts *electrical energy* to *mechanical energy*. Essentially, a motor is a generator operating in reverse. Instead of generating a current by rotating a loop, a current is supplied to the loop, and the torque acting on the current-carrying loop causes it to rotate.

As with a generator, a simple motor consists of a coil that is continuously turning about its axis.

What are the directions of the magnetic forces on each section of the loop? What will happen to the loop?

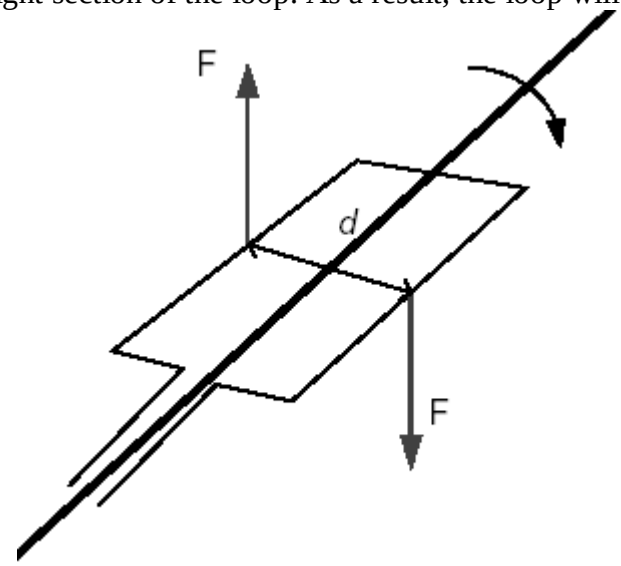


The current is

-parallel to the magnetic field in the front section of the loop and

-anti -parallel to the magnetic field in the top section of the loop .

When the current and the magnetic field are parallel (anti-parallel), the magnetic force is zero. No force on these part of the loop.Using the right hand rule, we see that the force is up on the left side of the loop and down on the right section of the loop. As a result, the loop will rotate clockwise



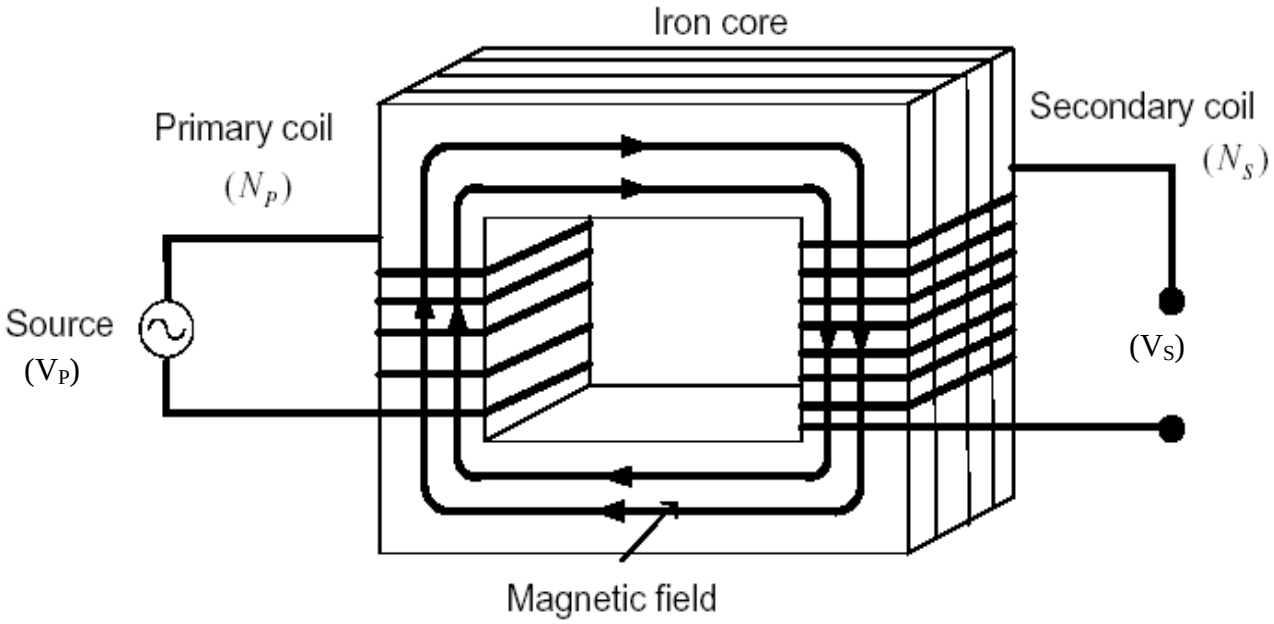
Useful mechanical work can be done by attaching the rotating armature to some external device.

We can increase the speed of the electric motor by increasing

- the current.
- the magnetic field.
- the number of windings of the armature.

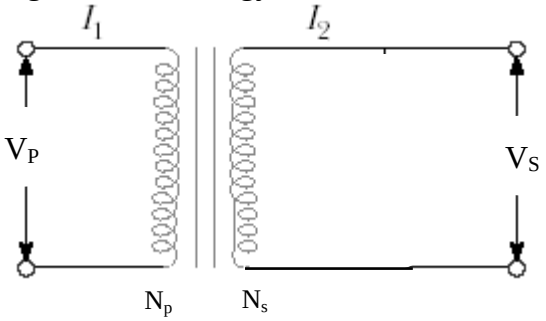
Transformer

Transformer:- is a device used to change a given ac voltage into a larger or smaller ac voltage. It consists of two coils, *primary* and *secondary*; electrically insulated from each other and wound on the same iron core. The purpose of the iron core is to increase the magnetic flux through the coil and to provide a medium in which nearly all the magnetic field lines through one coil pass through the other coil.



The iron core is laminated —that is, they are built up in thin layers separated by a non-conducting material to reduced energy loss due to eddy-current. This layered structure increases the resistance of eddy current paths and effectively confines the currents to individual layers. Such a laminated structure is used in transformer cores and motors to minimize eddy currents and thereby increase the efficiency of these devices.

Eddy current:- is an induced electric current set up in an iron cores of a transformers, electromagnets or and other electrical apparatuse due to changing magnetic fields. They produce heating effects in conductors leading to wastage of useful energy.



Circuit symbol for a transformer.

If the *secondary coil has more turns than the primary coil*, it is known as a **step-up transformer.**
One in which the *primary coil has more turns than the secondary* is known as **step-down transformer.**

*Step-up transformer ($N_S>N_P$):- is used to raise the input emf. $V_S>V_P$.

*Step-down transformer ($N_S<N_P$):- is used to lower the input emf. $V_S<V_P$.

The changing flux through the secondary coil gives rise to an induced emf (V_S)

$$V_S = -N_S \frac{d\Phi}{dt}$$

The induced emf in the primary (V_P)

$$V_P = -N_P \frac{d\Phi}{dt}$$

The flux through the primary equals the flux through the secondary
Taking the ratio we get

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

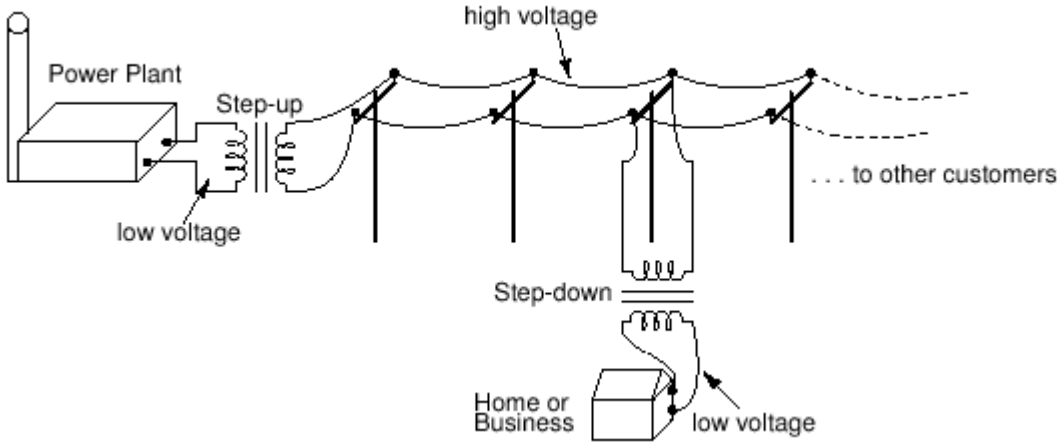
In an ideal transformer, where there are no losses,

$$P_P = P_S$$

$$I_P V_P = I_S V_S \qquad \frac{I_P}{I_S} = \frac{V_S}{V_P} \quad \text{OR} \quad \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Typical transformers have power efficiencies from 90% to 99%. In the discussion that follows, we assume an *ideal transformer*, one in which the energy losses in the windings and core are zero.
We can now also understand why transformers are useful for transmitting power over long distances. Because the generator voltage is stepped up, the current in the transmission line is reduced, and hence I^2R losses are reduced. In practice, the voltage is stepped up to around 230 000V at the generating station, stepped down to around 20 000 V at a distributing station, then to 4 000V for delivery to residential areas, and finally to 220 V at the customer's site.

There is a practical upper limit to the voltages that can be used in transmission lines. Excessive voltages could ionize the air surrounding the transmission lines, which could result in a conducting path to ground or to other objects in the vicinity. This, of course, would present a serious hazard to any living creatures. For this reason, a long string of insulator is used to keep high-voltage wires away from their supporting metal towers. Other insulators are used to maintain separation between wires.

