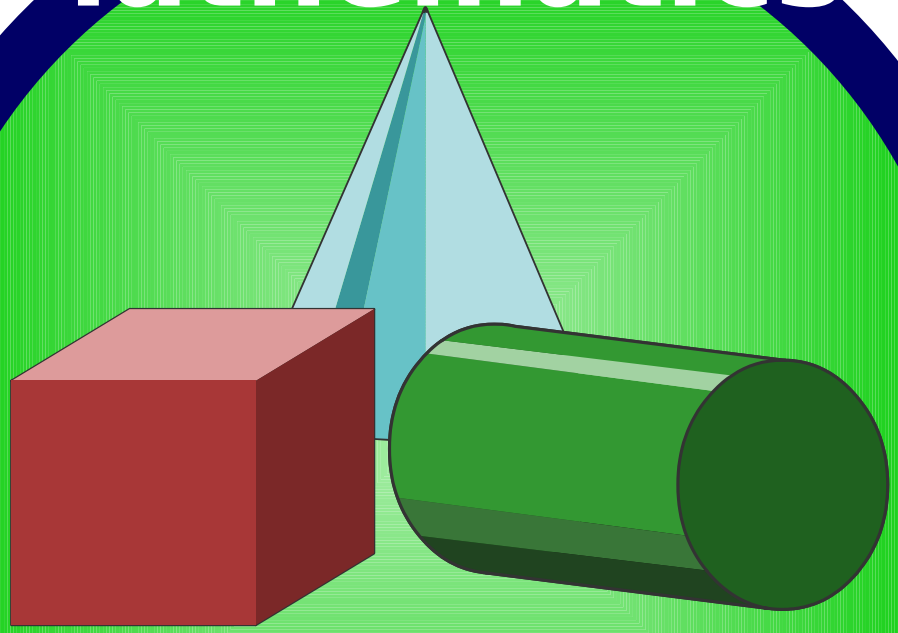
KS4 Mathematics



S10 Length, area and volume

© Boardworks Ltd 2005

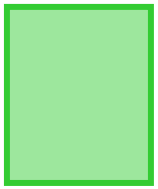
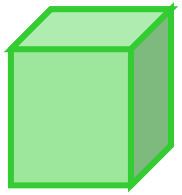
Contents

S10 Length, area and volume

- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Dimensions of length, area and volume

length		one dimension	length	mm, cm, m, km, inch, foot
area		two dimensions	length × length	mm^2, cm^2, m^2, hectare, km^2, square inch, square foot
volume		three dimensions	length × length × length	mm^3, cm^3, m^3, km^2, cubic inch, cubic foot

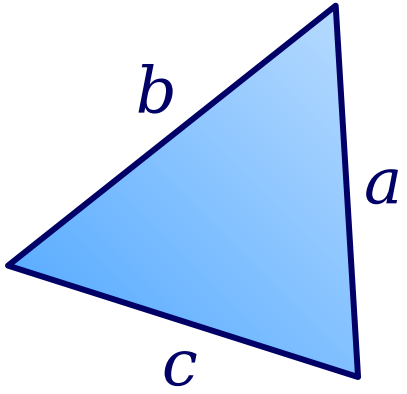
© Boardworks Ltd 2005

Using dimensions to check formulae

We can use dimensions to check whether a given formula is correct and to find out whether it represents a length, an area or a volume.

When we analyze the dimensions of a formula we often use a capital L in brackets, [L], to represent a dimension of length.

Suppose we have a triangle with sides of length a, b and c.



Perimeter = $a + b + c$

Analyzing the dimensions we have:

a length + a length + a length = a length

or [L] + [L] + [L] = [L]

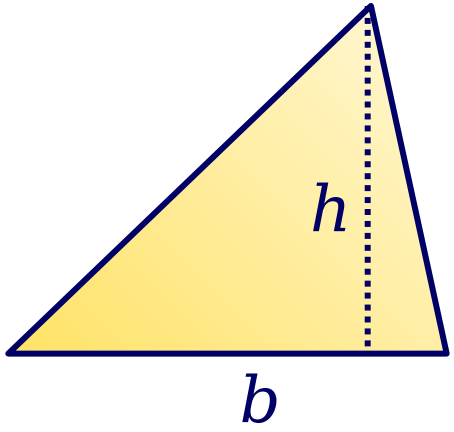
© Boardworks Ltd 2005

Using dimensions to check formulae

Area is two-dimensional. It is derived from multiplying two lengths together.

$$[L] \times [L] = [L^2]$$

Suppose we have a triangle with base b and height h.



$$\text{Area} = \tfrac{1}{2}bh$$

Analyzing the dimensions we have:

a number × a length × a length = an area

or $[L] \times [L] = [L^2]$

In a formula numbers, including π, do not have dimensions

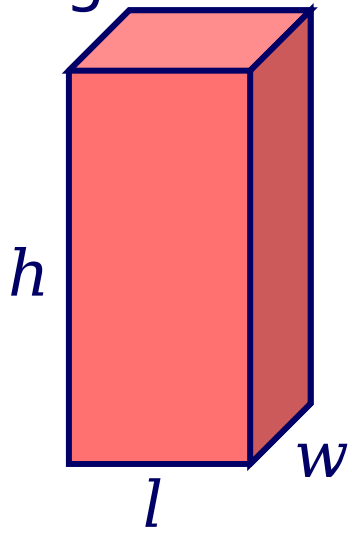
© Boardworks Ltd 2005

Using dimensions to check formulae

If we add two or more areas together, for example when calculating surface area, the result is an area.

$$[L^2] + [L^2] = [L^2]$$

Suppose we have a cuboid with length l width w and height h.



Surface area = $2(lw + wh + lh)$

Analyzing the dimensions we have:

2 × (an area + an area + an area) = an are

or

$[L^2] + [L^2] + [L^2] = [L^2]$

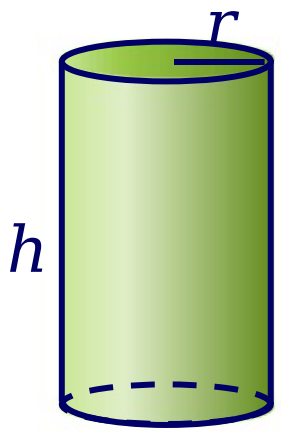
© Boardworks Ltd 2005

Using dimensions to check formulae

Volume is three-dimensional. It is derived from multiplying three lengths together.

$$[L] \times [L] \times [L] = [L^3]$$

Suppose we have a cylinder with height h and radius r.



$$\text{Volume} = \pi r^2 h$$

Analyzing the dimensions we have:

a number × an area × a length = a volum

or $[L^2] \times [L] = [L^3]$

We ignore π because it is a number and so has no dimension

© Boardworks Ltd 2005

Consistency in formulae

Remember,

- Adding or subtracting lengths gives us a length.
- Adding or subtracting areas gives us an area.
- Adding or subtracting volumes gives us a volume.

What happens if we add an area to a volume?

This is impossible

If a formula involved an expression with 2 dimensions added to an expression with 3 dimensions it would be **inconsistent**.

We would conclude that a mistake had been made.




© Boardworks Ltd 2005

Consistency in formulae

Show that the formula $C = 2\pi r^2 + \pi r^3$ is inconsistent and therefore incorrect.

Analyzing the dimensions we have:

an area + a volume

or $[L^2] + [L^3]$

This is inconsistent and so the formula is incorrect.





© Boardworks Ltd 2005

Length, area or volume?

All the letters except π represent lengths.
What does this expression represent?

$ab + \pi r$

- Length
- Volume
- Area
- None of these

© Boardworks Ltd 2005

Contents

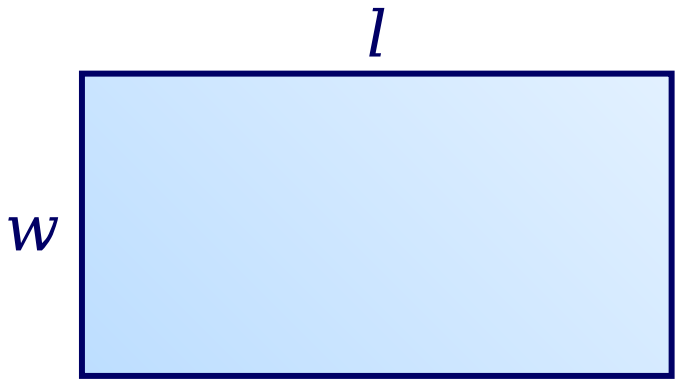
S10 Length, area and volume

- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Rectangles

The perimeter of a rectangle with length l and width w can be written as:



$$\text{Perimeter} = 2l + 2w$$

or

$$\text{Perimeter} = 2(l + w)$$

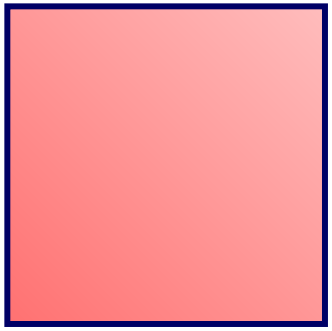
The area of a rectangle is given as:

$$\text{Area} = lw$$

© Boardworks Ltd 2005

Squares

When the length and the width of a rectangle are equal we call it a square. A square is just a special type of rectangle.
The perimeter of a square with length l is given as:

l 

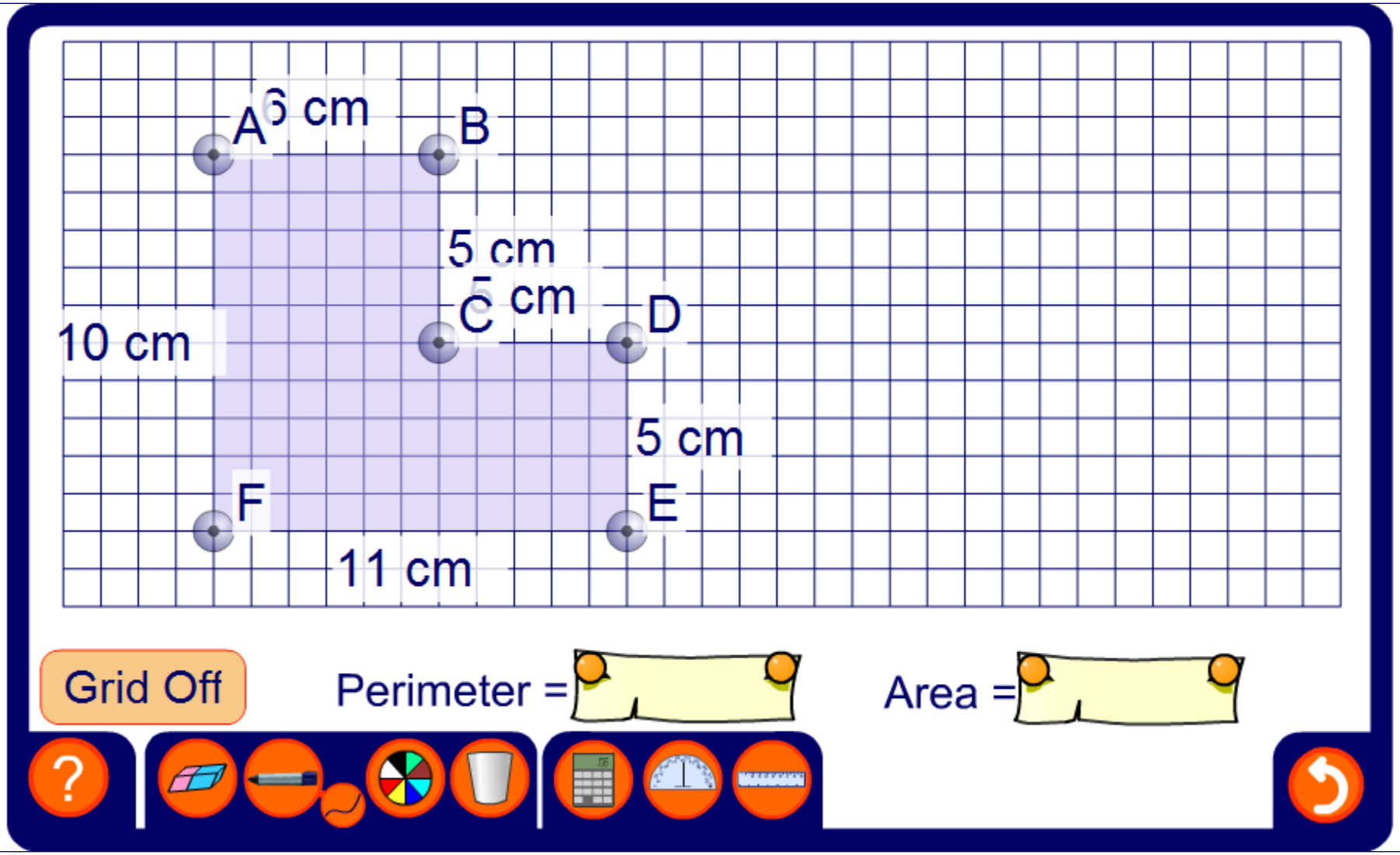
$$\text{Perimeter} = 4l$$

The area of a square is given as:

