

Chapter Two

APPROXIMATIONS AND ERRORS

2.1: Computing using Calculators

A calculator is an electronic device which can be used to carry out mathematical computations. These are two types of calculators, namely;

- (i) simple calculators, e.g., those used by shopkeepers and cashiers.
- (ii) scientific calculators which can be used for virtually all operations. In this course, the scientific calculator will be used.

On a calculator, the keypad consists of different operations and digits 0 to 9. To accommodate all operations, the same button may have two functions (one written on top, the other beside). The key which is written beside can be accessed by pressing the inverse (shift or second function) key followed by the button containing the key. The mode of operation varies from one calculator to another, thus the use of the manual is important.

Figure 2.1 shows part of the keypad of a calculator.

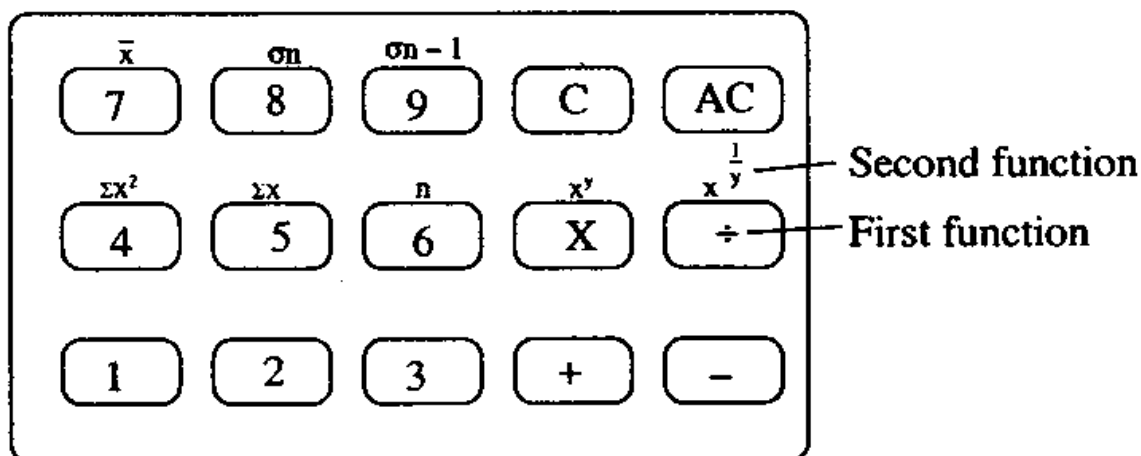


Fig. 2.1

A calculator can display as many digits as will fit on its screen. For example, to display a single digit number, e.g., 5, key in 5. To display a two-digit number like 12, key in 1 followed by 2. A negative number such as -9 can be displayed as follows, key in 9 followed by $\boxed{+/-}$.

Use a calculator to display the following:

- | | | |
|----------|-----------|------------|
| (i) - 8 | (ii) 14 | (iii) - 41 |
| (vi) 426 | (v) - 624 | (vi) 5 272 |

The Four Basic Operations using Calculators***Example 1***

$$5 + 8 = ?$$

Solution

Step I Key in 5.

Step II Key in +

Step III Key in 8

Step IV Key in =

The screen displays 13.

Therefore, $5 + 8 = 13$

Example 2

$$-6 + 15 = ?$$

Solution

Step I Key in - 6

Step II Key in +

Step III Key in 15

Step IV Key in =

The screen displays 9

$$-6 + 15 = 9$$

Example 3

$$28 - 13 = ?$$

Step I Key in 28

Step II Key in -

Step III Key in 13

Step IV Key in =

The screen displays 15. Thus, $28 - 13 = 15$

Example 4

$$4 \times 9 = ?$$

Solution

Step I Key in 4

Step II Key in x

Step III Key in 9

Step IV Key in =

The screen displays 36. So, $4 \times 9 = 36$

Example 5

$$35 \div 7?$$

Solution

Step I Key in 35

Step II Key in $\div$

Step III Key in 7

Step IV Key in $=$

The screen displays 5. Therefore, $35 \div 7 = 5$

Example 6

$$\frac{3.51 + 4.51}{7.5 \times 1.28}$$

Solution

Step I Key in 3.51

Step II Key in $+$

Step III Key in 4.51

Step IV Key in $=$

Step V Key in $\div$

Step VI Key in (

Step VII Key in 7.5

Step VIII Key in $\times$

Step IX Key in 1.28

Step X Key in)

