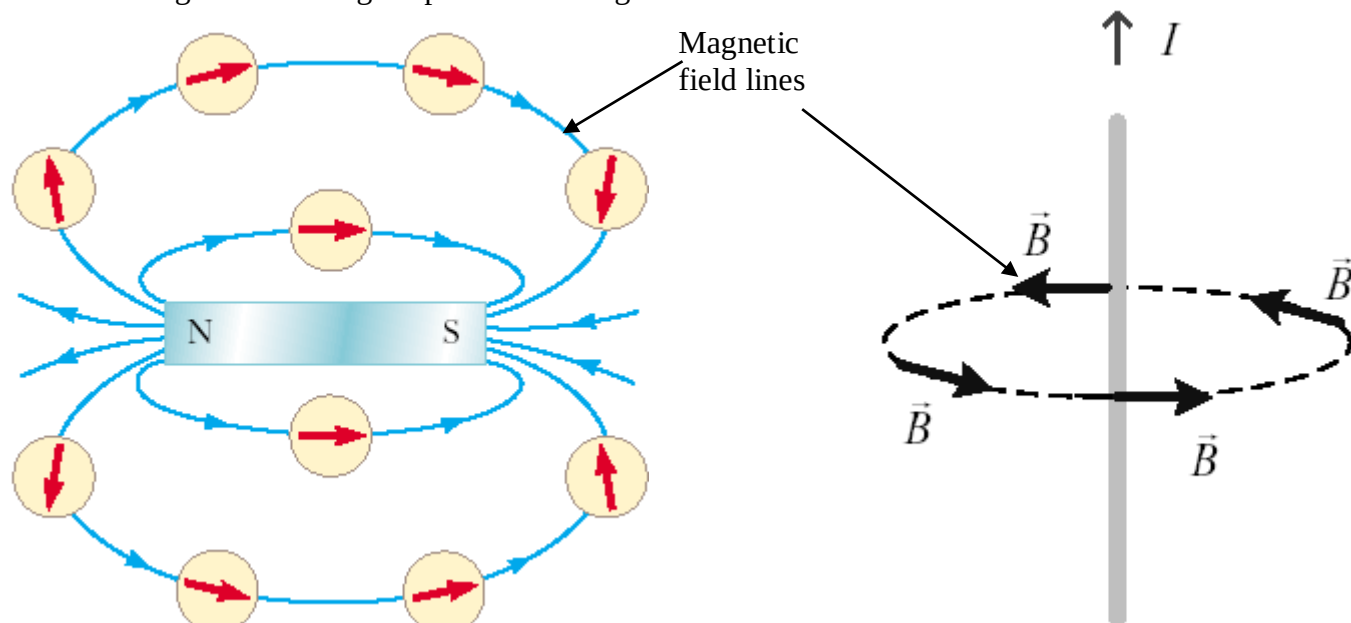


UNIT 6: MAGNETIC FIELD AND FORCE

Magnetic Field:- is a region near a permanent magnet or a current carrying conductor in which

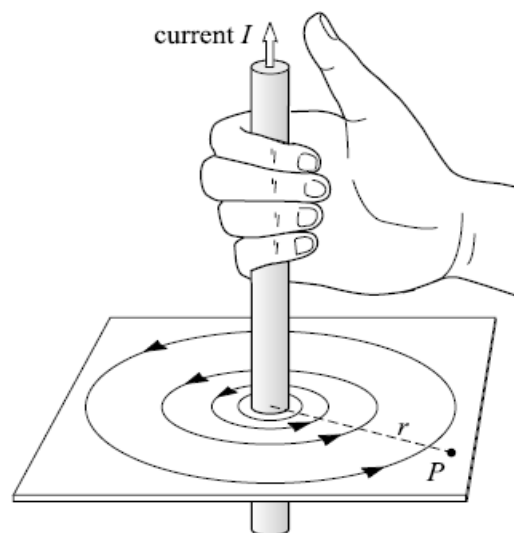
- a small pivoted magnet detects a magnetic effect, or
- a moving electric charge experiences a magnetic force.



Magnetic field lines (magnetic lines of force):- are imaginary lines drawn in a region to show the direction of a magnetic field.

The direction of magnetic field lines for a current carrying wire is found using the right hand rule.

“Grasp the wire with your right hand in such a way that your thumb points in the direction of the current, your fingers encircle the wire in the same direction as the magnetic field lines.”

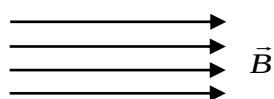


The field lines are:- **closely spaced for strong magnetic field.**

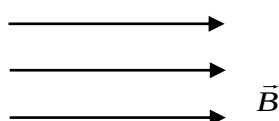
- **widely spaced for weak magnetic field.**

- **equally spaced for uniform magnetic field.**

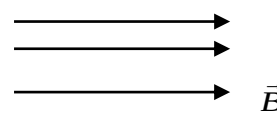
- **not equally spaced for non-uniform magnetic field.**



Strong and uniform field



Weak and uniform field



non-uniform field

Magnetic flux

The number of magnetic lines of force crossing a given surface perpendicularly is defined as magnetic flux.

$$\Phi_B = \vec{B} \cdot \vec{A}$$

$$= BA \cos\theta \quad \text{Its unit is Weber } 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

Where θ is the angle between $\vec{B}$ and a vector perpendicular to the area $\hat{n}$

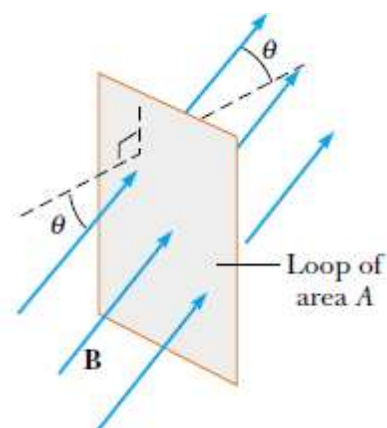
$$\vec{A} = \hat{n}A, \hat{n} \text{ is a unit vector normal to the surface.}$$

If $\vec{B}$ is uniform and perpendicular to A then

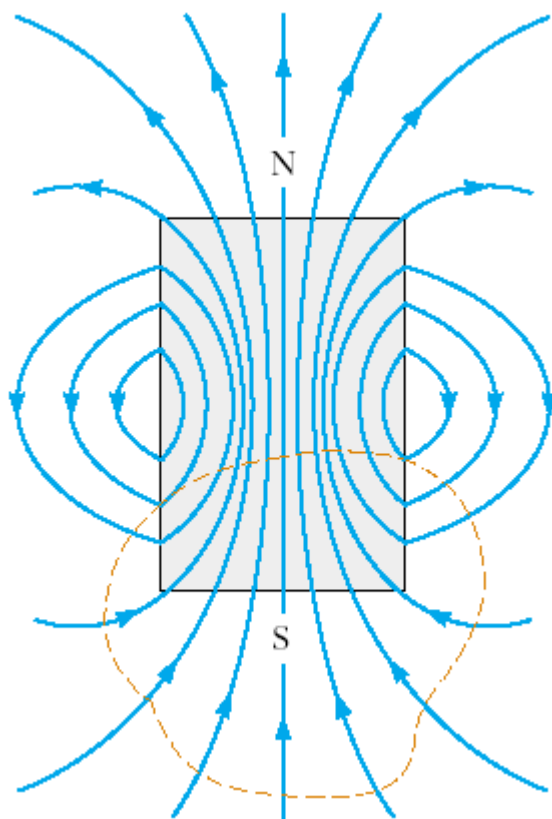
$$\Phi = BA \quad \text{and}$$

$$B = \frac{\Phi}{A} \quad \text{is magnetic flux density } 1 \text{ T} = 1 \text{ Wb/m}^2$$

$$1 \text{ Gauss (G)} = 10^{-4} \text{ Tesla (T)}$$

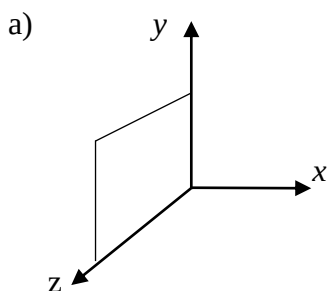


Note that for any closed surface such as the one outlined by the dashed line in figure below, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero.

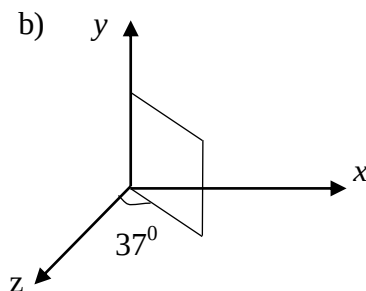


Example

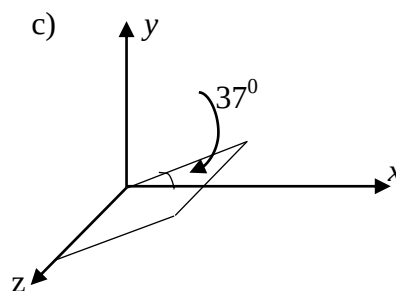
1. If a magnetic field of $\vec{B} = 0.25T$ is in the positive x -direction, find the flux through a square of side 4cm in each orientation shown.



Ans. $\Phi = 4 \times 10^{-4} \text{ wb}$



Ans. $\Phi = 3.2 \times 10^{-4} \text{ wb}$



Ans. $\Phi = 2.4 \times 10^{-4} \text{ wb}$

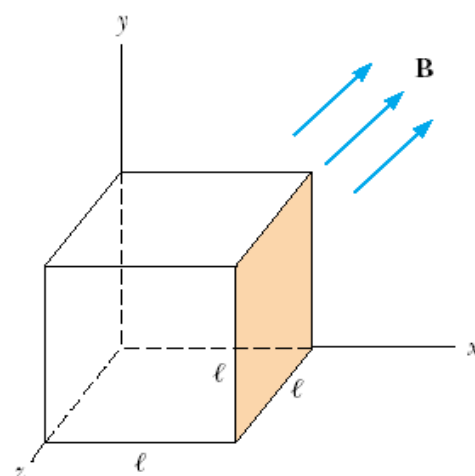
2. A cube of edge length $\ell = 2.50 \text{ cm}$ is positioned as shown in figure below. A uniform magnetic field given by $\vec{B} = (5\hat{i} + 4\hat{j} + 3\hat{k})T$ exists throughout the region.

- a) Calculate the flux through the shaded face.

Ans. $\Phi = 3.125 \times 10^{-3} \text{ Wb}$

- b) What is the total flux through the six faces?

Ans. $\Phi = 0$



3. A square flat loop of wire is placed in a uniform magnetic field that is in the y direction. The magnetic flux through the loop is a maximum if the plane of the loop is in the
- A) xy plane. C) yz plane.
 B) xz plane. D) *inclined 45° with respect to the xy plane.*
4. What is the change in magnetic flux in a coil of area 5m^2 as its orientation relative to the perpendicular of a uniform $3T$ magnetic field changes from 45° to 90° ?
- A) 0Wb B) -11Wb C) 7.5Wb D) 11Wb
5. A magnetic field of strength $5 \times 10^{-5}T$ passes through an area of 20 cm^2 that is at an angle of 60° to the normal to the area. What is the magnetic flux?
- (A) 0 Wb (B) $5 \times 10^{-8}T$ (C) $2.5 \times 10^{-8}T$ (D) $8.7 \times 10^{-8}T$

Magnetic properties of matter

Magnetism:- is a phenomenon produced by the motion of electric charge, which results in attractive and repulsive forces between objects.

Depending on their magnetic properties, substances can be classified in to three categories.

Ferromagnetic materials

These substances contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed.

Paramagnetic substances:- have a small magnetic moments. These moments interact only weakly with each other and are randomly oriented in the absence of an external magnetic field. When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field, but not retain permanent magnetism.

Diamagnetic substance:- When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field. This causes diamagnetic substances to be weakly repelled by a magnet.

Magnetic force on a moving charged particle

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. ***In addition to containing an electric field, the region of space surrounding any moving electric charge also contains a magnetic field.***

A magnetic field also surrounds a magnetic substance making up a permanent magnet.

The magnetic force on a moving charged particle in a magnetic field is

$$\vec{F}_B = q\vec{V} \times \vec{B} \quad \text{Where } \vec{V} \text{ is velocity of the particle.}$$

$$\text{OR} \quad F = qVB \sin\theta$$

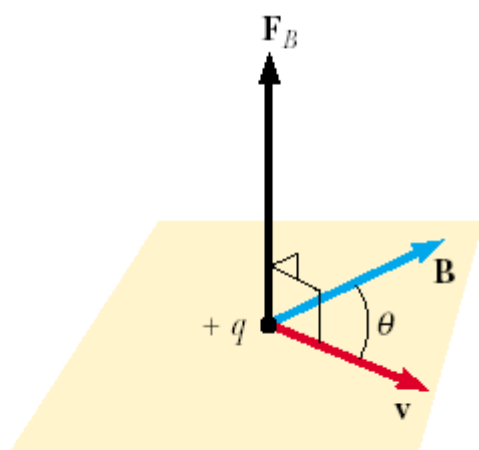
$\vec{B}$ is the magnetic field.

q is the charge on the particle.

