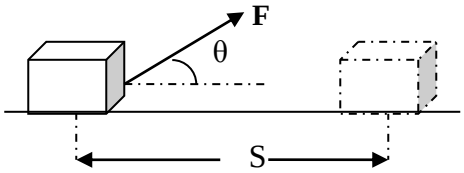
UNIT 5: WORK, ENERGY AND POWER

The *work done on a body* by a constant force **F** to displace it through a distance S is:

$W = \mathbf{F} \bullet \mathbf{S}$

OR $W = FS\cos\theta$ Where θ is the angle between **F** and **S**



In the presence of frictional force, the work done by

i) the force **F** is

$W_F = \mathbf{F} \bullet \mathbf{S}$

$= FS\cos\theta$

ii) the frictional force f is

$W_f = f\,S\cos 180^0$

$= -f\,S$

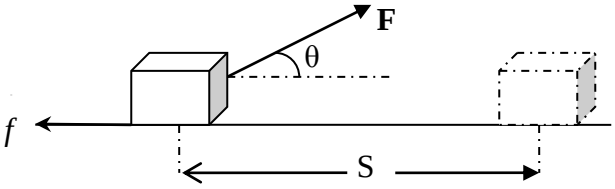
iii) the net force F_{net} is

$W_{net} = F_{net}\,S$

$= (F\cos\theta - f)\,S$

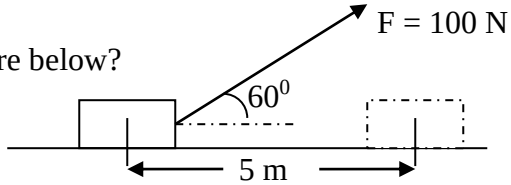
$= FS\cos\theta - f\,S$

$= W_F + W_f$



Example

1. What is the work done by the 100 N force in the figure below?



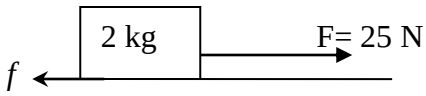
Given	Required	Soln
F = 100 N	W =?	$W = FS\cos\theta$
$\theta = 60^0$		$= 100\ N \times 5\ m \cos 60^0$
S = 5 m		$= 100\ N \times 5\ m \times 0.5$
		$= 250\ J$

2. A 2 kg block is pulled through 5 m along a level surface by a horizontal force of 25 N.

If $\mu_k = 0.4$, what is the

a) work done by the applied force? c) net work done?

b) work done by the frictional force?



Given	Required	Soln
m = 2 kg	a) W_F = ?	a) $W_F = FS$
$\mu_k = 0.4$	b) W_f = ?	$= 25\ N \times 5\ m$
S = 5 m	c) W_{net} = ?	$= 125\ J$

b) $W_f = -f\,S$

$= -F_N\,S$

$= -\mu\,mg\,S$

$= -0.4 \times 2\ kg \times 10\ m/s^2 \times 5\ m$

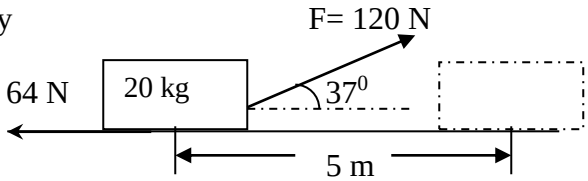
$= -40\ J$

c) $W_{net} = W_F + W_f$

$= 125\ J + (-40\ J)$

$W_{net} = 85\ J$

3. For the following figure, calculate the work done by
   a) the force F. **Ans**. $W_F = 480$ J
   b) the frictional force. **Ans**. $W_f = -320$ J
   c) the net force. **Ans**. $W_{net} = 160$ J



Given	Required	Soln
$F = 120$ N	a) $W_F = ?$	a) $W_F = FS\cos\theta$
$f = 64$ N	b) $W_f = ?$	$= 120\ \text{N} \times 5\ \text{m} \cos 37^0$
$S = 5$ m	c) $W_{net} = ?$	$= 120\ \text{N} \times 5\ \text{m} \times 0.8$
$m = 20$ kg		$= 480$ J
$\theta = 37^0$		b) $W_f = -F_f S$
		$= -64\ \text{N} \times 5\ \text{m}$
		$= -320$ J

c) $W_{net} = W_F + W_f$
$= 480\ \text{J} + (-320\ \text{J})$
$= 160$ J

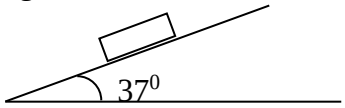
4. What is the coefficient of kinetic friction between the block and the surface for question 3 above?

Given	Required	Soln
$f = 64$ N	$\mu_k = ?$	$f = \mu_k F_N$
$m = 20$ kg		$f = \mu_k (mg - F\sin\theta)$

$$\mu_k = \frac{f}{mg - F\sin\theta}$$

$$= \frac{64}{20\times 10 - 120\cos 37^0} = \frac{64}{200 - 120\times 0.6} = 0.5$$

5. Calculate the net work done when a 10 kg block in the figure below slides down through 2 m. ($\mu_k = 0.5$)



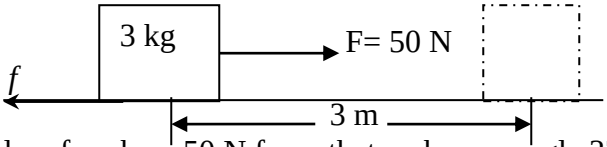
Given	Required	Soln
$m = 10$ kg	$W_{net} = ?$	$W_{net} = W_{grav} + W_f$
$\mu_k = 0.5$		$= (mg\sin\theta - \mu F_N) S$
$\theta = 37^0$		$= (mg\sin\theta - \mu mg\cos\theta) S$
		$= mg(\sin\theta - \mu\cos\theta) S$
		$= 10\ \text{kg} \times 10\ \text{m/s}^2 (\sin 37^0 - 0.5\cos 37^0) \times 2\ \text{m}$
		$= 40$ J

6. A force $\vec{F} = (7\hat{i} + 5\hat{j} - 2\hat{k})$ N displaces an object through $\vec{S} = (4\hat{i} - 4\hat{j} + \hat{k})$ m. What is the work done?

