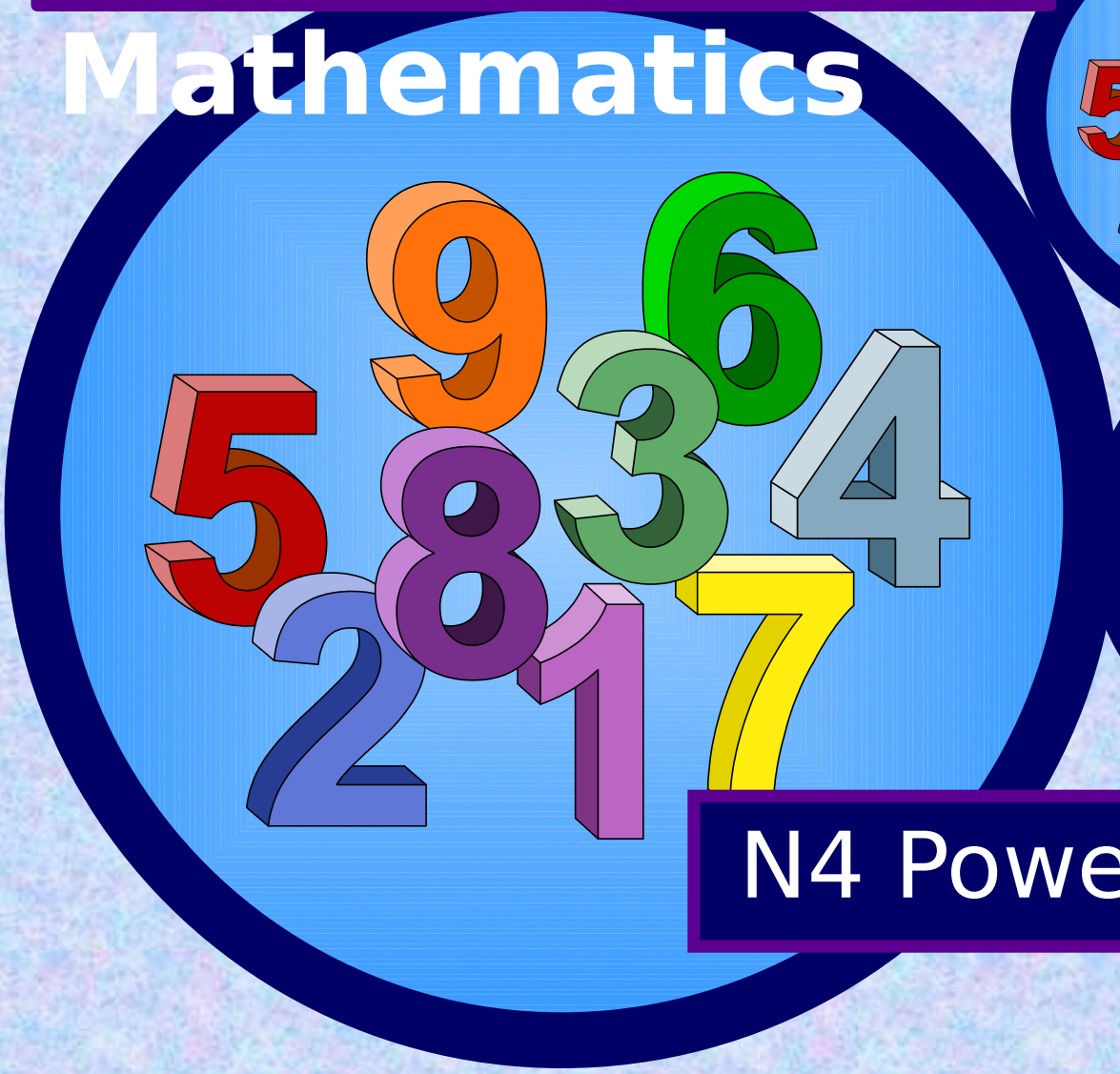
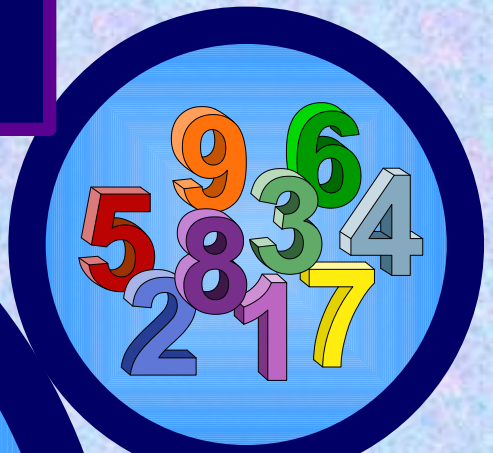
KS3 Mathematics







N4 Powers and roots

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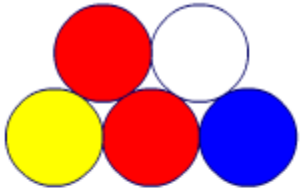
N4 Powers and Roots

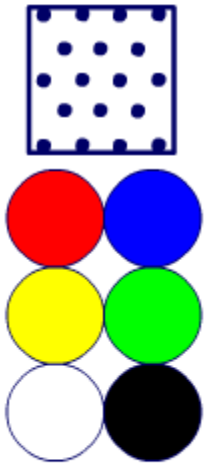
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Making triangles