The screen displays 0.8354166.

Be careful to press the equals key when you have completed the top line (numerator), so that the calculator displays 8.02. On most calculators, it is necessary to enclose the bottom line (denominator) in brackets, i.e., (7.5×1.28) .

Squares and Squares Roots of Numbers using a Calculator

Most calculators have the keys x^2 and $\sqrt{}$, which can be used to find the square and square root respectively.

Example 7

Find the square of 17.

Solution

Step I Key in 17

Step II Key in x^2

The screen displays 289, which is the square of 17.

The square of 17 can also be found as below:

Step I Key in 17

Step II Key in x^y or y^x

Step III Key in 2

Example 8

Find the square root of 324.

Solution

Step I Key in 324

Step II Key in $\sqrt{}$

The screen displays 18, which is the square root of 324.

This can be achieved as below:

Step I Key in 324

Step II Key in $\sqrt[y]{x}$ or $x^{\frac{1}{y}}$

Step III Key in 2

Cubes and Cube Roots of Numbers using a Calculator

Cubes and cube roots can be obtained in the same way as squares and square roots.

Example 9

Use a calculator to find 5^3 .

Solution

Step I Key in 5

Step II Key in x^3

The calculator displays 125, which is the cube of 5.

Alternatively;

Step I Key in 5.

Step II Key in x^y or y^x

Step III Key in 3

125 is displayed.

Example 10

Find the cube root of 64.

Solution

Step I Key in 64

Step II Key in $\sqrt[3]{}$

The screen displays 4, which is the cube root of 64.

Alternatively;

Step I Key in 64

Step II Key in $\sqrt[x]{y}$ or $x^{\frac{1}{y}}$

Step III Key in 3

The screen displays 4.

Exercise 2.1

1. Use a calculator to work out each of the following:

- | | | |
|-------------------------------------|-------------------------------|------------------------------|
| (a) (i) $26 + 53$ | (ii) $267.29 + 729.78$ | |
| (iii) $-879 + 641$ | (iv) $4\,693 + 74\,938$ | |
| (v) $-8\,926 + 5\,679 + (-993\,96)$ | (vi) $0.003792 + 8.3698$ | |
| (b) (i) $72\,938 - 127696$ | (ii) $-0.0082 - 0.0017$ | |
| (iii) $-0.00293 - 2.695$ | (iv) $824 - 749$ | |
| (v) $2\,463 - 9\,361 + 2\,693$ | (vi) $577 - 805 - 415$ | |
| (c) (i) 13×8.5 | (ii) -25×78 | |
| (iii) 0.0045×267 | (iv) -34×-18 | |
| (v) 0.0078×0.00856 | (v) 529×894 | |
| (d) (i) $-536 \div 789$ | (ii) $-5.0067 \div 19$ | |
| (iii) $-0.0082 \div (-0.0017)$ | (iv) $52 \div 16$ | |
| (v) $6\,213 \div 923$ | (vi) $359 \div 0.00394$ | |
| (e) (i) 89^2 | (ii) $(-6.003)^2$ | (iii) $(7.09)^2$ |
| (iv) 349^2 | (v) 46^2 | (vi) $(-429)^2$ |
| (f) (i) $\sqrt{869}$ | (ii) $\sqrt{89\,361}$ | (iii) $\sqrt{726}$ |
| (iv) $\sqrt{0.0092}$ | (v) $\sqrt{0.00317}$ | (vi) $\sqrt{82}$ |
| (g) (i) 68^3 | (ii) $(-341)^3$ | (iii) $(0.06867)^3$ |
| (iv) $(1\,671)^3$ | (v) $(0.0003947)^3$ | (vi) 13^3 |
| (h) (i) $(52)^{\frac{1}{3}}$ | (ii) $\sqrt[3]{692}$ | (iii) $(-673)^{\frac{1}{3}}$ |
| (iv) $\sqrt[3]{0.0082}$ | (v) $(0.00731)^{\frac{1}{3}}$ | (vi) $\sqrt[3]{-0.00321}$ |

