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UNITY UNIVERSITY

1. Total cost equation in terms of quantity:

Total cost = fixed cost + variable cost

Fixed cost = $200,000 (cost to construct the factory building)

Variable cost per unit of production = $20

Total variable cost = $20 x quantity

Total cost equation:

Total cost = $200,000 + ($20 x quantity)

To determine the break-even quantity (revenue), we need to find the quantity at which the total revenue equals the total cost:

Total revenue = unit price x quantity

Total revenue = $50 x quantity

Set total revenue equal to total cost and solve for the quantity:

$50 x quantity = $200,000 + ($20 x quantity)

$30 x quantity = $200,000

Quantity = 6,667 units

Therefore, the break-even quantity for the company is 6,667 units.

2. TR, TC, and profit functions for Helen Trading:

Total revenue (TR) = unit selling price x quantity

TR = 100x

Total cost (TC) = fixed cost + total variable cost

Total variable cost (TVC) = unit variable cost x quantity

TVC = 60x

TC = 400,000 + 60x

Profit (π) = TR - TC

π = 40x - 400,000

To find the breakeven point in terms of quantity, set TR equal to TC and solve for x:

100x = 400,000 + 60x

40x = 400,000

x = 10,000 units

Therefore, the breakeven point for Helen Trading is 10,000 units.

To find the sales volume corresponding to a profit of 500,000, set the profit equation equal to 500,000 and solve for x:

40x - 400,000 = 500,000

40x = 900,000

x = 22,500 units

To find the revenue, multiply the quantity by the unit selling price:

Revenue = 22,500 x 100 = 2,250,000

If sales are 2,000,000, the profit can be found by substituting the sales volume into the profit function:

π = 40x - 400,000

π = 40(20,000) - 400,000

π = 200,000

Therefore, the profit would be 200,000 if sales are 2,000,000.

3. Number of units of each product to be produced in a month to use up all the available resources:

Let x, y, and z be the number of units of Metakeb, Mewesson, and Metekem, respectively.

The time required in the machining department is 1200x + 1800y + 3000z minutes.

The time required in the assembly department is 2400x + 1200y + 2400z minutes.

The time required in the inspection department is 600x + 3000y + 1200z minutes.

Converting the available hours to minutes:

Machining department: 1050 x 60 = 63,000 minutes

Assembly department: 1160 x 60 = 69,600 minutes

Inspection department: 830 x 60 = 49,800 minutes

We can then write the following system of equations:

1200x + 1800y + 3000z ≤ 63,000

2400x + 1200y + 2400z ≤ 69,600

600x + 3000y + 1200z ≤ 49,800

To solve this system of equations using Gaussian elimination, we can start by writing the augmented matrix:

[1200 1800 3000 | 63000]

[2400 1200 2400 | 69600]

[600 3000 1200 | 49800]

Performing row operations to put the matrix in row echelon form, we get:

[1 3/2 5/3 | 525]

[0 1 -1 | 400]

[0 0 1 | 10]

Using back substitution, we can solve for z = 10, y = 410, and x = 320. Therefore, the company should produce 320 units of Metakeb, 410 units of Mewesson, and 10 units of Metekem to use up all the available resources.

4. Number of South Koreans who attended the opening ceremony at the first and second class:

Let x be the number of people who attended the first class and y be the number of people who attended the second class.

The total number of people who attended the opening ceremony is 80,000, so x + y = 80,000.

The total ticket receipts were $3,500,000, so 62.5x + 37.5y = 3,500,000.

We can solve this system of equations using substitution. Solving the first equation for y, we get y = 80,000 - x. Substituting this into the second equation, we get:

62.5x + 37.5(80,000 - x) = 3,500,000

62.5x + 3,000,000 - 37.5x = 3,500,000

25x = 500,000

x = 20,000

Therefore, 20,000 people attended the opening ceremony at the first class, and 60,000 people attended at the second class.

5. Number of customers of Colget and Akuafresh after two transitions:

Let x and y be the number of customers of Colget and Akuafresh, respectively, after two transitions.

After the first transition, we have:

Colget customers = 0.4x + 0.25y

Akuafresh customers = 0.6x + 0.75y

After the second transition, we have:

Colget customers = 0.4(0.4x + 0.25y) + 0.25(0.6x + 0.75y) + 10,000

Akuafresh customers = 0.6(0.4x + 0.25y) + 0.75(0.6x + 0.75y) + 10,000

Simplifying these expressions, we get:

Colget customers = 0.31x + 0.22y + 10,000

Akuafresh customers = 0.69x + 0.78y + 10,000

If the current number of Colget customers is 6,000, we have:

x = 6,000

y = 10,000 - x = 4,000

After two transitions, the number of customers of Colget and Akuafresh would be:

Colget customers = 0.31(6,000) + 0.22(4,000) + 10,000 = 12,420

Akuafresh customers = 0.69(6,000) + 0.78(4,000) + 10,000 = 17,580

Therefore, there would be 12,420 customers of Colget and 17,580 customers of Akuafresh after two transitions.

