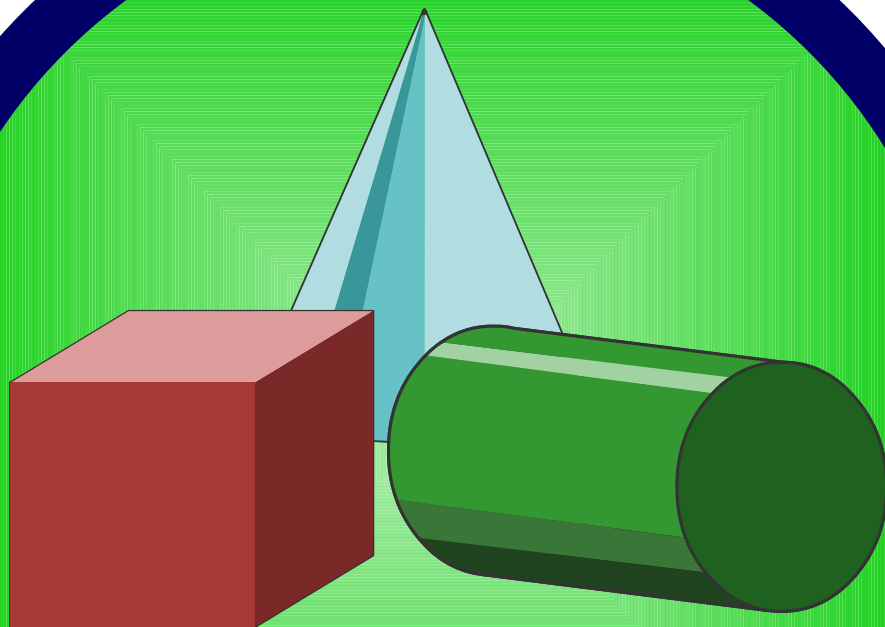
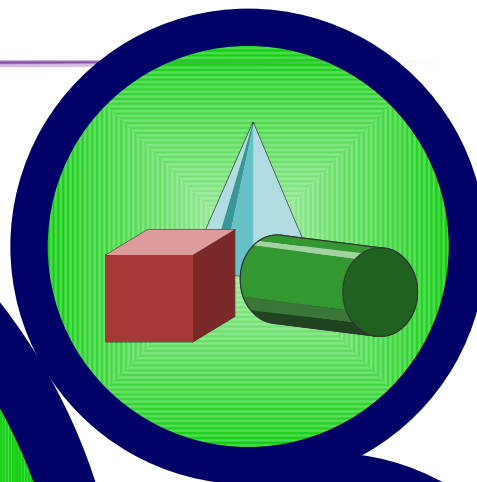
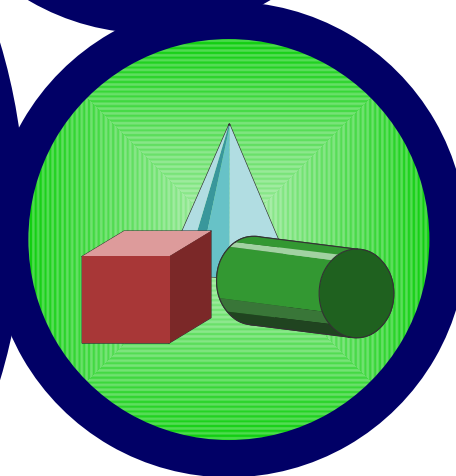




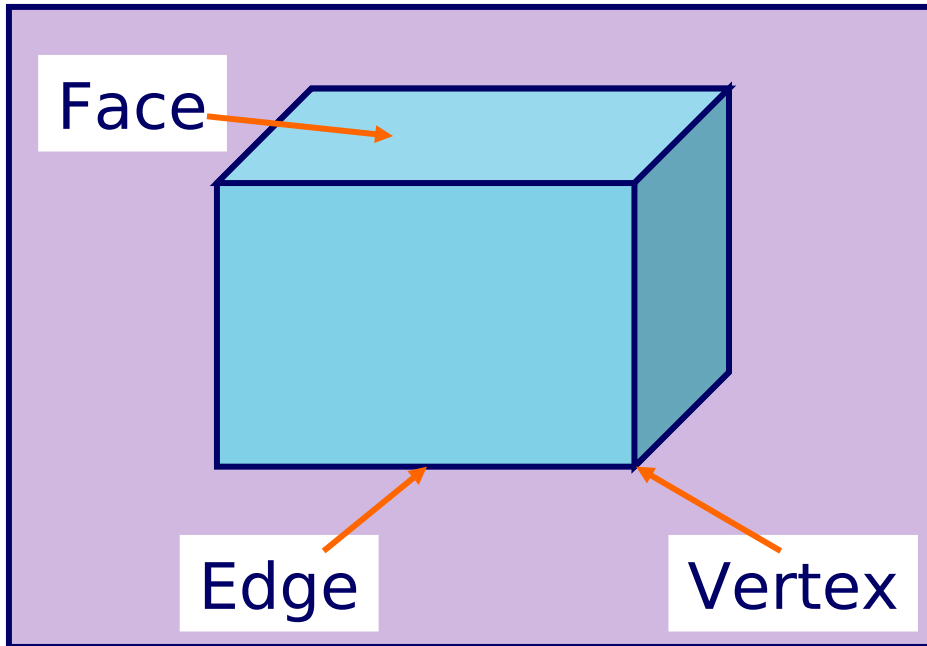
VOLUME

- **S10.3 Cubes and cuboids**
- **S10.4 Prisms and pyramids**
- **S10.5 Cylinders, cones and spheres**
- **S10.6 Lengths, areas and volumes of similar shapes**



# Cubes and cuboids

A **cuboid** is a 3-D shape with edges of different lengths. All of its faces are rectangular or square.



How many **faces** does a cuboid have **6**?

How many **edges** does a cuboid have **12**?

How many **vertices** does a cuboid have **8**?

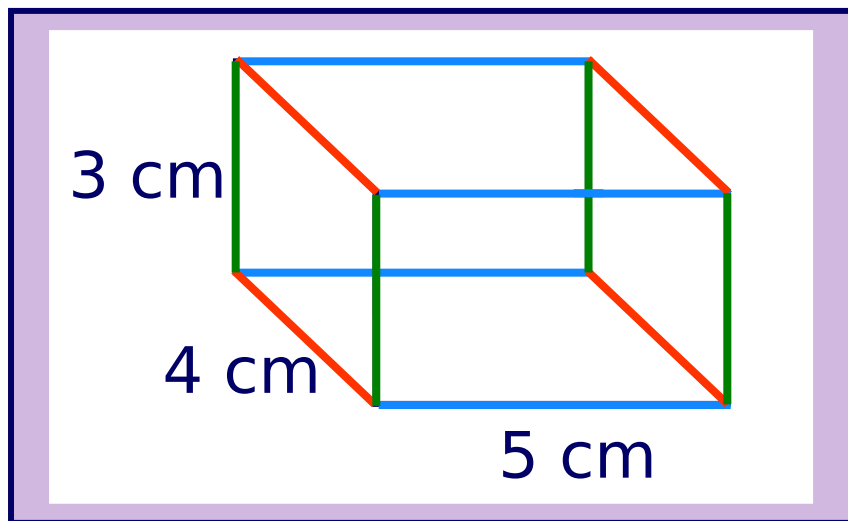
A **cube** is a special type of cuboid with edges of equal length. All of its faces are square.



# Length around the edges

Suppose we have a cuboid of length 5 cm, width 4 cm and height 3 cm. What is the total length around the edges?