$$\begin{array}{ll}
 \text{(i)} \quad \text{(i)} \quad \sqrt{\frac{693 \times 31}{44 \times 673}} & \text{(ii)} \quad \left(\frac{536}{437 \times 620}\right)^{\frac{1}{3}} \\
 \text{(iii)} \quad \frac{1}{\sqrt{82.49}} + \frac{3}{(0.089)^2} & \text{(iv)} \quad \sqrt{\frac{34^3 - 257}{97 \times 1243}}
 \end{array}$$

2.2: Approximation

There are a number of cases where exact figures may not be required. For instance, in a certain centre, the number of aids orphans may be given as 250. The actual number could fluctuate between 240 and 260. In this case, 250 is an approximation. Approximation involves rounding off and truncating.

Rounding Off

In rounding off, the place value to which a number is to be rounded must be stated. The digit occupying the next lower place value is considered. The number is rounded up if the digit is equal or greater than 5 and rounded down if it is less than 5.

Example 10

Round off 395.184 to:

- (a) the nearest hundreds.
- (b) four significant figures.
- (c) the nearest whole number.
- (d) two decimal places.

Solution

- (a) 400 (b) 395.2 (c) 395 (d) 395.18

Truncation

Truncation means cutting off numbers to the given decimal places or significant figures, ignoring the rest.

Example 11

Truncate 3.246 to 2 decimal places.

Solution

3.246 is 3.24 truncated to 2 d.p.

Example 12

Truncate 561.7 to:

- (a) 3 significant figures.
- (b) 2 significant figures.

Solution

- (a) 561.7 is 561 truncated to 3 s.f.
(b) 561.7 is 560 truncated to 2 s.f.

Estimation

Estimation involves rounding off numbers in order to carry out a calculation faster to get an approximate answer. This acts as a useful check on the actual answer.

Example 13

Estimate the answer to $\frac{152 \times 269}{32}$

Solution

The answer should be close to $\frac{150 \times 270}{30} = 1\,350$

The exact answer is 1277.75. 1277.75 written to 2 significant figures is 1 300, which is close to the estimated answer.

Exercise 2.2

1. Round off the following numbers to the nearest:

- (a) thousands.
(b) hundreds.
(c) tens.

(i) 79 546

(ii) 65 324

(iii) 67 238

(iv) 4 031

(v) 379 876

(vi) 13 188

2. Round off the following to the nearest:

- (a) 1 decimal place.
(b) 2 significant figures.

(i) 7.583

(ii) 8.00695

(iii) 17.366

(iv) 0.0898

(v) 4.2039

(vi) 0.00283

3. Round off the following to the nearest:

- (a) tenths.
(b) hundredths.
(c) thousandths.

(i) 67.246

(ii) 1.2578

(iii) 0.03791

(iv) 0.0369

(v) 32.9263

(vi) 9.5396

4. Truncate the following numbers to the accuracy given in brackets:

(a) 709.398 (1 d.p.)

(b) 3.321 (2 s.f.)

(c) 16 988 (4 s.f.)

(d) 56.615 (2 s.f.)

(e) 0.06291 (3 d.p.)

(f) 56.605 (2 d.p.)

