

MATHEMATICS NOTES

FORM 3 SIMPLIFIED TUTORIAL

QUICK REVISION NOTES

*An Updated Well-Organized Detailed Revision Notes for the
Current Form 3 Syllabus.*

SERIES 1

THIS IS A FREE SAMPLE OF THE
ORIGINAL NOTES

CONTACT US FOR FULL VERSION OF THE NOTES

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MWALIMU CONSULTANCY

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Printed in Kenya

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QUADRATIC EXPRESSION AND EQUATIONS

Specific Objective

By the end of the topic the learner should be able to:

- (a) Factorize quadratic expressions;
- (b) Identify perfect squares;
- (c) Complete the square;
- (d) Solving quadratic equations by completing the square;
- (e) Derive the quadratic formula;
- (f) Solve quadratic equations using the formula;
- (g) Form and solve quadratic equations from roots and given situations;
- (h) Make tables of values from a quadratic relation;
- (i) Draw the graph of a quadratic relation;
- (j) Solve quadratic equations using graphs;
- (k) Solve simultaneous equations (one linear and one quadratic) analytically and graphically;
- (l) Apply the knowledge of quadratic equations to real life situations.

Content

- (a) Factorization of quadratic expressions
- (b) Perfect squares
- (c) Completion of the squares
- (d) Solution of quadratic equations by completing the square
- (e) Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- (f) Solution of quadratic equations using the formula.
- (g) Formation of quadratic equations and solving them
- (h) Tables of values for a given quadratic relation
- (i) Graphs of quadratic equations
- (j) Simultaneous equation - one linear and one quadratic
- (k) Application of quadratic equation to real life situation.

Perfect square

Expressions which can be factorized into two equal factors are called perfect squares.

Completing the square

Any quadratic expression can be simplified and written in the form $ax^2 + bx + c$ where a , b and c are constant and a is not equal to zero. We use the expression $\left(\frac{b}{2a}\right)^2$ to make a perfect square.

We are first going to look for expression where coefficient of $x = 1$

Example

What must be added to $x^2 + 10x$ to make it a perfect square?

Solution

- Let the number to be added be a constant c .
- Then $+ 10x + c$ is a perfect square.
- Using
- $(10 / 2 = c$
- $C = 25$ (25 must be added)

Example

What must be added to $+ _ + 36$ to make it a perfect square

Solution

- Let the term to be added be bx where b is a constant
- Then $+ bx + 36$ is a perfect square.
- Using
- $b = 12x$ or $-12x$

We will now consider the situations where $a \neq 1$

In the above you will notice that ac . We use this expression to make perfect squares where a is not one and its not zero.

Example

What must be added to $+ _ + 9$ to make it a perfect square?

Solution

- Let the term to be added be bx .
- Then, $+ bx + 9$ is a perfect square.
- The term to be added is thus.

Example

What must be added to $_ - 40x + 25$ to make it a perfect square?

Solution

- Let the term to be added be a
- Then $- 40x + 25$ is a perfect square.
- Using

Solutions of quadratic equations by completing the square methods

Example

Solve $+ 5x + 1 = 0$ by completing the square.

Solution

- $+ 5x + 1 = 0$ Write original equation.
- $+ 5x = -1$ Write the left side in the form $+ bx$.
- $+ 10x + (= ($ Add to both sides
- $+ 10x + =$
- $=$ Take square roots of each side and factorize the left side

=Solve for x.

Simplify

Therefore $x = -0.2085$ or 4.792

Cannot be solved by factorization.

Example

Solve $+4x + 1 = 0$ by completing the square

Solution

$+4x = -1$ **make coefficient of one by dividing both sides by 2**

$+2x = -1/2$

$+2x + 1 = - + 1$

Adding 1 to complete the square on the LHS

The quadratic formula

Example

Using quadratic formula solve

Solution

Comparing this equation to the general equation we get; $a=2$ $b=-5$ $c=-5$

Substituting in the quadratic formulae

Formation of quadratic equations

Peter travels to his uncle's home, 30 km away from his place. He travels for two thirds of the journey before the bicycle developed mechanical problems and he had to push it for the rest of the journey. If his cycling speed is 10 km/h faster than his walking speed and he completes the journey in 3 hours 30 minutes, determine his cycling speed.

Solution

Let Peter's cycling speed be x km/h, then his walking speed is $(x-10)$ km/h.

Time taken in cycling

Time taken in walking = $(30 - 20)(x - 10)$

Total time h

Therefore

$$60(x-10) + 30(x) = 10(x)(x-10)$$

$$-190x + 600 = 0$$

$$-19x + 60 = 0$$

If his cycling speed is 4 km/h , then his walking speed is (4 -10) km/h, which gives – 6 km/h. Thus, 4 is not a realistic answer to this situation. therefore his cycling speed is 15 km/h.

Example

A positive two digit number is such that the product of the digits is 24. When the digits are reversed , the number formed is greater than the original number by 18. Find the number

Solution

Let the ones digit of the number be y and the tens digit be x ,

Then , $xy = 24$ 1

When the number is reversed, the ones digit is x and the tens digit is y .

Therefore;

$$(10y + x) - (10x + y) = 18$$

$$9y - 9x = 18$$

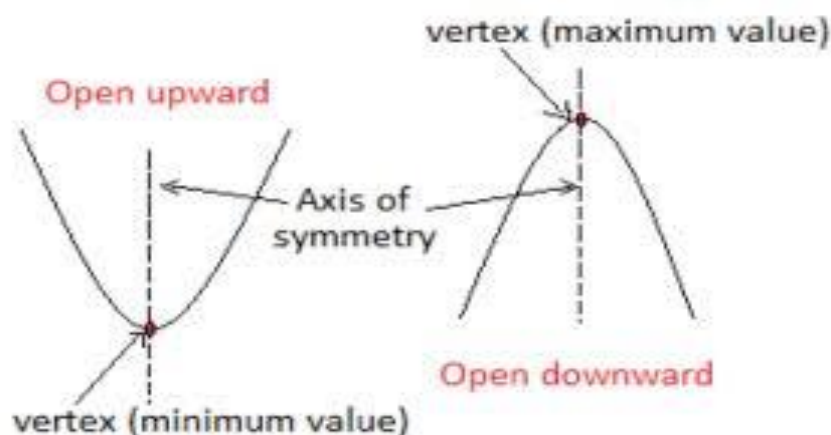
Substituting 2 in equation 1 gives

Since the required number is positive $x = 4$ and $y = 4 + 2 = 6$

Therefore the number is 46

Graphs of quadratic functions

A quadratic function has the form $y = ax^2 + bx + c$ where $a \neq 0$. The graph of a quadratic function is U-shaped and is called a parabola. For instance, the graphs of $y = x^2$ and $y = -x^2$ are shown below. The origin (0, 0) is the lowest point on the graph of $y = x^2$ and the highest point on the graph of $y = -x^2$. The lowest or highest point on the graph of a quadratic function is called the vertex. The graphs of $y = x^2$ and $y = -x^2$ are symmetric about the y -axis, called the *axis of symmetry*. In general, the axis of symmetry for the graph of a quadratic function is the vertical line through the vertex..