$$\text{Area} = l^2$$

© Boardworks Ltd 2005

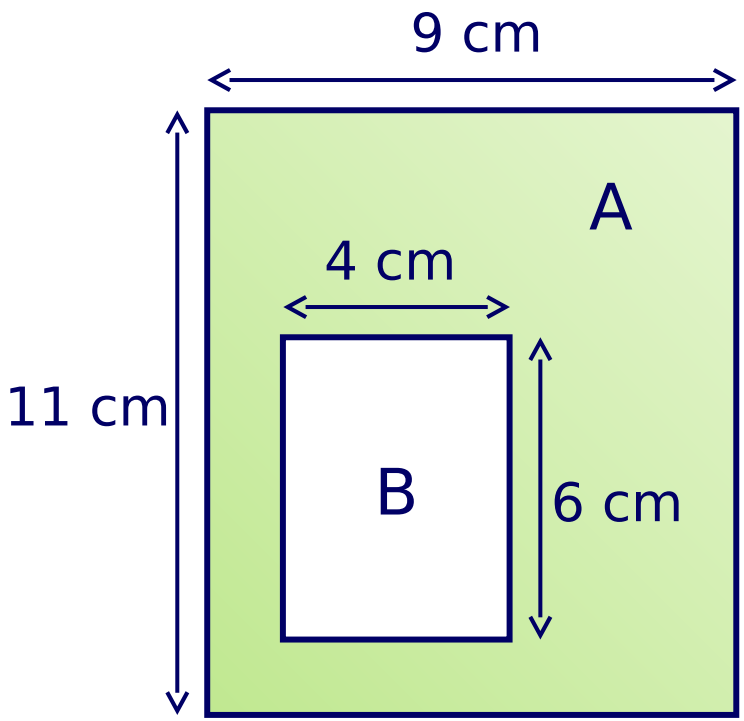
Shapes made from rectangles



© Boardworks Ltd 2005

Shapes made from rectangles

How can we find the area of the shaded shape?



We can think of this shape as being made up of one rectangle cut out of another rectangle.

Label the rectangles A and B.

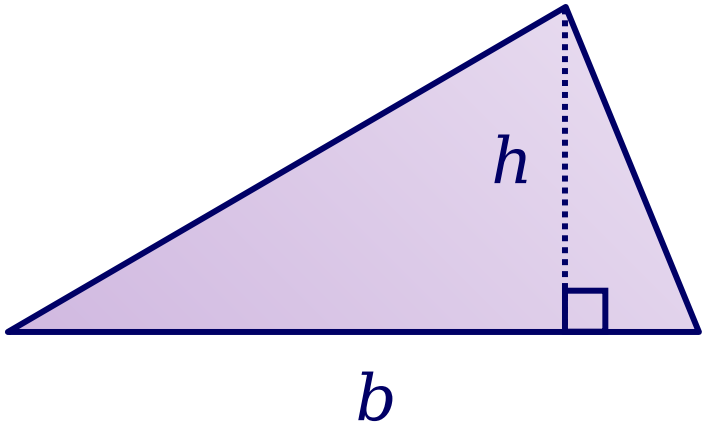
Area A = $9 \times 11 = 99$ cm^2

Area B = $4 \times 6 = 24$ cm^2

Total area = $99 - 24 =$ **75 cm^2**

© Boardworks Ltd 2005

The area of a triangle

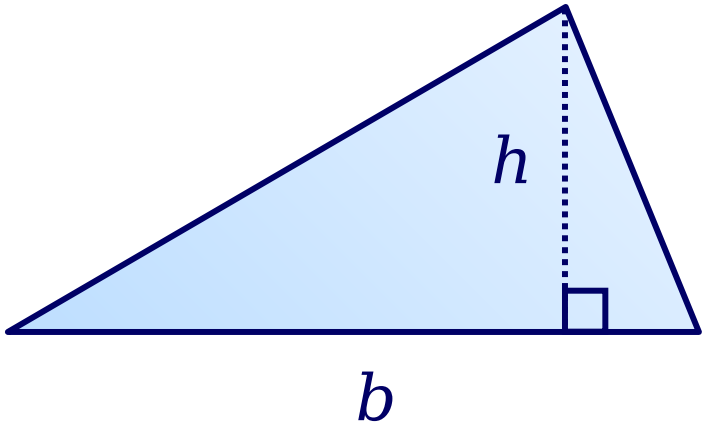


The area of a triangle with base b and perpendicular height h is given by:

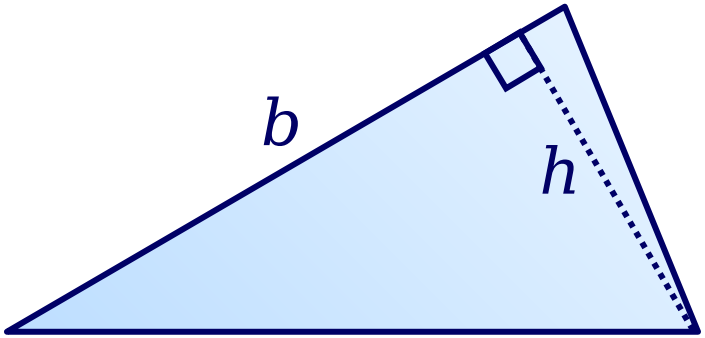
$$\text{Area of a triangle} = \frac{1}{2}bh$$

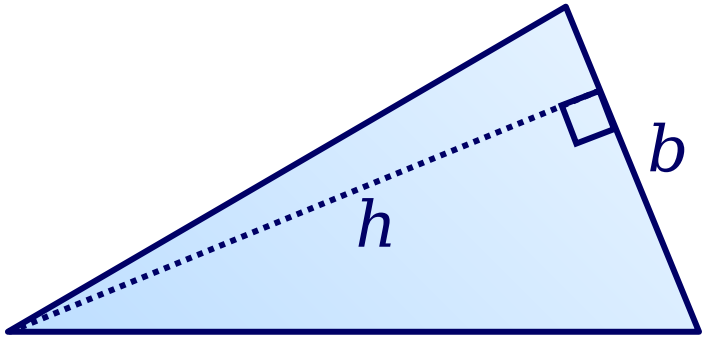
© Boardworks Ltd 2005

The area of a triangle



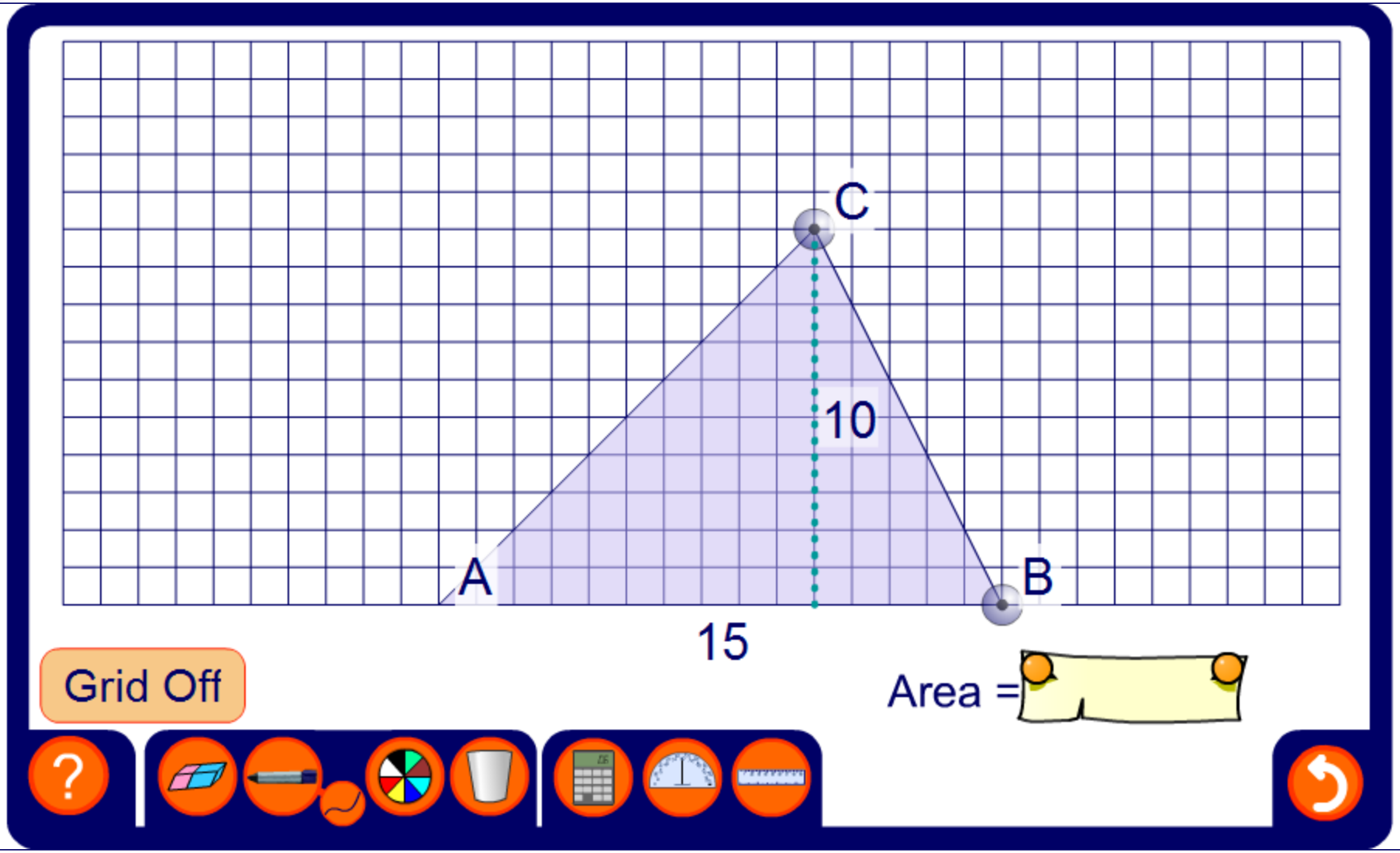
Any side of the triangle can be taken as the base, as long as the height is perpendicular to it:





© Boardworks Ltd 2005

The area of a triangle

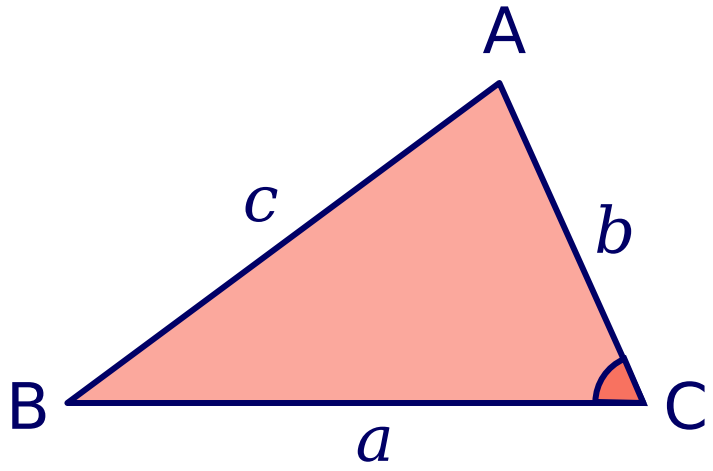


© Boardworks Ltd 2005

The area of a triangle

Suppose that instead of the height of a triangle, we are given the base, one of the sides and the included angle.