5. Estimate the answers to:

- (a) (i) $369 + 51$ (ii) $3\,489 + 6\,699$ (iii) $3\frac{1}{8} + 7\frac{8}{9}$
 (b) (i) $1\frac{3}{4} - \frac{1}{2}$ (ii) $459.4 - 137.4$ (iii) $4572.6 - 178.2 + 79.8$
 (c) (i) 3.3×0.9 (ii) 79.1×151 (iv) $2\,367 \times 19\,879$
 (d) (i) $25.4 \div 2.4$ (ii) $1\frac{7}{8} \div \frac{3}{4}$ (iii) $2\,492 \div 32$
 (e) (i) $\frac{125 \times 11.2}{9.8}$ (ii) $\frac{396.4 \times 7.15}{4.2}$ (iii) $\frac{523.4 \div 13.3}{8.2 \times 5.13}$

6. A motorist travelled for $2\frac{2}{3}$ hours and covered a distance of 569 km. Estimate his speed.

7. Estimate the time which a bus travelling at 79.67 km/h will take to cover a distance of 3 598 km.

2.3: Accuracy and Errors

In everyday life, we encounter measurements such as:

- (i) the capacity of a water tank.
- (ii) the length of a rope.
- (iii) the mass of a baby.
- (iv) room temperature, and so on.

Such quantities are expected to have actual values. However, when they are measured, there is always a small difference between the actual value and the measured value no matter how accurate the measuring instrument is. Hence the measured values are approximations (estimates) of the actual values. The difference between the actual value and the measured value is referred to as the error.

Accuracy of Instruments

An ordinary ruler can be used to measure length up to the nearest millimetre. When a length is given as 3.6 cm, it means it is nearer 3.6 cm, than 3.5 or 3.7 cm. The measurement lies between 3.55 cm and 3.65 cm. Therefore, 3.55 cm is the lower limit while 3.65 cm is the upper limit.

A micrometer screw gauge or vernier calipers can measure length up to the nearest thousandths of a centimetre. When a length is measured as 2.348 cm, it means it is nearer 2.348 cm than 2.347 cm or 2.349 cm. The measurement lies between 2.3475 cm and 2.3485 cm, where the two are lower and upper limits respectively. Thus, a micrometer screw gauge or vernier calipers measures lengths to a higher degree of accuracy than a ruler. Similarly, a digital stop watch will measure time to a higher degree of accuracy than a wall clock, an

electronic balance gives a higher level of accuracy in measuring masses than an ordinary balance. Can you think of more accurate instruments that measure capacity, temperature and electric current?

Absolute Error

When a measurement is stated as 3.6 cm to the nearest millimetre, it lies between 3.55 cm and 3.65 cm. The least unit of measurement in this case is the millilitre, or 0.1 cm.

The greatest possible error is $3.55 - 3.6 = -0.05$ or $3.65 - 3.6 = +0.05$.

Since we are only interested in the size of the possible error, we ignore the sign and say that the absolute error is 0.05. Thus, $|-0.05| = |+0.05| = 0.05$.

If l centimetres is the exact length, then $3.55 \leq l \leq 3.65$ or $l = 3.6 \pm 0.05$.

When a measurement is stated as 2.348 cm to the nearest thousandths of a centimetre (0.001), then the absolute error is $\frac{1}{2} \times 0.001 = 0.0005$.

Generally, the absolute error of a stated measurement is half of the least unit of measurement used, unless otherwise stated.

