

MATHEMATICS NOTES

FORM 4 SIMPLIFIED TUTORIAL

QUICK REVISION NOTES

*An Updated Well-Organized Detailed Revision Notes for the
Current Form 4 Syllabus.*

SERIES 1

THIS IS A FREE SAMPLE OF THE
ORIGINAL NOTES

CONTACT US FOR FULL VERSION OF THE NOTES

Mr Isaboke 0746 222 000

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Printed in Kenya

00100

Nairobi

+254 746-222-000

mwalimuconsultancy@gmail.com

Mwalimu Consultancy Ltd.

MATRIX AND TRANSFORMATION

Specific Objectives

By the end of the topic the learner should be able to:

- (a) Relate image and object under a given transformation on the Cartesian Plane;
- (b) Determine the matrix of a transformation;
- (c) Perform successive transformations;
- (d) Determine and identify a single matrix for successive transformation;
- (e) Relate identity matrix and transformation;
- (f) Determine the inverse of a transformation;
- (g) Establish and use the relationship between area scale factor and determinant of a matrix;
- (h) Determine shear and stretch transformations;
- (i) Define and distinguish isometric and non-isometric transformation;
- (j) Apply transformation to real life situations.

Content

- (a) Transformation on the Cartesian plane
- (b) Identification of transformation matrix
- (c) Successive transformations
- (d) Single matrix of transformation for successive transformations
- (e) Identity matrix and transformation
- (f) Inverse of a transformations
- (g) Area scale factor and determinant of a matrix
- (h) Shear and stretch (include their matrices)
- (i) Isometric and non-isometric transformations
- (j) Application of transformation to real life situations.

Matrices of transformation

A transformation change the shape, position or size of an object as discussed in book two.

Pre –multiplication of any 2 x 1 column vector by a 2 x 2 matrix results in a 2 x 1 column vector

Example

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 17 \\ -9 \end{bmatrix}$$

If the vector $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ is thought of as apposition vector that is to mean that it is representing the points with coordinates (7, -1) to the point (17, -9).

Note;

The transformation matrix has an effect on each point of the plan. Let's make T a transformation matrix $T \begin{bmatrix} 7 \\ -1 \end{bmatrix}$ Then T maps points (x, y) onto image points x^1, y^1

$$T \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$$

$$= \begin{bmatrix} 3x + 4y \\ -1x + 2y \end{bmatrix}$$

Finding the Matrix of transformation

The objective is to find the matrix of given transformation.

Examples

Find the matrix of transformation of triangle PQR with vertices P (1, 3) Q (3, 3) and R (2, 5). The vertices of the image of the triangle are $P^1(1, -3)$, $Q^1(3, -3)$ and $R^1(2, -5)$.

Solution

Let the matrix of the transformation be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} P & Q & R \\ 1 & 3 & 2 \\ 3 & 3 & 5 \end{pmatrix} = \begin{pmatrix} P^1 & Q^1 & R^1 \\ 1 & 3 & 2 \\ -3 & -3 & -5 \end{pmatrix}$$

$$\begin{pmatrix} a + 3b & 3a + 3b & 2a + 5b \\ c + 3d & 3c + 3d & 2c + 5d \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ -3 & -3 & -5 \end{pmatrix}$$

Equating the corresponding elements and solving simultaneously

$$a + 3b = 1$$

$$\underline{3a + 3b = 3}$$

$$2a = 2$$

$$a = 1 \text{ and } b = 0$$

$$c + 3d = -3$$

$$\underline{3c + 3d = -3}$$

$$2c = 0$$

$$c = 0 \text{ and } d = -1$$

Therefore the transformation matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Example

A trapezium with vertices A (1, 4) B(3,1) C (5,1) and D(7,4) is mapped onto a trapezium whose vertices are $A^1(-4,1)$, $B^1(-1, 3)$, $C^1(-1,5)$, $D^1(-4, 7)$. Describe the transformation and find its matrix

Solution

Let the matrix of the transformation be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A & B & C & D \\ 1 & 3 & 5 & 7 \\ 4 & 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} A^1 & B^1 & C^1 & D^1 \\ -4 & -1 & -1 & -4 \\ 1 & 3 & 5 & 7 \end{pmatrix}$$

Equating the corresponding elements we get;

$$\begin{aligned} a + 4b &= -4 \\ 3a + b &= -1 \end{aligned}$$

$$\begin{aligned} c + 4d &= 1 \\ 3c + d &= 3 \end{aligned}$$

Solve the equations simultaneously

$$3a + 12b = -12$$

$$\underline{3a + b = -1}$$

$$11b = -11 \quad \text{hence } b = -1 \text{ or } a = 0$$

$$3c + 12d = 3$$

$$3c + d = 3$$

$$11d = 0$$

$$d = 0 \quad c = 1$$

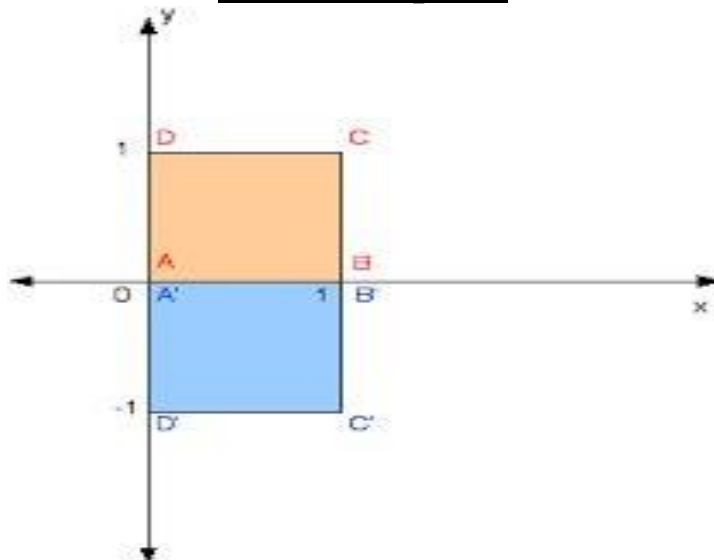
The matrix of the transformation is therefore $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

The transformation is positive quarter turn about the origin

Note;

Under any transformation represented by a 2×2 matrix, the origin is invariant, meaning it does not change its position. Therefore if the transformation is a rotation it must be about the origin or if the transformation is reflection it must be on a mirror line which passes through the origin.

The unit square



The unit square ABCD with vertices A (0,0), B(1,0), C(1,1) and D(0,1) helps us to get the transformation of a given matrix and also to identify what transformation a given matrix represent.

Example

Find the images of I and J under the transformation whose matrix is;

a) $\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 6 \\ 4 & 5 \end{pmatrix}$

Solution

a) $\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} I & J \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} I^1 & J^1 \\ 2 & 3 \\ 5 & 4 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 6 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} I & J \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} I^1 & J^1 \\ -1 & 6 \\ 4 & 5 \end{pmatrix}$

NOTE;

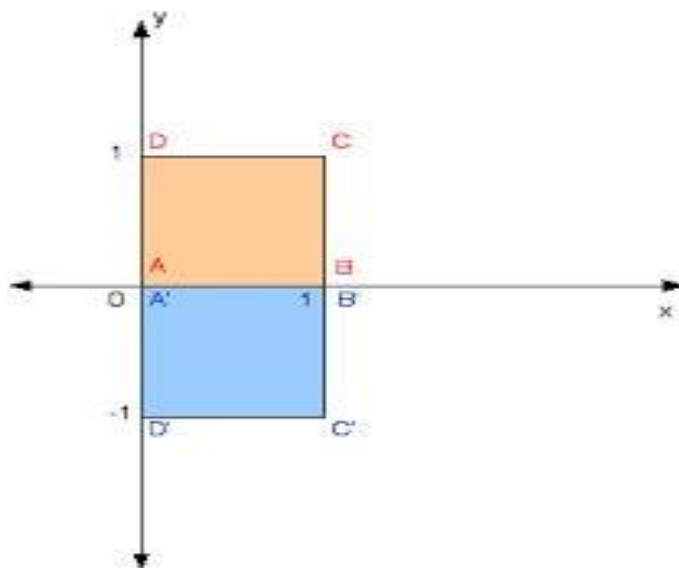
The images of I and J under transformation represented by any 2×2 matrix i.e., $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are $I^1(a, c)$ and $J^1(b, d)$

Example

Find the matrix of reflection in the line $y = 0$ or x axis.

