

COMMERCIAL ARITHMETIC (II)

6.1: Simple Interest

Interest is the money charged for the use of borrowed money for a specific period of time. If a sum of money is deposited in, or borrowed from a financial institution for a period of time, it earns interest. The sum of money borrowed or deposited is called the **principal (P)**. The ratio of interest earned in a given period of time to the principal is called the **rate (R) of interest**. The rate of interest is normally expressed as a percentage of the principal per annum (p.a.)

When interest is calculated using only the initial principal at a given rate and time, it is called **simple interest (I)**.

Example 1

Find the simple interest earned on sh. 2 000 at 12% per annum for:

- (a) 1 year. (b) 2 years. (c) 3 years.

Solution

$$\begin{aligned} \text{(a)} \quad I &= \text{sh. } \frac{12}{100} \times 2\,000 \times 1 \\ &= \text{sh. } 240 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad I &= \text{sh. } \frac{12}{100} \times 2\,000 \times 2 \\ &= \text{sh. } 480 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad I &= \text{sh. } \frac{12}{100} \times 2\,000 \times 3 \\ &= \text{sh. } 720 \end{aligned}$$

We notice that in simple interest situations, the interest earned is constant for every year. In the example above, the interest earned in the first year is sh. 240, in the second year an additional sh. 240, and so on.

Thus if sh. P is borrowed at R% per annum for T years, then, interest for 1 year sh. $\frac{R}{100} \times P \times 1$ is given by;

$$\text{For T years it is } = \frac{R \times P \times T}{100}$$

This can be abbreviated as $I = \frac{PRT}{100}$

This is the **simple interest formula**. The sum of money at the end of a given period of time is referred to as the **amount (A)**.

Thus, Amount (A) = principal (P) + interest (I)

$$A = P + I$$

Example 2

Calculate the simple interest and the amount on sh. 16 000 for $1\frac{1}{2}$ years at 14% per annum.

Solution

$$I = \frac{PRT}{100}$$

$$= \text{sh. } 16\,000 \times \frac{14}{100} \times \frac{3}{2}$$

$$= \text{sh. } 3\,360$$

$$\text{Amount} = P + I$$

$$= \text{sh. } 16\,000 + \text{sh. } 3\,360$$

$$= \text{sh. } 19\,360$$

Example 3

Calculate the rate of interest if sh. 4 500 earns sh. 500 after $1\frac{1}{2}$ years.

Solution

From the simple interest formula;

$$I = \frac{PRT}{100}, R = \frac{100I}{PT}$$

$$P = \text{sh. } 4\,500$$

$$I = \text{sh. } 500$$

$$T = 1\frac{1}{2} \text{ years}$$

$$\text{Therefore, } R = \frac{100 \times 500}{4\,500 \times 1\frac{1}{2}}$$

$$= 7.4\%$$

Example 4

Juma invested a certain amount of money in a bank which paid 12% p.a. simple interest. After 5 years, his total savings were sh. 5 600. Determine the amount of money he invested initially.

Solution

Let the amount invested be sh. P

$$T = 5 \text{ years}$$

$$R = 12\% \text{ p.a.}$$

$$A = \text{sh. } 5\,600$$

$$\text{But } A = P + I$$

$$\begin{aligned}\text{Therefore } 5\,600 &= P + P \times \frac{12}{100} \times 5 \\ &= P + 0.60 P \\ &= 1.6 P\end{aligned}$$

$$\begin{aligned}\therefore P &= \frac{5\,600}{1.6} \\ &= \text{sh. } 3\,500\end{aligned}$$

Exercise 6.1

1. Calculate the simple interest on each of the following:
 - (a) sh. 15 300 at $11\frac{1}{2}\%$ p.a. for 3 years.
 - (b) sh. 15 600 at 13% p.a. for 4 years.
 - (c) sh. 23 700 at $12\frac{1}{4}\%$ p.a. for 5 years.
 - (d) sh. 18 000 at 9% p.a. for $1\frac{1}{2}$ years.
 - (e) sh. 15 000 at 10% p.a. for 9 months.
 - (f) sh. 7 800 at $13\frac{1}{2}\%$ p.a. for 17 months.
2. Calculate the rate of interest in each of the following:
 - (a) sh. 48 500 earning sh. 7 500 in $2\frac{1}{2}$ years.
 - (b) sh. 12 000 earning sh. 4 000 in 2 years.
 - (c) sh. 13 200 earning sh. 3 300 in $2\frac{1}{2}$ years.
 - (d) sh. 20 400 earning sh. 2 975 in $2\frac{1}{3}$ years.
 - (e) sh. 4 760 earning sh. 1 870 in $1\frac{1}{2}$ years.
 - (f) sh. 16 400 earning sh. 3 280 in 16 months.
3. Find the time during which simple interest on:
 - (a) sh. 7 200 at $8\frac{1}{2}\%$ p.a. accrues to sh. 1 224.
 - (b) sh. 24 800 at $6\frac{1}{4}\%$ p.a. accrues to sh. 4 650.
 - (c) sh. 9 500 at 5% p.a. accrues to sh. 750.
 - (d) sh. 20 000 at $12\frac{1}{2}\%$ p.a. accrues to sh. 4 000.
 - (e) sh. 50 000 at 10% p.a. accrues to sh. 12 500.
 - (f) sh. 14 700 at $2\frac{2}{9}\%$ p.a. accrues to sh. 3 675.