We can use trigonometry to find the area of the triangle:



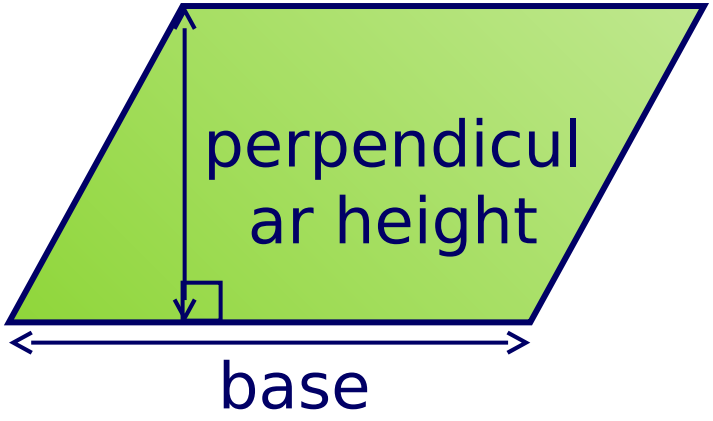
$$\text{Area of triangle ABC} = \frac{1}{2}\ ab \sin C$$

© Boardworks Ltd 2005

The area of a parallelogram

The area of any **parallelogram** can be found using the formula:

Area of a parallelogram = base × perpendicular height



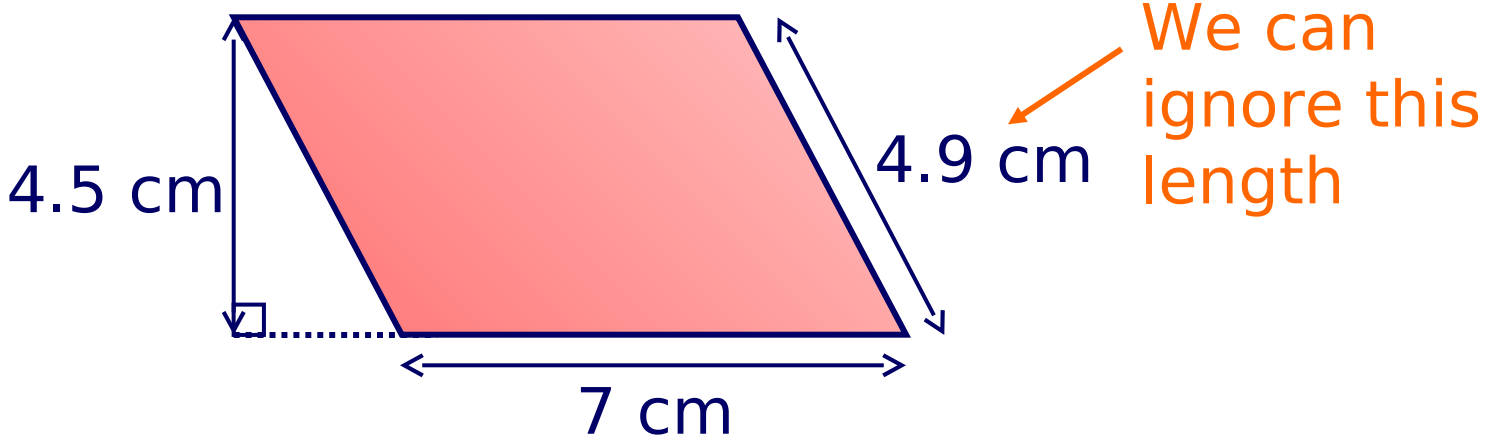
Or using letter symbols,

$$A = bh$$

© Boardworks Ltd 2005

The area of a parallelogram

What is the area of this parallelogram?



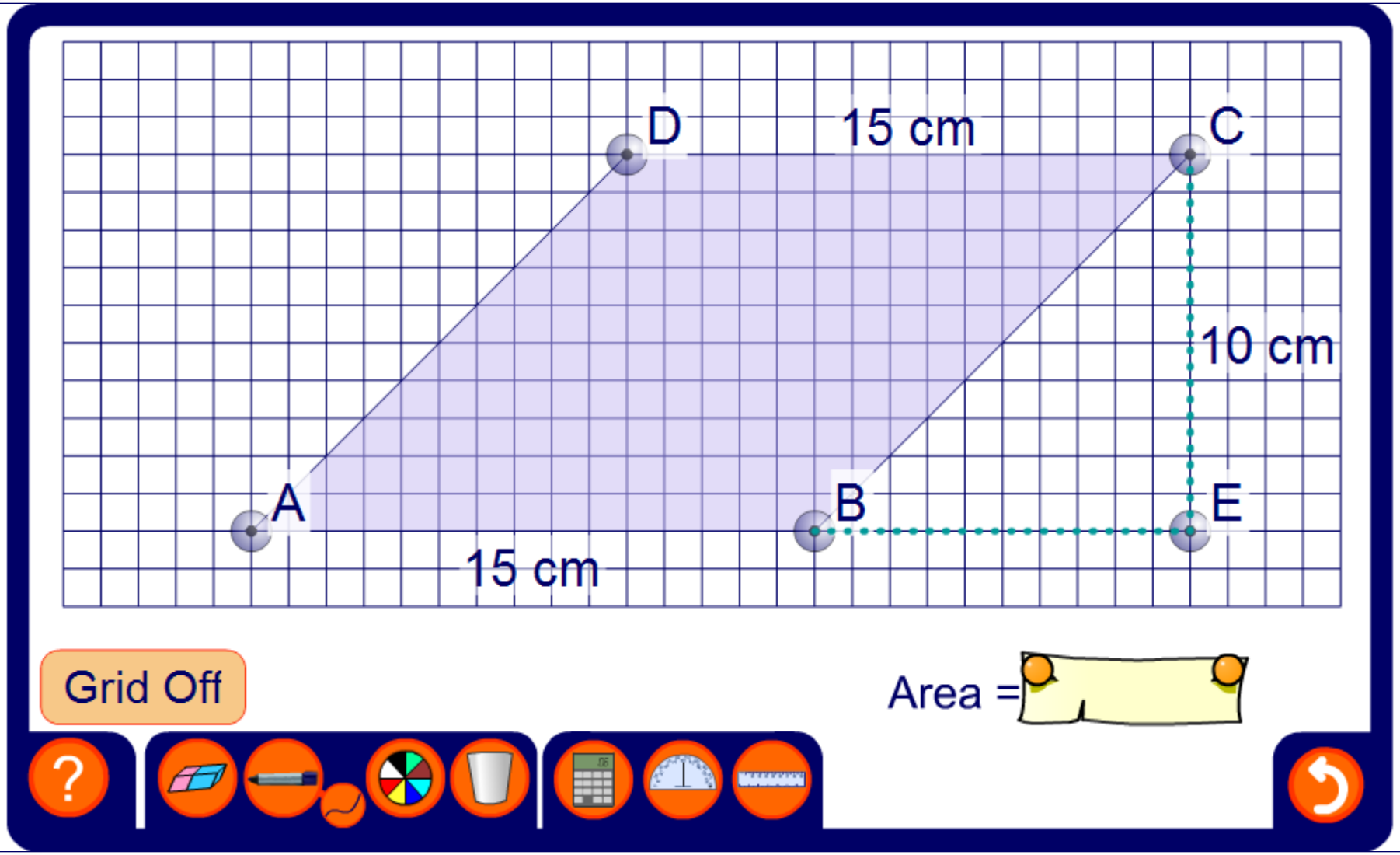
Area of a parallelogram = bh

$= 7 \times 4.5$

$= \mathbf{31.5\ cm^2}$

© Boardworks Ltd 2005

The area of a parallelogram

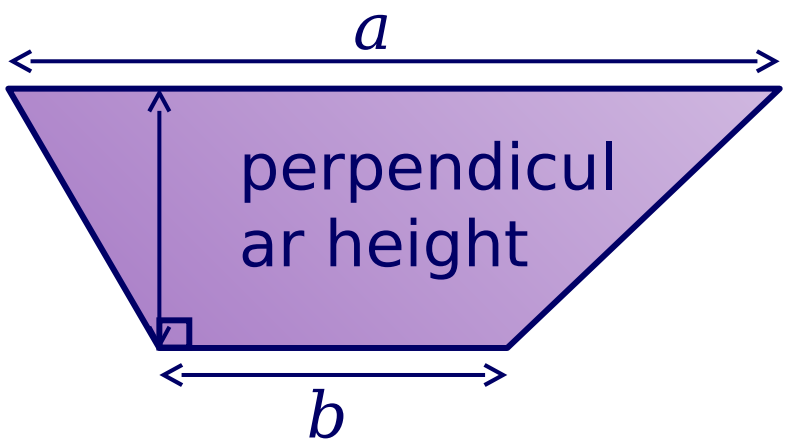


© Boardworks Ltd 2005

The area of a trapezium

The area of any **trapezium** can be found using the formula:

$$\text{Area of a trapezium} = \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$$



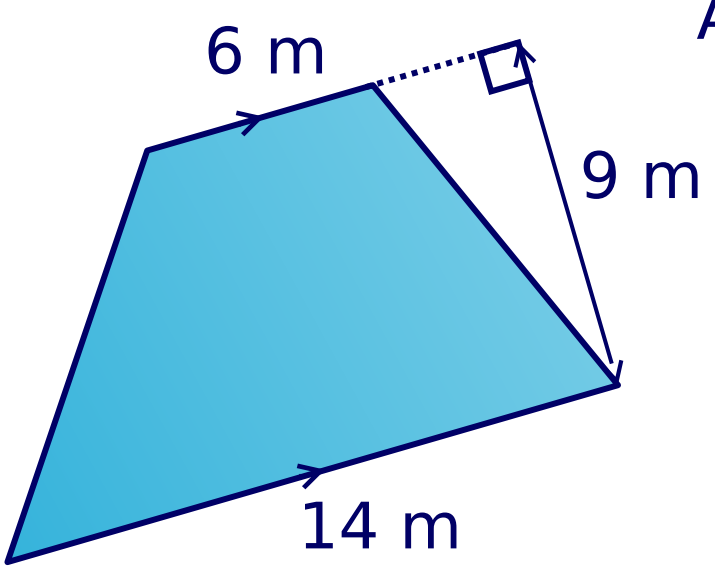
Or using letter symbols,

$$A = \frac{1}{2}(a + b)h$$

© Boardworks Ltd 2005

The area of a trapezium

What is the area of this trapezium?



$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(6 + 14) \times 9$$

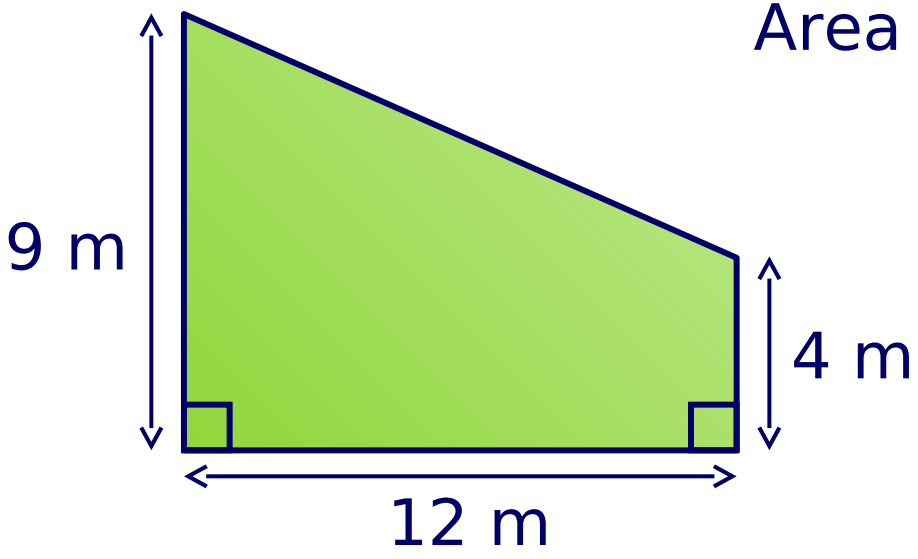
$$= \frac{1}{2} \times 20 \times 9$$

$$= \mathbf{90\ m^2}$$

© Boardworks Ltd 2005

The area of a trapezium

What is the area of this trapezium?