Imagine the cuboid as a hollow wire frame:



The cuboid has 12 edges.

4 edges are 5 cm long.

4 edges are 4 cm long.

4 edges are 3 cm long.

$$\begin{aligned} \text{Total length around the edges} &= 4 \times 5 + 4 \times 4 + 4 \times 3 \\ &= 20 + 16 + 12 \\ &= 48 \text{ cm} \end{aligned}$$



# Length around the edges

To find the length around the edges of a cuboid of length  $l$ , width  $w$  and height  $h$  we can use the formula:

$$\text{Length around the edges} = 4l + 4w + 4h$$

or

$$\text{Length around the edges} = 4(l + w + h)$$

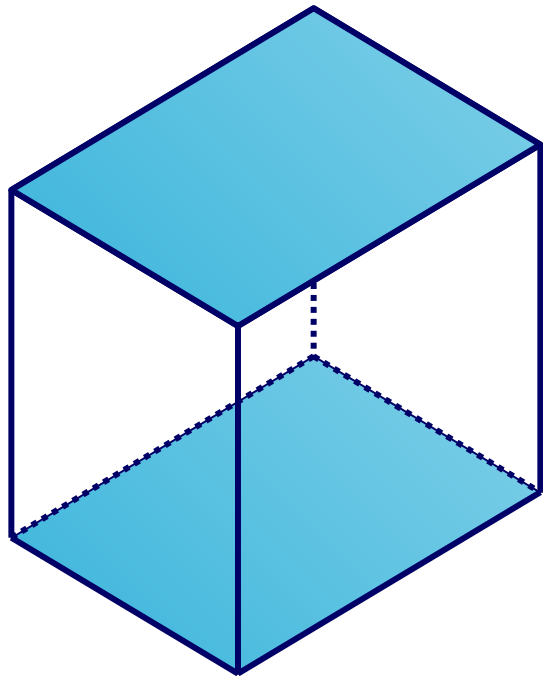
To find the length around the edges of a cube of length  $l$  we can use the formula:

$$\text{Length around the edges} = 12l$$



# Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



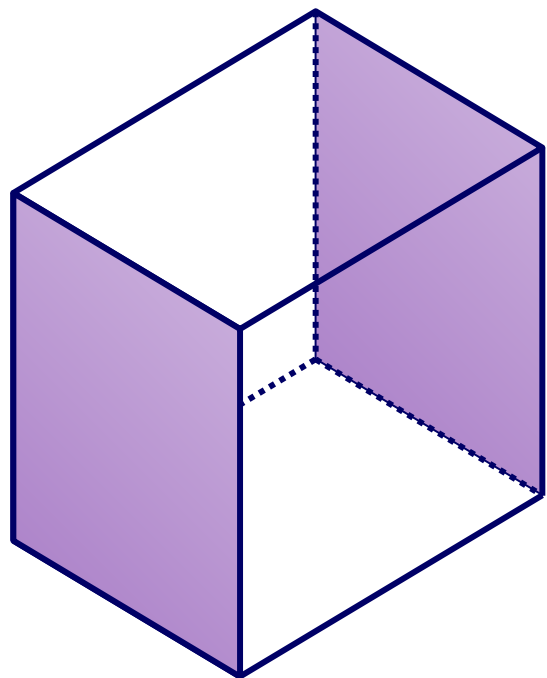
A cuboid has 6 faces.

The top and the bottom of the cuboid have the same area.



# Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



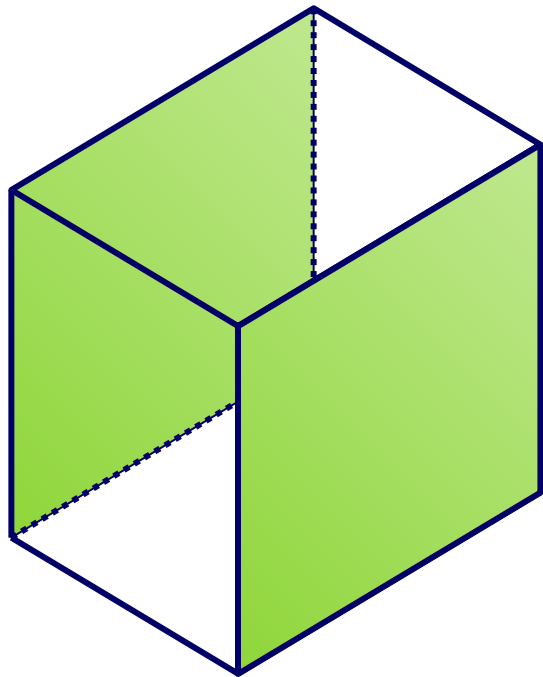
A cuboid has 6 faces.

The front and the back of the cuboid have the same area.



# Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



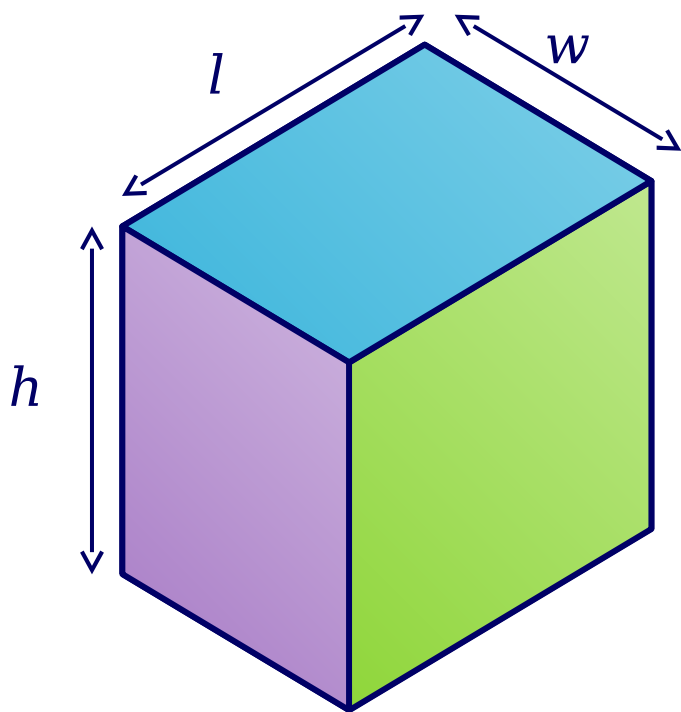
A cuboid has 6 faces.

The left hand side and the right hand side of the cuboid have the same area.



# Formula for the surface area of a cuboid

We can find the formula for the surface area of a cuboid as follows.



Surface area of a cuboid

$2 \times lw$       Top and bottom

$+ 2 \times hw$       Front and back

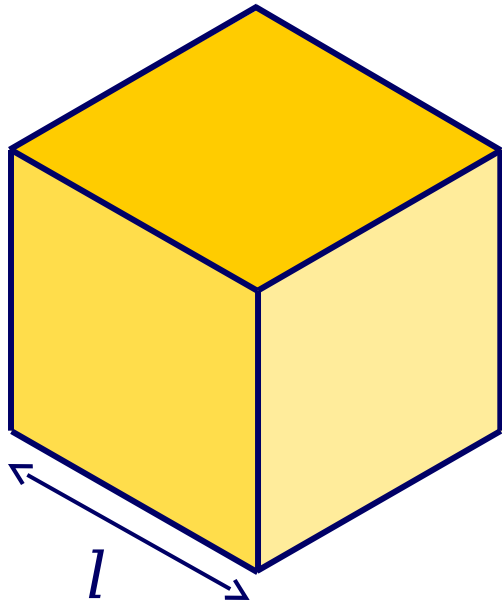
$+ 2 \times lh$       Left and right side

$$\text{Surface area of a cuboid} = 2lw + 2hw + 2lh$$



# Surface area of a cube

How can we find the surface area of a cube of length  $l$ ?