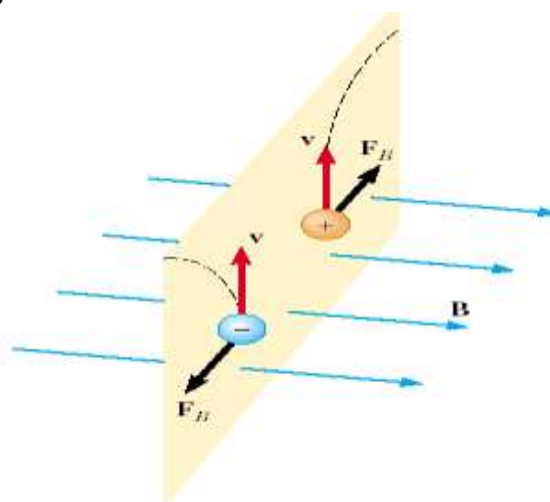
θ is the smaller angle between $\vec{V}$ and $\vec{B}$.

The direction of force is

- **parallel** to $\vec{V} \times \vec{B}$ for **positive charges**
- **anti-parallel** to $\vec{V} \times \vec{B}$ for **negative charges**



(a)



(b)

The direction of the magnetic force $\mathbf{F}_B$ acting on a charged particle moving with a velocity $\mathbf{v}$ in the presence of a magnetic field $\mathbf{B}$.

(a) The magnetic force is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. (b) Oppositely directed magnetic forces $\mathbf{F}_B$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field. The dashed lines show the paths of the particles.

There are several important differences between electric and magnetic forces:

- The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

When a charged particle moves with a velocity $\mathbf{v}$ through a magnetic field, the field can change the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.

The direction of magnetic force can also be determined using the right hand rule. “Curl the fingers of your right hand from the direction of $\vec{v}$ toward $\vec{B}$, then the extended thumb, which is at a right angles to the fingers, points in the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ ”

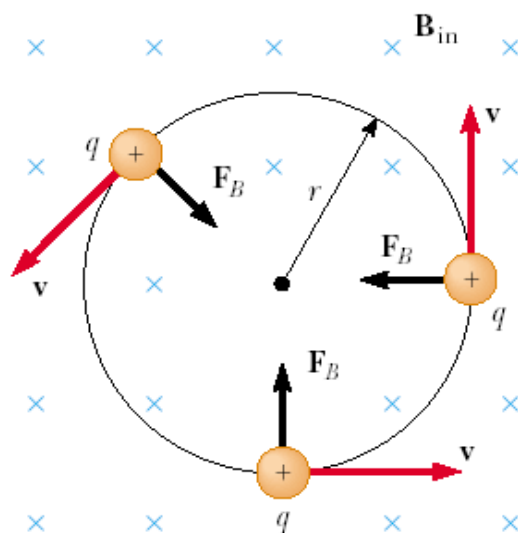
Consider a particle of mass m with charge q moving at a velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$. The magnetic force on the charge is

$$F = qvB \quad \text{which is always perpendicular to both } \mathbf{v} \text{ and } \mathbf{B}.$$

The effect of the force is to change the direction of $\mathbf{v}$ without changing its magnitude. This results in a uniform circular motion.

$$\begin{aligned} F_B &= qvB \\ F_c &= \frac{mV^2}{r} \\ F_B &= F_c \\ qvB &= \frac{mV^2}{r} \\ r &= \frac{mV}{qB} \end{aligned}$$

The radius of the circle followed by the charged particle



The radius of the path is directly proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field.

The period of the particle is

$$T = \frac{2\pi r}{V} = \frac{2\pi}{V} \frac{mV}{qB}$$

$$T = \frac{2\pi m}{qB} \quad \text{and its frequency is}$$

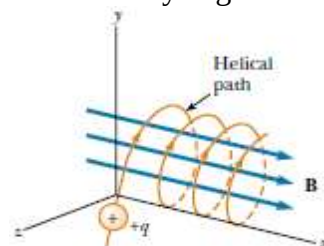
$$f = \frac{qB}{2\pi m}$$

The angular velocity is

$$\omega = \frac{qB}{m}$$

These results show that *the angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit.*

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to B , its path is a helix.



Example

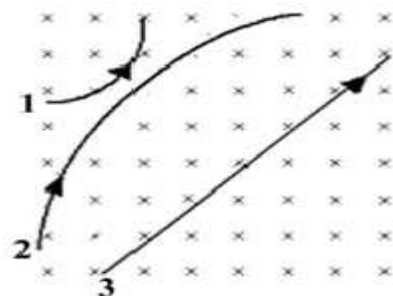
1. $q = 2\mu\text{C}$, $V = 100\text{ m/s}$, $B = 1.5\text{ T}$ and $\theta = 30^\circ$. Calculate F_B . **Ans.** $F_B = 1.5 \times 10^{-4}\text{ N}$.
2. A proton moving at $5 \times 10^6\text{ m/s}$ through a magnetic field of 2 T experiences a force of $9.6 \times 10^{-13}\text{ N}$. What is the angle between V and B ? **Ans.** $\theta = 37^\circ$
3. An electron is moving at 10^8 m/s east in a uniform magnetic field of 0.8 T directed vertically upward. Find the magnetic force on the electron. **Ans.** $F = 1.28 \times 10^{-11}\text{ N}$ north.
4. If the direction of the velocity and magnetic field are interchanged, what is the magnetic force on the electron in question 3 above? **Ans.** $F = 1.28 \times 10^{-11}\text{ N}$ south.
5. A particle with $m = 10^{-16}\text{ kg}$, $q = 2\mu\text{C}$ is projected at $4 \times 10^6\text{ m/s}$ into a uniform magnetic field of 0.1 T at an angle 30° with the direction of the field. What is the radius of its path? **Ans.** $R = 4 \times 10^{-3}\text{ m}$
6. Three particles of equal mass follow the paths numbered 1, 2 and 3, through a uniform magnetic field directed into the page.

- a) What are the signs of the charge on the particles?

Ans. Particle 1. Positive
Particle 2. Negative
Particle 3. No charge

- b) Compare the magnitude of their velocities if $q_1 = q_2$

Ans. $V_2 > V_1$



7. An electron with speed V_0 moves in a circle of radius R_0 in uniform magnetic field. The period of revolution is T_0 . If the speed of the electron is doubled, what will be
 - a) radius of the circle? **Ans.** $R = 2R_0$.
 - b) period of revolution? **Ans.** $T = T_0$
8. An electron that enters perpendicularly into a magnetic field B with a speed V moves in a circular path of radius R . If the magnetic field is doubled, the radius of the circular path will be:
 - A) $2R$
 - B) R
 - C) $\sqrt{2} R$
 - D) $\frac{1}{2} R$
9. A particle with $q = 4 \times 10^{-9}\text{ C}$ moving at $\vec{V}_1 = 3 \times 10^4 (\hat{i} + \hat{j})\text{ m/s}$, in a uniform magnetic field experiences a force $\vec{F}_1$ along the z -axis. When the particle moves at $\vec{V}_2 = 2 \times 10^4\text{ m/s } \hat{k}$, a force $\vec{F}_2 = 4 \times 10^{-5}\text{ N } \hat{i}$ acts on the particle. What is
 - a) the magnetic field? **Ans.** $\vec{B} = -0.5\text{ T } \hat{j}$.
 - b) the force $\vec{F}_1$? **Ans.** $\vec{F}_1 = -6 \times 10^{-5}\text{ N } \hat{k}$

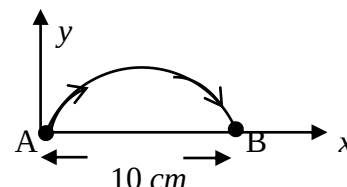
Exercise

1. In a magnetic field directed along positive z-axis, a particle initially moving in the positive y-axis is deflected toward the negative x-axis. What is the sign of the charge on the particle? **Ans.** Negative
2. A particle is shot horizontally toward north in a region where there is a magnetic field directed vertically upward. What is the direction of the magnetic force on the particle if it is
 - a) positively charged? **Ans.** East.
 - b) negatively charged? **Ans.** West.
3. A proton moving along the positive x-axis experiences a magnetic deflection in the positive z-axis. What is the direction of the magnetic field? **Ans.** In the positive y-direction.
4. An electron ($m_e = 9 \times 10^{-31}$ kg) at point A has a velocity $V = 10^7$ m/s.

What is

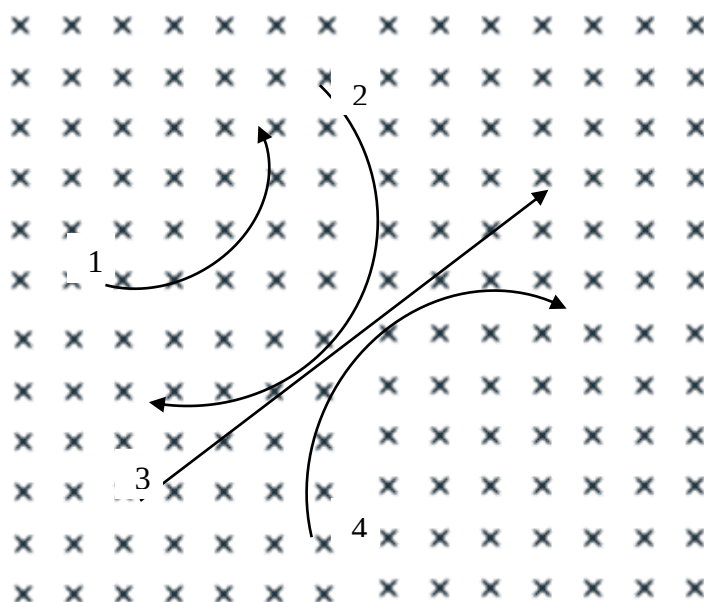
- a) the magnetic field that will cause the electron follow the semicircular path from A to B? **Ans.** $B = 1.125$ mT
- b) the time required to move from A to B?

Ans. $t = 1.57 \times 10^{-8}$ sec.

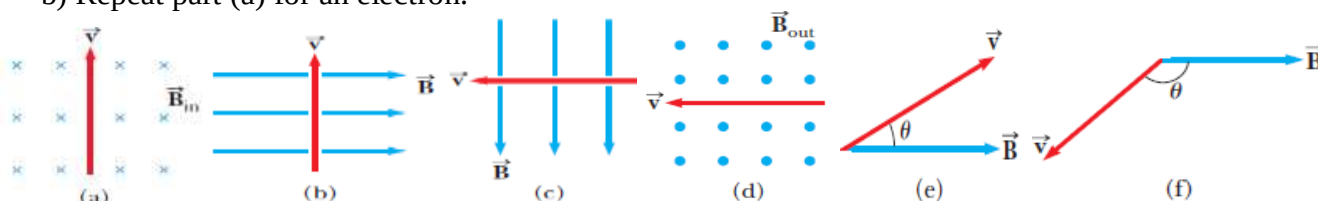


5. Four particles of equal mass follow the paths shown in the figure below. What is the sign of the charge on each particle?

Ans. Particle 1: Positive
Particle 2: Negative
Particle 3: No charge
Particle 4: Negative



6. a) Find the direction of the force on a proton moving through the magnetic fields in figure below.
b) Repeat part (a) for an electron.

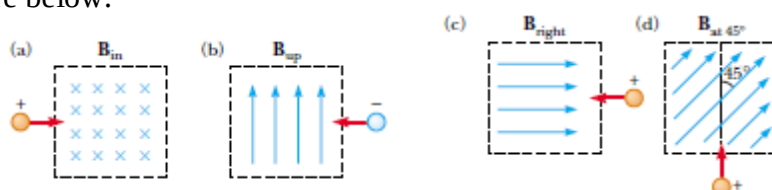


7. A particle with a charge of 2×10^{-12} C moving at 2.5×10^6 m/s $\hat{k}$ enters a region of uniform magnetic field. It experiences a magnetic force of $5 \times 10^{-6}(-0.1\hat{i} + 0.05\hat{j})$ N. When the velocity is redirected and becomes $4 \times 10^6(\hat{i} + 2\hat{j})$ m/s, the magnetic force is zero. What is the magnetic field?