$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

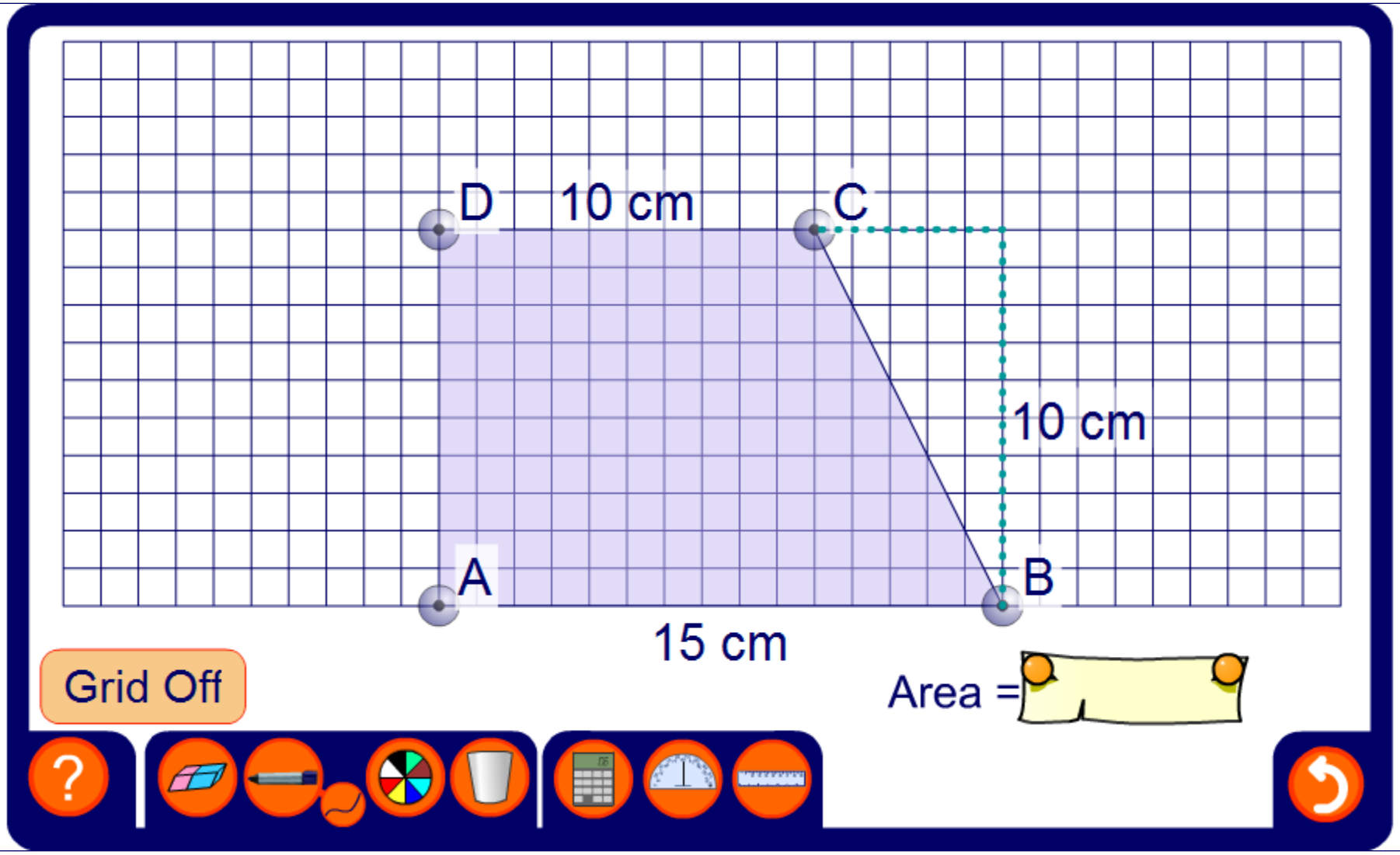
$$= \frac{1}{2}(9 + 4) \times 12$$

$$= \frac{1}{2} \times 13 \times 12$$

$$= \mathbf{78\ m^2}$$

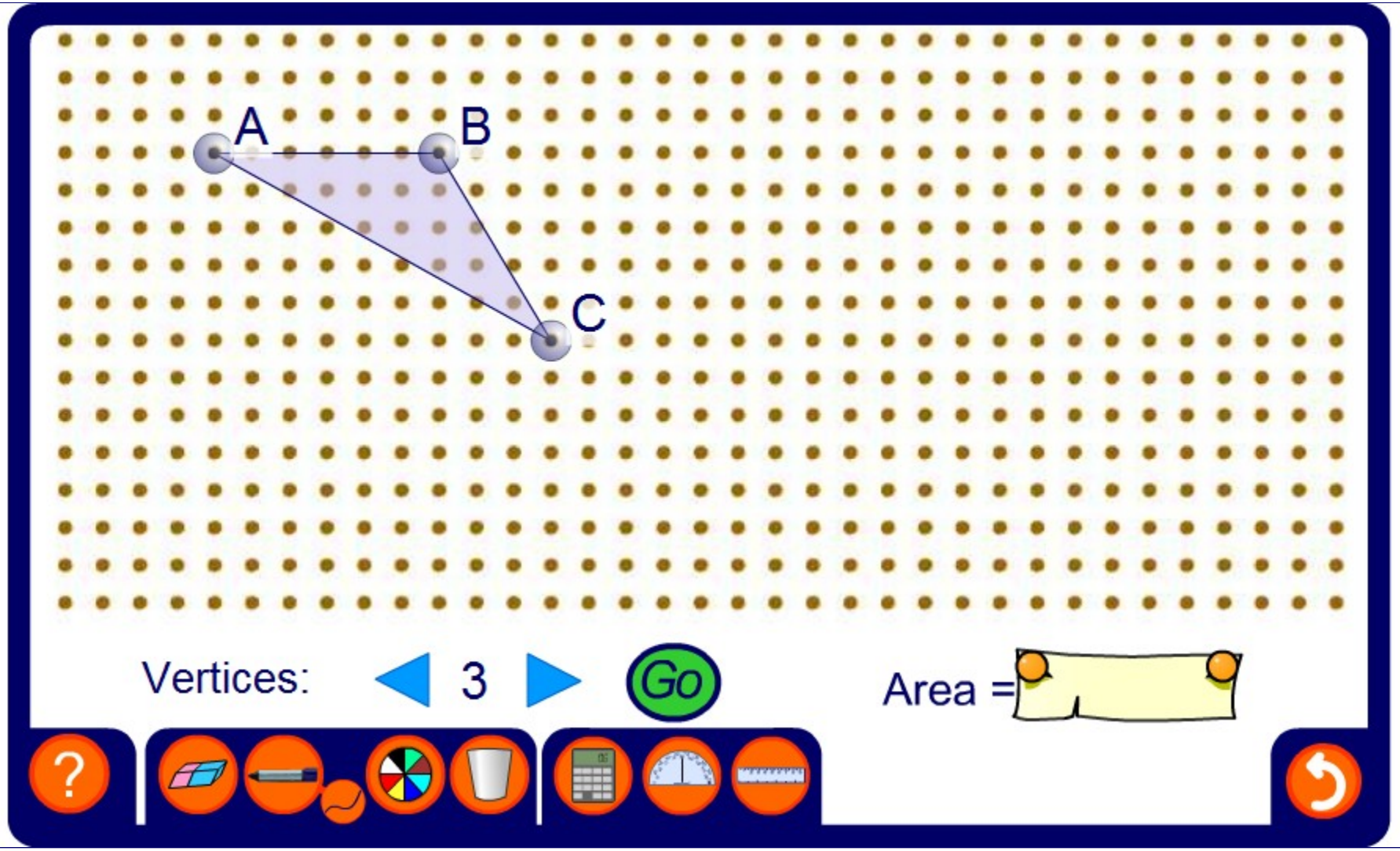
© Boardworks Ltd 2005

The area of a trapezium



© Boardworks Ltd 2005

Finding the area of irregular polyg



© Boardworks Ltd 2005

Contents

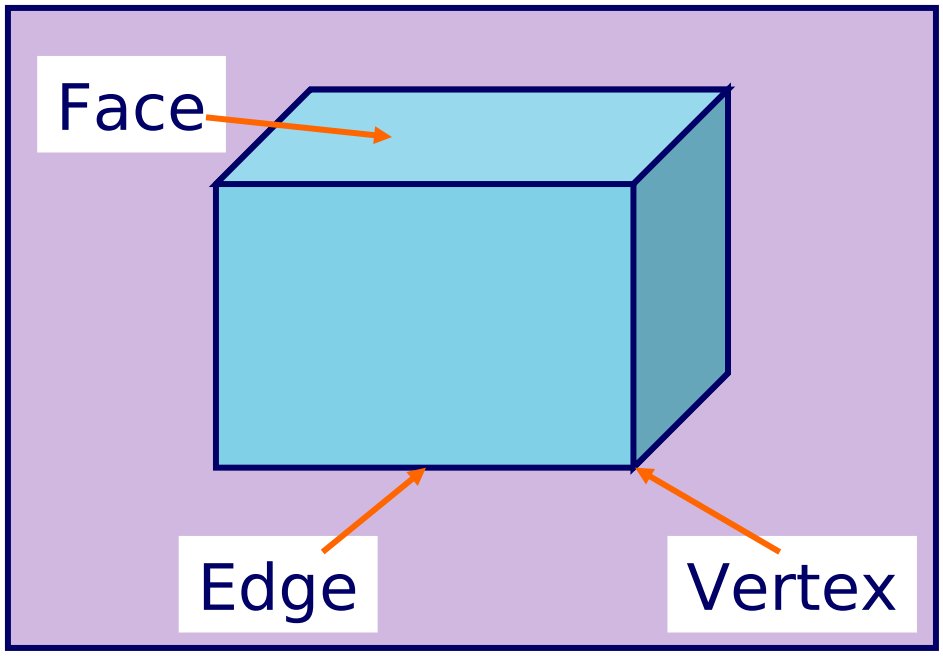
S10 Length, area and volume

- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Cubes and cuboids

A **cuboid** is a 3-D shape with edges of different lengths. All of its faces are rectangular or square.



How many **faces** does a cuboid have? **6**

How many **edges** does a cuboid have? **12**

How many **vertices** does a cuboid have? **8**

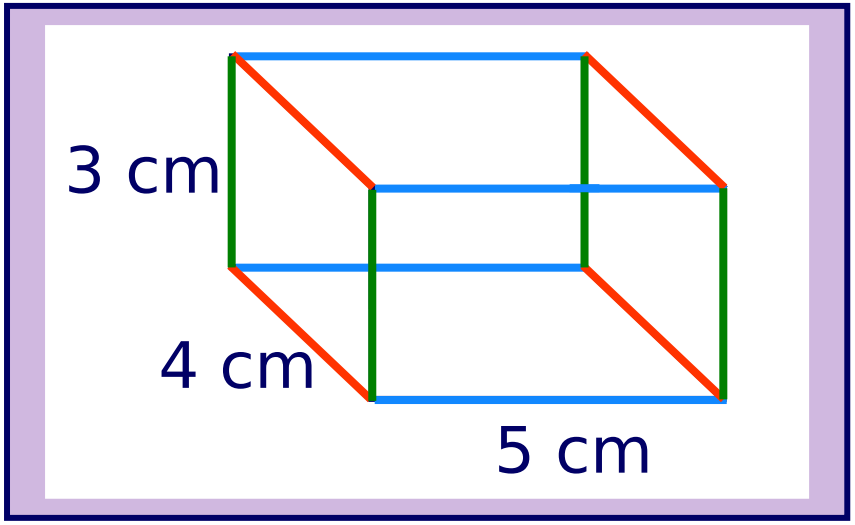
A **cube** is a special type of cuboid with edges of equal length. All of its faces are square.

© Boardworks Ltd 2005

Length around the edges

Suppose we have a cuboid of length 5 cm, width 4 cm and height 3 cm. What is the total length around the edges?

Imagine the cuboid as a hollow wire frame:



The cuboid has 12 edges.

4 edges are 5 cm long.

4 edges are 4 cm long.

4 edges are 3 cm long.