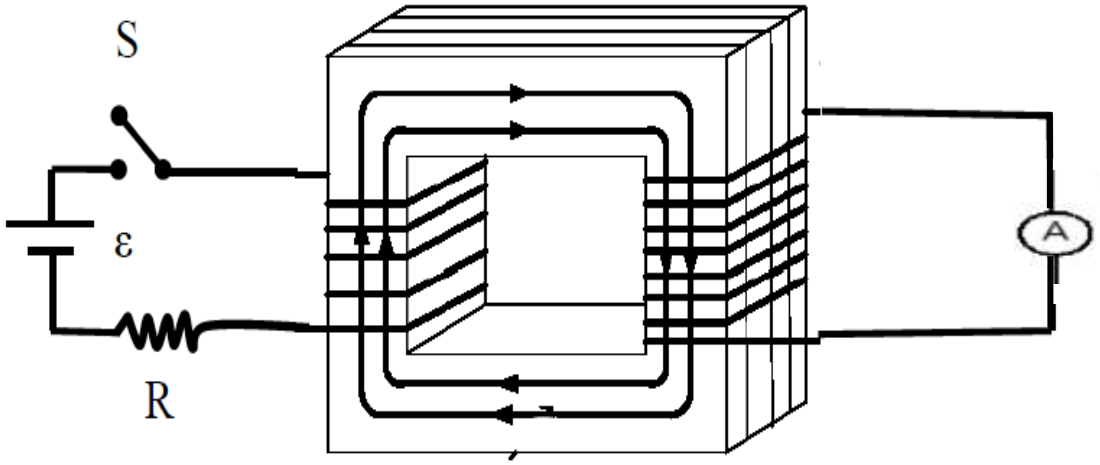


Example

1. A transformer is used to convert 120V to the 9V required for a radio.
   a) Is a step-up or a step-down transformer used?
   b) If the primary has 480 turns, how many turns are on the secondary? **Ans**. N_S = 36 turns
   c) Determine the current in the primary coil if the radio operates with 400 *m*A. **Ans**. I_P = 30 *m*A
   d) What is the power in the transformer? **Ans**. P = 3.6W
2. A generator at a hydroelectric station operates at 12 kV and 12 A. A step-up transformer is used to step up the transmission line voltage to 120 kV. Determine the current in the transmission line. **Ans**. I_S=1.2A
3. A transformer is used to step down 220 V to 6 V. If the primary coil has 440turns, how many turns are there in the secondary? **Ans**. N_S = 12
4. A step-down transformer is used to light a 12 V, 27 W lamp from the 240 V main. The current through the primary is 0.125 A. What is efficiency of the transformer? **Ans**. efficiency = 90%

Questions from national exam

1. A step-up transformer is used to:
   A) increase the power. C) increase the voltage.
   B) decreases the power. D) decreases the voltage.
2. A 6 V source is connected across the primary coil of a transformer having 50 turns. If the secondary coil of the transformer has 100 turns, what voltage appears across the secondary?
   A) 24 V B**)** 12 V C) 6 V D) 3 V
3. Which conversion process forms the basis for the function of an electric generator?
   A) alternating current to direct. C) electrical energy to mechanical energy.
   B) mechanical energy to electrical. D) low voltage to high voltage or vice versa.
4. Why is the core in a transformer laminated?
   A) To increase the induced current in the primary coil.
   B) To increase the induced current in the secondary coil.
   C) To avoid heat exchange between the primary and secondary coils.
   D) To reduce the loss in power in the core due to eddy currents.
5. The primary coil of a transformer is connected to a battery, a resistor and a switch. The secondary coil is connected to an ammeter. What does the ammeter show when the switch is close



   A) Zero current as transformer is not working on DC.
   B**)** A nonzero current for a short instant.
   C) A steady current.
   D) Emf will be induced but not current.

Alternating current circuit

Alternating circuit, ac, is a varying current that changes both in magnitude and direction with time. An ac meter measures the average of the oscillating values, not the instantaneous values. The root-mean-square (rms) describes a procedure for finding the average values for the ac voltage or current.

AC source produces a voltage that varies with time as

$$V = V_o \sin \omega t$$

The ac current in the circuit is also given by

$$I = I_o \sin \omega t$$ Where V_o and I_o are peak (maximum values)

$$I^2 = I_o^2 \sin^2 \omega t$$ V and I are instantaneous values

The *root-mean-square (rms),* or *effective value* is defined as *the square root of the average of the square of the instantaneous values* in one period of oscillation.

$$\left(I^2\right)_{av} = I_o^2 \left(\sin^2 \omega t\right)_{av}$$

$$\left(\sin^2 \omega t\right)_{av} + \left(\cos^2 \omega t\right)_{av} = 1$$

For one complete cycle $\left(\sin^2 \omega t\right)_{av} = \left(\cos^2 \omega t\right)_{av}$

Therefore, $\left(\sin^2 \omega t\right)_{av} + \left(\cos^2 \omega t\right)_{av} = 1$ becomes

$$2\left(\sin^2 \omega t\right)_{av} = 1 \quad \Longrightarrow \quad \left(\sin^2 \omega t\right)_{av} = \frac{1}{2}$$

$\left(I^2\right)_{av} = I_o^2 \left(\sin^2 \omega t\right)_{av}$ becomes

$$\left(I_o^2\right)_{av} = I_o^2 \left(\frac{1}{2}\right) \Longrightarrow \quad I_{rms} = \sqrt{\left(I^2\right)_{av}}$$

$$I_{rms} = I_o \left(\frac{1}{\sqrt{2}}\right) = 0.707 I_o$$

The rms value of the voltage of an ac source is also given by

$$V_{rms} = V_o \left(\frac{1}{\sqrt{2}}\right) = 0.707 V_o$$

Circuit containing a resistor, a capacitor or an inductor

*i)*Resistor in an ac circuit

Consider a circuit that consists of ac generator, represented by a symbol, ⏦ and a resistor.

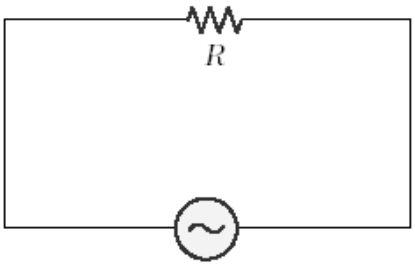
According to Ohm's law

$V = V_0 \sin \omega t$

$$I = \frac{V}{R} = \frac{V_o \sin \omega t}{R}$$

$I = I_o \sin \omega t$

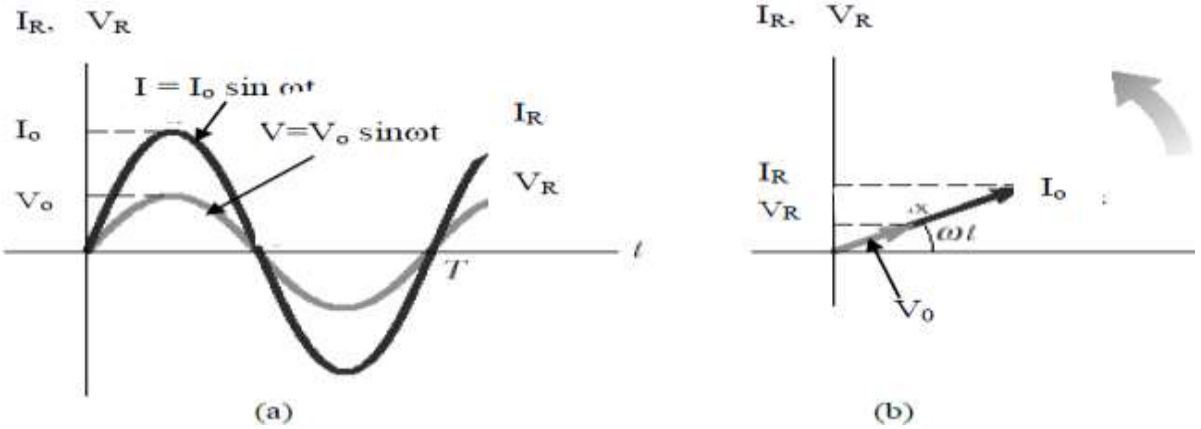
That is $I_o = \frac{V_o}{R}$



For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor. For resistors in AC circuits, there are no new concepts to learn. Resistors behave essentially the same way in both DC and AC circuits. This will not be the case for capacitors and inductors.

To simplify our analysis of circuits containing two or more elements, we use graphical constructions called *phasor diagrams.* A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents (V_o for voltage and I_o for current in the present discussion) and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.



(a) Graphs of the instantaneous current I_R and instantaneous voltage V_R across a resistor as functions of time. The current in a resistor is always in phase with the voltage across the resistor.

(b) Phasor diagram for the resistive circuit showing that the current is always in phase with the voltage across the resistor.

The average power (the rate at which energy is delivered to a resistor) is

$$P_{av} = (VI)_{av}$$
$$= [(V_o \sin \omega t)(I_o \sin \omega t)]_{av}$$
$$= V_o I_o (\sin^2 \omega t)_{av}$$
$$= V_o I_o (½)$$
$$= \frac{V_o}{\sqrt{2}} \frac{I_o}{\sqrt{2}}$$
$$P_{av} = V_{rms} I_{rms}$$
$$P_{av} = \frac{V_{rms}^2}{R} \quad \text{OR} \quad P_{av} = I_{rms}^2 R$$

Example

1. The peak value of a sinusoidal current is 25 *m*A. Calculate its I_{rms}. **Ans**. I_{rms} = 17.7 *m*A
2. An ac generator produces voltage given by V =100V sin (ωt). What is its rms value? **Ans**. V_{rms}= 70.7V
3. An ac source has an output given by V = (200 V) sin($2\pi f t$). This source is connected to a 100 Ω resistor. What is
   a) V_{rms}? **Ans**. = 141.4 V, b) I_{rms}? **Ans**. I_{rms} = 1.41 A
4. A rms voltage of 100 V is applied to a device that has a resistance of 5 Ω. Calculate
   a) V_o. **Ans**. V_o = 141.4 V, b) I_{rms}. **Ans**. I_{rms} = 20 A, c) I_o. **Ans**. I_o = 28.3A