Notes;

The graph of $y = ax^2 + bx + c$ and $y = -ax^2 - bx - c$ are symmetric about the y -axis.

Example

Draw the graph of $y = x^2 - 4x + 4$

Solution

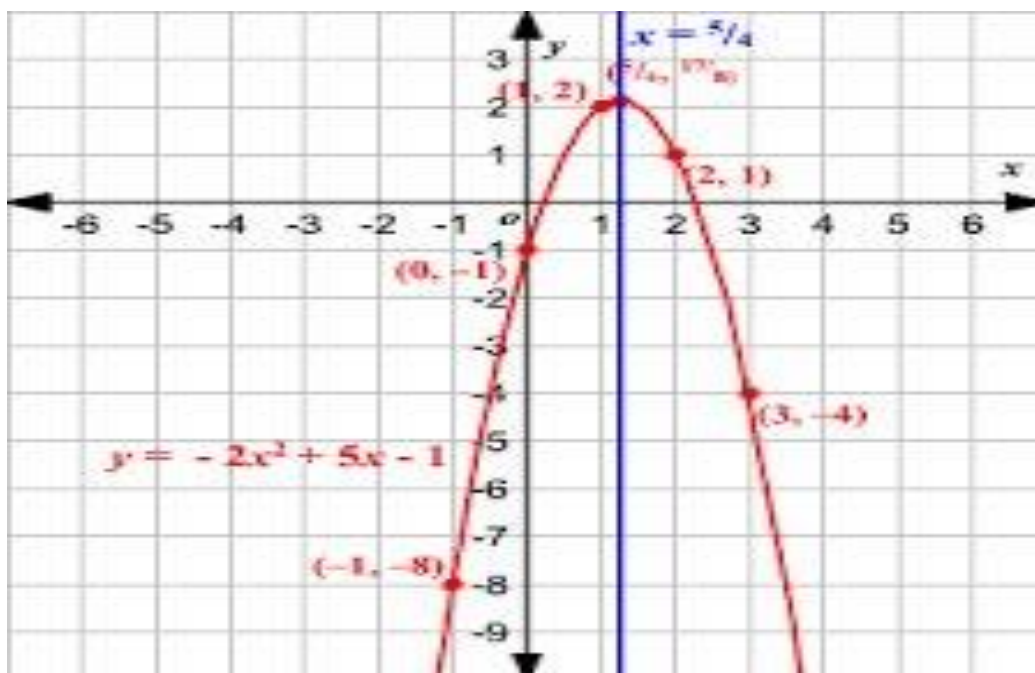
Make a table showing corresponding value of x and y .

X	-1	0	1	2	3	
Y	-8	-1	2	1	-4	

Note ; To get the values replace the value of x in the equation to get the corresponding value of x

E. g $y = -2(-1$

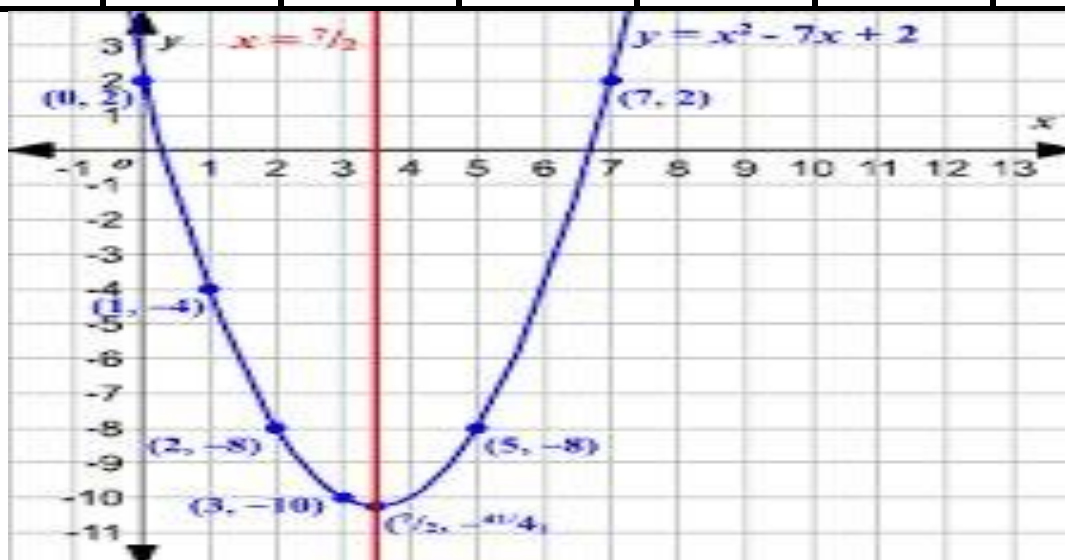
$y = -2(0$



Example

Draw the graph of $y =$

x	0	1	2	3	5	7
y	2	-4	-8	-10	-8	2



Graphical solutions of simultaneous equations

We should consider simultaneous equation one of which is linear and the other one is quadratic.

Example

Solve the following simultaneous equations graphically:

Solution

Corresponding values of x and y

x	-2	-1	0	1	2	3	4	x
y	9	4	1	0	1	4	9	y

We use the table to draw the graph as shown below, on the same axis the line $y = 5 - 2x$ is drawn. Points where the line $y = 5 - 2x$ and the curve intersect give the solution. The points are $(-2, 9)$ and $(2, 1)$. Therefore, when $x = -2$, $y = 9$ and when $x = 2$, $y = 1$

Past KCSE Questions on the topic.

1. The table shows the height metres of an object thrown vertically upwards varies with the time t seconds

The relationship between s and t is represented by the equations $s = at^2 + bt + 10$ where b are constants.

t	0	1	2	3	4	5	6	7	8	9	10
s		45.1									

- (a) (i) Using the information in the table, determine the values of a and b
(2 marks)
- (ii) Complete the table (1 mark)
- (b)(i) Draw a graph to represent the relationship between s and t (3 marks)
- (ii) Using the graph determine the velocity of the object when $t = 5$ seconds
2. (a) Construct a table of value for the function $y = x^2 - x - 6$ for $-3 \leq x \leq 4$
- (b) On the graph paper draw the graph of the function
 $Y = x^2 - x - 6$ for $-3 \leq x \leq 4$
- (c) By drawing a suitable line on the same grid estimate the roots of the equation $x^2 + 2x - 2 = 0$
3. (a) Draw the graph of $y = 6 + x - x^2$, taking integral value of x in $-4 \leq x \leq 5$. (The grid is provided. Using the same axes draw the graph of $y = 2 - 2x$
- (b) From your graphs, find the values of X which satisfy the simultaneous equations $y = 6 + x - x^2$
 $y = 2 - 2x$
- (c) Write down and simplify a quadratic equation which is satisfied by the values of x where the two graphs intersect.
4. (a) Complete the following table for the equation $y = x^3 - 5x^2 + 2x + 9$

x	-2	-1.5	-1	0	1	2	3	4	5
x^2		-3.4	-1	0	1		27	64	125
$-5x^2$	-20	-11.3	-5	0	-1	-20	-45		
2x	-4	-3		0	2	4	6	8	10
9	9	9	9	9	9	9	9	9	99
		-8.7			9	7		-3	

