

Chapter Four

SURDS

4.1: Rational and Irrational Numbers

A **rational number** is a number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. The integers p and q must not have common factors other than 1. Numbers such as 2 , $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $\sqrt{4}$, $\sqrt[3]{8}$ and $\sqrt{9}$ are examples of rational numbers. Terminating and recurring decimals are also rational numbers since they can be expressed in the form $\frac{p}{q}$.

Irrational Numbers

Numbers such as π , $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $3\sqrt{10}$ cannot be written in the form $\frac{p}{q}$.

Such numbers are called **irrational numbers**.

Classify the following numbers as rational or irrational:

- | | | | |
|--------------------|---------------------|-------------------|-----------------------|
| (i) $\sqrt{11}$ | (ii) $\sqrt{8}$ | (iii) $\sqrt{16}$ | (iv) $\sqrt{25}$ |
| (v) $\sqrt[4]{32}$ | (vi) $\sqrt[4]{16}$ | (vii) $\sqrt{21}$ | (viii) $\sqrt[3]{27}$ |

Surds

Consider expressions such as $\sqrt{9}$, $\sqrt{\frac{16}{9}}$, $16^{\frac{1}{2}}$, $\sqrt{0.09}$ and $\sqrt[3]{8}$. These numbers have exact square roots or cube roots.

Thus, $\sqrt{9} = \pm 3$, $16^{\frac{1}{2}} = \pm 4$, $\sqrt{0.09} = \pm 0.3$ and $\sqrt[3]{8} = 2$.

However, expressions such as $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{36}$, $\sqrt[3]{16}$ do not have exact square roots or cube roots. Such numbers are called **surds**. All surds are irrational numbers. Although a surd is an irrational number, the number under the root sign is a rational number.

The product of a surd and a rational number is called a mixed surd. Examples are;

$$2\sqrt{3}, 4\sqrt{7} \text{ and } \frac{1}{3}\sqrt{2}$$

4.2: Order of Surds

$\sqrt{3}$, $\sqrt{6}$, $\sqrt{10}$, $\sqrt{12}$ are surds of order 2.

$\sqrt[3]{2}$, $\sqrt[3]{6}$, $\sqrt[3]{10}$, $\sqrt[3]{30}$ are surds of order 3.

$\sqrt[4]{2}$, $\sqrt[4]{64}$, $\sqrt[4]{100}$ are surds of order 4.

In general, if $\sqrt[n]{a}$ is a surd, then the order of the surd is n .

4.3: Simplification of Surds

A surd can be reduced to its lowest term possible, as follows;

Example 1

Simplify: (a) $\sqrt{18}$ (b) $\sqrt{48}$ (c) $\sqrt{72}$

Solution

$$\begin{aligned} \text{(a)} \quad \sqrt{18} &= \sqrt{9 \times 2} \\ &= \sqrt{9} \times \sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \sqrt{48} &= \sqrt{16 \times 3} \\ &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \sqrt{72} &= \sqrt{36 \times 2} \\ &= \sqrt{36} \times \sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

Simplify each of the following to the lowest term:

$$\text{(i)} \quad \sqrt{50} \qquad \text{(ii)} \quad \sqrt{200} \qquad \text{(iii)} \quad \sqrt{12} \qquad \text{(iv)} \quad \sqrt{125} \qquad \text{(v)} \quad \sqrt{54}$$

4.4: Operations on Surds

The four basic operation of addition, subtraction, division and multiplication apply to surds in the same way as in algebra.

Addition and Subtraction

Surds can only be added or subtracted if they are of the same order and the number under the root sign is the same. Otherwise, the surd is in its simplest form.

Example 2

Evaluate:

(a) $\sqrt{2} + \sqrt{2}$ (b) $3\sqrt{3} + 5\sqrt{3}$ (c) $5\sqrt{3} - 2\sqrt{3}$ (d) $3\sqrt{6} + 4\sqrt{10}$

Solution

(a) $\sqrt{2} + \sqrt{2}$

Let $a = \sqrt{2}$

$$\begin{aligned}\therefore \sqrt{2} + \sqrt{2} &= a + a \\ &= 2a\end{aligned}$$

But $a = \sqrt{2}$

$$\therefore \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

(b) $3\sqrt{3} + 5\sqrt{3}$

Let $a = \sqrt{3}$

$$\begin{aligned}\therefore 3\sqrt{3} + 5\sqrt{3} &= 3a + 5a \\ &= 8a\end{aligned}$$