Questions from national exam

1. When an ac voltage source with peak value 100 V is applied across a resistor, a peak current of 2A flows through the resistor. What is the power dissipated in the resistor?
   A) 100 W B) 200 W C) 400 W D) 500 W
2. The voltage output of a generator is given by V = (300 V) sin ωt. What is the rms current in the circuit when this generator is connected to a 100 Ω resistor?
   A) 1.50A B**)** 2.12 A C) 1.12 A D) 1.41 A
3. The peak voltage of an alternating emf is 141 V. The effective value of the voltage in volts is
   A) 70.5 B**)** 100 C) 141 D) 200
4. An alternating current is represented by I(t) = (10.0 A) sin (100 πt). What are the amplitude and frequency of the alternating current?
   A) 100 A, 75 Hz B) 10.0A, 100Hz C) 10.0A**,** 50Hz D) 5.0A, 25 Hz

ii) Capacitors in an ac circuits (capacitive ac circuit):-

The charge on a capacitor is related to the p.d between its plates by

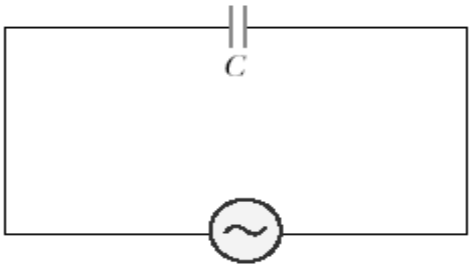
$$Q = CV$$

Where $V = V_0 \sin \omega t$

$$Q = C V_o \sin \omega t$$

Because $I = \frac{dQ}{dt}$

$$I = \frac{d(CV)}{dt}$$
$$I = \frac{d}{dt} \frac{(CV_o \sin \omega t)}{dt}$$



$I = \omega C V_o \cos \omega t$ (1)

$= I_o \cos \omega t$ (2)

$= I_o \sin (\omega t + \pi/2)$ Because $\cos \omega t = \sin (\omega t + \pi/2)$

Comparing (1) and (2) we get $\omega C V_o = I_o$

$$X_c = \frac{V_o}{I_o} = \frac{1}{\omega C}$$ Capacitive reactance

Comparing $V = V_o \sin \omega t$ with $I = I_o \sin(\omega t + \pi/2)$, we see that the *voltage across the capacitor lags behind current by $-\pi/2 rad = -90^0$* .

Or we can say, current leads voltage across the capacitor by $\pi/2$ rad = 90^0.

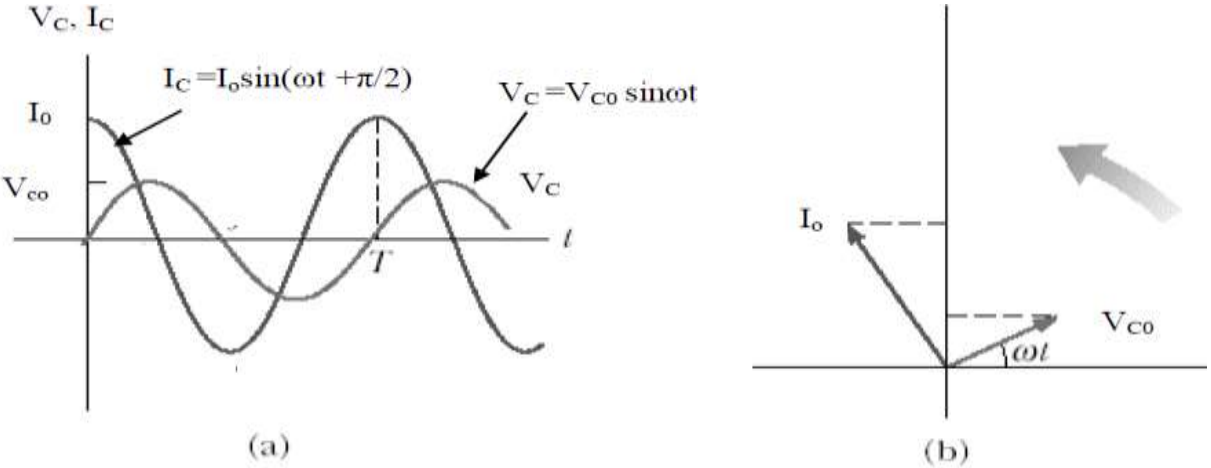
The phase angle, *the angle by which voltage across the capacitor lags behind current*, is

$\phi = -\pi/2$ rad = -90^0

$V_{rms} = I_{rms} X_c$

X_c stands for the amount with which the capacitor impedes the flow of current through the circuit. It is called **capacitive reactance**.

Reactance:- is the equivalent quantity to resistance when we are talking about capacitors or inductors.



(a) Graphs of the instantaneous current I_C and instantaneous voltage V_C across a capacitor as functions of time. The voltage lags behind the current by 90°.

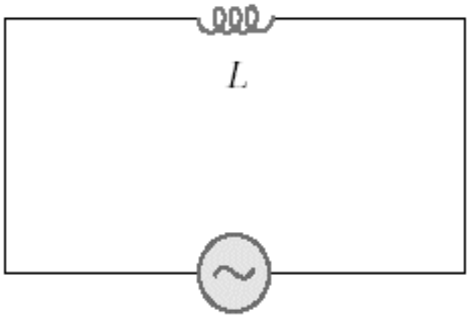
(b) Phasor diagram for the capacitive circuit showing that the current leads the voltage by 90°.

Example

1. An 8 μF capacitor is connected to a 60 Hz generator whose rms voltage is 150 V. What is
   a) X_C ? **Ans**. $X_C = 332\ \Omega$ c) peak voltage? **Ans**. $V_o = 212.2$V
   b) the rms current? **Ans**. $I_{rms} = 0.452$ A d) peak current? **Ans**. $I_o = 0.64$ A
2. What is the capacitive reactance of a 10 μ F capacitor at
   a) 100 Hz? **Ans**. $X_C = 159.2\ \Omega$ b) 10 000 Hz? **Ans**. $X_C = 1.592\ \Omega$
3. A capacitor of capacitance 1000 μF is connected to a 16 V peak value ac supply of 50 Hz. Calculate
   a) the ac peak current. **Ans**. $I_o = 5$ A
   b) the ac current through the capacitor. **Ans**. $I = (5A) \sin(314\ t + \pi/2)$
4. A certain capacitor in a circuit has a capacitive reactance of 30 Ω when the frequency is 120 Hz. What a reactance does the capacitor have at a frequency of 10,000 Hz? **Ans**. $X_C = 0.36\ \Omega$
5. An ac generator with rms voltage output of 30 V is connected to a 4 μ capacitor, and a rms current of 0.3 A flows. What is the frequency of the source? **Ans**. $f = 3.98$ Hz
6. What value of capacitor must be inserted in a 60 Hz circuit in series with a generator of peak output voltage 85 V to produce a rms current output of 0.75A? **Ans**. $C = 1.66 \times 1o^{-5}$F
7. The impedance of an 8 μF capacitor at 1000 Hz is:
   A) 600 Ω B) 22.7 Ω C**)** 19.9 Ω D) 0.44 Ω

iii) Inductors in an ac circuits (inductive ac circuits)

The self-induced instantaneous voltage across the inductor is



$$V_L = L\frac{dI}{dt}$$

Where $I = I_o \sin \omega t$

$$V_L = L\frac{d}{dt}(I_o \sin \omega t)$$

$$= \omega L I_o \cos \omega t$$

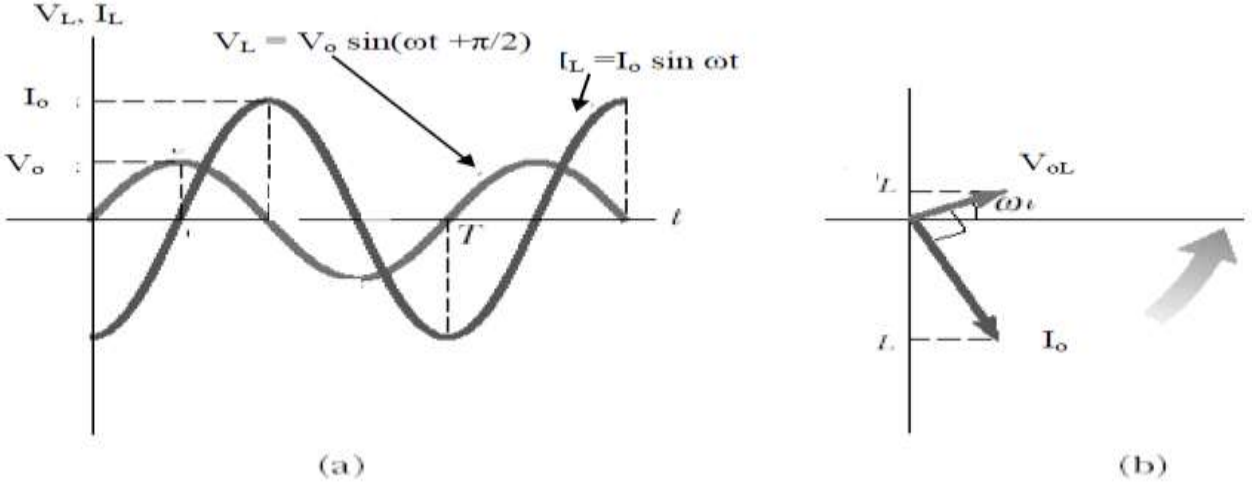
$$V_L = \omega L I_o \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{Where } V_o = \omega L I_0$$

$$X_L = \frac{V_o}{I_o} = \frac{\omega L I_o}{I_o}$$

$X_L = \omega L$ Inductive reactance

$V_{rms} = I_{rms} X_L$

The phase angle, *the angle by which voltage across the inductor leads the current*, is $\phi = \pi/2$ rad $= 90^0$



(a) Graphs of the instantaneous current I_L and instantaneous voltage V_L across an inductor as functions of time. The current lags behind the voltage by 90 °.
(b) Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by 90 °.