- (b) On the grid provided draw the graph of $y = x^3 - 5x^2 + 2x + 9$ for $-2 \leq x \leq 5$
- (c) Using the graph estimate the root of the equation $x^3 - 5x^2 + 2x + 9 = 0$ between $x = 2$ and $x = 3$
- (d) Using the same axes draw the graph of $y = 4 - 4x$ and estimate a solution to the equation $x^2 - 5x^2 + 6x + 5 = 0$
5. (a) Complete the table below, for function $y = 2x^2 + 4x - 3$

x	-4	-3	-2	-1	0	1	2
$2x^2$	32		8	2	0	2	
$4x - 3$			-11		-3		5
y			-3			3	13

- (b) On the grid provided, draw the graph of the function $y = 2x^2 + 4x - 3$ for $-4 \leq x \leq 2$ and use the graph to estimate the roots of the equation $2x^2 + 4x - 3 = 0$ to 1 decimal place. (2mks)
- (c) In order to solve graphically the equation $2x^2 + x - 5 = 0$, a straight line must be drawn to intersect the curve $y = 2x^2 + 4x - 3$. Determine the equation of this straight line, draw the straight line hence obtain the roots. $2x^2 + x - 5$ to 1 decimal place.
6. (a) (i) Complete the table below for the function $y = x^3 + x^2 - 2x$ (2mks)

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	2	2.5
x^3		15.63				-0.13			1		
x^2			4					0.25			6.25
$-2x$						1			-2		
y				1.87				0.63			16.88

- (ii) On the grid provided, draw the graph of $y = x^3 + x^2 - 2x$ for the values of x in the interval $-3 \leq x \leq 2.5$
- (iii) State the range of negative values of x for which y is also negative
- (b) Find the coordinates of two points on the curve other than (0, 0) at which x- coordinate and y- coordinate are equal

7.

The table shows some corresponding values of x and y for the curve represented by $Y = \frac{1}{4}x^3 - 2$

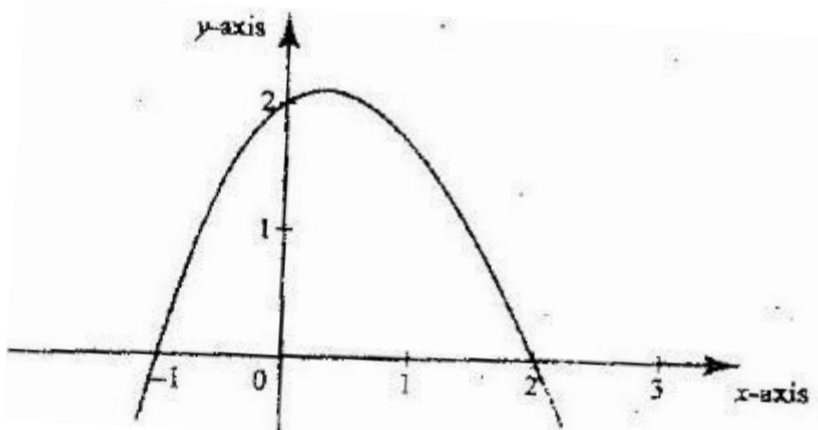
X	-3	-2	-1	0	1	2	3
Y	-8.8	-4	-2.3	-2	-1.8	0	4.8

On the grid provided below, draw the graph of $y = \frac{1}{4}x^2 - 2$ for $-3 \leq x \leq 3$. Use the graph to estimate the value of x when $y = 2$

8. A retailer planned to buy some computers from a wholesaler for a total of Kshs 1,800,000. Before the retailer could buy the computers the price per unit was reduced by Kshs 4,000. This reduction in price enabled the retailer to buy five more computers using the same amount of money as originally planned.

- Determine the number of computers the retailer bought
- Two of the computers purchased got damaged while in store, the rest were sold and the retailer made a 15% profit. Calculate the profit made by the retailer on each computer sold

9. The figure below is a sketch of the graph of the quadratic function $y = k$



$$(x+1)(x-2)$$

Find the value of k

- Draw the graph of $y = x^2 - 2x + 1$ for values $-2 \leq x \leq 4$
 - Use the graph to solve the equations $x^2 - 4 = 0$ and line $y = 2x + 5$
- Draw the graph of $y = x^3 + x^2 - 2x$ for $-3 \leq x \leq 3$ take scale of 2cm to represent 5 units as the horizontal axis
 - Use the graph to solve $x^3 + x^2 - 6x - 4 = 0$ by drawing a suitable linear graph on the same axes.
- Solve graphically the simultaneous equations $3x - 2y = 5$ and $5x + y = 17$

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CHAPTER FOURTY TWO

APPROXIMATION AND ERROR

Specific Objectives

By the end of the topic the learner should be able to:

- (a) Perform various computations using a calculator;
- (b) Make reasonable approximations and estimations of quantities in computations and measurements;
- (c) Express values to a given number of significant figures;
- (d) Define absolute, relative, percentage, round-off and truncation errors;
- (e) Determine possible errors made from computations;
- (f) Find maximum and minimum errors from operations.

Content

- (a) Computing using calculators
- (b) Estimations and approximations
- (c) Significant figures
- (d) Absolute, relative, percentage, round-off (including significant figures) and truncation errors
- (e) Propagation of errors from simple calculations
- (f) Maximum and minimum errors.

Approximation

Approximation involves rounding off and truncating numbers to give an estimation

Rounding off

In rounding off the place value to which a number is to be rounded off must be stated. The digit occupying the next lower place value is considered. The number is rounded up if the digit is greater or equal to 5 and rounded down if it's less than 5.

Example

Round off 395.184 to:

- a. The nearest hundreds
- b. Four significant figures
- c. The nearest whole number
- d. Two decimal places

Solution

- a. 400
- b. 395 .2
- c. 395
- d. 395.18

Truncating

Truncating means cutting off numbers to the given decimal places or significant figures, ignoring the rest.

Example

Truncate 3.2465 to

- a. 3 decimal places
- b. 3 significant figures

Solution

- a. 3.246
- b. 3.24

Estimation

Estimation involves rounding off numbers in order to carry out a calculation faster to get an approximate answer. This acts as a useful check on the actual answer.

Example

Estimate the answer to

Solution

The answer should be close to

The exact answer is 1277.75. 1277.75 written to 2 significant figures is 1300 which is close to the estimated answer.

ACCURACY AND ERROR

Absolute error

The absolute error of a stated measurement is half of the least unit of measurement used. When a measurement is stated as 3.6 cm to the nearest millimeter, it lies between 3.55 cm and 3.65 cm. The least unit of measurement is milliliter, or 0.1 cm. The greatest possible error is $3.55 - 3.6 = -0.05$ or $3.65 - 3.6 = +0.05$.

To get the absolute error we ignore the sign. So the absolute error is 0.05 thus, $|-0.05| = |+0.05| = 0.05$. When a measurement is stated as 2.348 cm to the nearest thousandths of a centimeters (0.001) then the absolute error is .