Ans. $\vec{B} = (0.05\hat{i} + 0.1\hat{j})$ T

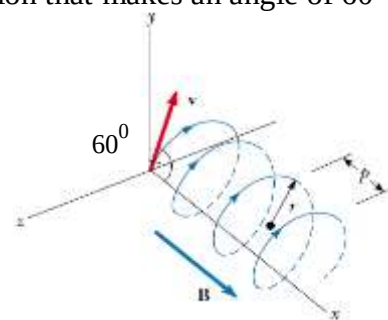
8. What is the rate at which work is done by a magnetic force on a charged particle? Hint: $P = \vec{F} \cdot \vec{V}$
9. A 0.5 g particle having a charge of 2.5×10^{-8} C is given a velocity of 8×10^4 m/s east. What is the magnetic field that keeps the particle move in its original direction? **Ans.** $B = 2.5$ T North.

10. Consider an electron near the Earth's equator. In which direction does it tend to deflect if its velocity is directed
 a) downward, b) northward, c) westward, d) south- eastward?
11. Determine the initial direction of the deflection of charged particles as they enter the magnetic fields as shown in figure below.



12. An electron is projected horizontally into a region where there is a uniform magnetic field pointing vertically upward, the electron moves along a circular path
 A) in a clockwise direction.
 B) whose radius is inversely proportional to the mass of the electron.
 C) whose radius is directly proportional to the charge of the electron.
 D) in the counter clockwise direction.
13. A uniform magnetic field of magnitude 0.15T is directed along the positive x axis. An electron ($m_e = 9 \times 10^{-31}$ kg) moving at 5×10^6 m/s enters the field along a direction that makes an angle of 60° with the x axis. The motion of the particle is expected to be a helix as in figure below. Calculate the pitch p of the trajectory.

Ans. $p = 0.6\text{mm}$



Questions from national exam

- An electron projected at right angles into a uniform magnetic field of 1.5×10^{-3} T moves in a circular path of radius 9 cm. What is speed of the electron? ($m_e = 9 \times 10^{-31}$ kg)
 A) 2.4×10^7 m/s B) 2.4×10^6 m/s C) 2.4×10^9 m/s D) 2.4×10^8 m/s
- An electron is projected at right angles into a uniform magnetic field of 1.2 T with a velocity of 5×10^7 m/s. What is the magnetic force on the electron?
 A) 3.6×10^{-12} N B) 7.2×10^{-12} N C) 4.8×10^{-12} N D) 9.6×10^{-12} N
- An electron and a proton are projected at right angles to a uniform magnetic field with the same linear momentum. Which one of the following statements is correct?
 A) The electron's trajectory will be less curved than the proton's trajectory.
 B) The proton's trajectory will be less curved than the electron's trajectory.
 C) Both trajectories will be equally spaced.
 D) Both particles will move along a straight line.
- A charged particle is moving in a uniform magnetic field. If the direction of motion of the charged particle is parallel to the magnetic field, the shape of the charged particle's path will be
 A) circular B) straight C) spiral D) elliptical
- Find the magnitude of the force felt by an electron traveling at an angle of 60° to the earth's magnetic field at 1.4×10^7 m/s. The magnitude of the earth's magnetic field is 5×10^{-5} T. ($\sin 60^\circ = 0.866$, $\cos 60^\circ = 0.5$)
 A) 0 N B) 9.7×10^{-17} N C) 1.1×10^{-16} N D) 8.6×10^{-17} N
- A particle of mass m is fired into a magnetic field of strength B at a speed v . The particle travels in a circular path inside the field with a radius r . Which of the following expressions gives the magnitude of the charge on the particle?
 A) $\frac{mr}{v^2 B}$ B) $\frac{mv^2}{Br}$ C) $\frac{vB}{mr}$ D) $\frac{mv}{Br}$

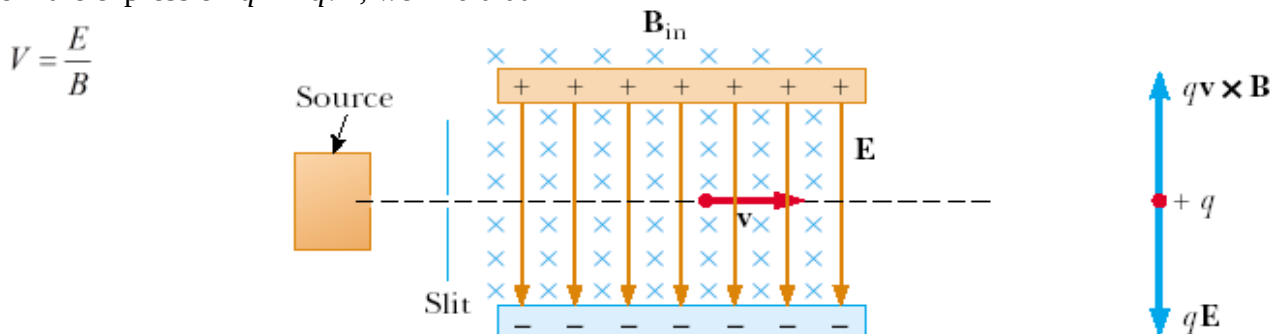
7. An electron moving at a speed of 1.5×10^7 m/s enters a region of uniform magnetic field of magnitude 2×10^{-3} T, in a direction perpendicular to the field. The path followed by the electron is circular. What is the radius of the circular path?
 A) 0.43 cm B) 4.3 cm C) 43 cm D) 430 cm
8. A charged particle moves perpendicular to a uniform magnetic field. Which statement is correct about the motion of the particle?
 A) Both kinetic energy and linear momentum of the particle change.
 B) The kinetic energy of the particle changes but not its linear momentum.
 C) The linear momentum of the particle changes but not its kinetic energy.
 D) Both kinetic energy and linear momentum do not change.
9. A positively charged particle of charge $q = 3.2 \times 10^{-19}$ C is moving parallel to x-axis with a speed 1000 m/s in a magnetic field of 0.5 T parallel to y-axis. What is the magnetic force on the particle?
 A) 8.0×10^{-16} N in -ve z. C) 8.0×10^{-17} N in +ve x.
 B) 3.2×10^{-16} N in -ve x. D) 1.6×10^{-16} N in +ve z.
10. If a charged particle enters into a uniform magnetic field perpendicular to its direction, then what will happen?
 (A) It moves in an elliptical path.
 (B) The radius of its path is proportional to the magnetic field strength.
 (C) The charged particle will not be affected by magnetic field.
 (D) The period of the motion is inversely related to the magnetic field strength.
11. What is the expression of the centripetal acceleration of a particle of charge q and mass m moving in a uniform magnetic field $\vec{B}$ on a circle of radius r if $\vec{B}$ is perpendicular to the particle's direction of motion?
 A) $\frac{qB}{m}$ B) $\frac{2qB}{mr}$ C) $\frac{q^2 B^2}{m^2} r$ D) $\frac{q^2 B^2}{m} r$

Motion of charged particles in electric and magnetic fields

Velocity Selector

There are useful devices those use a combination of electric and magnetic fields. An example is a velocity selector. A uniform electric field is directed vertically downward (in the plane of the page), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page). If q is positive and the velocity V is to the right, the magnetic force $qv \times B$ is upward and the electric force qE is downward. When the magnitudes of the two fields are chosen so that $qE = qvB$, the particle moves in a straight horizontal line through the region of the fields.

From the expression $qE = qvB$, we find that



Only those particles having speed V pass undeflected through the mutually perpendicular electric and magnetic fields.

The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward. Those moving at speeds less than this are deflected downward.

The Mass Spectrometer

A mass spectrometer separates ions according to their charge-to-mass ratio. A beam of ions first passes through a velocity selector and then enters a second uniform magnetic field. Upon entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector array at P .

If the ions are positively charged, the beam deflects upward, as in the figure below. If the ions are negatively charged, the beam deflects downward.

The centripetal force on the ion is

$$F_c = \frac{mV^2}{r}$$

The magnetic force on the ion is

$$F_B = qVB$$

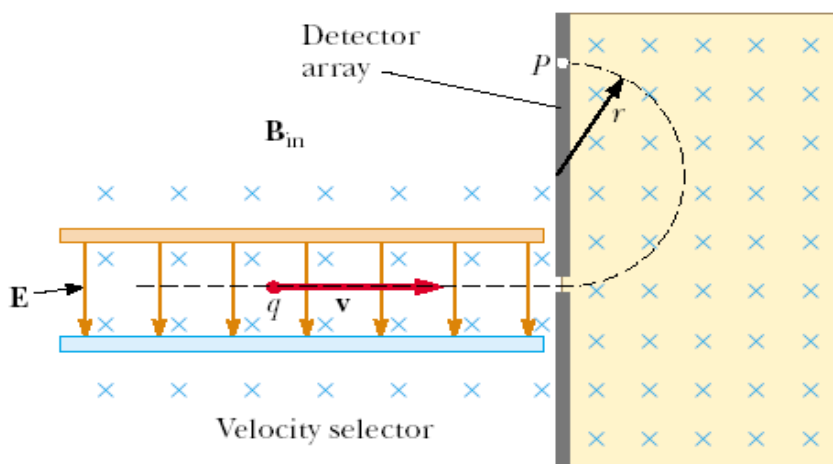
Equating F_c and F_B , we get

$$\frac{q}{m} = \frac{V}{rB}$$

Substituting $V = \frac{E}{B}$ into the

above equation, we get

$$\frac{q}{m} = \frac{E}{rB^2}$$



Therefore, we can determine $\frac{q}{m}$ by measuring the radius of curvature and knowing the field magnitudes B and E .

Example

1. Find the speed of an electron moving perpendicular to both electric and magnetic fields in a velocity selector operating with $E = 3 \times 10^3$ V/m and $B = 3$ T. **Ans.** $v = 1000$ m/s
2. An electron travels in a straight line through a crossed magnetic and electric fields in a velocity selector. If $B = 0.05$ T and $E = 2000$ N/C. Calculate the speed of the electron. **Ans.** $V = 4 \times 10^4$ m/s

Magnetic Force on a Current-Carrying Conductor

When a current carrying conductor is placed in a magnetic field, a magnetic force exerted on a conductor of length ℓ is

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

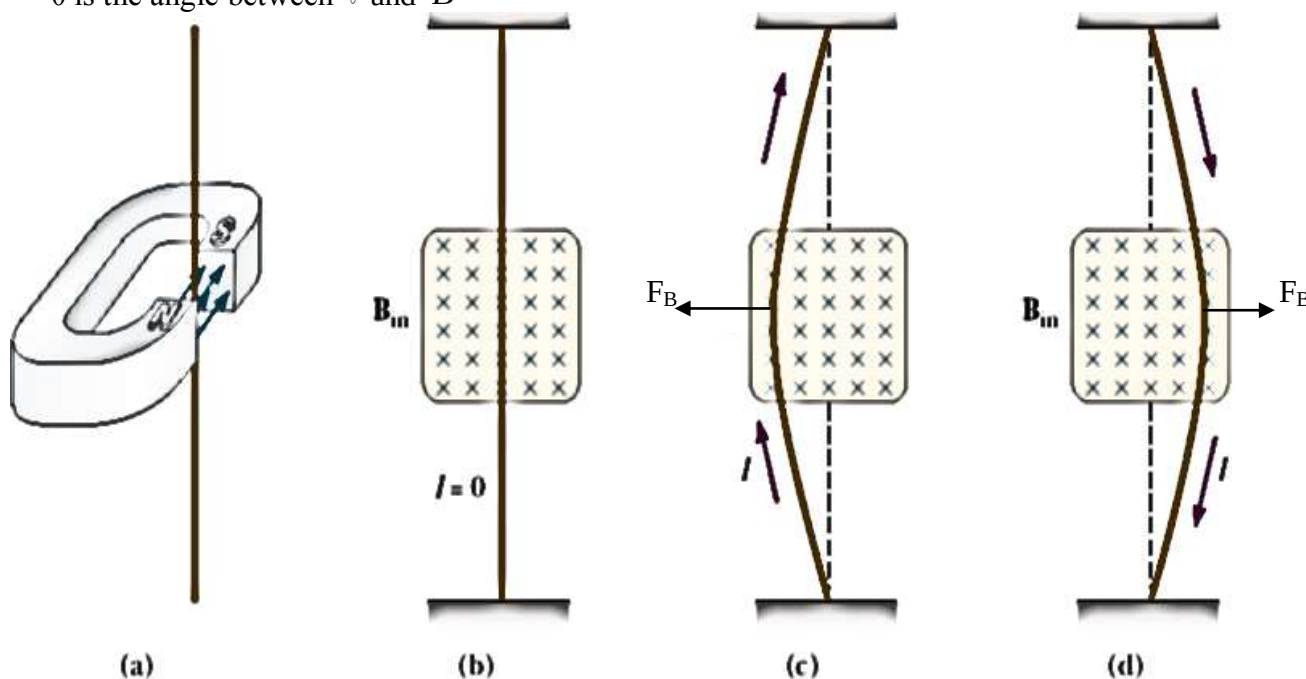
OR $F = I\ell B \sin \theta$

Where $\vec{\ell}$ is along the direction of current flow.