All six faces of a cube have the same area.

The area of each face is  $l \times l = l^2$

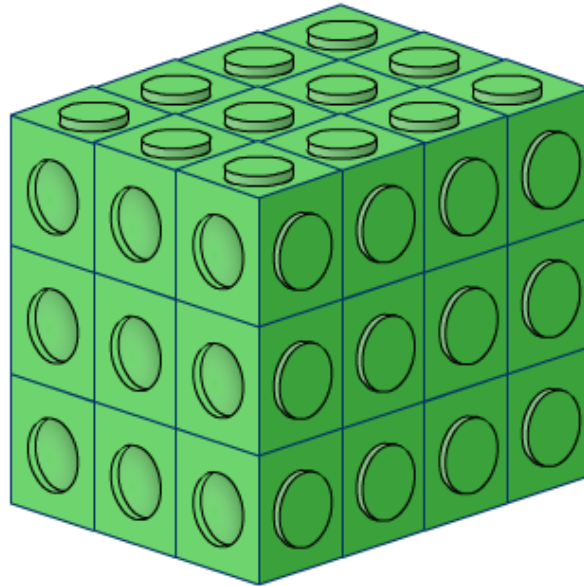
Therefore,

Surface area of a cube =  $6l^2$



# Volume of a cuboid

The following cuboid is made out of interlocking cubes.

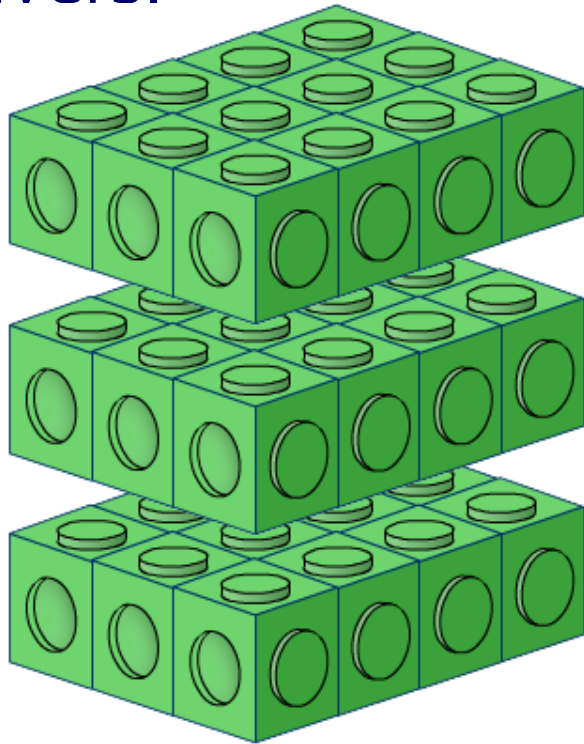


How many cubes does it contain?



# Volume of a cuboid

We can work this out by dividing the cuboid into layers.



The number of cubes in each layer can be found by multiplying the number of cubes along the length by the number of cubes along the width.

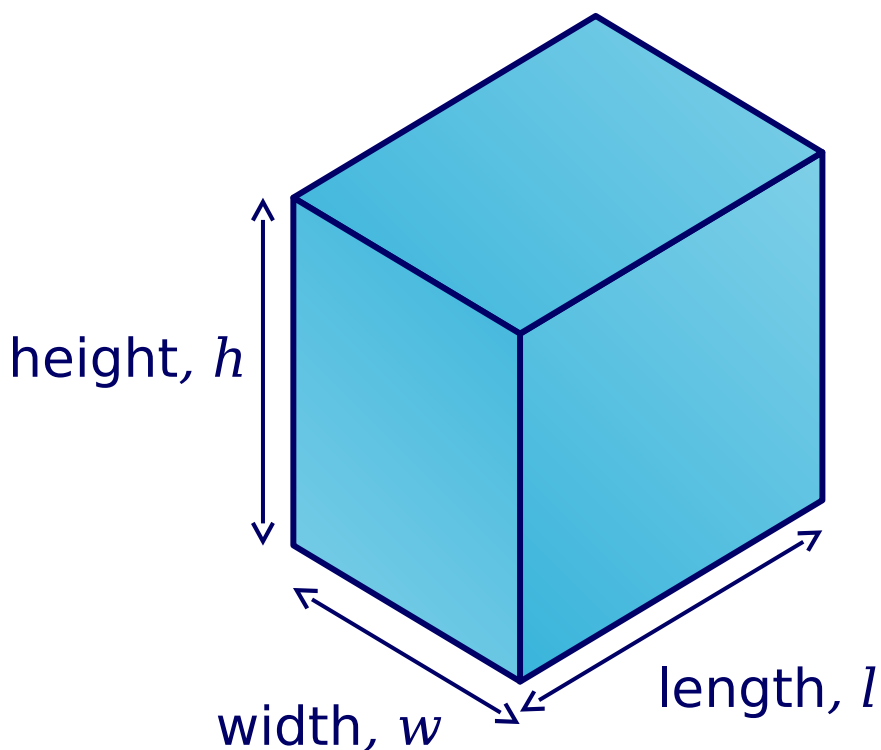
$$3 \times 4 = 12 \text{ cubes in each layer}$$

There are three layers altogether so the total number of cubes in the cuboid  
 $= 3 \times 12 = \mathbf{36 \text{ cubes}}$



# Volume of a cuboid

We can find the volume of a cuboid by multiplying the area of the base by the height.



The area of the base  
 $= \text{length} \times \text{width}$

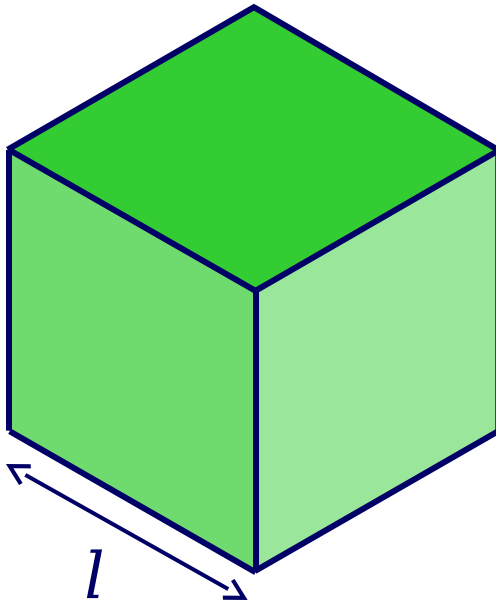
So,

$$\begin{aligned}\text{Volume of a cuboid} \\ &= \text{length} \times \text{width} \times \text{height} \\ &= lwh\end{aligned}$$



# Volume of a cube

How can we find the volume of a cube of length  $l$ ?