Given	Required	Soln
$\vec{F} = (7\hat{i} + 5\hat{j} - 2\hat{k})$ N	W=?	$W = \vec{F} \bullet \vec{S}$
$\vec{S} = (4\hat{i} - 4\hat{j} + \hat{k})$ m		$W = (7\hat{i} + 5\hat{j} - 2\hat{k})\ \text{N} \bullet (4\hat{i} - 4\hat{j} + \hat{k})\ \text{m}$
		$= (28 - 20 - 2)$ J
		$= 6$ J

Exercise

1. Calculate the net work done if $\mu_k = 0.5$
   Ans. $W_{net} = 105$ J

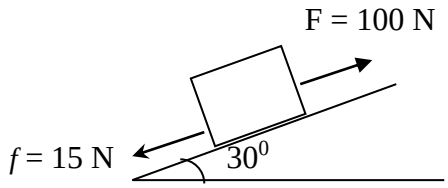


2. A 15 kg block is pulled along a horizontal surface by a 50 N force that makes an angle 37^0 with the horizontal. If $\mu_k = 0.1$, what is the net work done on the block as it moves a distance of 20 m?
   Ans. $W_{net} = 560$ J
3. What is the work done if?

a) a particle undergoes a displacement of $\vec{S} = (3\hat{i} + 4\hat{j})$ m when a force $\vec{F} = (12\hat{i} + 5\hat{j})$ N acts on it.
Ans. W = 56 J

b) box is dragged through $\vec{S} = (5\hat{i} + \hat{j})$ m by a force of $\vec{F} = (50\hat{i} + 25\hat{j})$ N. **Ans**. W= 275 J

4. Calculate the net work done on a 5 kg box as it moves 2 m up the plane.
Ans. W_{net} = 120 J



Questions from national exams

1. A man pushes a trolley loaded with goods with a force of 40 N a distance of 25 m. The mass of the loaded trolley is 75 kg. What is the net work done on the loaded trolley, if μ_k = 0.8? (g = 9.8 m/s^2)
Ans. No work done
A) 588 J B) 412 J C) 1000 J D) 668 J

2. A body is moved from initial position having a coordinate (0, 1, 1.5) m to a final position with a coordinate (1.2, 2, 1.5) m under the influence of a constant force given by $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})$ N. What is the work done by the force in moving the body from its initial position to its final position?
A) 8 J B) 0.8 J C) 4 J D) 0.4 J

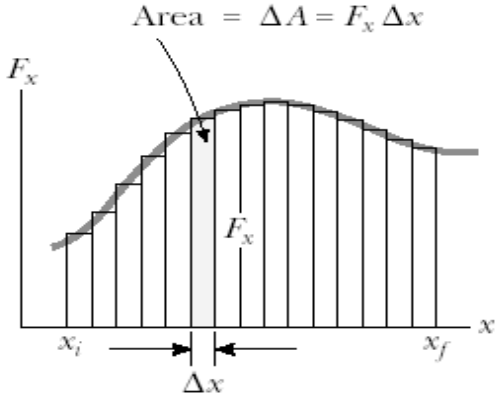
Work done by a variable force

When a force changes in magnitude or in its direction or in both, the force is known as ***variable force***. If an object is displaced from $x = x_i$ to $x = x_f$, we cannot use $W = F\cos\theta$ to calculate the work done by the force because this relationship applies only when F is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, as shown in the figure below, the x component F_x of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done by the force as

$$W \approx F_x \Delta x$$

This is just the ***area of the shaded rectangle*** in the figure. The total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W_{Tot} = \sum_{x_i}^{x_f} F_x \Delta x$$

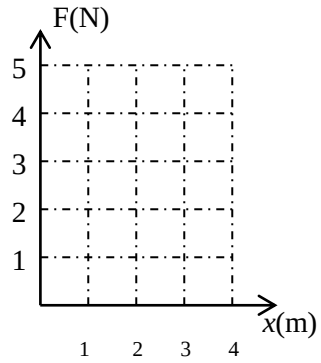


The *work done* by a *variable force that is increasing linearly* with x from x_i =0 to $x = x_f$ can also be obtained using *algebraic method*.

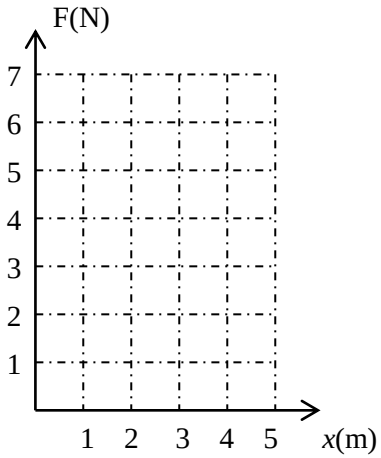
$W = F_{av}x$ OR $W = \left[\frac{F(0) + F(x)}{2}\right]x$