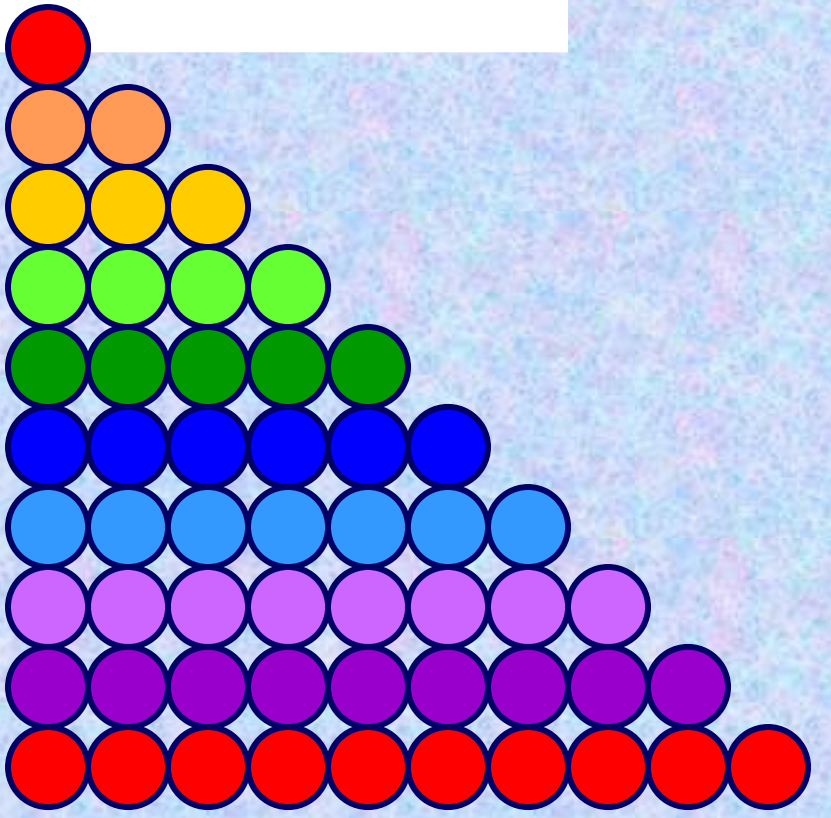


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Triangular numbers

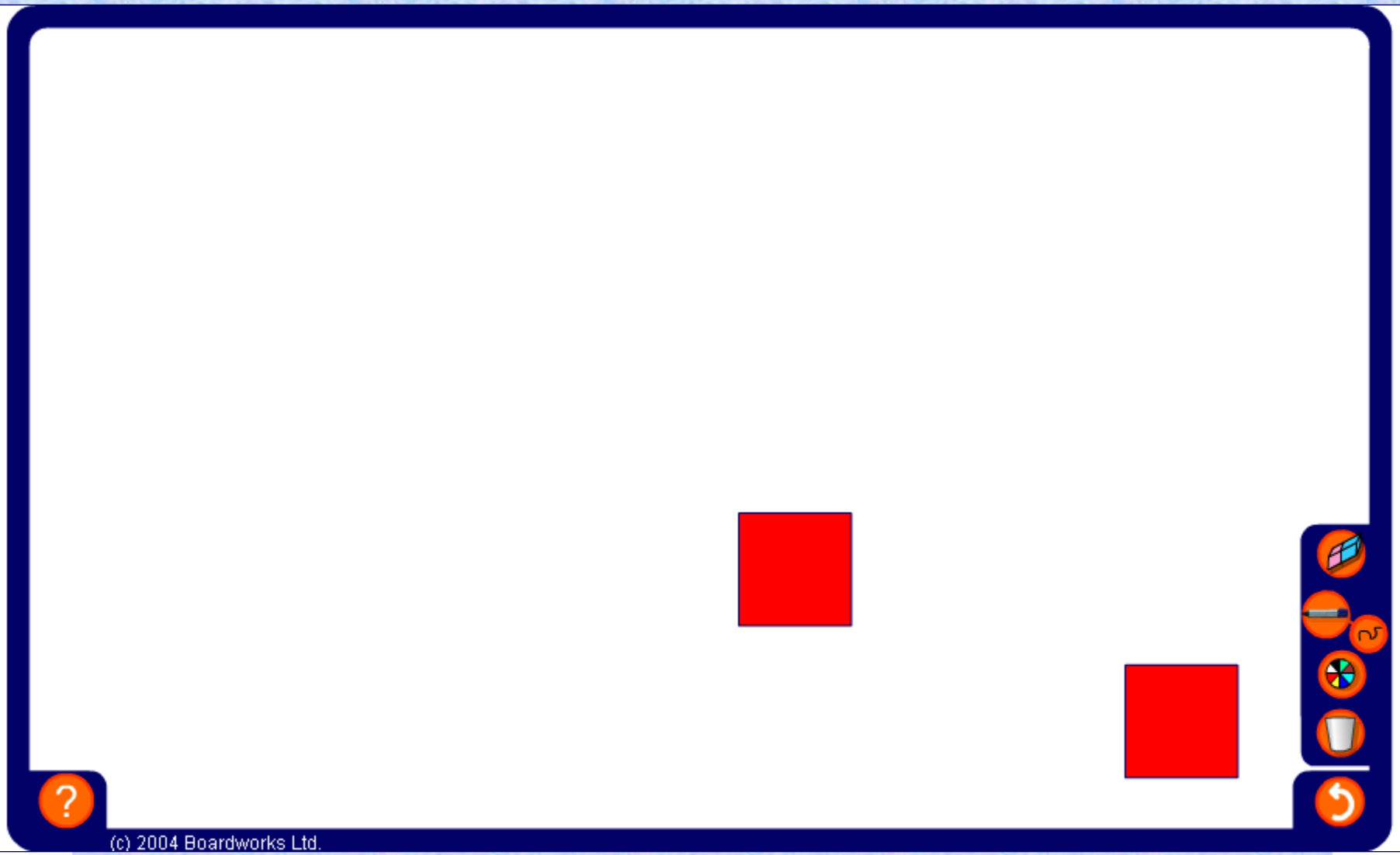
The tenth triangular number is 55.

1.



1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

Making squares



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Square numbers

When we multiply a number by itself we say that we are **squaring** the number.

To square a number we can write a small ^{2} after it.

For example, the number 3 multiplied by itself can be written as

Three squared

3×3 or 3^2

The value of three squared is 9.

The result of any *whole* number multiplied by itself is called a **square number**.