4. Calculate the principal that will earn the following simple interests:
 - (a) sh. 7 200 in 2 years at 12% p.a.
 - (b) sh. 1 050 in 6 months at 15% p.a.
 - (c) sh. 2 000 in 4 months at $12\frac{1}{2}\%$ p.a.
 - (d) sh. 11 200 in 3 years at $9\frac{1}{3}\%$ p.a.
 - (e) sh. 16 000 in 2 years at 15% p.a.
 - (f) sh 3 629.50 in 10 years at 16% p.a.
5. Calculate the amount earned on each of the following:
 - (a) sh. 12 000 for 3 years at 12% p.a.
 - (b) sh. 50 000 for $1\frac{1}{2}$ years at $12\frac{1}{2}\%$ p.a.
 - (c) sh. 150 000 for 5 years at 14% p.a.
 - (d) sh. 450 000 for 5 years at $7\frac{1}{2}\%$ p.a.
 - (e) sh. 60 000 for 12 months at 14% p.a.
 - (f) sh 8 670 for $\frac{1}{4}$ year at $13\frac{1}{3}\%$ p.a.
6. Find the simple interest that would accrue when a welfare group lends to a member sh. 15 000 at the rate of 12% in one year.
7. If a customer deposits sh. 26 000 in a commercial bank which pays simple interest at the rate of 14% p.a., find the amount after 3 years.
8. The simple interest on a given sum of money borrowed for 4 years at 10% p.a. exceeds the simple interest on the same sum borrowed for $2\frac{1}{2}$ years at 12% p.a. by sh. 12 960. What was the sum of money borrowed?
9. A man deposited some money in a savings bank for $2\frac{3}{4}$ years and found that the money had earned sh. 8 600 simple interest. If the rate was $8\frac{1}{2}\%$ p.a., how much money did he have in his account at the end of the period?
10. A farmer borrowed sh. 12 460 from a financial institution. The simple interest rate was $12\frac{1}{2}\%$ p.a. After 6 months, he paid back sh. 8 460. How much did he still owe the bank including interest?
11. A butcher obtained a loan on which simple interest was charged at 14% p.a. He cleared his loan by paying sh. 24 805 at the end of $1\frac{1}{2}$ years. Find the sum borrowed.
12. A sum of sh. 7 600 was partly lent at 10% p.a. simple interest and partly at $12\frac{1}{2}\%$ p.a. simple interest. The total interest after 2 years was sh. 1 672. How much money was lent at $12\frac{1}{2}\%$?

6.2: Compound Interest

If money is borrowed from or deposited in a financial institution, it earns interest after a specified period of time at a stated rate of interest. Instead of this interest being paid back to the owner, it may be added to (compounded with) the principal and thereafter also earns interest. Such interest is called **compound interest**. The period after which it is compounded to the principal is referred to as the **interest period**.

The compound interest may be calculated annually, semi-annually, quarterly, monthly, etc.

If the rate of compound interest is $R\%$ p.a. and the interest is calculated n times per year, then the rate of interest per period is $\left(\frac{R}{n}\right)\%$. For example a rate of 20% p.a. compounded quarterly would give a rate of $(20 \div 4)\% = 5\%$ per interest period.

After every interest period, the principal increases and therefore the interest earned also increases. The sum of the principal and the compound interest is called the **accumulated amount** (A).

Example 5

Find the amount after 2 years if sh. 10 000 is invested at 15% p.a. compound interest.

Solution

$$\begin{aligned}\text{1st year interest} &= \text{sh. } \frac{15}{100} \times 10\,000 \times 1 \\ &= \text{sh. } 1\,500\end{aligned}$$

$$\begin{aligned}\text{Amount after 1st year} &= \text{sh.}(10\,000 + 1\,500) \\ &= \text{sh. } 11\,500\end{aligned}$$

$$\begin{aligned}\text{2nd year interest} &= \text{sh. } \frac{15}{100} \times 11\,500 \\ &= \text{sh. } 1\,725\end{aligned}$$

$$\begin{aligned}\text{Amount after 2nd year} &= \text{sh. } (11\,500 + 1\,725) \\ &= \text{sh. } 13\,225\end{aligned}$$

Example 6

A customer deposited sh. 14 000 in a savings account. Find the accumulated amount after one year if interest was paid at 12% p.a. compounded quarterly.

Solution

Since in one year there are 4 interest periods, the rate per period is $12 \div 4 = 3\%$.