Relative error

Relative error =

Example

An error of 0.5 kg was found when measuring the mass of a bull. If the actual mass of the bull was found to be 200 kg. Find the relative error

Solution

Relative error =

Percentage error

Percentage error = relative error x 100%

Example

The thickness of a coin is 0.20 cm.

- a. The percentage error
- b. What would be the percentage error if the thickness was stated as 0.2 cm ?

Solution

The smallest unit of measurement is 0.01

Absolute error

Percentage error

The smallest unit of measurement is 0.1

Absolute error

Percentage error

$$= 25 \%$$

Rounding off and truncating errors

An error found when a number is rounded off to the desired number of decimal places or significant figures, for example when a recurring decimal 1. is rounded to the 2 significant figures, it becomes 1.7 the rounded off error is;

1.7 - 1.

Note;

1.6 converted to a fraction.

Truncating error

The error introduced due to truncating is called a truncation error. In the case of 1.6 truncated to 2 S.F., the truncated error is; $|1.6 - 1.5| =$

Propagation of errors

Addition and subtraction

What is the error in the sum of 4.5 cm and 6.1 cm, if each represents a measured measurement.

Solution

The limits within which the measurements lie are 4.45, i.e. 4.45 to 4.55 and 6.05 to 6.15, i.e. 6.1.

The maximum possible sum is 4.55 + 6.15 = 10.7 cm

The

minimum possible sum is $4.45 + 6.1 = 10.55$ cm

The working sum is $4.5 + 6.1 = 10.6$

The absolute error = maximum sum – working sum

$$= |10.55 - 10.6|$$

$$= 0.05$$

Example

What is the error in the difference between the measurements 0.72 g and 0.31 g?

Solution

The measurements lie within and respectively the maximum possible difference will be obtained if we subtract the minimum value of the second measurement from the maximum value of the first, i.e ;

$$0.725 - 0.305 \text{ cm}$$

The minimum possible difference is $0.715 - 0.315 = 0.400$. the working difference is $0.72 - 0.31 = 0.41$, which has an absolute error of $|0.420 - 0.41|$ or $|0.400 - 0.41| = 0.01$. Since our working difference is 0.41, we give the absolute error as 0.01 (to 2 s.f)

Note:

In both addition and subtraction, the absolute error in the answer is equal to the sum of the absolute errors in the original measurements.

Multiplication

Example

A rectangular card measures 5.3 cm by 2.5 cm. find

- a. The absolute error in the area of the card
- b. The relative error in the area of the card

Solution

- a.) The length lies within the limits
- b.) The length lies within the limits

The maximum possible area is $5.35 \times 2.55 = 13.6425$

The minimum possible area is $5.25 \times 2.45 = 12.8625$

The working area is $5.3 \times 2.5 = 13.25$

Maximum area – working area = $13.6425 - 13.25 = 0.3925$.

Working area – minimum area = $13.25 - 12.8625 = 0.3875$

We take the absolute error as the average of the two.

Thus, absolute error

$$= \frac{0.3925 + 0.3875}{2} = 0.3900$$

The same can also be found by taking half the interval between the maximum area and the minimum area

The relative error in the area is :

Division

Given 8.6 cm .Find:

- a. The absolute error in the quotient
- b. The relative error in the quotient

Solution

a. 8.6 cm has limits 8.55 cm and 8.65 cm. 3.4 has limits 3.35 cm and 3.45 cm. The maximum possible quotient will be given by the maximum possible value of the numerator and the smallest possible value of the denominator, i.e.,

$$= 2.58 \text{ (to 3 s.f)}$$

The minimum possible quotient will be given by the minimum possible value of the numerator and the biggest possible value of the denominator, i.e.

$$= 2.48 \text{ (to 3 s.f)}$$

The working quotient is; $= 2.53 \text{ (to 3 .f.)}$

The absolute error in the quotient is;

$$X 0.10$$

b. Relative error in the working quotient ;

$$= 0.0197$$

$$= 0.020 \text{ (to 2 s.f)}$$

Alternatively

Relative error in the numerator is

Relative error in the denominator is

Sum of the relative errors in the numerator and denominator is

$$0.00581 + 0.0147 = 0.02051s$$

$$= 0.021 \text{ to 2 S.F}$$

Past KCSE Questions on the topic.

1. (a) Work out the exact value of $R = \frac{1}{0.003146 - 0.003130}$
- (b) An approximate value of R may be obtained by first correcting each of the decimal in the denominator to 5 decimal places
 - (i) The approximate value
 - (ii) The error introduced by the approximation
2. The radius of circle is given as 2.8 cm to 2 significant figures
 - (a) If C is the circumference of the circle, determine the limits between which C/π lies

(b) By taking π to be 3.142, find, to 4 significant figures the line between which the circumference lies.

3. The length and breadth of a rectangular floor were measured and found to be 4.1 m and 2.2 m respectively. If possible error of 0.01 m was made in each of the measurements, find the:

- (a) Maximum and minimum possible area of the floor
- (b) Maximum possible wastage in carpet ordered to cover the whole floor

4. In this question Mathematical Tables should not be used

The base and perpendicular height of a triangle measured to the nearest centimeter are 6 cm and 4 cm respectively.

Find

(a) The absolute error in calculating the area of the triangle

(b) The percentage error in the area, giving the answer to 1 decimal place

5. By correcting each number to one significant figure, approximate the value of 788×0.006 . Hence calculate the percentage error arising from this approximation.

6. A rectangular block has a square base whose side is exactly 8 cm. Its height measured to the nearest millimeter is 3.1 cm

Find in cubic centimeters, the greatest possible error in calculating its volume.

7. Find the limits within the area of a parallelogram whose base is 8cm and height is 5 cm lies. Hence find the relative error in the area

8. Find the minimum possible perimeter of a regular pentagon whose side is 15.0cm.

9. Given the number 0.237

(i) Round off to two significant figures and find the round off error

(ii) Truncate to two significant figures and find the truncation error

10. The measurements $a = 6.3$, $b = 15.8$, $c = 14.2$ and $d = 0.00173$ have maximum possible errors of 1%, 2%, 3% and 4% respectively. Find the maximum possible percentage error in $\frac{ad}{bc}$ correct to 1sf.