$\vec{B}$ is magnetic field

I is current in the conductor

θ is the angle between $\vec{\ell}$ and $\vec{B}$



(a) A wire suspended vertically between the poles of a magnet. (b) The setup shown in part (a) as seen looking at the south pole of the magnet, so that the magnetic field is directed into the page. When there is no current in the wire, it remains vertical. (c) When the current is upward, the wire deflects to the left. (d) When the current is downward, the wire deflects to the right.

Example

1. A wire of length 50 cm is placed in uniform magnetic field of 2 T. If a current of 10 A flows in the wire, that makes an angle of 53° with the magnetic field, what is the magnetic force on the wire?
Ans. $F=8\text{N}$
2. A 0.5 m long wire carrying a current of 10 A along positive y-axis is placed in a uniform magnetic field which has components $B_x = 0.3\text{ T}$, $B_y = -1.2\text{ T}$ and $B_z = 0.5\text{ T}$. What is the magnetic force on the wire? **Ans.** $\vec{F} = (2.5\hat{i} - 1.5\hat{k})\text{N}$
3. What is the direction of magnetic force, due to the Earth's magnetic field at the equator, on a wire carrying current
 - a) vertically downward? **Ans.** horizontally toward east.
 - b) vertically upward? **Ans.** horizontally toward west.
4. A wire with linear density of $5 \times 10^{-2}\text{ kg/m}$ is placed in a uniform magnetic field of 1 T north perpendicular to the direction of magnetic field. What should be the magnitude and direction of the current for the wire to be suspended against gravity by the magnetic force? **Ans.** $I = 0.5\text{ A}$ east
5. A straight, horizontal wire with $m/\ell = 40\text{ g/m}$, carries $I = 5\text{ A}$ east which is perpendicular to magnetic field. What is the magnetic field needed to float the wire to balance its weight? **Ans.** $\vec{B} = 0.08\text{ T}$ north

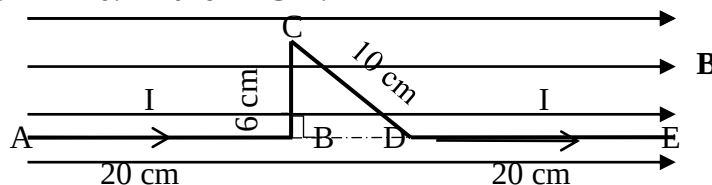
6. Find the force on each segment of the wire if $B = 0.2 \text{ T}$ and $I = 5 \text{ A}$.

Ans.

In sections AB and BC: $F = 0$

In section BC : $F = 0.06 \text{ N}$

In section CD : $F = 0.06 \text{ N}$

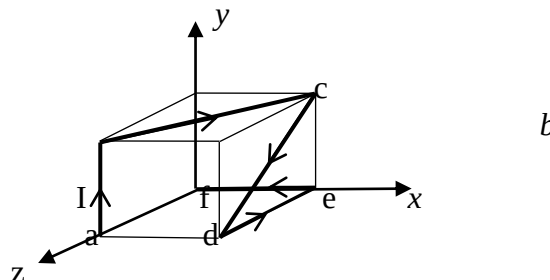


7. A cube, 0.5 m on a side, is in a uniform magnetic field of $\vec{B} = 0.6T\hat{i}$. The wire $abcdef$ carries a current of 4 A in the direction indicated. Determine the magnetic force on the segments ab , bc , cd , de and ef . **Ans.**

$$\mathbf{F}_{ab} = 1.2 \text{ N } (-k), \quad \mathbf{F}_{de} = 1.2 \text{ N } (-j)$$

$$\mathbf{F}_{bc} = 1.2 \text{ N } (-j), \quad \mathbf{F}_{ef} = 0$$

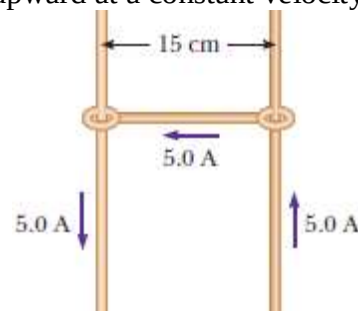
$$\mathbf{F}_{cd} = 1.2 \text{ N } (j + k)$$



Exercise

1. A 15-cm length of conductor that is free to move is held in place between two thin conductors. When a 5.0-A current is directed as shown in the figure, the wire segment moves upward at a constant velocity. If the mass of the wire is 15 g, find the magnitude and direction of the minimum magnetic field that is required to move the wire. (The wire slides without friction on the two vertical conductors.)

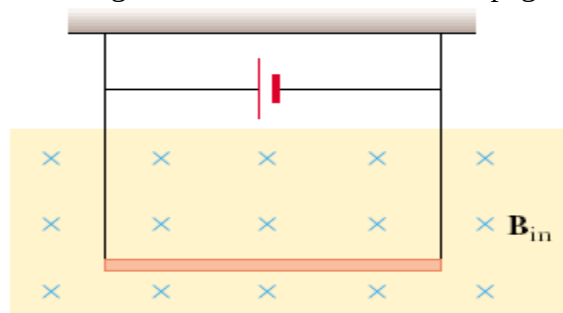
Ans. $B = 0.2 \text{ T}$ out of page



2. A conductor suspended by two flexible wires as shown in figure below has a mass per unit length of 0.040 kg/m . What is the magnitude and direction of current that must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.20 T into the page?

Ans. $I = 0.125 \text{ A}$

From left to right (counter clockwise)



3. Assume that in each case the velocity vector shown is replaced with a wire carrying a current in the direction of the velocity vector. For each case, find the direction of the magnetic field that will produce the magnetic force shown.



(a)



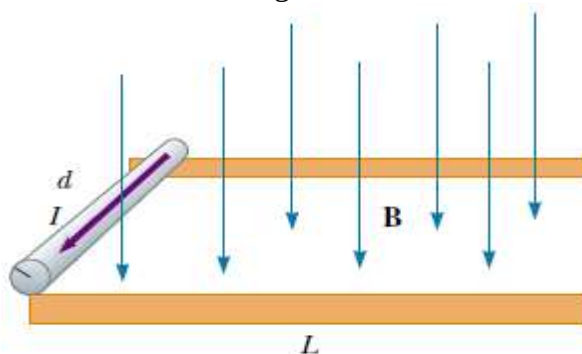
(b)



(c)

4. A rod with a mass of 0.75kg rests on two parallel rails (fig.bellow) that are 20cm apart and 64cm long. The rod carries a current of 30A in the direction shown and slides along the rails. If it starts from rest, what is the speed of the rod as it leaves the rails if a uniform magnetic field of magnitude 0.25T is directed perpendicular to the rod and the rails?

Ans. $V = 1.6\text{m/s}$



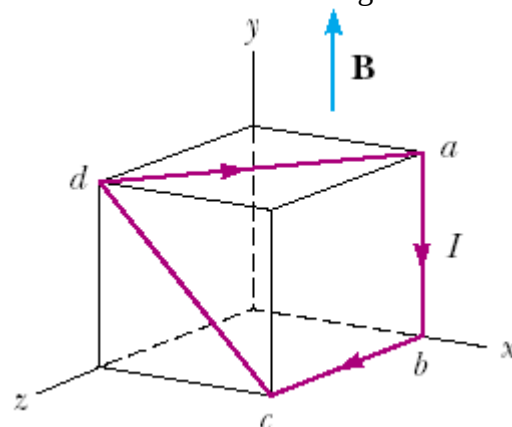
5. In the following figure, the cube is 40 cm on each edge. Four straight segments of wire ab , bc , cd , and da form a closed loop that carries a current $I = 5\text{ A}$, in the direction shown. A uniform magnetic field $B = 0.020\text{ T}$ is in the positive y direction. Determine the magnitude and direction of the magnetic force on each segment.

Ans. $\vec{F}_{ab} = 0$

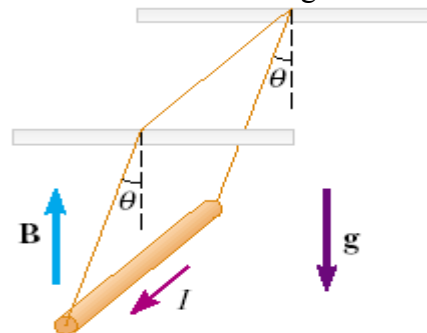
$$\vec{F}_{bc} = -0.04N\hat{i}$$

$$\vec{F}_{cd} = -0.04N\hat{k}$$

$$\vec{F}_{da} = (0.04\hat{i} + 0.04\hat{k})N$$

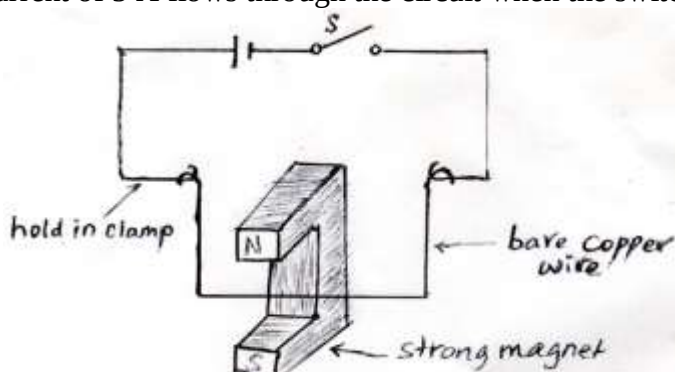


6. A metal rod having a mass per unit length λ carries a current I . The rod hangs from two vertical wires in a uniform vertical magnetic field as shown in the figure below. The wires make an angle θ with the vertical when in equilibrium. Determine the magnitude of the magnetic field. **Ans.** $B = \frac{\lambda g}{I} \tan \theta$



Questions from national exam

1. A 2m long wire carries a current of 5 A in a region where there is a uniform magnetic field of 1.5 T. What magnetic force is exerted on the wire if the direction of the current makes an angle of 37° with the direction of the magnetic field?
 A) 6N B) 7N C) 8N D) 9N
2. A long straight wire carrying a steady current of 5 A flowing in the positive z direction is placed in a magnetic field $\vec{B} = (0.4\hat{i} + 0.5\hat{j}) \text{ T}$. What is the magnetic force acting on 2 m section of the wire?
 A) $(4i-5j) \text{ N}$ B) $(-4i-5j) \text{ N}$ C) $(-5i+4j) \text{ N}$ D) $(5i+4j) \text{ N}$
3. A long straight wire carrying a steady current of 3 A flowing in a direction parallel to a unit vector $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ is placed in a magnetic field $\vec{B} = (0.75\hat{i} + 0.4\hat{k}) \text{ T}$. What is the magnetic force per unit length of the wire? **Ans.** $\frac{\vec{F}}{|\ell|} = (0.69\hat{i} + 0.61\hat{j} - 1.3\hat{k}) \text{ N/m}$.
 A) $(0.69i - 0.61j - 1.3k) \text{ N/m}$ C) $(0.49i - 0.51j - 1.37k) \text{ N/m}$
 B) $(0.39i - 0.41j - 2.3k) \text{ N/m}$ D) $(0.59i - 0.71j - 0.13k) \text{ N/m}$
4. A wire with linear mass density of 0.5 g/cm carries a 2 A current horizontally towards the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?
 A) 0.25 T towards west C) 0.50 T towards west
 B) 0.25 T towards east D) 0.50 T towards east
5. A wire carries a steady current of 2.4 A. A straight section of the wire is 0.75 m long and lies along the x axis within a uniform magnetic field, $\vec{B} = 1.6\text{T}$ along +z axis. If the current is in the +ve x direction, what is the magnetic force on the section of the wire?
 A) $2.88 \hat{j} \text{ N}$ B) $1.62 \hat{j} \text{ N}$ C) $-1.62 \hat{j} \text{ N}$ D) $-2.88 \hat{j} \text{ N}$
6. A wire carries a steady current of 2.4 A. A straight section of the wire is 0.75 m long and lies in xy-plane along the x axis within a uniform magnetic field, $\vec{B} = 1.6\text{T} \hat{k}$. If the current is in the positive x direction, what is the magnetic force on the section of the wire?
 A) 2.88 N, in the negative y direction. C) 3.84 N, in the negative y direction.
 B) 2.88 N, in the positive y direction. D) 3.84 N, in the positive y direction.
7. If a wire carrying current in the plane of this paper toward the top of the page experiences a magnetic force toward the right edge of the page, what will be the direction of the magnetic field causing this force?
 A) In the plane of the page and toward the left edge.
 B) In the plane of the page and toward the bottom edge.
 C) Perpendicular and directed out of the plane of the page.
 D) Perpendicular and directed into the plane of the page.
8. A current of 5 A flows through the circuit when the switch in the figure below is closed.