Example

1. I_o=250 μA, V_o=3.6 V and f =1.6 MHz. What is a) X_L? **Ans**. X_L=14400 Ω. b) L? **Ans**. L= 1.433×10^{-3} H
2. In a purely inductive ac circuit L= 25 *m*H and the rms voltage is 150 V. If the frequency is 60 Hz, what is a) the inductive reactance? **Ans**. X_L = 9.42Ω. b) the rms current? **Ans**. I_{rms} = 15.9A
3. Calculate the inductive reactance of 3 *m*H inductor at a frequency of
   a) 10 Hz. **Ans**. X_L = 0.188 Ω. b) 10^6 Hz. **Ans**. X_L = 18840 Ω
4. A solenoid of inductance 10 *m*H is connected to a 16 V peak ac supply with a frequency of 50 Hz. Calculate: a) the peak current. **Ans**. I_o = 5.1 A. b) the rms current. **Ans**. V_{rms} = 3.6 A
5. An inductor is connected to a 20 Hz power supply that produces a 50 V rms voltage. What inductance is needed to keep the maximum current of 80 *m*A? **Ans**. L = 0.142 H
6. In a purely inductive circuit, the rms voltage is 120 V.
   a) If the rms current is 10 A at a frequency of 60 Hz, calculate L. **Ans**. L = 0.032 H
   b) At what frequency will the rms current be reduced to 5 A? **Ans**. f = 119.4 Hz

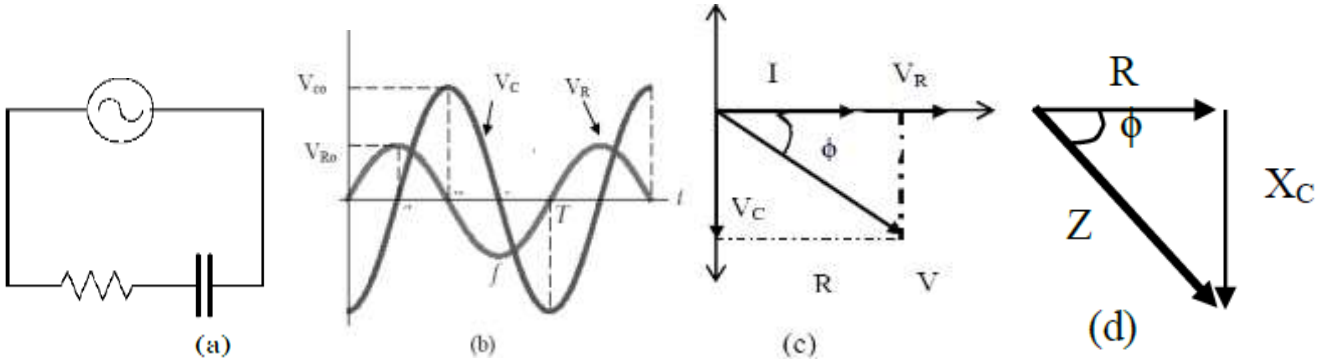
7. In a purely inductive ac circuit L = 35 *m*H and the rms voltage is 165 V. What is the inductive reactance in Ω and the rms current in ampere in the circuit if the frequency is 40 Hz, respectively?
   A**)** 8.79, 18.77 B) 7.79, 20.77 C) 7.69, 20.77 D) 9.89, 18.77
8. Which statement is correct about a purely inductive circuit?
   A) The current leads the voltage by π/2.
   B) The current is in phase with the voltage.
   C) The current lags behind the voltage by π/2.
   D) The current lags by π/2 at low frequency and leads by π/2 at high frequency.

R-C, R-L and R-L-C series circuit

i) Series R-C circuit

Across the resistor, V_R and current I are in phase.

Across the capacitor, *V_c lags behind* I *by -π/2rad/sec=-90^0*. Or, I leads V_C by π/2 rad.



(a) A series circuit consisting of a resistor and a capacitor connected to an AC source.
(b) Phase relationships for instantaneous voltages in the series R-C circuit.
(c) Phasor diagram for the series R-C circuit.
(d) An impedance triangle for a series R-C circuit.

The phase angle ϕ, *the angle between the current and the total voltage phasor*, is

$$\tan\phi = \frac{V_C}{V_R}$$

Impedance (Z), *the total resistance to the flow of current in an ac circuit*, is:

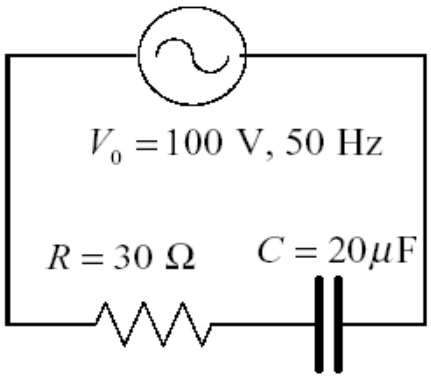
$$Z = \frac{V}{I} = \frac{\sqrt{V_R^2 + V_C^2}}{I} = \frac{\sqrt{(IR)^2 + (IX_C)^2}}{I} \qquad Z = \frac{\sqrt{I^2\left(R^2 + X_C^2\right)}}{I}$$

$$Z = \sqrt{R^2 + X_C^2} \qquad \tan\phi = \frac{X_C}{R}$$

OR $\tan\phi = \dfrac{1}{\omega CR}$ $\qquad \cos\phi = \dfrac{R}{Z}$

Example

1. For the circuit show below, calculate
   a) the total impedance. **Ans**. Z = 162 Ω
   b) the peak current. **Ans**. I_o = 0.617 A.
   c) the phase angle. **Ans**. ϕ = $\tan^{-1}$ (5.3) = 79.3^o



2. A 60 μF capacitor and a 20 Ω resistor are connected in series with a peak voltage of 200 V, 50 Hz power supply. Calculate the

   a) impedance. **Ans**. Z = 56.7 Ω. b) peak current. **Ans**. I_o = 3.7 A.

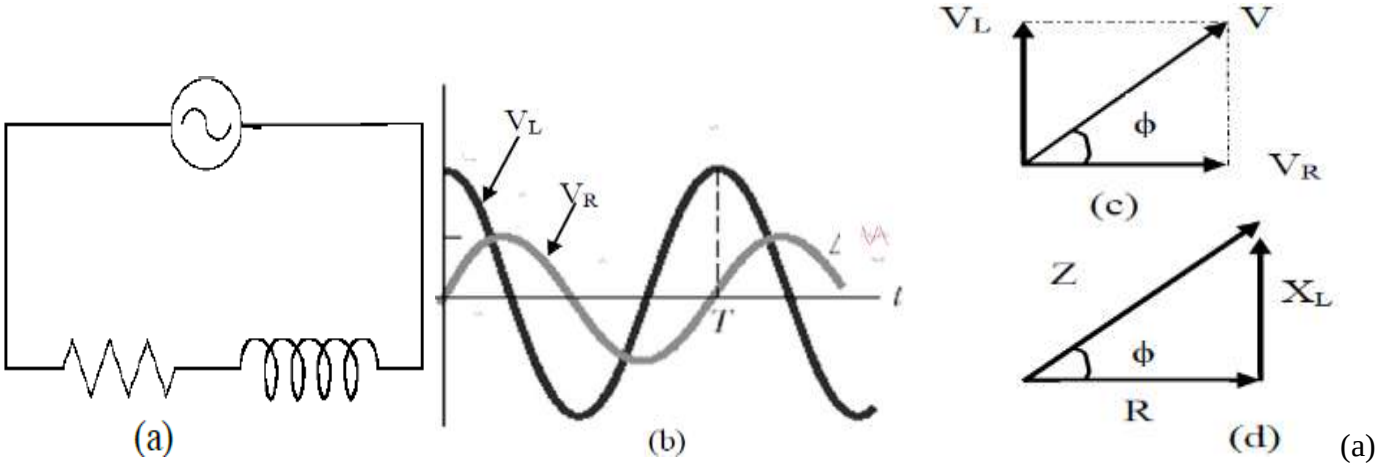
   c) phase angle. **Ans**. $\phi = \tan^{-1}(2.65) = 69.3^0$

3. A 40 μF capacitor is connected in series to a 50 Ω resistor and a generator whose rms output is 30 V at 60 Hz. What is the

   a) impedance? **Ans**. Z = 83 Ω. b) rms current? **Ans**. I_o = 0.36 A. **Ans**. phase angle? **Ans**. $\phi = 53^o$

ii) Series R-L circuits

Across the inductor, V_L leads the current by π/2 rad.



(a) A series circuit consisting of a resistor and an inductor connected to an AC source.

(b) Phase relationships for instantaneous voltages in the series R-L circuit.

(c) Phasor diagram for the series R-L circuit.

