

10.1 Tangents to Circles

Geometry

Mrs. Spitz

Spring 2005



Objectives/Assignment

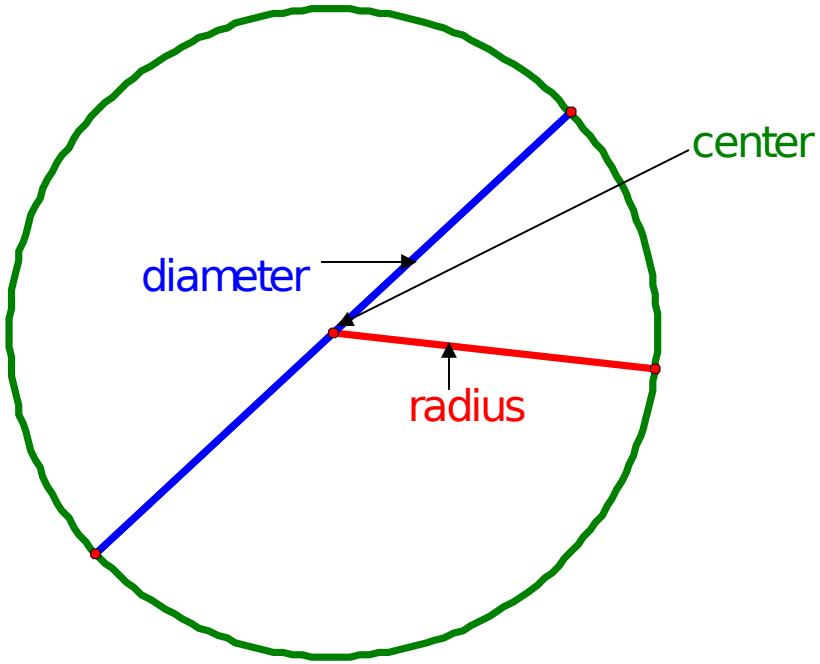
- Identify segments and lines related to circles.
- Use properties of a tangent to a circle.
- Assignment:
  - Chapter 10 Definitions
  - Chapter 10 Postulates/Theorems
  - pp. 599-601 #5-48 all

Some definitions you need

- Circle – set of all points in a plane that are equidistant from a given point called a center of the circle. A circle with center P is called "circle P", or P.
- The distance from the center to a point on the circle is called the radius of the circle. Two circles are congruent if they have the same radius.

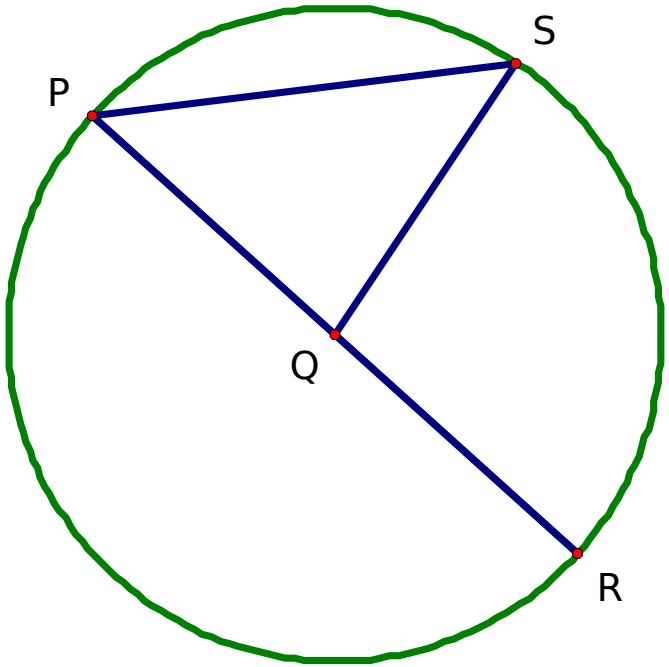
Some definitions you need

- The distance across the circle, through its center is the diameter of the circle. The diameter is twice the radius.
- The terms radius and diameter describe segments as well as measures.



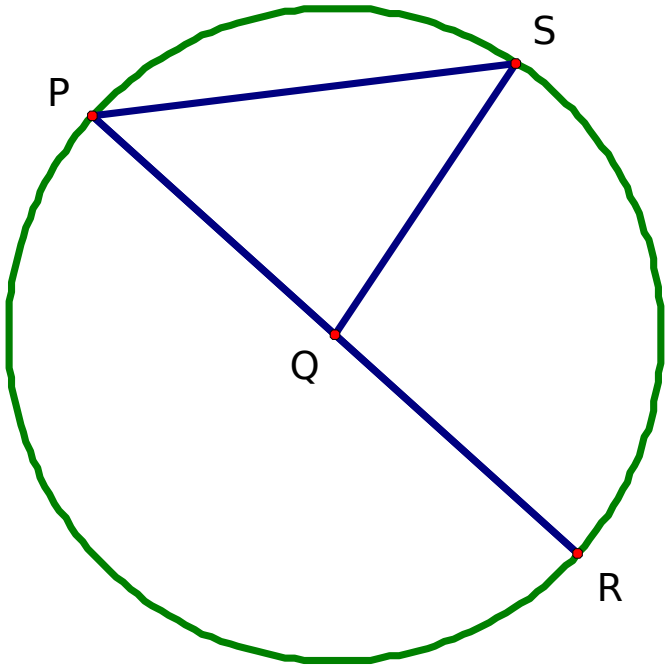
Some definitions you need

- A radius is a segment whose endpoints are the center of the circle and a point on the circle.
- ~~QP~~, ~~QR~~, and ~~QS~~ are radii of ⊙Q. All radii of a circle are congruent.