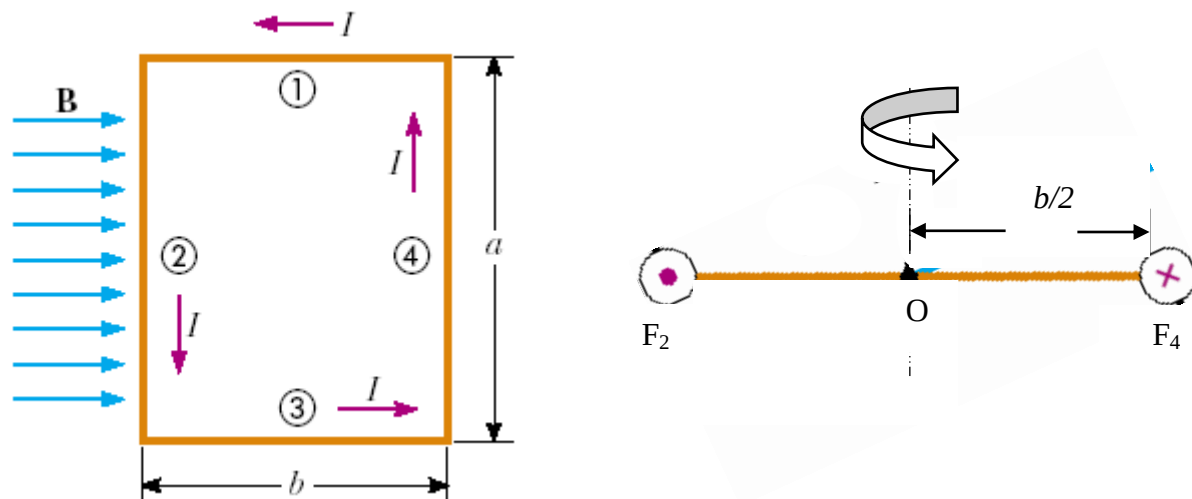
What will happen to the copper wire suspended on the clamp?

- A) deflected outward. C) lifted upward.
 B) deflected inward. D) experiences no force.

Torque on a current loop in a uniform magnetic field

In the preceding section, we showed how a magnetic force is exerted on a current carrying conductor placed in a magnetic field. With this as a starting point, we now show that a torque is exerted on a current loop placed in a magnetic field. The results of this analysis will be of great value when we discuss electric motors.

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field directed parallel to the plane of the loop, as shown in the next figure.



No magnetic forces act on sides ① and ③ because these wires are parallel to the field; hence,

$\vec{L} \times \vec{B} = 0$ for these sides. However, magnetic forces act on sides ② and ④ because these sides are oriented perpendicular to the field.

The magnitude of these forces on sides ② and ④ is

$$F_2 = F_4 = IaB$$

The direction of F_2 , on wire ②, is out of the page and that of F_4 , on wire ④ is into the page. If the loop is pivoted so that it can rotate about point O, these two forces produce a torque that rotates the loop clockwise.

The magnitude of this torque $\tau_{\max}$ is

$$\begin{aligned}\tau_{\max} &= F_2 \frac{b}{2} + F_4 \frac{b}{2} \\ &= IaB \frac{b}{2} + IaB \frac{b}{2} \\ &= IabB\end{aligned}$$

Where the moment arm about O is $b/2$ for each force. Because the area enclosed by the loop is $A = ab$, we can express the maximum torque as

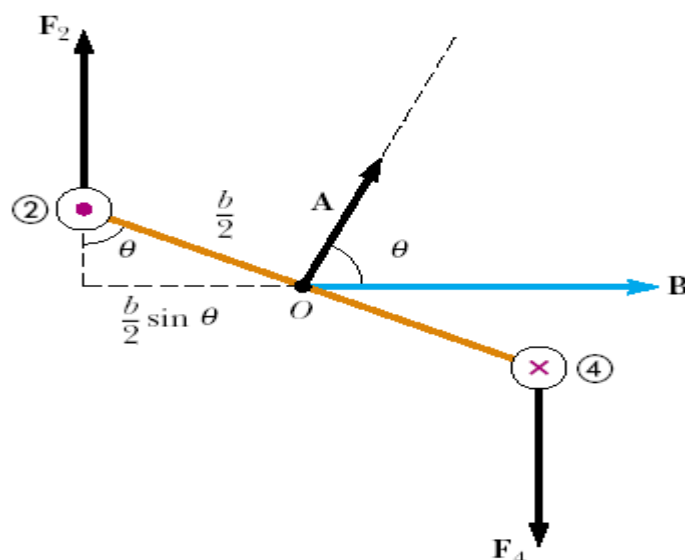
$$\tau_{\max} = IAB$$

The sense of the rotation is *clockwise* when *viewed from side ③*. If the current direction were reversed, the force directions would also reverse, and the rotational tendency would be counterclockwise.

Now suppose that the uniform magnetic field makes an angle $\theta < 90^\circ$ with a line perpendicular to the plane of the loop, as in the following figure, the moment arm of F_2 about the point O is equal to $(b/2) \sin \theta$. Likewise, the moment arm of F_4 about O is also $(b/2) \sin \theta$.

Because $F_2 = F_4 = IaB$, the magnitude of the net torque about O is

$$\begin{aligned}\tau &= F_2 \frac{b}{2} \sin \theta + F_4 \frac{b}{2} \sin \theta \\ &= IaB \frac{b}{2} \sin \theta + IaB \frac{b}{2} \sin \theta \\ &= IabB \sin \theta \\ &= IAB \sin \theta\end{aligned}$$



This result shows that the torque has its maximum value IAB when the field is perpendicular to the normal to the plane of the loop ($\theta = 90^\circ$), and is zero when the field is parallel to the normal to the plane of the loop ($\theta = 0$).

A convenient expression for the torque exerted on a loop placed in a uniform magnetic field $\mathbf{B}$ is

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Where $\vec{A}$, is a vector perpendicular to the plane of the loop and has a magnitude equal to the area of the loop. We determine the direction of $\vec{A}$ using the right-hand rule described in figure below.

The product $I\vec{A}$ is defined to be the **magnetic dipole moment** μ (often simply called the “magnetic moment”) of the loop:

$$\vec{\mu} = I\vec{A}$$

Its direction is determined using the right-hand rule.

“Curl the fingers of your right hand in the direction of the current, the extended thumb points in the direction of $\vec{\mu}$.”

The SI unit of magnetic dipole moment is ampere- meter² (A.m²).

The torque exerted on a current-carrying loop in a magnetic field $\mathbf{B}$ is

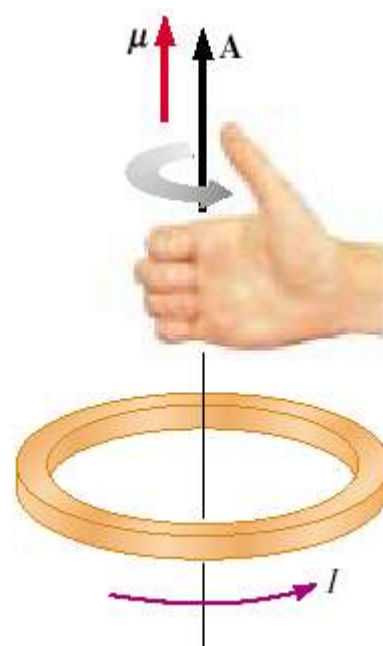
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Although we obtained the torque for a particular orientation of $\vec{B}$ with respect to the loop, the equation $\vec{\tau} = \vec{\mu} \times \vec{B}$ is valid for any orientation.

Furthermore, although we derived the torque expression for a rectangular loop, the result is valid for a loop of any shape.

If the loop has N turns :

$$\vec{\mu} = N I \vec{A}$$



When a current carrying loop is placed in a magnetic field, it is rotated so as to align its magnetic moment vector with the magnetic field vector.

Example

1. A plane of rectangular loop of wire, 5 cm by 8 cm, is parallel to a magnetic field of 0.15 T. If the loop carries a current of 10 A, what is
 - a) the magnetic moment of the loop? **Ans.** $\mu = 0.04 \text{ A}\cdot\text{m}^2$. b) the torque on the loop? **Ans.** $\tau = 6 \times 10^{-3} \text{ N}\cdot\text{m}$
2. A circular coil of wire 8 cm in diameter has 12 turns and carries a current of 5 A. The coil is in a magnetic field of 0.16 T.
 - a) What is the maximum torque on the coil? **Ans.** $\tau_{\max} = 0.048 \text{ Nm}$
 - b) In what direction would the torque be one-half as great as in (a)? **Ans.** $\theta = 30^\circ$
3. A square coil, 10 cm on a side, $N = 10$ turns and $I = 10 \text{ A}$, is placed in $B = 0.05 \text{ T}$. Calculate the torque if the angle between the normal and the direction of the field is
 - a) 0° . **Ans.** $\tau = 0$. c) 45° . **Ans.** $\tau = 3.54 \times 10^{-3} \text{ Nm}$. e) 90° . **Ans.** $\tau = 5 \times 10^{-3} \text{ Nm}$
 - b) 30° . **Ans.** $\tau = 2.5 \times 10^{-3} \text{ Nm}$. d) 60° . **Ans.** $4.33 \times 10^{-3} \text{ Nm}$
4. A rectangular coil of 15 cm by 12 cm has 25 turns. If a magnetic field of 1.2 T which is perpendicular to the magnetic dipole moment of the coil produces a torque of 5.4 Nm, what is the current in the coil? **Ans.** $I = 10 \text{ A}$
5. A circular loop, a square and a rectangular loop all have the same perimeter. Which one will have a maximum dipole moment when placed in a uniform magnetic field? **Ans.** $\mu_{\text{circle}} > \mu_{\text{square}} > \mu_{\text{rectangle}}$

Exercise

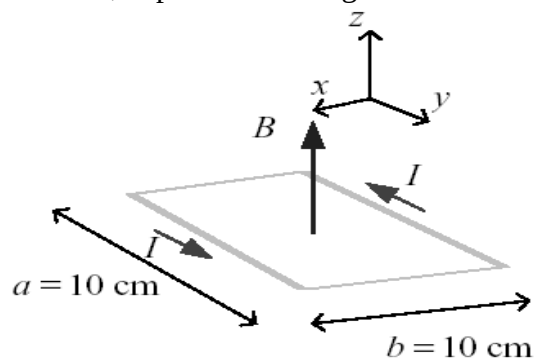
1. A square loop of dimensions 10 cm by 10 cm, that carries $I = 15 \text{ mA}$, is placed in a magnetic field $\vec{B} = 0.02 \text{ T } \hat{k}$ which is perpendicular to the plane of the loop. What is

a) the magnetic dipole moment?

Ans. $\vec{\mu} = 1.5 \times 10^{-4} \text{ A}\cdot\text{m}^2 \hat{k}$

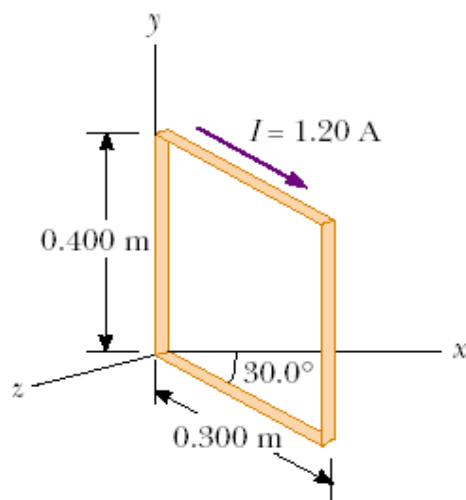
b) the magnetic torque on the loop?

Ans. $\tau = 0$



2. A rectangular coil, consists of a 100 closely wrapped turns, has dimensions $a = 0.40 \text{ m}$ and $b = 0.30 \text{ m}$. The coil is hinged along the y axis, and its plane makes an angle $\theta = 30^\circ$ with the x axis. What is the torque on the coil by a uniform magnetic field $B = 0.8 \text{ T}$ directed along the x axis when the current is $I = 1.20 \text{ A}$ in the direction shown?