Square numbers

Here are the first 10 square numbers:

$1^2 = 1 \times 1 = 1$

+ 3

$2^2 = 2 \times 2 = 4$

+ 5

$3^2 = 3 \times 3 = 9$

+ 7

$4^2 = 4 \times 4 = 16$

+ 9

$5^2 = 5 \times 5 = 25$

+ 11

$6^2 = 6 \times 6 = 36$

+ 13

$7^2 = 7 \times 7 = 49$

+ 15

$8^2 = 8 \times 8 = 64$

+ 17

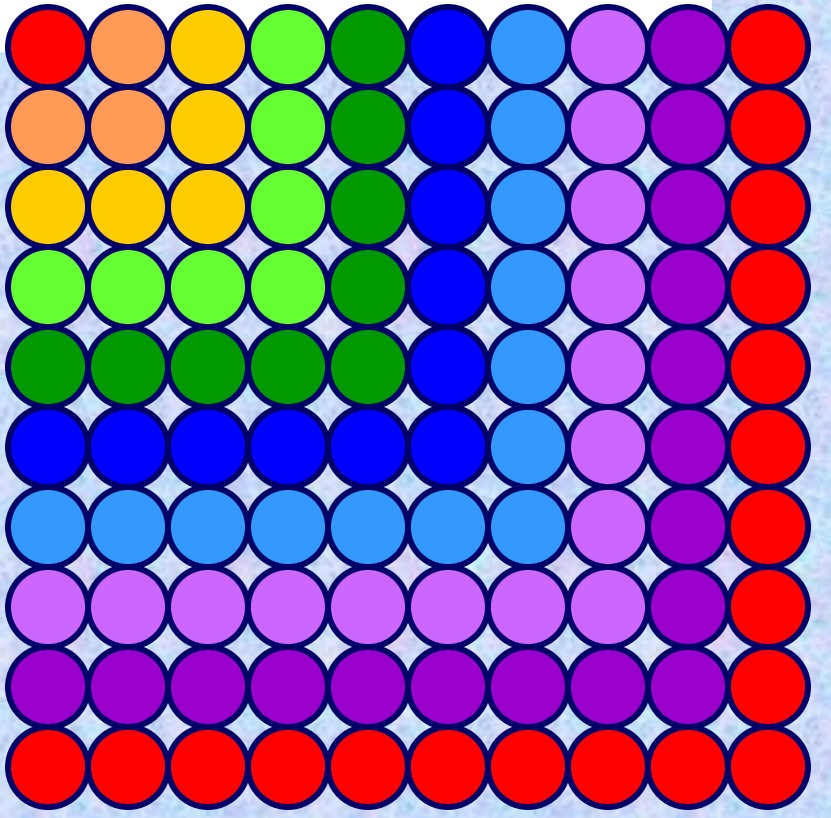
$9^2 = 9 \times 9 = 81$

+ 19

$10^2 = 10 \times 10 = 100$

Adding consecutive odd numbers

The tenth square number is 100.



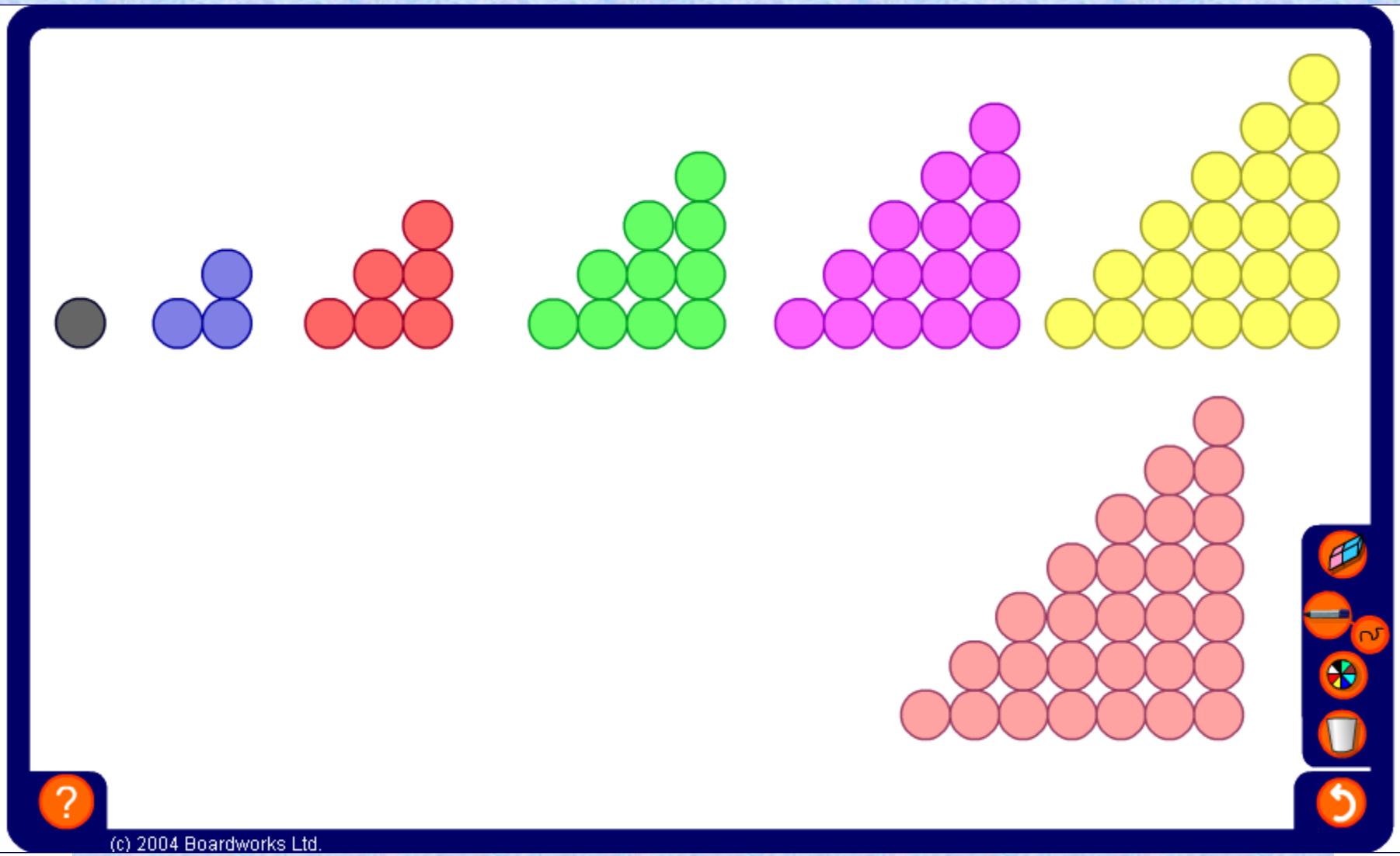
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

Making square numbers

There are several ways to generate a sequence of square numbers.