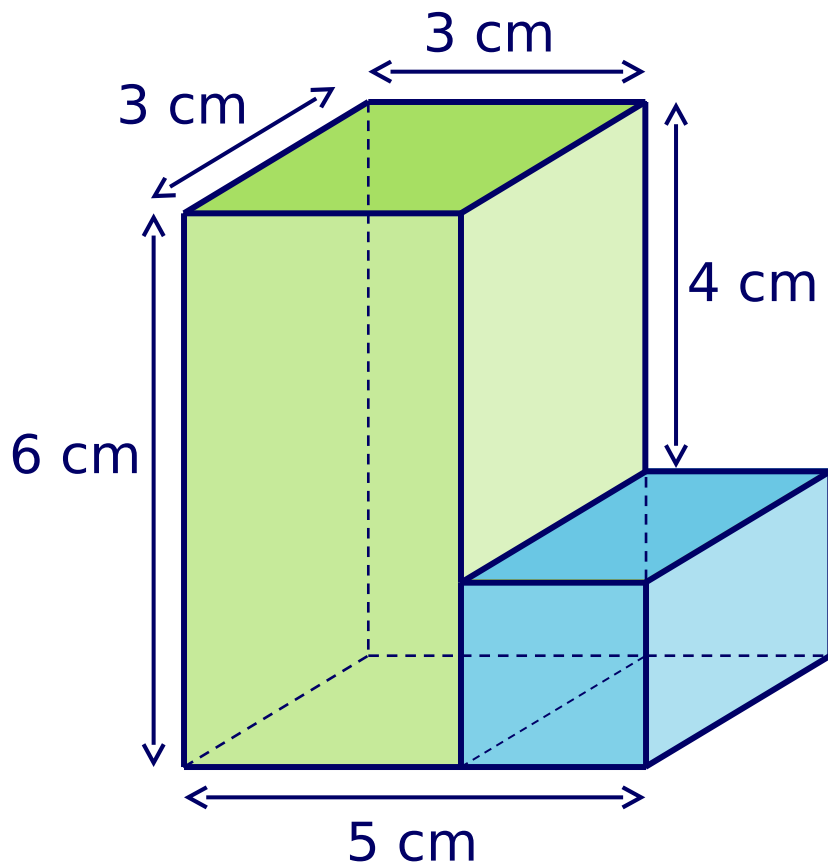
The length, width and height of a cube are the same.

Therefore:

$$\begin{aligned}\text{Volume of a cube} \\ &= (\text{length of one edge})^3 \\ &= l^3\end{aligned}$$

# Volume of shapes made from cuboids

What is the volume of this 3-D shape?



We can think of this shape as two cuboids joined together.

Volume of the green cuboid  
 $= 6 \times 3 \times 3 = 54 \text{ cm}^3$

Volume of the blue cuboid  
 $= 3 \times 2 \times 2 = 12 \text{ cm}^3$

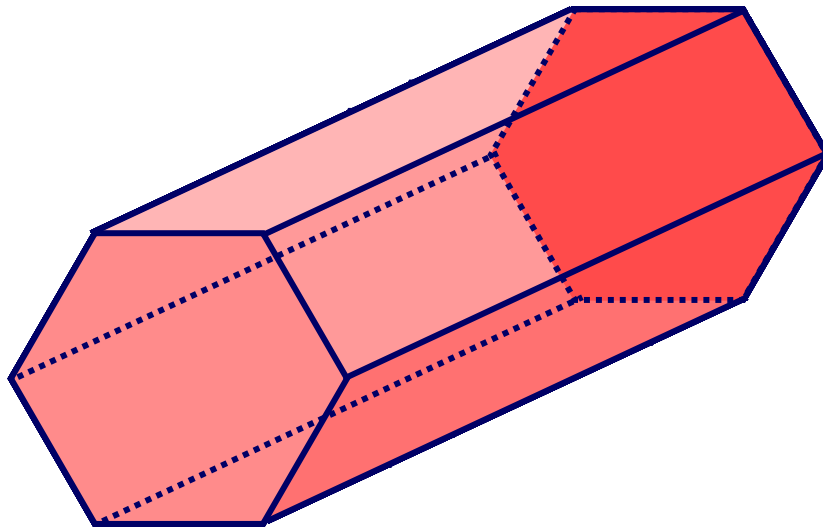
Total volume  
 $= 54 + 12 = 66 \text{ cm}^3$



# Prisms

A **prism** is a 3-D shape that has a constant cross-section along its length.

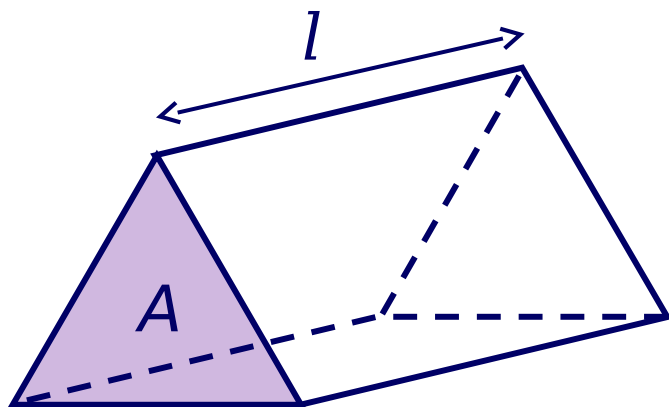
For example, this hexagonal prism has the same hexagonal cross-section throughout its length.



This is called a hexagonal prism because its cross-section is a hexagon.

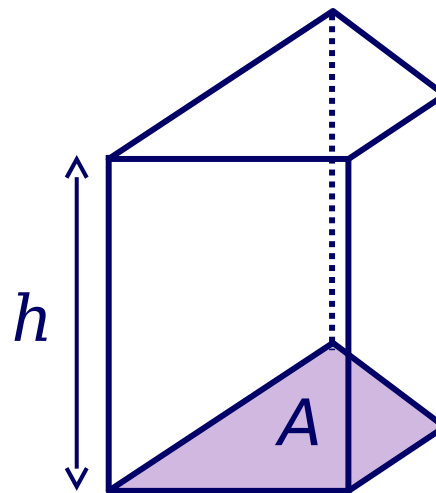
# Volume of a prism

The volume of a prism is found by multiplying the area of its cross-section  $A$  by its length  $l$  (or by its height if it is standing on its cross-section).



$$V = Al$$

or

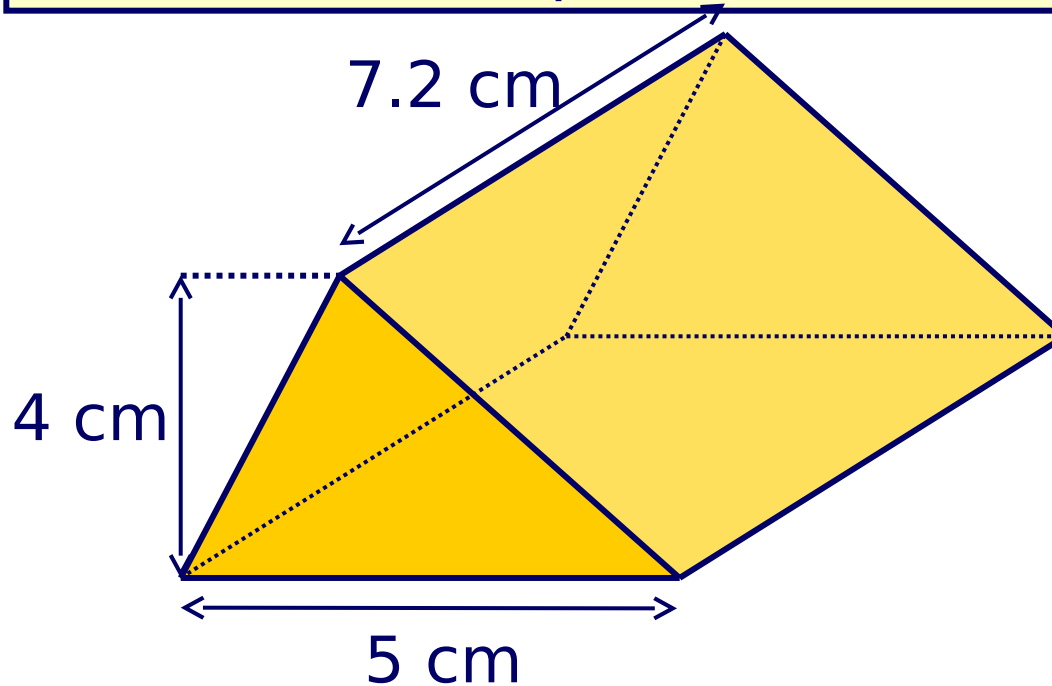


$$V = Ah$$



# Volume of a prism

What is the volume of this triangular prism?



