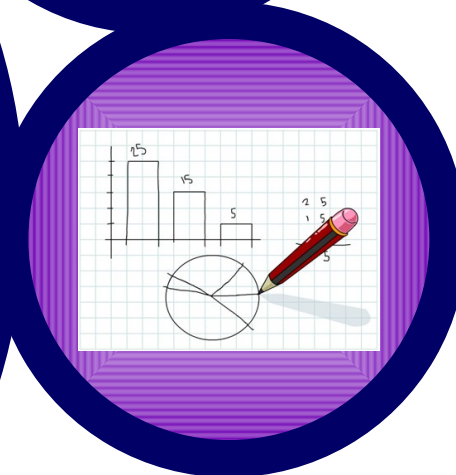
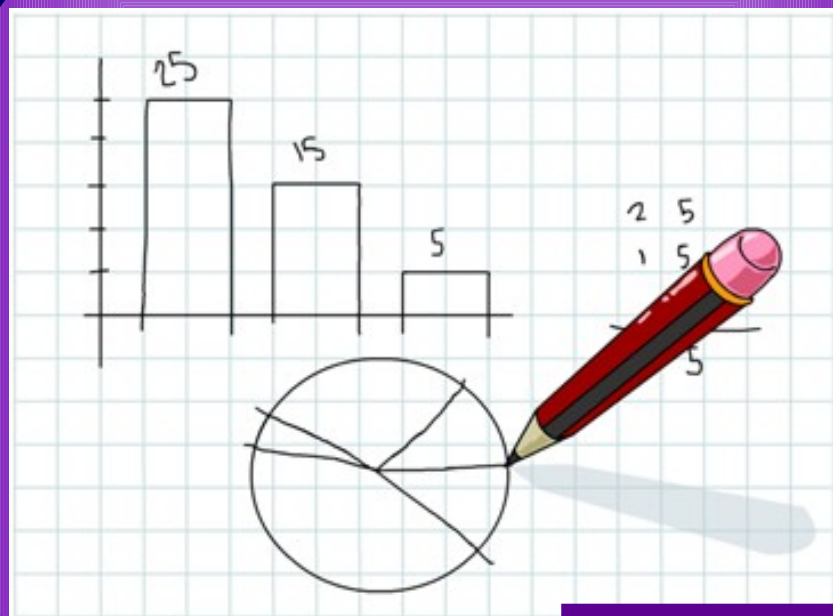




PROBABILITY



D6

Probability

- D6.1 The language of probability
- D6.2 Probabilities of single events
- D6.3 Probabilities of combined events
- D6.4 Tree diagrams
- D6.5 Experimental probability



The language of probability



"I've definitely failed that exam. I didn't answer any of the questions!"



The language of probability



Use this word in a sentence about probability:

even chance





Outcomes and events

An **event** can have several **outcomes**.

What are the possible outcomes when throwing a ten-sided dice?

If a dice has ten faces, then there are ten possible outcomes, one for each face of the dice.

Can you think of an event that has two outcomes?

A simple example of an event that has two outcomes is flipping a coin.

The two outcomes are heads and tails.





Outcomes and events

Each outcome of a given event has a **probability** or a **chance** of **occurring**.

What are the chances of each outcome from throwing a ten-sided dice?

Assuming that the dice is **fair**, the chances of each outcome occurring is $\frac{1}{10}$.

Can you think of an event that has two outcomes which have probabilities that are not equal?

One example is that a randomly chosen person will be right- or left-handed.





Two outcomes

In which of these scenarios does each outcome have an equal chance of occurring?

- It will rain tomorrow.
- A child born will be a boy.
- A coin will show tails when it is flipped.
- A number selected at random from 1 to 100 will be even.
- When a dice is thrown, it will show a square number.
- The next person to walk into the room will be right handed.
- The bus will be on time tonight.
- The bus driver will be female.



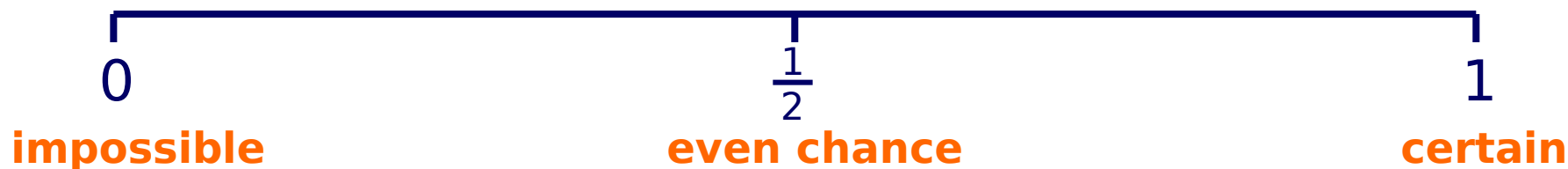
The probability scale

We measure probability on a scale from 0 to 1.

If an event is **impossible** or has no chance of occurring, then it has a probability of **0**.

If an event is **certain** it has a probability of **1**.

This can be shown on the probability scale as:



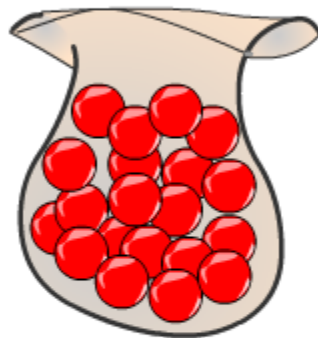
Probabilities are written as fractions, decimals and, less often, as percentages.