(d) An impedance triangle for a series R-L circuit.

$$Z = \frac{V}{I} = \frac{\sqrt{V_R^2 + V_L^2}}{I}$$

$$Z = \frac{\sqrt{I^2\left(R^2 + X_L^2\right)}}{I} \qquad \text{OR} \qquad Z = \sqrt{R^2 + X_L^2}$$

$$\tan\phi = \frac{V_L}{V_R}$$

$$\tan\phi = \frac{IX_L}{IR}$$

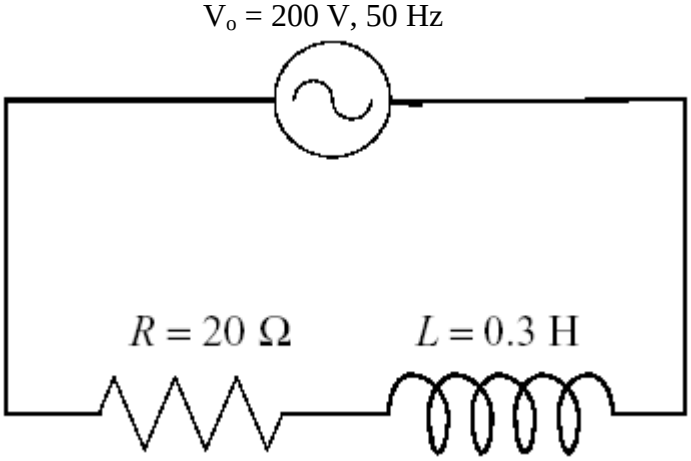
$$\tan\phi = \frac{X_L}{R} \qquad \text{OR} \qquad \tan\phi = \frac{\omega L}{R} \qquad \cos\phi = \frac{R}{Z}$$

Phase and Amplitude Relations for Alternating Currents and Voltages

Current Element	Symbole	Resistance or Reactance	Phase of the Current	Phase Constant ϕ	Amplitude Relation
Resistor	R	R	In phase with V_R	0^o = 0 rad	$V_R = I_R R$
Capacitor	C	$X_C = \frac{1}{\omega C}$	Leads V_C by $90^o = \pi/2$ rad	$-90^o = -\pi/2$ rad	$V_C = I_C X_C$
Inductor	L	$X_L = \omega C$	Lags V_L by $90^o = \pi/2$ rad	$+90^o = +\pi/2$ rad	$V_L = I_L X_L$

Example

1. For the following circuit, calculate

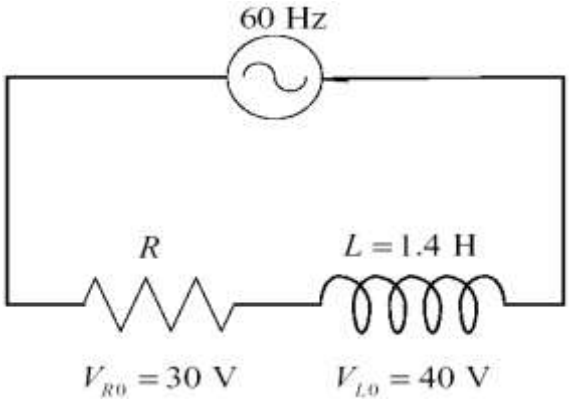


a) the impedance. **Ans**. Z= 96.3 Ω.
b) the peak current. **Ans**. I_o= 2.08A.
c) the phase angle. **Ans**. $\phi = \tan^{-1}(4.71) = 78^o$

2. An inductor of inductance 0.2 H and a resistor of resistance 20 Ω are connected in series to a peak voltage of 200V, 50 Hz power supply. Calculate.
a) the impedance. **Ans**. Z = 65.9 Ω. b) the peak current. **Ans**. I_o=3A. c) the phase angle. **Ans**. $\phi = 72.3^0$

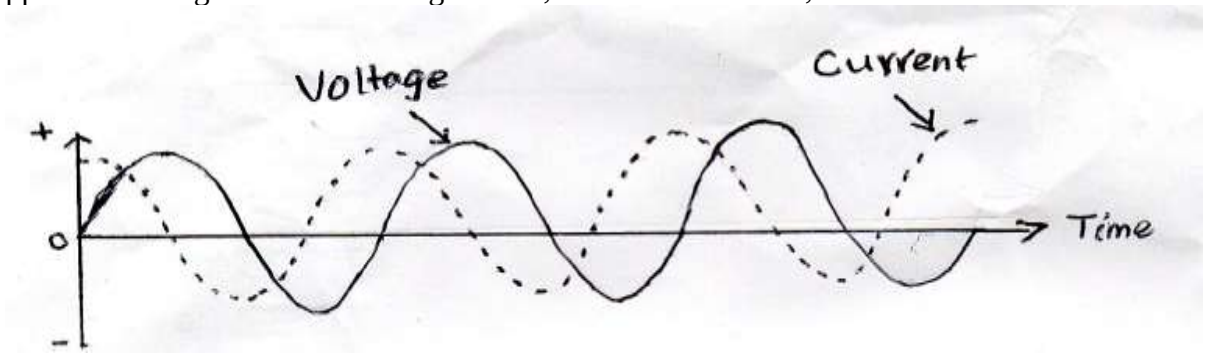
3. What is

a) the resistance? **Ans**. R = 394.7 Ω
b) the impedance? **Ans**. Z = 659.1Ω
c) the peak source voltage?
Ans. V_o = 50.1 V



4. An inductive coil has a resistance of 100 Ω. When an ac source of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45^0. What is the inductance of the coil?
A) 10 *m*H B) 12 *m*H C**)** 16 *m*H D) 20 *m*H

5. The applied ac voltage and the resulting current, in a circuit element, is shown below.



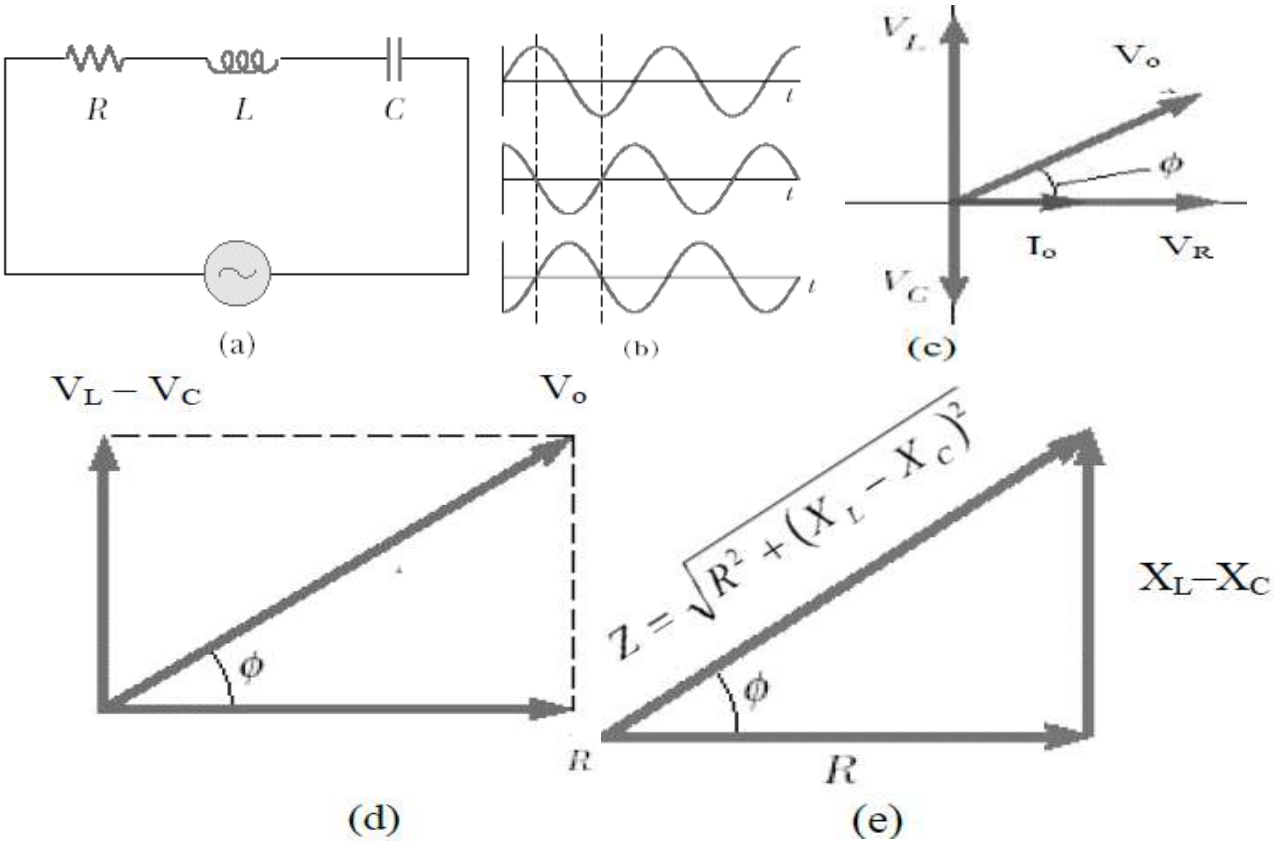
What is the circuit element?
A) Inductor B) Diode C) Resistor D) Capacitor

iii) Series R-L-C circuits

Across the resistor, V_R and I are in phase.

Across the inductor, V_L leads I by π/2 rad.

Across the capacitor, V_C lags I by π/2 rad.



(a) A series circuit consisting of a resistor, an inductor and a capacitor, connected to an AC source.
(b) Phase relationships for instantaneous voltages in the series *R-L-C* circuit.
(c) Phasor diagram for the series *R-L-C* circuit.
(d) Simplified version of the phasor diagram shown in part (c).
(e) An impedance triangle for a series *R-L-C* circuit.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

$$Z = \frac{V}{I} = \frac{\sqrt{I^2R^2 + I^2(X_L - X_C)^2}}{I} \quad \text{OR}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle ϕ is

$$\tan\phi = \frac{V_L - V_C}{V_R} \quad \text{OR}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

The phase angle is +ve if $X_L > X_C$, –ve if $X_C > X_L$

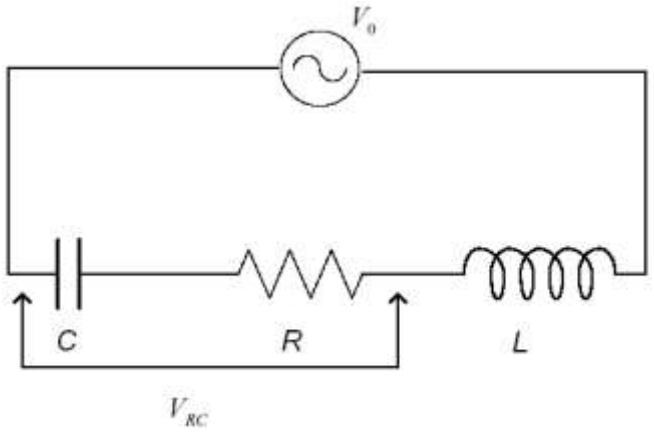
The quantity $X = X_L - X_C$ is called *the reactance of the circuit*.

If $X_L >> X_C$, the R-L-C series circuit acts like an inductive circuit and if $X_L << X_C$, the R-L-C series circuit acts like a capacitive circuit.