Relative Error

An error of 0.5 kg in measuring the mass of a cock is much more serious than an error of 0.5 kg in measuring the mass of a bull. If the actual mass of a cock is 2 kg and that a bull is 200 kg, then the relative error with respect to the cock is;

$$\frac{0.5 \text{ kg}}{2 \text{ kg}} = 0.25, \text{ while that with respect to the bull is;}$$

$$\frac{0.5 \text{ kg}}{200 \text{ kg}} = 0.0025$$

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual measurement}}$$

Percentage Error

$$\text{Percentage error} = \text{relative error} \times 100 \%$$

$$= \frac{\text{absolute error}}{\text{actual measurement}} \times 100 \%$$

In the examples above:

$$\begin{aligned} \text{(i) for the cock, percentage error} &= \frac{0.5 \text{ kg}}{2 \text{ kg}} \times 100 \% \\ &= 25 \% \end{aligned}$$

$$\begin{aligned} \text{(ii) for the bull, percentage error} &= \frac{0.5 \text{ kg}}{200 \text{ kg}} \times 100 \% \\ &= 0.25 \% \end{aligned}$$

A quantity which has 25 % error is less reliable than that which is 0.25% in error.

Example 14

The capacity of a gourd is stated as 2 000 millilitres to the nearest 10 millilitres.

Find: (a) the limits within which the capacity lies.

(b) the percentage error.

Solution

(a) Absolute error is $\frac{1}{2}(10) = 5 \text{ ml}$

Lowest possible capacity is $2\,000 \text{ ml} - 5 \text{ ml} = 1\,995 \text{ ml}$.

Highest possible capacity is $2\,000 \text{ ml} + 5 \text{ ml} = 2\,005 \text{ ml}$.

(b) Percentage error = $\frac{\text{absolute error}}{\text{actual measurement}} \times 100 \%$
 $= \frac{5 \text{ ml}}{2\,000 \text{ ml}} \times 100 \%$
 $= 0.25 \%$

Work through this example by taking the capacity of the gourd as 2 litres to the nearest 0.01 litres.

Example 15

The thickness of a coin is 0.20 cm.

(a) What is the percentage error?

(b) What would be the percentage error if its thickness was stated as 0.2 cm?

Solution

(a) The smallest unit of measurement is 0.01.

$$\begin{aligned}\text{Absolute error} &= \frac{1}{2} \times 0.01 \\ &= 0.005\end{aligned}$$

$$\begin{aligned}\text{Percentage error} &= \frac{0.005}{0.20} \times 100 \% \\ &= 2.5 \%\end{aligned}$$

(b) The smallest unit of measurement is 0.1.

$$\text{Absolute error} = \frac{1}{2} \times 0.1 = 0.05 \text{ cm}$$

$$\begin{aligned}\text{Percentage error} &= \frac{0.05}{0.2} \times 100 \% \\ &= 25 \%\end{aligned}$$

Note:

The zero in the second decimal place in (a) is significant and tells us that measurement is accurate to 2 decimal places while that in (b) is accurate to one decimal place.

Round Off and Truncation Errors**Rounding Off Error**

If a number is rounded off to a desired number of decimal places or significant figures, an error is introduced in so far as the new number is different from the original one. Such an error is called a **round off error**.

For instance, when a recurring decimal $1.\dot{6}$ is rounded to 2 significant figures, it becomes 1.7. The round off error is;

$$\begin{aligned} 1.7 - 1.\dot{6} &= \frac{17}{10} - \frac{5}{3} \\ &= \frac{1}{30} \end{aligned}$$

Note:

1.6 converted to a fraction is $\frac{5}{3}$.

Truncation Error

The error introduced due to truncation is called a **truncation error**. In the case

$$\begin{aligned} \text{of } 1.\dot{6} \text{ truncated to 2 s.f., the truncation error is; } |1.\dot{6} - 1.6| &= |1.\dot{6} - 1\frac{6}{10}| \\ &= \frac{1}{15} \end{aligned}$$