Solution

Using a unit square the image of B is $(1, 0)$ and D is $(0, -1)$. Therefore, the matrix of the transformation is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



Example

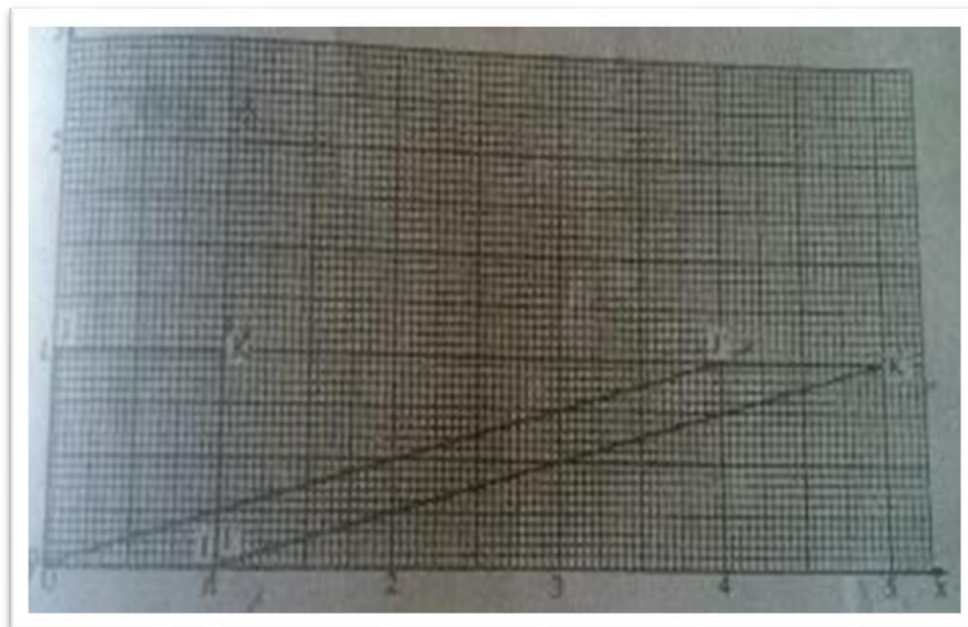
Show on a diagram the unit square and its image under the transformation represented by the matrix $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

Solution

Using a unit square, the image of I is (1, 0), the image of J is (4, 1), the image of O is (0, 0) and

that of K is $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} k \\ 1 \end{pmatrix} = \begin{pmatrix} K^1 \\ 5 \end{pmatrix}$

Therefore, K^1 , the image of K is (5, 1)



Successive transformations

The process of performing two or more transformations in order is called successive transformation eg performing transformation H followed by transformation Y is written as follows YH or if A, b and C are transformations; then ABC means perform C first, then B and finally A, in that order.

The matrices listed below all perform different rotations/reflections:

This transformation matrix is the identity matrix. When multiplying by this matrix, the point matrix is unaffected and the new matrix is exactly the same as the point matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 1) + (3 \times 0) \\ (4 \times 0) + (3 \times 1) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

This transformation matrix creates a reflection in the x-axis. When multiplying by this matrix, the x co-ordinate remains unchanged, but the y co-ordinate changes sign.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 1) + (3 \times 0) \\ (4 \times 0) + (3 \times -1) \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

This transformation matrix creates a reflection in the y-axis. When multiplying by this matrix, the y co-ordinate remains unchanged, but the x co-ordinate changes sign.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times -1) + (3 \times 0) \\ (4 \times 0) + (3 \times 1) \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

This transformation matrix creates a rotation of 180 degrees. When multiplying by this matrix, the point matrix is rotated 180 degrees around (0, 0). This changes the sign of both the x and y co-ordinates.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times -1) + (3 \times 0) \\ (4 \times 0) + (3 \times -1) \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

This transformation matrix creates a reflection in the line y=x. When multiplying by this matrix, the x co-ordinate becomes the y co-ordinate and the y-ordinate becomes the x co-ordinate.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 0) + (3 \times 1) \\ (4 \times 1) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

This transformation matrix rotates the point matrix 90 degrees clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees clockwise around (0, 0).

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 0) + (3 \times 1) \\ (4 \times -1) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

This transformation matrix rotates the point matrix 90 degrees anti-clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees anti-clockwise around (0, 0).

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 0) + (3 \times -1) \\ (4 \times 1) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

This transformation matrix creates a reflection in the line y=-x. When multiplying by this matrix, the point matrix is reflected in the line y=-x changing the signs of both co-ordinates and swapping their values.

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4 \times 0) + (3 \times -1) \\ (4 \times -1) + (3 \times 0) \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$$

Inverse matrix transformation

A transformation matrix that maps an image back to the object is called an inverse of matrix.

Note;

If A is a transformation which maps an object T onto an image T^1 , then a transformation that can map T^1 back to T is called the inverse of the transformation A, written as image A^{-1} .

If R is a positive quarter turn about the origin the matrix for R is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and the matrix for R^{-1} is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ hence $R^{-1}R = R^{-1}R = 1$

Example

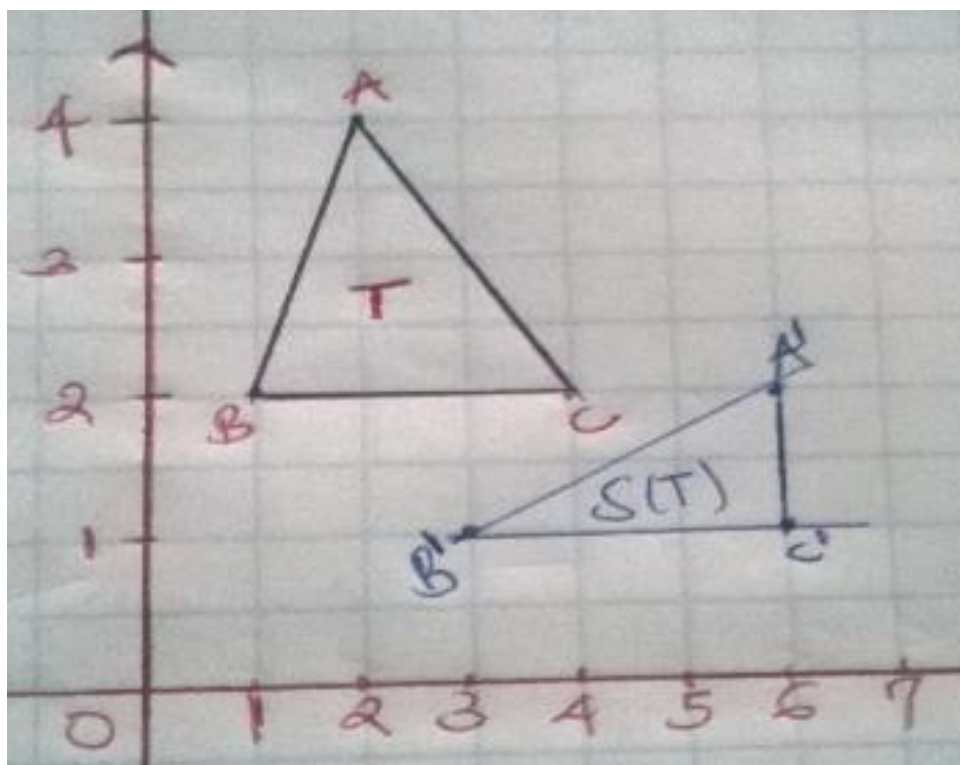
T is a triangle with vertices A (2, 4), B (1, 2) and C (4, 2). S is a transformation represented by the matrix $\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$