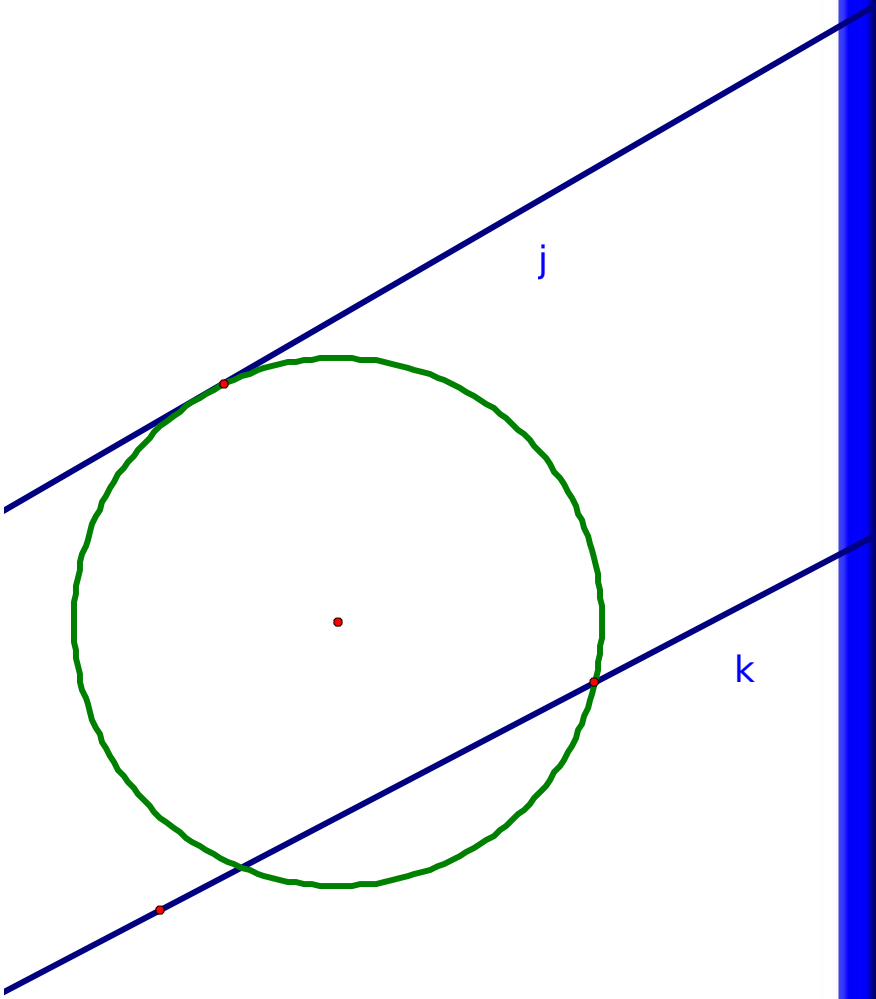
Some definitions you need

- A chord is a segment whose endpoints are points on the circle. $\overline{PS}$ and $\overline{PR}$ are chords.
- A diameter is a chord that passes through the center of the circle. $\overline{PR}$ is a diameter.



Some definitions you need

- A secant is a line that intersects a circle in two points. Line k is a secant.
- A tangent is a line in the plane of a circle that intersects the circle in exactly one point. Line j is a tangent.



Ex. 1: Identifying Special Segments and Lines

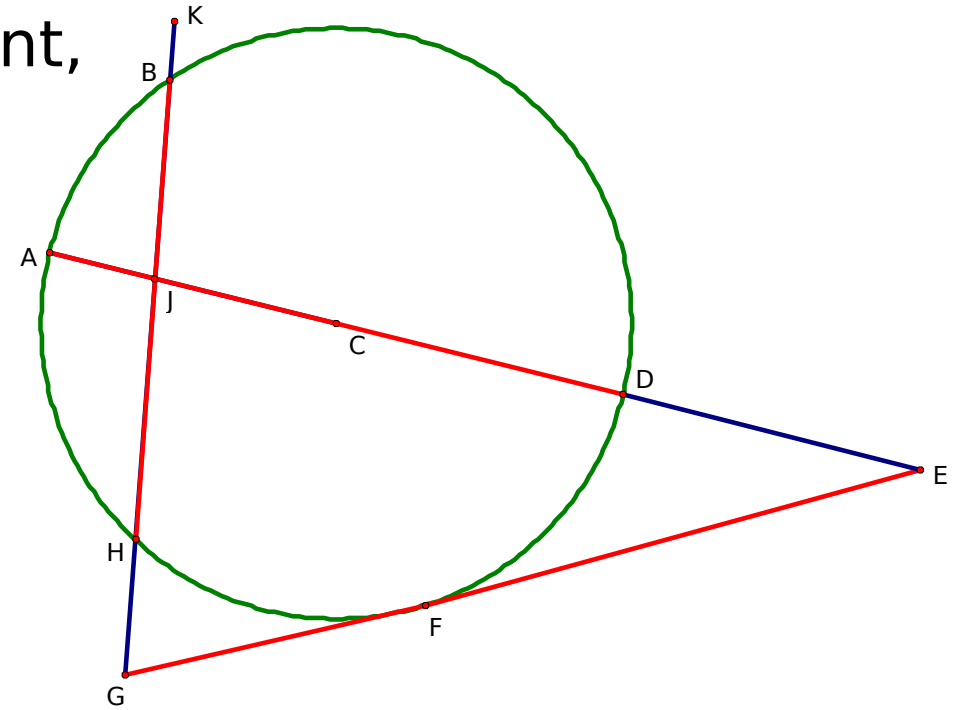
Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of ⊙C.

a. $\overline{AD}$

b. $\overline{CD}$

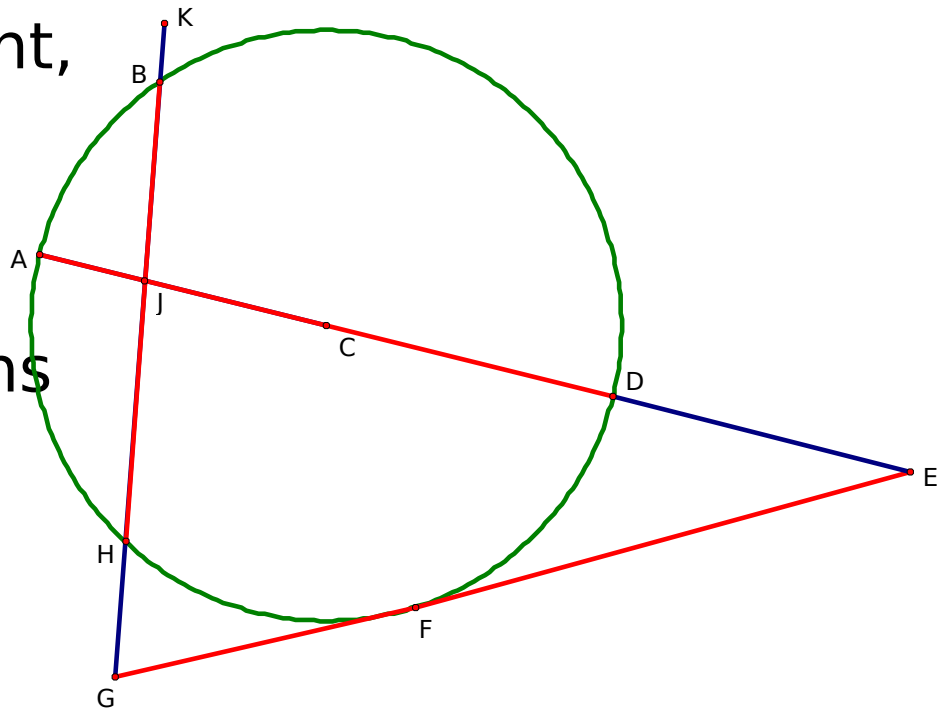
c. $\overleftrightarrow{EG}$

d. $\overline{HB}$



Ex. 1: Identifying Special Segments and Lines

Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of ⊙C.



a. $\overline{AD}$ – Diameter because it contains the center C.

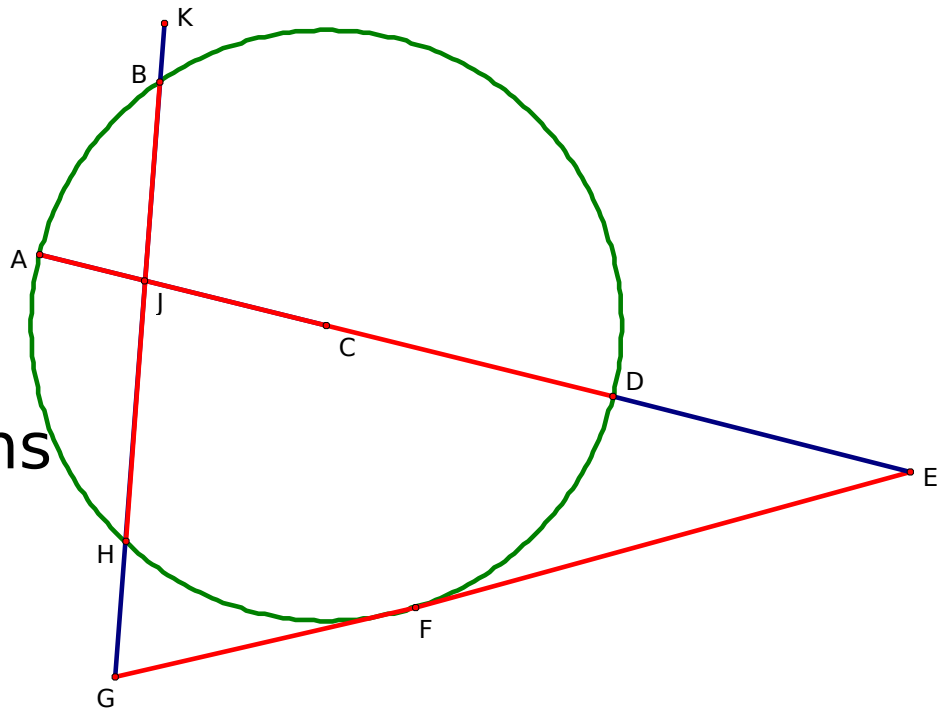
b. CD

c. EG

d. HB

Ex. 1: Identifying Special Segments and Lines

Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of ⊙J.