1st interest period:

$$\begin{aligned}\text{Interest} &= \text{sh. } \frac{3}{100} \times 14\,000 \times 1 \\ &= \text{sh. } 420\end{aligned}$$

$$\begin{aligned}\text{Amount after 1st period} &= \text{sh. } (14\,000 + 420) \\ &= \text{sh. } 14\,420\end{aligned}$$

2nd interest period:

$$\begin{aligned}\text{Interest} &= \text{sh. } \frac{3}{100} \times 14\,420 \times 1 \\ &= \text{sh. } 432.60\end{aligned}$$

$$\begin{aligned}\text{Amount after 2nd period} &= \text{sh. } (14\,420 + 432.60) \\ &= \text{sh. } 14\,852.60\end{aligned}$$

3rd interest period:

$$\begin{aligned}\text{Interest} &= \text{sh. } \frac{3}{100} \times 14\,852.60 \times 1 \\ &= \text{sh. } 445.60\end{aligned}$$

$$\begin{aligned}\text{Amount after 3rd period} &= \text{sh. } (14\,852.60 + 445.60) \\ &= \text{sh. } 15\,298.20\end{aligned}$$

4th interest period

$$\begin{aligned}\text{Interest} &= \frac{3}{100} \times 15\,298.20 \\ &= \text{sh. } 458.95\end{aligned}$$

$$\begin{aligned}\text{Amount after 4th period} &= \text{sh. } (15\,298.20 + 458.95) \\ &= \text{sh. } 15\,757.15\end{aligned}$$

$\therefore$ The accumulated amount is sh. 15 757.15.

Note:

The interest period is taken to be annual, unless otherwise stated.

Compound Interest Formula

Suppose sh. P is invested at $r\%$ per period compound interest. The accumulated amount after 4 interest periods can be calculated as follows:

$$\begin{aligned}\text{Amount after 1st period} &= \text{sh. } \left(P + \frac{rP}{100}\right) \\ &= \text{sh. } P\left(1 + \frac{r}{100}\right)\end{aligned}$$

2nd period:

$$\text{Interest} = \text{sh. } \frac{r}{100} \times P(1 + \frac{r}{100})$$

$$\text{Amount} = \text{sh. } P(1 + \frac{r}{100}) + \text{sh. } \frac{r}{100} \times P(1 + \frac{r}{100})$$

Factoring out $P(1 + \frac{r}{100})$;

$$P(1 + \frac{r}{100})\{1 + \frac{r}{100}\} = P(1 + \frac{r}{100})^2$$

3rd period:

$$\text{Interest} = \text{sh. } \frac{r}{100} \times P(1 + \frac{r}{100})^2$$

$$\text{Amount} = P(1 + \frac{r}{100})^2 + \frac{r}{100} \times P(1 + \frac{r}{100})^2$$

Factoring out $P(1 + \frac{r}{100})^2$;

$$P(1 + \frac{r}{100})^2(1 + \frac{r}{100}) = P(1 + \frac{r}{100})^3$$

4th period:

$$\text{Interest} = \frac{r}{100} \times P(1 + \frac{r}{100})^3$$

$$\text{Amount} = P(1 + \frac{r}{100})^3 + \frac{r}{100} \times P(1 + \frac{r}{100})^3$$

$$= P(1 + \frac{r}{100})^3(1 + \frac{r}{100})$$

$$= P(1 + \frac{r}{100})^4$$

We can notice that for n interest periods, the accumulated amount would be

$$A = P(1 + \frac{r}{100})^n$$

This is the **compound interest formula**.

Example 7

Find the amount at the end of the fourth year if sh. 30 000 is deposited at 15% p.a. compound interest.

Solution

The interest period is 1 year, the rate per period (r) is 15% and the number of periods is 4.

$$\begin{aligned} \text{Amount at the end of the fourth year} &= \text{sh. } 30\,000(1 + \frac{15}{100})^4 \\ &= \text{sh. } 30\,000 \times 1.15^4 \\ &= \text{sh. } 52\,470 \end{aligned}$$

Example 8

What would sh. 15 000 amount to after 3 years at 16% p.a. compounded quarterly?

Solution

For every year, we have 4 interest periods. Thus, in 3 years we will have 12 interest periods, i.e., $n = 12$

Rate per period (r) = $16 \div 4 = 4\%$

$P = \text{sh. } 15\,000$

$$\begin{aligned}\therefore A &= \text{sh. } 15\,000 \left(1 + \frac{4}{100}\right)^{12} \\ &= \text{sh. } 15\,000 \times 1.04^{12} \\ &= \text{sh. } 24\,015.50\end{aligned}$$

Example 9

Find the accumulated amount if sh. 20 000 is deposited for $3\frac{1}{3}$ years at 10% p.a. compound interest.

Solution

$$\begin{aligned}\text{Amount after 3 years} &= \text{sh. } 20\,000 \left(1 + \frac{10}{100}\right)^3 \\ &= \text{sh. } 20\,000 \times 1.1^3 \\ &= \text{sh. } 26\,620\end{aligned}$$

$$\begin{aligned}\text{Interest for the last } \frac{1}{3} \text{ of the year} &= \text{sh. } 26\,620 \times \frac{10}{100} \times \frac{1}{3} \\ &= \text{sh. } 887.35\end{aligned}$$

$$\begin{aligned}\therefore \text{Amount after } 3\frac{1}{3} \text{ years} &= \text{sh. } (26\,620 + 887.35) \\ &= \text{sh. } 27\,507.35\end{aligned}$$

Example 10

A man wants to have sh. 30 000 in 8 years. How much money must he invest now if the rate of interest is 7.5% p.a. compound interest?