- Draw T and its image T^1 under the transformation S
- Find the matrix of the inverse of the transformation S

Solution

- Using transformation matrix $S = \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} A & B & C \\ 2 & 1 & 4 \\ 4 & 2 & 2 \end{pmatrix} = \begin{pmatrix} A^1 & B^1 & C^1 \\ 6 & 3 & 6 \\ 2 & 1 & 1 \end{pmatrix}$$



Let the inverse of the transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. This can be done in the following ways

I. $S^{-1}S = 1$

Therefore $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Equating corresponding elements and solving simultaneously;

$$a = 1, b = -2, c = 0 \text{ and } d = 2$$

Therefore $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$

$$S^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$$

II. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A^1 & B^1 & C^1 \\ 6 & 3 & 6 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} A & B & C \\ 2 & 1 & 4 \\ 4 & 2 & 2 \end{pmatrix}$

$$a = 1, b = -2, c = 0 \text{ and } d = 2$$

$$S^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$$

Area Scale Factor and Determinant of Matrix

The ratio of area of image to area object is the area scale factor (A.S.F)

$$\text{Area scale factor} = \frac{\text{area of image}}{\text{area of object}}$$

Area scale factor is numerically equal to the determinant. If the determinant is negative you simply ignore the negative sign.

Example

Area of the object is 4 cm and that of image is 36 cm find the area scale factor.

Solution

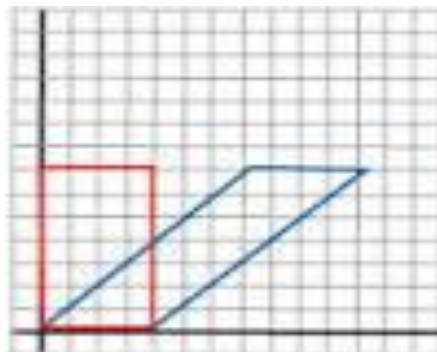
$$\frac{36}{4} = 9$$

If it has a matrix of $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ the determinant is $9 - 0 = 9$ hence equal to A.S.F

Shear and stretch

Shear

The transformation that maps an object (in orange) to its image (in blue) is called a shear



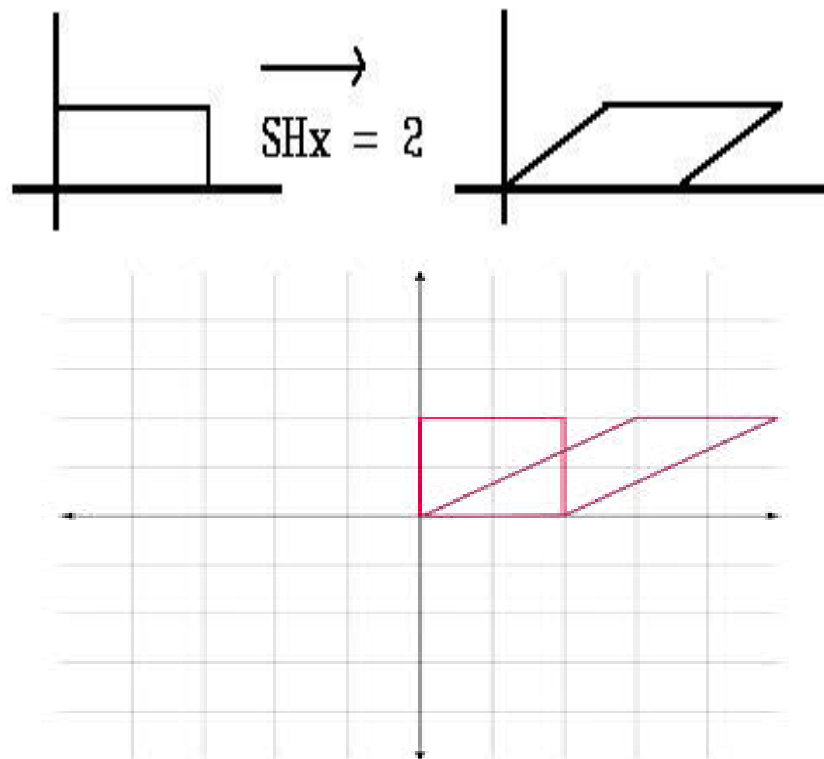
The object has same base and equal heights. Therefore, their areas are equal. Under any shear, area is always invariant (fixed)

A shear is fully described by giving;

- a.) The invariant line
- b.) A point not on the invariant line, and its image.

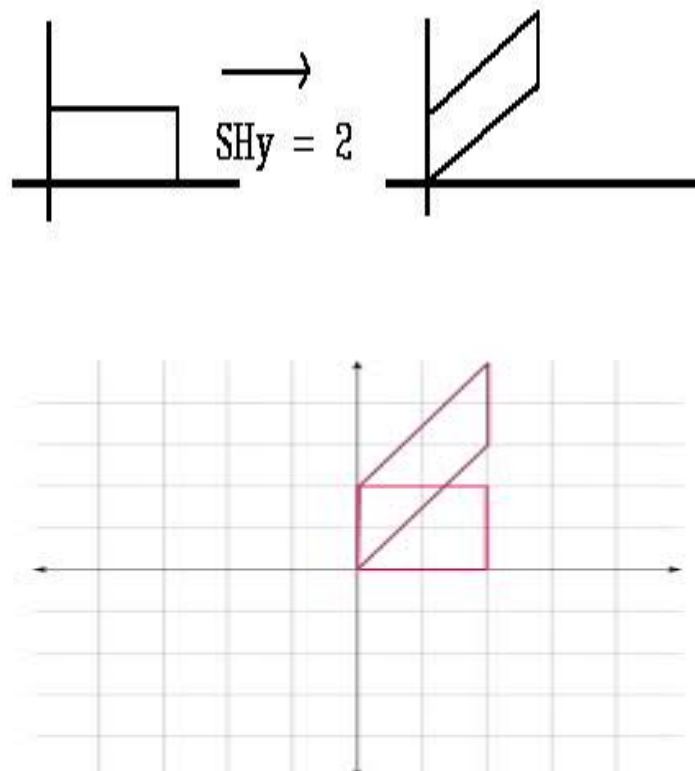
Example

A shear X axis invariant



Example

A shear Y axis invariant



Note;

Shear with x axis invariant is represented by a matrix of the form $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ under this transformation, $J(0, 1)$ is mapped onto $J^1(k, 1)$.

Likewise a shear with y – axis invariant is represented by a matrix of the form $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$. Under this transformation, $I(0,1)$ is mapped onto $I^1(1,k)$.

Stretch

A stretch is a transformation which enlarges all distance in a particular direction by a constant factor.