Example

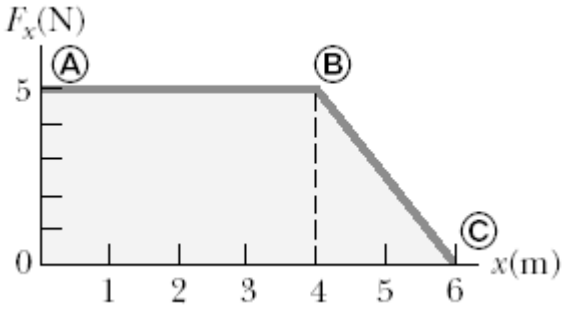
1. If a force varying with position as F= (x + 1) N acts on a body and displaces it by x = 4 m,
   a) draw a graph of F vs x.
   b) calculate the area under F. **Ans**. Area= 12 J
   c) calculate the average force between x= 0 and x= 4 m. **Ans**. F_{av}= 3 N
   d) calculate work done by the variable force between x=0 and x= 4 m



2. a) Plot the graph of F= (x + 2) N vs x between x = 0 and x = 5 m.
   b) Calculate the area between the origin and x = 5 m.
   Ans. Area= 22.5 J
   c) Calculate the average force between x = 0 and x = 5 m.
   Ans. F_{av} = 4.5 N
   d) Calculate the work done. **Ans**. W= 22.5 J



3. The force acting on a particle varies with x, as shown in the figure below. Calculate the work done by the force as the particle moves from x =0 to x = 6.0 m. **Ans**. 25 J



4. A body is moved from its initial position at the origin along the x-axis to a final position x = 5 m under the influence of a force given by $\vec{F} = (4x)\hat{i}$ N. What is the work done by the force in moving the body from its initial position to its final position?

   A) 100 J B) 50 J C) 25 J D) 20 J

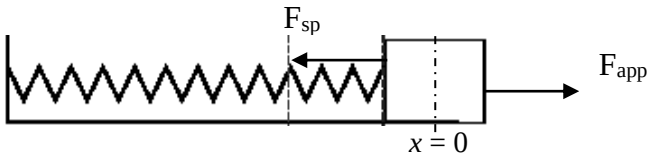
Work done by spring

When a force stretches a spring by x from its equilibrium position (x=0), the applied force F_{app} is given by

$F_{app} = kx$

The spring force F_{sp} is

$F_{sp} = -kx$



The negative sign indicates that F_{sp} is always directed *opposite* to the displacement from equilibrium.

The *work done by a variable force* that obeys Hook's law can be obtained using *algebraic method* or *graphically*.

i) Algebraic method

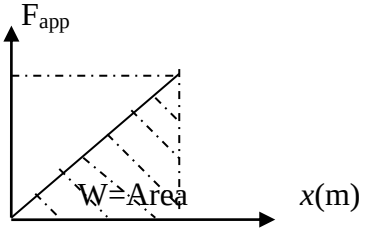
$W = F_{av}\,x$

$$W = \frac{F(0)+F(x)}{2}x$$

$$= \left(\frac{0+kx}{2}\right)x$$

$$W = \frac{1}{2}kx^2$$

ii) Graphical method



W = Area

$$= \frac{1}{2}\times(base)\times(height)$$

$$= \frac{1}{2}x(kx) = \frac{1}{2}kx^2$$

Example

1. A spring of force constant 200 N/m is compressed by 10 cm. What is work done?

Given	Required	Soln
K = 200 N/m	W=?	W = ½ K x^2
x = 10 cm		= 1/2×200×0.1×0.1
= 0.1 m		= 1 J

2. A 100 g mass is attached to one end of a spring of force constant 160 N/m. If the spring is compressed by 10 cm and released, what is

a) work done by external force? b) the maximum speed of the mass?

Given	Required	Soln
m = 100 g	a) W_F =?	a) W_F = ½ K x^2
= 0.1 kg	b) V_{max}= ?	= ½ ×160×0.1×0.1= 0.8 J
K= 160 N/m		½ mV^2_{max} = ½ Kx^2
x =10 cm		0.1V^2_{max} = 160×0. 1×0. 1
= 0.1 m		V_{max} = 4 m/s

3. A 20 N force is used to compress a spring of force constant 200 N/m. How much energy is stored?

Given	Required	Soln
F = 20 N	U =?	F = Kx and x = F/K = 20/200= 0.1 m
K = 200 N/m		U = ½ (Kx) x = ½ ×200×0.1×0.1= 1 J

Questions from national exams

1. A force of 100 N is required to stretch a spring that obeys Hook's law by 50 cm. What is the work done in stretching the spring?

A) 500 J B) 50 J C) 25 J D) 0 J

2. A spring has a force constant of 125 N/m. How much work is required to extend the spring 2 cm from its equilibrium position?

A) 0.5 J B) 0.05 J C) 0.25 J D) 0.025 J

Work-Energy Theorem

Energy is defined as *the ability to do work*.

Mechanical energies are classified as:

1. *kinetic* energy:- energy of a body *due to its motion.*
   - *i*) *translational* kinetic energy:- due to *linear motion.*
   - *ii*) *rotational* kinetic energy:- due to *rotational motion*.
2. *potential* energy:- energy of a body *due to its position* or *shape*.
   - *i*) *gravitational* potential energy:- due to its *position*.
   - *ii*) *elastic* potential energy :- due to its *shape*.

Work and kinetic energy

Consider a constant force F that acts on a body of mass m.

$W = F\,S$

$= maS$

$= m\left(\frac{V-u}{t}\right)\left(\frac{V+u}{2}\right)t$

$W = \frac{1}{2}mV^2 - \frac{1}{2}mu^2$

$W = KE_f - KE_i$ Work - kinetic energy theorem

Example

A 0.6 kg particle has a speed of 2 m/s at point A. The total work done on the particle as it moves from point A to B is 6.3 J. What is the speed of the particle at point B?