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CHAPTER FOURTY THREE

TRIGONOMETRY

Specific Objectives

By the end of the topic the learner should be able to:

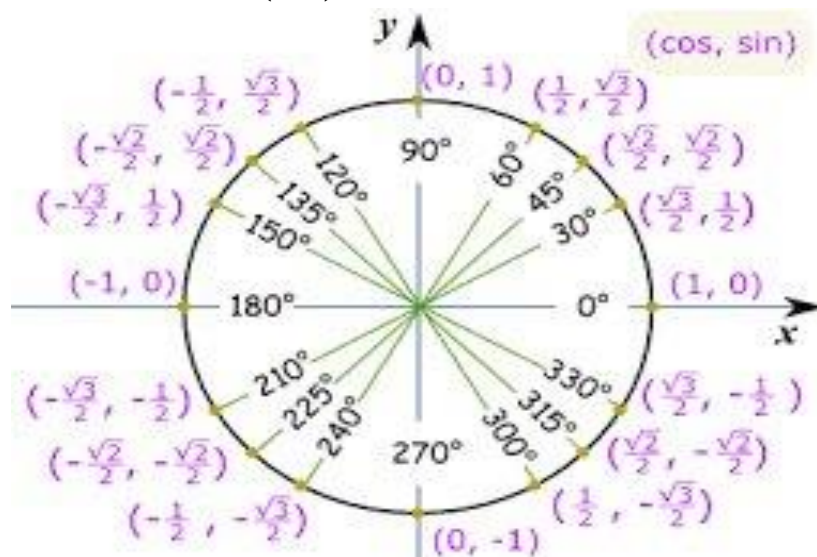
- (a) Define and draw the unit circle;
- (b) Use the unit circle to find trigonometric ratios in terms of co-ordinates of points for $0 < \theta < 360^\circ$;
- (c) Find trigonometric ratios of negative angles;
- (d) Find trigonometric ratios of angles greater than 360° using the unit circle;
- (e) Use mathematical tables and calculators to find trigonometric ratios of angles in the range $0 < \theta < 360^\circ$;
- (f) Define radian measure;
- (g) Draw graphs of trigonometric functions; $y = \sin x$, $y = \cos x$ and $y = \tan x$ using degrees and radians;
- (h) Derive the sine rule;
- (i) Derive the cosine rule;
- (j) Apply the sine and cosine rule to solve triangles (sides, angles and area),
- (k) Apply the knowledge of sine and cosine rules in real life situations.

Content

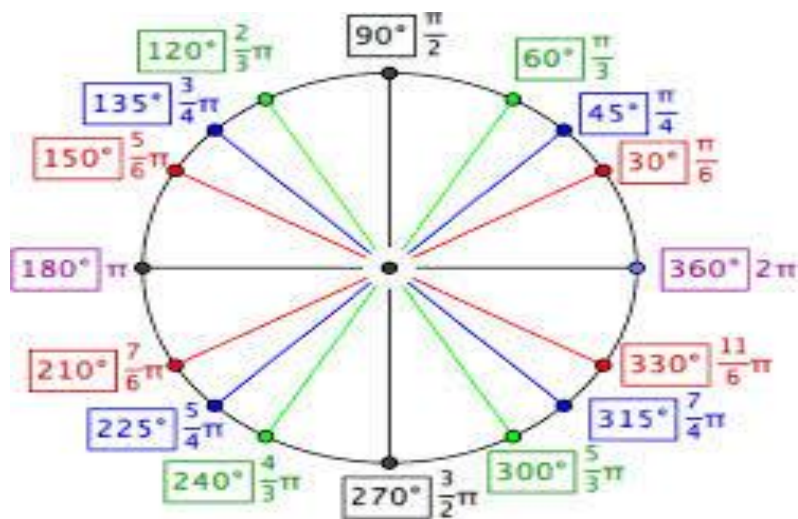
- (a) The unit circles
- (b) Trigonometric ratios from the unit circle
- (c) Trigonometric ratios of angles greater than 360° and negative angles
- (d) Use of trigonometric tables and calculations
- (e) Radian measure
- (f) Simple trigonometric graphs
- (g) Derivation of sine and cosine rule
- (h) Solution of triangles
- (i) Application of sine and cosine rule to real situation.

The unit circle

It is circle of unit radius and centre O (0, 0).

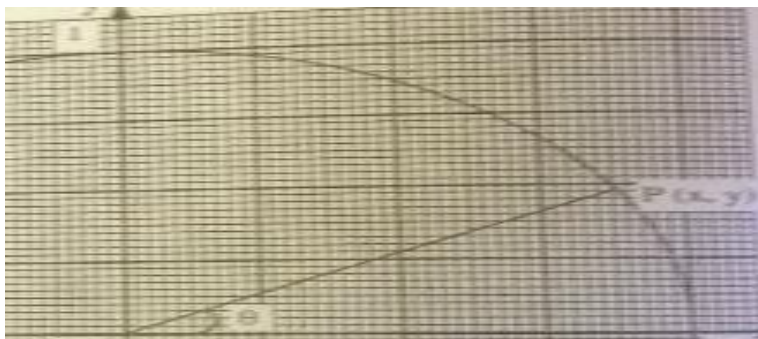


An angle measured anticlockwise from positive direction of x – axis is positive. While an angle measured clockwise from negative direction of x – axis is negative.



In general, on a unit circle

- I.
- II.
- III.