Ans. $\tau = 9.98 \text{ N}\cdot\text{m}$ clockwise



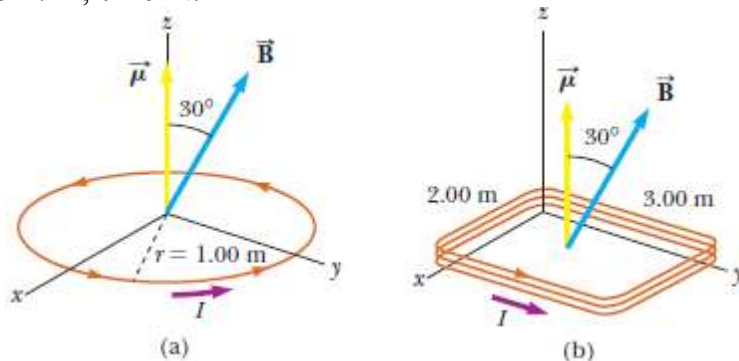
3. A circular wire loop of radius 1m is placed in a magnetic field of magnitude 0.5T. The normal to the plane of the loop makes an angle of 30° with the magnetic field (fig.a). The current in the loop is 2A in the direction shown.

a) Find the magnetic moment of the loop and the magnitude of the torque at this instant.

Ans. $\mu = 6.28 \text{ A}\cdot\text{m}^2$, $\tau = 1.57 \text{ N}\cdot\text{m}$

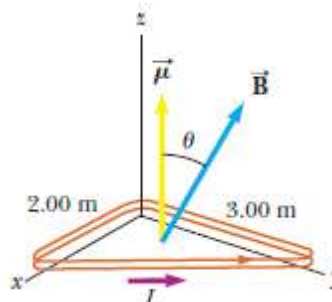
- b) The same current is carried by the rectangular 2m by 3m coil with three loops shown in fig.b. Find the magnetic moment of the coil and the magnitude of the torque acting on the coil at that instant.

Ans. $\mu = 36 \text{ A}\cdot\text{m}^2$, $\tau = 9 \text{ N}\cdot\text{m}$



4. Suppose a right triangular coil with base of 2m and height 3 m having two loops carries a current of 2A as shown in figure bellow. Find the magnetic moment and the torque on the coil. The magnetic field is again 0.5T and makes an angle of 30° with respect to the normal direction.

Ans. $\mu = 12 \text{ A}\cdot\text{m}^2$, $\tau = 3 \text{ N}\cdot\text{m}$



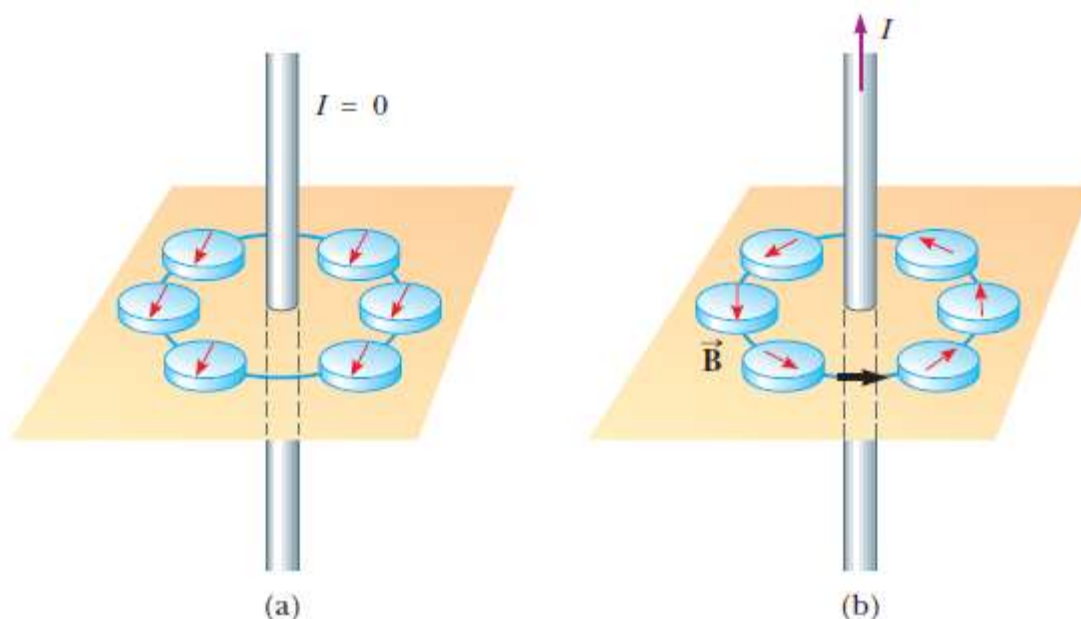
5. The torque experienced by a current carrying loop in an external magnetic field is:

- A) independent of the number of turns of the loop.
- B) independent of the current in the loop.
- C) dependent on the orientation of the loop.
- D) dependent on magnetic field generated according to Lenz's law.

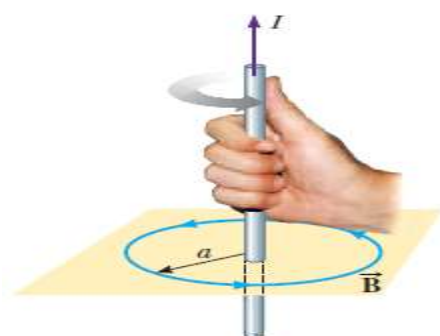
Magnetic field of a long straight conductor

During a lecture demonstration in 1819, the Danish scientist Hans Christian Oersted (1777–1851) found that an electric current in a wire deflected a nearby compass needle. This very important discovery, linking a magnetic field with an electric current for the first time, was the beginning of our understanding of the origin of magnetism. A simple experiment first carried out by Oersted in 1820 clearly demonstrates that a current-carrying conductor produces a magnetic field.

When there is no current in the wire, all needles point in the same direction (that of Earth's field), as one would expect as in figure (a). However, when the wire carries a strong, steady current, the needles all deflect in directions tangent to the circle, as in figure (b).



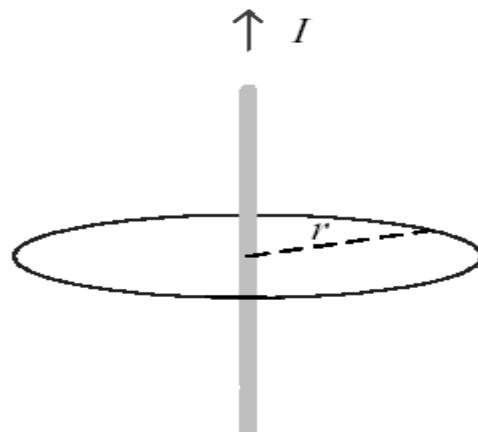
These observations show that the direction of $\vec{B}$ is consistent with the **right-hand rule**:
Point the thumb of your right hand in the direction of current, as in figure below. Your fingers then naturally curl in the direction of the magnetic field .



The magnetic field strength B at point P a perpendicular distance r from a wire carrying a current I is

$$B = \frac{\mu_0 I}{2\pi r}$$

Where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is a constant of proportionality.

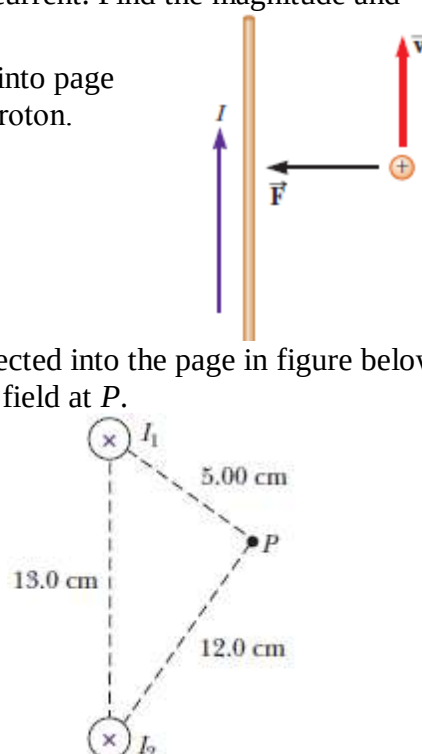


Example

1. A long conductor is carrying a current of 5 A. What is the magnitude of B at a perpendicular distance of 5 cm from the conductor? **Ans.** $B = 2 \times 10^{-5} \text{ T}$
2. At a perpendicular distance of 10 cm from a long conductor $B = 2 \times 10^{-5} \text{ T}$. What is the current in the conductor? **Ans.** $I = 10 \text{ A}$
3. The magnetic field at a distance of 0.4 m from a long straight wire is $2.5 \times 10^{-6} \text{ T}$. What is the magnetic field at a distance of 2 m from the same conductor? **Ans.** $B = 5 \times 10^{-7} \text{ T}$
4. By what factor will the magnetic field strength change if the distance from a long straight conductor carrying current I increases four times? **Ans.** $\frac{1}{4}$
5. A magnetic field of $5 \times 10^{-5} \text{ T}$ is produced at a distance of 5 cm from a long straight wire.
 - a) What current is required to produce this field? **Ans.** $I = 12.5 \text{ A}$
 - b) With the current found in (a), what is the magnetic field at
 - i) 10 cm from the wire? **Ans.** $B = 2.5 \times 10^{-5} \text{ T}$
 - ii) 20 cm from the wire? **Ans.** $B = 1.25 \times 10^{-5} \text{ T}$

Exercise

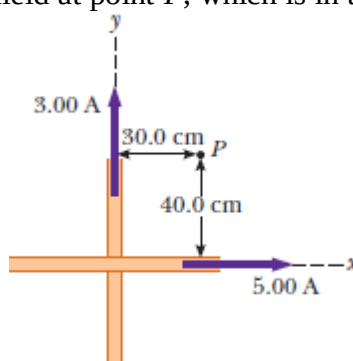
1. A long, straight wire carries a current of 5 A. At one instant, a proton, 4 mm from the wire, travels at $1.5 \times 10^3 \text{ m/s}$ parallel to the wire and in the same direction as the current. Find the magnitude and direction of
 - a) the magnetic field created by the wire. **Ans.** $B = 2.5 \times 10^{-4} \text{ T}$ into page
 - b) the magnetic force the wire's magnetic field exerts on the proton.
Ans. $F = 6 \times 10^{-20} \text{ N}$ toward left
2. Two long parallel conductors carry currents $I_1 = I_2 = 3 \text{ A}$, both directed into the page in figure below. Determine the magnitude and direction of the resultant magnetic field at P .



Ans. $B_{\text{resultant}} = 1.3 \times 10^{-5} \text{ T}$ toward the bottom of the page

3. Two long, straight wires cross each other at right angles, as shown in figure bellow. Find the magnitude and direction of the magnetic field at point P , which is in the same plane as the two wires.

Ans. $B_{\text{res}} = 5 \times 10^{-7}$ T out of page



4. Three long parallel conductors carry currents of $I = 2\text{A}$. The following figure is an end view of the conductors, with each current coming out of the page. Given that $a = 1.0\text{ cm}$, determine the magnitude and direction of the magnetic field at points A , B , and C .

Ans.

At A :

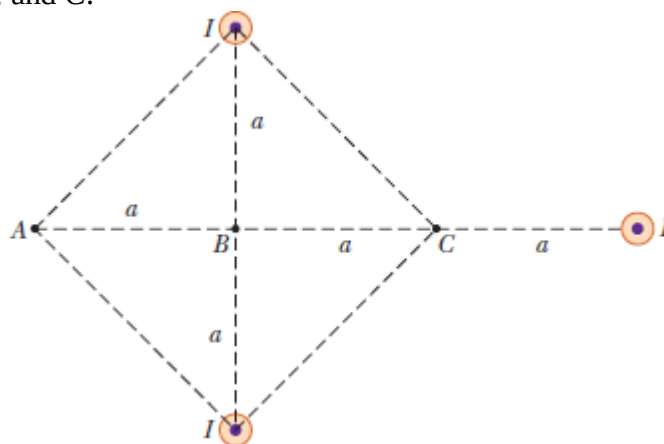
$B = 5.33 \times 10^{-5}\text{T}$ toward the bottom of the page

At B :

$B = 2 \times 10^{-5}\text{T}$ toward the bottom of the page

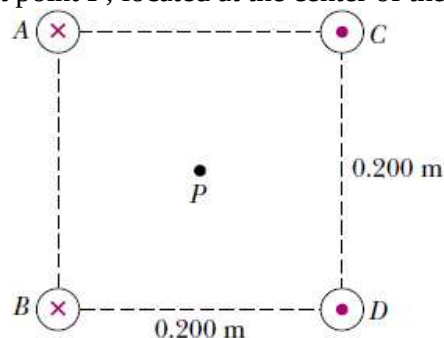
At C :

$B = 0$



5. Four long, parallel conductors carry equal currents of $I = 5\text{A}$. The figure bellow is an end view of the conductors. The direction of the current is into the page at points A and B and out of the page at C and D . Calculate the magnitude and direction of the magnetic field at point P , located at the center of the square with edge of length 0.2m .