Given	Required	Soln
m = 0.6 kg	V_B = ?	V_A ●——————● V_B=?
V_A= 2 m/s		
W = 6.3 J		

$W = ½\ m\ V^2_B - 1/2m\ V^2_B$

$6.3 = 1/2 \times 0.6\ V^2_B - 1/2 \times 0.6 \times 2 \times 2$ ⟹ $6.3 = 0.3V^2_B - 1.2$

$6.3+1.2 = 0.3V^2_B$ ⟹ $V_B = 5$m/s

Exercise

1. A 2 kg block, initially at rest, is pushed by a horizontal force of 12 N through a distance of 3 m.
   What is : a) the final speed of the block? **Ans**. V= 6 m/s
   b) acceleration of the block ? **Ans**. a = 6 m/s^2
   c) the time taken? **Ans**. t= 1 sec
2. What is the final speed of the block in question 2 above if μ_k = 0.2? **Ans**. V = 4.9 m/s

3. A frictional force of 10 N slows a 3 kg body to half its initial velocity after moving a distance of 10 m. What is

a) the change in kinetic energy? **Ans**. $\Delta KE = -100$ J

b) the initial velocity? **Ans**. $u = \frac{20}{3}\sqrt{2}\ m/s$

c) the final velocity? **Ans**. $V = \frac{10}{3}\sqrt{2}\ m/s$

Questions from national exams

1. A 2 kg block is accelerated uniformly from rest to a speed of 12 m/s in 3 sec. What is the work done on the block during this time?

A) 36 J B) 90 J C) 72 J D**)** 144 J

2. A 1 kg ball is thrown with a speed of 6 m/s. How much work was done on the ball to give it that speed?

A) 3 J B) 6 J C) 12 J D**)** 18 J

3. A net force of 12 N acts on a 3 kg block initially at rest. What is the work done by the force during the third second?

A) 216 J B) 96 J C**)** 120 J D) 180 J

4. How much work is required to change the speed of 1000 kg car from 5 m/s to 8 m/s?

A) 195 J B) 1950 J C**)** 19500 J D) 390 J

5. A 500 kg car, working against a resistance of 500 N, accelerates from rest to 20 m/s in 100 m. What is the work done in this process and what is the final kinetic energy of the car?

A**)** 10^5 J, 10^5 J C) 5×10^4 J, 2×10^5 J

B) 1.5×10^5 J, 1.5×10^5 J D) 1.05×10^5 J, 8.5×10^4 J

6. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. What is the original speed of the man in m/s?

A) $\frac{1}{\sqrt{2}+1}$ B**)** $\frac{1}{\sqrt{2}-1}$ C) $\frac{1}{\sqrt{2}+2}$ D) $\frac{1}{\sqrt{2}-2}$

7. The speed of a mass is increased by two fold. By what factor does the kinetic energy change?

A) 2 B**)** 4 C) 8 D) 16

8. A force acts on a 3 kg particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2$. What is the work done by the force during the first 4 seconds?

A) 1218 J B) 1228 J C**)** 1248 J D) 1258 J

9. A car of mass 1500 kg is travelling at a uniform velocity of 30 m/s due east. The driver applies the brakes to slow down the car to a velocity of 10 m/s due east. What is the work done in slowing down the car?

A) 15 kJ B) 300 kJ C**)** 600 kJ D) 750 kJ

10. A block of mass 10 kg is pulled up a rough inclined plane with a force of 100 N parallel to the inclined plane. The inclined plane makes an angle of 37^0 with the horizontal. At the instant the force is applied the block has a speed of 1.5 m/s. If the coefficient of kinetic friction is 0.40, and the block is pulled up 5 m, what is the change in the kinetic energy of the block?

A) 20 J B) 30 J C**)** 40 J D) 45.5 J

Work and potential energy

Work and *gravitational potential energy*

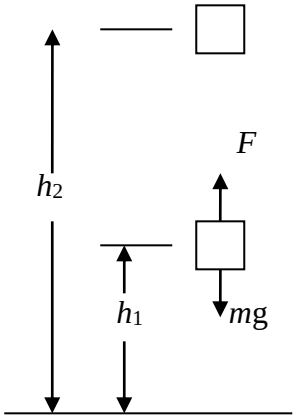
• When an object of mass m is lifted by a force F at constant speed through a displacement of $h_2 - h_1$, the gravitational force does work, W, on the object.

$W_{grav} = -mg(h_2 - h_1)$ because mg and displacement are in opposite direction.

$= -(mgh_2 - mgh_1)$

$= -(U2 - U_1)$

$= -\Delta U$



• Consider the upward force F that lifts the object upward with a constant speed.

The work done by this force is:

$W_F = mg(h_2 - h_1)$ Because F and $(h_2 - h_1)$ are in the same direction.

$= mgh_2 - mgh_1$

$= U_2 - U_1$

$W_F = \Delta U$ Work – potential energy theorem

Example

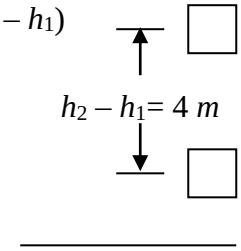
1. How much work is done by gravity in lifting a 3 kg object with a constant velocity through a vertical displacement of 2 m? Take g = 10 m/s^2

Given	Required	Soln
m = 3 kg	W_F =?	$W = -mg(h_2 - h_1)$
$h_2 - h_1$= 2 m		$= -3\ kg \times 10\ m/s^2 \times 2\ m$
g = 10 m/s^2		$= -60$ J

2. A 2 kg mass falls through 4 m.

a) How much work is done by gravitational force? b) What is the change in potential energy?