Total length around the edges $= 4 \times 5 + 4 \times 4 + 4 \times 3$

$= 20 + 16 + 12$

$= 48$ cm

© Boardworks Ltd 2005

Length around the edges

To find the length around the edges of a cuboid of length l, width w and height h we can use the formula:

$$\text{Length around the edges} = 4l + 4w + 4h$$

or

$$\text{Length around the edges} = 4(l + w + h)$$

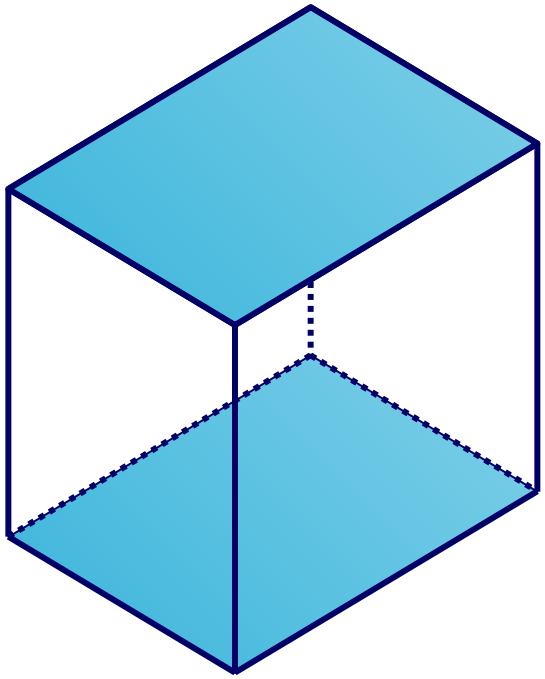
To find the length around the edges of a cube of length l we can use the formula:

$$\text{Length around the edges} = 12l$$

© Boardworks Ltd 2005

Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



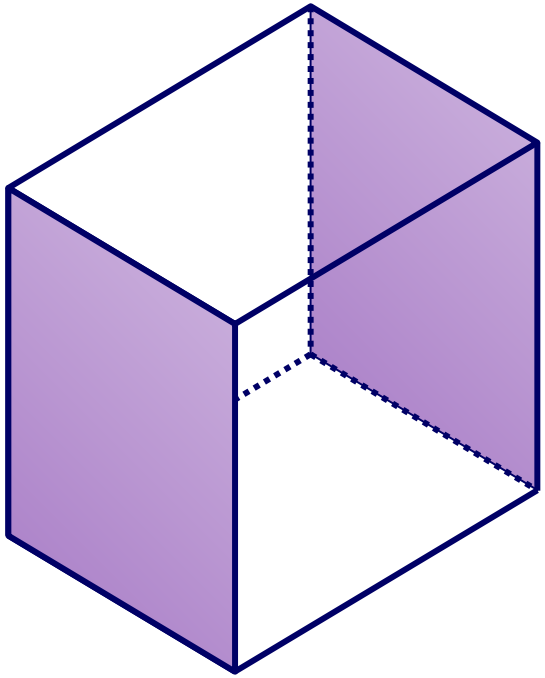
A cuboid has 6 faces.

The top and the bottom of the cuboid have the same area.

© Boardworks Ltd 2005

Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



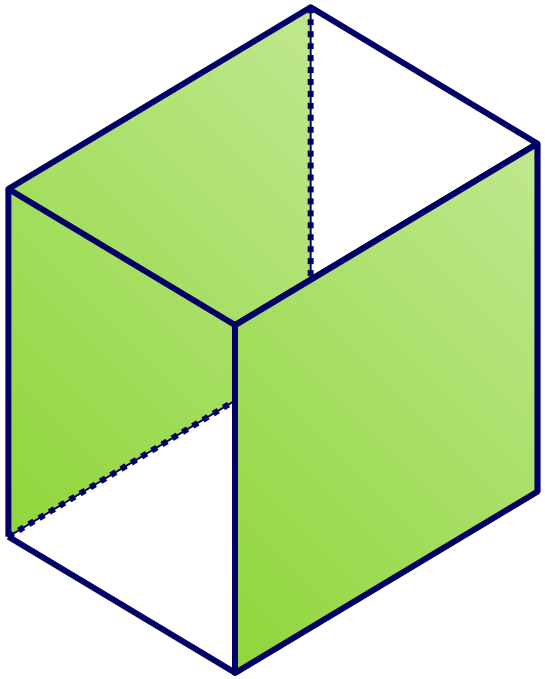
A cuboid has 6 faces.

The front and the back of the cuboid have the same area.

© Boardworks Ltd 2005

Surface area of a cuboid

To find the **surface area** of a cuboid, we calculate the total area of all of the faces.



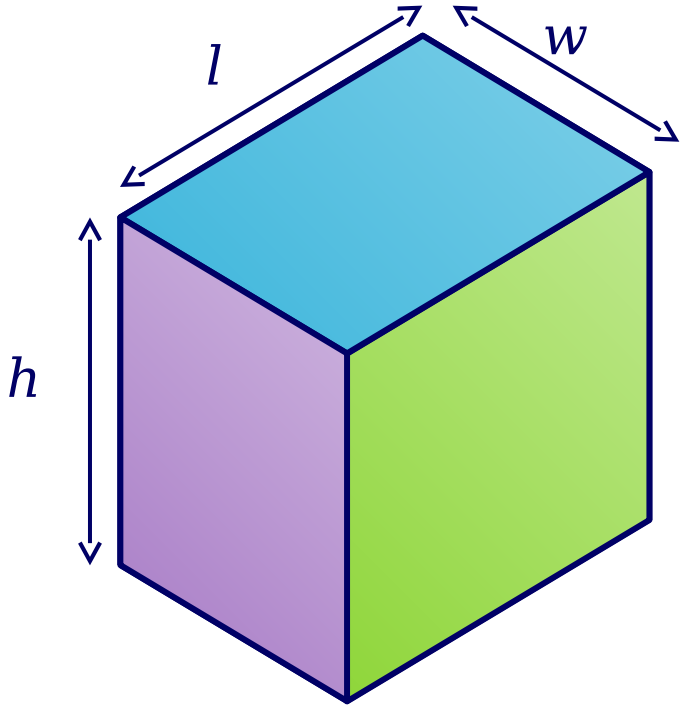
A cuboid has 6 faces.

The left hand side and the right hand side of the cuboid have the same area.

© Boardworks Ltd 2005

Formula for the surface area of a cuboid

We can find the formula for the surface area of a cuboid as follows.



Surface area of a cuboid

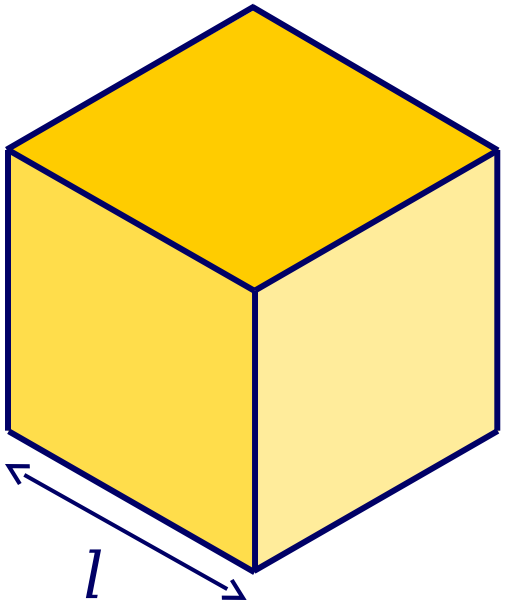
$2 \times lw$	Top and bottom
$+ 2 \times hw$	Front and back
$+ 2 \times lh$	Left and right side

Surface area of a cuboid $= 2lw + 2hw + 2lh$

© Boardworks Ltd 2005

Surface area of a cube

How can we find the surface area of a cube of length l?



All six faces of a cube have the same area.

The area of each face is $l \times l = l^2$

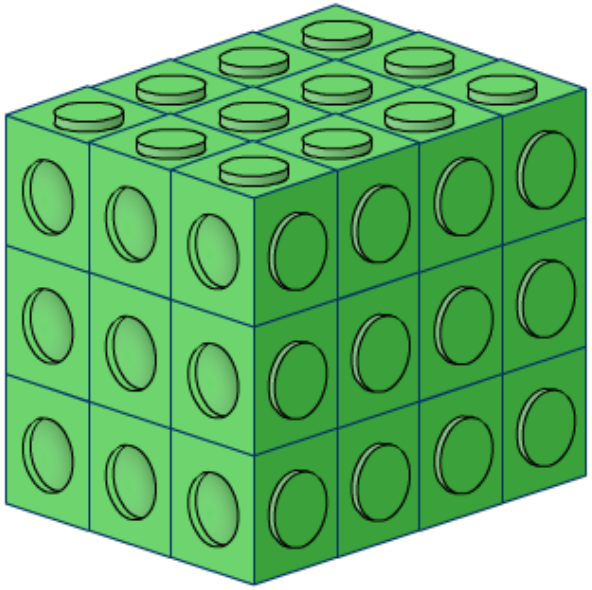
Therefore,

Surface area of a cube = $6l^2$

© Boardworks Ltd 2005

Volume of a cuboid

The following cuboid is made out of interlocking cubes.

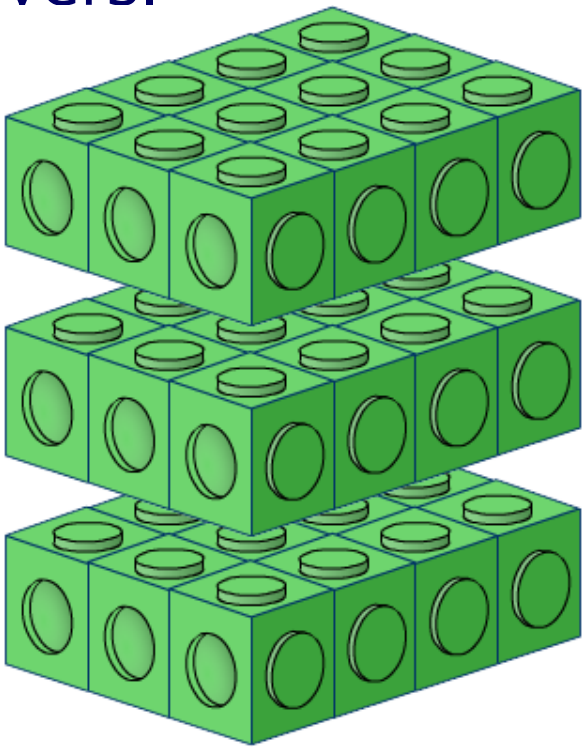


How many cubes does it contain?

© Boardworks Ltd 2005

Volume of a cuboid

We can work this out by dividing the cuboid into lavers.



The number of cubes in each layer can be found by multiplying the number of cubes along the length by the number of cubes along the width.

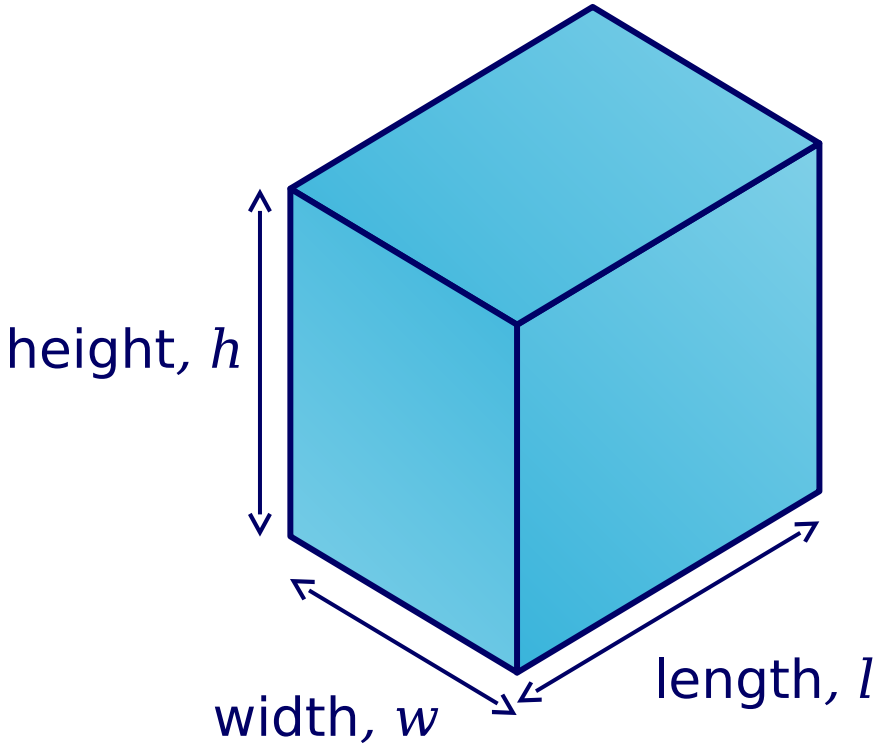
3 × 4 = 12 cubes in each layer

There are three layers altogether so the total number of cubes in the cuboid = 3 × 12 = **36 cubes**

© Boardworks Ltd 2005

Volume of a cuboid

We can find the volume of a cuboid by multiplying the area of the base by the height.



The area of the base

= length × width

So,

Volume of a cuboid

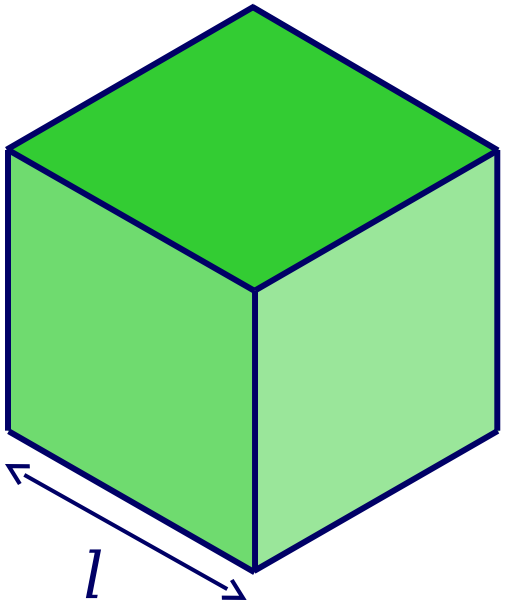
= length × width × height

= lwh

© Boardworks Ltd 2005

Volume of a cube

How can we find the volume of a cube of length l?