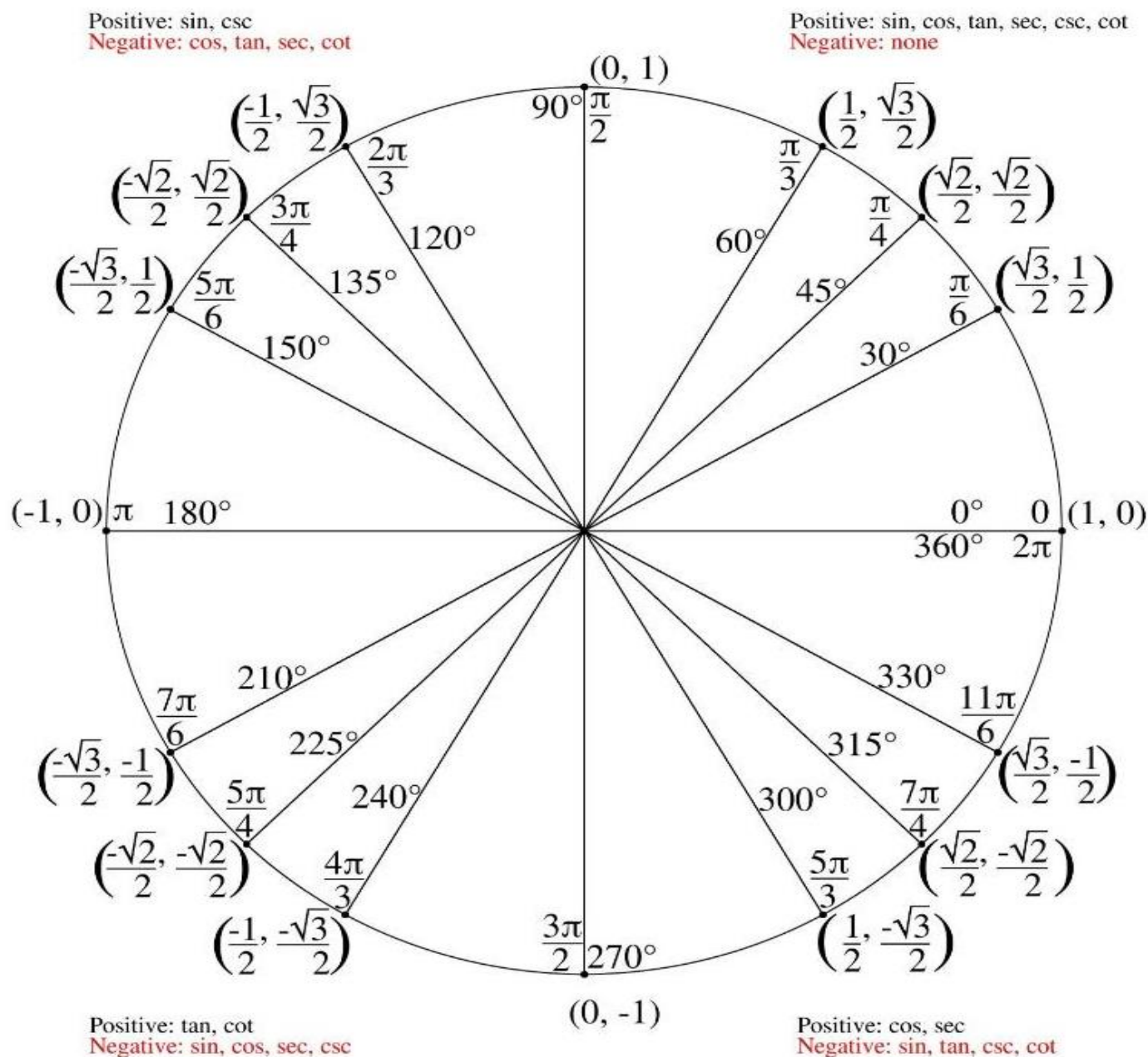


Trigonometric ratios of negative angles

In general

- I.
- II.
- III.

The Unit Circle



EmbeddedMath.com

Use of calculators

Example

Use a calculator to find

I. Tan

Solution

- Key in tan
- Key in 30
- Screen displays 0.5773502
- Therefore $\tan 30^\circ = 0.5774$

To find the inverse of sine cosine and tangent

- Key in shift
- Then either sine cosine or tangent
- Key in the number

Note;

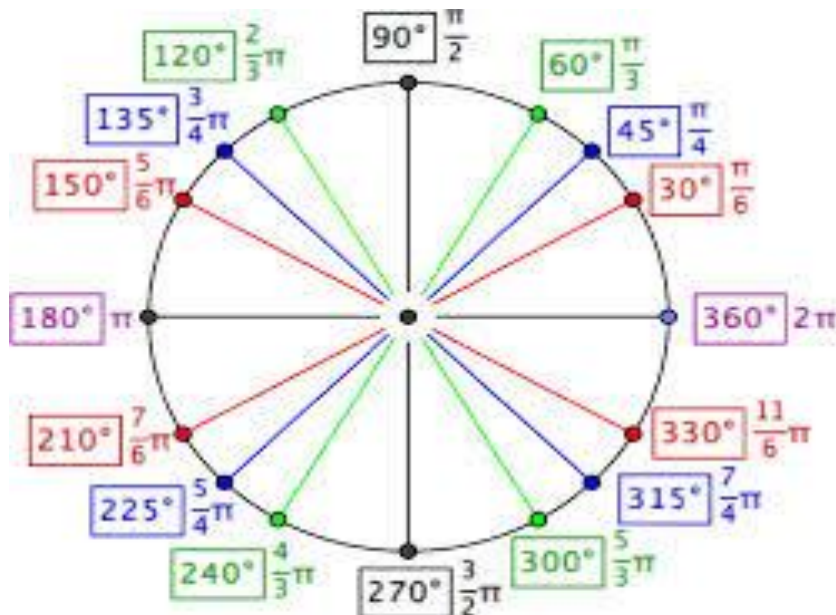
Always consult the manual for your calculator. Because calculators work differently

Radians

One radian is the measure of an angle subtended at the centre by an arc equal in length to the radius of the circle.

Because the circumference of a circle is $2\pi r$, there are 2π radians in a full circle. Degree measure and radian measure are therefore related by the equation $360^\circ = 2\pi$ radians, or $180^\circ = \pi$ radians.

The diagram shows equivalent radian and degree measures for special angles from 0° to 360° (0 radians to 2π radians). You may find it helpful to memorize the equivalent degree and radian measures of special angles in the first quadrant. All other special angles are just multiples of these angles.



Example

Convert into radians

Solution

If $\theta = 57.29$

Therefore $\theta = 2.182$ to 4 S.F

Example

Convert the following degrees to radians, giving your answer in terms

Solution

Therefore

Example

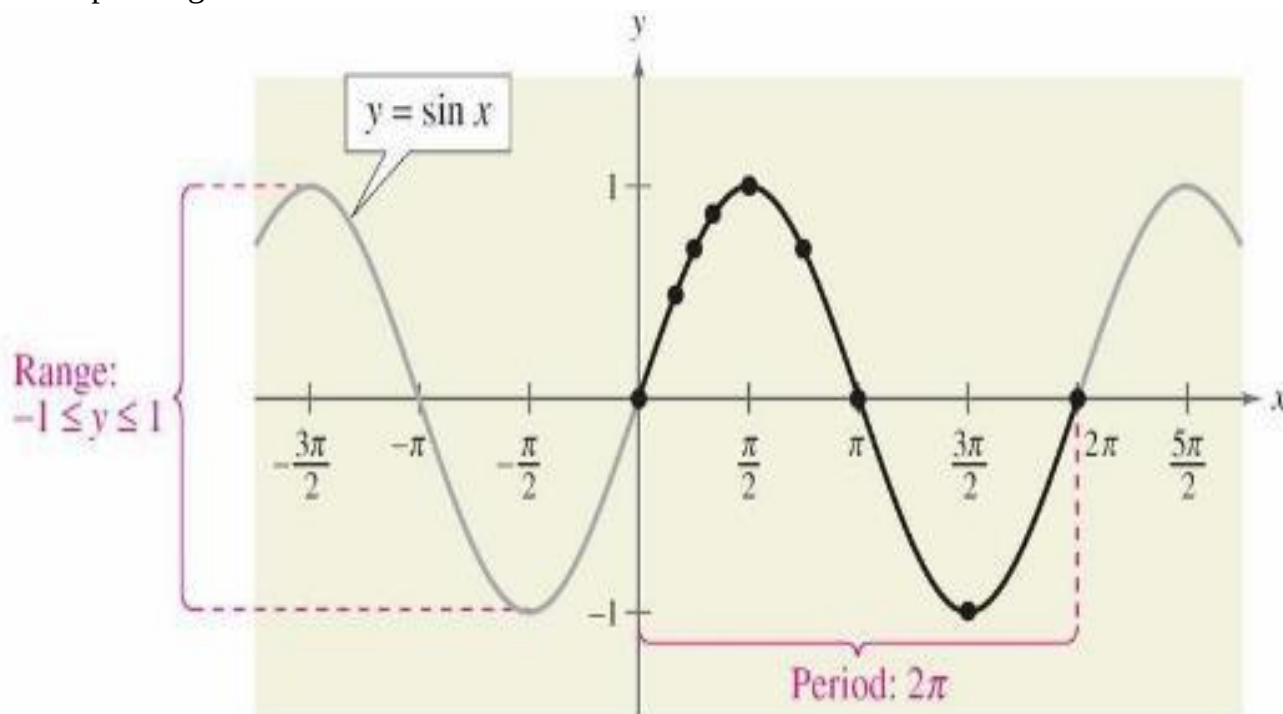
What is the length of the arc that subtends an angle of 0.6 radians at the centre of a circle of radius 20 cm.