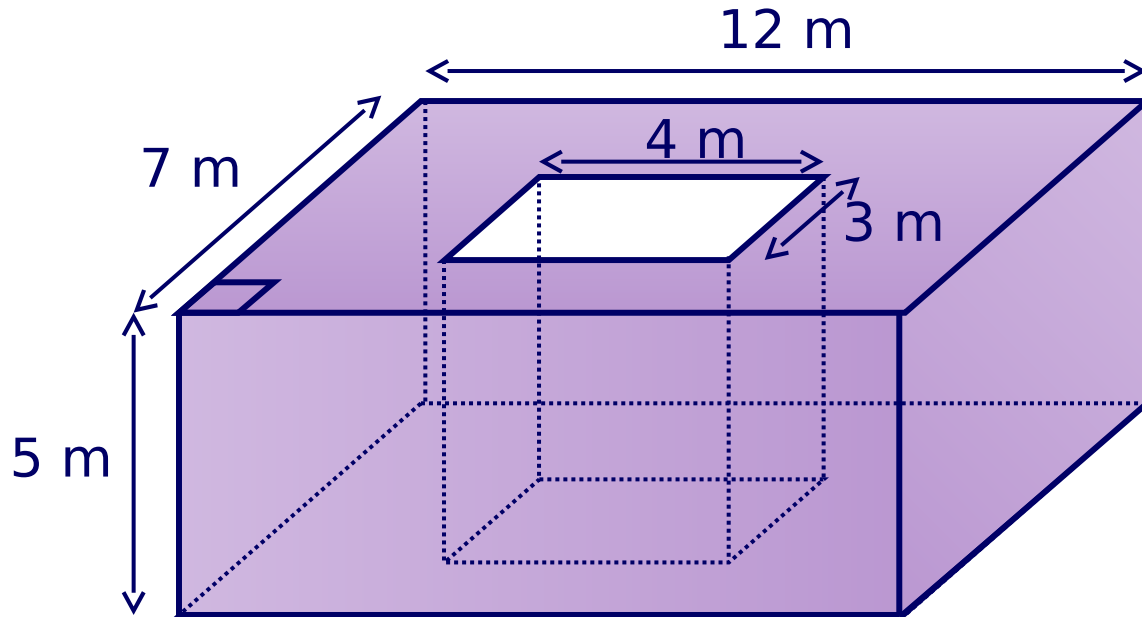
$$\text{Area of cross-section} = \frac{1}{2} \times 5 \times 4 = 10 \text{ cm}^2$$

$$\text{Volume of prism} = 10 \times 7.2 = 72 \text{ cm}^3$$



# Volume of a prism

What is the volume of this prism?



$$\text{Area of cross-section} = 7 \times 12 - 4 \times 3 = 84 - 12 = 72 \text{ m}^2$$

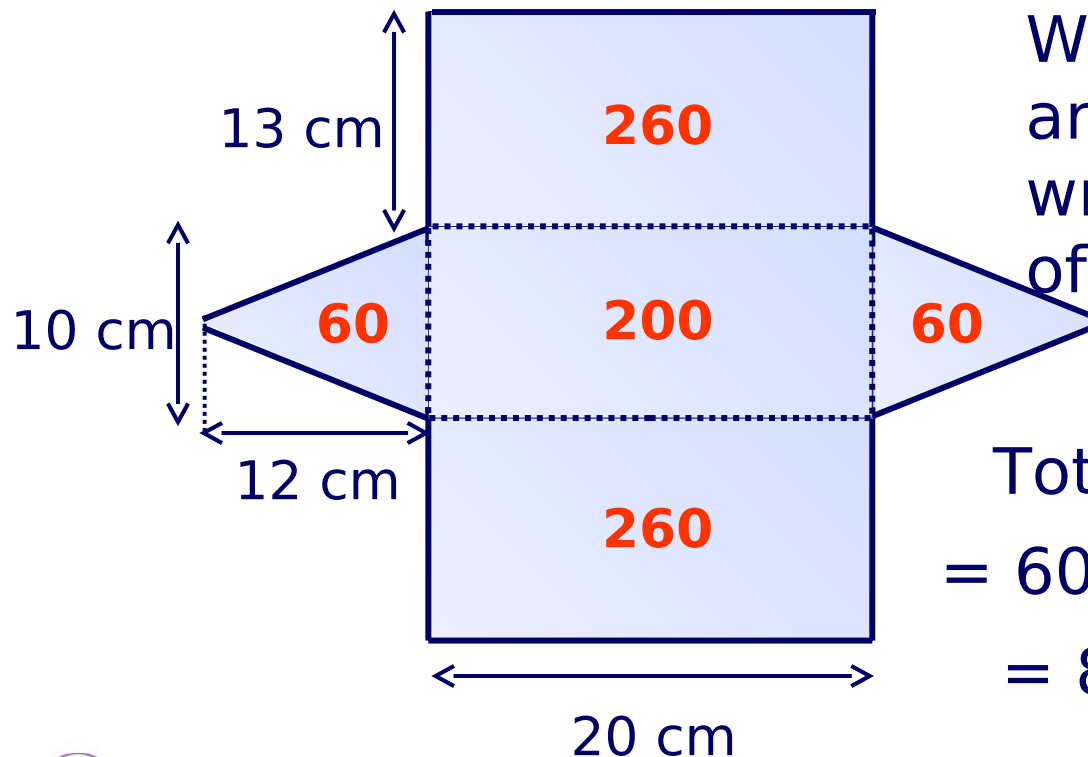
$$\text{Volume of prism} = 72 \times 5 = 360 \text{ m}^3$$



# Surface area of a prism

Here is the net of a triangular prism.

What is its surface area?



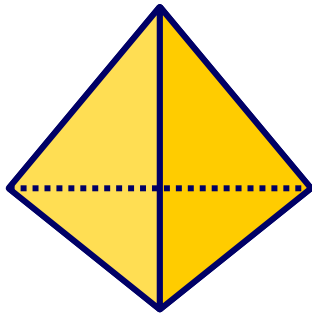
We can work out the area of each face and write it in the diagram of the net.

$$\begin{aligned}\text{Total surface area} &= 60 + 60 + 200 + 260 + 260 \\ &= 840 \text{ cm}^2\end{aligned}$$

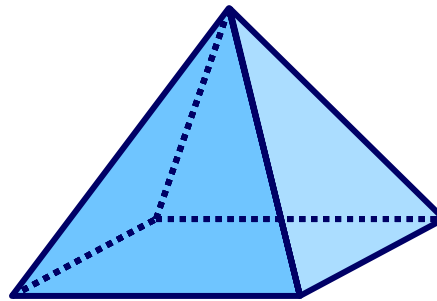


# Pyramids

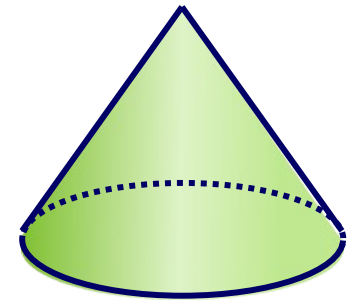
A **pyramid** is a 3-D shape whose base is usually a polygon but can also be a shape with curved edges. The faces rising up from the base meet at a common vertex or **apex**.  
The most common pyramids are:



A  
tetrahedron  
or triangular  
pyramid.