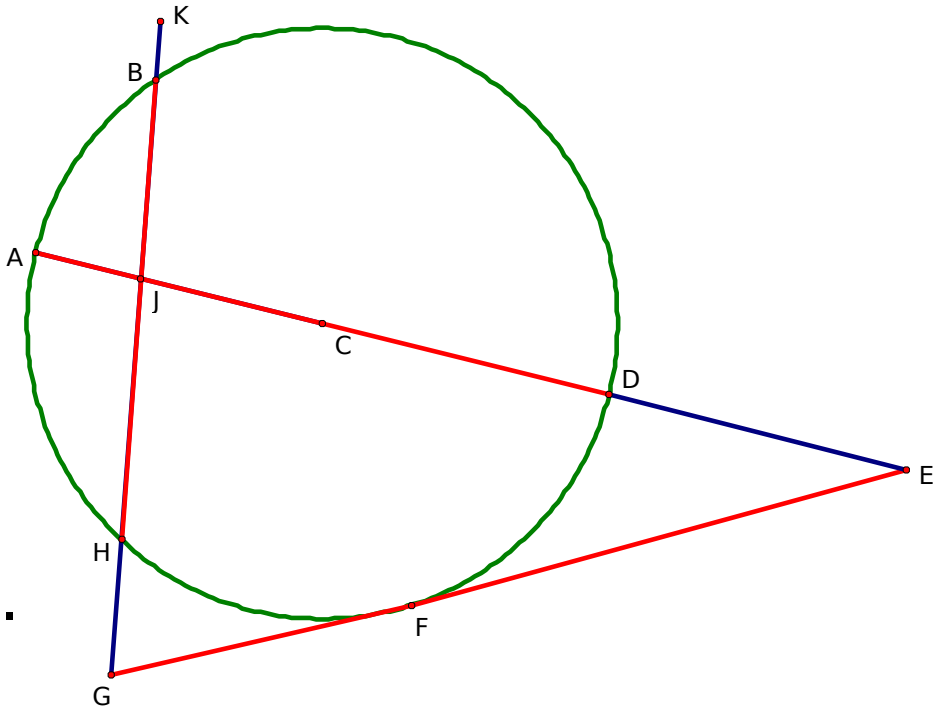
a. $\overline{AD}$ – Diameter because it contains the center C.

b. $\overline{CD}$– radius because C is the center and D is a point on the circle

Ex. 1: Identifying Special Segments and Lines

Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of ⊙C.

c. EG – a tangent because it intersects the circle in one point.

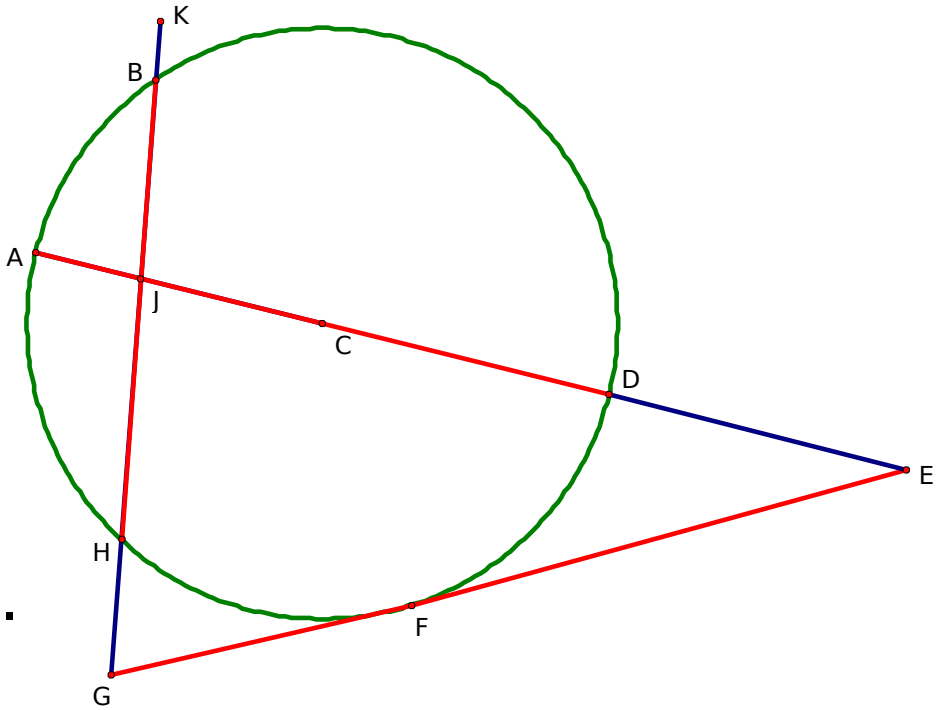


Ex. 1: Identifying Special Segments and Lines

Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of ⊙C.

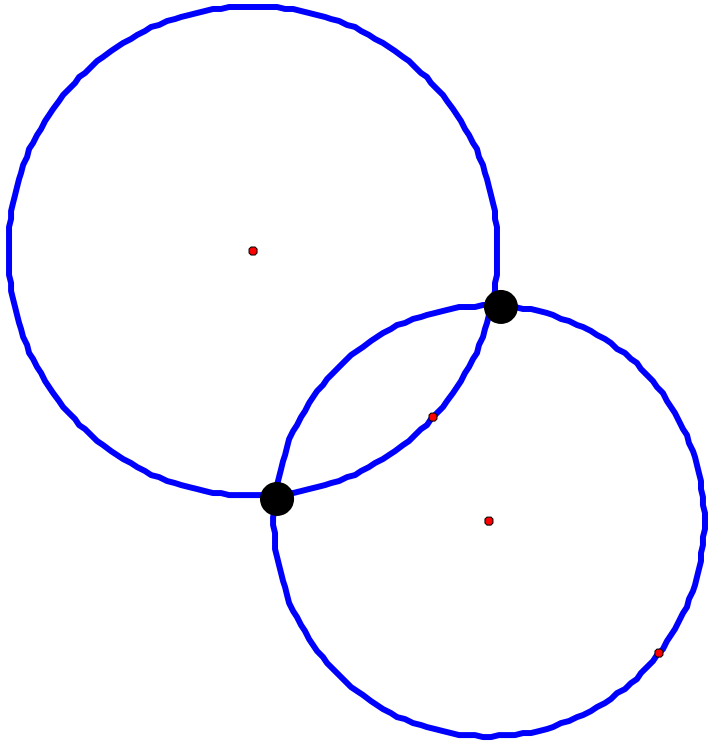
c. EG – a tangent because it intersects the circle in one point.