Solution

Using the compound interest formula;

$A = \text{sh. } 30\,000$

$r = 7.5\%$ per period

$n = 8$ interest periods

$$\text{Therefore, } P \left(1 + \frac{7.5}{100}\right)^8 = \text{sh. } 30\,000$$

$$\begin{aligned}
 P &= \text{sh. } \frac{30\,000}{(1.075)^8} \\
 &= \text{sh. } 16\,821
 \end{aligned}$$

Example 11

Find the rate per annum at which a certain amount of money doubles after being invested for a period of 5 years compounded annually.

Solution

Let the amount invested be sh. P and the interest rate per annum be $R\%$:

Amount after 5 years = sh. $2P$

$$\text{So, } 2P = P\left(1 + \frac{R}{100}\right)^5$$

$$2 = \left(1 + \frac{R}{100}\right)^5$$

Taking 5th roots of both sides

$$\left(1 + \frac{R}{100}\right) = 1.149 \text{ (logarithms or a calculator can be used to evaluate } \sqrt[5]{2} \text{)}$$

$$1 + \frac{R}{100} = 1.149$$

$$\frac{R}{100} = 0.149$$

$$R = 14.9\%$$

Exercise 6.2

1. Find the compound interest in each of the following:
 - (a) sh. 8 000 invested for 2 years at 13% p.a.
 - (b) sh. 15 000 invested for $3\frac{1}{2}$ years at 14% p.a.
 - (c) sh. 10 000 invested for 3 years at $8\frac{1}{4}$ p.a.
 - (d) sh. 110 000 invested for 5 years at 14% p.a.
 - (e) sh. 20 150 invested for 6 years at $5\frac{1}{2}\%$ p.a.
 - (f) sh. 1 000 000 invested for 4 years at 15% p.a.
2. Find the accumulated amount for each of the following:
 - (a) sh. 23 500 invested for 3 years at 10% p.a.
 - (b) sh. 8 500 invested for 2 years at 5% p.a. compounded semi-annually.
 - (c) sh. 17 250 invested for $3\frac{1}{2}$ years at 8% p.a.
 - (d) sh. 15 000 invested for 2 years at 12% p.a. compounded quarterly.
 - (e) sh. 10 000 invested for 2 years at 15% p.a. compounded monthly.
 - (f) sh. 550 000 invested for $5\frac{1}{2}$ years at 11% p.a.

3. Find the time taken for each of the principals below to accumulate to the given amounts at the given rates of compound interest:
 - (a) accumulated amount of sh. 112 896 from a principal of sh. 90 000 at a rate of 10% p.a.
 - (b) accumulated amount of sh. 18 816 from a principal of sh. 15 000 at 12% p.a.
 - (c) accumulated amount of sh. 314 175 from a principal of sh. 200 000 at 15% p.a.
 - (d) accumulated amount of sh. 71 105 from a principal of sh. 50 000 at 18% p.a. compounded quarterly.
 - (e) accumulated amount of sh. 17 908 from a principal of sh. 10 000 at a rate of 12% p.a. compounded semi-annually.
4. A farmer invested sh. 48 000 for 5 years. Calculate the rate per annum of compound interest if the accumulated amount was sh. 77 304.50.
5. A farmer sold his cows for sh. 25 000 and deposited this money in a savings account which paid 12% p.a. compound interest. After 2 years, he withdrew sh. 25 000 and left the rest for a further 2 years. What was the amount in the bank after the two periods?
6. A jua kali mechanic wants to accumulate sh. 400 000 in 5 years. How much money must he invest now at 10% p.a. compound interest?
7. A man sold a plot of land for sh. 80 000 and invested the money in a building society which pays 12% p.a. compounded semi-annually. After 2 years, he withdrew sh. 50 000 and left the rest for a further 3 years. Calculate the total interest he earned in the 5-year period.
8. How much more interest does sh. 10 000 earn after 5 years invested at 5% p.a. compound interest than if it is invested for the same period and rate at simple interest?
9. A businessman borrowed £1 500 at a rate of 9% p.a. compound interest. He paid £450 at the end of each year for 2 years. How much did he have to pay at the end of the third year in order to settle his debt?
10.
 - (a) What rate of interest (p.a.) is equivalent to:
 - (i) 1% per month.
 - (ii) 3% every 3 months.
 - (iii) 6% every 6 months.
 - (b) What rate of interest p.a. compounded semi-annually is equivalent to 10% p.a. compounded quarterly?
 - (c) What rate of interest p.a. compounded semi-annually is equivalent to 12% p.a. compounded monthly?

6.3: Appreciation and Depreciation

Cars and clothes in use experience wear and tear with time. Consequently, they lose their value. The loss in value of an asset with time is called **depreciation**. If, however, an asset gains in value with time, it is said to have **appreciated**. For example the value of land increases with time due to high demand.