Given	Required	Soln
m = 2 kg	a) W_{grav} =?	a) $W_{grav} = mg(h_2 - h_1)$ because mg and $(h_2 - h_1)$
$(h_2 - h_1)$ = 4 m	b) ΔU=?	are in the same direction
g = 10 m/s^2		$= 2\ kg \times 10\ m/s^2 \times 4\ m = 80$ J
		b) $\Delta U = -mg(h_2 - h_1)$
		$= -2\ kg \times 10\ m/s^2 \times 4\ m = -80$ J



Conservation of mechanical energy

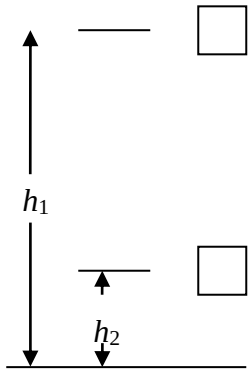
When a mass m is falling from a height h_1 to h_2, the work done by the gravitational force is

$W_{grav} = mg\,(h_1 - h_2)$

$W_{grav} = mgh_1 - mgh_2$ . . . . . (1)

From the work – kinetic energy theorem

$$W = \frac{1}{2}mV^2 - \frac{1}{2}mu^2 \quad \ldots\ldots (2)$$



Equating (1) and (2) gives

$$\frac{1}{2}mV^2 - \frac{1}{2}mu^2 = mgh_1 - mgh_2$$

$$KE_f - KE_i = PE_f - PE_i$$

$$\frac{1}{2}mV^2 + mgh_2 = \frac{1}{2}mu^2 + mgh_1$$

$$KE_f + PE_f = KE_i + PE_i$$

$$ME_f = ME_i$$

$$KE_f - KE_i = PE_f - PE_i$$

OR $\Delta KE = \Delta PE$

Where ME_f is final mechanical energy

ME_i is initial mechanical energy

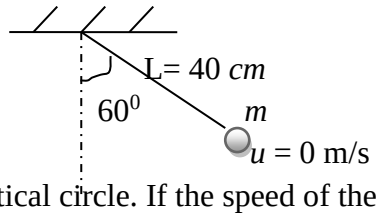
Example

1. A ball is dropped from a height of 12 m above the ground. Calculate the speed of the ball

a) when it is at a height of 7 m above the ground. **Ans**. V= 10 m/s

b) at 7 m above the ground if it is thrown down $u = 2\sqrt{11}\ m/s$. **Ans**. V= 12 m/s

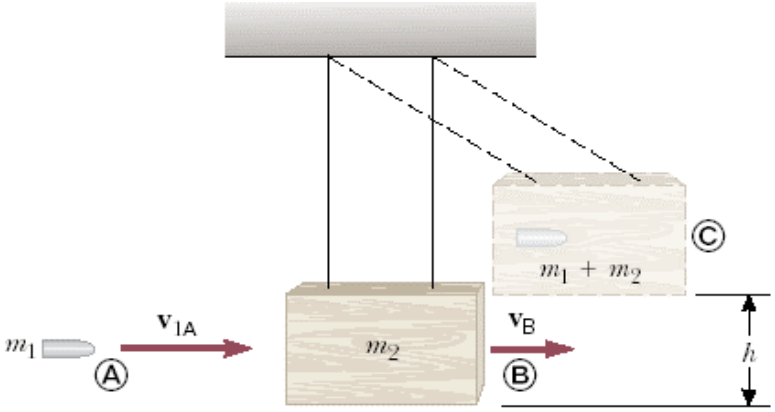
2. What is the speed of the mass m in the figure when it is at its lowest point? **Ans**. V= 2 m/s



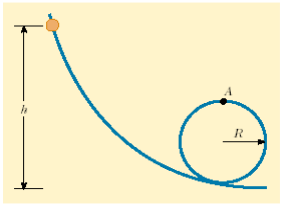
Exercise

1. A boy swings a ball attached to the end of a 0.5 m long string in a vertical circle. If the speed of the ball at the highest point is 4 m/s, what is the tangential speed at the lowest point? **Ans**. V= 6 m/s

2. A bullet of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. Show that the initial speed of the bullet is given by $v_{1A} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$



3. A bead slides without friction around a loop- the - loop. Find the minimum height *h*, in terms of R for the block to be in contact with the track at A.

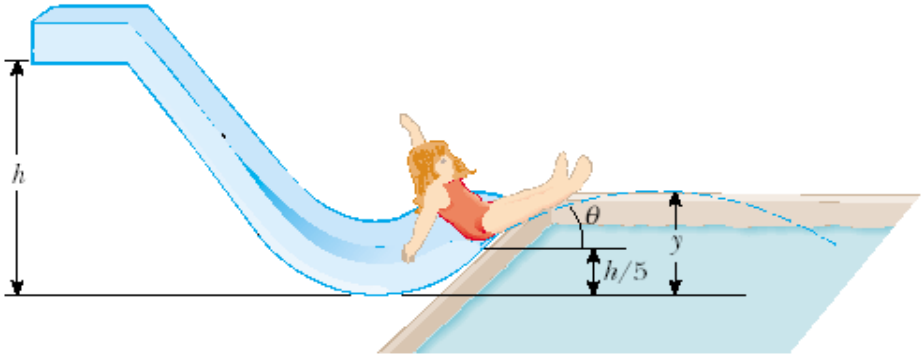


Hint: For the bead to be in contact with the track at A,

$$\frac{mV_A^2}{R} = mg$$

Ans. $h = 2.5R$

4. A child slides without friction from a height *h* along a curved water slide as shown in the figure below. She is launched from a height *h*/5 into the pool. Show that her maximum height *y* is given by. **Ans**. $y = 1/5\ h\ (1 + 4\sin^2\theta)$



Questions from national exams

1. A ball rolls down an inclined plane of length L starting from rest at the top. Where along the plane would it have half the velocity it attains at the bottom?