d. HB is a chord because its endpoints are on



More information you need--

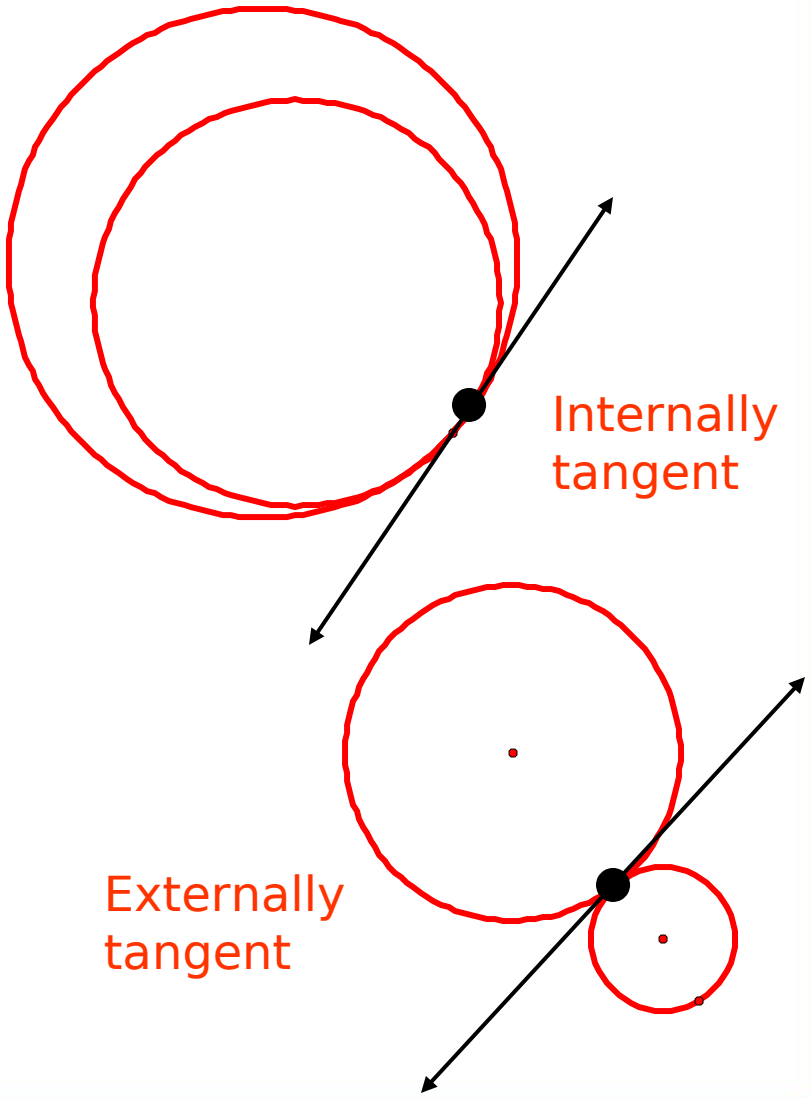
- In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called tangent circles. Coplanar circles that have a common center are called concentric.



2 points of intersection.

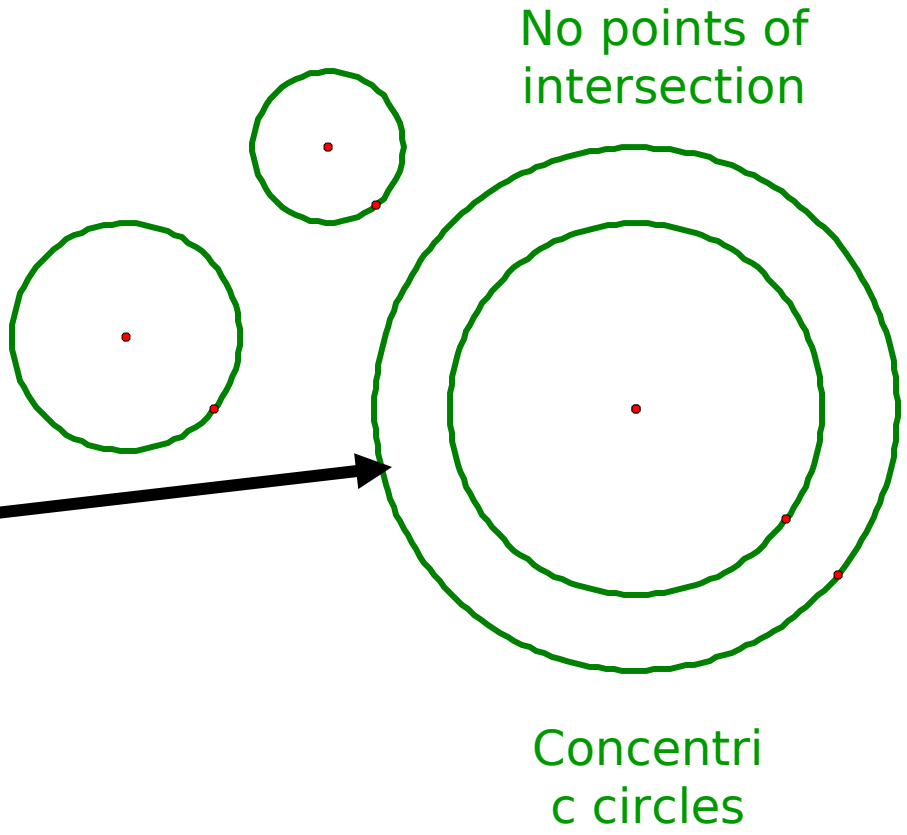
Tangent circles

- A line or segment that is tangent to two coplanar circles is called a common tangent. A common internal tangent intersects the segment that joins the centers of the two circles. A common external tangent does not intersect the segment that joins the center of the two circles.