Example 12

An iron box costs sh. 500. Every year it depreciates by 10% of its value at the beginning of that year. What will its value be after 4 years?

Solution

$$\begin{aligned}\text{Value after the first year} &= \text{sh. } (500 - \frac{10}{100} \times 500) \\ &= \text{sh. } 450\end{aligned}$$

$$\begin{aligned}\text{Value after the second year} &= \text{sh. } (450 - \frac{10}{100} \times 450) \\ &= \text{sh. } 405\end{aligned}$$

$$\text{Value after the third year} = \text{sh. } (405 - \frac{10}{100} \times 405)$$

$$\begin{aligned}\text{Value after the fourth year} &= \text{sh. } (364.50 - \frac{10}{100} \times 364.50) \\ &= \text{sh. } 328.05\end{aligned}$$

Show that the value of the iron box after the four years is $500(1 - \frac{10}{100})^4$.

In general, if P is the initial value of an asset, A the value after depreciation for n periods and r the rate of depreciation per period;

$$A = P(1 - \frac{r}{100})^n$$

Example 13

A minibus costs sh. 400 000. Due to wear and tear, it depreciates in value by 2% every month. Find its value after one year.

Solution

$$A = P(1 - \frac{r}{100})^n$$

Substituting $P = 400\,000$, $r = 2$, and $n = 12$ in the formula;

$$\begin{aligned}A &= \text{sh. } 400\,000(1 - 0.02)^{12} \\ &= \text{sh. } 400\,000(0.98)^{12}\end{aligned}$$

No.	Log.
0.98 ¹²	$\bar{1}.9912 \times 12$
	= $\bar{1}.8944$
400 000	5.6021
3.137×10^5	5.4965

Therefore, A = sh. 313 700

Example 14

The initial cost of a ranch is sh. 5 000 000. At the end of each year, the land value increases by 2%. What will be the value of the ranch at the end of 3 years?

Solution

$$\begin{aligned}\text{Value after 1st year} &= \text{sh.}(5\,000\,000 + \frac{2}{100} \times 5\,000\,000) \\ &= \text{sh. } 5\,100\,000\end{aligned}$$

$$\begin{aligned}\text{Value after 2nd year} &= \text{sh.}(5\,100\,000 + \frac{2}{100} \times 5\,100\,000) \\ &= \text{sh. } 5\,202\,000\end{aligned}$$

$$\begin{aligned}\text{Value after 3rd year} &= \text{sh.}(5\,202\,000 + \frac{2}{100} \times 5\,202\,000) \\ &= \text{sh. } 5\,306\,040\end{aligned}$$

Alternatively;

$$\begin{aligned}\text{The value of the ranch after 3 years} &= \text{sh. } 5\,000\,000(1 + \frac{2}{100})^3 \\ &= \text{sh. } 5\,000\,000(1.02)^3 \\ &= \text{sh. } 5\,306\,040.\end{aligned}$$

Therefore the value of the ranch after the 3 years is sh. 5 306 040.

In general, if P is the initial value of an asset, A the value after appreciation for n periods and r the appreciation rate per period;

$$A = P(1 + \frac{r}{100})^n$$

Note:

With a slight change of meaning to A and P in the formulae for appreciation and depreciation, they can be used to solve problems involving situations where there is a uniform rate of change over a period of time, e.g., growth of animal or town populations.

Exercise 6.3

1. A school van was bought for K£75 000. Each year, its value decreases by 10%. Find its value after the second and third year.
2. A piece of land was valued at sh. 150 000. The value of the land appreciates every year by 12%. What will be the value of the land after 4 years?
3. A coffee hullery depreciates in value at the rate of 8% p.a. If the original cost of the hullery was sh. 98 000, find its value after 5 years.
4. The population of a town increases at 4% p.a. If the present population of the town is 500 000, what will it be after 5 years?
5. A car which costs sh. 200 000 depreciates in value at the rate of 5% every 6 months. Find its value after 4 years.
6. The cost of an article is sh. 1 200. Find the rate of inflation if the cost of the article after 5 years is sh. 1 932.60.
7. A seedling is 4 cm high. If its average growth rate is 20% per week, how long will it take for it to be 9.953 cm high?
8. At the end of the year 2000, the population of a centre was 30 000. What was the population of this centre at the beginning of 1997 if the growth rate was 3% per year?
9. What was the population of a town A four years ago if the present population is 800 000 and the growth rate is 5% p.a.?
10. The area covered by a desert is 40 000 km² at present. If the desertification rate is estimated to be 2% every 10 years, calculate the area of the desert in 30 years time.
11. Statistics from a town in Kenya indicate that 160 000 people are HIV positive. It has also been established that the rate of its spread in this town is 8% p.a. How many people will be carriers in 6 years time?
12. A commercial plot was valued at sh. 80 000. The plot appreciates in value at 20% in the first year, 15% in the second year and 10% in the third year. Find the value of the plot at the end of the third year.
13. The present cost of a ranch is sh. 4 072 000. It cost 10 years ago was sh. 2 500 000. If this ranch has been appreciating in value uniformly, determine the annual rate of appreciation.
14. A building was valued at sh. 500 000 in January 1999. Due to high demand, the building appreciated by 15% every year. What was its value at the end of December 2001?