- We can multiply a whole number by itself.
- We can add consecutive odd numbers starting from 1.
- We can add together two consecutive triangular numbers.

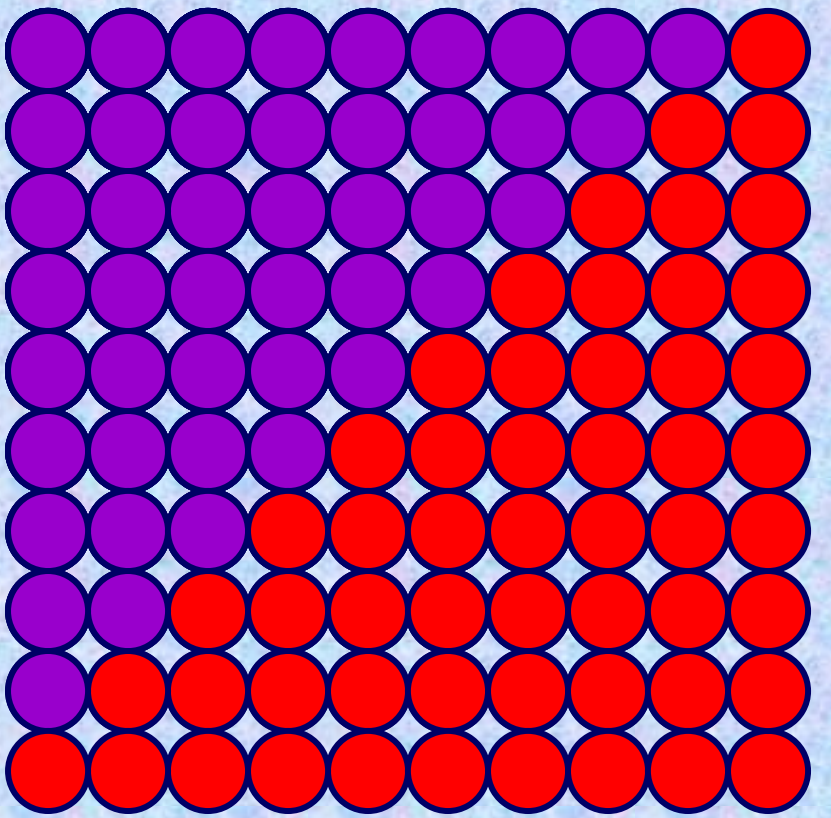
Adding consecutive triangular numb



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Adding two consecutive triangular numbers

We can make square numbers by adding two consecutive triangular numbers.



$45 + 55 = 100$

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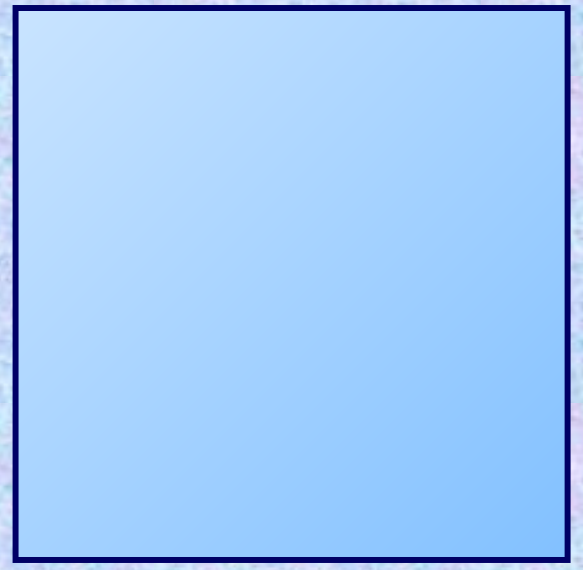
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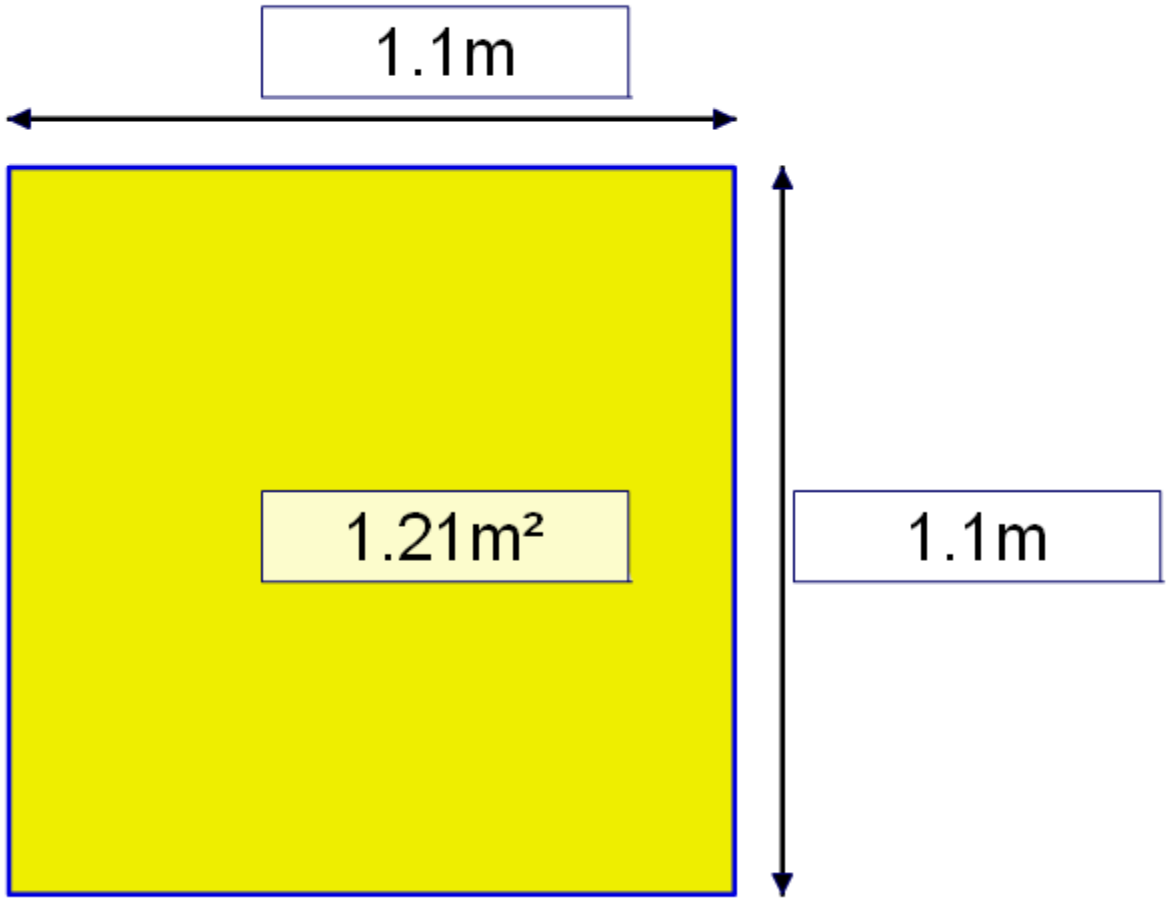
Square roots