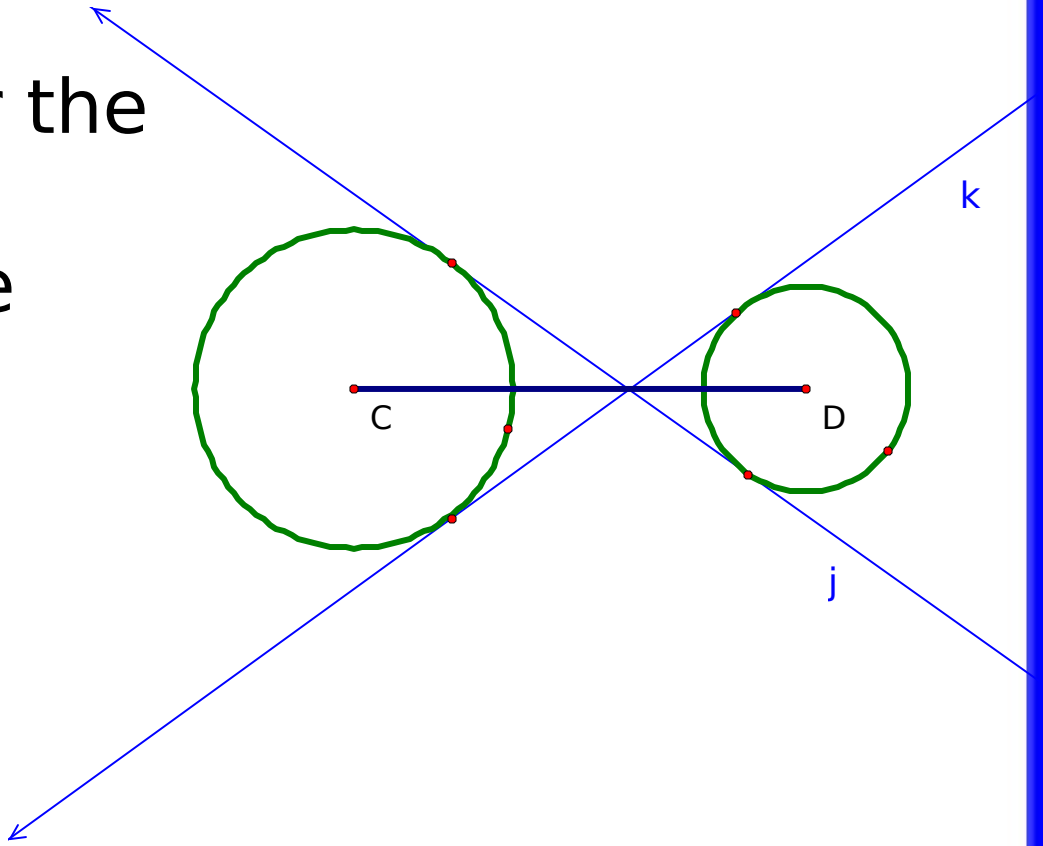
Concentric circles

- Circles that have a common center are called concentric circles.



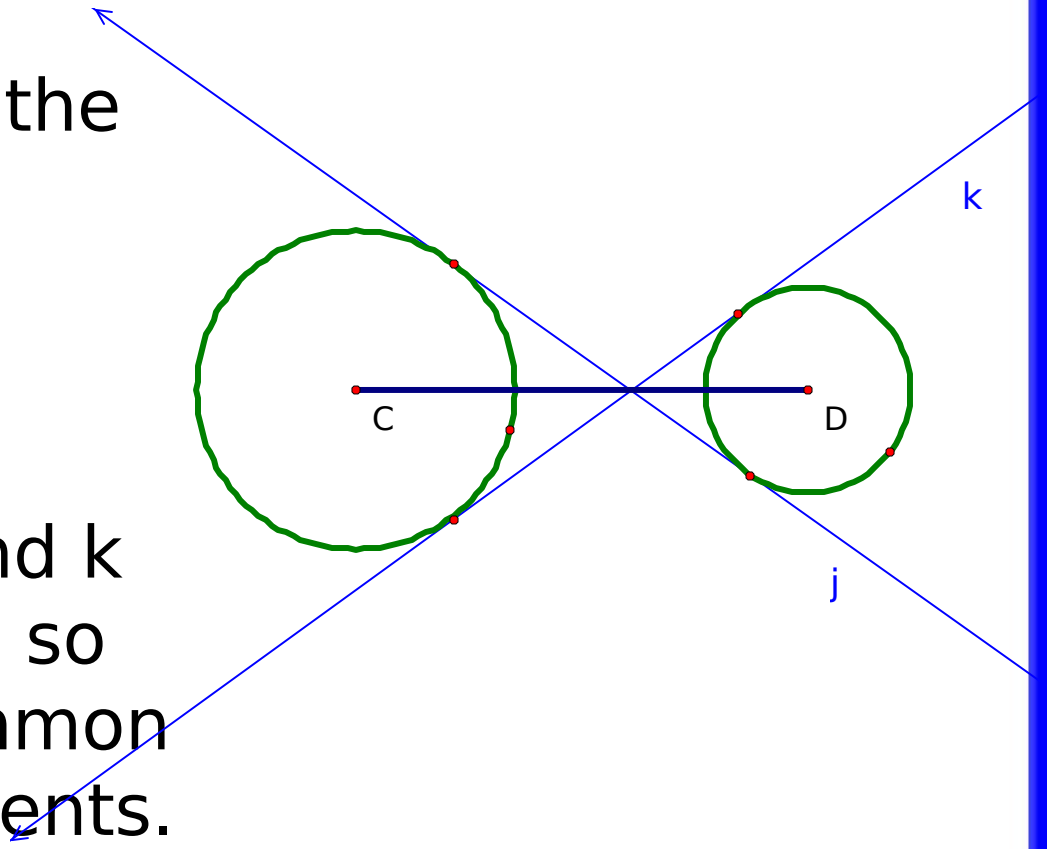
Ex. 2: Identifying common tangents

- Tell whether the common tangents are internal or external.



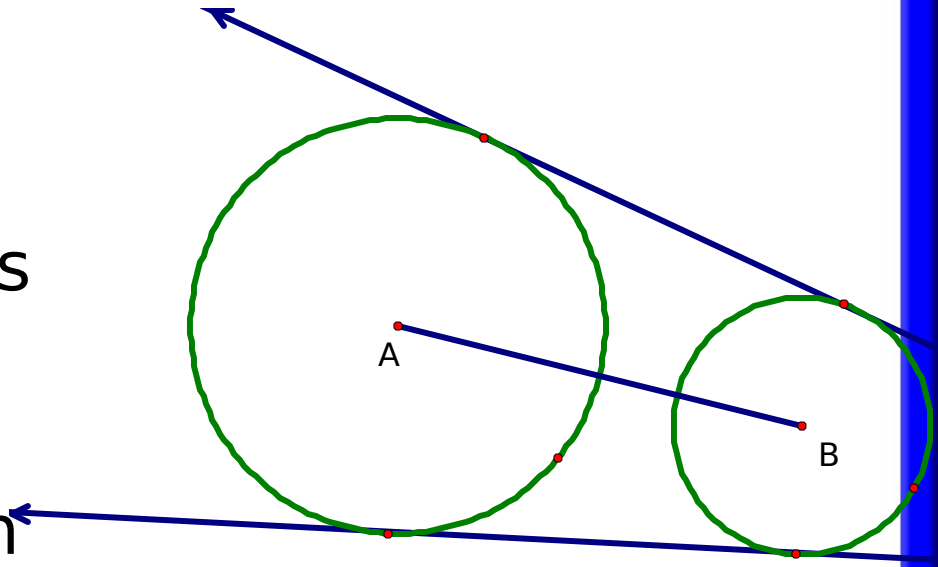
Ex. 2: Identifying common tangents

- Tell whether the common tangents are internal or external.
- The lines j and k intersect CD, so they are common internal tangents.



Ex. 2: Identifying common tangents

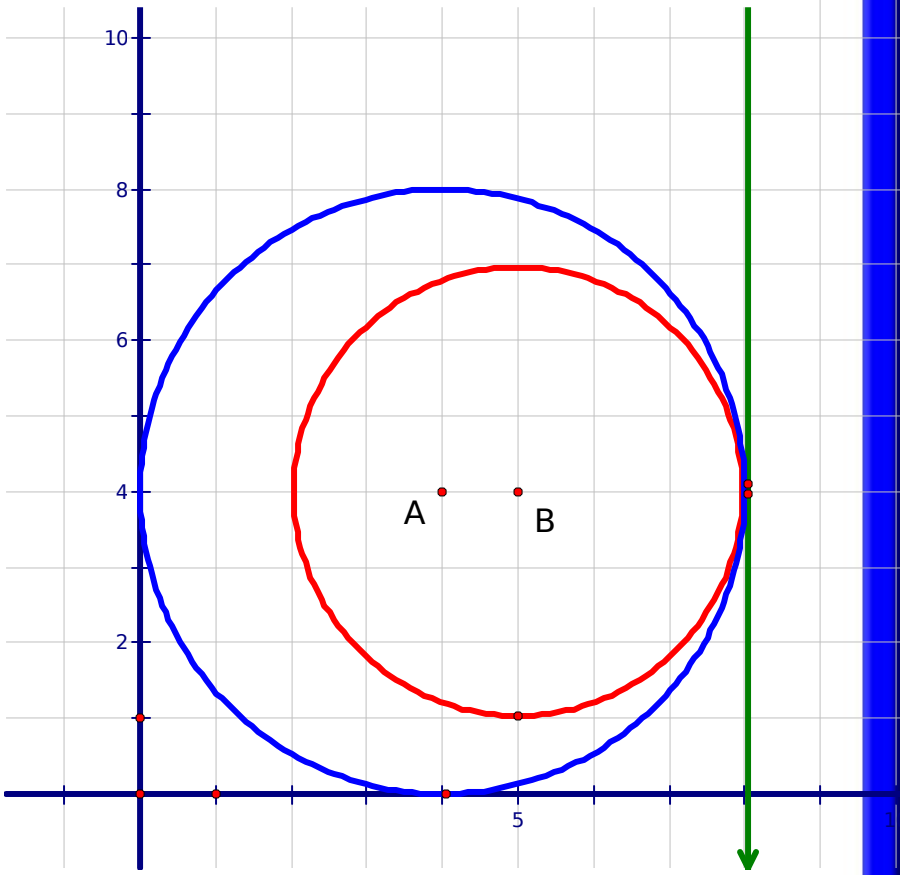
- Tell whether the common tangents are internal or external.
- The lines m and n do not intersect AB, so they are common external tangents.