6.4: Hire Purchase

Many a time, individuals find themselves compelled to spend huge sums of money on basic needs and emergencies. As a result, they have to acquire certain items on credit. To buy on credit implies making a down payment (deposit)

then paying the balance by installments (usually regular).

In this method, interest has to be against the borrowing. The rate of interest charged depends on the time, the amount and the lending body.

Credit traders, allow customers to buy such items as TV sets, sewing machines, fridges, cookers etc., this way. The following examples illustrate how the loans are repaid.

Example 15

Achieng' wants to buy a sewing machine on hire purchase. It has a cash price of sh. 7 500. She can pay the cash price or make a down payment of sh. 2 250 and 15 monthly instalments of sh. 550 each. How much interest does she pay under the instalment plan?

Solution

$$\begin{aligned}
 \text{Total amount of instalments} &= \text{sh. } 550 \times 15 \\
 &= \text{sh. } 8\,250 \\
 \text{Down payment (deposit)} &= \text{sh. } 2\,250 \\
 \text{Total payment} &= \text{sh. } (8\,250 + 2\,250) \\
 &= \text{sh. } 10\,500 \\
 \text{Amount of interest charged} &= \text{sh. } (10\,500 - 7\,500) \\
 &= \text{sh. } 3\,000
 \end{aligned}$$

Note that Achieng' takes the sewing machine as soon as she pays the down payment.

The interest charged for buying goods/services on credit is called the **carrying charge** and is calculated as a compound interest. This method of buying goods/services by instalments is called **hire purchase**.

Example 16

Calculate the rate of interest charged per month in example 15.

Solution

When a down payment is made, the compound interest is charged on the balance from the cash price of the given item.

$$\begin{aligned}
 \text{Here, amount borrowed} &= \text{sh. } (7\,500 - 2\,250) \\
 &= \text{sh. } 5\,250 \text{ (this is the new principal)}
 \end{aligned}$$

Let the rate of interest be $r\%$ p.m.

$$\text{Then, } A = P\left(1 + \frac{r}{100}\right)^n$$

A is the principal and the carrying charge.

P – is the money borrowed (new principal). Where no deposit is paid this is the same as the cash price.

$$\text{Thus, } 8\,250 = 5\,250\left(1 + \frac{r}{100}\right)^{15}$$

Therefore, $r = 3.059\%$

Exercise 6.4

1. The cash price of a bed is sh. 6 500. A man buys it on hire purchase by making a down payment of sh. 2 300 followed by 12 monthly instalments of sh. 490. Calculate the carrying charge.
2. The hire purchase price of an iron box is sh. 840. If a down payment of sh. 160 is made and the balance paid in 10 equal monthly instalments, calculate the size of each instalment.
3. The cash price of a cooker is sh. 9 000. A customer bought the cooker by paying 15 monthly instalments of sh. 950 each. Calculate:
 - (a) the carrying charge.
 - (b) the rate of interest.
4. A bicycle can be bought for sh. 3 500 cash or by paying 10 monthly instalments of sh. 490 each. Find the carrying charge and the rate of interest charged per annum.
5. A colour television whose cash price is sh. 30 000 can be sold by two methods:
 - (a) 15 monthly instalments of sh. 2 350 each.
 - (b) Paying $\frac{1}{4}$ of the cash price as deposit and 14 monthly instalments of sh. 1 875 each.

Which of the two methods would be cheaper, and by how much?
6. A businessman offers a photocopying machine for either a down payment of sh. 5 000 and 15 monthly instalments of sh. 1 050 each, or a down payment of sh. 7 500 and 10 monthly instalments of sh. 1 250 each.
 - (a) Which plan is cheaper and by how much?
 - (b) If the machine can be bought for sh. 18 000 cash, which of the two plans carries a higher rate of interest?

6.5: Income Tax

A person who is employed receives payments in some form for work done. Such payments may include salaries, housing allowances, medical allowances, hardship allowances, etc.

Each person getting such payments is expected to pay back part of it to the government in the form of taxes. Such a tax on personal income is known as **income tax**.

An individual's **gross income** is the total amount of money due to the individual at the end of the month or year. This includes the salary and any other allowances or benefits due to the individual.

Taxable income is the amount on which tax is levied. This is the gross income less any special benefits on which taxes are not levied. Such benefits include refunds for expenses incurred while one is on official duty.

In order to calculate the income tax that one has to pay, we convert the taxable income into Kenya pounds (K£) per annum or per month as dictated by the table of rates given. We then divide the taxable income into a slab of K£ 5 808 and the rest in slabs of K£ 5 472 as the table below starting from the lowest rate. The remainder is taxed at the next tax bracket.

The rates of income tax are given by the government and keep on changing from time to time. The table below gives the rate applicable in Kenya in the year 2003.