Using the probability scale



What's the probability of picking a pink counter?



0

0.5

1





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Probability

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Calculating probability

If the **outcomes** of an event are **equally likely** then we can calculate the probability using the formula:

$$\text{Probability of an event} = \frac{\text{Number of possible successful outcomes}}{\text{Total number of possible outcomes}}$$



For example, a bag contains 1 yellow, 3 green, 4 blue and 2 red

What is the probability of pulling a green marble from the bag without looking?

$$P(\text{green}) = \frac{3}{10} \text{ or } 0.3 \text{ or } 30\%$$



Calculating probability

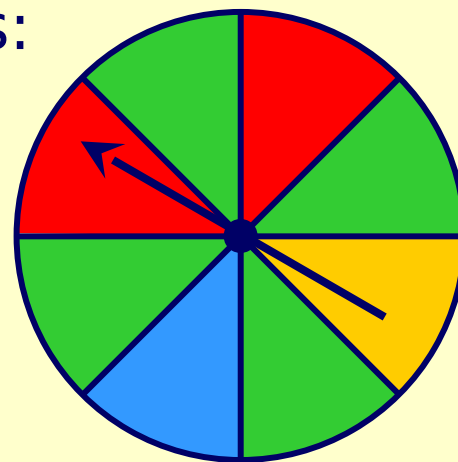
This spinner has 8 equal divisions:

What is the probability of the spinner

a) landing on a blue sector?

b) landing on a red or green sector?

c) not landing on a green sector?



a) $P(\text{blue}) = \frac{1}{8}$

b) $P(\text{red or green}) = \frac{6}{8} = \frac{3}{4}$

c) $P(\text{not green}) = \frac{4}{8} = \frac{1}{2}$

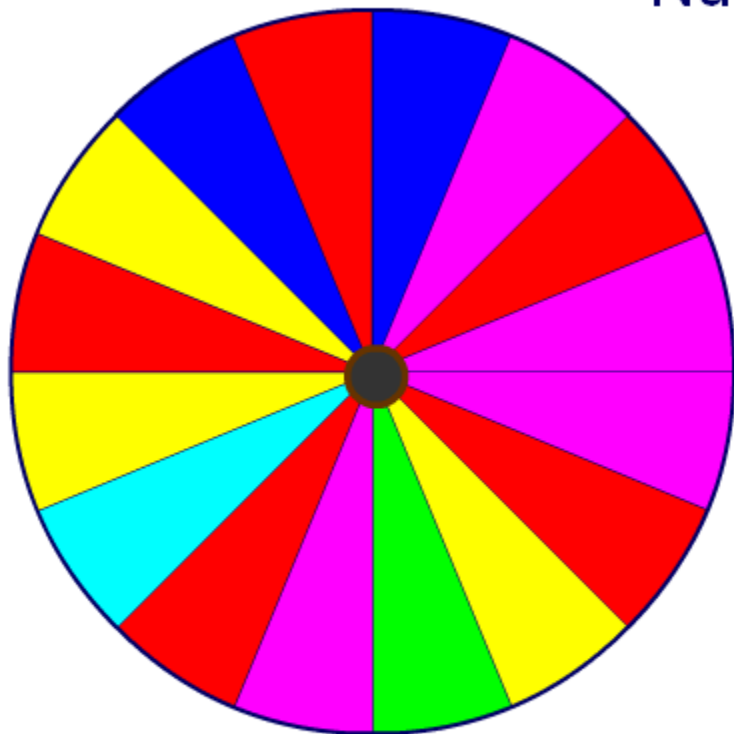




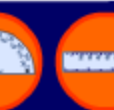
Calculating probabilities with spinners



Number of sections: ◀ 16 ▶



What is the probability of this spinner landing on dark blue?





Calculating probabilities

The probability of a spinner landing on yellow is

0.2

What is the probability of **not** landing on yellow?

$$1 - 0.2 = 0.8$$

If the probability of an event occurring is p then the probability of it *not* occurring is $1 - p$.

A spinner has green, red and blue sections. Landing on red is twice as likely as landing on green. Fill in the missing probabilities:

Green	Red	Blue
0.26	0.52	0.22



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Combined events: horse race

You are going to take part in a simulation of a horse race.

There are twelve horses, numbered 1 to 12.

1	2	3	4	5	6	7	8	9	10	11	12

- Throw two dice and add the numbers together. Each total represents a horse.
- Fill in a square in the table each time the horse's number comes up. This represents the horse moving forward one place.
- The first horse to reach the top wins the race.
- Before you start, place a bet on a horse to win.





Horse race

Play the game a few times. You can bet on different horses each time, or stick with the same horse.

Discuss these questions with a partner:

- 1) Which horses won in your races?
- 2) Which horses are unlikely to win? Explain.
- 3) Are there any horses you wouldn't bet on? If so, why?
- 4) Which horses are the best ones to bet on? Why?
- 5) Why are some horses more likely to win than others?
- 6) How would the game change if you used ten-sided dice?