But $a = \sqrt{3}$

$$\therefore 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$$

(c) $5\sqrt{3} - 2\sqrt{3}$

Let $a = \sqrt{3}$

$$\begin{aligned}\therefore 5\sqrt{3} - 2\sqrt{3} &= 5a - 2a \\ &= 3a\end{aligned}$$

But $a = \sqrt{3}$

$$5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$$

(d) $3\sqrt{6} + 4\sqrt{10} = 3\sqrt{6} + 4\sqrt{10}$; Expression already in its simplest form.

In general, (i) $a\sqrt{n} + b\sqrt{n} = (a + b)\sqrt{n}$

(ii) $a\sqrt{n} - b\sqrt{n} = (a - b)\sqrt{n}$

Multiplication and Division

Surds of the same order can be multiplied or divided irrespective of the number under the root sign.

Example 3

Evaluate:

- (a) $\sqrt{2} \times \sqrt{2}$ (b) $\sqrt{2} \times \sqrt{3}$ (c) $2\sqrt{6} \times 7\sqrt{10}$
 (d) $\sqrt{10} \div \sqrt{5}$ (e) $2\sqrt{15} \div \sqrt{3}$

Solution

- (a) $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2}$
 $= \sqrt{4}$
 $= 2$
 (b) $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3}$
 $= \sqrt{6}$
 (c) $2\sqrt{6} \times 7\sqrt{10} = (2 \times 7)\sqrt{6 \times 10}$
 $= 14\sqrt{60}$
 $= 14(\sqrt{4} \times \sqrt{15})$
 $= 14 \times 2\sqrt{15}$
 $= 28\sqrt{15}$
 (d) $\sqrt{10} \div \sqrt{5} = \sqrt{\frac{10}{5}}$
 $= \sqrt{2}$
 (e) $2\sqrt{15} \div \sqrt{3} = 2\sqrt{\frac{15}{3}}$
 $= 2\sqrt{5}$

In general; (i) $\sqrt{m} \times \sqrt{n} = \sqrt{mn}$

(ii) $\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{\frac{m}{n}}$

Note:

$$\sqrt{(a \pm b)} \neq \sqrt{a} \pm \sqrt{b}$$

Example 4

Simplify:

- (a) $(\sqrt{2} + \sqrt{3})^2$ (b) $(2 - \sqrt{5})^2$
 (c) $(2 - \sqrt{3})(2 + \sqrt{3})$ (d) $(2\sqrt{3} + \sqrt{7})(\sqrt{3} + 4\sqrt{11})$

Solution

$$\begin{aligned}\text{(a)} \quad (\sqrt{2} + \sqrt{3})^2 &= (\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3}) \\&= \sqrt{2}(\sqrt{2} + \sqrt{3}) + \sqrt{3}(\sqrt{2} + \sqrt{3}) \\&= \sqrt{4} + \sqrt{6} + \sqrt{4} + \sqrt{6} \\&= 2 + 2\sqrt{6} + 2 \\&= 4 + 2\sqrt{6}\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad (2 - \sqrt{5})^2 &= (2 - \sqrt{5})(2 - \sqrt{5}) \\&= 2(2 - \sqrt{5}) - \sqrt{5}(2 - \sqrt{5}) \\&= 4 - 2\sqrt{5} - 2\sqrt{5} + \sqrt{25} \\&= 4 - 4\sqrt{5} + 5 \\&= 9 - 4\sqrt{5}\end{aligned}$$

$$\begin{aligned}\text{(c)} \quad (2 - \sqrt{3})(2 + \sqrt{3}) &= 2(2 + \sqrt{3}) - \sqrt{3}(2 + \sqrt{3}) \\&= 4 + 2\sqrt{3} - 2\sqrt{3} - \sqrt{9} \\&= 4 - 3 \\&= 1\end{aligned}$$

$$\begin{aligned}\text{(d)} \quad (2\sqrt{3} + \sqrt{7})(\sqrt{3} + 4\sqrt{11}) &= 2\sqrt{3}(\sqrt{3} + 4\sqrt{11}) + \sqrt{7}(\sqrt{3} + 4\sqrt{11}) \\&= 2\sqrt{9} + 8\sqrt{33} + \sqrt{21} + 4\sqrt{77} \\&= 2 \times 3 + 8\sqrt{33} + \sqrt{21} + 4\sqrt{77} \\&= 6 + 8\sqrt{33} + \sqrt{21} + 4\sqrt{77}\end{aligned}$$

