# CIRCLE THEOREMS

# Circle Theorems

Lesson Objectives - Students will:

- Apply your knowledge of circle properties to solving problems.
- Identify and apply circle theorems.

| | |
|---|---|
| Level 1<br>LEARN | Complete “Circle Theorem” lesson |
| Level 2<br>REVISE | Revise Circle Properties page 408 – 409<br>Theorem 1 – Subtended angle at centre (p. 411),<br>Theorem 2 – Angle in a semi-circle (p. 412),<br>Theorem 3 – Angle in same segment (p. 415)<br>Theorem 4 – Angles in cyclic quadratic (p. 415)<br>(GCSE Math TEXT – red and blue)<br>Try these: Page 410 #2,3 , Page 413 #1,5,6, Page 417 #1,2,4,10 |
| Level 3<br>TEST | Circle Theorem TEST<br>Go to<br>http://www.bbc.co.uk/schools/gcsebitesize/maths/shapes/<br>Scroll to “Circle - Higher” then click “Test” |

# Contents

## S5 Circles

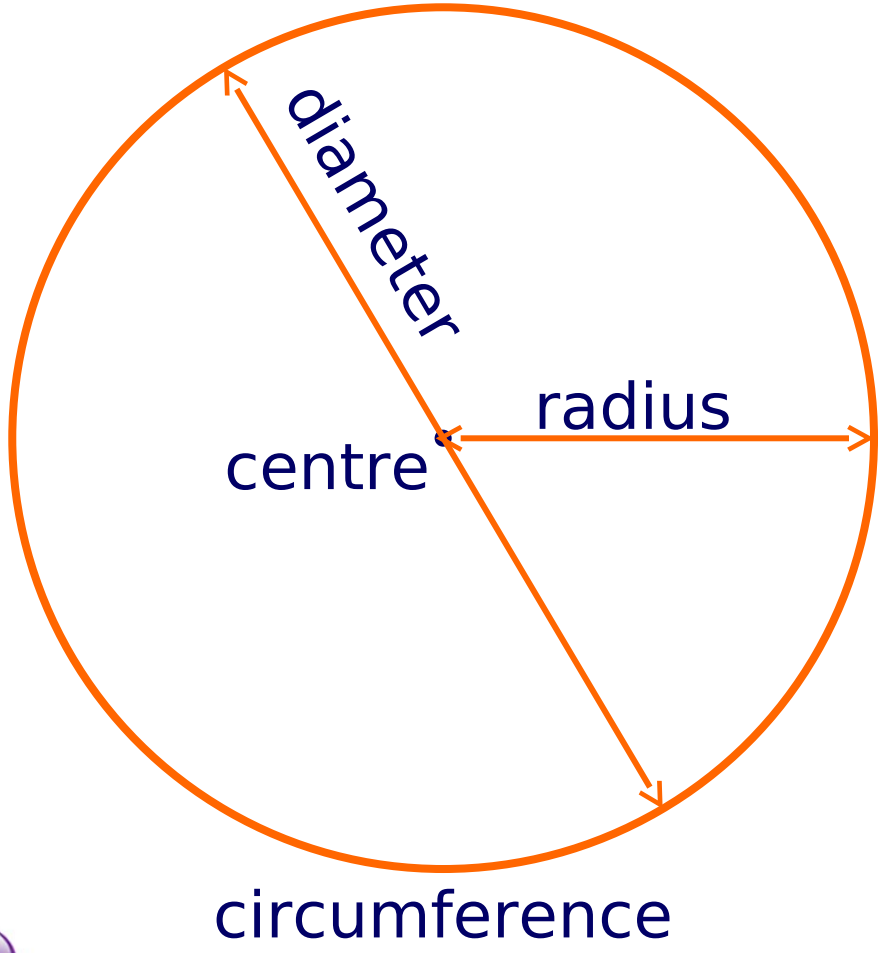

- S5.1 Naming circle parts
- S5.2 Angles in a circle
- S5.3 Tangents and chords
- S5.4 Circumference and arc length
- S5.5 Areas of circles and sectors






# Naming the parts of a circle

A **circle** is a set of points **equidistant** from its **centre**.

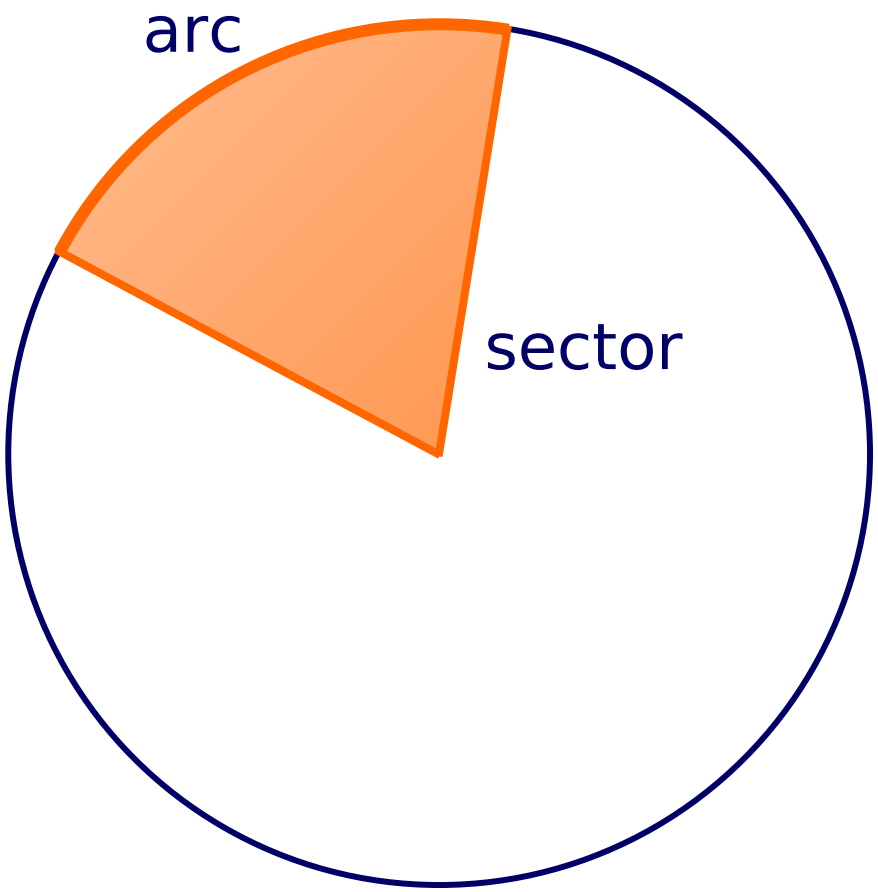



The distance around the outside of a circle is called the **circumference**.

The **radius** is the distance from the centre of the circle to the circumference.

The **diameter** is the distance across the width of the circle through the centre.

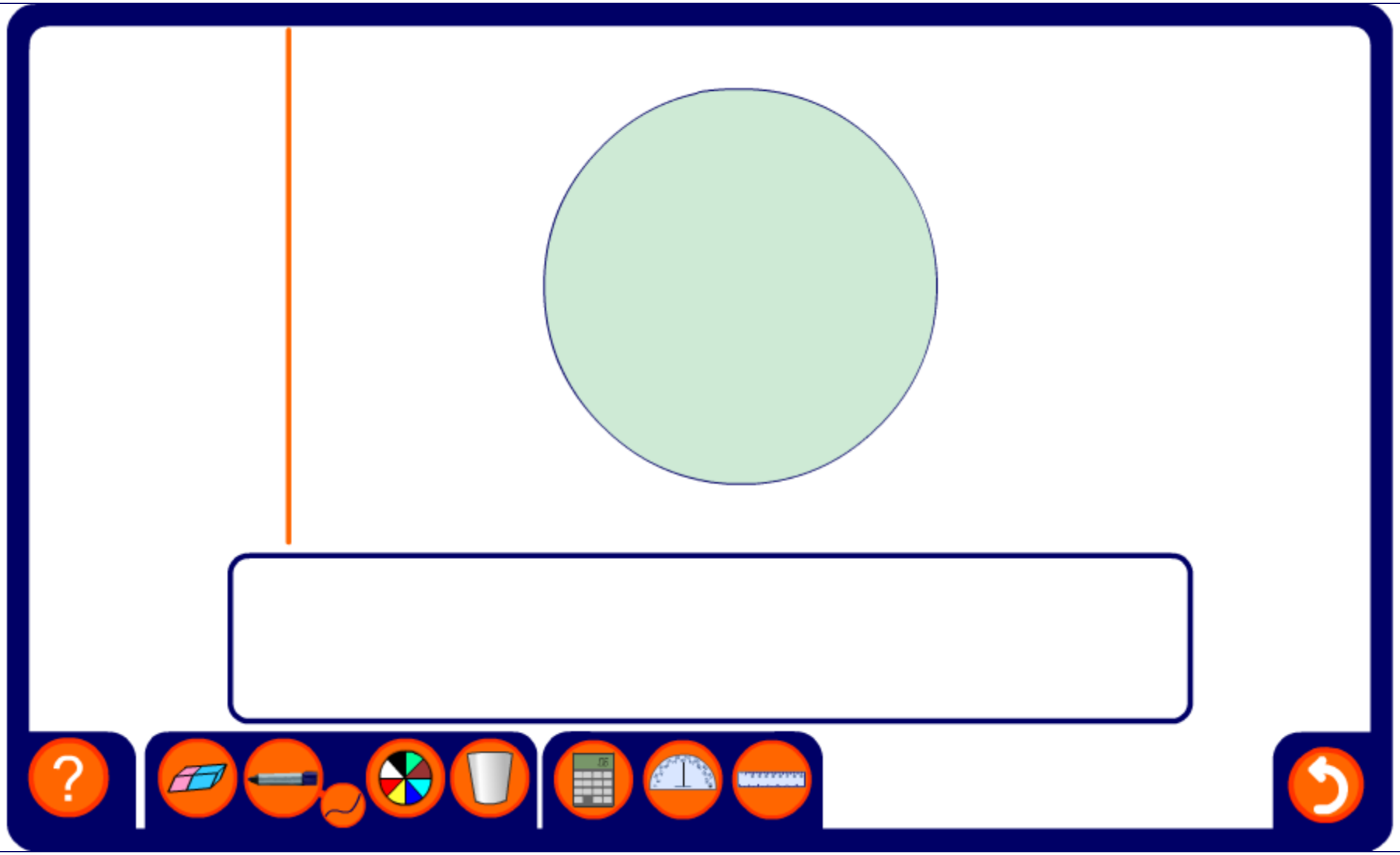
# Arcs and sectors




An **arc** is a part of the circumference.