To get a total of 8, you can throw a 3 and a 5 or a 5 and a 3. What other combinations produce a



Horse race evaluation

Horse	Combinations	Ways	Probabilit
1		0	~~0~~
2	1,1	1	$\frac{1}{36}$
3	1,2 2,1	2	$\frac{2}{36} = \frac{1}{18}$
4	1,3 3,1 2,2	3	$\frac{3}{36} = \frac{1}{12}$
5	1,4 4,1 2,3 3,2	4	$\frac{4}{36} = \frac{1}{9}$
6	1,5 5,1 2,4 4,2 3,3	5	$\frac{5}{36}$
7	1,6 6,1 2,5 5,2 3,4 4,3	6	$\frac{6}{36} = \frac{1}{6}$
8	2,6 6,2 3,5 5,3 4,4	5	$\frac{5}{36}$
9	3,6 3,6 4,5 5,4	4	$\frac{4}{36} = \frac{1}{9}$
10	4,6 4,6 5,5	3	$\frac{3}{36} = \frac{1}{12}$
11	5,6 6,5	2	$\frac{2}{36} = \frac{1}{18}$
12	6,6	1	$\frac{1}{36}$



Sample space diagrams

This table is another way of displaying all the outcomes from throwing two dice and adding them together. It is called a **sample space diagram**.

		Second die					
First die	+	1	2	3	4	5	6
	1		3				
	2						
	3					8	
	4						
	5						
	6			9			

Fill in the rest of the cells in the table.

Colour in all the twos one colour, the threes another colour etc.

What patterns do you notice in the table?



Sample space diagrams

		Second die					
First die	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Now make up
your own
questions!

- 1) What is the probability of getting a total more than 5?
- 2) What is the probability of getting a total less than 10?
- 3) What is the probability of getting a total that is a square number?
- 4) What is the probability of getting an even total?
- 5) What is the probability of getting an odd total?
- 6) What is the probability of getting a total less



Combined events with coins and dice

A coin and a ten-sided die are thrown and the outcomes recorded in the two-way table below.

		Dice									
Coin	+	1	2	3	4	5	6	7	8	9	10
	H		2, H					7, H			
	T								8,T		

Complete the table to show all the possible outcomes.



Combined events

		Die									
Coin	+	1	2	3	4	5	6	7	8	9	10
	H	1, H	2, H	3, H	4, H	5, H	6, H	7, H	8, H	9, H	10, H
	T	1,T	2,T	3,T	4,T	5,T	6,T	7,T	8,T	9,T	10, T

- 1) How many outcomes are there?
- 2) What is the probability of getting a head and an even number?
- 3) What is the probability of getting a tail and a square number?

Make up a game where you win points if you get certain outcomes from the table above.



Other combined events

Two four-sided dice are thrown and the numbers added together.

Construct a sample space diagram to show all the outcomes

		Second die			
First die	+	1	2	3	4
	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

What is the probability of getting:

- 1) a total more than 4?
- 2) a total less than 8?
- 3) a prime number total?
- 4) a total that is at least 3?
- 5) a total of 4 or 5?
- 6) the same number on both dice?

7) a lower number on the first

Calculating the number of outcome



die 1



6



die 2



6



+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						



Using fractions to find probabilities

		Second die			
First die	+	1	2	3	4
	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

Use the table to find the probability of getting a score of 3 or 4.

The probability of getting a score of 3 or 4 can be written as $P(3 \text{ or } 4)$.

This is the same as the probability of getting a 3 added to the probability of getting a 4.

$$P(3 \text{ or } 4) = P(3) + P(4) = \frac{2}{16} + \frac{3}{16} = \frac{5}{16}$$

=

$$P(5 \text{ or } 6) = P(5) + P(6) = \frac{4}{16} + \frac{3}{16} = \frac{7}{16}$$

=

$$P(2 \text{ or } 7) = P(2) + P(7) = \frac{1}{16} + \frac{2}{16} = \frac{3}{16}$$

=



Using fractions to find probabilities

Two six-sided dice are thrown.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Work out $P(3 \text{ or } 4)$
by adding fractions.

$$P(3 \text{ or } 4) = P(3) + P(4)$$

$$= \frac{2}{36} + \frac{3}{36} = \frac{5}{36}$$

Work out $P(5 \text{ or } 6)$
by adding fractions.

$$P(5 \text{ or } 6) = P(5) + P(6) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36} = \frac{1}{4}$$



Using fractions to find probabilities

Shakil works out that the probability of getting a prime number or an even number is $\frac{33}{36}$.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Explain why Shakil is wrong.

Let's colour all of the even numbers yellow...

...and all the prime numbers blue.

2 is both even and prime.

$P(\text{a prime number or an even number}) =$

$$P(\text{prime}) + P(\text{even}) - P(2) = \frac{15}{36} + \frac{18}{36} - \frac{1}{36} = \frac{32}{36} = \frac{8}{9}$$





Mutually exclusive events

When throwing two six-sided dice, if one die lands on 1, it is impossible to get a total score of 8.

The events “rolling a 1” and “getting a total score of 8” are said to be **mutually exclusive**.

Mutually exclusive events cannot occur at the same time.

The events “getting a prime number” and “getting an even number” are not mutually exclusive since 2 is both prime and even.

When events are mutually exclusive, the probabilities of each event can be added together.

When they are not, the “overlap” must be subtracted.





Independent events

Peter rolls an unbiased six-sided die fifty times and doesn't roll a six once. He says "I must get a six soon!"

Do you agree?

Each roll of the die is unaffected by the previous outcomes.