Exercise 4.1

1. Express each of the following surds in its simplest form:

- | | | | |
|---------------------|------------------|------------------|-----------------|
| (a) $\sqrt{8}$ | (b) $\sqrt{12}$ | (c) $\sqrt{32}$ | (d) $\sqrt{52}$ |
| (e) $\sqrt{1\,000}$ | (f) $\sqrt{250}$ | (g) $\sqrt{128}$ | (h) $\sqrt{54}$ |

2. Copy and complete the table:

$4\sqrt{3}$	$\sqrt{4^2 (\sqrt{3})^2}$	$\sqrt{16 \times 3}$	$\sqrt{48}$
$2\sqrt{5}$	$\sqrt{2^2 (\sqrt{5})^2}$	$\sqrt{4 \times 5}$	$\sqrt{20}$
$6\sqrt{3}$	—	—	$\sqrt{108}$
—	$\sqrt{7^2 (\sqrt{2})^2}$	—	$\sqrt{98}$
$13\sqrt{10}$	—	$\sqrt{169 \times 10}$	—
$20\sqrt{10}$	—	—	—

3. Simplify:

- (a) $2\sqrt{3} + 5\sqrt{3}$ (b) $3\sqrt{7} + 10\sqrt{7}$ (c) $2\sqrt{7} + \frac{1}{2}\sqrt{7}$
 (d) $5\sqrt{11} + \sqrt{11}$ (e) $3\sqrt{23} + 5\sqrt{23} - 14\sqrt{23}$
 (f) $2\sqrt{x} + 3\sqrt{x}$ (g) $2\sqrt{9x} + 5\sqrt{3x}$
 (h) $4\sqrt{18} + 5\sqrt{72} - \sqrt{108}$ (i) $3\sqrt{x^2} + 7\sqrt{(4x^2)} - 21\sqrt{(16x^2)}$
 (j) $3\sqrt{19} - 2\sqrt{19}$ (k) $6\sqrt{12} - 3\sqrt{10}$ (l) $7\sqrt{24} - 9\sqrt{24}$
 (m) $16\sqrt{40}$ (n) $8\sqrt{32} - 2\sqrt{32}$ (p) $\frac{1}{2}\sqrt{6} - \frac{1}{3}\sqrt{6}$
 (q) $\frac{4}{3}\sqrt{125} - 3\sqrt{5}$ (r) $10\sqrt{216} - \frac{3}{4}\sqrt{6}$ (s) $\frac{4}{3}\sqrt{27} - \frac{1}{2}\sqrt{3}$
 (t) $2\sqrt{343} - 10\sqrt{7}$

4. Simplify:

- (a) $\sqrt{5} \times \sqrt{2}$ (b) $2\sqrt{3} \times 5\sqrt{7}$ (c) $\sqrt{11} \times \sqrt{9}$
 (d) $3\sqrt{13} \times 5\sqrt{2}$ (e) $6\sqrt{4} \times \frac{1}{2}\sqrt{3}$ (f) $8\sqrt{x} \times 2\sqrt{y}$
 (g) $2\sqrt{3} \times \frac{1}{5}\sqrt{5} \times 8\sqrt{23}$ (h) $(\sqrt{5})^2$ (i) $\sqrt{24} \times \sqrt{6}$

- (j) $2\sqrt{8} \times 5\sqrt{3}$ (k) $\sqrt{24} \times \sqrt{6}$ (l) $\sqrt{72} \times \sqrt{2}$
 (m) $\sqrt{81} \times \sqrt{7}$ (n) $\sqrt{225} \times \sqrt{15}$ (p) $\sqrt{169} \times \sqrt{14}$
 (q) $\sqrt{196} \times \sqrt{5}$ (r) $\sqrt{625} \times \sqrt{7}$ (s) $2\sqrt{28} \times 3\sqrt{42}$