When an arc is bounded by two **radii** a **sector** is formed.

# A line moving through a circle



# Naming the parts of a circle

centre

radius

circumference

diameter

arc

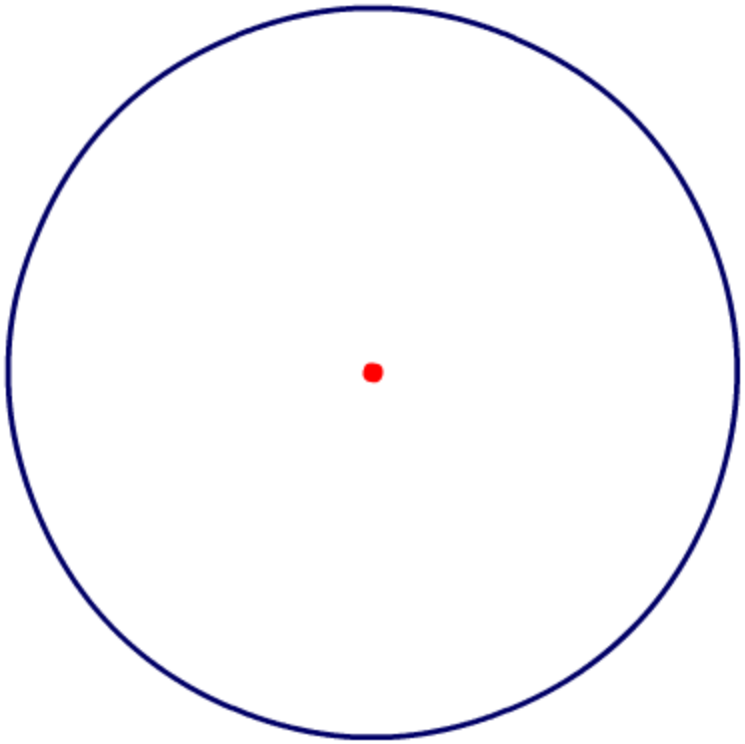
sector

segment

semicircle

tangent

chord



# Contents

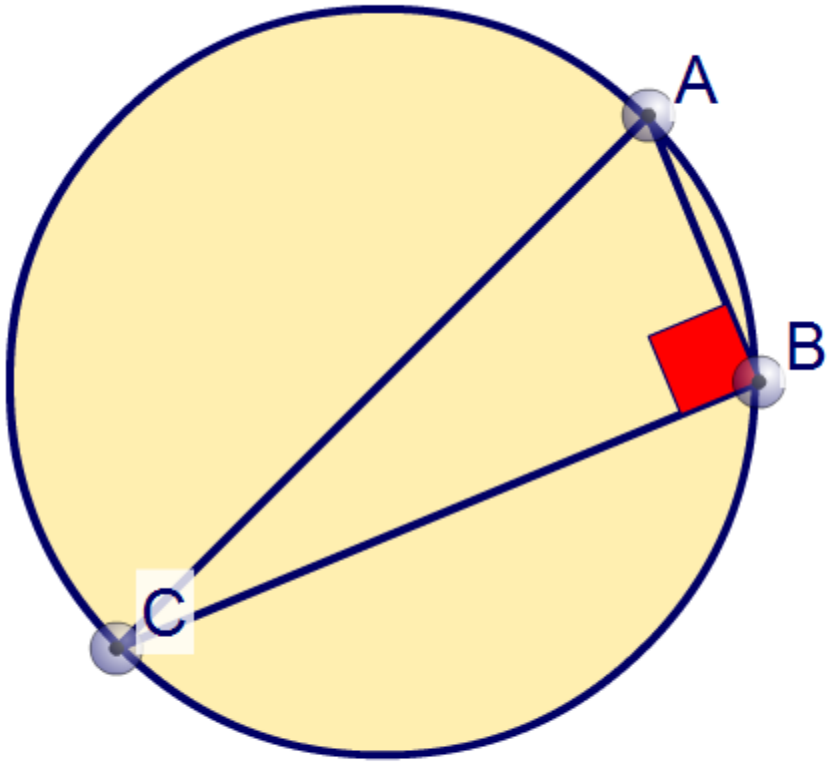
## S5 Circles


- S5.1 Naming circle parts
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# Right angles in a semicircle

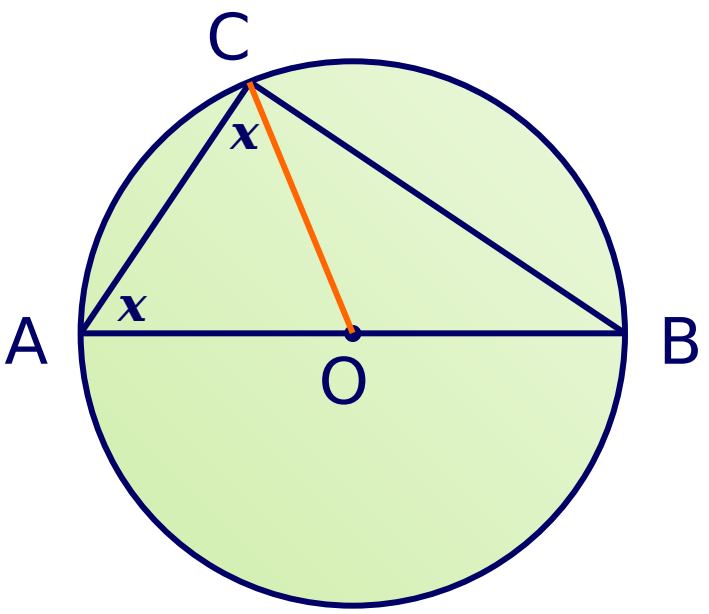



The angle in a semicircle is always 90°

# Right angles in a semicircle

We have just seen a demonstration that the angle in a semicircle is always a right angle.

We can **prove** this result as follows:




Draw a line from C to O. This line is a radius of the circle.

In triangle AOC, OA = OC (both radii)

So, angle OAC = angle OCA

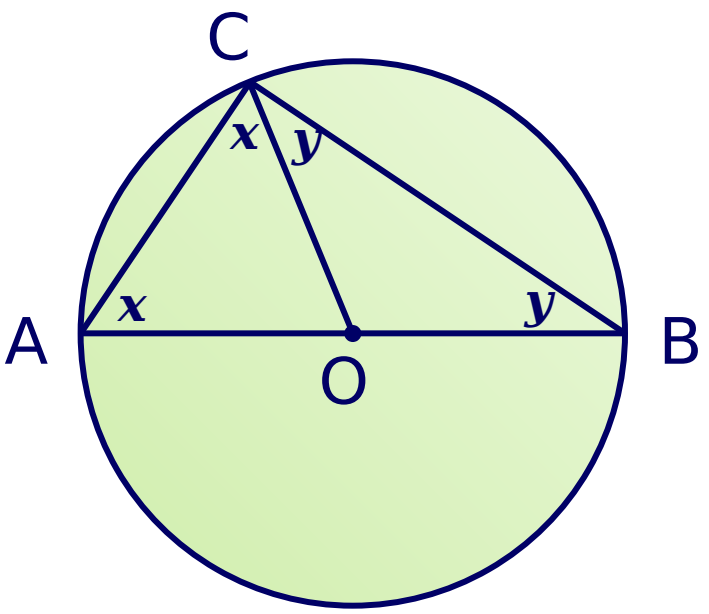
(angles at the base of an isosceles triangle)

Let's call these angles $x$.

# Right angles in a semicircle

We have just seen a demonstration that the angle in a semicircle is always a right angle.

We can **prove** this result as follows:




In triangle BOC,

OB = OC (both radii)

So, angle OBC = angle OCB

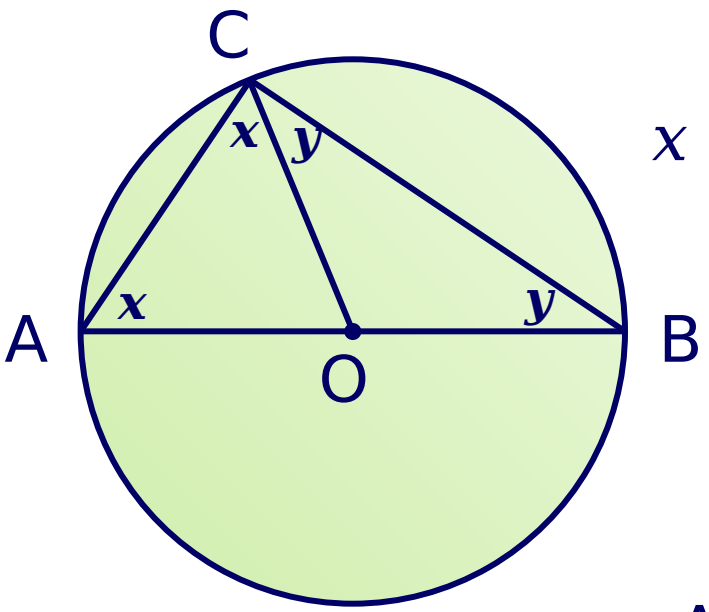
(angles at the base of an isosceles triangle)

Let's call these angles $y$.

# Right angles in a semicircle

We have just seen a demonstration that the angle in a semicircle is always a right angle.