The length, width and height of a cube are the same.

Therefore:

Volume of a cube

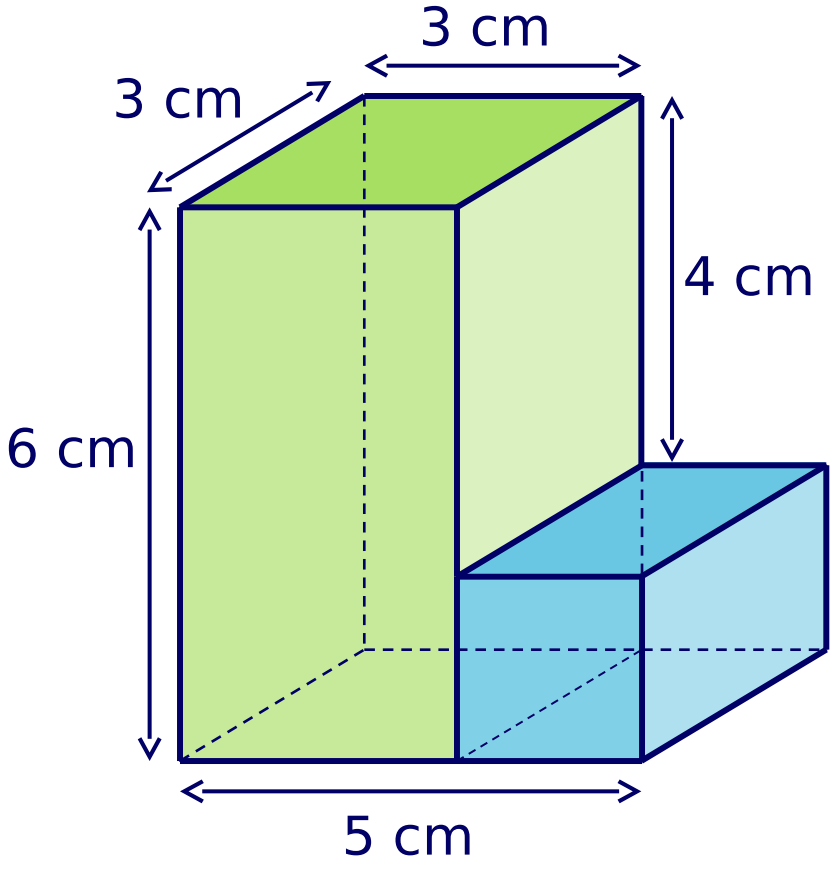
= (length of one edge)^3

= l^3

© Boardworks Ltd 2005

Volume of shapes made from cuboids

What is the volume of this 3-D shape?



We can think of this shape as two cuboids joined together.

Volume of the green cuboid

$= 6 \times 3 \times 3 = 54 \text{ cm}^3$

Volume of the blue cuboid

$= 3 \times 2 \times 2 = 12 \text{ cm}^3$

Total volume

$= 54 + 12 =$ **66 cm³**

© Boardworks Ltd 2005

Contents

S10 Length, area and volume

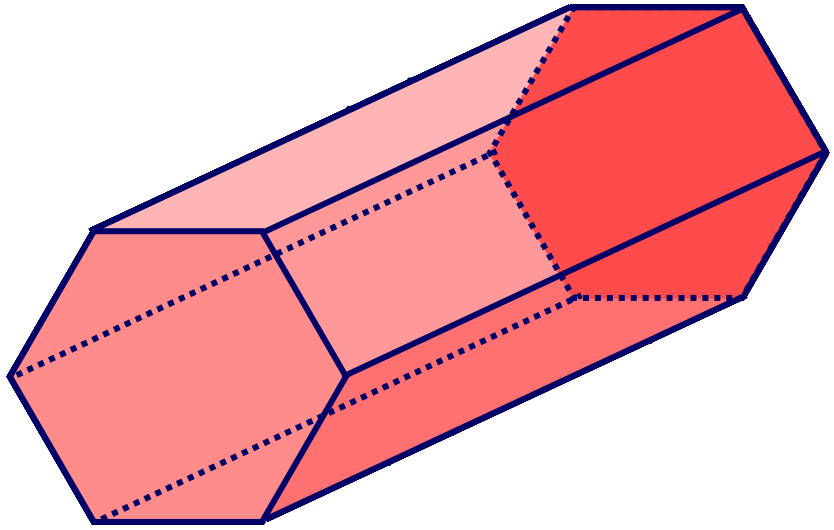
- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Prisms

A **prism** is a 3-D shape that has a constant cross-section along its length.

For example, this hexagonal prism has the same hexagonal cross-section throughout its length.

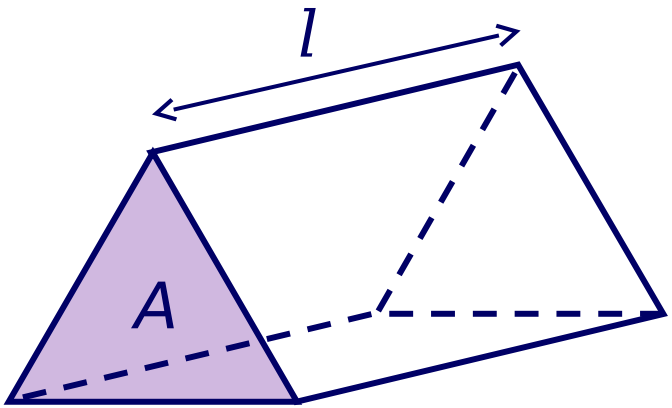


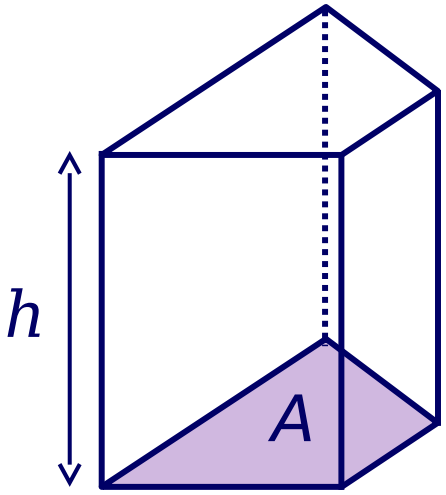
This is called a hexagonal prism because its cross-section is a hexagon.

© Boardworks Ltd 2005

Volume of a prism

The volume of a prism is found by multiplying the area of its cross-section A by its length l (or by its height if it is standing on its cross-section).



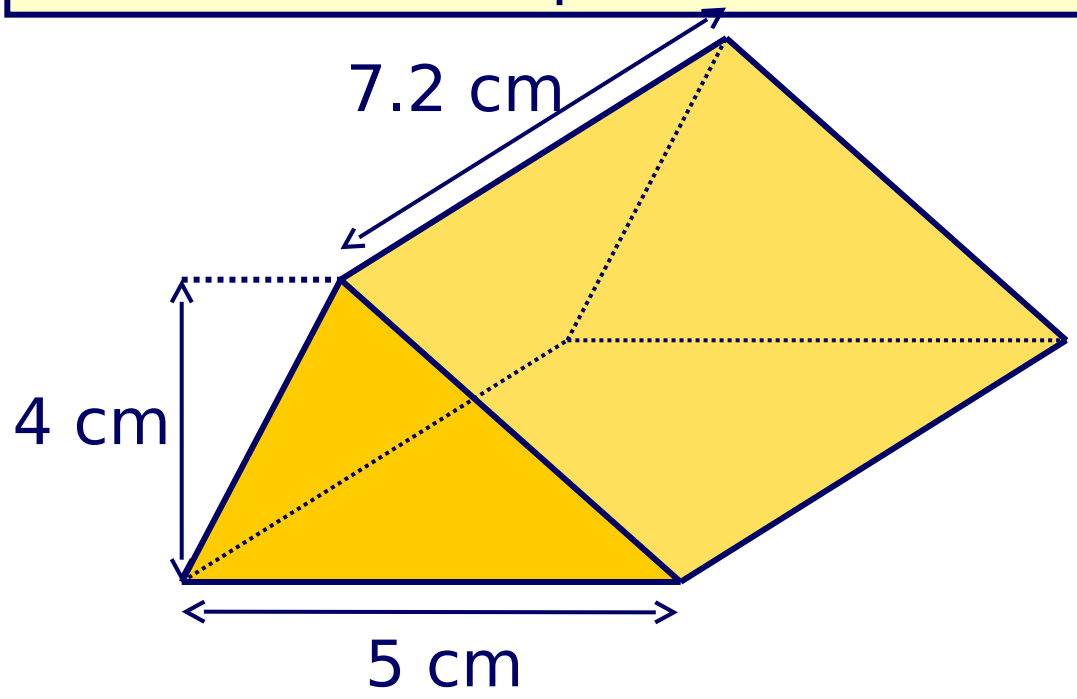


$V = Al$ or $V = Ah$

© Boardworks Ltd 2005

Volume of a prism

What is the volume of this triangular prism?



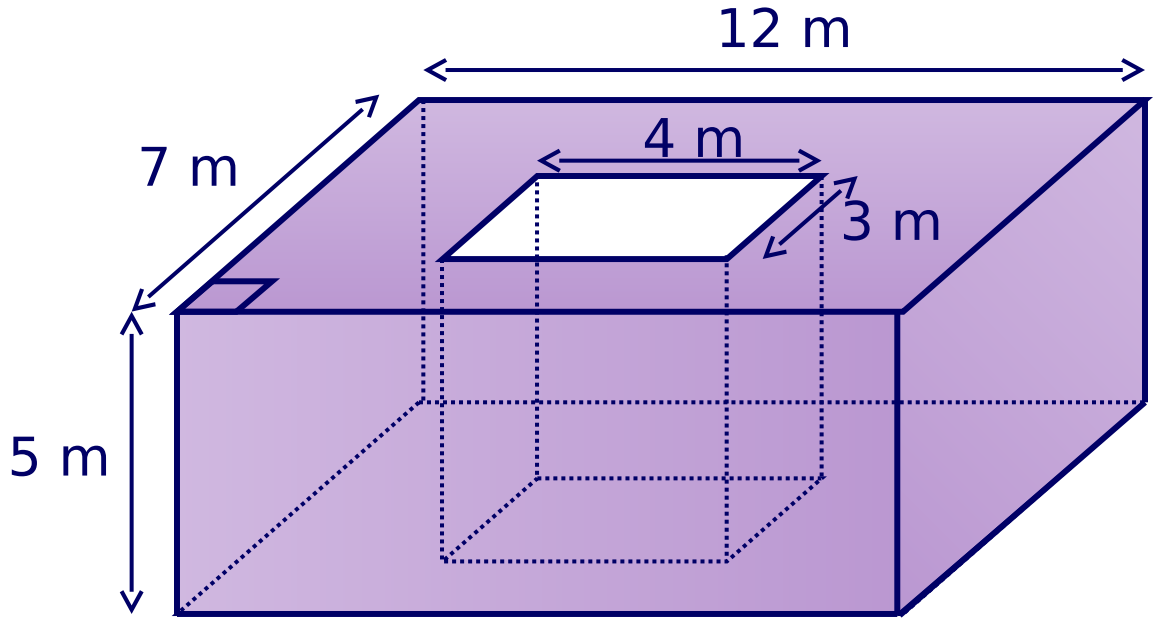
Area of cross-section = ½ × 5 × 4 = 10 cm²

Volume of prism = 10 × 7.2 = 72 cm³

© Boardworks Ltd 2005

Volume of a prism

What is the volume of this prism?