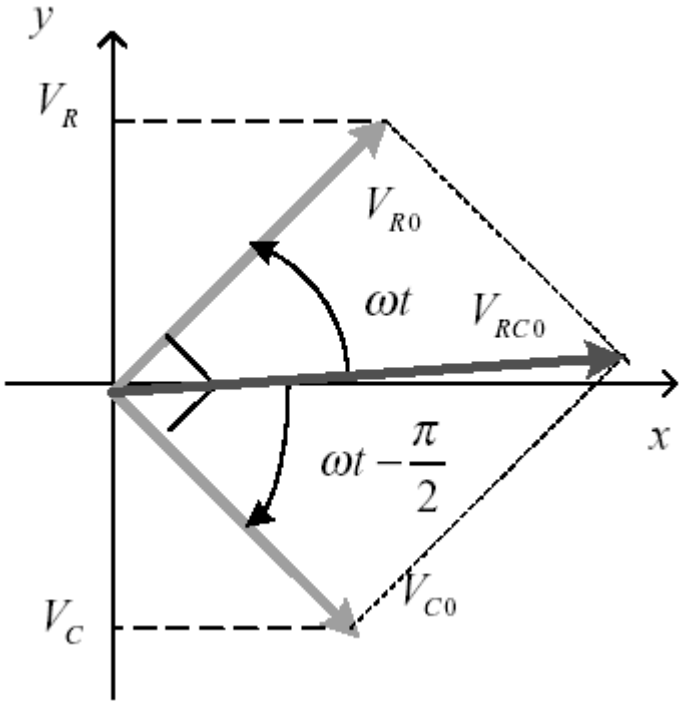
For a series R-L-C circuit shown below, the current I and voltage across the resistor V_{R0} are in phase. Therefore, the current phasor vector and the resistor voltage phasor vector are parallel. The voltage across the capacitor V_{C0} lags the current I by $\pi/2$ rad, so the capacitor voltage phasor vector is perpendicular to the resistor voltage and current phasor vectors.

The magnitude of the sum of the phasors V_{R0} and V_{C0} are given by

$$V_{RC0} = \sqrt{V_{R0}^2 + V_{C0}^2}$$

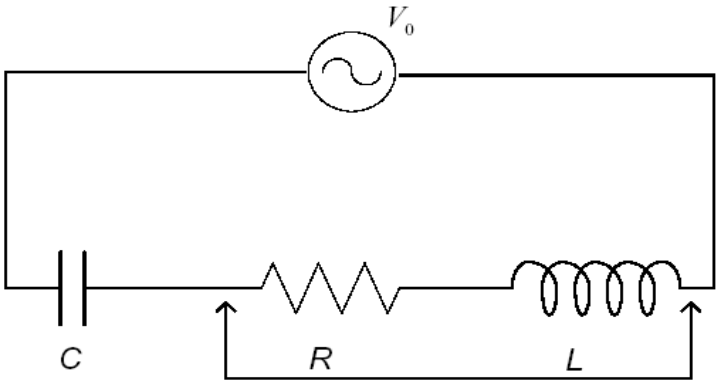


It is clear from the diagram that V_{RC0} lags the current as shown below.

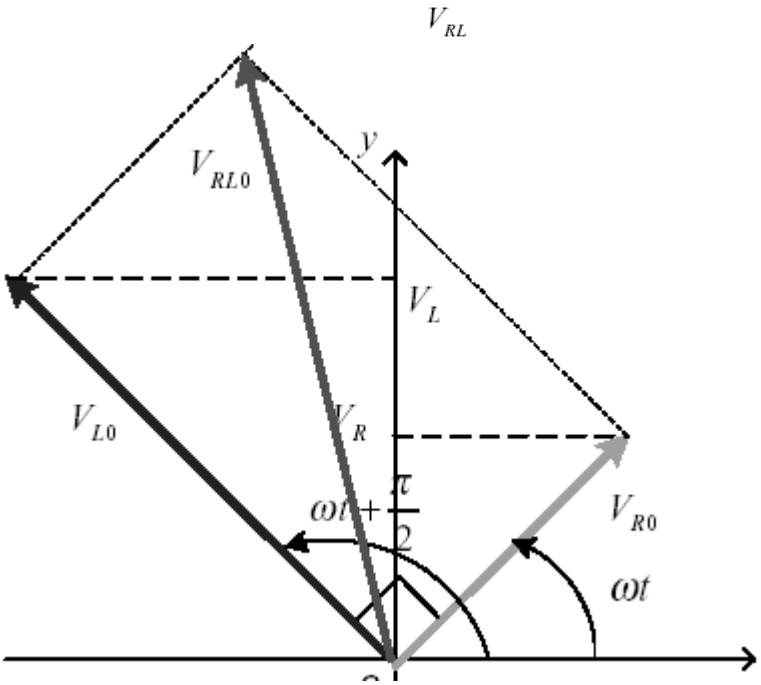


Find the expression for the V_{RLO} voltage across the resistor and inductor.

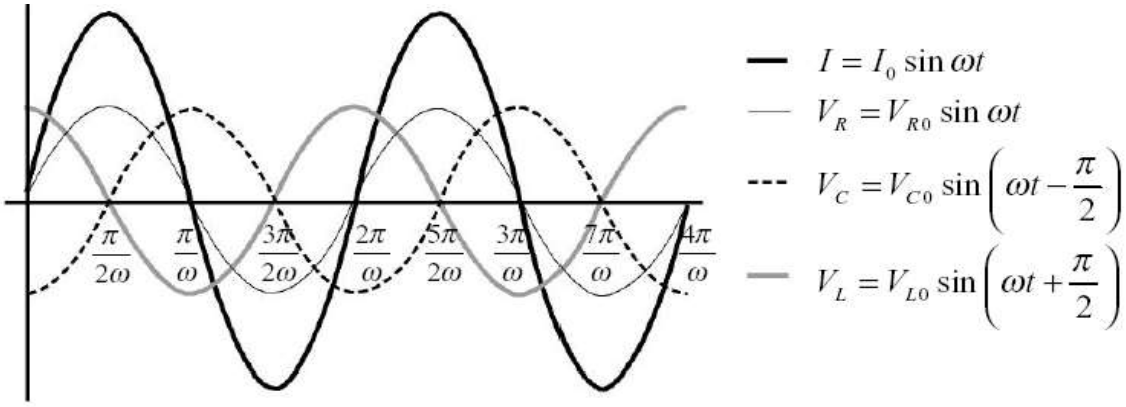
Ans. $V_{RL0} = \sqrt{V_{R0}^2 + V_{L0}^2}$



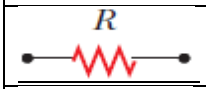
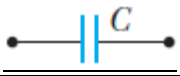
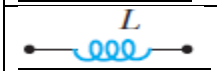
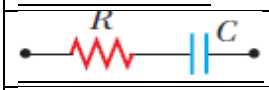
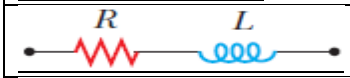
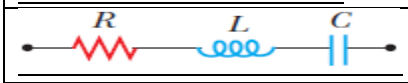
The V_{RL0} leads current.



For a series R-L-C circuit connected to a sinusoidal source, show that the current, resistor voltage, capacitor voltage and inductor voltage time graphs drawn on the same axes is as shown as below.

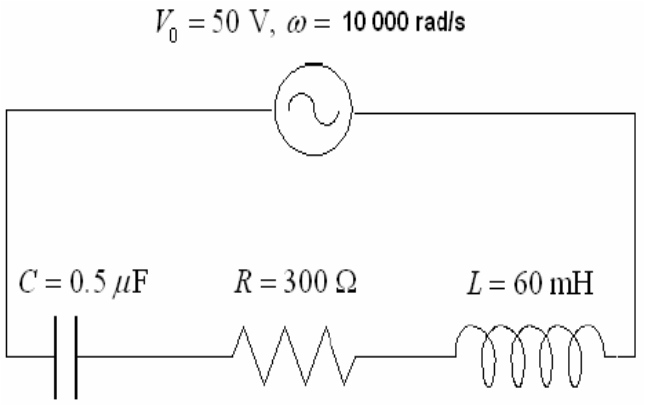


Impedance Values and Phase Angles for Various Circuit-Element Combinations

Circuit element	Impedance Z	Phase Angle ϕ
	R	0^0
	X_c	-90^0
	X_L	$+90^0$
	$\sqrt{R^2 + X_C^2}$	Negative, between -90^0 and 0^0
	$\sqrt{R^2 + X_L^2}$	Positive, between 0^0 and 90^0
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

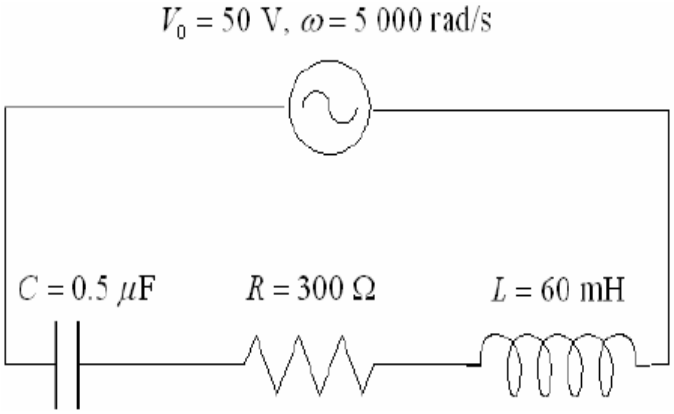
Example

1. For the circuit shown below, calculate:



a) X_C. **Ans**. $X_C = 200\ \Omega$
b) X_L. **Ans**. $X_L = 600\ \Omega$
c) Z. **Ans**. $Z = 500\ \Omega$
d) the rms voltages across the resistor.
Ans. $V_{Rrms} = 21.2$ V
e) the rms voltage across capacitor.
Ans. $V_{Crms} = 14.2$ V
f) the rms voltage across the inductor.
Ans. $V_{Lrms} = 42.5$V
g) the peak voltages across each circuit element.
Ans. $V_{R0} = 30$V, $V_{L0} = 60$V and $V_{C0} = 20$V
h) the phase angle. **Ans**. $\phi = 53^0$.
The voltage *leads* current by 53^0
i) the V_{RC0}. **Ans**. $V_{RC0} = 36.06$ V
the V_{RL0}. **Ans**. $V_{RL0} = 67.08$ V