We can **prove** this result as follows:




In triangle ABC,

$x + x + y + y = 180°$ (angles in a triangle)

$2x + 2y = 180°$

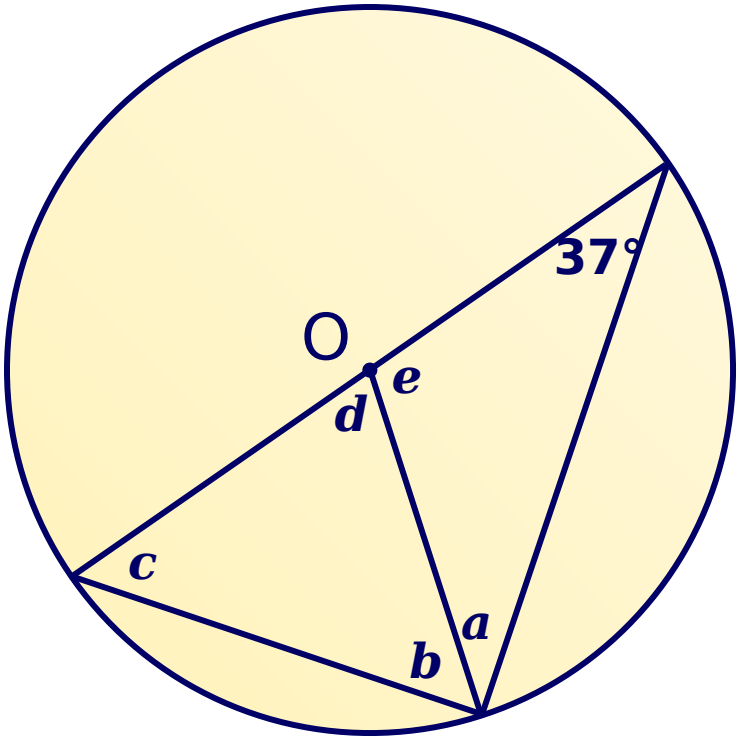
$2(x + y) = 180°$

$x + y = 90°$

Angle ACB = $x + y = 90°$

# Calculating the size of unknown angles

Calculate the size of the labeled angles in the following diagram:




$a = 37°$ (angles at the base of an isosceles triangle)

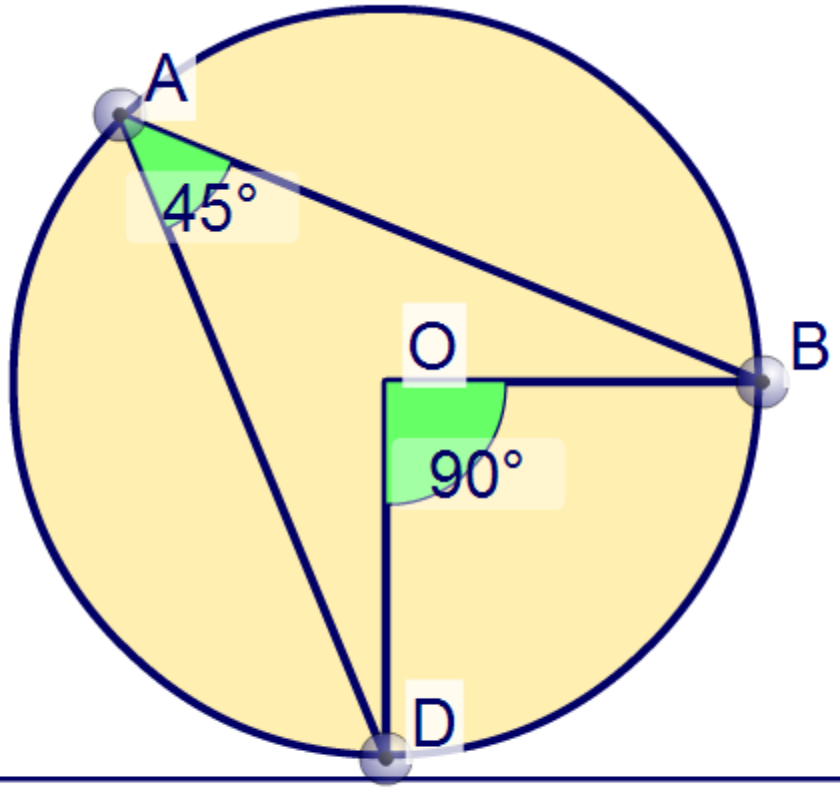
$b = 90° - 37°$
$= 53°$ (angle in a semi-circle)

$c = 53°$ (angles at the base of an isosceles triangle)

$d = 180° - 2 \times 53°$
$= 74°$ (angles in a triangle)

$e = 180° - 74°$
$= 106°$ (angles on a line)

# The angle at the centre

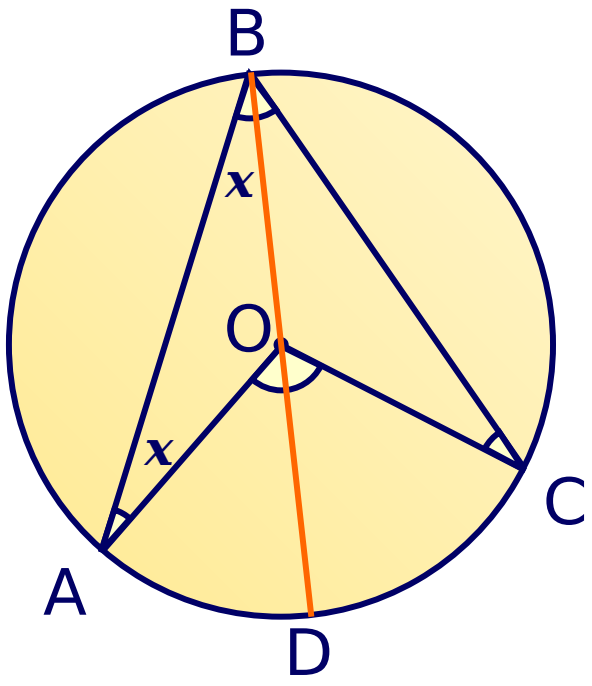



The angle at the centre is twice the angle at the circumference

# The angle at the centre

We have just seen a demonstration that the angle at the centre of a circle is twice the angle at the circumference.

We can prove this result as follows:




Draw a line from B, through the centre O, and to the other side D.

In triangle AOB, OA = OB (both radii)

So, angle OAB = angle OBA

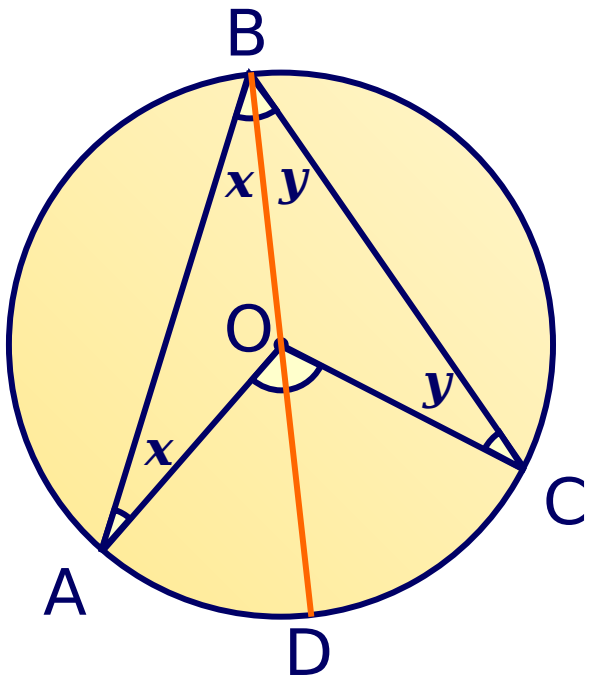
(angles at the base of an isosceles triangle)

Let's call these angles $x$.

# The angle at the centre

We have just seen a demonstration that the angle at the centre of a circle is twice the angle at the circumference.

We can prove this result as follows:




In triangle BOC,

OB = OC (both radii)

So, angle OBC = angle OCB

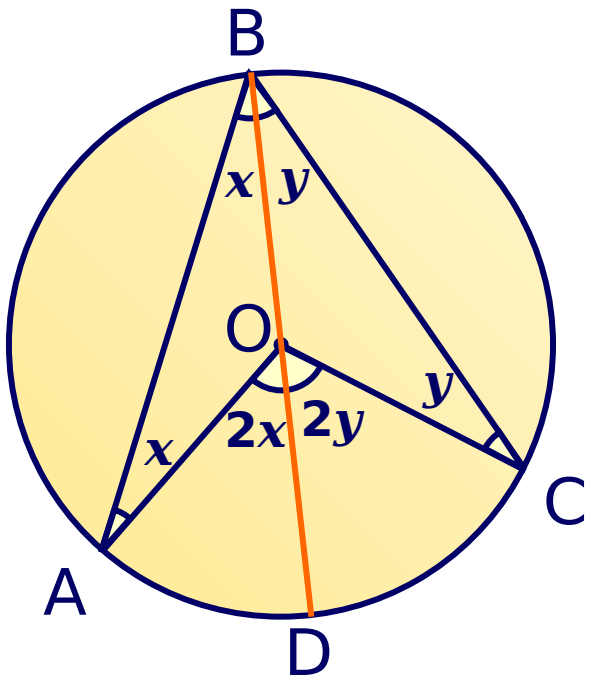
(angles at the base of an isosceles triangle)

Let's call these angles $y$.

# The angle at the centre

We have just seen a demonstration that the angle at the centre of a circle is twice the angle at the circumference.

We can prove this result as follows:




angle AOD = $2x$

and angle COD = $2y$

(the exterior angle in a triangle is equal to the sum of the opposite interior angles)

angle AOC = $2x + 2y$

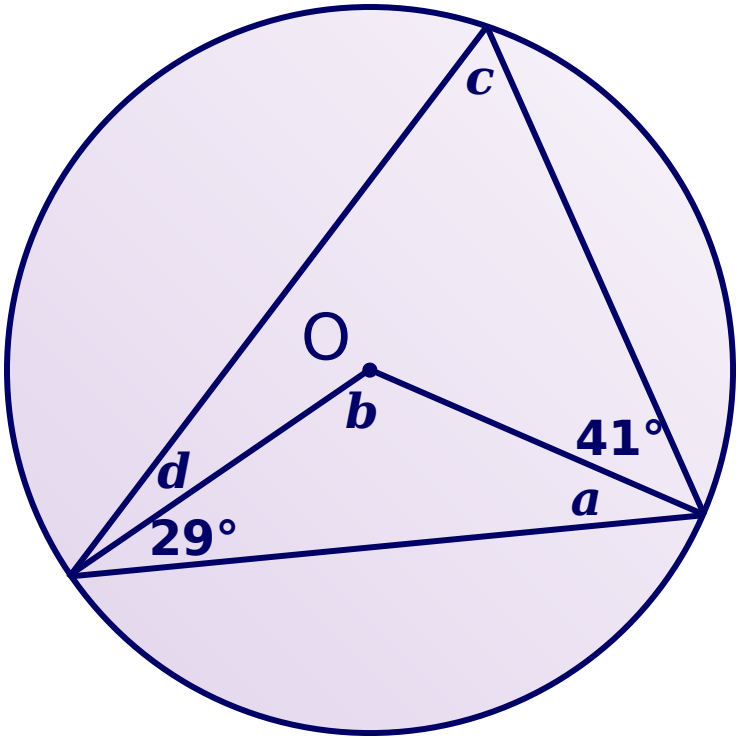
$= 2(x + y)$

angle ABC = $x + y$

$\therefore$ angle AOC = 2 × angle ABC

# Calculating the size of unknown angles

Calculate the size of the labeled angles in the following diagram:



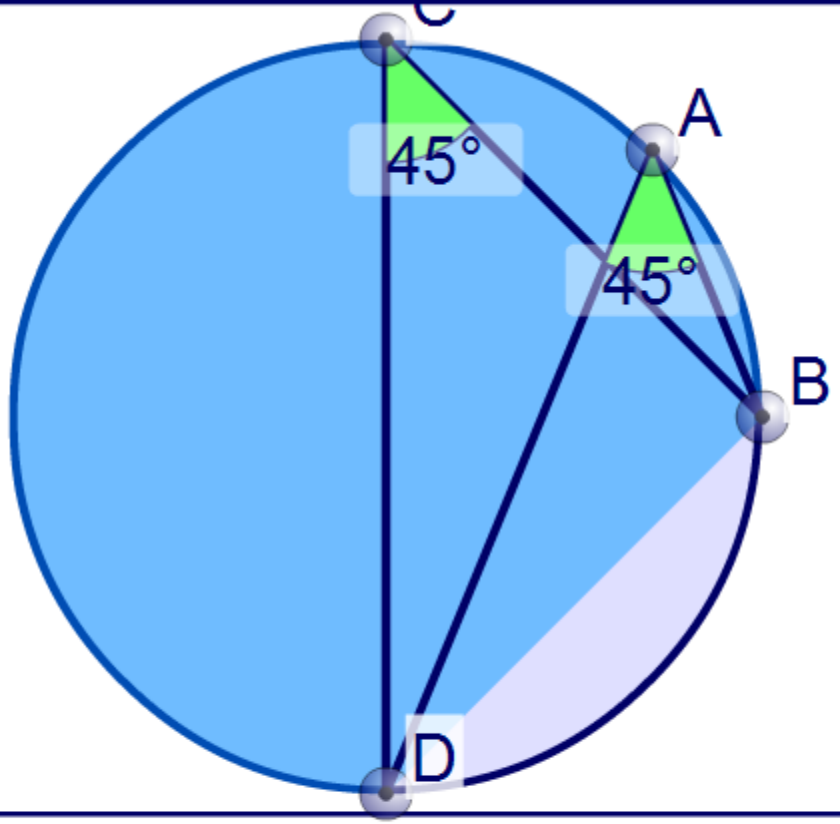

$a = 29°$ (angles at the base of an isosceles triangle)

$b = 180° - 2 \times 29°$
$= 122°$ (angles in a triangle)

$c = 122° \div 2$
$= 61°$ (angle at the centre is twice angle on the circumference)

$d = 180° - (29° + 29° + 41° + 61°)$
$= 20°$ (angles in a triangle)

# Angles in the same segment

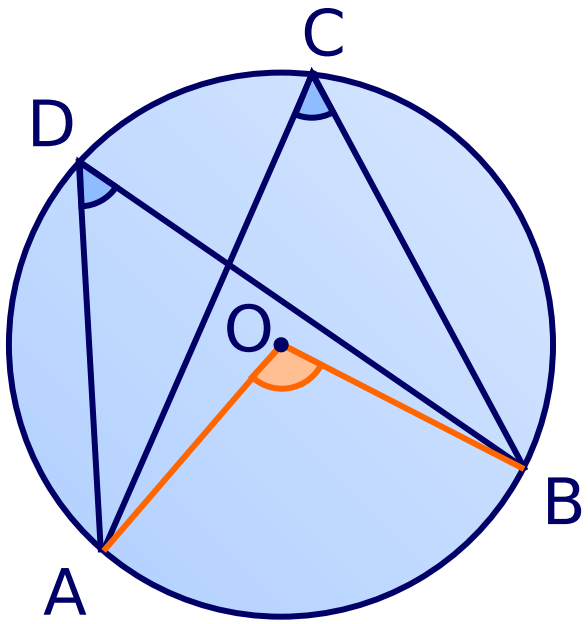



Angles in the same segment are equal.

# Angles in the same segment

We have just seen a demonstration that the angles in the same segment are equal.

We can prove this result as follows:




Mark the centre of the circle O and angle AOB.

angle ADB = ½ of angle AOB

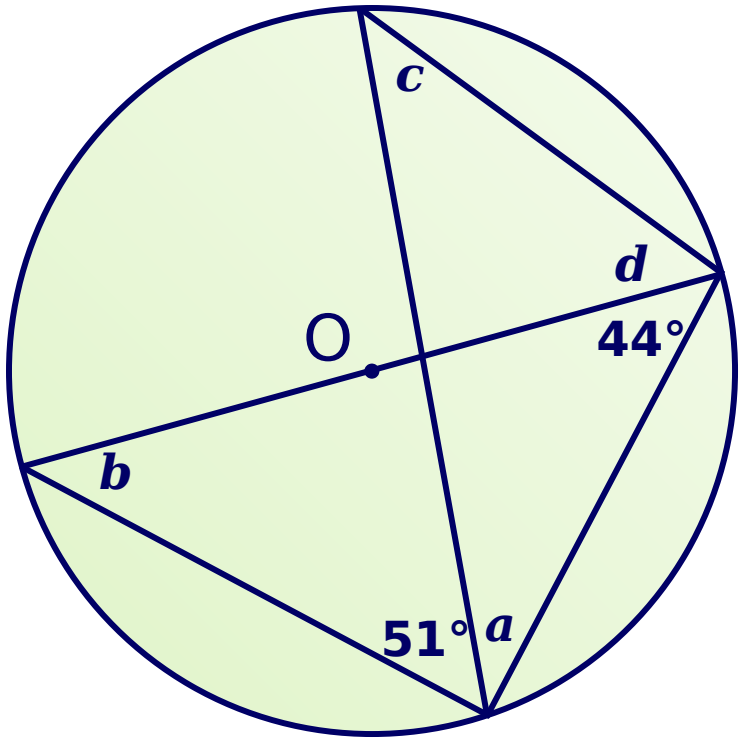
and angle ACB = ½ of angle AOB

(the angle at the centre of a circle is twice the angle at the circumference)

∴ angle ADB = angle ACB

# Calculating the size of unknown angles

Calculate the size of the labeled angles in the following diagram:



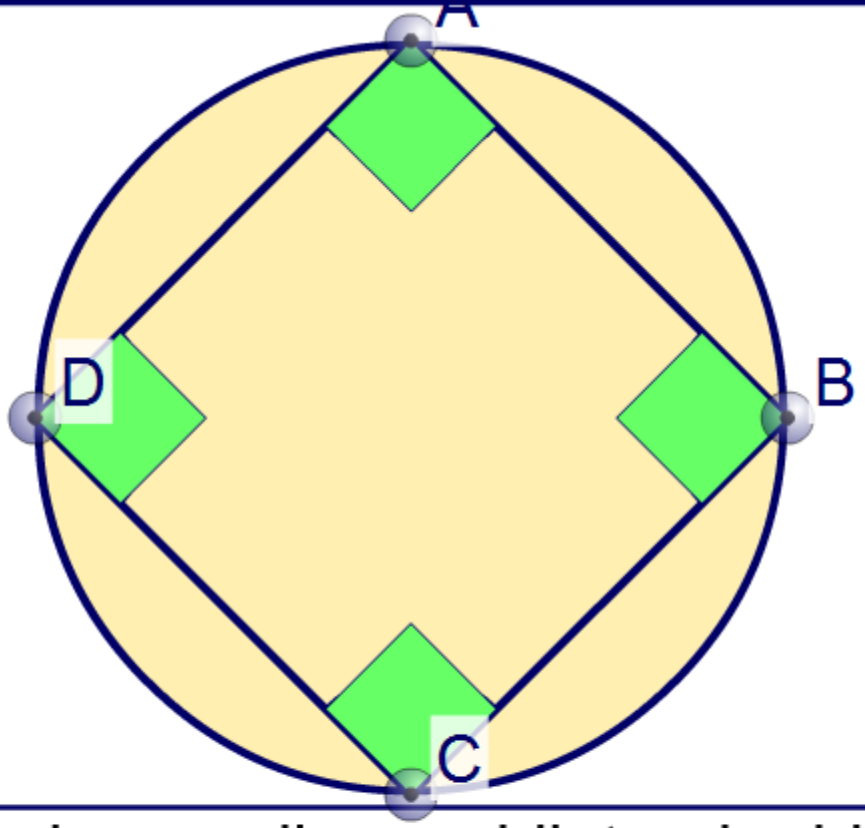

$a = 90° - 51°$
$= 39°$ (angle in a semi-circle)

$b = 180° - (90° + 44°)$
$= 46°$ (angles in a triangle)

$c = 46°$ (angles in the same segment)

$d = 51°$ (angles in the same segment)

# Angles in a cyclic quadrilateral

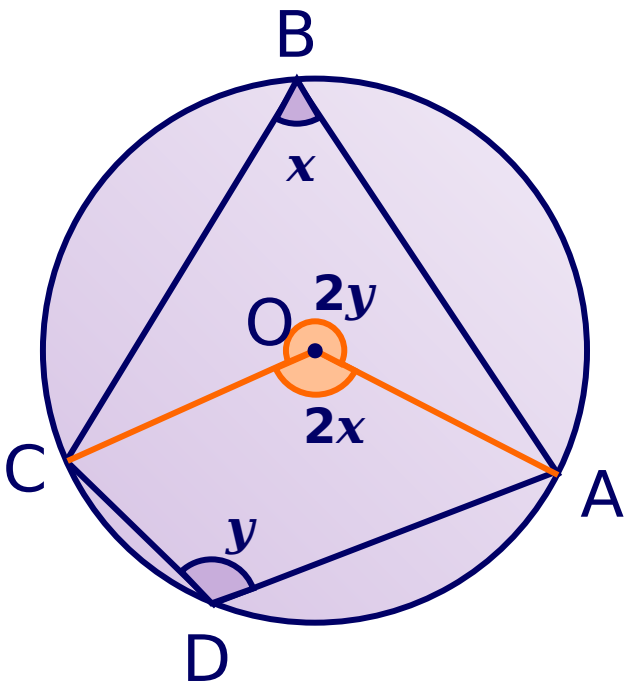



Opposite angles in a cyclic quadrilateral add up to 180°

# Angles in a cyclic quadrilateral

We have just seen a demonstration that the opposite angles in a cyclic quadrilateral add up to 180°.