A stretch is described fully by giving;

- The scale factor
- The invariant line

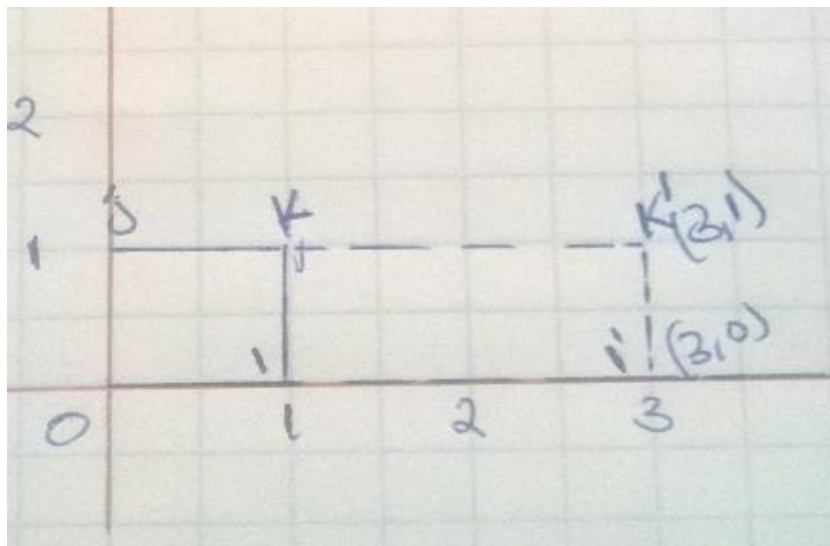
Note;

- i.) If k is greater than 1, then this really is a stretch.
- ii.) If k is less than one 1, it is a squish but we still call it a stretch
- iii.) If $k = 1$, then this transformation is really the identity i.e. it has no effect.

Example

Using a unit square, find the matrix of the stretch with y axis invariant ad scale factor 3

Solution



The image of I is $I^1(1, 0)$ and the image of J is $(0,1)$ therefore the matrix of the stretch is $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

Note;

The matrix of the stretch with the y-axis invariant and scale factor k is $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ and the matrix of a stretch with x – axis invariant and scale factor k is $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$

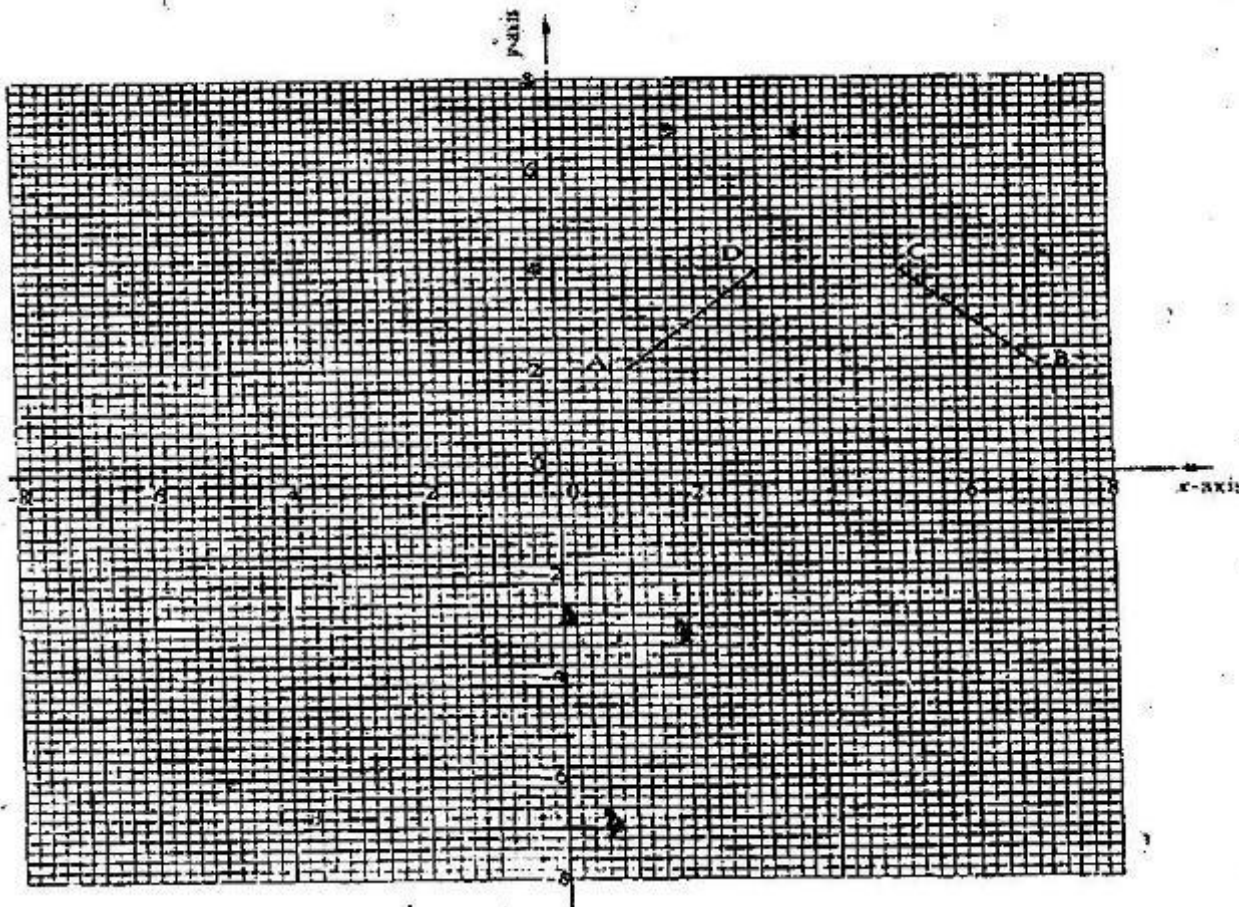
Isometric and Non- Isometric Transformation

Isometric transformations are those in which the object and the image have the same shape and size (congruent) e.g. rotation, reflection and translation

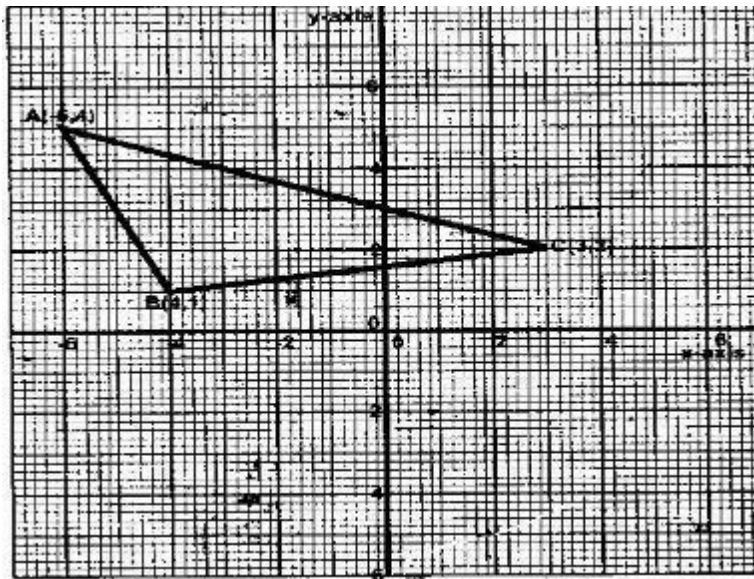
Non- isometric transformations are those in which the object and the image are not congruent e.g., shear stretch and enlargement

Past KCSE Questions on the topic.