A square-based  
pyramid

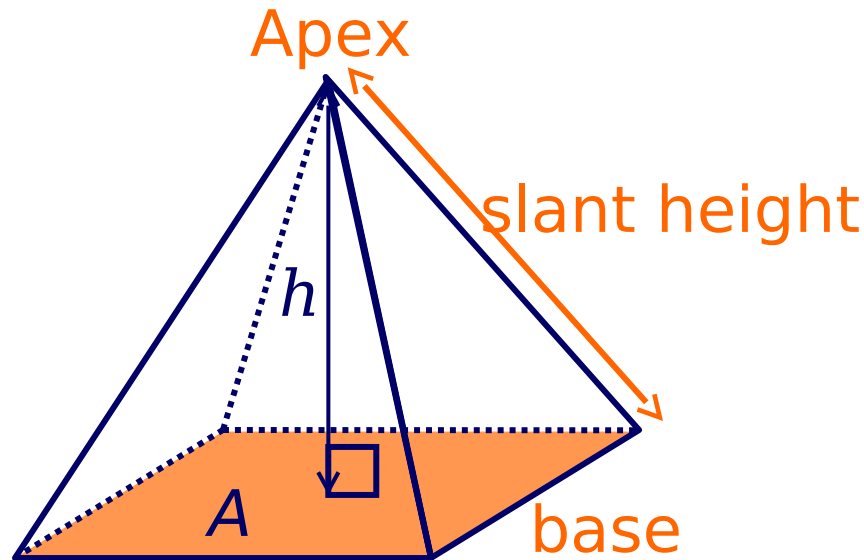


A cone



# Volume of a pyramid

The volume of a pyramid is found by multiplying the area of its base  $A$  by its perpendicular height  $h$  and dividing by 3.



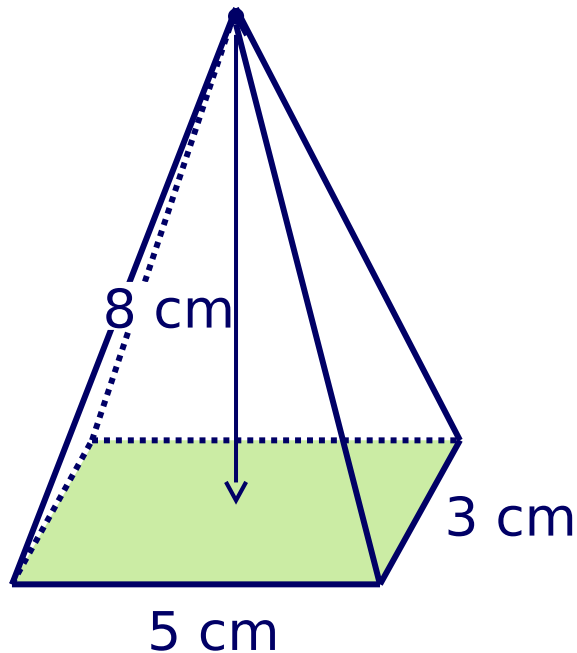
Volume of a pyramid =  $\frac{1}{3} \times \text{area of base} \times \text{height}$

$$V = \frac{1}{3} Ah$$



# Volume of a pyramid

What is the volume of this rectangle-based pyramid?



$$\begin{aligned}\text{Area of the base} &= 5 \times 3 \\ &= 15 \text{ cm}^2\end{aligned}$$

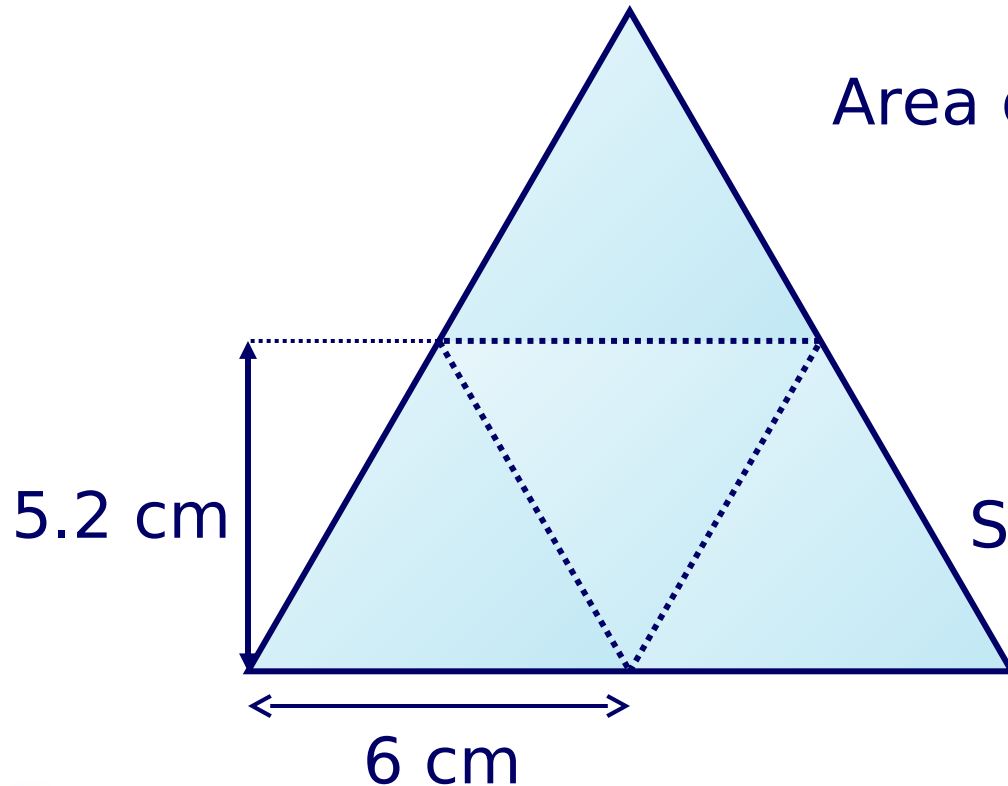
$$\begin{aligned}\text{Volume of pyramid} &= \frac{1}{3} Ah \\ &= \frac{1}{3} \times 15 \times 8 \\ &= 40 \text{ cm}^3\end{aligned}$$



# Surface area of a pyramid

Here is the net of a regular tetrahedron.

What is its surface area?



Area of each face =  $\frac{1}{2}bh$

$$= \frac{1}{2} \times 6 \times 5.2$$

$$= 15.6 \text{ cm}^2$$

Surface area =  $4 \times 15.6$

$$= \mathbf{62.4 \text{ cm}^2}$$



## S10 Length, area and volume

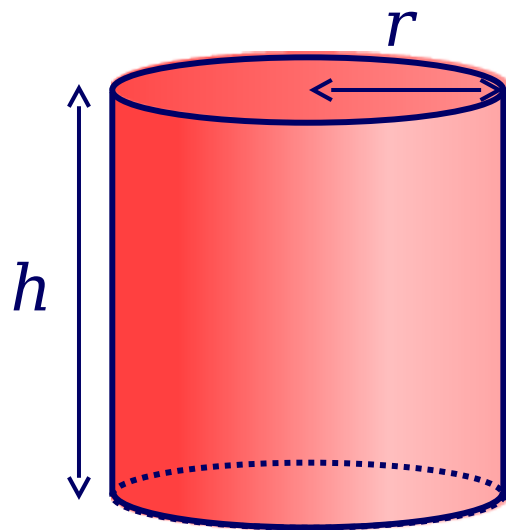
- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes



# Volume of a cylinder

A **cylinder** is a special type of prism with a circular cross-section.