The next roll of the die is no more likely to be a six than any of the other rolls. The probability is still $\frac{1}{6}$.

Two events are said to be **independent** if the outcome of one has no effect on the outcome of the other.



Independent events

		Second dice					
First dice	+	1	2	3	4	5	6
	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

What is the probability of getting a 1 followed by a 2 when two dice are thrown?

You can use the table to find the number of successful results and the number of possible results...

...or you can multiply the probabilities of the scores on each dice, $P(1)$ and $P(2)$:

$$P(1) \text{ and } P(2) = P(1) \times P(2) =$$

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

It is important to remember that this method can only be used when the two events are independent.



Independent events

		Second dice					
First dice	+	1	2	3	4	5	6
	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

What is the probability of getting a 1 on the first die and an even number on the second die?

$$P(1) = \frac{1}{6}$$

$$P(\text{even}) = \frac{1}{2}$$

$$P(1) \text{ and } P(\text{even}) = P(1) \times P(\text{even}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

Compare this with the result from the table: $\frac{3}{36} = \frac{1}{12}$





Combining probabilities using multiplication

The probability of two independent events happening at the same time is:

$$P(A \text{ and } B) = P(A) \times P(B)$$

This method applies when there are more than two events. For example:

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$





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Tree diagrams

You have a pack of fifteen playing cards.

Six of the cards are red and the rest are black.

Write down the decimal probability of choosing a red card at random from the pack.

$$P(\text{red card}) = \frac{6}{15} = 0.4$$

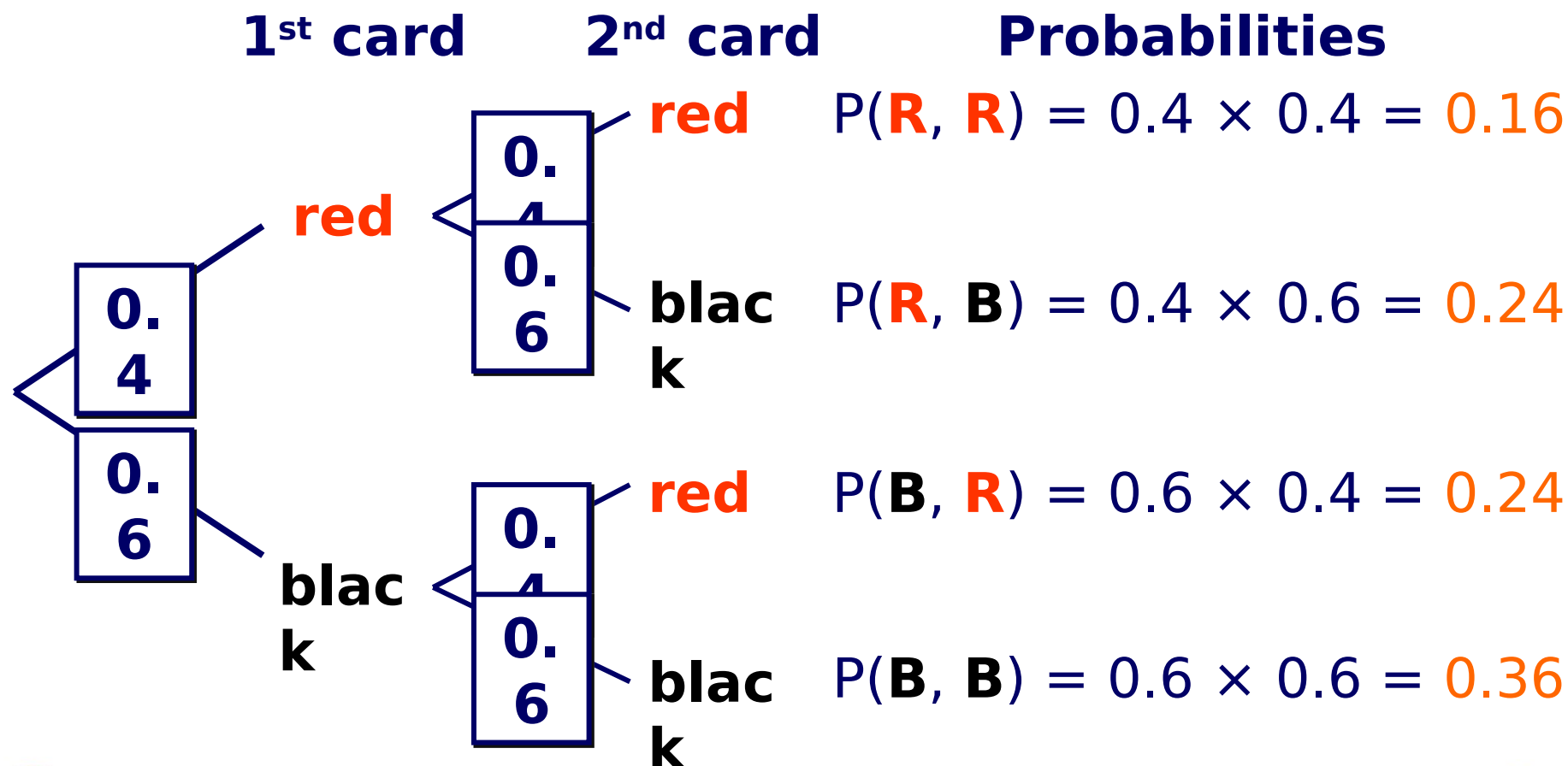
You decide to pick a card at random from the pack, replace it and then pick another.