We can prove this result as follows:




Mark the centre of the circle O and label angles ABC and ADC $x$ and $y$.

The angles at the centre are $2x$ and $2y$

(the angle at the centre of a circle is twice the angle at the circumference)

$$2x + 2y = 360°$$

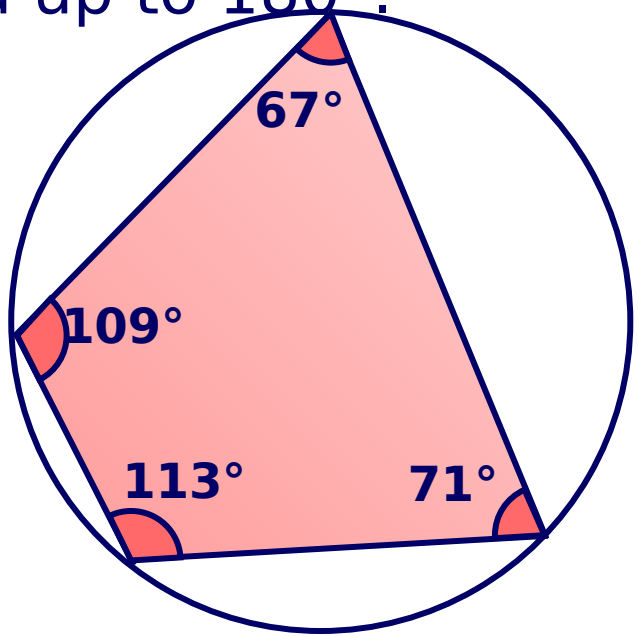
$$2(x + y) = 360°$$

$$x + y = 180°$$

# Angles in a cyclic quadrilateral

As a result of this theorem we can conclude that is the opposite angle of a quadrilateral add up to 180° a circle can be drawn through each of its vertices.

For example, the opposite angles in this quadrilateral add up to 180°.

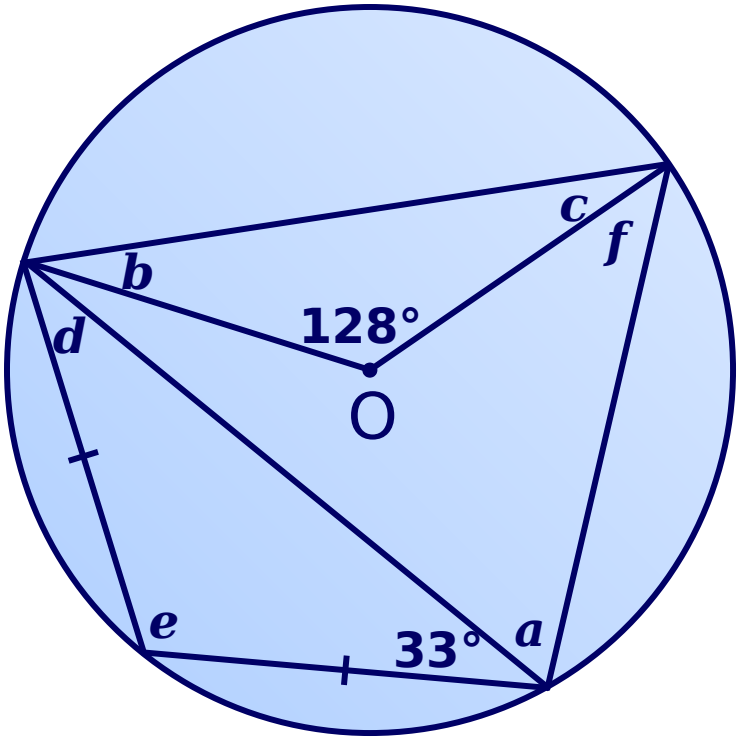



It is a cyclic quadrilateral.

Remember that when two angles add up to 180° they are often called **supplementary angles**.

# Calculating the size of unknown angles

Calculate the size of the labeled angles in the following diagram:




$a = 64°$ (angle at the centre)

$b = c = (180° - 128°) \div 2$

$= 26°$ (angles at the base of an isosceles triangle)

$d = 33°$ (angles at the base of an isosceles triangle)

$e = 180° - 2 \times 33°$

$= 114°$ (angles in a triangle)

$f = 180° - (e + c)$

$= 180° - 140°$

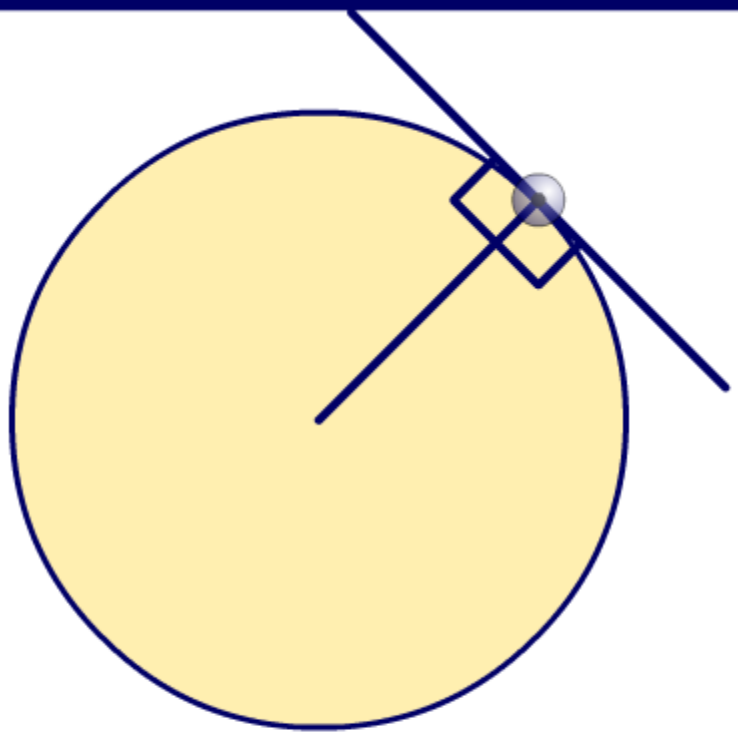
$= 40°$ (opposite angles in a cyclic quadrilateral)

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## S5 Circles

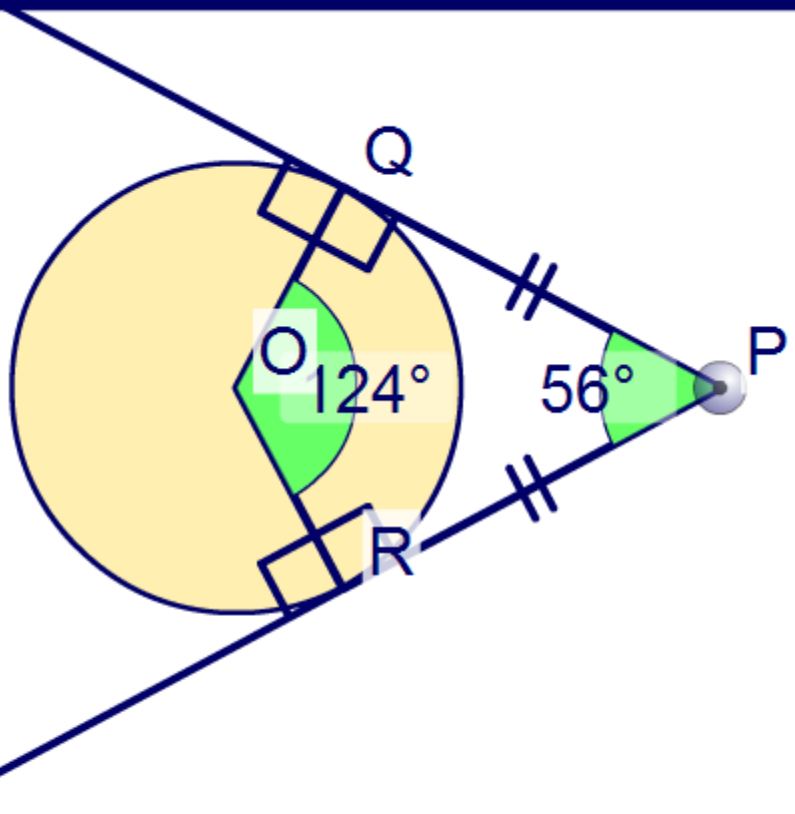

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# The tangent and the radius



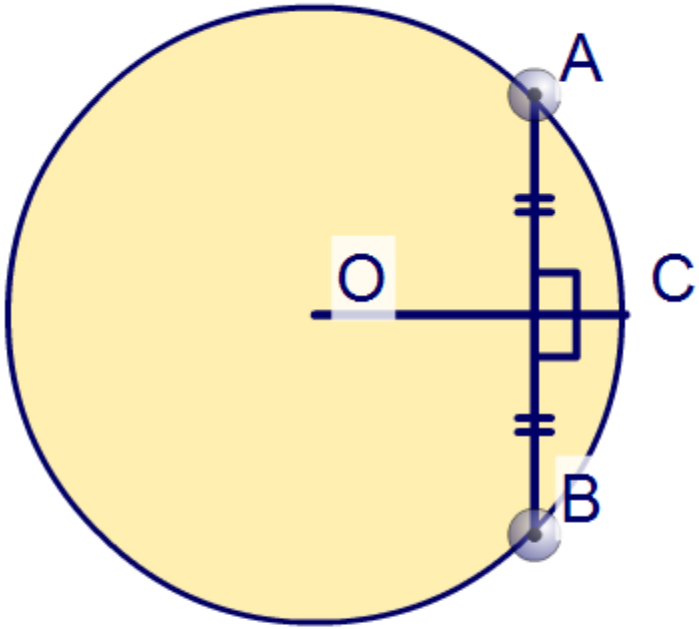
The angle between the tangent and the radius is 90° at the point of contact.