Remember, the volume of a prism can be found by multiplying the area of the cross-section by the height of the prism.



The volume of a cylinder is given by:

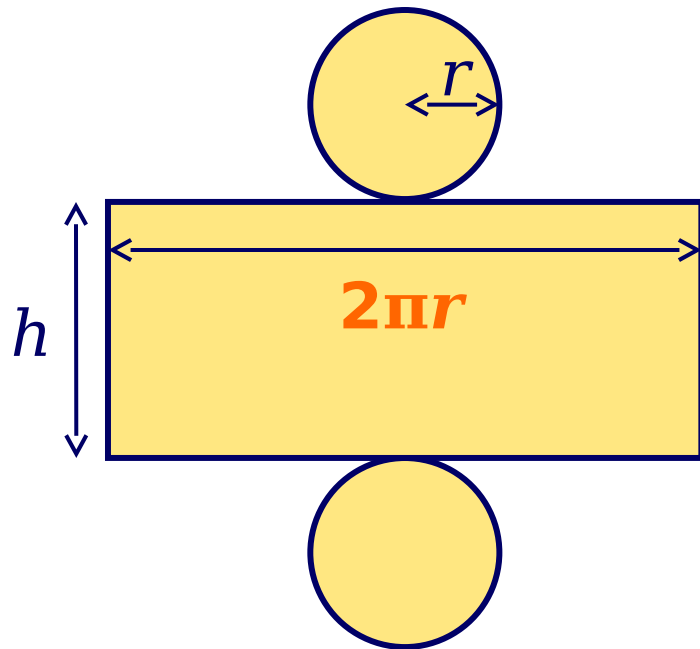
Volume = area of circular base  $\times$  height

or

$$V = \pi r^2 h$$

# Surface area of a cylinder

To find the formula for the surface area of a cylinder we can draw its net



How can we find the width of the curved face?

The width of the curved face is equal to the circumference of the circular base,  $2\pi r$ .  
Area of curved face =  $2\pi rh$

Area of 2 circular faces =  $2 \times \pi r^2$

Surface area of a cylinder =  $2\pi rh + 2\pi r^2$

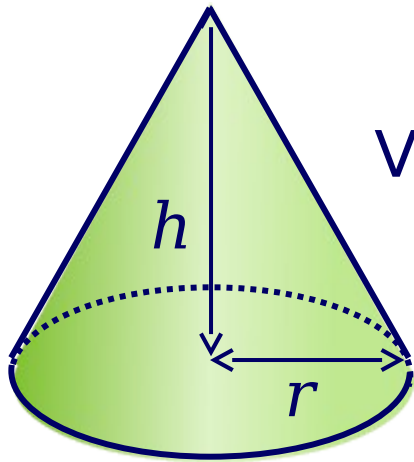
or Surface area =  $2\pi r(h + r)$



# Volume of a cone

A **cone** is a special type of pyramid with a circular base.

Remember, the volume of a pyramid can be found by multiplying the area of the base by the height and dividing by 3.



The volume of a cone is given by:

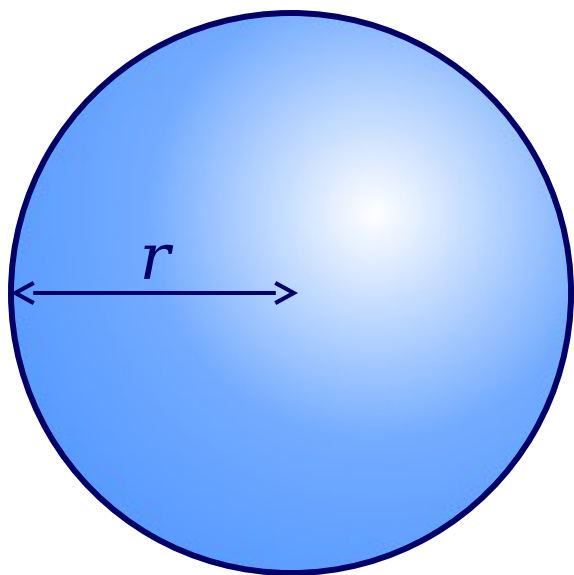
$$\text{Volume} = \frac{1}{3} \times \text{area of circular base} \times \text{height}$$

or

$$V = \frac{1}{3} \pi r^2 h$$

# Volume and surface area of a sphere

A **sphere** is a 3-D shape whose surface is always the same distance from the centre. This fixed distance is the radius of the sphere.



For a sphere of radius  $r$  :

$$\text{Volume} = \frac{4}{3} \pi r^3$$

and

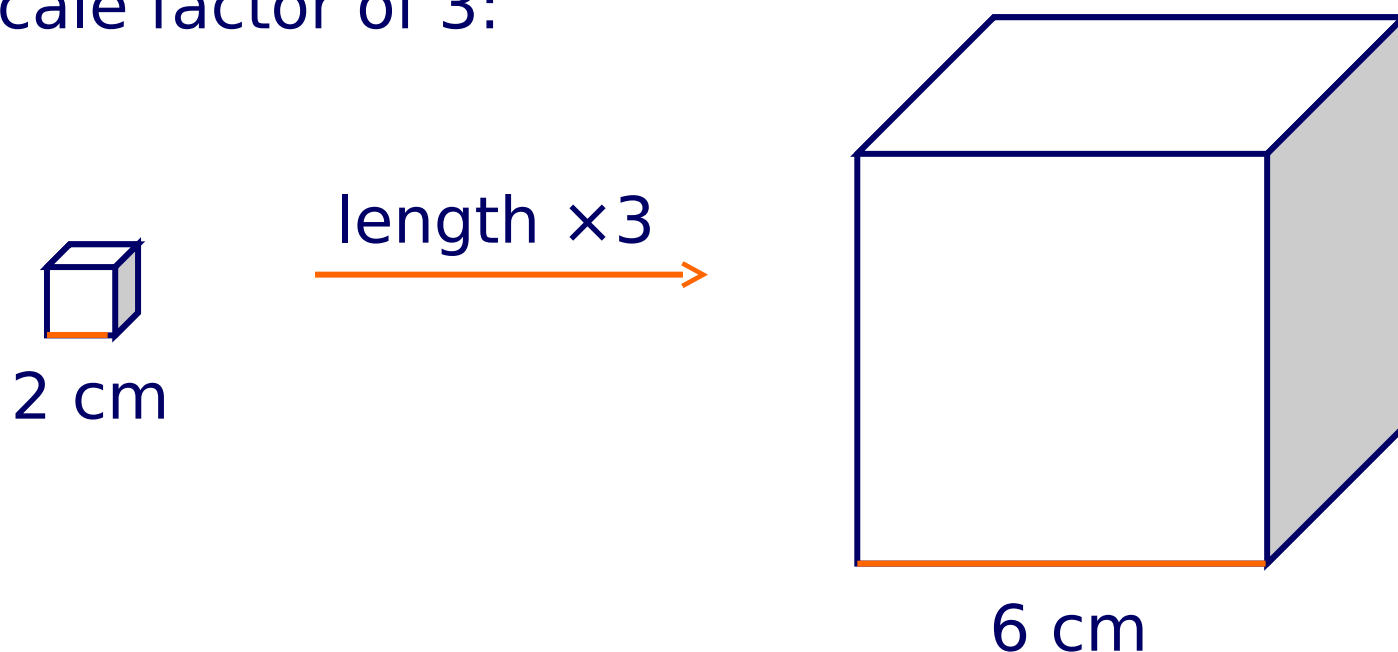
$$\text{Surface area} = 4\pi r^2$$



# Lengths in similar shapes

Remember, when a shape is enlarged every length in the shape is multiplied by a given scale factor.