Ans. $B = 2 \times 10^{-5}\text{T}$ toward the bottom of the page



Questions from national exam

- The magnetic field at a distance of 20 cm from a long straight wire is $5\mu\text{ T}$. What is the current in the wire?
A) 3A B) 4A C) 5A D) 6A
- If the magnitude of the current carried by a long straight copper wire is 10A , then what would be the magnitude of the magnetic field at a distance of 5cm from the copper wire?
A) 0.16 G B) 0.32 G C) 0.40 G D) 0.52 G
- Which one of the following statements is correct about the effect and formation of a magnetic field?
A) A magnetic field exerts a nonzero force on a stationary charged object.
B) A magnetic field exerts a nonzero force on a conducting wire placed in the field.
C) A uniform magnetic field can be created at all points around a long current carrying wire.
D) A uniform magnetic field can be created in the space surrounding any charged object that is in motion.

The Biot-Savart Law

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{S}$ of a wire carrying a steady current I

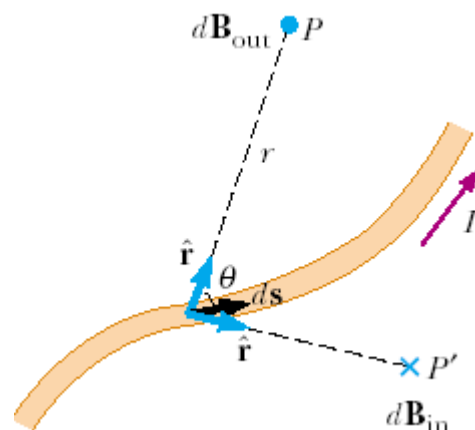
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \left(\frac{d\vec{S} \times \hat{r}}{r^2} \right)$$

OR
$$dB = \frac{\mu_0 I}{4\pi} \frac{dS \sin \theta}{r^2}$$

Where $\hat{r}$ is a unit vector pointing from the current element to point P .

θ is the angle between $d\vec{S}$ and $\hat{r}$.

$\mu_0 = 4\pi \times 10^{-7}$ T. m/A permeability constant.



The resultant magnetic field due to a current distribution is

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{S} \times \hat{r}}{r^2}$$

Where the line integral extends along the entire wire distribution.

The magnetic force between two parallel conductors

When a current carrying conductor is placed in a magnetic field, a magnetic force is exerted on the conductor. Because a current in a conductor sets up its own magnetic field, it is easy to understand that two current-carrying conductors exert magnetic forces on each other.

Consider two long, straight, parallel wires separated by a distance r and carrying currents I_1 and I_2 in the same direction, as in figure below.

The force exerted on wire 1 by wire 2 is:

$$F_1 = I_1 L B_2$$

$$F_1 = I_1 L \left(\frac{\mu_0 I_2}{2\pi r} \right)$$

$$F_1 = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

B_2 is magnetic field produced by conductor 2 due to I_2 .

The force exerted on wire 2 by wire 1 is:

$$F_2 = I_2 L B_1$$

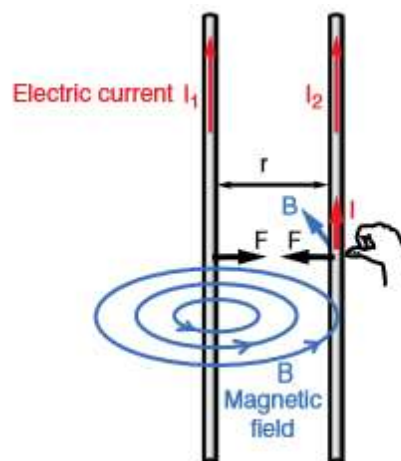
$$F_2 = I_2 L \left(\frac{\mu_0 I_1}{2\pi r} \right)$$

$$F_2 = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

B_1 is magnetic field produced by conductor 1 due to I_1 .

The force per unit length exerted on the wire is

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$



If the current in the wires are

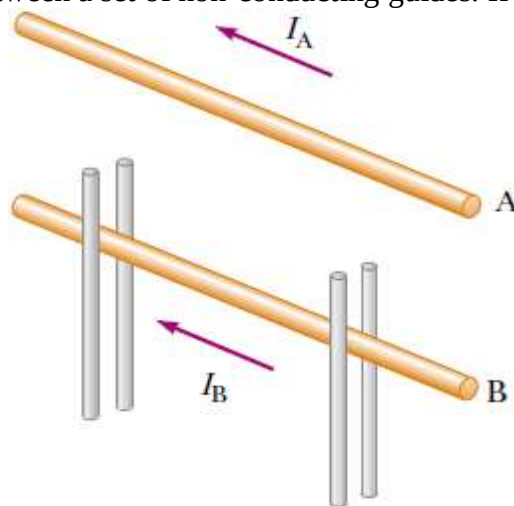
i) in **the same direction**, the force is **attractive**.

ii) in **opposite direction**, the force is **repulsive**.

Example

- Two parallel 50 cm long rods, 1.5 mm apart, carry currents of 15 A in opposite directions. What is the force between the wires? **Ans.** $F = 1.5 \times 10^{-2}$ N repulsive.
- Two parallel conductors are 10 cm apart with $I_1 = 5$ A and $I_2 = 10$ A in the same direction. Calculate the force per unit length between the conductors. **Ans.** $F/L = 10^{-4}$ N/m attractive
- Two long, parallel conductors carry currents in the same direction as shown in figure bellow. Conductor A carries a current of 100A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. If the mass per unit length of conductor B is 0.2g/cm, what value of current I_B will result in equilibrium when the distance between the two conductors is 2.50 cm?

Ans. $I_B = 250$ A

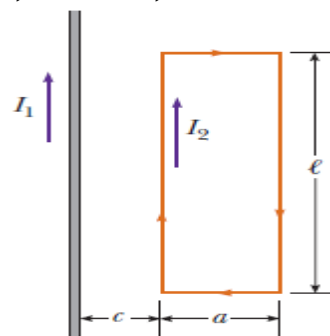


Exercise

- In the figure bellow, the current in the long, straight wire is $I_1 = 5$ A, and the wire lies in the plane of the rectangular loop, which carries $I_2 = 10$ A. The dimensions shown are $c = 0.1$ m, $a = 0.15$ m, and $\ell = 0.45$ m.

Find the magnitude and direction of the net force exerted by the magnetic field due to the straight wire on the loop.

Ans. $F_{\text{net}} = 2.7 \times 10^{-5}$ N toward the left

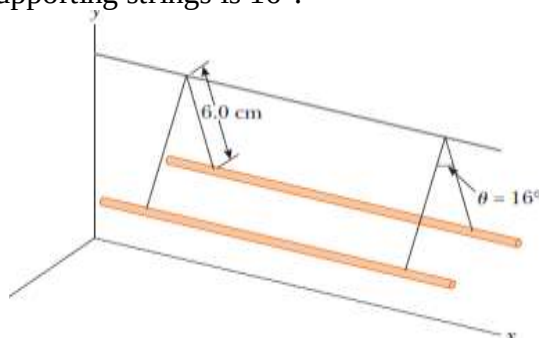


- A wire with a weight per unit length of 0.08 N/m is suspended directly above a second wire. The top wire carries a current of 30 A and the bottom wire carries a current of 60 A. Find the separation between the wires so that the top wire will be held in place by magnetic repulsion. **Ans.** 4.5 mm
- Two long parallel wires, each with a mass per unit length of 40 g/m, are supported in a horizontal plane by 6 cm long strings, as shown in figure bellow. Each wire carries the same current I , causing the wires to repel each other so that the angle between the supporting strings is 16° .

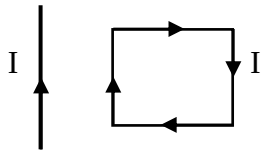
- Are the currents in the same or opposite directions?
- Determine the magnitude of each current.

($\sin 8^\circ = 0.14$, $\sin 16^\circ = 0.276$, $\tan 8^\circ = 0.14$, $\tan 16^\circ = 0.287$)

Ans. $I = 68.6$ A



Questions from national exam

- Two long parallel conductors, separated by a distance of 3 cm, carry current in opposite directions. If $I_1 = 6 \text{ A}$ and $I_2 = 5 \text{ A}$, what is the force per unit length exerted on each conductor by the other?
 A) $2 \times 10^{-4} \text{ N/m}$, repulsive. C) 10^{-4} N/m , attractive.
 B) $2 \times 10^{-4} \text{ N/m}$, attractive. D) 10^{-4} N/m , repulsive.
 - Two parallel copper wires are 1 cm apart. Lightning sends a 10,000 A pulse of current along each conductor. What is the force per unit length on one conductor?
 A) 1000 N/m B) 200 N/m C) 20 N/m D) 2000 N/m
 - A rectangular loop carrying a current is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and it is in a plane of the loop. If a steady current is situated in the wire as shown in figure, the loop will
 A) remain stationary.
 B) move toward the wire.
 C) move away from the wire.
 D) rotate about an axis parallel to the wire.
- 
- Two long wires of the same length and cross-sectional area carry the same amount of current. Which one of the following statements is correct about the magnetic field and the magnetic forces on the wires?
 A) The magnetic field is zero at the midpoint between the wires when the currents are in the same direction.
 B) The forces between the wires is attractive when they carry current in opposite direction.
 C) The magnetic fields between the wires reinforce each other when the currents are in the same direction.
 D) The magnetic field due to one is proportional to the current through the other wire.
 - Two parallel wires each carrying 5 A current in the same direction are separated by 2.5 cm. What is magnitude of the force per unit length acting on the wires?
 (A) $2 \times 10^{-4} \text{ N/m}$, attractive (C) $8 \times 10^{-4} \text{ N/m}$, repulsive
 (B) $4 \times 10^{-4} \text{ N/m}$, repulsive (D) $5 \times 10^{-4} \text{ N/m}$, attractive
 - Two long wires, 10 m in length, each carries a current of 2 A and 0.5 A in the same direction. The wires are separated by 4 cm. What is the magnetic force that the wires exerted on each other?
 A) $4 \times 10^{-6} \text{ N}$ B) $5 \times 10^{-6} \text{ N}$ C) $5 \times 10^{-7} \text{ N}$ D) $5 \times 10^{-5} \text{ N}$

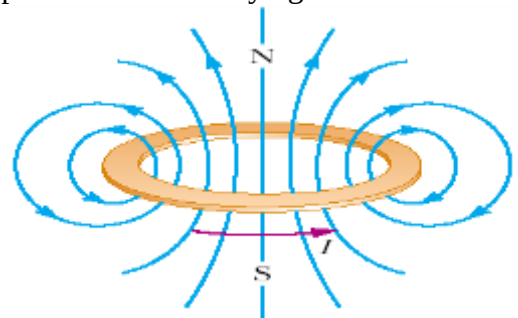
The magnetic field of a circular current loop

The magnetic field at the center of a single circular current loop with radius r carrying current I is

$$B = \frac{\mu_0 I}{2r}$$

For a coil of N number of turns

$$B = \frac{N\mu_0 I}{2r}$$



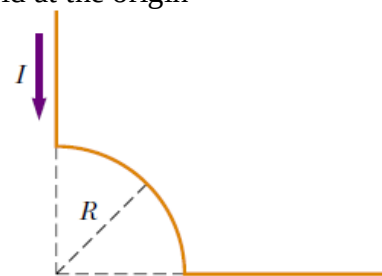
Example

- A conductor consists of a circular loop of radius $R = 0.1 \text{ m}$ and two straight, long sections, as shown in figure bellow. The wire lies in the plane of the paper and carries a current of $I = 7 \text{ A}$. Determine the magnitude and direction of the magnetic field at the center of the loop. **Ans. $B = 14\pi \times 10^{-6} \text{ T}$ into page**



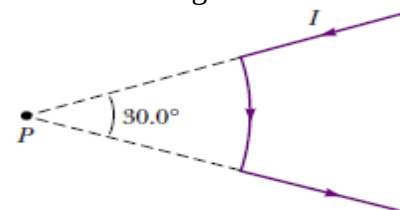
2. The segment of wire in figure bellow carries a current of $I = 5\text{A}$, where the radius of the circular arc is $R = 3\text{cm}$. Determine the magnitude and direction of the magnetic field at the origin

Ans. $\mathbf{B} = \frac{5}{6} \pi \times 10^{-5} \text{T}$ into page



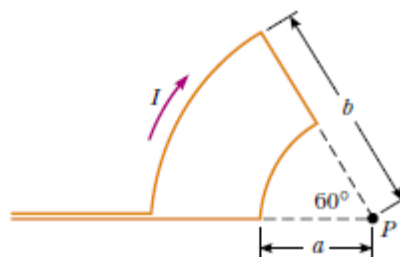
3. A current path shaped as shown in figure bellow produces a magnetic field at P , the center of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.4 m , what are the magnitude and direction of the field produced at P if the current is 3A ?