A) At a distance L/2 from the top. C**)** At a distance L/4 from the top.
B) At a distance L/3 from the top. D) At a distance 3L/4 from the top.

2. A girl swings a ball attached to the end of a string in a vertical circle of radius 1 m. If the speed of the ball at the highest point is 6 m/s, what is the magnitude of the acceleration of the ball at the lowest point?

A) 18 m/s^2 B) 27 m/s^2 C) 36 m/s^2 D**)** 76 m/s^2

3. A bullet of mass m_1 is fired into a wooden block of mass m_2 suspended from some light string. The bullet remains embedded into the block and the entire system swings through a height *h*. Initial velocity of the bullet, expressed in terms of the given masses and height is

A) $\frac{m_1}{m_1+m_2}\sqrt{2gh}$ B) $\frac{m_2}{m_1+m_2}\sqrt{2gh}$ C**)** $\frac{(m_1+m_2)}{m_1}\sqrt{2gh}$ D) $\frac{(m_1+m_2)}{m_2}\sqrt{2gh}$

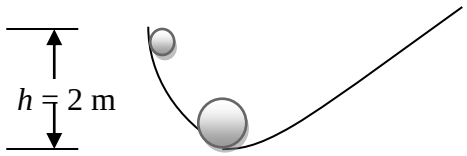
4. Abebe swings on a rope which is 10 m long initially inclined at an angle of 60^0 to the vertical. If his friend pushes Abebe with a speed of 10m/s, Abebe's speed at the bottom of the swing is about, (g= 9.8 m/s^2)

A) 20.05 m/s B**)** 14.07 m/s C) 18.06 m/s D) 12.03 m/s

5. A ball of mass 1 kg slides down a frictionless curved surface as shown in the figure below. It starts from rest at a height of 2 m. At the lowest point of the curve, the ball collides elastically with another ball which is at rest and is twice its mass. The height the heavier ball moves up the curve before it comes to stop is about

A) 1.28 m C) 0.89 m

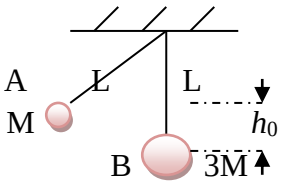
B) 0.59 m D) 0.68 m



6. Two spherical masses, A and B, of mass M and 3M respectively hang from the ceiling on strings of length L. Sphere A is drawn aside so that is raised to a height shown in the figure below, and then released. Sphere A collides with sphere B; they stick together and swing to a maximum height *h*. What would be the value of this height?

A) $h_0/16$ C) $h_0/4$

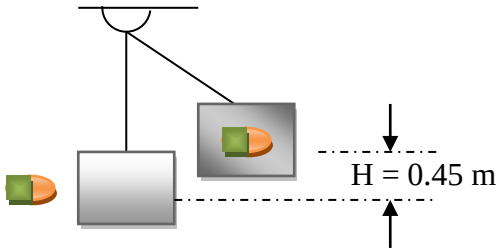
B) $h_0/8$ D) $h_0/2$



7. A bullet of mass 10 g, moving horizontally strikes and embeds itself in a box of 1 kg suspended from a light string as shown in the figure below. If the composite mass rises to a height, H= 0.45 m, then what is the speed of the bullet before collision?

A) 303 m/s C) 90.9 m/s

B) 300 m/s D) 90 m/s



Elastic Potential Energy

When a spring with force constant K is compressed or stretched by a distance *x* from its equilibrium position:

The work done by external force is

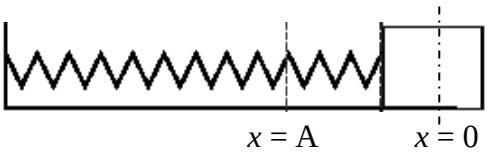
$$W = \tfrac{1}{2} K x^2$$

The work done by the spring force is

$$W = -\tfrac{1}{2} K x^2$$

The elastic potential energy is

$$U = \tfrac{1}{2} K x^2$$



If the mass has velocity v_1 at position x_1 and velocity v_2 at position x_2, the total mechanical energy, ME, of the spring-mass system is expressed as

$$ME_1 = ME_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}Kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}Kx_2^2$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2 = 0 \quad \Longrightarrow \quad \Delta KE + \Delta PE = 0$$

In the absence of retarding force like friction, total mechanical energy of the system is conserved.

It follows that at the maximum displacement x = A, v= 0

ME = ½ KA^2

Similarly, at the equilibrium position x = 0, the speed is maximum v_{max} .

$$ME = \frac{1}{2}mv_{max}^2$$

From this we see that **maximum potential energy at the amplitude position is equal to the maximum kinetic energy at the equilibrium position**.

½ KA^2 = ½ m v_{max}^2

Example

1. A 1 kg mass is attached to a spring of K = 400 N/m and oscillates with an amplitude of 10 *cm*. What is the maximum speed of the mass?

Given	Required	Soln
m = 1 kg K= 400 N/m A = 10 *cm* = 0.1 *m*	v_{max} =?	$\frac{1}{2}mv_{max}^2 = \frac{1}{2}KA^2$ OR $v_{max} = A\sqrt{\frac{K}{m}}$ $v_{max} = 0.1\times\sqrt{\frac{400}{1}}$ Or v_{max} = 2 m/sec

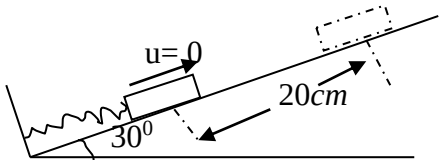
2. A 2 kg block moving at 12 m/s strikes a spring of K = 160 N/m. What will be the speed of the block at the instant the spring is compressed by 1 m?

