Warm Up



1)Simplify the formula:

$a = 5 + (n-1)6$

2)Solve the system:

$2x + y = 9$

$9x + 4y = 10$

Math II

UNIT QUESTION: How is a geometric sequence like an exponential function?

Standard: MM2A2, MM2A3

Today's Question:
How do you calculate the sum of an arithmetic sequence?

Standard: MM2A3d,e

Sequences and Series

Sequence

- There are 2 types of Sequences

  Arithmetic:

  You add a common difference each time.

  Geometric:

  You multiply a common ratio each time.

Arithmetic Sequences

Example:

- **{2, 5, 8, 11, 14, …}**
  Add 3 each time
- **{0, 4, 8, 12, 16, …}**
  Add 4 each time
- **{2, -1, -4, -7, -10, …}**
  Add –3 each time

Arithmetic Sequences

- Find the 7th term of the sequence:
  2,5,8,...

Determine the pattern:

Add 3 (known as the common difference)

Write the new sequence:

2,5,8,11,14,17,20

So the 7th number is 20

Arithmetic Sequences

- When you want to find a large sequence, this process is long and there is great room for error.
- To find the 20^{th}, 45^{th}, etc. term use the following formula:

$$\mathbf{a_n = a_1 + (n - 1)d}$$

Arithmetic Sequences

$$a_n = a_1 + (n - 1)d$$

Where:

a_1 is the first number in the sequence

n is the number of the term you are looking for

d is the common difference

a_n is the value of the term you are looking for

Arithmetic Sequences

- Find the 15^{th} term of the sequence:
  34, 23, 12,...

Using the formula $\mathbf{a_n = a_1 + (n - 1)d,}$

$\mathbf{a_1 = 34}$

$\mathbf{d = -11}$

$\mathbf{n = 15}$

$\mathbf{a_n = 34 + (n-1)(-11) = -11n + 45}$

$\mathbf{a_{15} = -11(15) + 45}$

$\mathbf{a_{15} = -120}$

Arithmetic Sequences

Melanie is starting to train for a swim meet. She begins by swimming 5 laps per day for a week. Each week she plans to increase her number of daily laps by 2. How many laps per day will she swim during the 15^{th} week of training?

Arithmetic Sequences

- What do you know?

$$\mathbf{a_n = a_1 + (n - 1)d}$$

$a_1 = 5$

$d = 2$

$n = 15$

$\mathbf{t_{15}} = ?$

Arithmetic Sequences

- $t_n = t_1 + (n - 1)d$
- $t_n = 5 + (n - 1)2$
- $t_n = 2n + 3$
- $t_{15} = 2(15) + 3$
- $t_{15} = 33$

During the 15^{th} week she will swim 33 laps per day.

Arithmetic Series

- The sum of the terms in a sequence are called a series.
- There are two methods used to find arithmetic series:

  Formula

  Sigma Notation

Arithmetic Series

Sequence	Sum	Average	Avg. x n
2, 5, 8, 11, 14	40	8	40
1, 8, 15, 22	46	23/2	46
-1,1, 3	3	1	3

What's another way to get the average without adding all the numbers and dividing by n?

Arithmetic Series

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

- a_1 = first term
- n = number of terms
- a_n = value of the nth term

In many problems, you will first have to use the arithmetic sequence formula.

Arithmetic Series

- Find the sum of the first forty terms in an arithmetic series in which $t_1 = 70$ and $d = -21$

So:

- $n = 40$
- $a_1 = 70$
- $a_n = ?$
- $S_n = ?$

Arithmetic Series

Use the sequence formula to find t_{40}.

$\mathbf{a_n = 70 + (n - 1)(-21)}$

$\mathbf{a_n = -21n + 91}$

$\mathbf{a_{40} = -749}$

Arithmetic Series

- Now use the series formula:

$$S_{40} = 40\,[(70 + -749)/2]$$

$$S_{40} = 40(-339.5)$$

$$S_{40} = -13{,}580$$

So the sum of the first 40 terms is -13,580

Arithmetic Series

- Find a formula for the partial sum of this series:

  1+3+5+….+(2n-5)+(2n-3)+(2n-1)

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S_n = n\left(\frac{1 + 2n - 1}{2}\right)$$

$$S_n = n^2$$