1. Matrix p is given by $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
 (a) Find P^{-1}
 (b) Two institutions, Elimu and Somo, purchase beans at Kshs. B per bag and maize at Kshs m per bag. Elimu purchased 8 bags of beans and 14 bags of maize for Kshs 47,600. Somo purchased 10 bags of beans and 16 of maize for Kshs. 57,400
 (c) The price of beans later went up by 5% and that of maize remained constant. Elimu bought the same quantity of beans but spent the same total amount of money as before on the two items. State the new ratio of beans to maize.
2. A triangle is formed by the coordinates $A(2, 1)$ $B(4, 1)$ and $C(1, 6)$. It is rotated clockwise through 90° about the origin. Find the coordinates of this image.
3. On the grid provided on the opposite page $A(1, 2)$ $B(7, 2)$ $C(4, 4)$ $D(3, 4)$ is a trapezium



- (a) ABCD is mapped onto A'B'C'D' by a positive quarter turn. Draw the image A'B'C'D' on the grid
- (b) A transformation $\begin{pmatrix} -2 & -1 \\ 0 & 1 \end{pmatrix}$ maps A'B'C'D' onto A''B''C''D''. Find the coordinates of A''B''C''D''
4. A triangle T whose vertices are A (2, 3) B (5, 3) and C (4, 1) is mapped onto triangle T¹ whose vertices are A¹ (-4, 3) B¹ (-1, 3) and C¹ (x, y) by a Transformation M = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- a) Find the: (i) Matrix M of the transformation
(ii) Coordinates of C₁
- b) Triangle T² is the image of triangle T¹ under a reflection in the line y = x. Find a single matrix that maps T and T₂
5. Triangles ABC is such that A is (2, 0), B (2, 4), C (4, 4) and A''B''C'' is such that A'' is (0, 2), B'' (-4, -10) and C '' is (-4, -12) are drawn on the Cartesian plane Triangle ABC is mapped onto A''B''C'' by two successive transformations R = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Followed by P = $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
- (a) Find R
- (b) Using the same scale and axes, draw triangles A'B'C', the image of triangle ABC under transformation R
- Describe fully, the transformation represented by matrix R
6. Triangle ABC is shown on the coordinate's plane below



- (a) Given that A (-6, 5) is mapped onto A (6,-4) by a shear with y- axis invariant
- (i) Draw triangle A'B'C', the image of triangle ABC under the shear
- (ii) Determine the matrix representing this shear

(b)

Triangle A B C is mapped on to A'' B'' C'' by a transformation defined by the matrix

-1

$$\begin{pmatrix} 0 & 1\frac{1}{2} & -1 \end{pmatrix}$$

(i) Draw triangle A'' B'' C''

(ii) Describe fully a single transformation that maps ABC onto A''B'' C''

7. Determine the inverse T^{-1} of the matrix $\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$

Hence find the coordinates to the point at which the two lines

$$x + 2y = 7 \text{ and } x - y = 1$$

8. Given that $A \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$ and $B \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$

Find the value of x if

(i) $A - 2x = 2B$

(ii) $3x - 2A = 3B$

(iii) $2A - 3B = 2x$

9. The transformation R given by the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ maps } \begin{pmatrix} 17 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 15 \\ 8 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 17 \end{pmatrix} \text{ to } \begin{pmatrix} -8 \\ 15 \end{pmatrix}$$

(a) Determine the matrix A giving a, b, c and d as fractions

(b) Given that A represents a rotation through the origin determine the angle of rotation.

(c) S is a rotation through 180 about the point (2, 3). Determine the image of (1, 0) under S followed by R.

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CHAPTER FIFTY SEVEN

STATISTICS II

Specific Objectives

By the end of the topic the learner should be able to:

- (a) State the measures of central tendency;
- (b) Calculate the mean using the assumed mean method;
- (c) Make cumulative frequency table,
- (d) Estimate the median and the quartiles by
 - Calculation and
 - Using ogive;
- (e) Define and calculate the measures of dispersion: range, quartiles, interquartile range, quartile deviation, variance and standard deviation
- (f) Interpret measures of dispersion

Content

- (a) Mean from assumed mean:
- (b) Cumulative frequency table
- (c) Ogive
- (d) Median
- (e) Quartiles
- (f) Range
- (g) Interquartile range
- (h) Quartile deviation
- (i) Variance
- (j) Standard deviation

These statistical measures are called measures of central tendency and they are mean, mode and median.

Mean using working (Assumed) Mean

Assumed mean is a method of calculating the arithmetic mean and standard deviation of a data set. It simplifies calculation.

Example

The masses to the nearest kilogram of 40 students in the form 3 class were measured and recorded in the table below. Calculate the mean mass

Mass kg	47	48	49	50	51	52	53	
Number of employees	2	0	1	2	3	2	5	

54	55	56	57	58	59	60
6	7	5	3	2	1	1

Solution

We are using assumed mean of 53

Mass x kg	t= x - 53	f	ft
47	-6	2	-12
48	-5	0	0
49	-4	1	-4
50	-3	2	-6
51	-2	3	-6
52	-1	2	-2
53	0	5	0
54	1	6	6
55	2	7	14
56	3	5	15
57	4	3	12
58	5	2	10
60	7	1	6
		1	7
		$\Sigma f = 40$	$\Sigma ft = 40$

$$\text{Mean of } t = \frac{\Sigma f t}{\Sigma f} = \frac{40}{40} = 1$$

$$\begin{aligned} \text{Mean of } x &= 53 + \text{mean of } t \\ &= 53 + 1 \\ &= 54 \end{aligned}$$

Mean of grouped data

The masses to the nearest gram of 100 eggs were as follows

Marks	100- 103	104- 107	108- 111	112-115	116-119	120-123
Frequency	1	15	42	31	8	3

Find the mean mass

Solution

Let use a working mean of 109.5.

class	Mid-point x	t= x - 109.5	f	f t
100-103	101.5	-8	1	- 8
104-107	105.5	-4	15	- 60
108-111	109.5	0	42	0
112-115	113.5	4	31	124
116- 119	117.5	8	8	64
120 -123	121.5	12	3	36
			$\Sigma f = 100$	$\Sigma ft = 156$

$$\text{Mean of } t = \frac{156}{100} = 1.56$$

$$\begin{aligned}\text{Therefore, mean of } x &= 109.5 + \text{mean of } t \\ &= 109.5 + 1.56 \\ &= 111.06 \text{ g}\end{aligned}$$

To get the mean of a grouped data easily, we divide each figure by the class width after subtracting the assumed mean. In order to obtain the mean of the original data from the mean of the new set of data, we will have to reverse the steps in the following order;

- Multiply the mean by the class width and then add the working mean.

Example

The example above to be used to demonstrate the steps

class	Mid-point x	$t = \frac{x-109.5}{4}$	f	f t
100-103	101.5	-2	1	- 2
104-107	105.5	-1	15	- 15
108-111	109.5	0	42	0

112-115	113.5	1	31	31
116- 119	117.5	2	8	16
120 -123	121.5	3	3	9
			$\Sigma f = 100$	$\Sigma ft = 39$

$$\bar{t} = \frac{\Sigma ft}{\Sigma f} = \frac{39}{100} = 0.39$$

Therefore $\bar{x} = 0.39 \times 4 + 109.5$
 $= 1.56 + 109.5$
 $= 111.06 \text{ g}$

Quartiles, Deciles and Percentiles

A median divides a set of data into two equal part with equal number of items.

Quartiles divides a set of data into four equal parts. The lower quartile is the median of the bottom half. The upper quartile is the median of the top half and the middle coincides with the median of the whole set of data

Deciles divides a set of data into ten equal parts. Percentiles divides a set of data into hundred equal parts.

Note;

For percentiles deciles and quartiles the data is arranged in order of size.