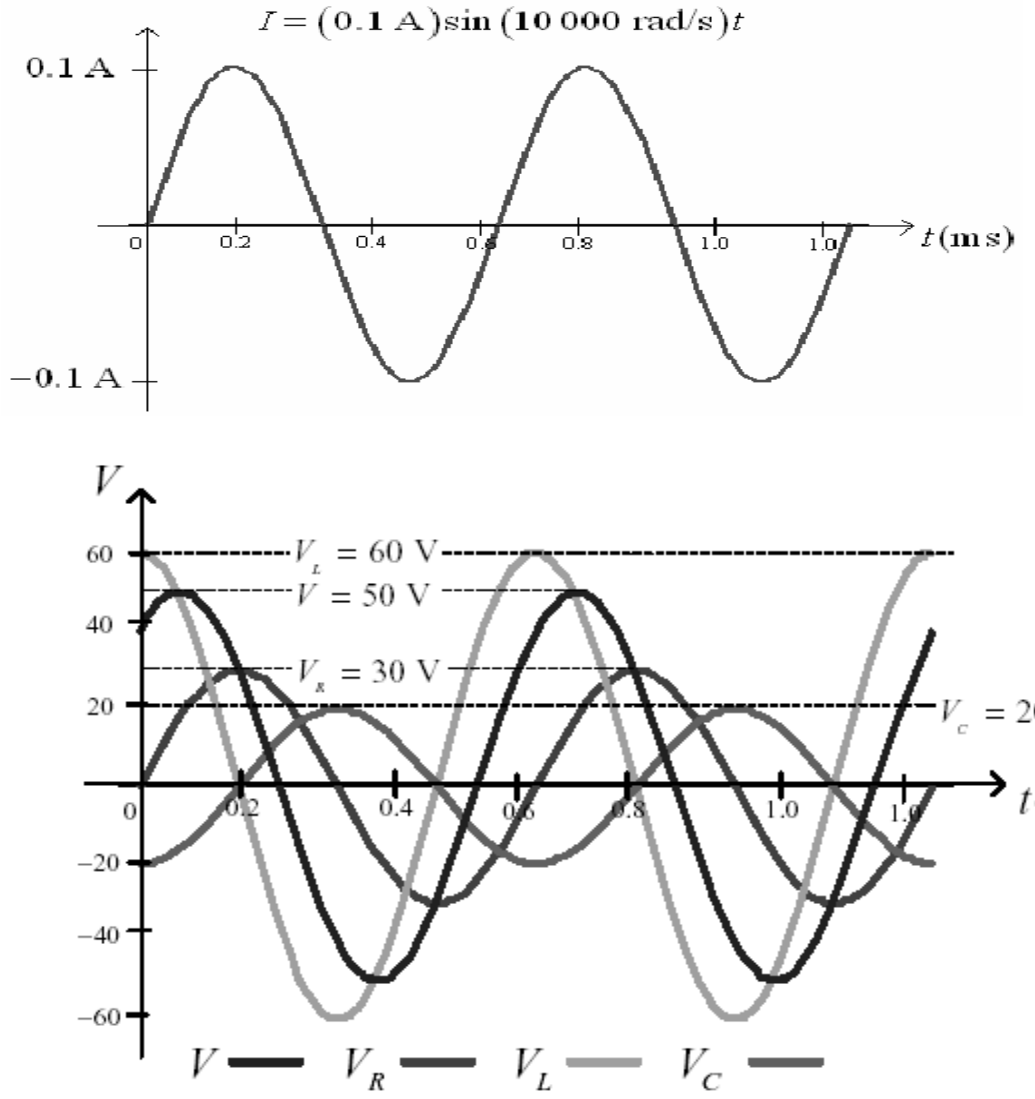
2. Consider the same circuit as before, but with $\omega = 5\ 000$ rad/s. Calculate



a) the peak current. **Ans**. I = 0.16 A
b) the impedance. **Ans**. $Z = 316.2\Omega$
c) the phase angle. **Ans**. $\phi = -18.4^0$
The –ve sign indicates that the circuit is more capacitive than inductive.
*The **voltage lags the current** by 18.4^0 or the **current leads voltage** by 18.4^0*

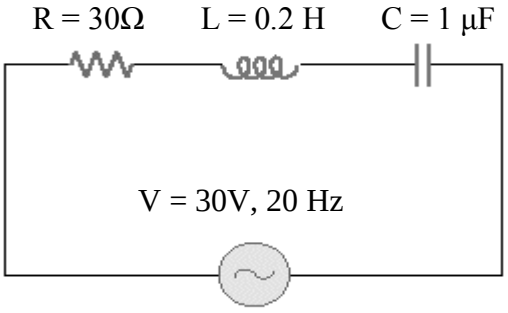
We can conclude that the angular frequency affects the current as well as the phase angle.

3. Consider the same circuit as question 1. Show that the instantaneous
   a) current is given by: I(t) = (0.1 A) sin (10 000rad/s) t
   b) voltage across the resistor is given by: V_R (t) = (30V) sin (10000rad/s) t
   c) voltage across the inductor is given by: $V_L(t)$ = (60V) sin [(10000rad/s) t + π/2rad]
   d) voltage across the capacitor is given by: $V_C(t)$ = (20V) sin[(10000rad/s) t – π/2rad]



Questions from national exam

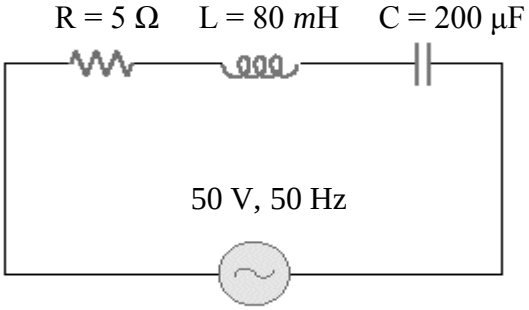
1. What is the voltage drop across the RL combination of the series circuit below?



A) 1.125V
B**)** 0.149 V
C) 0.750 V
D) 2.50 V

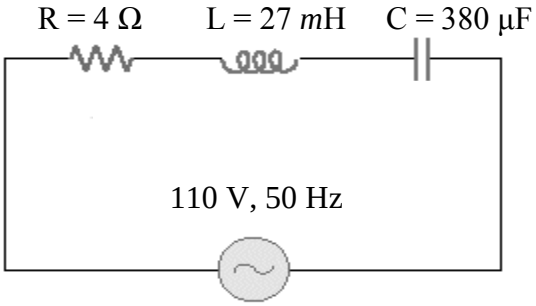
2. The phase angle in an alternating current is zero degree when the circuit contains:
   A) resistor only.
   B) capacitor only.
   C) inductor only.
   D) series combination of resistor and inductor.

3. Find the impedance for the RLC circuit shown below. **Ans**. $Z = 10.3\ \Omega$



A) 1.96 Ω
B) 19.2 Ω
C) 25.5 Ω
D) 648.1 Ω

4. Determine the total impedance for the circuit shown below.



A) 0 Ω
B**)** 4 Ω
C) 27 Ω
D) 31 Ω

5. Which of the following is correct about the characteristic of current and voltage in a.c circuit?
   (A) If the current leads the voltage, its capacitive reactance is greater than inductive reactance.
   (B) If the current is in phase with the voltage, the circuit is inductive.
   (C) If the current leads the voltage, it is inductive circuit.
   (D) If the current lags the voltage, it is capacitive circuit.

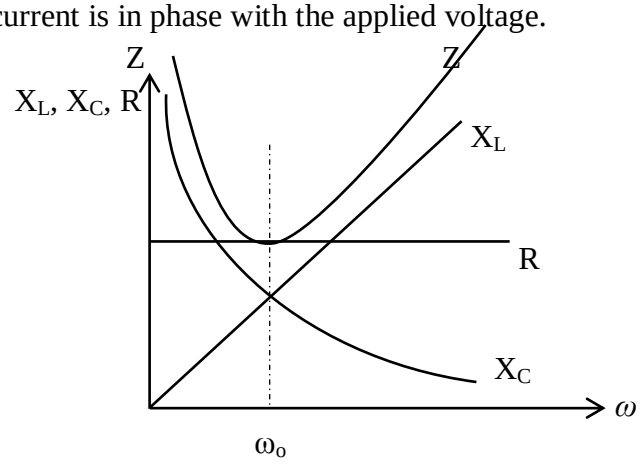
Series R-L-C circuits, Resonance

A series R-L-C circuit is said to be in *resonance* when $X_L = X_C$. In this case, the impedance of the circuit will be only the resistance R. The frequency ω_o at which $X_L - X_C = 0$ is called the **resonance frequency** of the circuit.

To find ω_0, we use the condition $X_L = X_C$, from which we obtain $\omega_o L = 1/\omega_o C$, or

$$\omega_o = \frac{1}{\sqrt{LC}}$$ Resonance frequency.