Table 6.1

<i>Income (K£ per annum)</i>	<i>Rate (sh. per pound)</i>
1 – 5 808	2
5 809 – 11 280	3
11 281 – 16 752	4
16 753 – 22 224	5
Excess over 22 224	6

Relief

- (i) Every employee in Kenya is entitled to an automatic personal tax relief of sh. 12 672 p.a. (sh. 1 056 per month)...
- (ii) An employee with a life insurance policy on his life, that of his wife or child, may make a tax claim on the premiums paid towards the policy at sh. 3 per pound subject to a maximum claim of sh. 3 000 per month.

Example 17

Mr Langat earns a total of K£12 300 p.a. Calculate how much tax he should pay per annum.

Solution

Langat's income lies between £1 and £12 300. The highest tax band is therefore the third band.

For the first £5 808, tax due is sh.	$5\,808 \times 2$	= sh. 11 616
For the next £5 472, tax due is sh.	$5\,472 \times 3$	= sh. 16 416
Remaining £1 020, tax due sh.	$1\,020 \times 4$	= <u>sh. 4 080 +</u>
Total tax due		sh. 32 112
Less personal relief of sh.	$1\,056 \times 12$	= <u>sh. 12 672 –</u>
		<u>sh. 19 440</u>

∴ Tax payable p.a. is sh. 19 440.

Example 18

Mr. Masimbwa earns a basic salary of sh. 15 000 per month. In addition, he gets a medical allowance of sh. 2 400 and a house allowance of sh. 12 000. Use the tax brackets in table 6.1 to calculate the tax he pays in a year.

Solution

$$\begin{aligned}\text{Taxable income per month} &= \text{sh.}(15\,000 + 2\,400 + 12\,000) \\ &= \text{sh. } 29\,400\end{aligned}$$

$$\begin{aligned}\text{Converting to K£ p.a.} &= \text{K£ } 29\,400 \times \frac{12}{20} \\ &= \text{K£ } 17\,640\end{aligned}$$

Tax due

First £5 808 = sh.	$5\,808 \times 2$	= sh. 11 616
Next £5 472 = sh.	$5\,472 \times 3$	= sh. 16 416
Next £5 472 = sh.	$5\,472 \times 4$	= sh. 21 888
Remaining £888 = sh.	888×5	= <u>sh. 4 440 +</u>
Total tax due		sh. 54 360
Less personal relief		<u>sh. 12 672 –</u>
Therefore, tax payable p.a.		sh. 41 688

PAYE

In Kenya, every employer is required by law to deduct income tax from the monthly earnings of his employees every month and to remit the money to the income tax department. This system is called Pay As You Earn (PAYE).

To assist employers, PAYE tables are published by the income tax department showing the various levels of monthly income, in pounds and the corresponding tax payable.

Housing

If an employee is provided with a house by the employer (either freely or for a nominal rent) then 15% of his salary is added to his salary (less rent paid) for

purposes of tax calculation. If the tax payer is a director and is provided with a free house, then 15% of his basic salary is added to his salary before taxation.

Example 19

Mr. Mwashighadi who is a civil servant lives in a government house for which he pays a rent of sh. 500 per month. If his salary is £9 000 p.a., calculate how much PAYE he remits monthly.

Solution

Basic salary		£9 000
Housing	$\text{£ } \frac{15}{100} \times 9\,000$	= £1 350
Less rent paid	= £300	<u>£1 050 +</u>
Taxable income	=	£10 050

Tax charged;

First £5 808, the tax due is sh. 5 808 × 2	= sh. 11 616
Remaining £4 242, the tax due is sh. 4 242 × 3	= <u>sh. 12 726 +</u>
	sh. 24 342
Less personal relief	<u>sh. 12 672 –</u>
	<u>sh. 11 670</u>

$$\begin{aligned}\text{PAYE} &= \text{sh. } \frac{11\,670}{12} \\ &= \text{sh. } 972.50\end{aligned}$$

Example 20

Mr Odhiambo is a senior teacher on a basic monthly salary of Ksh. 16 000. On top of his salary, he gets a house allowance of Ksh. 12 000, a medical allowance of Ksh. 3 060 and a hardship allowance of Ksh. 4 635. He has a life insurance policy for which he pays Ksh. 800 per month and claims insurance relief.

(a) Use the tax table below to calculate his PAYE.

Table 6.2

Income in £ per month	Rate (%)
1 – 484	10
485 – 940	15
941 – 1 396	20
1 397 – 1 852	25
Excess over 1 852	30

- (b) In addition to the PAYE, the following deductions are made on his pay every month:
- (i) WCPS at 2% of basic salary.
 - (ii) NHIF Ksh. 400
 - (iii) Co-operative shares and loan recovery Ksh. 4 800.
- Calculate his net pay.