Exercise 2.3

- Find the limits within which the measurements below lie:
 (a) 26 cm (b) 26.0 cm (c) 26.07 cm
- For each of the following, state the upper and lower limits of the measurements and the absolute error:
 (a) 1.025 kg (b) 2.50 m (c) 158 km
 (d) 12.10 s (e) 12.1 s (f) 12 s
- State the greatest possible error in each of the following measurements:
 (a) 10 s (b) 11.0 s (c) 3.14 cm (d) 98.4 °F (e) 40 min
- The temperature of a body is measured and recorded as 29.5 °C. Find:
 (a) the absolute error.
 (b) the relative error.
 (c) the percentage error.
- An angle was measured to the nearest degree as 40°. Calculate:

- (a) the maximum possible value of the stated angle.
 - (b) the minimum possible value of the stated angle.
 - (c) the percentage error.
6. The best time for 100 m dash on a school sports day is given as 10.7 s. State within which limits the actual time lies.
7. Find the absolute error, the relative error and percentage error in each of the following measurements:
- (a) 45.38 s (b) 0.45 g (c) 7.62 l
 - (d) 12.004 cm (e) 2 505 tonnes
8. The number 2.5 has been rounded off to one decimal place. Calculate:
- (a) the range within which the exact value lies.
 - (b) the relative error.
9. The thickness of a book was measured to the nearest 0.02 cm as 8.0 cm. What is the percentage error?
10. What is the percentage error in using:
- (a) 0.67 (b) 0.66
- as an estimate of $\frac{2}{3}$?
11. For each of the following numbers:
- (a) round off to 2 significant figures and calculate the percentage error.
 - (b) truncate to 2 significant figures and calculate the percentage error.
- (i) 0.327 (ii) 4.05 (iii) 2.718
 - (iv) 3.14 (v) 10.36 (vi) 395
12. The following numbers are rounded to the degree of accuracy in brackets. Find the upper and lower bounds:
- (a) -186.00 (2 d.p.)
 - (b) 8 000 (to the nearest 100)
 - (c) -43 000 (to the nearest thousand)
 - (d) 694.0 (to the nearest 0.1)

2.4: Propagation of Errors

When approximate values are used in addition, subtraction, multiplication and division, the resultant values are approximations of the actual value. This is illustrated in examples below.

Addition and Subtraction

Example 16

What is the error in the sum of 4.5 cm and 6.1 cm, if each represents a measurement?

Solution

The limits within which the measurements lie are 4.45, i.e., 4.55 or 4.5 ± 0.005 and 6.05 to 6.15, i.e., 6.1 ± 0.05 .

The maximum possible sum is $4.55 + 6.15 = 10.7$ cm.

The minimum possible sum is $4.45 + 6.05 = 10.5$ cm.

The working sum is $4.5 + 6.1 = 10.6$

$$\begin{aligned}\text{The absolute error} &= \text{maximum sum} - \text{working sum} \\ &= |10.7 - 10.6| \\ &= 0.10\end{aligned}$$

Example 17

What is the error in the difference between the measurements 0.72 g and 0.31 g?

Solution

The measurements lie within 0.72 ± 0.005 and 0.31 ± 0.005 respectively. The maximum possible difference will be obtained if we subtract the minimum value of the second measurement from the maximum value of the first, i.e.; $0.725 - 0.305$ cm see figure 2.2.

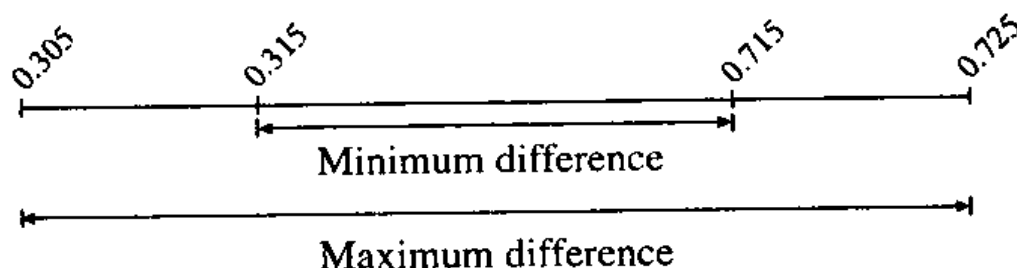


Fig. 2.2

The minimum possible difference is $0.715 - 0.315 = 0.400$. The working difference is $0.72 - 0.31 = 0.41$, which has an absolute error of $|0.420 - 0.41|$ or $|0.400 - 0.41| = 0.10$. Since our working difference is 0.41, we give the absolute error as 0.01 (to 2 s.f.).