If you pick two red cards, you will stay in and do your homework. If you don't, you will go out to a party.



Tree diagrams

All of the possible outcomes can be shown using a probability tree diagram:



Tree diagrams

Probabilities

$$P(\mathbf{R}, \mathbf{R}) = 0.4 \times 0.4 = 0.16$$

$$P(\mathbf{R}, \mathbf{B}) = 0.4 \times 0.6 = 0.24$$

$$P(\mathbf{B}, \mathbf{R}) = 0.6 \times 0.4 = 0.24$$

$$P(\mathbf{B}, \mathbf{B}) = 0.6 \times 0.6 = 0.36$$

1) What do the probabilities add up to?

$$0.16 + 0.24 + 0.24 + 0.36 = 1$$

2) What are the chances of going to the party?

$$P(\mathbf{R}, \mathbf{R}) = 0.4 \times 0.4 = 0.16$$

3) What are the chances of your doing your homework?

$$P(\mathbf{not\ R}, \mathbf{R}) = 1 - 0.16 = 0.84$$





Probability without replacement

The light bulb in your room has blown. You are in a rush.

There are twenty socks in a drawer. Fourteen of them are blue and the rest are green.

What is the probability of randomly picking a blue sock out of the drawer?

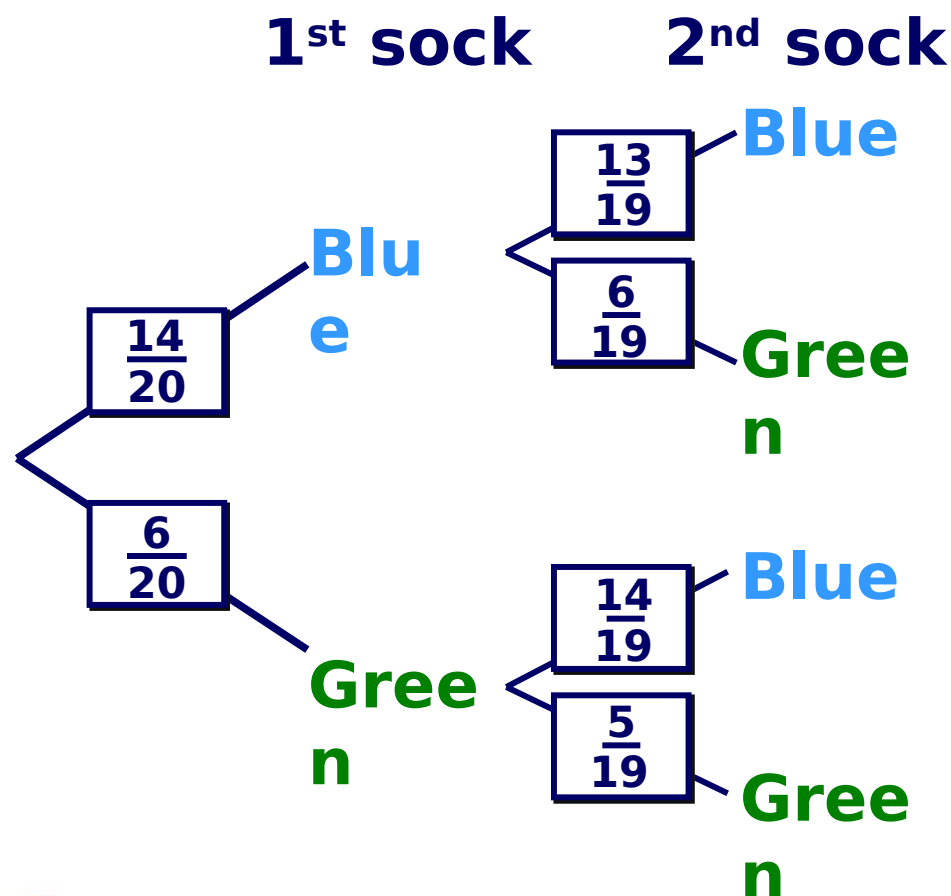
$$P(\text{blue sock}) = \frac{14}{20} = \frac{7}{10}$$

You pick out two socks, one after the other, in the hope of getting a matching pair.



Tree diagrams

All of the possible outcomes can be shown using a probability tree diagram:



Probabilities

$$P(\text{B, B}) = \frac{14}{20} \times \frac{13}{19} = \frac{182}{380} = \frac{91}{190}$$

$$P(\text{B, G}) = \frac{14}{20} \times \frac{6}{19} = \frac{84}{380} = \frac{21}{95}$$

$$P(\text{G, B}) = \frac{6}{20} \times \frac{14}{19} = \frac{84}{380} = \frac{21}{95}$$

$$P(\text{G, G}) = \frac{6}{20} \times \frac{5}{19} = \frac{30}{380} = \frac{3}{38}$$



Tree diagrams

Probabilities

$$P(\text{B}, \text{B}) = \frac{91}{190}$$

$$P(\text{B}, \text{G}) = \frac{21}{95}$$

$$P(\text{G}, \text{B}) = \frac{21}{95}$$

$$P(\text{G}, \text{G}) = \frac{3}{38}$$

1) What do the probabilities add up to?

$$\frac{91}{190} + \frac{21}{95} + \frac{21}{95} + \frac{3}{38} = \frac{182 + 84 + 84 + 30}{380} = 1$$

2) What is the probability of getting two socks the same colour?

$$P(\text{B}, \text{B}) + P(\text{G}, \text{G}) = \frac{91}{190} + \frac{3}{38} = \frac{106}{190} = \frac{53}{95}$$

3) What is the probability of getting two socks that don't match?

$$P(\text{B}, \text{G}) + P(\text{G}, \text{B}) = \frac{21}{95} + \frac{21}{95} = \frac{42}{95}$$



Probability without replacement activity



$$\frac{2}{6}$$

blue

green

blue

green

blue

green

Total number of socks: $\frac{2}{6}$

?