# Two tangents from a point



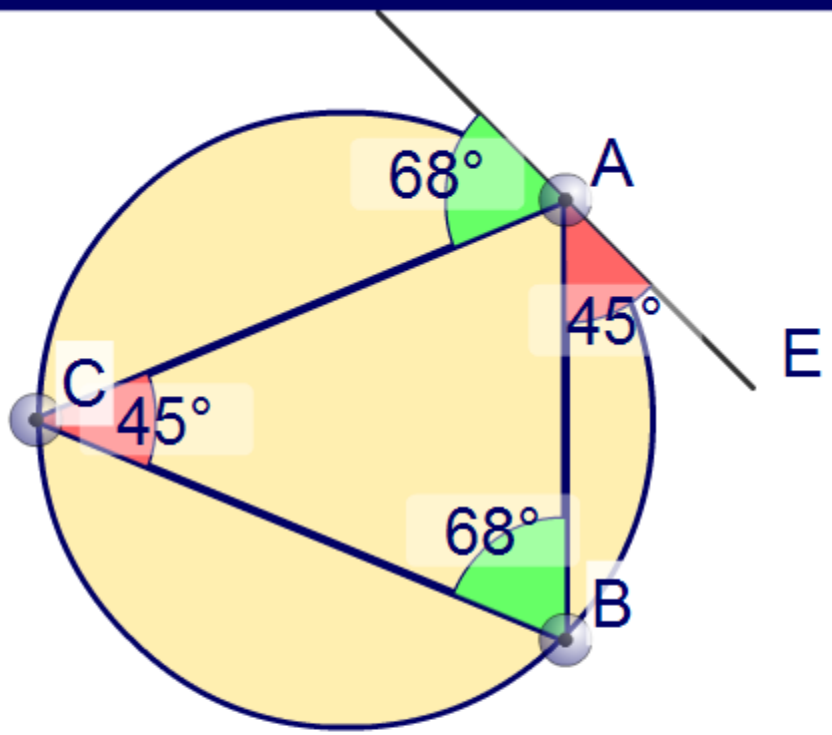

Two tangents from the same point to a circle are of equal length.

# The perpendicular from the centre to a chord




The perpendicular from the centre of a circle to a chord bisects the chord.

# The alternate segment theorem




The angle between a tangent and a chord is equal to the angle in the alternate segment

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# The value of $\pi$

For any circle the circumference is always just over three times bigger than the radius.

The exact number is called π (pi).

We use the symbol π because the number cannot be written exactly.

π = 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196 (to 200 decimal places)!

# Approximations for the value of $\pi$

When we are doing calculations involving the value $\pi$ we have to use an approximation for the value.

For a rough approximation we can use 3.

Better approximations are 3.14 or $\frac{22}{7}$.

We can also use the $\pi$ button on a calculator.

Most questions will tell you what approximations to use.

When a calculation has lots of steps we write $\pi$ as a symbol throughout and evaluate it at the end, if necessary.





# The circumference of a circle

For any circle,

$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

or,

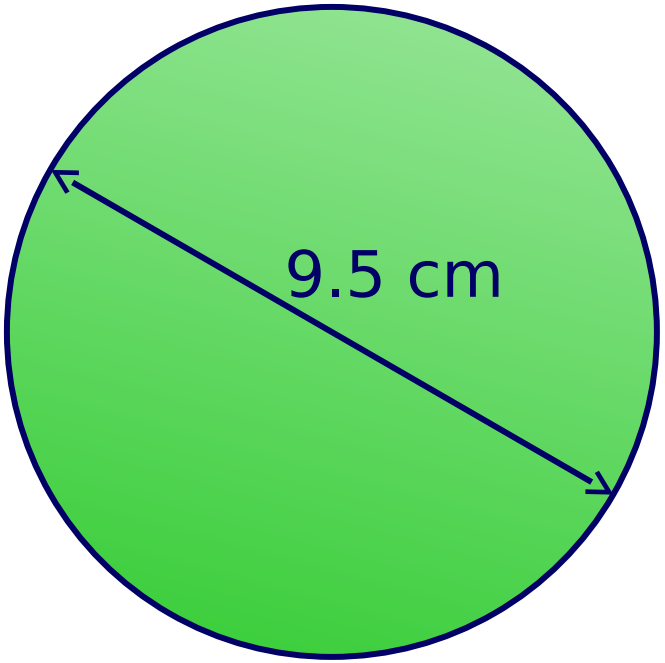
$$\pi = \frac{C}{d}$$

We can rearrange this to make an formula to find the circumference of a circle given its diameter.

$$C = \pi d$$

# The circumference of a circle

Use $\pi = 3.14$ to find the circumference of this circle.




$C = \pi d$

$= 3.14 \times 9.5$

$= \mathbf{29.83\ cm}$

# Finding the circumference given the radius

The diameter of a circle is two times its radius, or

$$d = 2r$$

We can substitute this into the formula

$$C = \pi d$$

to give us a formula to find the circumference of a circle given its radius.

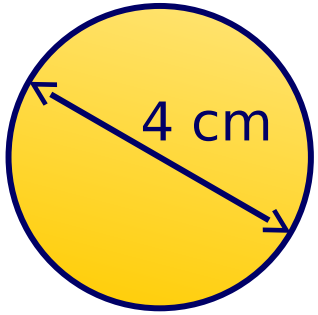
$$C = 2\pi r$$





# The circumference of a circle

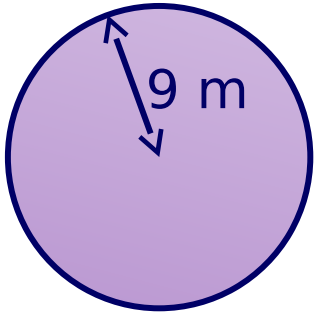
se π = 3.14 to find the circumference of the following circles




$C = \pi d$

$= 3.14 \times 4$

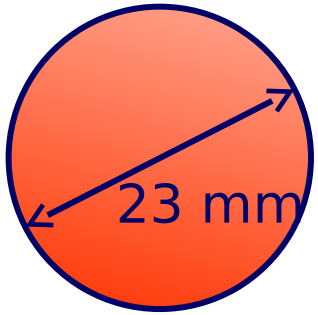
$=$ **12.56 cm**




$C = 2\pi r$

$= 2 \times 3.14 \times 9$

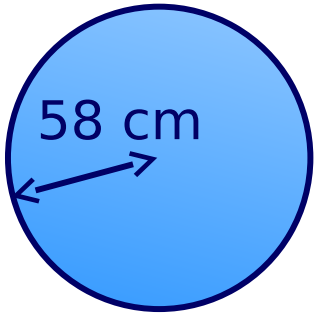
$=$ **56.52 m**




$C = \pi d$

$= 3.14 \times 23$

$=$ **72.22 mm**



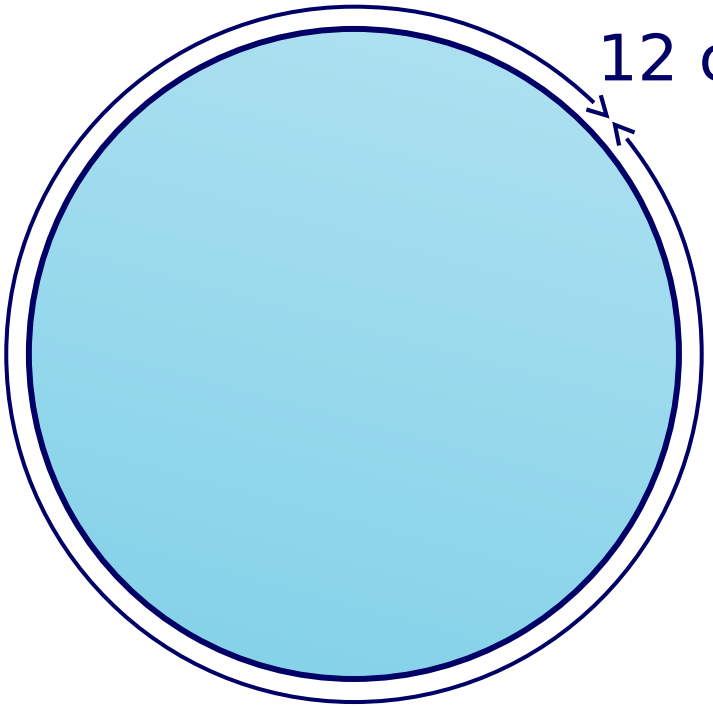

$C = 2\pi r$

$= 2 \times 3.14 \times 58$

$=$ **364.24 cm**

# Finding the radius given the circumference

Use $\pi = 3.14$ to find the radius of this circle.




$C = 2\pi r$

How can we rearrange this to make $r$ the subject of the formula?

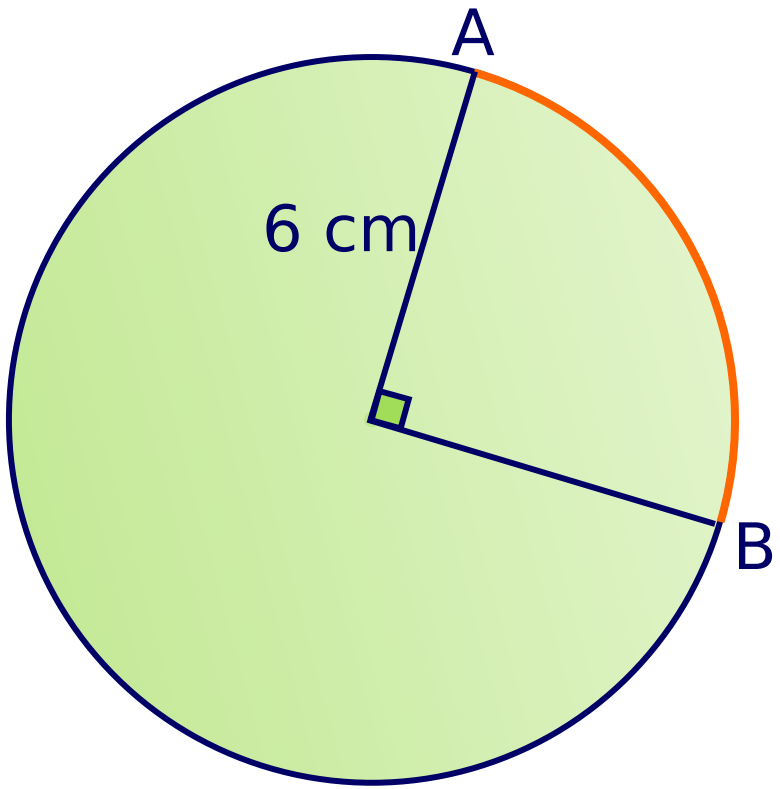
$$r = \frac{C}{2\pi}$$

$$= \frac{12}{2 \times 3.14}$$

$$= \mathbf{1.91 \text{ cm (to 2 d.p.)}}$$

# Finding the length of an arc

What is the length of arc AB?