Note:

In both addition and subtraction, the absolute error in the answer is equal to the sum of the absolute errors in the original measurements.

Find the relative errors in examples 16 and 17.

Multiplication**Example 18**

A rectangular card measures 5.3 cm by 2.5 cm. Find:

- the absolute error in the area of the card.
- the relative error in the area of the card.

Solution

- (a) The length lies within the limits 5.3 ± 0.05 cm.
 The breadth lies within the limits 2.5 ± 0.05 cm.
 The maximum possible area is $2.55 \times 5.35 = 13.6425 \text{ cm}^2$.
 The minimum possible area is $2.45 \times 5.25 = 12.8625 \text{ cm}^2$.
 The working area is $5.3 \times 2.5 = 13.25 \text{ cm}^2$.
 Maximum area – working area = $13.6425 - 13.25 = 0.3925$.
 Working area – minimum area = $13.25 - 12.8625 = 0.3875$.
 Note that the working area (product) is not midway between the lower and upper limits.
 We may take the absolute error as the average of the two.

$$\begin{aligned}\text{Thus, absolute error} &= \frac{0.3925 + 0.3875}{2} \\ &= 0.3900\end{aligned}$$

The error in the area is therefore 0.39. The same can also be found by taking half the interval between the maximum area and the minimum area, i.e.;

$$\frac{1}{2}(13.6425 - 12.8625) = 0.39$$

- (b) The relative error in the area is;

$$\frac{0.39}{13.25} = 0.029 \text{ (to 2 s.f.)}$$

Alternatively;

$$\begin{aligned}\text{Relative error in length is } \frac{0.05}{5.3} &= \frac{5}{530} \\ &= 0.0094\end{aligned}$$

$$\begin{aligned}\text{Relative error in width is } \frac{0.05}{2.5} &= \frac{5}{250} \\ &= 0.02\end{aligned}$$

$$\begin{aligned}\text{Sum of the relative errors in length and width} &= 0.0094 + 0.02 \\ &= 0.0294 \\ &= 0.029 \text{ (to 2 s.f.)}\end{aligned}$$

Generally, the relative error in a product is approximately equal to the sum of the relative errors in the individual values.

Division**Example 19**

Given $8.6 \text{ cm} \div 3.4 \text{ cm}$, find :

- (a) the absolute error in the quotient.
 (b) the relative error in the quotient .

Solution

- (a) 8.6 cm has limits 8.55 cm and 8.65 cm. 3.4 has limits 3.35 cm and 3.45 cm. The maximum possible quotient will be given by the maximum possible value of the numerator and the smallest possible value of the denominator, i.e.,

$$\frac{8.65}{3.35} = 2.58 \text{ (to 3 s.f.)}$$

The minimum possible quotient will be given by the minimum possible value of the numerator and the biggest possible value of the denominator, i.e.;

$$\frac{8.55}{3.45} = 2.48 \text{ (to 3 s.f.)}$$

The working quotient is; $\frac{8.6}{3.4} = 2.53 \text{ (to 3 s.f.)}$

The absolute error in the quotient is;

$$\begin{aligned} \frac{2.58 - 2.48}{2} &= \frac{1}{2} \times 0.10 \\ &= 0.050 \text{ (to 2 s.f.)} \end{aligned}$$

- (b) Relative error in the working quotient is;

$$\begin{aligned} \frac{0.05}{2.53} &= \frac{5}{253} \\ &= 0.0197 \\ &= 0.020 \text{ (to 2 s.f.)} \end{aligned}$$

Alternatively;

Relative error in the numerator is $\frac{0.05}{8.6} = 0.00581$

Relative error in the denominator is $\frac{0.05}{3.4} = 0.0147$

Sum of the relative errors in the numerator and denominator is

$$\begin{aligned} 0.00581 + 0.0147 &= 0.02051 \\ &= 0.021 \text{ (to 2 s.f.)} \end{aligned}$$

Again, we see that the relative error in the quotient is approximately equal to the sum of the relative errors in the numerator and denominator.