The area of this square is 64 cm^2.



What is the length of the sides?

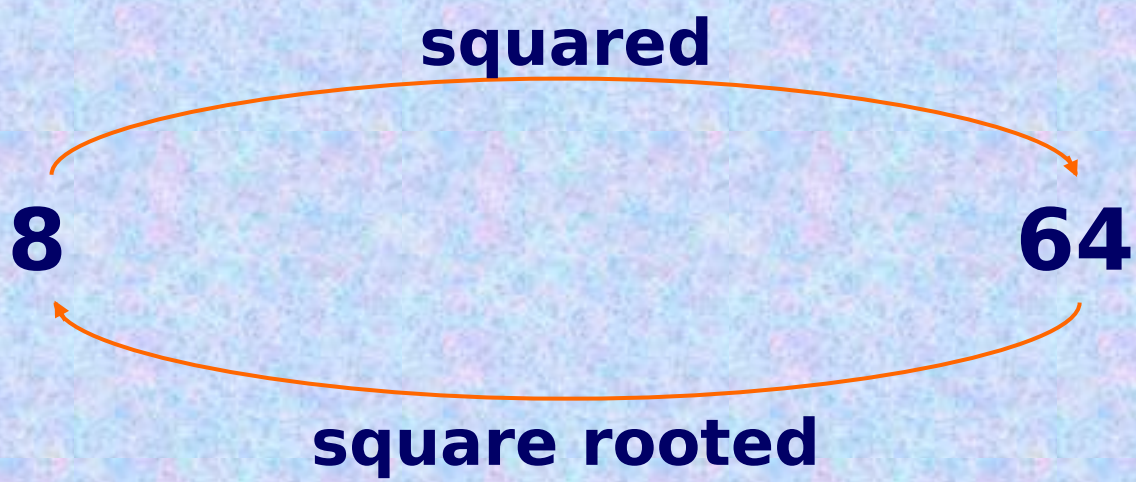
Square roots



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Square roots

inding the **square root** is the inverse of finding the square:



We write

$$\sqrt{64} = 8$$

The square root of 64 is 8.

Square roots

We can easily find the square root of a square number.

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

The product of two square numbers

The product of two square numbers is always another square number.

For example,

$$4 \times 25 = 100$$

because

$$2 \times 2 \times 5 \times 5 = 2 \times 5 \times 2 \times 5$$

and

$$(2 \times 5)^2 = 10^2$$

We can use this fact to help us find the square roots of larger square numbers.





Using factors to find square roots

If a number has factors that are square numbers then we can use these factors to find the square root.

For example,

Find $\sqrt{400}$

$\sqrt{400} = \sqrt{4 \times 100}$

$= 2 \times 10$

$= 20$

Find $\sqrt{225}$

$\sqrt{225} = \sqrt{9 \times 25}$

$= 3 \times 5$

$= 15$

Finding square roots of decimals

If a number can be made be dividing two square numbers then we can find its square root.

For example,

Find $\sqrt{0.09}$

$$\sqrt{0.09} = \sqrt{9 \div 100}$$

$$= 3 \div 10$$

$$= 0.3$$

Find $\sqrt{1.44}$

$$\sqrt{1.44} = \sqrt{144 \div 100}$$

$$= 12 \div 10$$

$$= 1.2$$

Approximate square roots

If a number cannot be written as a product or quotient of two square numbers then its square root cannot be found exactly.

Use the  key on your calculator to find out $\sqrt{2}$.

The calculator shows this as 1.414213562

This is an approximation to 9 decimal places.

The number of digits after the decimal point is infinite.

Estimating square roots

What is √10?

10 lies between 9 and 16.

Therefore,

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$

So,

$$3 < \sqrt{10} < 4$$

10 is closer to 9 than to 16, so √10 will be about 3.2

Use the √ key on you calculator to work out the answer.

√10 = 3.16 (to 2 decimal places.)

Trial and improvement

Suppose our calculator does not have a 

key.

Find √40

$\sqrt{36} < \sqrt{40} < \sqrt{49}$

40 is closer to 36 than to 49, so √40 will be about 6.3

So,

$6 < \sqrt{40} < 7$

$6.3^2 = 39.69$ **too small!**

$6.4^2 = 40.96$ **too big!**

Trial and improvement

$6.33^2 = 40.0689$ **too big!**

$6.32^2 = 39.9424$ **too small!**

Suppose we want the answer to 2 decimal places.

$6.325^2 = 40.005625$ **too big!**

Therefore,

$$6.32 < \sqrt{40} < 6.325$$

$\sqrt{40} = 6.32$ (to 2 decimal places)

Trial and improvement

Find $\sqrt{57}$ to 2 decimal places.

0●0		

Answer to 2 decimal places: 0●0 0

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Negative square roots

$5 \times 5 = 25$ **and** $-5 \times -5 = 25$

Therefore, the square root of 25 is 5 or –5.

When we use the √ symbol we usually mean the positive square root.

We can also write ±√ to mean both the positive and the negative square root.