An arc is a section of the circumference.

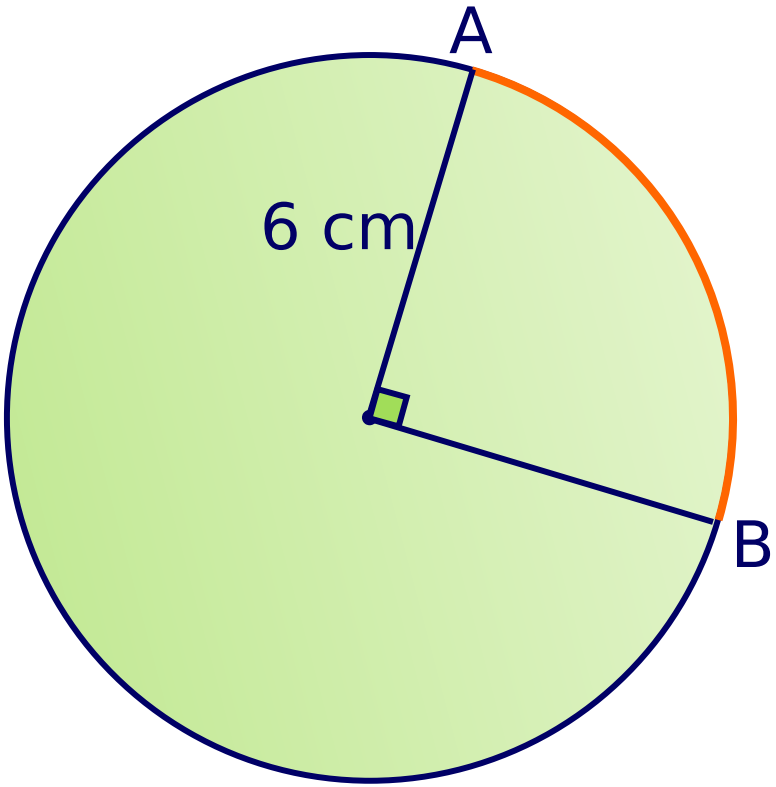
The length of arc AB is a fraction of the length of the circumference.

To work out what fraction of the circumference it is we look at the angle at the centre.

In this example, we have a 90° angle at the centre.

# Finding the length of an arc

What is the length of arc AB?




The arc length is $\frac{1}{4}$ of the circumference of the circle.

This is because,

$$\frac{90°}{360°} = \frac{1}{4}$$

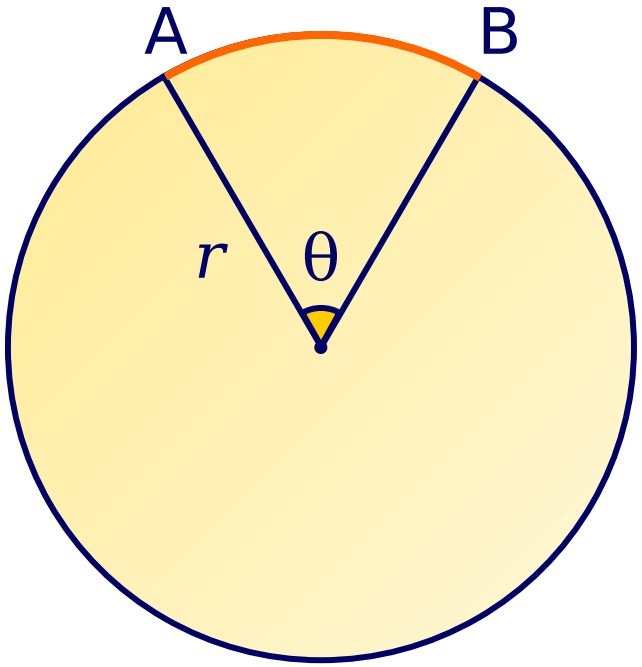
So,

$$\text{Length of arc AB} = \tfrac{1}{4} \times 2\pi r$$

$$= \tfrac{1}{4} \times 2\pi \times 6$$

Length of arc AB = 9.42 cm (to 2 d.p.)

# Finding the length of an arc



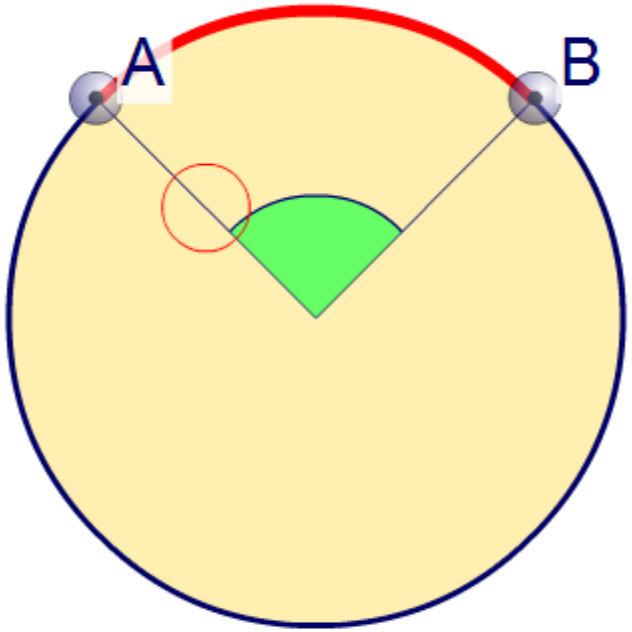

For any circle with radius $r$ and angle at the centre $\theta$,

$$\text{Arc length AB} = \frac{\theta}{360} \times 2\pi r$$

This is the circumference of the circle.

$$\text{Arc length AB} = \frac{2\pi r\theta}{360} = \frac{\pi r\theta}{180}$$

# Finding the length of an arc




$$\text{Arc AB} = \frac{2\pi r\theta}{360} = \frac{\pi r\theta}{180}$$

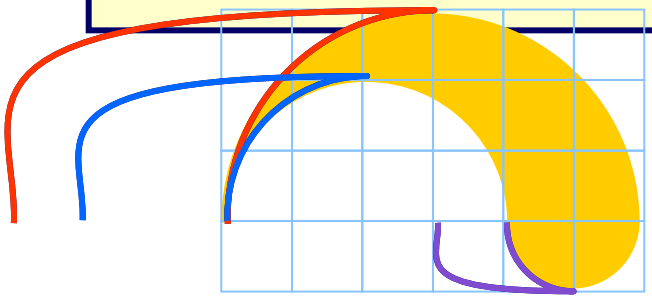
Arc AB =

=

5 cm

# The perimeter of shapes made from arcs

Find the perimeter of these shapes on a cm square grid:

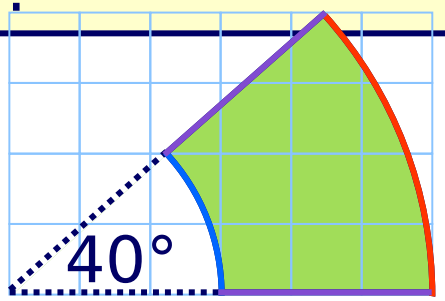


The perimeter of this shape is made from three semi-circles.

Perimeter $= \frac{1}{2} \times \pi \times 6 + \frac{1}{2} \times \pi \times 4 + \frac{1}{2} \times \pi \times 2$

$= 6\pi$ cm

$= 18.85$ cm (to 2 d.p.)




$\frac{40°}{360°} = \frac{1}{9}$

Perimeter $= \frac{1}{9} \times \pi \times 12 + \frac{1}{9} \times \pi \times 6 + 3 + 3$

$= 2\pi + 6$

$= 12.28$ cm (to 2 d.p.)

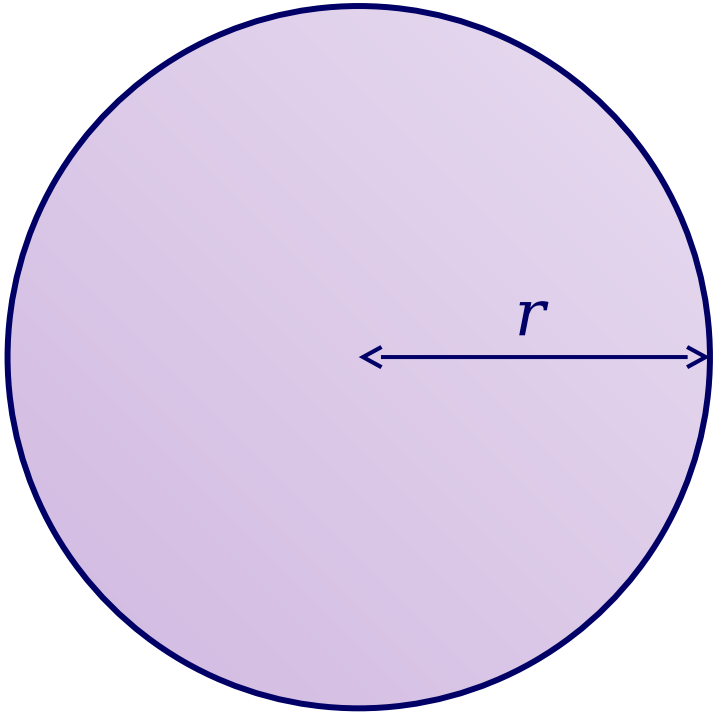
# Contents

## S5 Circles


- S5.1 Naming circle parts
- S5.2 Angles in a circle
- S5.3 Tangents and chords
- S5.4 Circumference and arc length
- S5.5 Areas of circles and sectors

# Formula for the area of a circle

We can find the area of a circle using the formula



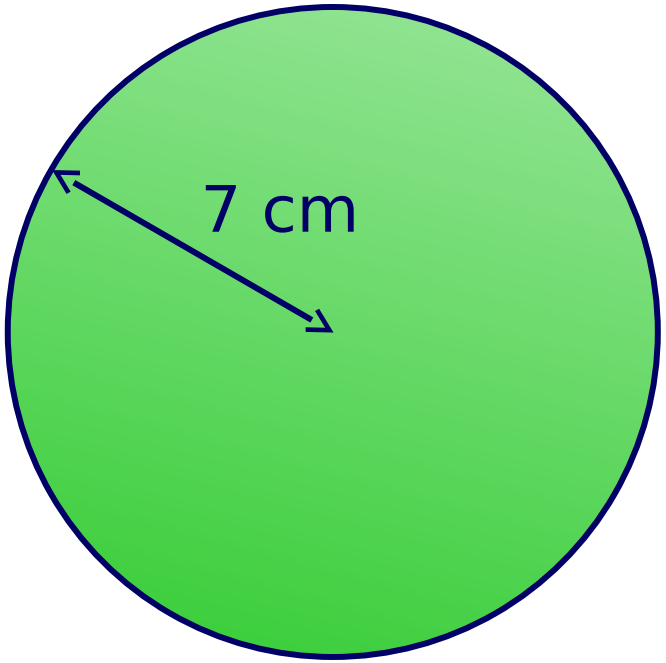

Area of a circle = $\pi \times r \times r$

or

Area of a circle = $\pi r^2$

# The area of a circle

Use $\pi = 3.14$ to find the area of this circle.