Given	Required	Soln
m = 2 kg v_{max} = 12 m/s K = 160 N/m X = 1 m	v =?	½ mv^2 + ½ Kx^2 = ½ mv^2_{max} OR v^2 = ($m\,v^2_{max}$ – KA^2)/m $v = \sqrt{\frac{mv_{max}^2 - KA^2}{m}} = \sqrt{\frac{2\times12^2 - 160\times1^2}{2}}$ v = 8 m/sec

Exercise

1. A 0.25 kg object vibrates with an amplitude of 1.6 m when connected to a spring of spring constant 25 N/m. What is the maximum speed of the object? **Ans**. V_{max} = 16 m/sec
2. A 10 kg block moving at 10 m/sec is stopped by a spring of force constant 10^5 N/m. What is compression of the spring? **Ans**. 0.1 m
3. A 2 kg box is pushed against a spring of K = 300 N/m so that the spring is compressed by 20 *cm*. The box is then released and moves up the plane. Calculate the velocity of the box at the instant the spring reaches its equilibrium position?

Ans. V_{max} = 2 m/sec



Questions from national exams

1. A 0.5 kg mass attached to a spring of force constant 8 N/m oscillates with an amplitude of 10 *cm*. What is the maximum speed of the mass?

   A) 0.3 m/s B**)** 0.4 m/s C) 0.5 m/s D) 0.6 m/s

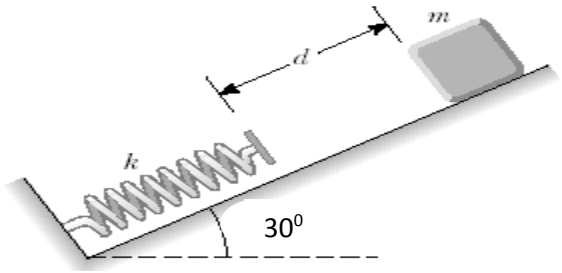
2. A 1 kg mass is attached to a spring of stiffness constant 100 N/m. What is the maximum displacement of the mass from its equilibrium position if its speed is 3 m/s at the equilibrium point?

   A) 0.1 m B**)** 0.3 m C) 3 m D) 10 m

3. The spring in the figure is compressed 1 m by a force of 100 N. A 10 kg mass is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring 2 m. Through what distance does the mass slide before coming to rest?

   A) 1 m C**)** 4 m

   B) 2 m D) 6 m



4. A block of mass 0.25 kg is placed on top of a light vertical spring of force constant 5000 N/m and pushed downward so that the spring is compressed by 0.1 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise? ($g = 9.8\ m/s^2$)

   A**)** 10.2 m B) 9.2 m C) 8.0 m D) 9.0 m

5. Which one of the following statements is true about a spring-mass system oscillating on a horizontal frictionless surface?

   A) The kinetic and potential energies are equal at all times.
   B) The kinetic and potential energies are both constant.
   C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
   D) The maximum kinetic energy and maximum potential energy are equal.

Conservative and non-conservative forces

As an object moves downward near the surface of the Earth, the work done by the gravitational force on the object does not depend on whether it falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. However, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to friction forces. We can use this varying dependence on path to classify forces as either **conservative** or **non-conservative**.

i) *Conservative force*:- is a force in which the work done in moving a body from one point to another is **independent of the path taken between them**.

Example: Gravitational force, electric force, spring force,

- Work done by conservative force is
  - independent of the path
  - equal to the difference between the final and initial values of a potential energy
  - completely recoverable

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

ii) *Non – conservative force*:- is force in which the work done in moving a body from one point to another is **dependent on the path followed**.

Example: Frictional force,

- Work done by a non-conservative force is
  - path dependent
  - not recoverable

**Work done by frictional force is equal to the difference between the final and initial mechanical energies.*

$W_{friction} = ME_{final} - ME_{final}$

Example

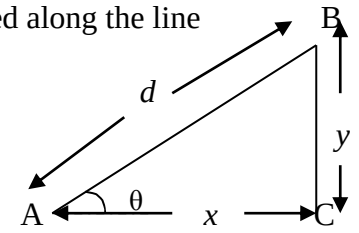
1. What is the work done by gravitational force on mass *m* if it is moved along the line

a) ACB? **Ans**. $W_{grav} = -mgy$

b) AB (a distance *d*)? **Ans**.

Hint:- $y = d\sin\theta$ and $\cos(90+\theta) = \cos 90^0 \cos\theta - \sin 90^0 \sin\theta$

$= -\sin\theta$



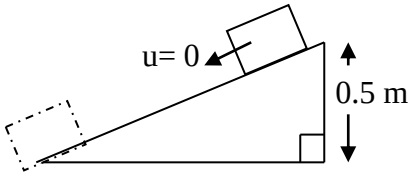
2. A 3 kg block slides down a 1m long inclined plane. If the block starts from rest and experiences a frictional force of 6 N, calculate

a) the initial mechanical energy. **Ans**. $ME_{initial} = 15$ J

b) the final mechanical energy. **Ans**. $ME_{final} = 9$ J

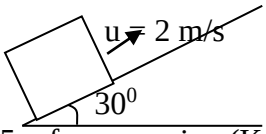
c) the change in mechanical energy. **Ans**. $\Delta ME = -6$ J

d) the work done by frictional force. **Ans**. $W_{fric} = -6$ J

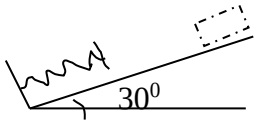


Exercise

1. A 1 kg object starts from rest at the top of an inclined plane and slides down the incline of height 4 m. If it reaches the bottom of the incline with a speed of 6 m/sec, what percentage of its total energy at the top of the incline is lost as a result of friction? **Ans**. 55%
2. A 1 kg block slides down a 20 m long inclined plane starting from rest. If $\mu_k = 0.25$ and $\theta = 37^0$ above the horizontal, calculate
   a) ME_{initi}. **Ans**. $ME_{initi} = 120$ J
   b) ME_{final}. **Ans**. $ME_{final} = 80$ J
   c) W_{fric}. **Ans**. $W_{fric} = -40$ J
   d) ΔME **Ans**.$\Delta ME = -40$ J
3. A 2 kg box is projected up the inclined plane with 2 m/s. Calculate how far the box gets up before coming to rest.
   a) if $\mu = 0$. **Ans**. S= 0.4 m
   b) if $f = 5$ N. **Ans**. S = 0.27 m