Ans. $\mathbf{B} = \frac{\pi}{8} \times 10^{-6} \text{T}$ into page



4. Consider the current-carrying loop shown in figure bellow, formed of radial lines and segments of circles whose centers are at point P . Find the magnitude and direction of $\mathbf{B}$ at P .

Ans. $\mathbf{B} = \frac{\mu_0 I}{12} \left(\frac{1}{a} - \frac{1}{b} \right)$ out of page



Ampere's Law

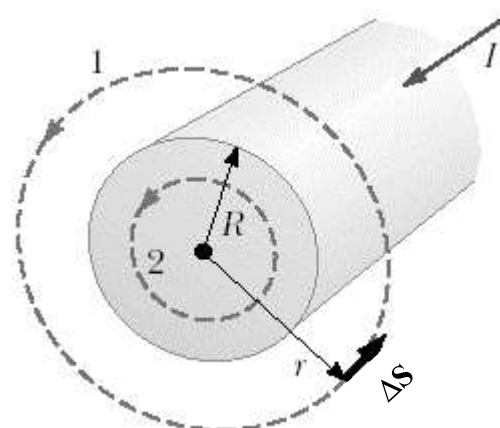
For a small length element $\Delta \vec{S}$ on the circular path, the vectors $\Delta \vec{S}$ and $\vec{B}$ are parallel at each point.

so $\vec{B} \cdot \Delta \vec{S} = B \Delta S$.

Furthermore, the magnitude of $\vec{B}$ is constant on this circle. Therefore, the circulation sum of the products $B \Delta S$ over the closed path is

$$\begin{aligned} \sum_c \vec{B} \cdot \Delta \vec{S} &= B \sum_c \Delta S \\ &= \frac{\mu_0 I}{2\pi r} (2\pi r) \\ &= \mu_0 I \end{aligned}$$

Where $\sum_c \Delta S = 2\pi r$ is the circumference of the circular path.



The general case, known as Ampère's law, can be stated as follows:

“The circulation sum of $\vec{B} \cdot \Delta \vec{S}$ around any closed path equals $\mu_0 I$, where, I is the total steady current passing through any surface bounded by the closed path.”

$$\sum_c \vec{B} \cdot \Delta \vec{S} = \mu_0 I \quad (1)$$

For a closed curve with radius r

$$\begin{aligned}\sum_C \vec{B} \cdot \Delta \vec{S} &= B \sum_C \Delta \vec{S} \\ &= B(2\pi r)\end{aligned}\quad (2)$$

Equate (1) and (2)

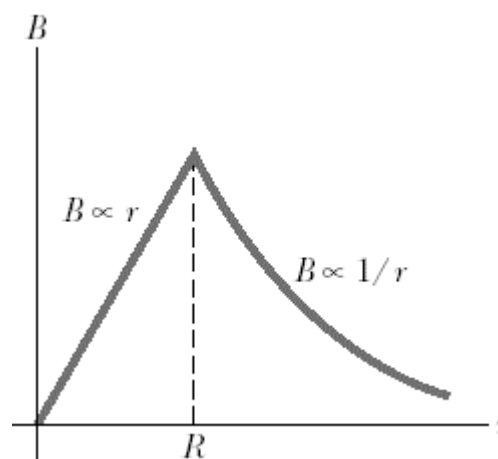
$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{For } r \geq R)$$

Now consider the interior of the wire, where $r < R$.

Here the current I' passes through the cross section of circle 2. Because the current is uniform over the cross section of the wire, the fraction of the current enclosed by circle 2 to total current I must equal the ratio of the area πr^2 to the cross-sectional area πR^2 of the wire:

$$\begin{aligned}\frac{I'}{I} &= \frac{\pi r^2}{\pi R^2} \\ I' &= \frac{r^2}{R^2} I\end{aligned}$$



Following the same procedure as for circle 1, we apply Ampère's law to circle 2:

$$B(2\pi r) = \mu_0 I'$$

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2} \right) I$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r$$

Note that this result is similar in form to the expression for the electric field inside a uniformly charged sphere.

Example

1. Calculate the magnetic field inside a conductor of radius 20 mm at a distance of 5 mm from the center of the conductor when a current of 3 A is passing through the conductor. **Ans.** $B = 7.5 \times 10^{-6}$ T.
2. What is the magnetic field inside a conductor of radius 10 mm at a distance of 3 mm from the center of the conductor when a current of 1.5 A is passing through the conductor? **Ans.** $B = 9 \times 10^{-6}$ T
3. A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross-section of the wire. What is the magnetic field a distance r from the center of the wire in the region $r > R$?

A) $\frac{\mu_0 I}{2\pi R^2} r$

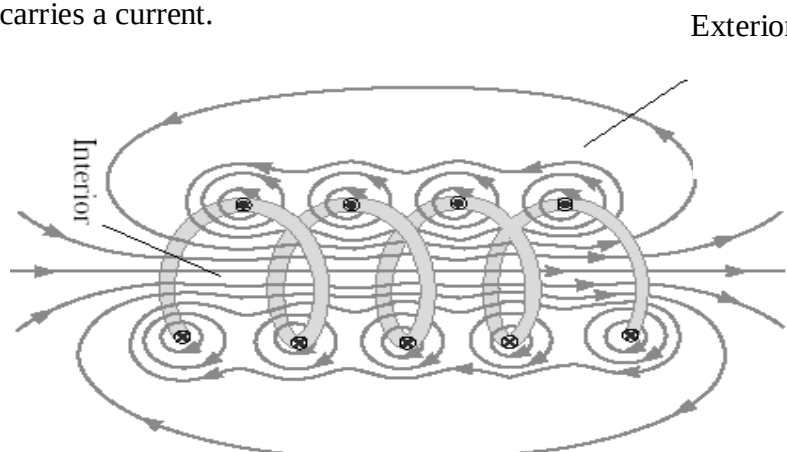
B) $\frac{\mu_0 I}{2\pi r^2} R$

C) $\frac{\mu_0 I}{\pi r}$

D) $\frac{\mu_0 I}{2\pi r}$

The magnetic field of a solenoid

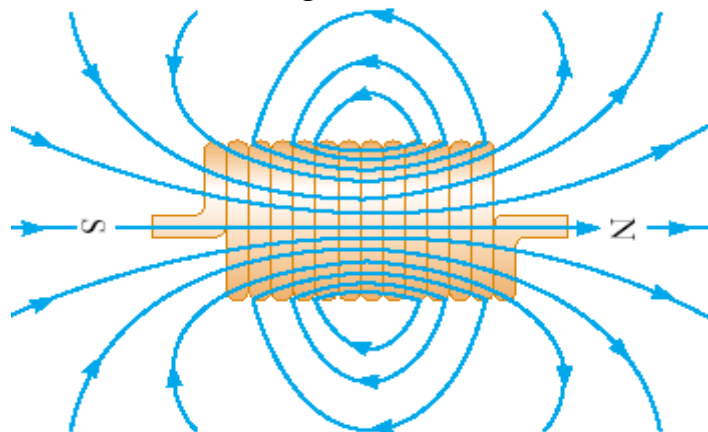
A **solenoid** is a long wire wound in the form of a helix. With this configuration, a uniform magnetic field can be produced when the solenoid carries a current.



The magnetic field lines for a loosely wound solenoid.

If the turns are closely spaced, the magnetic field lines are as shown in figure below.

This field line distribution is similar to that surrounding a bar magnet. Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the South Pole. As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker. In this case, the external field is close to zero, and the interior field is uniform over a great volume.



Magnetic field lines for a tightly wound solenoid.

We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in the solenoid.

Consider the rectangular path of length ℓ and width w . We can apply Ampère's law to this path by evaluating the circulation sum of $\vec{B} \cdot \Delta \vec{S}$ over each side of the rectangle.

- *The contribution along side 3 is zero because the magnetic field lines are perpendicular to the path in this region.
- *The contributions from sides 2 and 4 are both zero, again because $\vec{B}$ is perpendicular to $\Delta \vec{S}$ along these paths, both inside and outside the solenoid.
- *Side 1 gives a contribution to the circulation sum because along this path $\vec{B}$ is uniform and parallel to $\Delta \vec{S}$.

The circulation sum over the closed rectangular path is therefore

$$\begin{aligned}\sum_C \vec{B} \cdot \Delta \vec{S} &= \sum_{\text{side 1}} \vec{B} \cdot \Delta \vec{S} \\ &= B \sum_{\text{side 1}} \Delta S \\ &= B\ell\end{aligned}\quad (1)$$

If N is the number of turns in the length ℓ , the total current through the rectangle is NI .

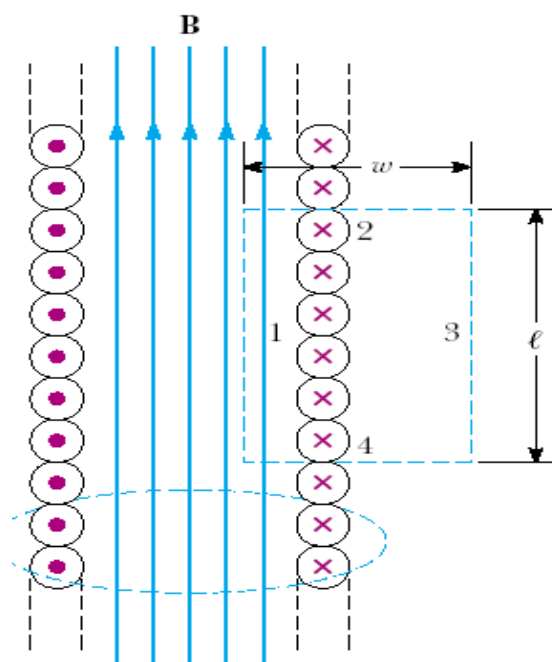
Therefore, Ampère's law applied to this path gives.

$$\sum_{\text{path 1}} \vec{B} \cdot \Delta \vec{S} = \mu_0 NI \quad (2)$$

Equating (1) and (2) gives

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI$$

Where $n = N/\ell$ number of turns per unit length



Cross-sectional view of a solenoid, where the interior magnetic field is uniform and the exterior field is close to zero. Ampère's law applied to the rectangular dashed path in the plane of the page can be used to calculate the magnitude of the interior field.

Example

1. Calculate the magnetic field at the center of a solenoid 50 cm long with 200 turns and a current of 2 A.

Ans. $B = 10^{-3} \text{ T}$

2. A solenoid is to be designed to produce a magnetic field of $0.1\pi \text{ T}$ at its center. Its length is to be 50 cm.

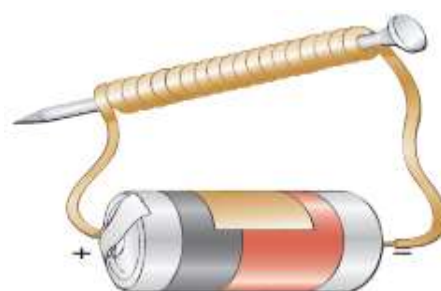
How many turns should this solenoid have? **Ans.** $N = 1.25 \times 10^4$ turns

3. A solenoid with 2000 turns is 50 cm long and carries 5 A of current through each turn. What is the magnetic field inside the solenoid? **Ans.** $B = 8\pi \times 10^{-3} \text{ T}$

4. If a solenoid of 0.4m long has 800 turns, what current is required to produce a magnetic field of $2 \times 10^{-3} \text{ T}$ at the center of the solenoid? **Ans.** $I = 2.5 \text{ A}$

Exercise: Construct an electromagnet by wrapping about 1 meter of small-diameter **insulated wire** around a steel nail as in figure bellow.

1. How many staples or paper clips can you pick up with your electromagnet?
2. How would you increase the magnetic field set up by the nail?
3. Disconnect the wire from the battery and test how much magnetism is retained by the nail by seeing

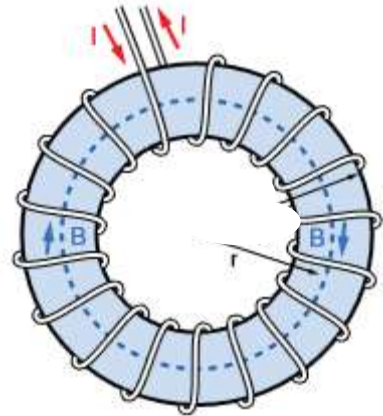


Magnetic field of a toroid

A device called a *toroid* has an interesting property that the magnetic field is confined entirely to the space enclosed by the windings; there is no field at all outside this region.

If there are N turns, the field magnitude at a point a distance R from the center is given by

$$B = \frac{\mu_0 NI}{2\pi r}$$



Calculate the magnetic field for a toroid of radius 4 cm with 200 turns carrying a current of 2 A.

Ans. $B = 2 \times 10^{-3} \text{ T}$