$A = \pi r^2$

$= 3.14 \times 7^2$

$= \mathbf{153.86\ cm^2}$

# Finding the area given the diameter

The radius of a circle is half of its radius, or

$$r = \frac{d}{2}$$

We can substitute this into the formula

$$A = \pi r^2$$

to give us a formula to find the area of a circle given its diameter.

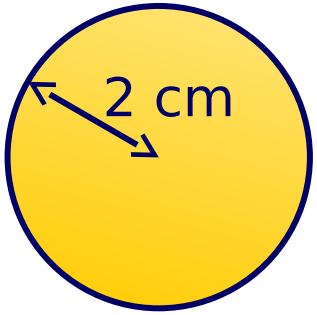
$$A = \frac{\pi d^2}{4}$$





# The area of a circle

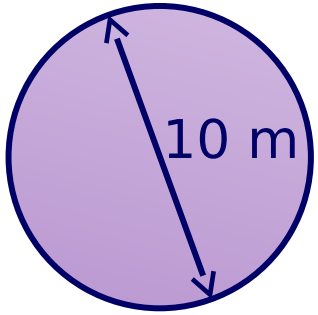
Use $\pi = 3.14$ to find the area of the following circles:




$A = \pi r^2$

$= 3.14 \times 2^2$

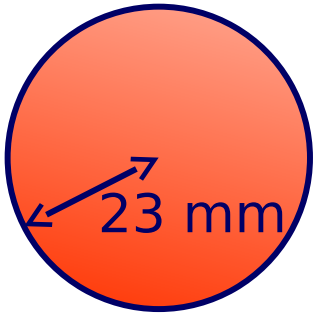
$= \mathbf{12.56\ cm^2}$




$A = \pi r^2$

$= 3.14 \times 5^2$

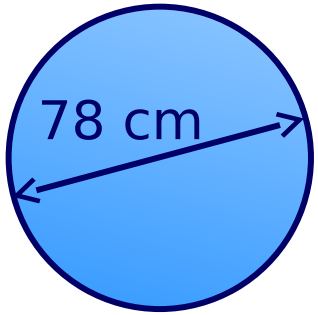
$= \mathbf{78.5\ m^2}$




$A = \pi r^2$

$= 3.14 \times 23^2$

$= \mathbf{1661.06\ mm^2}$



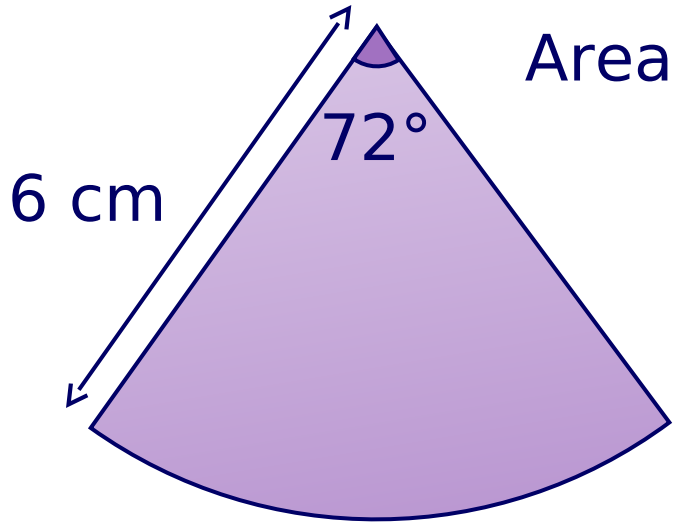

$A = \pi r^2$

$= 3.14 \times 39^2$

$= \mathbf{4775.94\ cm^2}$

# Finding the area of a sector

What is the area of this sector?




$$\text{Area of the sector} = \frac{72°}{360°} \times \pi \times 6^2$$

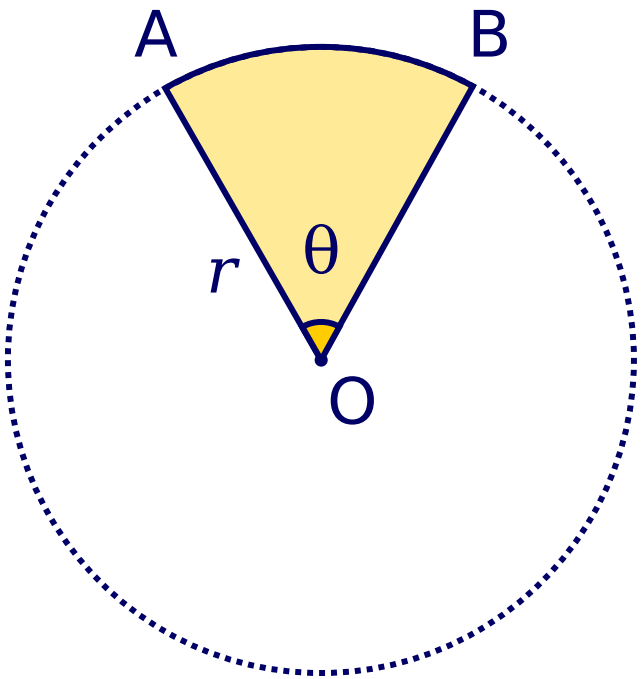
$$= \frac{1}{6} \times \pi \times 6^2$$

$$= \pi \times 6$$

$$= \mathbf{18.85\ cm^2\ (to\ 2\ d.p.}$$

We can use this method to find the area of any sector.

# Finding the area of a sector



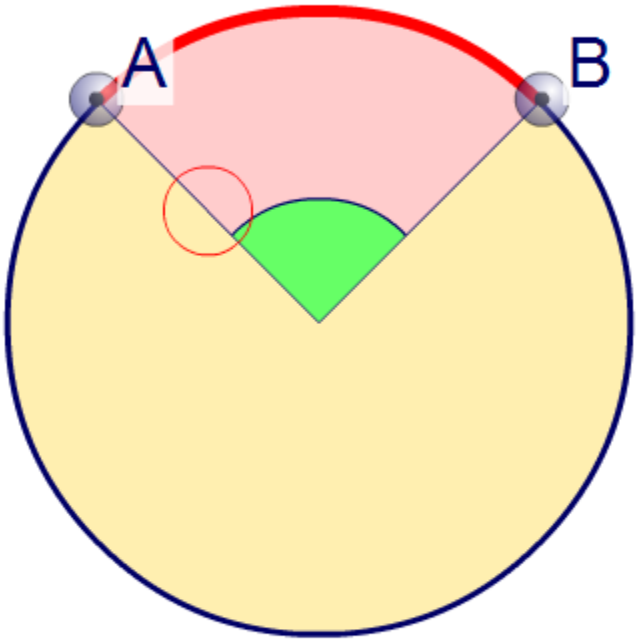

For any circle with radius $r$ and angle at the centre $\theta$,

$$\text{Area of sector AOB} = \frac{\theta}{360} \times \pi r^2$$

This is the area of the circle.

$$\text{Area of sector AOB} = \frac{\pi r^2 \theta}{360}$$

# Finding the area of a sector




Area of sector AOB = $\frac{\pi r^2 \theta}{360}$

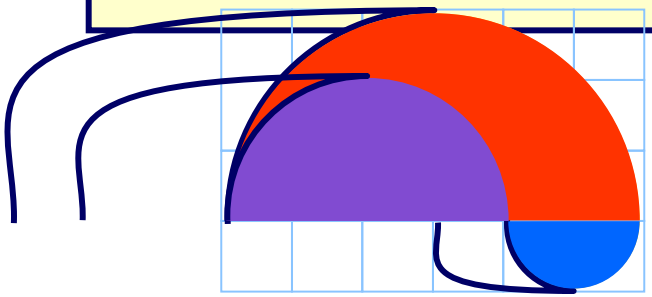
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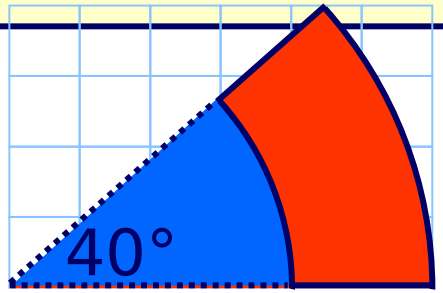
5 cm

# The area of shapes made from sectors

Find the area of these shapes on a cm square grid:



$$\text{Area} = \tfrac{1}{2} \times \pi \times 3^2$$
$$+ \tfrac{1}{2} \times \pi \times 1^2$$
$$- \tfrac{1}{2} \times \pi \times 2^2$$
$$= 3\pi \text{ cm}^2$$
$$= 9.42 \text{ cm}^2 \text{ (to 2 d.p.)}$$




$$\frac{40°}{360°} = \frac{1}{9}$$
$$\text{Area} = \tfrac{1}{9} \times \pi \times 6^2$$
$$- \tfrac{1}{9} \times \pi \times 4^2$$
$$= \tfrac{1}{9} \times \pi \times 20 \text{ cm}^2$$
$$= 6.98 \text{ cm}^2 \text{ (to 2 d.p.)}$$