Example

Height in cm	145- 149	150-154	155-159	160-164	165-169	170-174	175-179
frequency	2	5	16	9	5	2	1

Calculate the ;

- a.) Median height
- b.) I.) Lower quartile
- ii) Upper quartile
- c.) 80th percentile

Solution

- I. There are 40 students. Therefore, the median height is the average of the heights of the 20th and 21st students.

class	frequency	Cumulative frequency
145-149	2	2
150 - 154	5	7

155 - 159	16	23
160 - 164	9	32
165 - 169	5	37
170 - 174	2	39
175 - 179	1	40

Both the 20th and 21st students falls in the 155 -159 class. This class is called the median class. Using

$$\text{the formula } m = L + \frac{\left(\frac{n}{2} - c\right)i}{f}$$

Where L is the lower class limit of the median class

N is the total frequency

C is the cumulative frequency above the median class

I is the class interval

F is the frequency of the median class

Therefore;

$$\begin{aligned}\text{Height of the 20}^{\text{th}} \text{ student} &= 154.5 + \frac{13}{16} \times 5 \\ &= 154.5 + 4.0625 \\ &= 158.5625\end{aligned}$$

$$\begin{aligned}\text{Height of the 21}^{\text{st}} &= 154.5 + \frac{14}{16} \times 5 \\ &= 154.5 + 4.375 \\ &= 158.875\end{aligned}$$

$$\begin{aligned}\text{Therefore median height} &= \frac{158.5625 + 158.875}{2} \\ &= 158.7 \text{ cm}\end{aligned}$$

b.)

$$(I) \text{ lower quartile } Q_1 = L + \frac{\left(\frac{n}{4} - c\right)i}{f}$$

The 10th student fall in the in 155 – 159 class

$$\begin{aligned}Q_1 &= 154.5 + \frac{\left(\frac{40}{4} - 7\right)5}{16} \\ &= 154.5 + 0.9375 \\ &= 155.4375\end{aligned}$$

$$(ii) \text{ Upper quartile } Q_3 = L + \frac{\left(\frac{3}{4}n - c\right)i}{f}$$

The 10th student fall in the in 155 – 159 class

$$\begin{aligned}Q_3 &= 159.5 + \frac{\left(\frac{3}{4} \times 40 - 23\right)5}{9} \\ &= 159.5 + 3.888 \\ &= 163.3889\end{aligned}$$

Note;

The median corresponds to the middle quartile Q_2 or the 50th percentile

c.) $\frac{80}{100} \times 40 = 32$ the 32nd student falls in the
160 -164 class

$$\text{The 80th percentile} = L + \frac{\left(\frac{80}{100}n - c\right)i}{f}$$

$$= 159.5 + \frac{(32 - 23)5}{9}$$

$$= 159.5 + 5$$

$$= 164.5$$

Example

Determine the upper quartile and the lower quartile for the following set of numbers

5, 10, 6, 5, 8, 7, 3, 2, 7, 8, 9

Solution

Arranging in ascending order

2, 3, 5, 5, 6, 7, 7, 8, 8, 9, 10

The median is 7

The lower quartile is the median of the first half, which is 5.

The upper quartile is the median of the second half, which is 8.

Median from cumulative frequency curve

Graph for cumulative frequency is called an ogive. We plot a graph of cumulative frequency against the upper class limit.

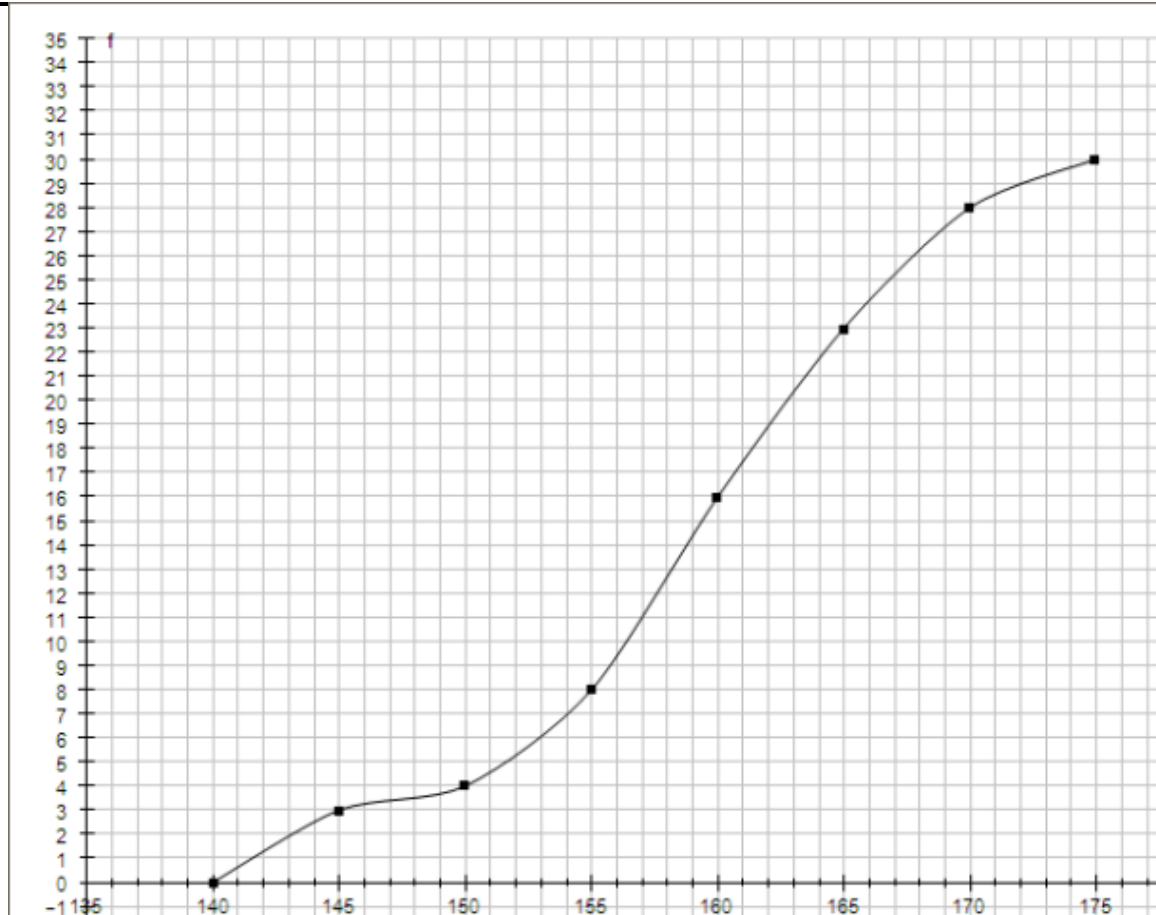
Example

Given the class interval of the measurement and the frequency, we first find the cumulative frequency as shown below.

Then draw the graph of cumulative frequency against upper class limit

Arm Span (cm)	Frequency (f)	Cumulative Frequency
$140 \leq x < 145$	3	3
$145 \leq x < 150$	1	4
$150 \leq x < 155$	4	8
$155 \leq x < 160$	8	16
$160 \leq x < 165$	7	23
$165 \leq x < 170$	5	28
$170 \leq x < 175$	2	30
Total:	30	

Solution



Example

The table below shows marks of 100 candidates in an examination

Marks

1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
4	9	16	24	18	12	8	5	3	1

FRCY

- Determine the median and the quartiles
- If 55 marks is the pass mark, estimate how many students passed
- Find the pass mark if 70% of the students are to pass
- Determine the range of marks obtained by
 - The middle 50 % of the students
 - The middle 80% of the students