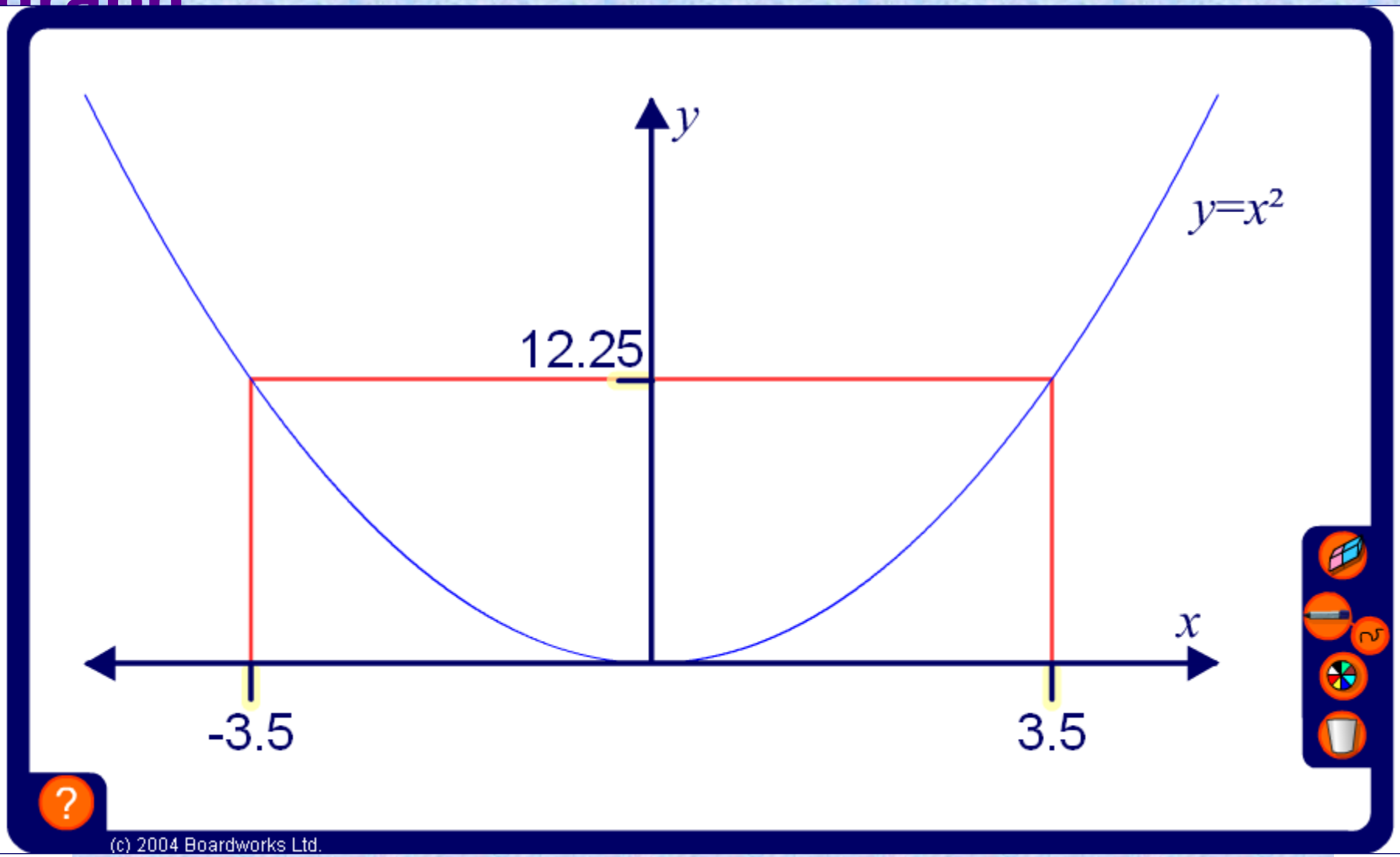
The equation,

$$x^2 = 25$$

has 2 solutions,

$x = 5$ **or** $x = -5$

Squares and square roots from a graph



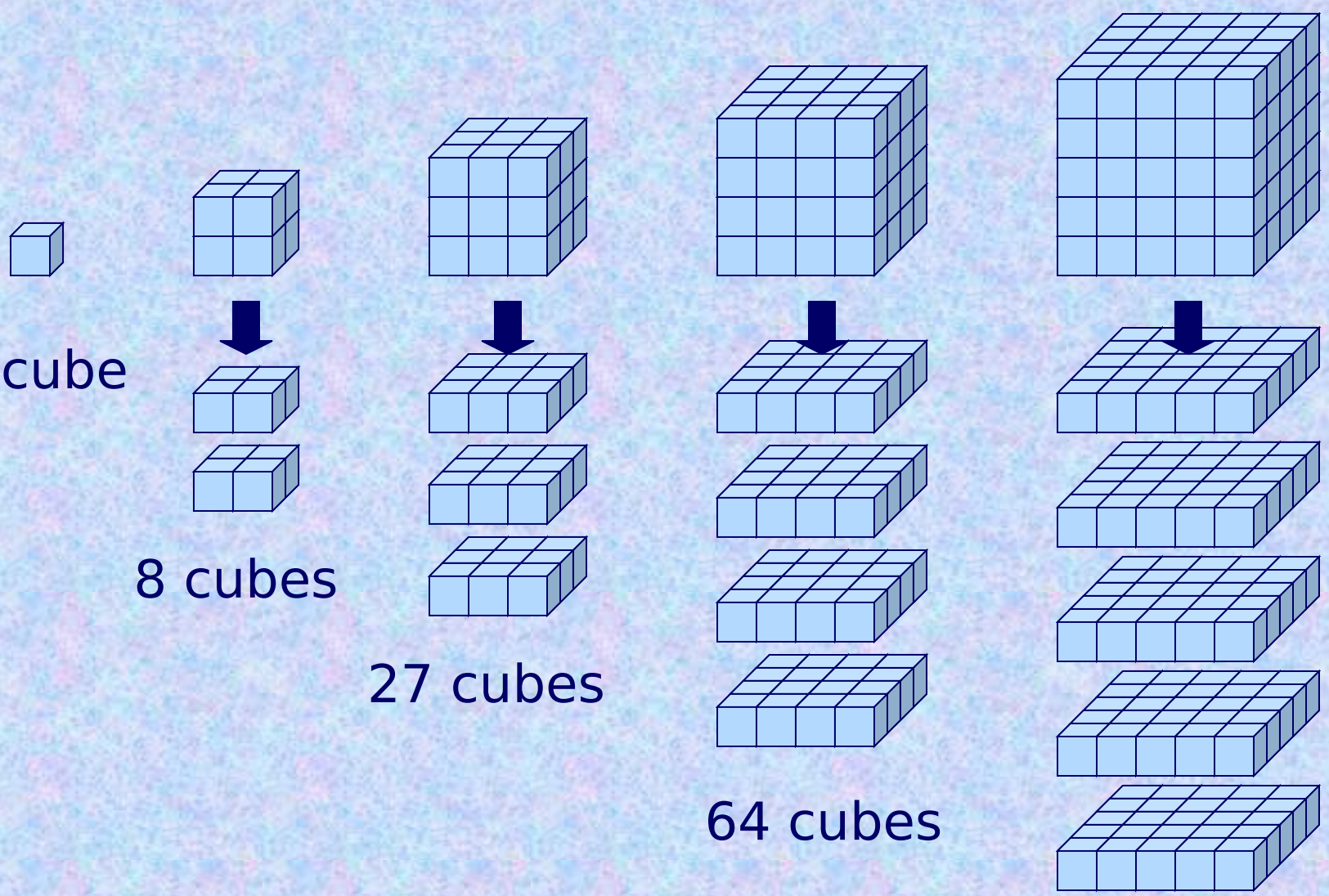
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Cubes