In a plane, the interior of a circle consists of the points that are *inside* the circle. The exterior of a circle consists of the points that are

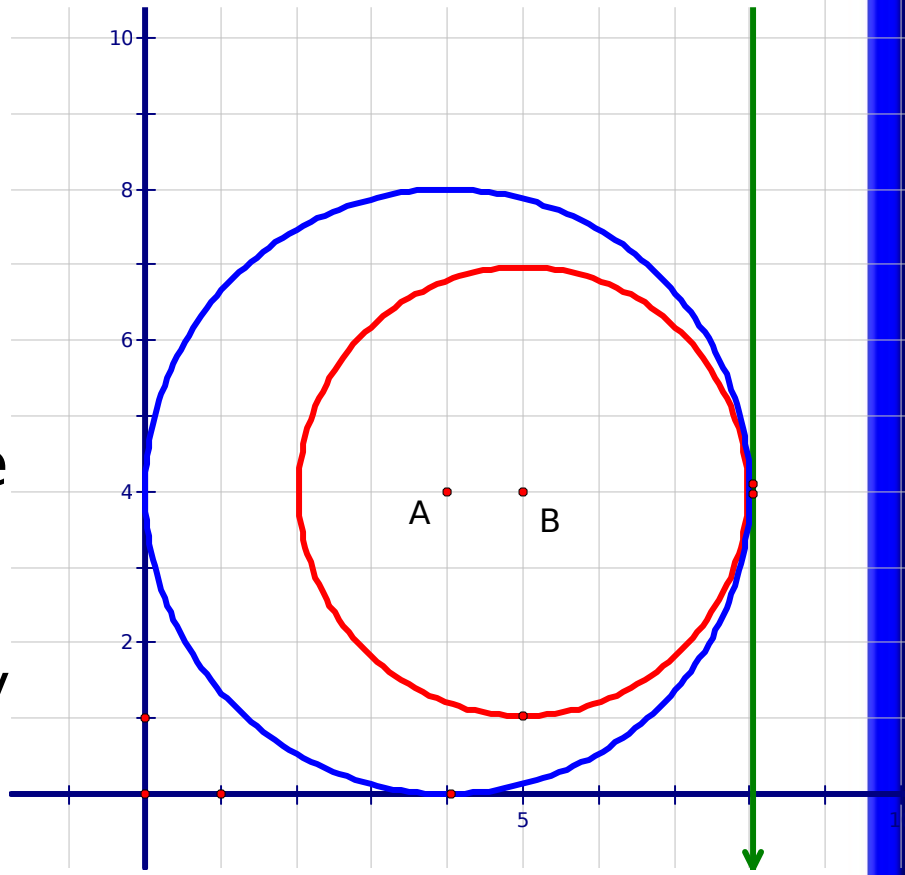
Ex. 3: Circles in Coordinate Geometry

- Give the center and the radius of each circle. Describe the intersection of the two circles and describe all common tangents.



Ex. 3: Circles in Coordinate Geometry

- Center of circle A is (4, 4), and its radius is 4. The center of circle B is (5, 4) and its radius is 3. The two circles have one point of intersection (8, 4). The vertical line x = 8 is the only common tangent of the two circles.



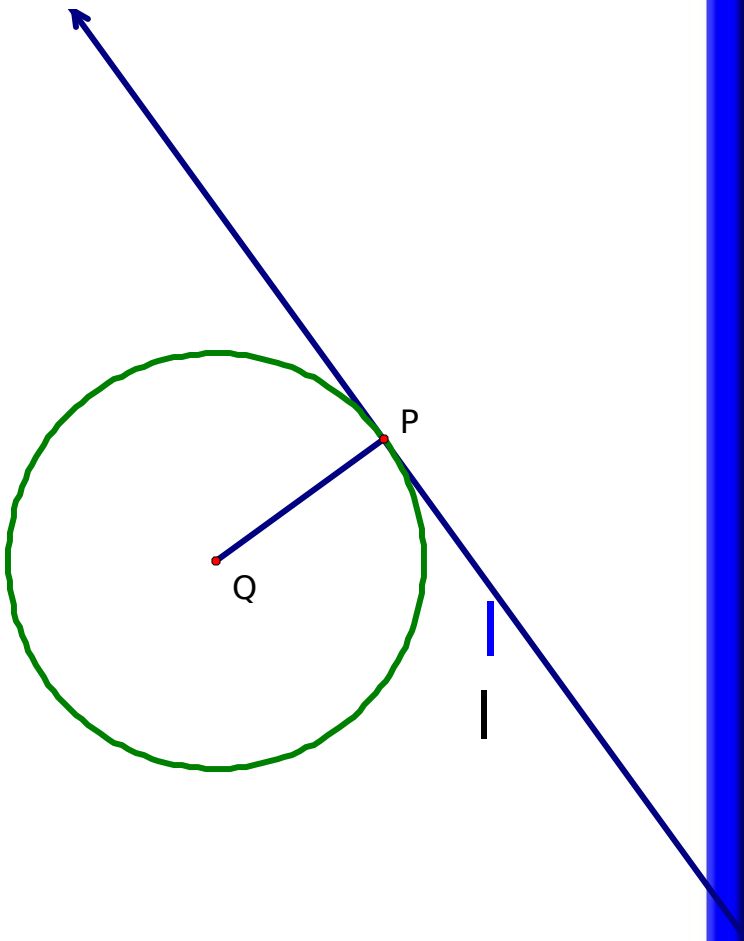


Using properties of tangents

- The point at which a tangent line intersects the circle to which it is tangent is called the *point of tangency*. You will justify theorems in the exercises.