Solution

1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
4	9	16	24	18	12	8	5	3	1

Marks

Frqcy

Cumulative 4 13 29 53 71 83 91 96 99 100

Frequency

Solution

a.) Reading from the graph

The median = 39.5

The Lower quartile $Q_1 = 28.5$

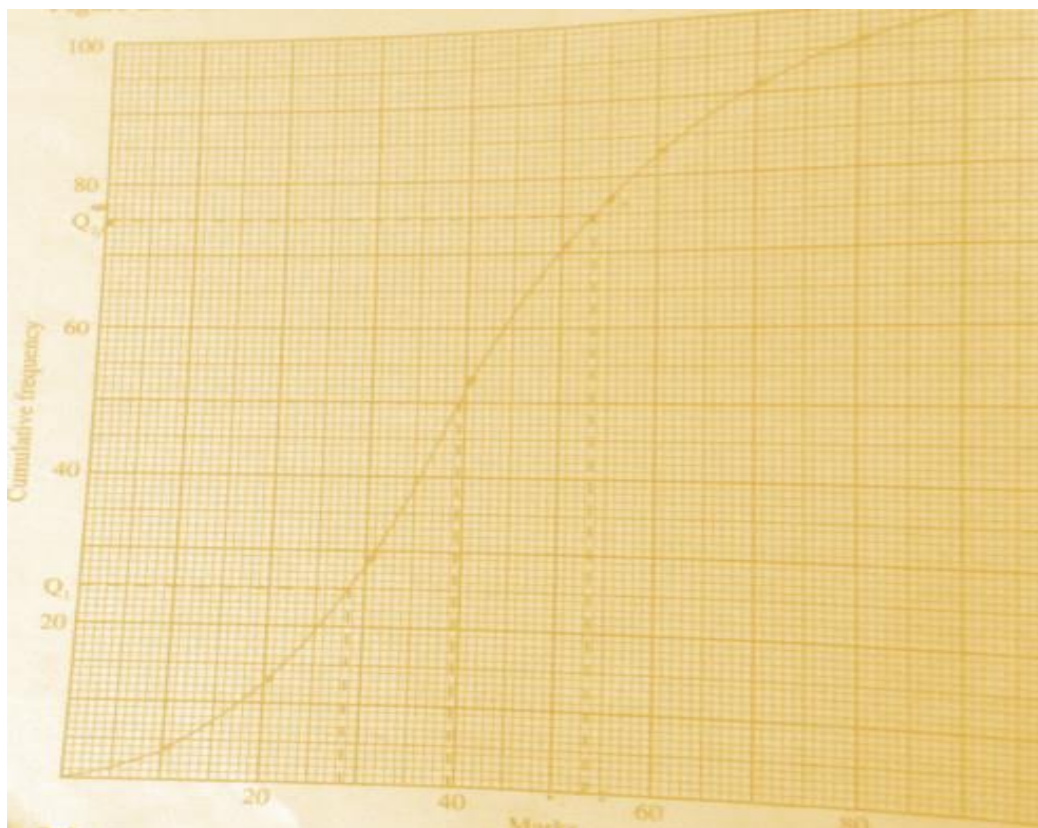
The upper quartile $Q_2 = 53.5$

b.) 23 candidates scored 55 and over

c.) Pass mark is 31 if 70% of pupils are to pass

d.) (I) The middle 50% include the marks between the lower and the upper quartiles i.e. between 28.5 and 53.5 marks.

(II) The middle 80% include the marks between the first decile and the 9th decile i.e between 18 and 69 marks



Measure of Dispersion

Range

The difference between the highest value and the lowest value

Disadvantage

It depends only on the two extreme values

Interquartile range

The difference between the lower and upper quartiles. It includes the middle 50% of the values

Semi quartile range

The difference between the lower quartile and upper quartile divided by 2. It is also called the quartile deviation.

Mean Absolute Deviation

If we find the difference of each number from the mean and find their mean, we get the mean Absolute deviation

Variance

The mean of the square of the square of the deviations from the mean is called variance or mean deviation.

Deviation from mean(d)	+1	-1	+6	-4	-2	-11	+1	10
f_i	1	1	36	16	4	121	1	100

Example

$$\text{Sum } d^2 = 1 + 1 + 36 + 16 + 4 + 121 + 1 + 100 = 280$$

$$\text{Variance} = \frac{\sum d^2}{N} = \frac{280}{8} = 35$$

The square root of the variance is called the standard deviation. It is also called root mean square deviation. For the above example its standard deviation $= \sqrt{35} = 5.9$

Example

The following table shows the number of children per family in a housing estate

Number of childred	0	1	2	3	4	5	6
Number of families	1	5	11	27	10	4	2

Calculate

- a.) The mean number of children per family
b.) The standard deviation

Solution

Number of children	Number of	fx	Deviations	d^2	fd^2
(x)	Families (f)		$d = x - m$		
0	1	0	-3	9	9
1	5	5	-2	4	20
2	11	22	-1	1	11
3	27	81	0	0	0
4	10	40	1	1	10
5	4	20	2	4	16
6	2	12	3	9	18
	$\Sigma f = 60$	$\Sigma fx = 180$			$\Sigma fd^2 = -40$

a.) Mean = $\frac{180}{60} = 3 \text{ children}$

b.) Variance = $\frac{\Sigma fd^2}{\Sigma f}$

$$= \frac{84}{60}$$

$$= 1.4$$

$$\text{standard deviation} = \sqrt{1.4} = 1.183$$

Example

The table below shows the distribution of marks of 40 candidates in a test

Marks	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
frequency	2	2	3	9	12	5	2	3	1	1

Calculate the mean and standard deviation.

Marks	Midpoint (x)	Frequency (f)	fx	$d = x - m$	d^2	fd^2
1-10	5.5	2	11.0	-39.5	1560.25	3120.5
11-20	15.5	2	31.0	-29.5	870.25	1740.5
21-30	25.5	3	76.5	-19.5	380.25	1140.75

31 -40	35.5	9	319.5	-9.5	90.25	812.25
41 -50	45.5	12	546.0	0.5	0.25	3.00
51-60	55.5	5	277.5	10.5	110.25	551.25
61- 70	65.5	2	131.0	20.5	420.25	840.5
71-80	75.5	3	226.5	30.5	930.25	2790.75
81 -90	85.5	1	85.5	40.5	1640.25	1640.25
91 -100	95.5	1	95.5	50.5	2550.25	2550.25
		$\Sigma f = 40$	$\Sigma f x = 1800$			$\Sigma f d^2 = 15190$

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1800}{40} = 45 \text{ marks}$$

$$\text{Variance} = \frac{\Sigma fd^2}{\Sigma f} = \frac{15190}{40} = 379.75$$

$$= 379.8$$

$$\text{Standard deviation} = \sqrt{379.8}$$

$$= 19.49$$

Note;

Adding or subtracting a constant to or from each number in a set of data does not alter the value of the variance or standard deviation.

More formulas

The formula for getting the variance s^2 of a variance x is

$$d^2 = \frac{\Sigma (x - \bar{x})^2 f}{\Sigma f}$$

$$= \frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2$$

Example

The table below shows the length in centimeter of 80 plants of a particular species of tomato

length	152-156	157-161	162-166	167-171	172- 176	177-181
frequency	12	14	24	15	8	7

Calculate the mean and the standard deviation

Solution

Let A = 169

Length	Mid-point x	x-169	$\frac{x-169}{5}$	f	ft	ft^2
152 -156	154	-15	-3	12	-36	108
157 -161	159	-10	-2	14	-28	56
162 -166	164	-5	-1	24	-24	24

167 -171	169	0	0	15	0	0
172-176	174	5	1	8	8	8
177-181	179	10	2	7	14	28
				$\Sigma f = 80$	$\Sigma ft = -66$	$\Sigma ft^2 = 224$

$$\bar{t} = \frac{-66}{80} = -0.825$$

$$\begin{aligned}\text{Therefore } \bar{x} &= -0.825 \times 5 + 169 \\ &= -4.125 + 169 \\ &= 164.875 \text{ (to 4 s.f)}\end{aligned}$$

$$\begin{aligned}\text{Variance of } t &= \frac{\Sigma ft^2}{\Sigma f} - \bar{t}^2 \\ &= \frac{224}{80} - (-0.825)^2 \\ &= 2.8 - 0.6806 \\ &= 2.119\end{aligned}$$

$$\begin{aligned}\text{Therefore , variance of } x &= 2.119 \times 5^2 \\ &= 52.975 \\ &= 52.98 \text{ (4 s.f)}\end{aligned}$$

$$\begin{aligned}\text{Standard deviation of } x &= \sqrt{52.98} \\ &= 7.279 \\ &= 7.28 \text{ (to 2 d.p)}\end{aligned}$$