5. Simplify:

- (a) $\sqrt{(56 \times 4)}$ (b) $\sqrt{72} \times \frac{1}{2}$ (c) $5\sqrt{(125 \times 6)}$
 (d) $\sqrt{400}$ (e) $\sqrt{124}$ (f) $\sqrt{96}$
 (g) $\sqrt{(92 \times 16)}$ (h) $\sqrt{9} \times \sqrt{24}$ (i) $\sqrt{(64 \times 13)}$
 (j) $16x^2\sqrt{9x^2}$ (k) $\sqrt{(6 \times 6)}$ (l) $\sqrt{(49 + 4)}$
 (m) $\sqrt{(625 \times 9)}$ (n) $\sqrt{(169 \times 18)}$ (p) $\sqrt{(156 \times 25)}$
 (q) $\sqrt{(72 \times 196)}$ (r) $\sqrt{(324 \times 3)}$ (s) $\sqrt{(150 \times 360)}$
 (t) $\sqrt{(125 \times 2)}$ (u) $\sqrt{(81 - 16)}$

6. Simplify:

- (a) $\sqrt{\frac{81}{36}}$ (b) $\sqrt{\frac{625}{100}}$ (c) $\sqrt{\frac{196}{81}}$ (d) $\sqrt{\frac{49}{16}}$
 (e) $\sqrt{\frac{243}{26}}$ (f) $\sqrt{\frac{64}{4}}$ (g) $\sqrt{\frac{128}{196}}$ (h) $\sqrt{\frac{360}{160}}$
 (i) $\sqrt{\frac{48}{36}}$ (j) $\frac{3\sqrt{24}}{4\sqrt{25}}$ (k) $\frac{\sqrt{72}}{\sqrt{32}}$ (l) $\frac{-2\sqrt{200}}{3\sqrt{27}}$
 (m) $\frac{17\sqrt{200}}{6\sqrt{400}}$ (n) $\frac{\sqrt{648}}{\sqrt{72}}$ (p) $\frac{16x^2y^6\sqrt{18x^8y^4}}{8x^3y^2\sqrt{6xy}}$ (q) $\sqrt{\frac{72 \times 18}{41 \times 16}}$
 (r) $\frac{\sqrt{\frac{49}{12}}}{\sqrt{\frac{96}{4}}}$ (s) $\frac{\sqrt{\frac{256}{72}}}{\sqrt{\frac{244}{16}}}$

7. Evaluate, leaving your answer in its simplest form:

- (a) $(2 + \sqrt{3})(2 + \sqrt{3})$ (b) $(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})$
 (c) $(\sqrt{5} - \sqrt{3})(\sqrt{3} + \sqrt{2})$ (d) $(\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2})$
 (e) $(\sqrt{6} - \sqrt{8})(\sqrt{7} - \sqrt{6})$ (f) $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{6})$
 (g) $(\sqrt{3} - \sqrt{5})(\sqrt{10} + \sqrt{2})$ (h) $(\sqrt{20} - \sqrt{15})(\sqrt{2} + \sqrt{5})$

(i) $(3\sqrt{2} - \sqrt{5})(\sqrt{10} + \sqrt{2})$

(j) $(2 + \sqrt{5})(2 + \sqrt{5})$

(k) $(\sqrt{7} + 4)(\sqrt{7} - 4)$

(l) $(4\sqrt{3} - 2\sqrt{3})(4\sqrt{3} + 6\sqrt{3})$

(m) $(\sqrt{125} + \sqrt{5})(\sqrt{3} - \sqrt{7})$

(n) $(\sqrt{149} + \sqrt{160})(\sqrt{3} - \sqrt{6})$

4.5: Rationalising the Denominator

The process of expressing a fraction having a surd in the denominator in such a way that there is no such a surd is called **rationalising** the denominator.

Example

Rationalise: (a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{7}}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{1}{\sqrt{5}} &= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}}{\sqrt{5 \times 5}} \\ &= \frac{\sqrt{5}}{\sqrt{25}} \\ &= \frac{\sqrt{5}}{5} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{1}{\sqrt{7}} &= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{\sqrt{7}}{\sqrt{7 \times 7}} \\ &= \frac{\sqrt{7}}{\sqrt{49}} \\ &= \frac{\sqrt{7}}{7} \end{aligned}$$

Consider the expressions;