Area of cross-section = $7 \times 12 - 4 \times 3 = 84 - 12 = 72$ m^2

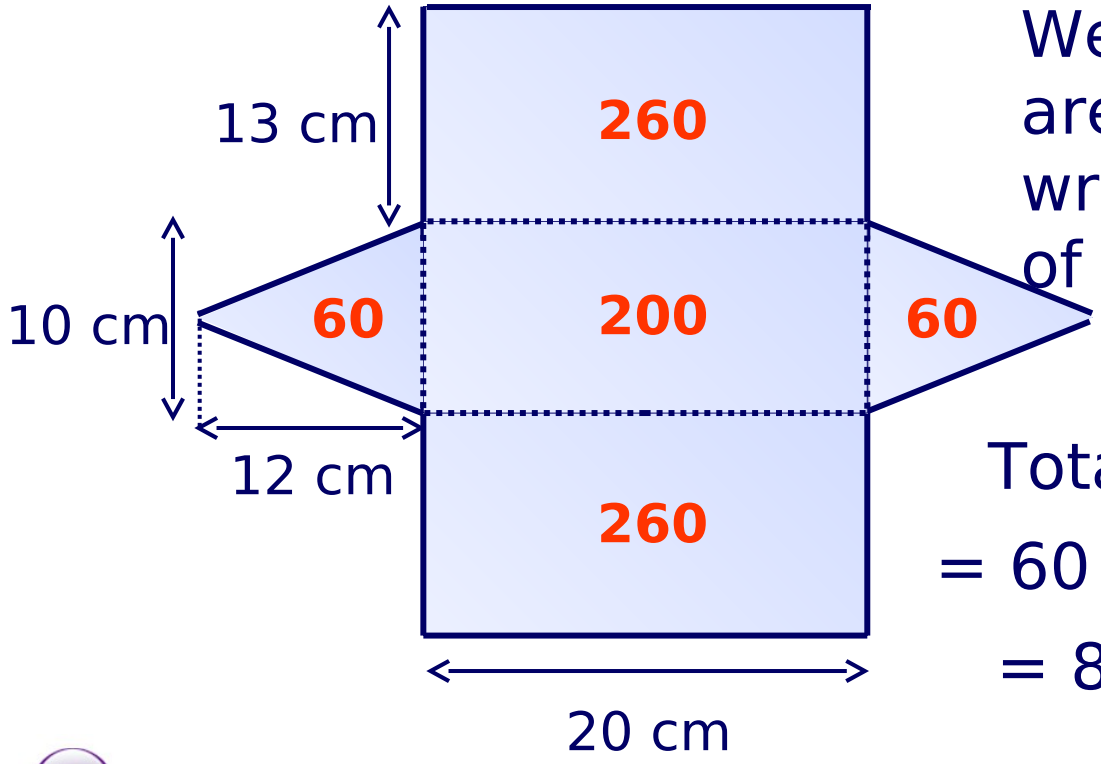
Volume of prism = $72 \times 5 = 360$ m^3

© Boardworks Ltd 2005

Surface area of a prism

Here is the net of a triangular prism.

What is its surface area?



We can work out the area of each face and write it in the diagram of the net.

Total surface area

$= 60 + 60 + 200 + 260 + 260$

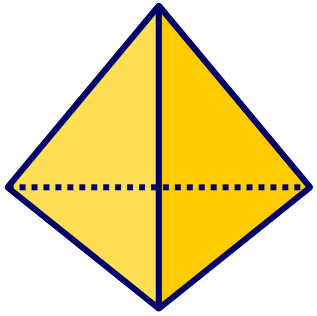
$= 840 \text{ cm}^2$

© Boardworks Ltd 2005

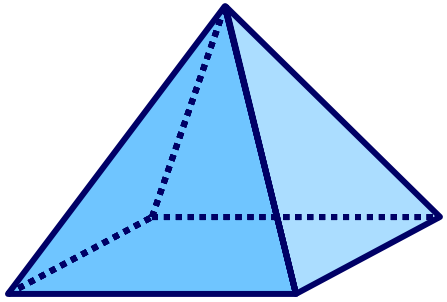
Pyramids

A **pyramid** is a 3-D shape whose base is usually a polygon but can also be a shape with curved edges. The faces rising up from the base meet at a common vertex or **apex**.

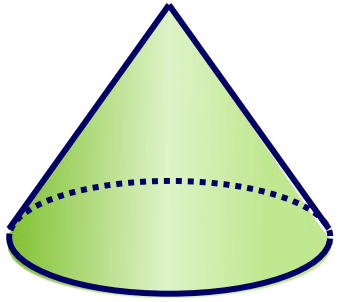
The most common pyramids are:



A tetrahedron or triangular pyramid.



A square-based pyramid

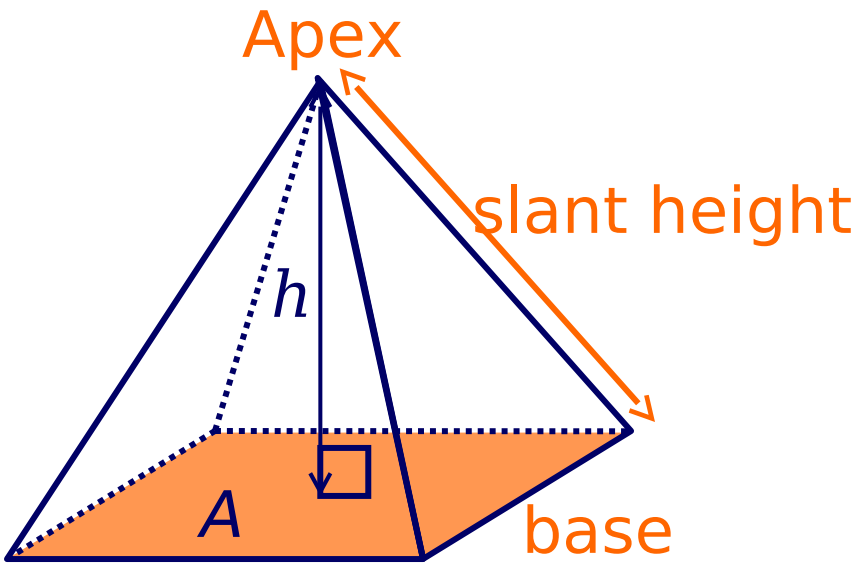


A cone

© Boardworks Ltd 2005

Volume of a pyramid

The volume of a pyramid is found by multiplying the area of its base A by its perpendicular height h and dividing by 3.



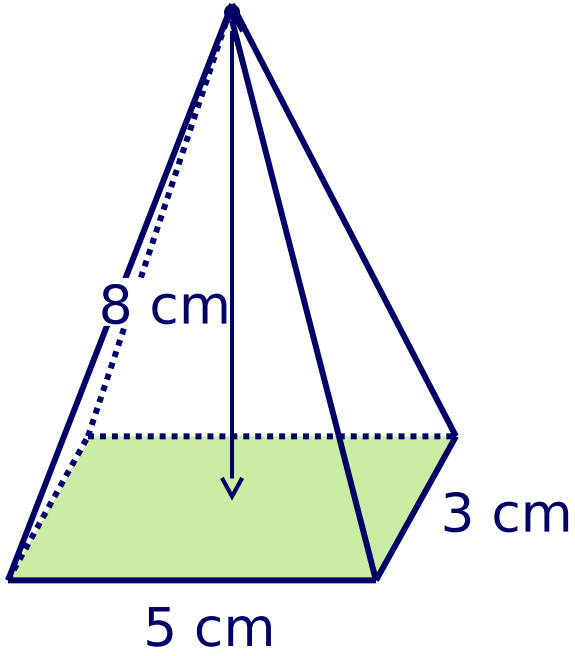
Volume of a pyramid = $\frac{1}{3}$ × area of base × height

$$V = \frac{1}{3}Ah$$

© Boardworks Ltd 2005

Volume of a pyramid

What is the volume of this rectangle-based pyramid?



Area of the base $= 5 \times 3$

$= 15 \text{ cm}^2$

Volume of pyramid $= \frac{1}{3} Ah$

$= \frac{1}{3} \times 15 \times 8$

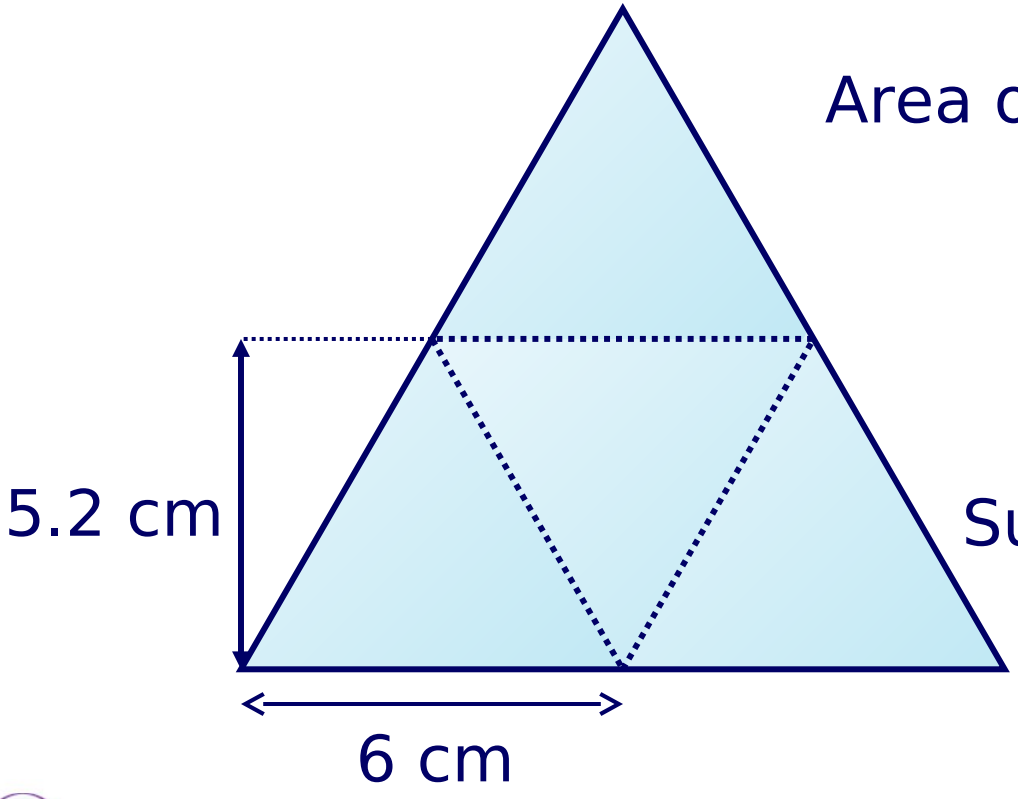
$= \mathbf{40 \text{ cm}^3}$

© Boardworks Ltd 2005

Surface area of a pyramid

Here is the net of a regular tetrahedron.

What is its surface area?



Area of each face = $\frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 5.2$

$= 15.6 \text{ cm}^2$

Surface area $= 4 \times 15.6$

$= \mathbf{62.4 \text{ cm}^2}$

© Boardworks Ltd 2005

Contents

S10 Length, area and volume

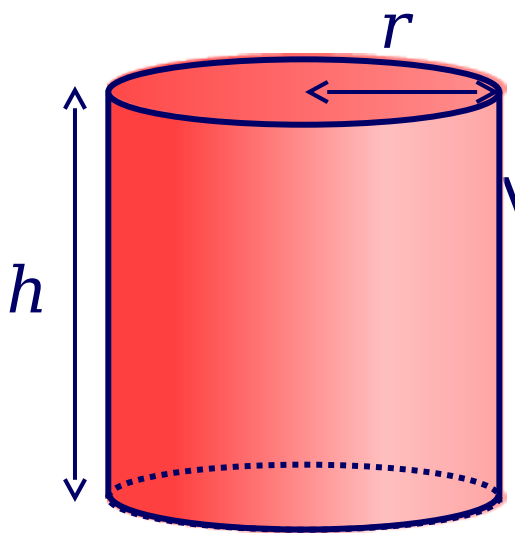
- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Volume of a cylinder

A **cylinder** is a special type of prism with a circular cross-section.

Remember, the volume of a prism can be found by multiplying the area of the cross-section by the height of the prism.