Past KCSE Questions on the topic

1. Every week the number of absentees in a school was recorded. This was done for 39 weeks these observations were tabulated as shown below

Number of absentees	0.3	4 -7	8 -11	12 - 15	16 - 19	20 - 23
(Number of weeks)	6	9	8	11	3	2

Estimate the median absentee rate per week in the school

2. The table below shows high altitude wind speeds recorded at a weather station in a period of 100 days.

Wind speed (knots)	0 - 19	20 - 39	40 - 59	60-79	80- 99	100- 119	120-139	140-159	160-179
Frequency (days)	9	19	22	18	13	11	5	2	1

- (a) On the grid provided draw a cumulative frequency graph for the data
- (b) Use the graph to estimate
- (i) The interquartile range
- (ii) The number of days when the wind speed exceeded 125 knots
3. Five pupils A, B, C, D and E obtained the marks 53, 41, 60, 80 and 56 respectively. The table below shows part of the work to find the standard deviation.

Pupil	Mark x	x - a	(x-a) ²
A	53	-5	
B	41	-17	
C	60	2	
D	80	22	
E	56	-2	

(a) Complete the table

(b) Find the standard deviation

4. In an agricultural research centre, the length of a sample of 50 maize cobs were measured and recorded as shown in the frequency distribution table below.

Length in cm	Number of cobs
8 – 10	4
11 – 13	7
14 – 16	11
17 – 19	15
20 – 22	8
23 - 25	5

Calculate

(a) The mean

(b) (i) The variance

(ii) The standard deviation

5. The table below shows the frequency distribution of masses of 50 new- born calves in a ranch

Mass (kg)Frequency

15 – 18 2

19- 22 3

23 – 26 10

27 – 30 14

31 – 34 13

35 – 38 6

39 – 42 2

(a) On the grid provided draw a cumulative frequency graph for the data

(b) Use the graph to estimate

(i) The median mass

(ii) The probability that a calf picked at random has a mass lying between 25 kg and 28 kg.

4. The table below shows the weight and price of three commodities in a given period

Commodity	Weight	Price Relatives
X	3	125
Y	4	164
Z	2	140

Calculate the retail index for the group of commodities.

7.

The number of people who attended an agricultural show in one day was 510 men, 1080 women and some children. When the information was represented on a pie chart, the combined angle for the men and women was 216° . Find the angle representing the children.

8. The mass of 40 babies in a certain clinic were recorded as follows:

Mass in Kg	No. of babies.
1.0 – 1.9	6
2.0 – 2.9	14
3.0 – 3.9	10
4.0 – 4.9	7
5.0 – 5.9	2
6.0 – 6.9	1

Calculate

(a) The inter – quartile range of the data.

(b) The standard deviation of the data using 3.45 as the assumed mean.

9. The data below shows the masses in grams of 50 potatoes

Mass (g)	25- 34	35-44	45 - 54	55- 64	65 - 74	75-84	85-94
No of potatoes	3	6	16	12	8	4	1

(a) On the grid provide, draw a cumulative frequency curve for the data

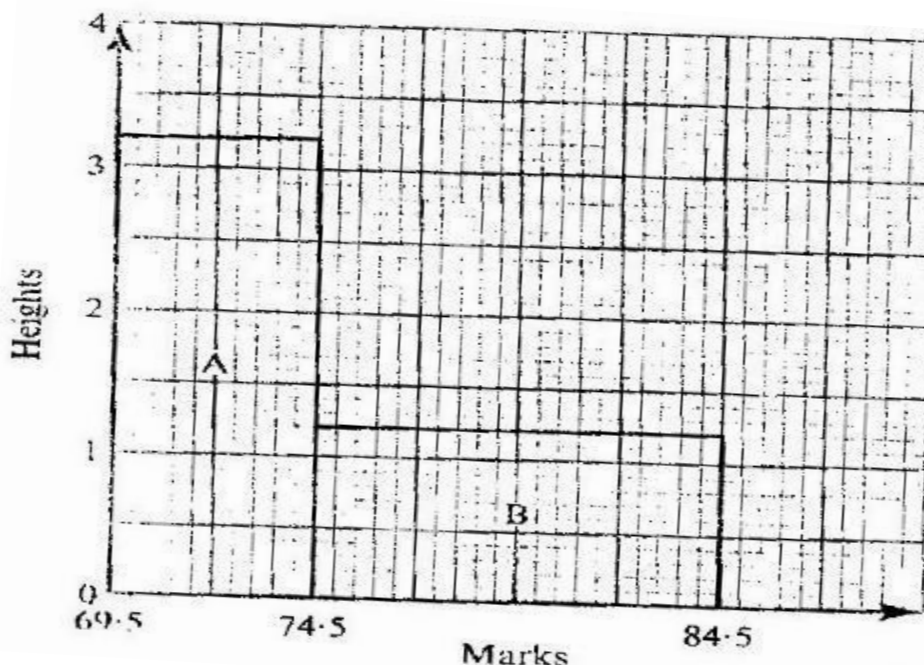
(b) Use the graph in (a) above to determine

(i) The 60th percentile mass

(ii) The percentage of potatoes whose masses lie in the range 53g to 68g

10. The histogram below represents the distribution of marks obtained in a test.

The bar marked A has a height of 3.2 units and a width of 5 units. The bar marked B has a height of 1.2 units and a width of 10 units



If the frequency of the class represented by bar B is 6, determine the frequency of the class represented by bar A.

11. A frequency distribution of marks obtained by 120 candidates is to be represented in a histogram. The table below shows the grouped marks. Frequencies for all the groups and also the area and height of the rectangle for the group 30 – 60 marks.

Marks	0-10	10-30	30-60	60-70	70-100
Frequency	12	40	36	8	24
Area of rectangle			180		
Height of rectangle			6		

- (a) (i) Complete the table
(ii) On the grid provided below, draw the histogram
- (b) (i) State the group in which the median mark lies
(ii) A vertical line drawn through the median mark divides the total area of the histogram into two equal parts

Using this information or otherwise, estimate the median mark

11. In an agriculture research centre, the lengths of a sample of 50 maize cobs were measured and recorded as shown in the frequency distribution table below

Length in cm	Number of cobs
8 – 10	4
11- 13	7
14 – 16	11
17- 19	15
20 – 22	8
23- 25	5

Calculate

- (a) The mean
(b) (i) The variance
(ii) The standard deviation
12. The table below shows the frequency distribution of masses of 50 newborn calves in a ranch.

Mass (kg)	Frequency
15 – 18	2
19- 22	3
23 – 26	10
27 – 30	14
31- 34	13
35 – 38	6
39 - 42	2

- (a) On the grid provided draw a cumulative frequency graph for the data

(b)

Use the graph to estimate

(i) The median mass

(ii) The probability that a calf picked at random has a mass lying between 25 kg and 28 kg

14. The table shows the number of bags of sugar per week and their moving averages

Number of bags per week	340	330	x	343	350	345
Moving averages		331	332	y	346	

(a) Find the order of the moving average

(b) Find the value of X and Y axis

CHAPTER FIFTY EIGHT
THREE DIMENSIONAL GEOMETRY

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