↺

Dependent events

When socks (or cards etc.) are not replaced, the events are called **dependent**. The probability of the second event is affected by the outcome of the first event.

Suppose you pick two cards at random from a pack of cards without replacing the first card.

The probability of the first card being a king is $\frac{4}{52} = \frac{1}{13}$.

Draw up a tree diagram to find the probability of getting

- (a) two Kings in a row
- (b) one King

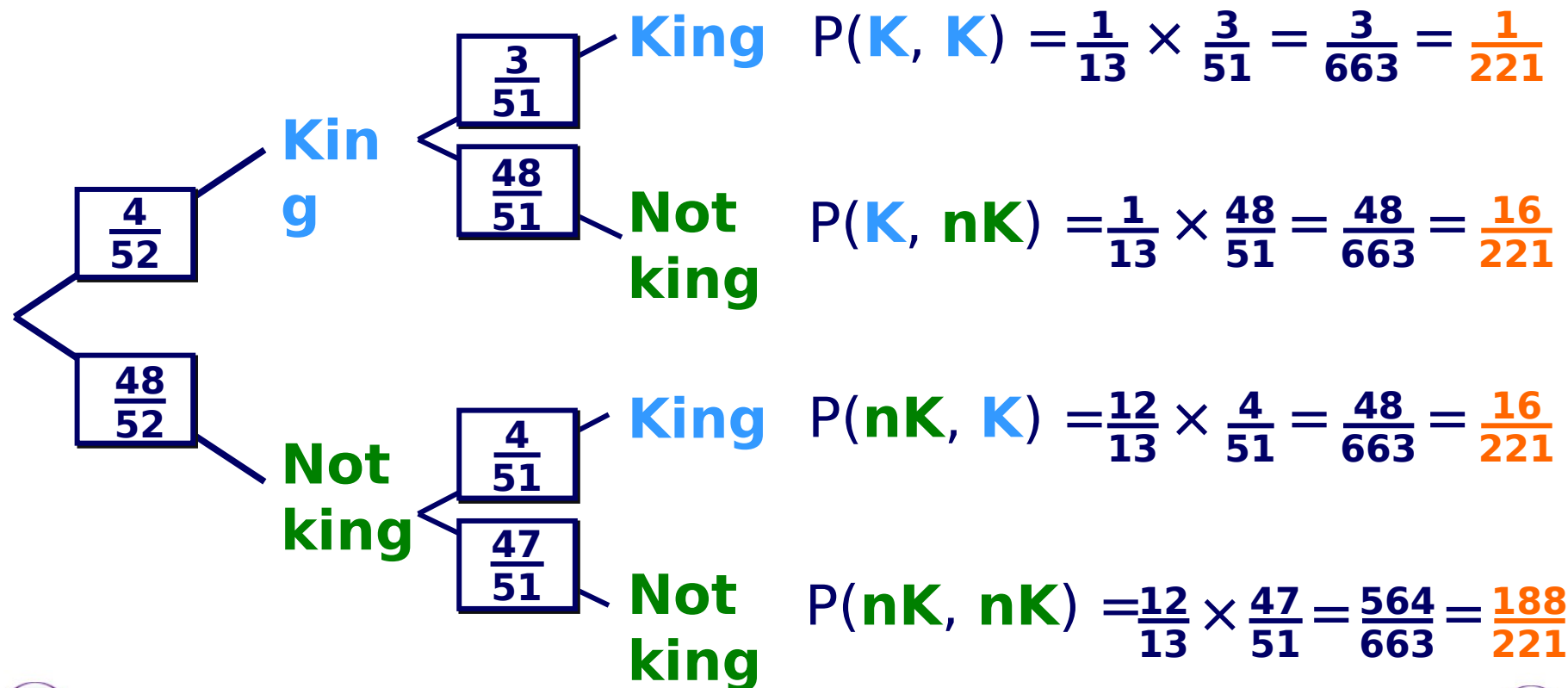
Tree diagrams

When drawing the tree diagram you only need to record “King” and “Not King”: other outcomes are irrelevant to the task

1st card

2nd card

Probabilities



Tree diagrams

Probabilities

$$P(\text{K}, \text{K}) = \frac{1}{221}$$

$$P(\text{K}, \text{nK}) = \frac{16}{221}$$

$$P(\text{nK}, \text{K}) = \frac{16}{221}$$

$$P(\text{nK}, \text{nK}) = \frac{188}{221}$$

1) What do the probabilities add up to?

$$\frac{1}{221} + \frac{16}{221} + \frac{16}{221} + \frac{188}{221} = 1$$

2) What is the probability of getting only one king?