Solution

$$\begin{aligned} \text{(a) Taxable income} &= \text{Ksh.}(16\,000 + 12\,000 + 3\,060 + 4\,635) \\ &= \text{Ksh. } 35\,695 \end{aligned}$$

$$\begin{aligned} \text{converting to K£} &= \frac{\text{K£ } 35\,695}{20} \\ &= \text{K£ } 1\,784.75 \end{aligned}$$

From table 6.2, tax charged is;

$$\text{First £ 484} = \text{£ } 484 \times \frac{10}{100} = \text{£ } 48.40$$

$$\text{Next £ 456} = \text{£ } 456 \times \frac{15}{100} = \text{£ } 68.40$$

$$\text{Next £ 456} = \text{£ } 456 \times \frac{20}{100} = \text{£ } 91.20$$

$$\text{Remaining £ 388} = \text{£ } 388 \times \frac{25}{100} = \text{£ } 97.00$$

$$\begin{aligned} \text{Total tax due} &= \text{£ } 305.00 \\ &= \text{sh. } 6\,100 \end{aligned}$$

$$\text{Insurance relief} = \text{sh. } \frac{800}{20} \times 3 = \text{sh. } 120$$

$$\text{Personal relief} = \text{sh. } 1\,056 +$$

$$\text{Total relief} = \underline{\text{sh. } 1\,176}$$

$$\begin{aligned} \text{Tax payable per month is sh. } 6\,100 \\ \underline{\text{sh. } 1\,176 -} \\ \text{sh. } 4\,924 \end{aligned}$$

Therefore, PAYE is sh. 4 924.

Note:

For calculation of PAYE, taxable income is rounded down (truncated) to the nearest whole number. For example, K£ 1 784.75 is taken as K£ 1 784.

- (b) Total deductions are.

$$\text{sh. } \left(\frac{2}{100} \times 16\,000 + 400 + 4\,800 + 800 + 4\,924 \right) = \text{sh. } 11\,244$$

$$\begin{aligned} \therefore \text{Net pay} &= \text{sh. } (35\,695 - 11\,244) \\ &= \text{sh. } 24\,451 \end{aligned}$$

If an employees' due tax is less than the relief allocated, then that employee is exempted from PAYE.

Exercise 6.5

Use table 6.1 for the rates of taxation and the given reliefs for this exercise.

1. Calculate the annual tax due for the following:
 - (a) Mr. Kemei with taxable income of K£ 8 000 p.a.
 - (b) Mrs. Pamba with taxable income of K£ 12 000 p.a.
 - (c) Mr. Kigosi with taxable income of K£ 18 460 p.a.
 - (d) Ms. Kuria with taxable income of K£ 40 000 p.a.
 - (e) Mrs. Mikisi with taxable income of K£ 22 300 p.a.
2. Calculate the PAYE payable by the following:
 - (a) Mrs. Ojunga on a monthly salary of sh. 12 400, a house allowance of sh. 8 000 per month and a medical allowance of sh. 2 400 per month.
 - (b) Mr. Kitur whose monthly earnings are; a salary of sh. 12 800, house allowance of sh. 6 000, hardship allowance of sh. 3 840 and a medical allowance of sh. 2 100.
 - (c) Ms. Okinyo with a salary of sh. 14 800, a house allowance of sh. 12 000 and a medical allowance of 2 400 all per month. She also pays for a life insurance policy at sh. 1 200 per month and claims insurance relief.
 - (d) Mr. Mwangi on a monthly salary of sh. 5 000, a medical allowance of sh. 900. He is housed by his employer and pays a nominal rent of sh. 300.
3. The annual income of a businesswoman is £ 8 000. She has a life insurance policy for which she pays a premium of £ 240 p.a. Calculate the income tax she has to pay.
4. The salary of a hotel manager is sh. 9 000 per month. He pays sh. 450 per month towards his life insurance policy and claims insurance relief. Find the income tax he pays in a year.
5. If a man pays £ 60 as income tax per year, find his annual income.
6. For what monthly income is the amount of PAYE chargeable equal to the personal relief?
7. Mr. Kemboi earns sh. 13 750 per month. He is given a monthly house allowance of sh. 5 800 and a car allowance of sh. 2 400 per month.
 - (a) How much PAYE does he pay?
 - (b) If he had a life insurance policy for which he paid premiums of sh. 900 per month, how much PAYE would he pay?
8. Mr. Kisaka's income is £12 840 p.a. and Mrs. Kisaka's income is £13 140 p.a.

They are housed by the employer and Mr. Kisaka pays a nominal rent of sh. 1 000 per month for the house. Find the total income tax they pay in a year.

9. Mr. Ahmed earns £13 636 p.a. and is housed by the employer. He pays a premium of £230 p.a. towards his life insurance policy and claims insurance relief. Find his PAYE.
10. Mr. Mambo's income in one year came from the following sources:
Rent from his houses sh. 150 000
Commissions sh. 48 000
Dividends on shares sh. 210 000
Calculate how much income tax he should pay.
11. Mr. Musasia is a director of a company and earns a salary of sh. 36 500 per month. He has a life insurance policy for which he pays sh. 2 000 per month. The company provides him with a free house, water, electricity and a 2 000 cc limousine. His other taxable benefits are as follows:
Car – £250 per month
Water – £20 per month
Electricity – £50 per month.
 - (a) How much PAYE does he pay?
 - (b) If apart from the insurance and PAYE in (a) other deductions on his pay add up to sh. 11 400 per month, calculate his net monthly pay.