(a) $(5 - 2\sqrt{7})(5 + 2\sqrt{7})$

(b) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$

Expanding and simplifying;

$$\begin{aligned} \text{(a)} \quad (5 - 2\sqrt{7})(5 + 2\sqrt{7}) &= 5(5 + 2\sqrt{7}) - 2\sqrt{7}(5 + 2\sqrt{7}) \\ &= 25 + 10\sqrt{7} - 10\sqrt{7} - 4\sqrt{49} \\ &= 25 - 4 \times 7 \\ &= 25 - 28 \\ &= -3 \end{aligned}$$

$$\text{(b)} \quad (\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = \sqrt{3}(\sqrt{3} + \sqrt{2}) - \sqrt{2}(\sqrt{3} + \sqrt{2})$$

$$\begin{aligned}
 &= (\sqrt{9} + \sqrt{6}) - (\sqrt{6} - \sqrt{4}) \\
 &= 3 - 2 \\
 &= 1
 \end{aligned}$$

Note that each expression reduces to a rational number. If the product of two surds gives a rational number, then the surds are called **conjugates** of each other or simply **conjugate surds**.

$(5 - 2\sqrt{7})$ is the conjugate of $(5 + 2\sqrt{7})$ and vice versa. Similarly, $(\sqrt{3} - \sqrt{2})$ is the conjugate of $(\sqrt{3} + \sqrt{2})$ and vice versa.

In general, the conjugate of $(\sqrt{a} - \sqrt{b})$ is $(\sqrt{a} + \sqrt{b})$.

Example 5

Simplify by rationalising the denominator:

$$(a) \quad \frac{8}{\sqrt{5} + \sqrt{2}} \qquad (b) \quad \frac{\sqrt{2} + \sqrt{3}}{\sqrt{5} - \sqrt{2}}$$

Solution

$$\begin{aligned}
 (a) \quad \frac{8}{\sqrt{5} + \sqrt{2}} &= \frac{8}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} \\
 &= \frac{8(\sqrt{5} - \sqrt{2})}{\sqrt{5}(\sqrt{5} - \sqrt{2}) + \sqrt{2}(\sqrt{5} - \sqrt{2})} \\
 &= \frac{8(\sqrt{5} - \sqrt{2})}{\sqrt{25} - \sqrt{10} + \sqrt{10} - \sqrt{4}} \\
 &= \frac{8(\sqrt{5} - \sqrt{2})}{5 - 2} \\
 &= \frac{8(\sqrt{5} - \sqrt{2})}{3}
 \end{aligned}$$

$$\begin{aligned}
 (a) \quad \frac{\sqrt{2} + \sqrt{3}}{\sqrt{6} - \sqrt{3}} &= \frac{(\sqrt{2} + \sqrt{3})}{(\sqrt{6} - \sqrt{3})} \times \frac{(\sqrt{6} + \sqrt{3})}{(\sqrt{6} + \sqrt{3})} \\
 &= \frac{\sqrt{2}(\sqrt{6} + \sqrt{3}) + \sqrt{3}(\sqrt{6} + \sqrt{3})}{\sqrt{6}(\sqrt{6} + \sqrt{3}) - \sqrt{3}(\sqrt{6} + \sqrt{3})} \\
 &= \frac{\sqrt{12} + \sqrt{6} + \sqrt{18} + \sqrt{9}}{\sqrt{36} + \sqrt{18} - \sqrt{18} - \sqrt{9}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt{4} \times \sqrt{3} + \sqrt{6} + \sqrt{9} \times \sqrt{2} + \sqrt{9}}{6 - 3} \\
 &= \frac{2\sqrt{3} + \sqrt{6} + 3\sqrt{2} + 3}{3}
 \end{aligned}$$

Note:

Fractional surds having a 'one term' expression in the denominator, e.g., $\frac{1}{\sqrt{2}}$, $\frac{3}{4\sqrt{6}}$ and $\frac{4}{9\sqrt{10}}$ are rationalised by multiplying both numerator and denominator by the denominator.

If the denominator is a 'two term' expression, e.g., $\frac{3}{\sqrt{2}+5}$, $\frac{4}{\sqrt{7}-\sqrt{5}}$ and $\frac{6}{\sqrt{x}-1}$

Then the fractional surd is rationalised by multiplying both numerator and denominator by the conjugate of the denominator.