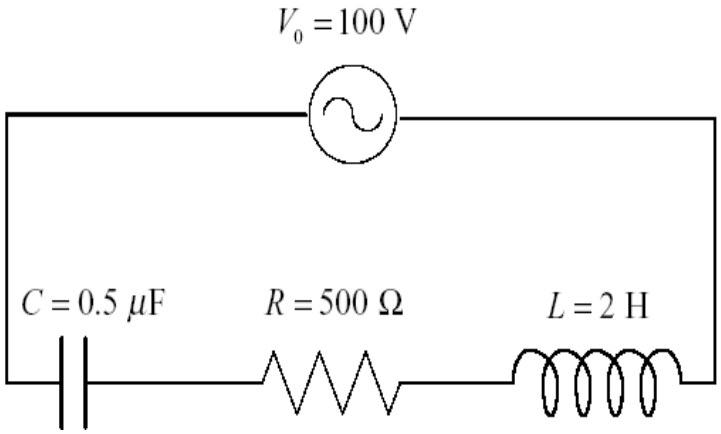
Because the impedance depends on the frequency of the source, the current in the R-L-C circuit also depends on the frequency. The current in a series R-L-C circuit reaches its maximum value when the frequency of the applied voltage matches the natural oscillator frequency, which depends only on *L* and *C*. Furthermore, at this frequency the current is in phase with the applied voltage.



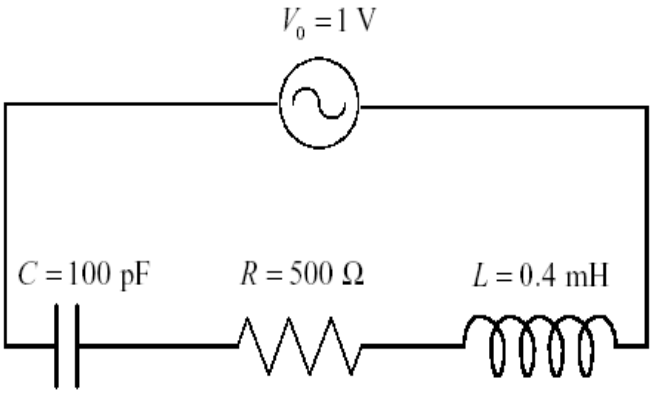
A graph of R, X_L, X_C and Z for series R-L-C circuit.

Example

1. For the following circuit, calculate
   a) the resonance angular frequency.
      Ans. ω_o = 1000 rad/s.
   b) the maximum current through the circuit.
      Ans. I_o = 0.2 A



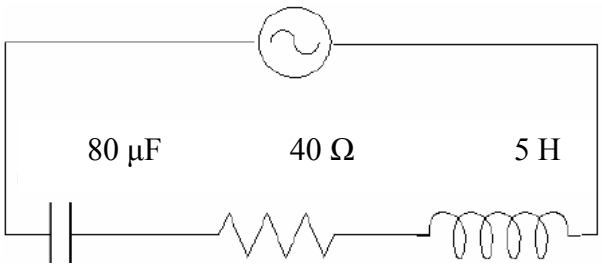
2. Consider the circuit below. Calculate,
   a) the resonance frequency. **Ans**. $\omega_o = 5\times10^6$rad/s
   b) the inductive reactance. **Ans**. $X_L = 2000\ \Omega$
   c) the capacitive reactance. **Ans**. $X_C = 2000\ \Omega$
   d) the impedance at resonance. **Ans**. $Z = 500\ \Omega$
   e) the peak current at resonance. **Ans**. I_o= 0.002A
   f) the peak voltages across all the circuit elements.
      Ans. $V_{R0} = 1$V, $V_{C0} = 4$ V, $V_{L0} = 4$ V



3. Consider a series *RLC* circuit for which $R = 150\Omega$, $L = 20$ *m*H, $V_{rms} = 20$ V, $\omega = 5000$ rad/s. Determine the value of the capacitance for which the current is a maximum. Hint: The current has its maximum value at the resonance frequency. **Ans**. C = 2 μF

Questions from national exam

1. At what frequency would the inductive reactance of a 2 *m*H inductor be equal to the capacitive reactance of a 5 μF capacitor?
   A**)** 10^4 rad/s B) 10^6 rad/s C) 10^3 rad/s D) 10^5 rad/s
2. The figure below shows a series LRC circuit connected to a variable frequency 230 V source. Which statement is NOT correct about the circuit?
   A) The source frequency that drives the circuit into resonance is 25/π.
   B) The impedance of the circuit is 40 Ω.
   C) The current amplitude at resonance is 8.1A.
   D) The rms p.d across R at resonance is zero.



3. What is the inductance of an LC circuit that resonates at 120 Hz when the capacitance is 8.0 μF?
   A) 0.22 μH B**)** 0.22 H C) 2.19 H D) 2.19 μH
4. In RLC circuit, given R = 7.6 Ω, L = 2.2 *m*H and C = 1.8 μF, what is the resonance frequency of the circuit?
   A**)** 2.5×10^3 Hz B) 15.72×10^3 Hz C) 12.5×10^3 Hz D) 23.0×10^3 Hz

Power in ac circuits

The average power delivered by a source to a resistor is:

$$P_{av} = I_{rms}V_{rms} \qquad P_{av} = I_{rms}^2 R \qquad \text{OR} \qquad P_{av} = \frac{V_{rms}^2}{R}$$

*At any instant, electromagnetic energy may be entering or leaving a capacitor, depending on whether it is charging or discharging. Since the circuit oscillates sinusoidally, the *energy that enters during the charging part of the cycle is equal to the energy that leaves during the discharging part of the cycle*.

Consequently, the average power for the capacitor is zero.

*As with the capacitor, the inductor is an energy storage device. *The average power for an inductor is also zero.*

A general expression for average power delivered to an RLC circuit is

$P_{av} = I_{rms}\, V_{rms} \cos\phi$ Where the quantity $\cos\phi = \frac{R}{Z}$ is called the **power factor**.

For a pure resistance, $\phi = 0$, $\cos 0^0 = 1$ and $P_{av} = I_{rma}\, V_{rms}$.

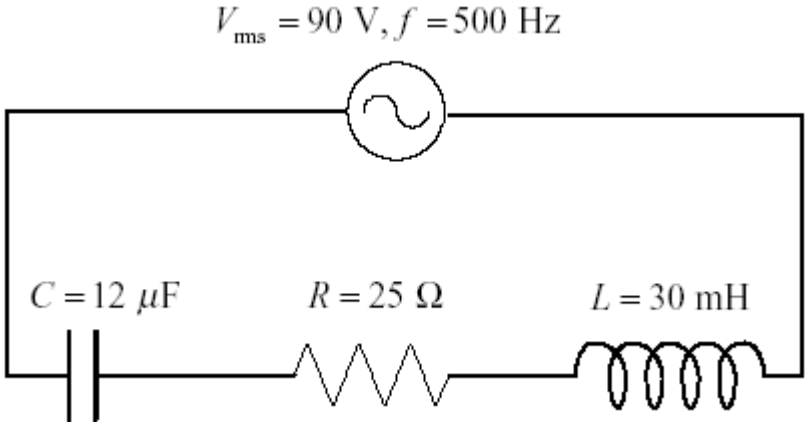
For a pure capacitor or inductor, $\phi = 90^0$, $\cos 90^0 = 0$ and $P_{av} = 0$.

We observe that *no power losses are associated with pure capacitors and pure inductors in an ac circuit*.

Example

1. Calculate the average power delivered to the series R-L-C circuit for which R = 250 Ω, L = 0.6 H, C = 3.5 μF capacitor, f = 60 Hz and V_{rms} = 150 V. **Ans**. P_{av} = 16.3 W
2. A series ac circuit contains a 50 Ω resistor, a 1.5 μF capacitor, a 0.2 H inductor and a 60 Hz generator that has a rms output of 90 V. Calculate the average power to the circuit. **Ans**. P_{av} = 0.14 W
3. A series R-L-C circuit is driven by an ac source with R = 870 Ω, L = 24 *m*H, C = 390 *n*F, Vo = 62 V and ω = 5.5 *k* rad/s. Calculate the average power delivered to the circuit. **Ans**. P_{av} = 1.9 W
4. A series ac circuit contains the following components: *R* =150Ω, *L* = 250 mH, *C* = 2 μF and a source with V_o = 210 V operating at 50 Hz. Calculate the
   a) inductive reactance. **Ans**. X_L = 78.5 Ω c) impedance. **Ans**. Z = 1520 Ω
   b) capacitive reactance. **Ans**. X_C = 1592 Ω d) maximum current. **Ans**. I_o = 0.138 A
   e) phase angle between current and source voltage. **Ans**. $\phi = -84.3^0$

5. Determine the average power supplied to the circuit. **Ans**. P_{av} = 38.7 W



Questions from national exam

1. In an RLC circuit, suppose R = 300 Ω, L = 60 *m*H, C = 0.5 μF, V_O = 50 V, and ω = 10,000 rad/s. What is the average power delivered to the entire circuit?
   A**)** 1.5 W B) 2.3 W C) 3.5 W D) 1.0 W
2. A capacitor C = 2 μF, inductor L = 10 H and coil resistance 5 Ω are in series in a circuit. When an alternating current of 2 A rms flows, the power in watts in the circuit is
   A) 100 B) 50 C) 20 D) 2
3. A series RLC circuit has a resistance of 45.0 Ω and an impedance of 75.0 Ω. What average power is delivered to this circuit when V_{rms} = 210 V?
   A) 573 W B**)** 353 W C) 255 W D) 247 W
4. A series RLC circuit has a total impedance of 75.0Ω. If the resistance is R = 45.0 Ω and the average power delivered to the circuit is P = 352.8 W, what is the root-mean square value of the ac voltage source?
   A) 83 V B) 116 V C**)** 210 V D) 297 V
5. An RLC circuit is designed to have a resistance of 400 Ω, capacitve reactance of 350 Ω and inductive reactance of 50 Ω. The circuit is connected to an alternative voltage source of effective (rms) value 50 V, and frequency 50 Hz. What is the output average power?
   (A) 62.5 W (B**)** 4 W (C) 400 W (D) 40 W
6. A series circuit consisting of 50 Ω resistor and capacitor of capacitancitive reactance 50 Ω are connected to alternating source of voltage(rms0 100 V. What is the average output and power factor of the circuit? **Ans**. P_{av} = 141.4 W, ϕ = 0.707
   A) 141.4 W, 0.8 B) 100 W, 0.707 C) 70.7 W, 0.707 D) 200 W, 0.8