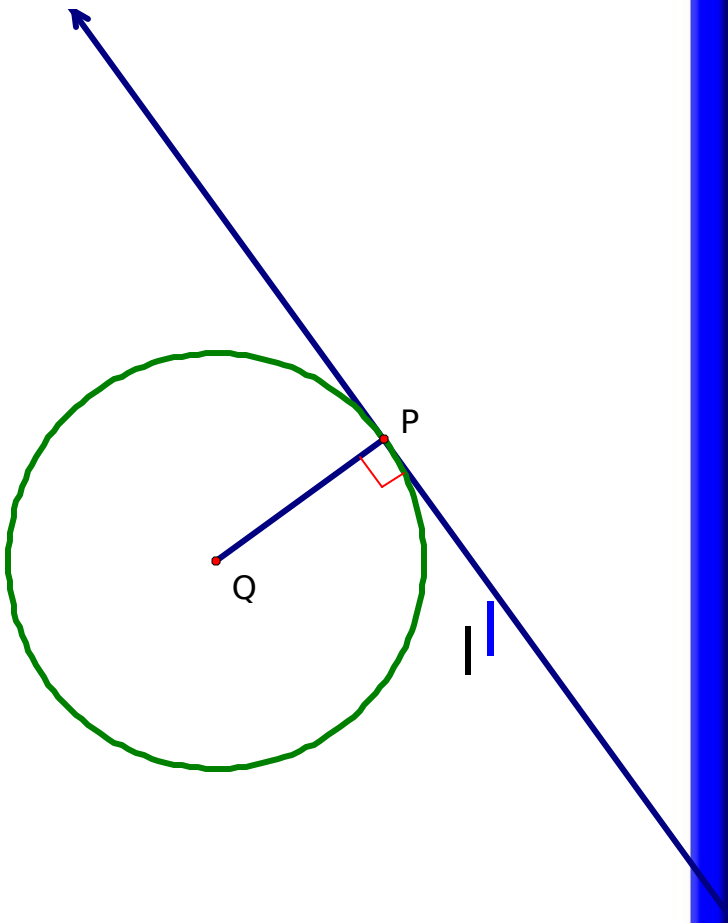
Theorem 10.1

- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. $\odot$
- If l is tangent to $\overline{Q}$ at point P, then l $\perp$QP.



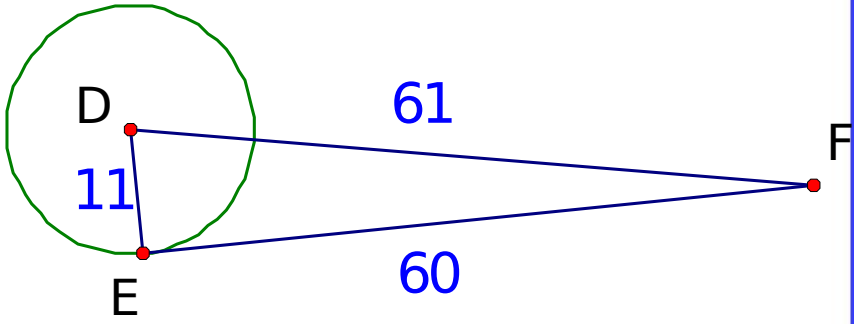
Theorem 10.2

- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on a circle, then the line is tangent to the circle.
- If l ⊥QP at P, then l is tangent to ⊙ Q.



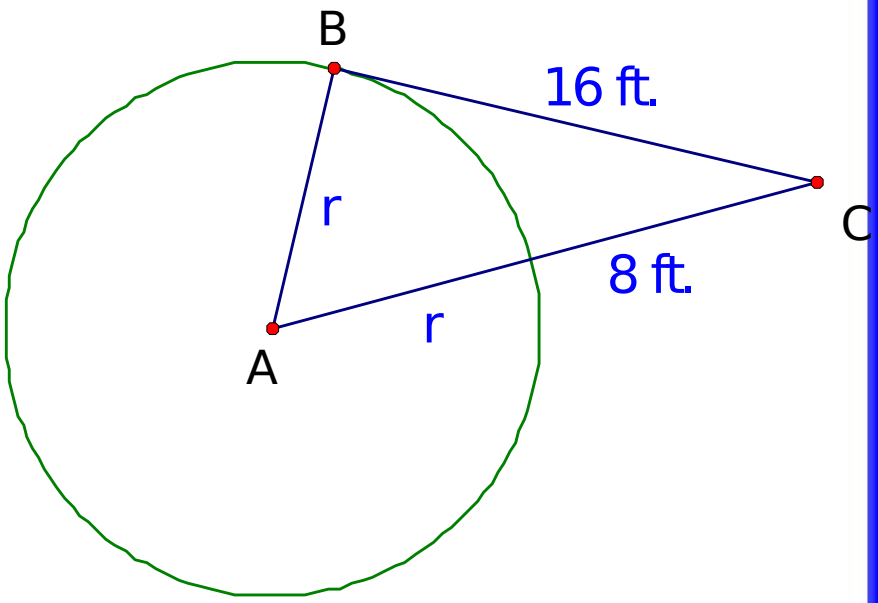
Ex. 4: Verifying a Tangent to a Circle

- You can use the Converse of the Pythagorean Theorem to tell whether EF is tangent to ⊙D.
- Because $11^2 + 60^2 = 61^2$, ΔDEF is a right triangle and $\overline{DE}$ is perpendicular to $\overline{EF}$. So by Theorem 10.2; $\overline{EF}$ is tangent to ⊙D.

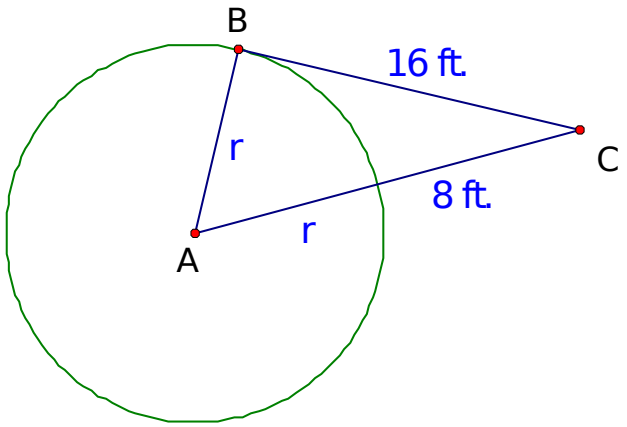


Ex. 5: Finding the radius of a circle

- You are standing at C, 8 feet away from a grain silo. The distance from you to a point of tangency is 16 feet. What is the radius of the silo?
- First draw it. Tangent BC is perpendicular to radius AB at B, so ΔABC is a right triangle; so you can use the Pythagorean theorem to solve.



Solution:



$c^2 = a^2 + b^2$	Pythagorean Thm.
$(r + 8)^2 = r^2 + 16^2$	Substitute values
$r^2 + 16r + 64 = r^2 + 256$	Square of binomial
$16r + 64 = 256$	Subtract r^2 from each side.
$16r = 192$	Subtract 64 from each side.
$r = 12$	Divide.

The radius of the silo is 12 feet.

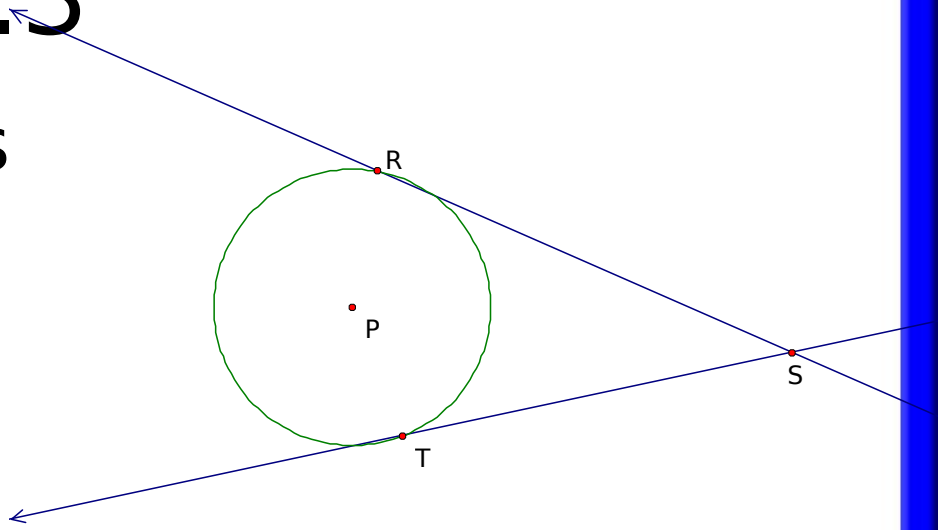


Note:

- From a point in the circle's exterior, you can draw exactly two different tangents to the circle. The following theorem tells you that the segments joining the external point to the two points of tangency are congruent.