Cubes

The numbers 1, 8, 27, 64, and 125 are called:

Cube numbers

$1^3 = 1 \times 1 \times 1 = 1$ ‘1 cubed’ or ‘1 to the power of 3’

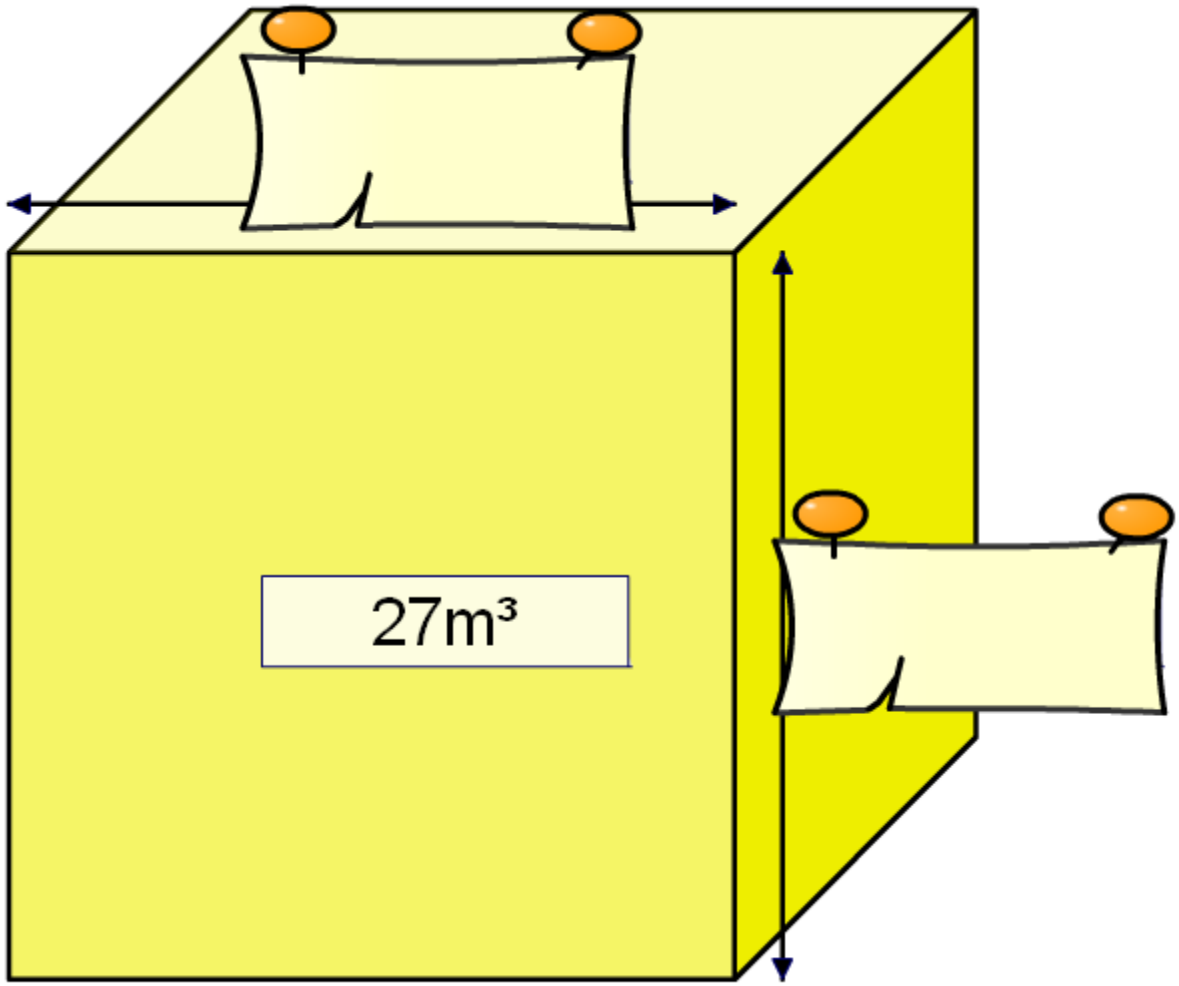
$2^3 = 2 \times 2 \times 2 = 8$ ‘2 cubed’ or ‘2 to the power of 3’

$3^3 = 3 \times 3 \times 3 = 27$ ‘3 cubed’ or ‘3 to the power of 3’

$4^3 = 4 \times 4 \times 4 = 64$ ‘4 cubed’ or ‘4 to the power of 3’

$5^3 = 5 \times 5 \times 5 = 125$ ‘5 cubed’ or ‘5 to the power of 3’