4. A 3 kg block on an inclined plane at a distance of 5m from a spring (K= 144 N/m) is released from rest. If the block compresses the spring by 1 m before coming to rest, find
   a) the frictional force acting on the block. **Ans**. $f = 3$ N
   b) speed of the block at equilibrium position. **Ans**. $V_{max} = 2\sqrt{10}$ m/s



Questions from national exams

1. A 4 kg block is given an initial speed of 8 m/s at the bottom of 30^0 inclined plane. The frictional force that retards its motion is 15 N. If the block is directed up the inclined plane, how far will it move up before it stops?
   A**)** 3.66 m B) 4.40 m C) 3.35 m D) 4.67 m
2. A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 2000 N/m. The spring is compressed 2.0 cm and is then released from rest. At what speed will the block pass through the equilibrium position if a constant friction force of 10.0 N retards its motion from the moment it is released?
   A) 70 m/s B) 50 m/s C) 70 cm/s D**)** 50 cm/s

Power

Power is the rate of doing work, or the work done per unit time.

$$\text{Power} = \frac{total\ work\ done}{total\ time\ taken} \Rightarrow P = \frac{W}{t} \quad \text{OR} \quad P = \vec{F} \bullet \vec{V}$$ Its SI unit is 1J/sec = 1Watt

Example

1. An automobile has a power of 75 kW and moves at a uniform speed of 25 m/s. What is the net force on the automobile?

Given	Required	Soln
P = 75 kW	F_{net} =?	From $P = F_{net}V\cos 0^0$ The direction of force is parallel to the direction of velocity.
V = 25 m/s		$F_{net} = P/V = 75000$ W/ 25 m/s = 3000 N

2. Calculate the power output of a machine that lifts a 500 kg object through a height of 18m in 60 sec.

Given	Required	Soln
$m = 500$ kg	P =?	$P = W/t$
$h = 18$ m		$= mgh/t = 500 \times 10 \times 18 / 60$
$t = 60$ sec		$= 1500$ W

3. A 1300 kg car accelerates from rest to a speed of 30 m/s in 12 sec. What is the power developed?

Given	Required	Soln
m = 1300 kg	P =?	P = W / t
u = 0		= Δ KE / t = (½ mv^2 – ½ mu^2)/t
v = 30 m/s		=(½ ×1300×30×30 – ½ ×1300×0×0) / 12
t = 12 sec		= 48750 W

4. A spring with K = 275 N/m is stretched by 20 cm in 2sec. What is the average power supplied?

Given	Required	Soln
K = 275 N/m	P_{av} =?	P_{av} = U / t
x = 20 *cm*		= ½ K x^2/ t = ½ ×275×0.2×0.2 / 2
t = 2 sec		= 2.75 W

5. A 2 kg mass is accelerated uniformly from rest to a speed of 12 m/s in 3 sec. Calculate the instantaneous power of the mass at 2 sec.

Given	Required	Soln
m = 2kg	P_{inst} = ?	a = (12 – 0) / 3= 4 m/s^2
u = 0		after 2sec, V= u + at ⟹ V = 0 + 4×2 = 8m/s
V_{3sec} = 12 m/s		P_{inst} = F_{inst} V_{inst} = ma_{inst} V_{inst}
		= 2×4×8 = 64W

Exercise

1. A machine raises 200 liters of water from a well of depth 9 m in 6 sec. Calculate the power delivered by the machine.(g = 10 m/s^2, ρ_w = 1000 kg/m^3) **Ans**. P = 3000 W
2. A man pushes a 100kg box across a level floor at a constant speed of 2m/s. If μ_k = 0.2, what is the power output? **Ans**. P = 400 W
3. A net force of 5N acts on a 15kg object initially at rest. Calculate the instantaneous power at t = 3sec. **Ans**. P_{inst} = 5 W
4. An engine having P = 30kW pulls a box along a horizontal rough surface at a constant speed of 20m/s. What is mass of the box if μ_k = 0.15? **Ans**. m_{box} = 1000 kg
5. A 1200m^3 of water passes over a water surface each second. The water fall is 100m high. If three-fourth of the kinetic energy gained by the water is converted to electrical energy by hydrostatic generator. Calculate the power output of the generator. **Ans**. P = 900 MW

Questions from national exams

1. Particle A has less mass than particle B. Both are pushed forward across a frictionless surface by equal forces for 1s. Which of the following statements is correct if both start from rest?
   A) The amount of work done on A is greater than the work done on B.
   B) The final speed of A is less than that of B.
   C) The impulses delivered to both particles are the same.
   D) The change in kinetic energy is the same for both particles.
2. If a single contact force F acts on a particle of mass m and the particle starts from rest, what is the instantaneous power delivered by the force at time t ?

   A) $\frac{2F}{m}t$ B) $\frac{F^2}{2m}t$ C) $\frac{F^2}{m}t$ D) $\frac{3F^2}{m}t$