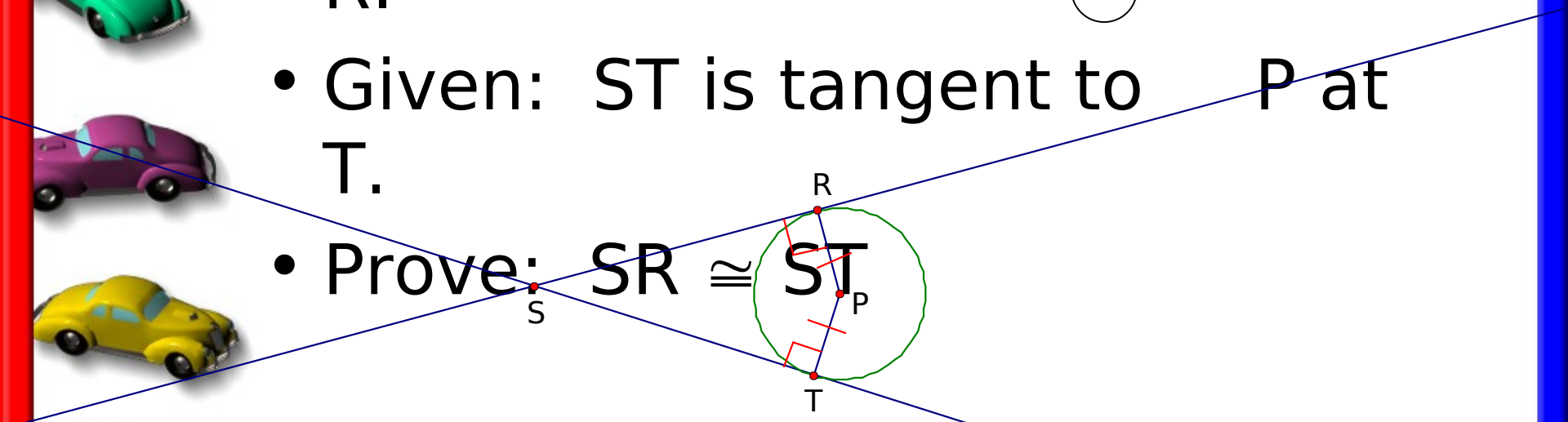
Theorem 10.3

- If two segments from the same exterior point are tangent to the circle, then they are congruent.
- IF $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $SR \cong ST$.

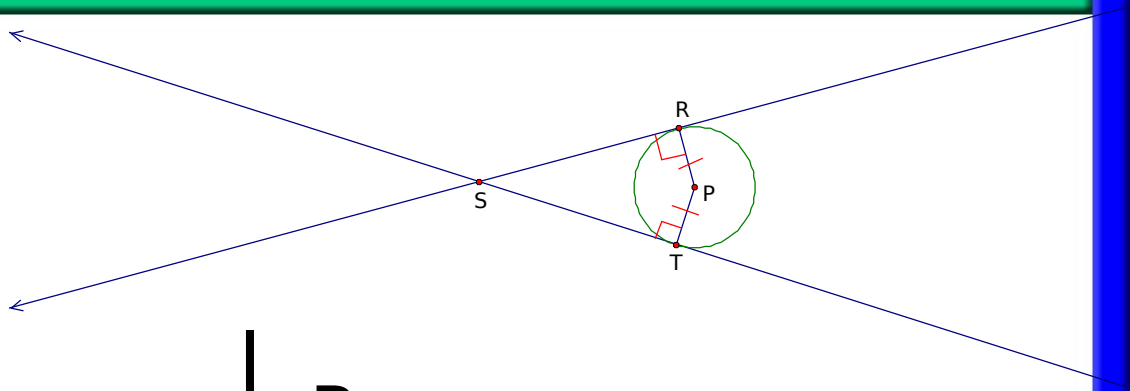


Proof of Theorem 10.3

- Given: SR is tangent to ⊙P at R.
- Given: ST is tangent to ⊙P at T.
- Prove: SR ≅ ST



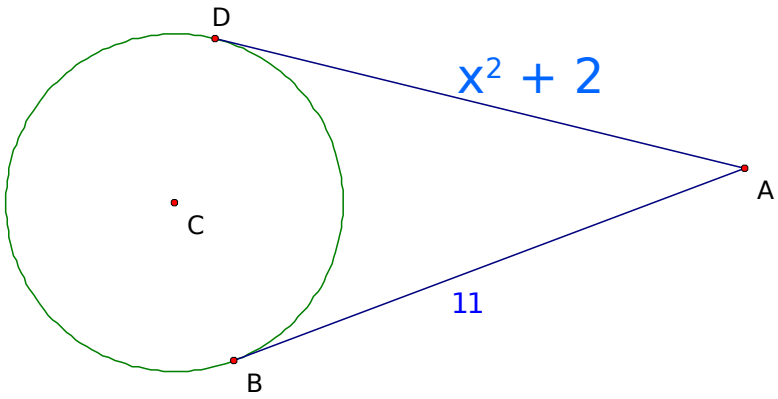
Proof



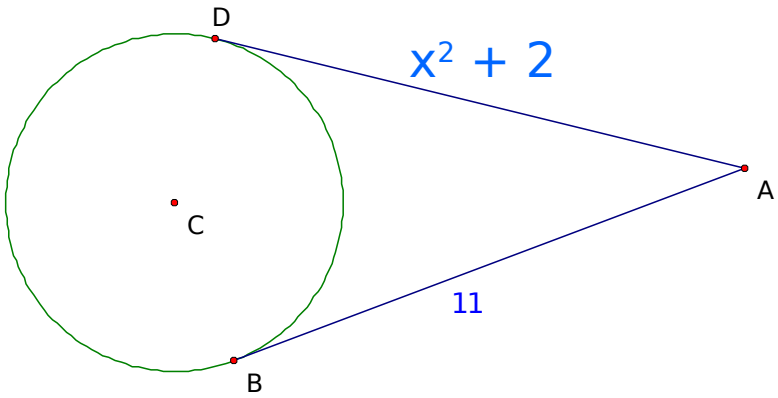
Statements:	Reasons:
SR and ST are tangent to ⊙P	Given
	Tangent and radius are ⊥.
SR ⊥ RP, ST⊥TP	Definition of a circle
RP = TP	Definition of congruence.
RP ≅ TP	Reflexive property
PS ≅ PS	HL Congruence Theorem
ΔPRS ≅ ΔPTS	CPCTC
SR ≅ ST	

Ex. 7: Using properties of tangents

- AB is tangent to ⊙C at B.
- AD is tangent to ⊙C at D.
- Find the value of x.



Solution:



$AB = AD$	Two tangent segments from the same point are $\cong$
$11 = x^2 + 2$	Substitute values
$9 = x^2$	Subtract 2 from each side.
$3 = x$	Find the square root of 9.

The value of x is 3 or -3.