Suppose we enlarge a cube with side length 2 cm by a scale factor of 3:

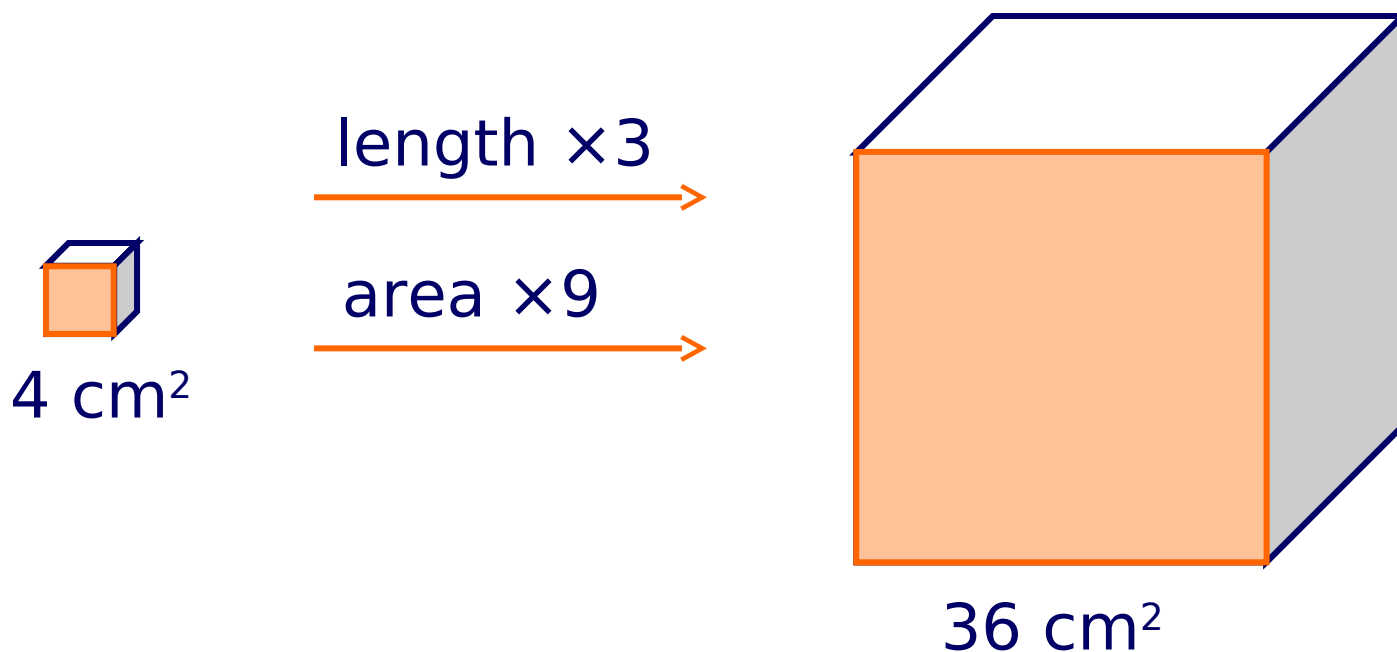


The length is enlarged by a scale factor of 3.



# Areas in similar shapes

Compare the area of one face of the original cube to the area of one face of the enlarged cube:



The area is enlarged by a scale factor of  $3^2$ .



# Volumes in similar shapes

Compare the volume of the original cube and the volume of the enlarged cube:



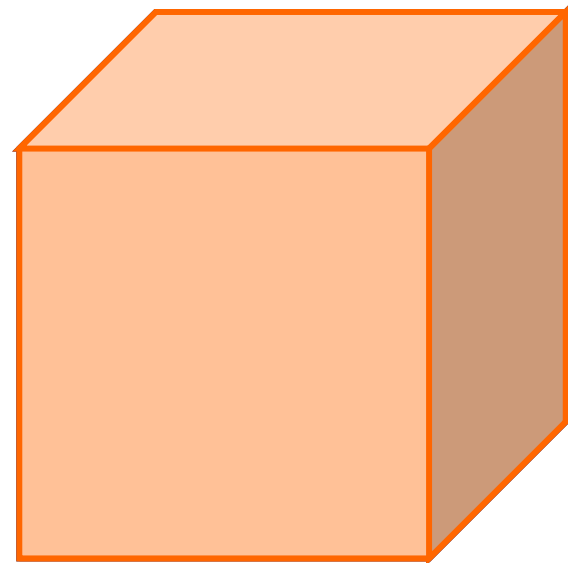
length  $\times 3$



area  $\times 9$



volume  $\times 27$



216 cm<sup>3</sup>

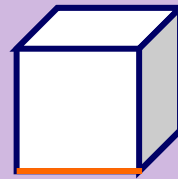
The volume is enlarged by a scale factor of  $3^3$ .



# Lengths in similar shapes

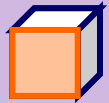


length = 1

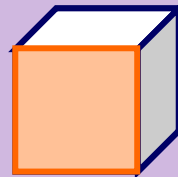


length =  $k$

In general, if an object is enlarged by a scale factor of  $k$ :



area = 1

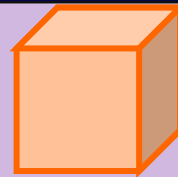


area =  $k^2$

its area or surface area is enlarged by a scale factor of  $k^2$



volume = 1



volume =  $k^3$

and its volume is enlarged by a scale factor of  $k^3$ .



# Could a giant survive on earth?

An average man:



Height: 1.7 m

Surface area: 1.8 m<sup>2</sup>

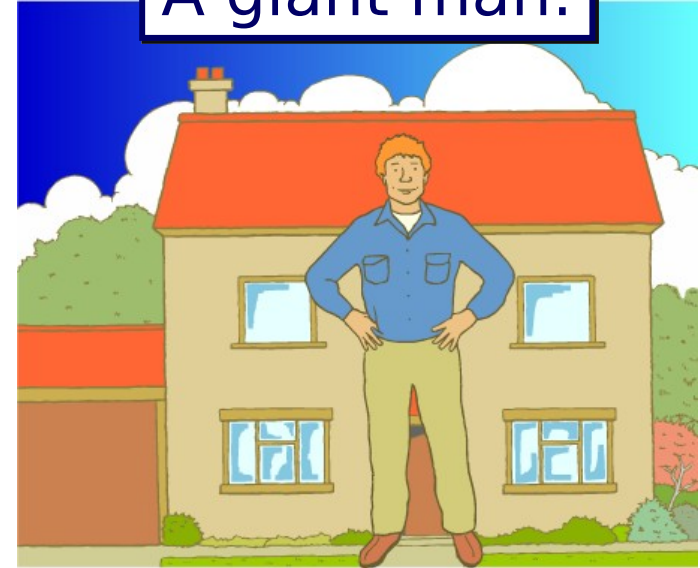
Volume: 0.07 m<sup>3</sup>

×5→

×25→

×125→

A giant man:



Height: 8.5 m

Surface area: 45 m<sup>2</sup>

Volume: 8.75 m<sup>3</sup>

Could this giant survive on earth?