Steady state market share is the long-run proportion of customers who purchase each brand. As the number of customers is constant, the steady state market share is simply the proportion of customers that each brand has in the long run. From the previous calculation, we have:

Total number of customers = 6,000 + 4,000 + 10,000 = 20,000

Steady state market share of Colget = 6,000/20,000 = 0.3 or 30%

Steady state market share of Akuafresh = 14,000/20,000 = 0.7 or 70%

6. Linear programming model for the problem of producing Hiwot and Durex tractors:

Let x and y be the number of Hiwot and Durex tractors produced, respectively.

The objective is to maximize total profit:

Maximize Z = 45x + 70y

Subject to the following constraints:

- Assembly time constraint: 9x + 12y ≤ 720 (in minutes)

- Engine constraint: x + y ≤ 75

- Packaging time constraint: 2x + 6y ≤ 300

The formulation of the linear programming model is complete.

To solve this problem using the simplex method, we can write the augmented matrix:

[9 12 1 | 720]

[1 1 1 | 75]

[2 6 0 | 300]

[-45 -70 0 | 0]

Performing row operations to put the matrix in row echelon form, we get:

[1 1 1 | 75]

[0 3 8 | 345]

[0 0 -2 | -90]

[0 0 -95 | -3,375]

Using back substitution, we can solve for y = 15, x = 60, and Z = 4,950. Therefore, the company should produce 60 units of Hiwot and 15 units of Durex to maximize total profit.

The interpretation of the result is that the company should allocate its resources to produce 60 units of Hiwot and 15 units of Durex to maximize its profit. The maximum profit that the company can earn is 4,950 birr.

7.

a) To meet the objective of the down payment, Bekele needs to determine the amount he should deposit at the end of every quarter.

The present value (PV) of the down payment is:

PV = 100,000

The future value (FV) of the sinking fund account after 5 years can be found using the formula:

FV = PV x (1 + r/n)^(n x t)

where r is the annual interest rate, n is the number of compounding periods per year, and t is the number of years.

Since the interest is compounded quarterly, we have:

r = 0.10/4 = 0.025 (quarterly interest rate)

n = 4 (number of compounding periods per year)

t = 5 (number of years)

Therefore, the future value of the sinking fund account is:

FV = 100,000 x (1 + 0.025/4)^(4 x 5) = 132,981.90

To determine the amount Bekele should deposit at the end of every quarter, we can use the formula for the present value of an annuity:

PV = A x [(1 - (1 + r/n)^(-n x t))/(r/n)]

where A is the amount of each deposit.

Solving for A, we get:

A = PV x (r/n)/[(1 - (1 + r/n)^(-n x t))]

Substituting the values, we get:

A = 132,981.90 x (0.025/4)/[(1 - (1 + 0.025/4)^(-4 x 5))] = 4,283.04

Therefore, Bekele should deposit Br. 4,283.04 at the end of every quarter to meet the objective of the down payment.

b) The amount of monthly mortgage payment to be made to discharge the mortgage within 30 years can be found using the formula for the present value of an annuity:

PV = PMT x [(1 - (1 + r/n)^(-n x t))/(r/n)]

where PMT is the monthly mortgage payment.

Solving for PMT, we get:

PMT = PV x (r/n)/[(1 - (1 + r/n)^(-n x t))]

The present value of the mortgage is:

PV = 500,000 - 100,000 = 400,000

Substituting the values, we get:

PMT = 400,000 x (0.01)/[(1 - (1 + 0.01/12)^(-12 x 30))] = 3,386.43

Therefore, the amount of monthly mortgage payment to be made to discharge the mortgage within 30 years is Br. 3,386.43.

c) The total interest paid on the mortgage can be found by subtracting the present value of the mortgage from the total amount paid over the entire period of the mortgage. The total amount paid over the entire period of the mortgage is:

PMT x 12 x 30 = 3,386.43 x 12 x 30 = 1,219,914.80

The total interest paid on the mortgage is:

1,219,914.80 - 400,000 = 819,914.80

Therefore, the total interest paid on the mortgage is Br. 819,914.80.

d) The total amount paid over the entire period of the mortgage is:

500,000 + 819,914.80 = 1,319,914.80

Therefore, the total amount paid over the entire period of the mortgage is Br. 1,319,914.80.

8.

a) The derivative of a constant function is zero. Therefore,

f'(X) = 0

b) The derivative of a power function is given by:

g'(X) = d/dX[X^n] = nX^(n-1)

Therefore,

g'(X) = d/dX[X^6] = 6X^(6-1) = 6X^5

c) The derivative of a sum of functions is the sum of the derivatives of the individual functions. Therefore,

h'(X) = d/dX[5X^3 + 3X^4] = d/dX[5X^3] + d/dX[3X^4]

Using the power rule, we get:

h'(X) = 15X^2 + 12X^3

d) The derivative of a product of functions is given by the product rule:

d/dX[f(X)g(X)] = f'(X)g(X) + f(X)g'(X)

Applying the product rule, we get:

g'(X) = (3X^2 - 3)(X + 5) +(3X^2 - 3)(d/dX[X+5])

Simplifying, we get:

g'(X) = (3X^2 - 3)(X + 5) + (3X^2 - 3)(1)

g'(X) = 3X^3 + 12X^2 - 3X - 15