Express the following in such a way that there is no surd in the denominator:

$$\begin{aligned}
 \text{(a)} \quad & \frac{1}{\sqrt{3}} & \text{(b)} \quad & \frac{\sqrt{2}+1}{\sqrt{7}} & \text{(c)} \quad & \frac{1}{\sqrt{3}+1} & \text{(d)} \quad & \frac{\sqrt{2}}{\sqrt{11}-4\sqrt{10}}
 \end{aligned}$$

Exercise 4.2

Rationalise and simplify the denominators in numbers 1 – 4.

- $\text{(a)} \quad \frac{1}{\sqrt{2}} \quad \text{(c)} \quad \frac{3}{\sqrt{3}} \quad \text{(e)} \quad \frac{5}{\sqrt{11}}$
 $\text{(d)} \quad \frac{1}{2\sqrt{10}} \quad \text{(f)} \quad \frac{2}{3\sqrt{2}} \quad \text{(g)} \quad \frac{3}{5\sqrt{3}}$
- $\text{(a)} \quad \frac{6}{3-\sqrt{3}} \quad \text{(b)} \quad \frac{26}{4-\sqrt{3}} \quad \text{(c)} \quad \frac{5}{3-\sqrt{2}} \quad \text{(d)} \quad \frac{2\sqrt{3}}{7-\sqrt{5}}$
- $\text{(a)} \quad \frac{5\sqrt{7}}{\sqrt{5}-\sqrt{2}} \quad \text{(b)} \quad \frac{2\sqrt{11}}{\sqrt{11}-\sqrt{5}} \quad \text{(c)} \quad \frac{\sqrt{2}+\sqrt{3}}{\sqrt{7}-\sqrt{5}}$
 $\text{(d)} \quad \frac{\sqrt{7}-\sqrt{5}}{\sqrt{5}+\sqrt{7}} \quad \text{(e)} \quad \frac{\sqrt{4}+1}{\sqrt{4}+\sqrt{5}}$
- $\text{(a)} \quad \frac{\sqrt{7}-2\sqrt{3}}{1+3\sqrt{2}} \quad \text{(b)} \quad \frac{3\sqrt{2}-5\sqrt{11}}{4\sqrt{15}-2\sqrt{7}} \quad \text{(c)} \quad \frac{\sqrt{3}-4}{\sqrt{4}-\sqrt{5}}$
 $\text{(d)} \quad \frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}} \quad \text{(e)} \quad \frac{\sqrt{5}}{3\sqrt{15}-2\sqrt{10}}$

5. Evaluate the following, given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.449$ and $\sqrt{7} = 2.646$.

(a) $\frac{1}{\sqrt{8}}$ (b) $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}$ (c) $\frac{3 + \sqrt{7}}{\sqrt{20}}$

(d) $\frac{2\sqrt{3}}{\sqrt{5} + \sqrt{6}}$ (e) $\frac{\sqrt{5}}{\sqrt{7} - \sqrt{6}}$

6. Given that $a = \sqrt{5}$ and $b = \sqrt{7}$, express the following in terms of a and b and simplify:

(a) $2\sqrt{5} - 6\sqrt{35}$ (b) $3\sqrt{5} + \sqrt{140}$

(c) $\frac{\sqrt{20} + \sqrt{112}}{\sqrt{5} + \sqrt{28}}$ (d) $\sqrt{320} - \sqrt{7}$

7. Simplify, leaving your answer in surd form:

(a) $\frac{1}{\sqrt{17} - 2\sqrt{5}} - \frac{1}{\sqrt{17} + 2\sqrt{5}}$ (b) $\frac{2}{\sqrt{6} + \sqrt{3}} - \frac{5}{\sqrt{7} - \sqrt{5}}$

8. Express in surd form and simplify by rationalising the denominator:

(a) $\frac{2}{1 - \cos 45^\circ}$ (b) $\frac{\sqrt{10}}{1 - \tan 60^\circ}$ (c) $\frac{1 + \cos 30^\circ}{1 - \sin 60^\circ}$

9. Express as a surd the area of an equilateral triangle of sides 5 cm.
10. Find the longest side of right-angled triangle whose other sides are: $(\sqrt{2} + 1)$ cm and $(\sqrt{2} - 1)$ cm long.

11. Solve the equations:

(a) $\sqrt{x + 5} = 5 - \sqrt{x}$ (b) $\sqrt{x} + \sqrt{x + 7} = 7$

Hint: square both sides.