$$P(\text{K}, \text{nK}) + P(\text{nK}, \text{K}) = \frac{16}{221} + \frac{16}{221} = \frac{32}{221}$$

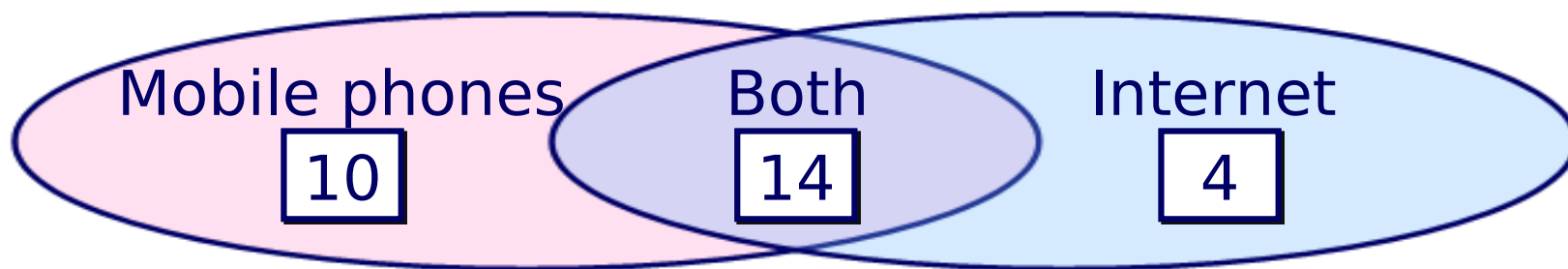
Remember, in tree diagrams, you **multiply** along the branches but **add** the outcomes in the final column that are relevant to a given event.



Harder problems

In a class of 30 students, 24 have mobile phones, 18 have internet access at home and 14 have both.

Fill in this diagram to show the number in each category:



What is the probability of selecting a pupil at random who has

- 1) only a mobile phone? $\frac{1}{3}$
- 2) a mobile phone or the internet but not both? $\frac{7}{15}$
- 3) neither? $\frac{1}{15}$
- 4) a mobile phone and/or the internet? $\frac{14}{15}$

Summary of methods

The “or” rule

- The word “or” often indicates that the probabilities need to be added together.
- $P(A \text{ or } B) = P(A) + P(B)$ if the two events are **mutually exclusive**.

The “and” rule

- The word “and” often indicates that the probabilities need to be multiplied together.
- $P(A \text{ and } B) = P(A) \times P(B)$ if the two events are **independent**.





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Expected probabilities

If you flipped a coin fifty times, how many heads would you expect to get?

- The chance of getting a head $\frac{1}{2}$ is $\frac{1}{2} \times 50 = 25$ heads.
- In reality, it would probably not be exactly 25: this is an estimate.
- When you throw a fair six-sided die 60 times, how many of each number would you expect to roll?
- There are 6 numbers on the die. $60 \div 6 = 10$.

	1	2	3	4	5	6
Throws	10	10	10	10	10	10





Expected probabilities

“When I throw a die sixty times, I will get 10 sixes because the probability of getting a six is $\frac{1}{6}$,” says Zarah.

“But I’ve just thrown an unbiased dice sixty times and these are my results!” protests Monica.

	1	2	3	4	5	6
Throws	12	10	8	14	5	11

Can you explain why Zarah is wrong?



Relative frequency

Monica's results can be expressed as fractions:

	1	2	3	4	5	6
Throws	12	10	8	14	5	11
Relative frequency	$\frac{12}{60}$	$\frac{10}{60}$	$\frac{8}{60}$	$\frac{14}{60}$	$\frac{5}{60}$	$\frac{11}{60}$

y

- These fractions are called the **relative frequencies**.
- They can also be called the **experimental probabilities**.
- The experimental probabilities for a die are not $\frac{1}{6}$ always
- Each time Monica repeats her experiment, the results will be slightly different.



Relative frequency

I keep a record of the weather while I am on holiday.
Here are my results:

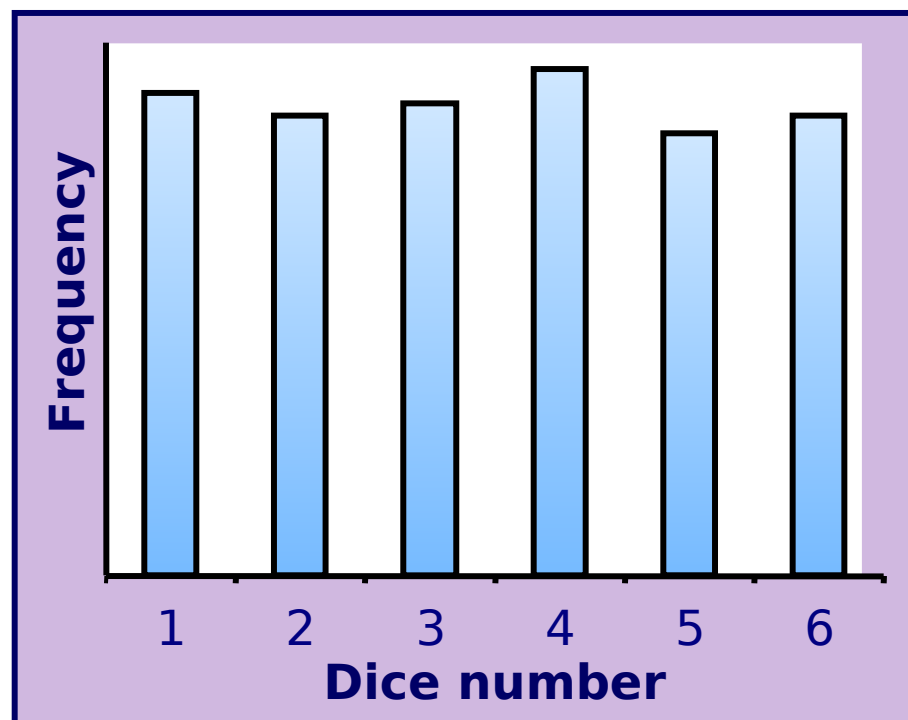
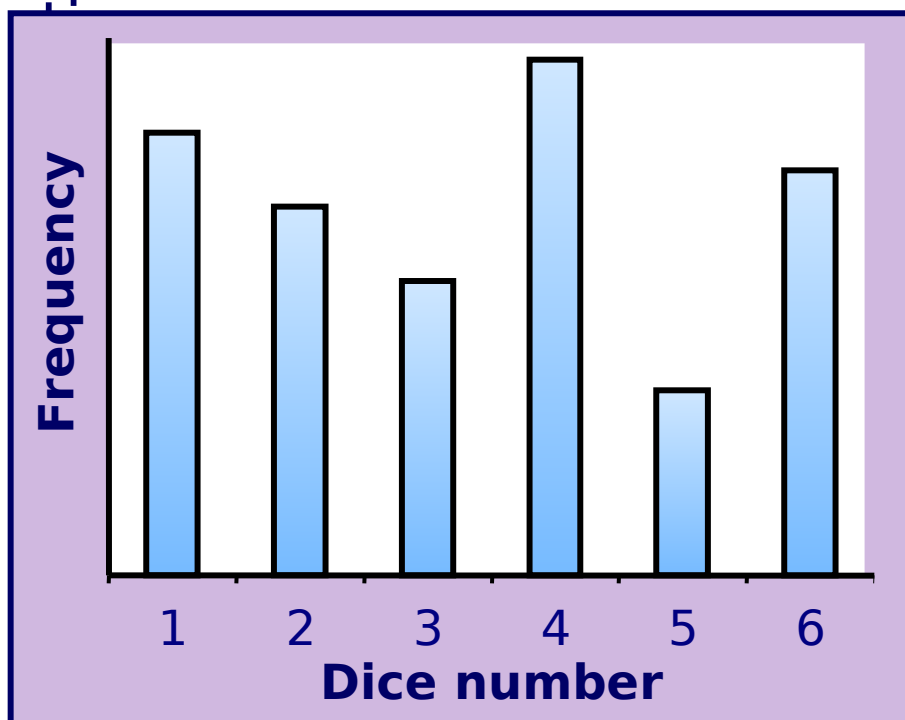
Rainy	Sunny	Cloudy
12 days	10 days	8 days
$\frac{12}{30} = \frac{2}{5}$	$\frac{10}{30} = \frac{1}{3}$	$\frac{8}{30} = \frac{4}{15}$



- What was the probability as a decimal that it rained on a particular day?
- What was the probability as a percentage that it was cloudy?
- Next year I am planning to go on holiday to the same place, at the same time of year, for 15 days. How many days is it likely to be sunny? Why is this only an estimate?

Experimental and theoretical probability

One of these graphs shows the results of Monica's experiment in which she threw a die 60 times. The other represents the results of throwing a die 6000



Which do you think is which?

Experimental and theoretical probability

- To work out the probability of throwing a six on a die, it is not necessary to do an experiment. The uniform properties of the cube provide us with enough information to calculate that it is $\frac{1}{6}$.
- This is called the **theoretical probability**. The theoretical probability of getting a six on a fair die is $\frac{1}{6}$.
- Can the **theoretical probability** of getting a head when you flip a coin be worked out?
- Yes – the symmetrical properties of the coin provide us with enough information to calculate that it is $\frac{1}{2}$.
- Can you work out the **theoretical probability** of a particular bus being on time?
- No – you would need to collect data and calculate an estimate.

Experimental and theoretical probability

- The probability worked out from carrying out an experiment or collecting data is called the **experimental probability**. This will vary each time you do an experiment.
- The theoretical probability of getting a six on a $\frac{1}{6}$ die is $\frac{1}{6}$. However, this does not mean every sixth throw of a die will be a six.
- The more times I throw the die, the closer to the theoretical probabilities the results will become.
- If I could throw the die an infinite number of times, the probabilities would be $\frac{1}{6}$ exactly.



Experimental and theoretical probability

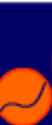


Does this refer to a probability you could establish theoretically or experimentally?

The probability of two numbers below 10, chosen at random, adding up to 8.

experimental

theoretical



- What are experimental and theoretical probabilities?
- Give examples of two mutually exclusive events.
- Give examples of two events that are not mutually exclusive.
- Give examples of two independent events.
- Give examples of two dependent events.
- A question asks for “the probability of event A or event B occurring”. Should you add or multiply the probabilities?
- A question asks for “the probability of event A followed by event B”. Should you add or multiply the probabilities?
- A question asks for “the probability of event A and event B occurring in any order”. Should you add or