

Matrices, Determinants and Inverses

1. Evaluating Determinants of 2×2 Matrices
2. Using Inverse Matrices to Solve Equations

1) Evaluating Determinants of 2x2 Matrices

When you multiply two matrices together, in the order AB or BA , and the result is the **identity matrix**, then matrices A and B are **inverses**.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



**Identity
matrix**

1) Evaluating Determinants of 2x2 Matrices

To show two matrices are **inverses**. You only have to prove ONE of these.

$$AB = I \quad \underline{\text{OR}} \quad BA = I$$

$$AA^{-1} = I \quad \underline{\text{OR}}$$

$$A^{-1}A = I$$

Inverse of A

Inverse of A

1) Evaluating Determinants of 2x2 Matrices

Example 1:

Show that B is the multiplicative inverse of A .

$$A = \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$$

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$$AB = I.$$

Therefore, B is
the inverse of A
and A is the
inverse of B .

$$AB = \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Check by multiplying BA ... answer
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$$AB = \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix}$$

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$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1) Evaluating Determinants of 2x2 Matrices

Example 2:

Show that the matrices are multiplicative inverses.

$$A = \begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} -8 & 5 \\ 3 & -2 \end{bmatrix}$$

1) Evaluating Determinants of 2x2 Matrices

Example 2:

Show that the matrices are multiplicative inverses.

$$A = \begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} -8 & 5 \\ 3 & -2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -8 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = I.$$

Therefore, B is
the inverse of A
and A is the
inverse of B .

1) Evaluating Determinants

of 2x2 Matrices

The **determinant** is used to tell us if an inverse exists.

If **$\det \neq 0$** , an inverse exists.

If **$\det = 0$** , no inverse exists.

1) Evaluating Determinants

of 2x2 Matrices

To calculate a **determinant**...

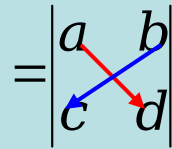
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

1) Evaluating Determinants

of 2x2 Matrices

To calculate a **determinant**...

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$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$


Multiply along the
diagonal

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of 2x2 Matrices

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Multiply along the
diagonal

$$= ad - bc$$

Equation to find
the determinant

1) Evaluating Determinants

of 2x2 Matrices

Example 1: Evaluate the determinant.

$$\det \begin{bmatrix} 7 & 8 \\ -5 & -9 \end{bmatrix}$$

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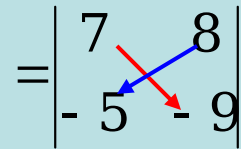
1) Evaluating Determinants

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1) Evaluating Determinants

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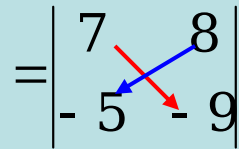
Example 1: Evaluate the determinant.

$$\det \begin{bmatrix} 7 & 8 \\ -5 & -9 \end{bmatrix}$$

$$\det = \begin{vmatrix} 7 & 8 \\ -5 & -9 \end{vmatrix}$$

$$\det = -23$$

Therefore, there
is an inverse.

$$= \begin{vmatrix} 7 & 8 \\ -5 & -9 \end{vmatrix}$$


$$= (7)(-9) - (8)(-5)$$

$$= -23$$

1) Evaluating Determinants

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Example 2: Evaluate the determinant.

$$\det \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix}$$

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Example 2: Evaluate the determinant.

$$\det \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix}$$

$$\det = \begin{vmatrix} 4 & 2 \\ 4 & 2 \end{vmatrix}$$

$$= (4)(2) - (4)(2)$$

$$= 0$$

1) Evaluating Determinants

of 2x2 Matrices

Example 2: Evaluate the determinant.

$$\det \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix} \quad \det = \begin{vmatrix} 4 & 2 \\ 4 & 2 \end{vmatrix}$$

$$= (4)(2) - (4)(2)$$

$$= 0$$

$$\det = 0$$

Therefore,
there is no
inverse.

1) Evaluating Determinants

of 2x2 Matrices

How do you know if a matrix has an inverse
AND what that inverse is?

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

**Equations to
find an inverse
matrix**

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$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1) Evaluating Determinants

of 2x2 Matrices

Example 1:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$M = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix}$$

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Step 1: Find $\det M$

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$$M = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix}$$

Step 1: Find $\det M$

$$\begin{aligned} ad - bc &= (-2)(-4) - (2)(5) \\ &= -2 \end{aligned}$$

$\det M = -2$, the inverse of M exists.

1) Evaluating Determinants

of 2x2 Matrices

Example 1:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$M = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix} \quad \text{Step 2: Rewrite the matrix in } \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ form.}$$

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Change signs

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Determine whether the matrix has an inverse. If an inverse exists, find it.

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Change signs

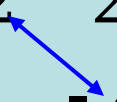
$$= \begin{bmatrix} ? & -2 \\ -5 & ? \end{bmatrix}$$

1) Evaluating Determinants

of 2x2 Matrices

Example 1:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$M = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix}$$


Step 2: Rewrite the matrix in

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

form.

Change
positions

$$= \begin{bmatrix} ? & -2 \\ -5 & ? \end{bmatrix}$$

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Example 1:

Determine whether the matrix has an inverse. If an inverse exists, find it.

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Step 2: Rewrite the matrix in $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **form.**

Change positions

$$= \begin{bmatrix} -4 & -2 \\ -5 & -2 \end{bmatrix}$$

1) Evaluating Determinants

of 2x2 Matrices

Example 1:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$M = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix} \quad \text{Step 3: Use the equation to find the inverse.}$$

$$M^{-1} = \frac{1}{-2} \begin{bmatrix} -4 & -2 \\ -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 \\ -5 & -2 \end{bmatrix}$$

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$$= \begin{bmatrix} -4 & -2 \\ -5 & -2 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 2 & 1 \\ 2.5 & 1 \end{bmatrix}$$

1) Evaluating Determinants

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Example 2:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

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Example 2:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$$

$$\begin{aligned} ad - bc &= (2)(3) - (4)(1) \\ &= 2 \end{aligned}$$

1) Evaluating Determinants

of 2x2 Matrices

Example 2:

Determine whether the matrix has an inverse. If an inverse exists, find it.

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1.5 & -2 \\ -0.5 & 1 \end{bmatrix}$$