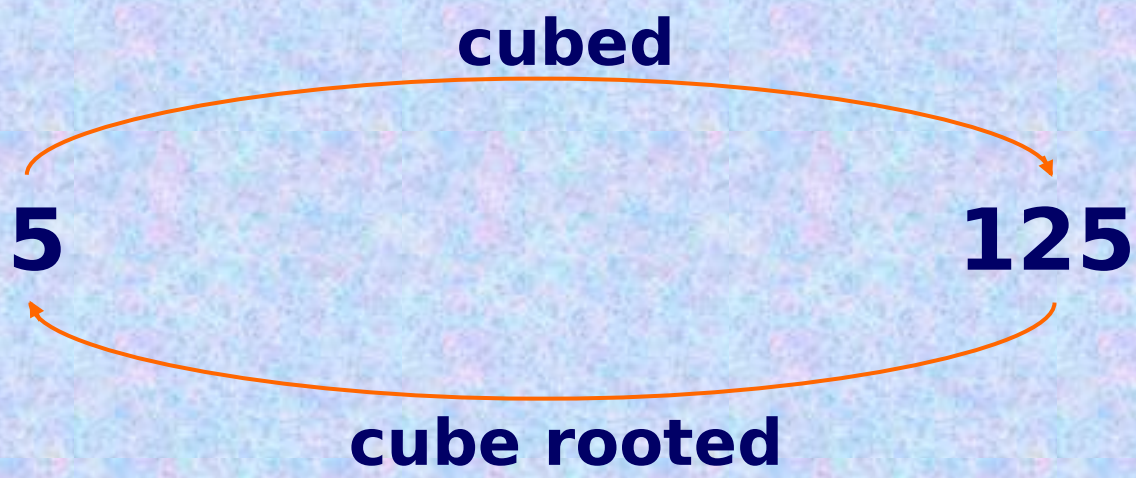
Cubes and cube roots



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Cube roots

Finding the cube root is the inverse of finding the cube:

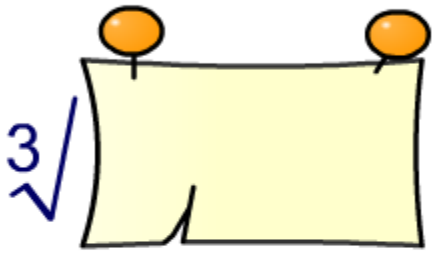


We write

$$\sqrt[3]{125} = 5$$

The cube root of 125 is 5.

Squares, cubes and roots



$= 30$

Squares 1

Square roots 2

Cubes 3

Cube roots 4

positive and negative

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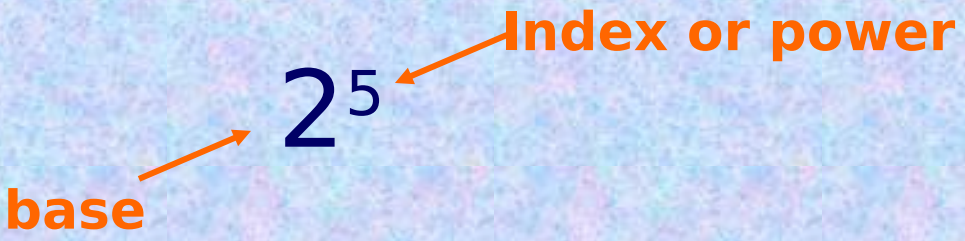


Index notation

We use index notation to show repeated multiplication by the same number.

For example,

e can use index notation to write **2 × 2 × 2 × 2 × 2** as



This number is read as ‘two to the power of five’.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Index notation

Evaluate the following:

$6^2 = 6 \times 6 = 36$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$(-5)^3 = -5 \times -5 \times -5 = -125$

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

$(-1)^5 = -1 \times -1 \times -1 \times -1 \times -1 = -1$

$(-4)^4 = -4 \times -4 \times -4 \times -4 = 64$

When we raise a negative number to an odd power the answer is negative.

When we raise a negative number to an even power the answer is positive.



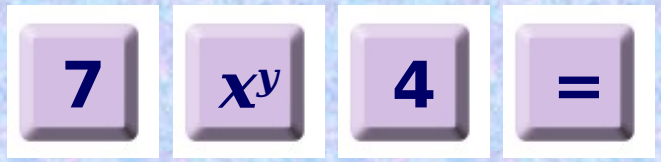


Calculating powers

We can use the x^y key on a calculator to find powers.

For example,

to calculate the value of 7^4 we key in:



The calculator shows this as 2401.

$7^4 = 7 \times 7 \times 7 \times 7 = 2401$

The first index law

When we multiply two numbers written in index form and with the same base we can see an interesting result.

For example,

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^6 = 3^{(4 + 2)}$

$7^3 \times 7^5 = (7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7)$

$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$= 7^8 = 7^{(3 + 5)}$

When we **multiply** two numbers with the **same base** the indices are **added**.

The second index law

When we divide two numbers written in index form and with the same base we can see another interesting result.

For example,

$$4^5 \div 4^2 = \frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4} = 4 \times 4 \times 4 = 4^3 = 4^{(5-2)}$$

$$5^6 \div 5^4 = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 5 \times 5 = 5^2 = 5^{(6-4)}$$

When we **divide** two numbers with the **same base** the indices are **subtracted**.





Zero indices

Look at the following division:

$6^4 \div 6^4 = 1$

Using the second index law

$6^4 \div 6^4 = 6^{(4-4)} = 6^0$

That means that

$6^0 = 1$

In fact, *any* number raised to the power of 0 is equal to 1.

For example,

$10^0 = 1$ $3.452^0 = 1$ $723\,538\,592^0 = 1$





Negative indices

Look at the following division:

$$3^2 \div 3^4 = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 3 \times 3} = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

Using the second index law

$$3^2 \div 3^4 = 3^{(2-4)} = 3^{-2}$$

That means that

$$3^{-2} = \frac{1}{3^2}$$

Similarly, $6^{-1} = \frac{1}{6}$ $7^{-4} = \frac{1}{7^4}$ and $5^{-3} = \frac{1}{5^3}$





Using algebra

We can write all of these results algebraically.

$$a^m \times a^n = a^{(m+n)}$$

$$a^m \div a^n = a^{(m-n)}$$

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

Using index laws

$$54^{-2} \div 54^{5} = 54^{-7}$$

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