The volume of a cylinder is given by:

Volume = area of circular base × height

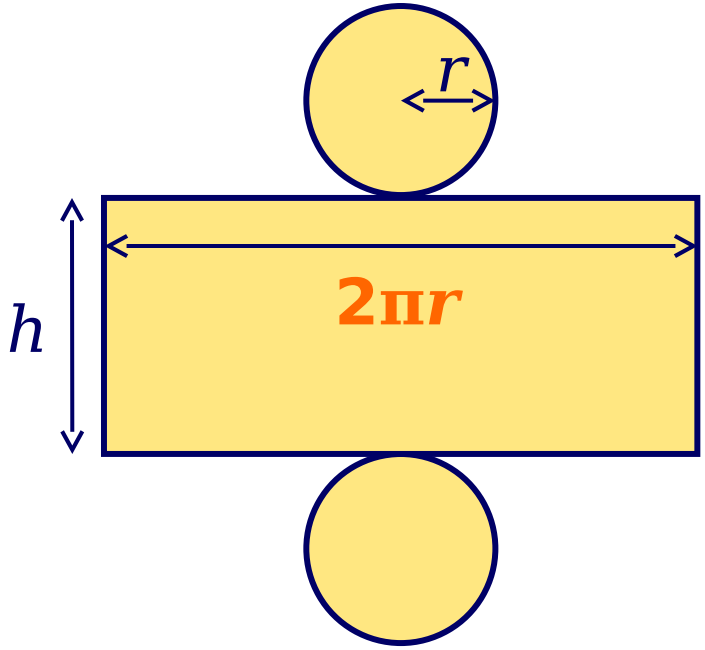
or

$$V = \pi r^2 h$$

© Boardworks Ltd 2005

Surface area of a cylinder

To find the formula for the surface area of a cylinder we can draw its net.



How can we find the width of the curved face?

The width of the curved face is equal to the circumference of the circular base, $2\pi r$.

Area of curved face $= 2\pi rh$

Area of 2 circular faces $= 2 \times \pi r^2$

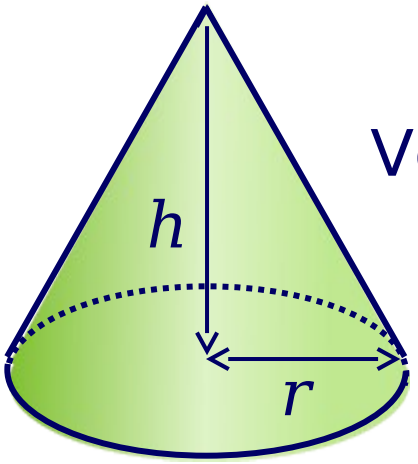
Surface area of a cylinder $= 2\pi rh + 2\pi r^2$

or $\boxed{\text{Surface area} = 2\pi r(h + r)}$

© Boardworks Ltd 2005

Volume of a cone

A **cone** is a special type of pyramid with a circular base.
Remember, the volume of a pyramid can be found by multiplying the area of the base by the height and dividing by 3.



The volume of a cone is given by:

Volume = $\frac{1}{3}$ × area of circular base × heigl

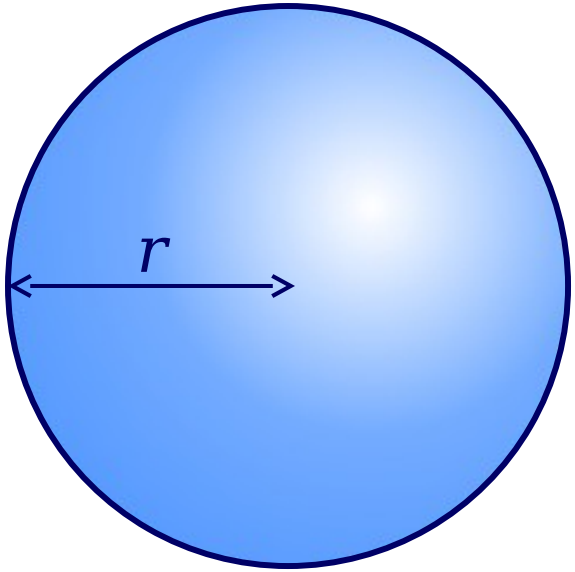
or

$$V = \frac{1}{3}\pi r^2 h$$

© Boardworks Ltd 2005

Volume and surface area of a sphere

A **sphere** is a 3-D shape whose surface is always the same distance from the centre. This fixed distance is the radius of the sphere.



For a sphere of radius r :

$$\text{Volume} = \frac{4}{3}\pi r^3$$

and

$$\text{Surface area} = 4\pi r^2$$

© Boardworks Ltd 2005

Contents

S10 Length, area and volume

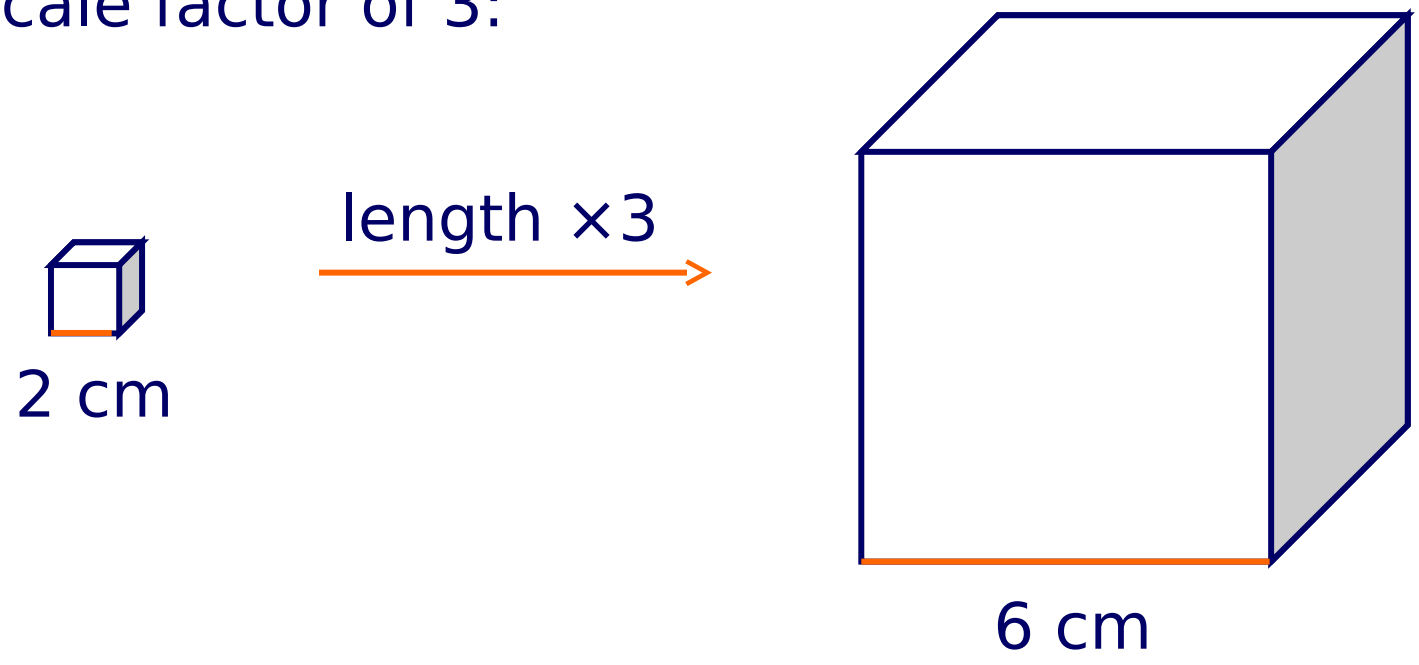
- S10.1 Dimensions of length, area and volume
- S10.2 Polygons
- S10.3 Cubes and cuboids
- S10.4 Prisms and pyramids
- S10.5 Cylinders, cones and spheres
- S10.6 Lengths, areas and volumes of similar shapes

© Boardworks Ltd 2005

Lengths in similar shapes

Remember, when a shape is enlarged every length in the shape is multiplied by a given scale factor.

Suppose we enlarge a cube with side length 2 cm by a scale factor of 3:

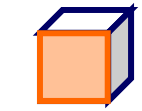


The length is enlarged by a scale factor of 3.

© Boardworks Ltd 2005

Areas in similar shapes

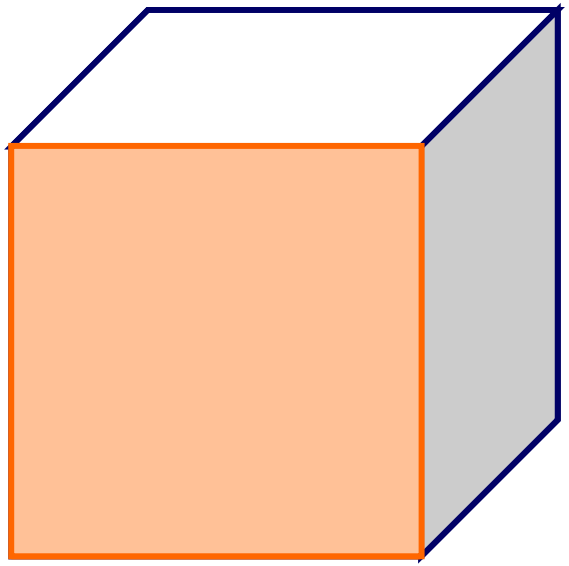
Compare the area of one face of the original cube to the area of one face of the enlarged cube:



4 cm²

length ×3

area ×9



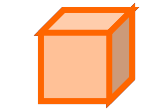
36 cm²

The area is enlarged by a scale factor of 3^2.

© Boardworks Ltd 2005

Volumes in similar shapes

Compare the volume of the original cube and the volume of the enlarged cube:

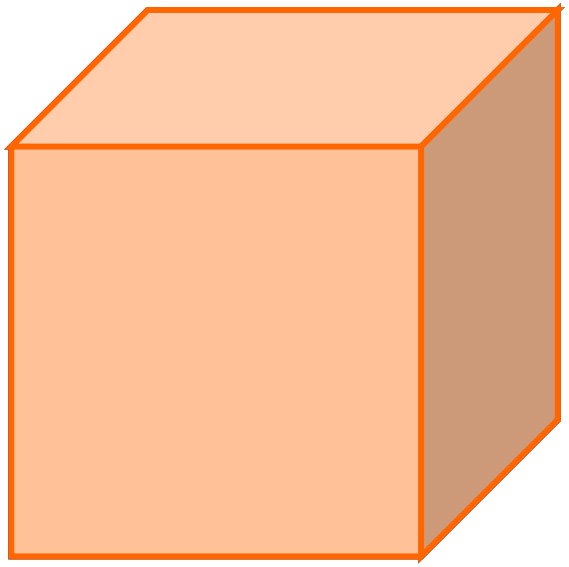


8 cm^3

length ×3

area ×9

volume ×27



216 cm^3

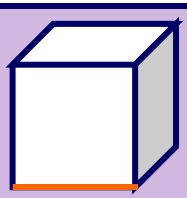
The volume is enlarged by a scale factor of 3^3.

© Boardworks Ltd 2005

Lengths in similar shapes



length = 1

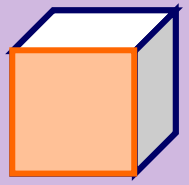


length = k

In general, if an object is enlarged by a scale factor of k:



area = 1

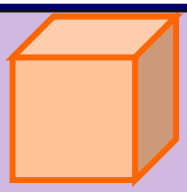


area = k^2

its area or surface area is enlarged by a scale factor of k^2



volume = 1



volume = k^3

and its volume is enlarged by a scale factor of k^3.

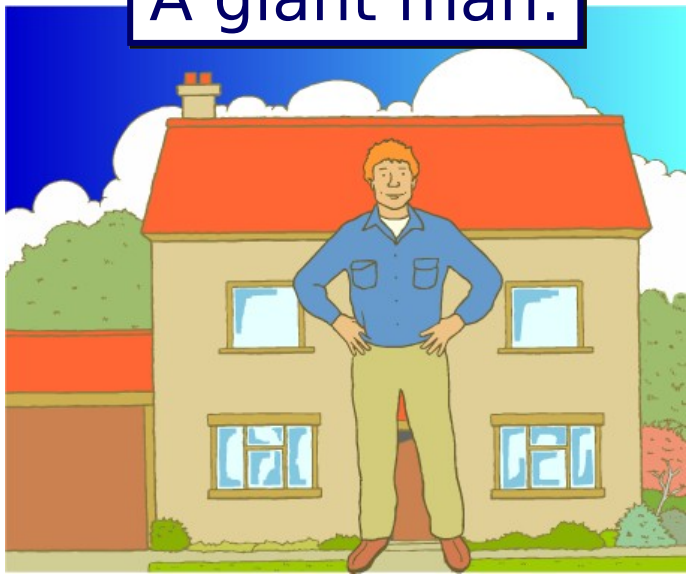
© Boardworks Ltd 2005

Could a giant survive on earth?

An average man:



A giant man:



Height: 1.7 m	×5 →	Height: 8.5 m
Surface area: 1.8 m^2	×25 →	Surface area: 45 m^2
Volume: 0.07 m^3	×125 →	Volume: 8.75 m^3

Could this giant survive on earth?

© Boardworks Ltd 2005