Exercise 2.4

- Find the minimum and maximum possible sums of the following measurements:
 - 10.34 cm and 0.84 cm
 - 8.63 l and 3.52 l
 - 450 m, 32.6 m and 0.39 m
- The sides of a triangle are 22.5 cm, 23 cm, 25.7 cm long. Within which limits does its perimeter lie?

- (b) A rectangular field measures 40 m by 32.5 m. Within which limits does its perimeter lie?
- The first and second athletes in a 200 m race were timed at 21.1 s and 21.3 s respectively. Calculate the minimum and maximum possible differences between their times.
 - A pipe 3.0 m long was cut into three pieces. The first and second pieces measured 1.3 m and 0.94 m respectively. Find the limits within which the length of the third piece lies.
 - Find the minimum possible perimeter of a regular pentagon whose side is 15.0 cm.
 - Find the lower and upper limits of the differences between:
 - 26.0 cm and 14.2 cm
 - 0.08 km and 0.093 km
 - 3.48 g and 1.29 g
 - 3.45 l and 1.25 l
 - Ten washers, each 3.7 mm in thickness, are piled one on top of the other. Find the maximum possible error in the calculation of the height of the pile.
 - The dimensions of a box of matches are stated as 5.2 cm $\times$ 3.8 cm $\times$ 1.5 cm. Within which limits does its:
 - volume, and,
 - surface area lie?
 - Temperatures, to the nearest 0.1 °C, are stated as $a = 2.7$ °C, $b = 3.4$ °C, $c = 9.8$ °C and $d = 3.0$ °C. Find the relative errors in each of the following expressions:

(a) $a + b$	(b) $d - c$	(c) $a - b + c$	(d) ac
(e) abc	(f) $a(d + c)$	(g) $bc - ad$	(h) $\frac{a}{c}$
(i) $\frac{a + d}{c}$	(j) $\frac{d}{ac}$	(k) $\frac{a + b}{c + d}$	(l) $\frac{b}{c} - \frac{c}{d}$
 - The radius and height of a cylindrical mug are 4.5 cm and 8.2 cm respectively. Find the limits within which its capacity lies (leave your answer in terms of π).
 - Find the limits within which the area of a parallelogram whose base is 8 cm and height is 5 cm lies. Hence, find the relative error in the area.
 - Six girls have masses given to the nearest 10 kg, as 40 kg, 50 kg, 60 kg, 60 kg, 70 kg and 80 kg. Calculate:
 - the maximum possible sum of their mass.
 - the minimum possible sum of their mass.
 - The dimensions of a rectangle are 10 cm and 15 cm. If there is an error of 5 % in each of the measurements, find the maximum and least possible area of the rectangle.

14. Two towns on the earth surface are 17 000 km apart to the nearest thousands. The average speed of a supersonic plane to the nearest five hundred is 3 500 km/h. Calculate the maximum possible time that the plane will take to cover the distance between the towns.
15. Find the percentage error in the calculation of the volume of a sphere radius 4.9 cm.
16. A triangle ABC is right angled at B. $AB = 5$ cm and $BC = 12$ cm. Find the relative error in the calculation of its area using each of the formulae:
 - (a) $A = \frac{1}{2}bh$
 - (b) $A = \frac{1}{2}ab\sin c$
 - (c) $A = \sqrt{s(s-a)(s-b)(s-c)}$