Solution

Simple trigonometric graphs

Graphs of $y = \sin x$

The graphs can be drawn by choosing a suitable value of x and plotting the values of y against the corresponding values of x .



The

black portion of the graph represents one period of the function and is called **one cycle** of the sine curve.

Example

Sketch the graph of $y = 2 \sin x$ on the interval $[-, 4]$.

Solution:

Note that $y = 2 \sin x = 2(\sin x)$ indicates that the y -values for the key points will have twice the magnitude of those on the graph of $y = \sin x$.

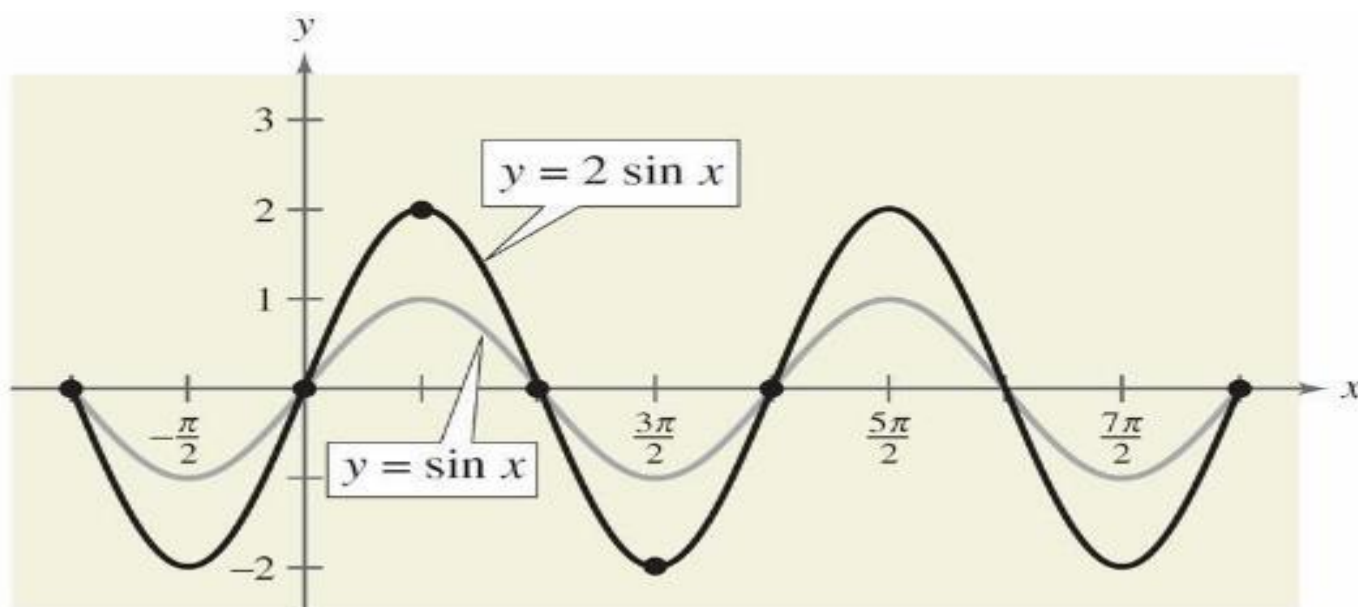
x			3	2
Y=2sin x	2	0	-2	0

To get the values of y substitute the values of x in the equation $y = 2 \sin x$ as follows

$y = 2 \sin (360)$ because 2 is equal to 36

Note;

- You can change the radians into degrees to make work simpler.
- By connecting these key points with a smooth curve and extending the curve in both directions over the interval $[-, 4]$, you obtain the graph shown in below.



Example

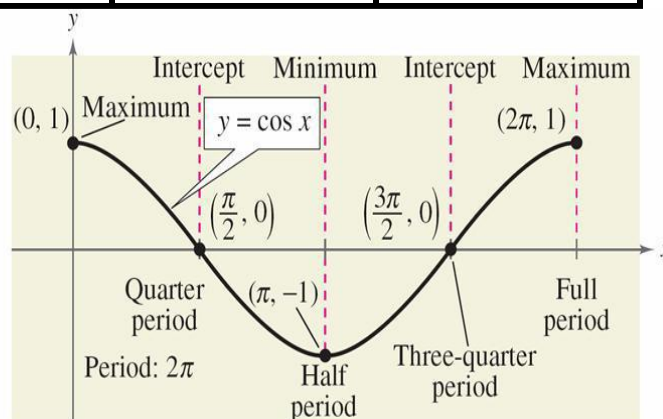
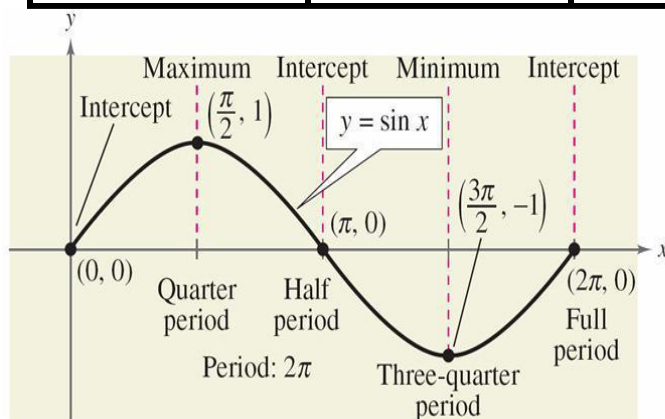
Sketch the graph of $y = \cos x$ for using an interval of

Solution:

The values of x and the corresponding values of y are given in the table below

x									
$Y = \cos x$	1	0.8660	0.5	0	-0.5	-0.8660	-1	-0.8660	-0.5
x									

x				
$Y = \cos x$	0	0.5	0.8660	1



Graph of tangents

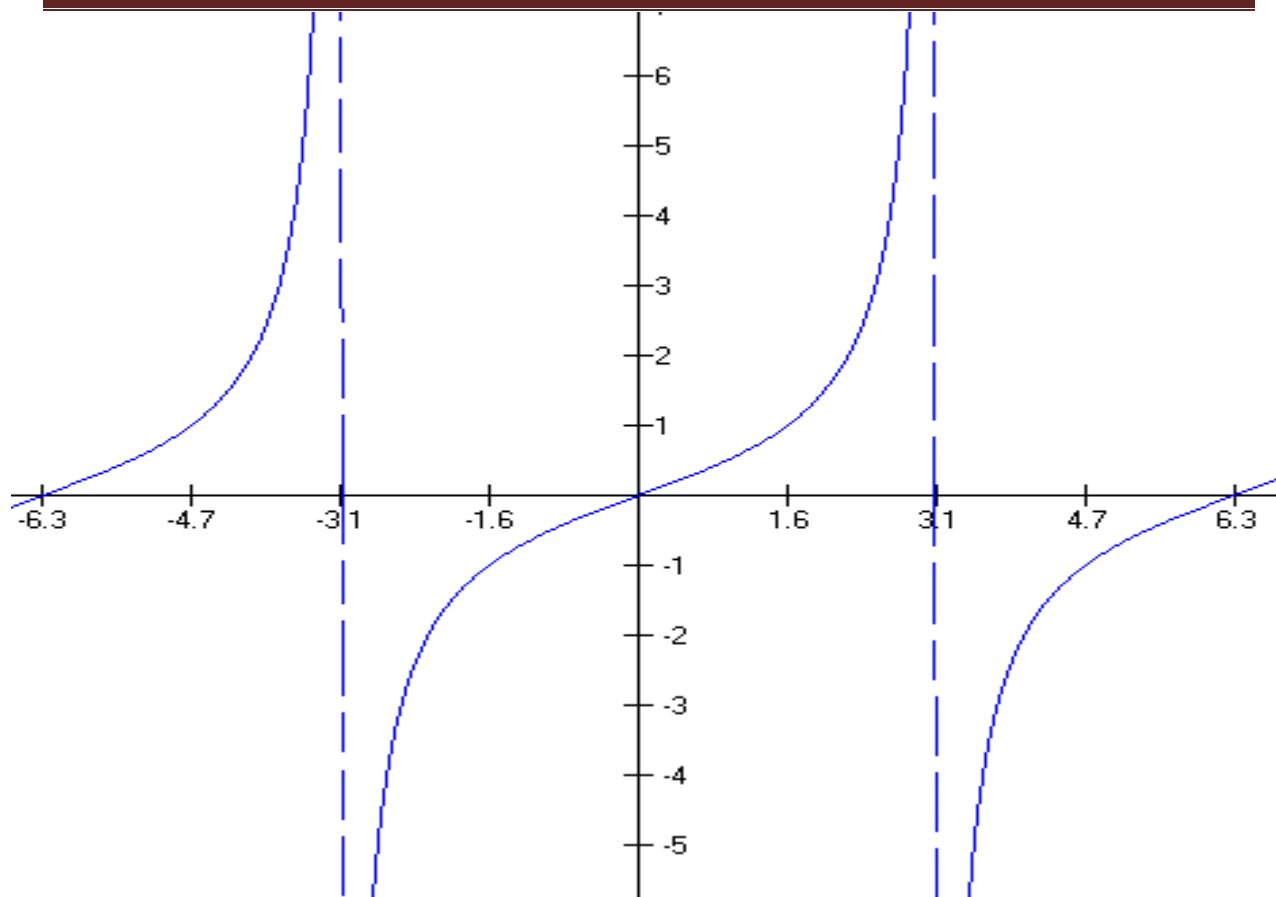
Note;

- As the value of x approaches $\frac{\pi}{2}$ and $27 \tan x$ becomes very large
- Hence the graph of $y = \tan x$ approaches the lines $x = \frac{\pi}{2}$ without touching them.
- Such lines are called asymptotes

Definition of Amplitude of Sine and Cosine Curves

The **amplitude** of $y = a \sin x$ and $y = a \cos x$ represents half the distance between the maximum and minimum values of the function and is given by

$$\text{Amplitude} = |a|.$$

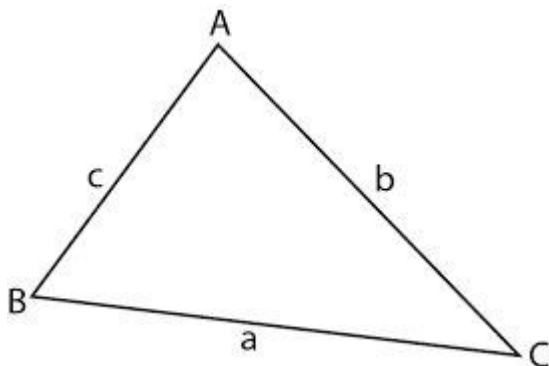


Solution of triangles

Sin rule

If a circle of radius R is circumscribed around the triangle ABC , then $a = 2R \sin A$.

The sine rule applies to both acute and obtuse-angled triangle.

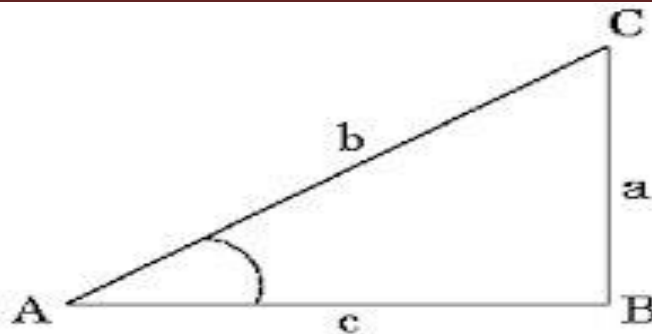


Example

Solve triangle ABC , given that $\angle CAB = 42^\circ$, $c = 14.6$ cm and $a = 11.4$ cm

Solution

To solve a triangle means to find the sides and angles not given



$$\sin c = 0.8720$$

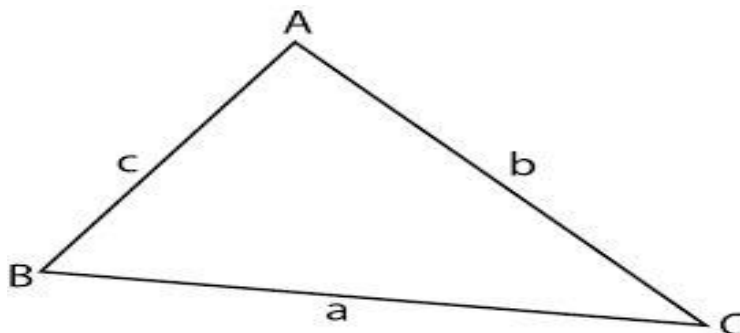
$$\text{Therefore } c = 60.6$$

Note;

The sin rule is used when we know

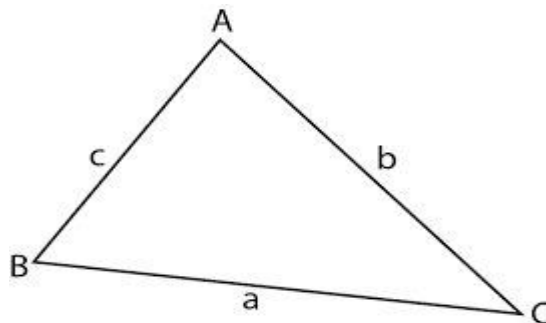
- Two sides and a non-included angle of a triangle
- All sides and at least one angle
- All angles and at least one side.

Cosine rule



Example

Find AC in the figure below, if $AB = 4 \text{ cm}$, $BC = 6 \text{ cm}$ and $\angle ABC = 7^\circ$



Solution

Using the cosine rule

$$\begin{aligned} &= 16 + 36 - 48 \\ &= 52 - 9.979 \\ &= 42.02 \text{ cm} \end{aligned}$$

Note;

The cosine rule is used when we know

- Two sides and an included angle
- All three sides of a triangle

Past KCSE Questions on the topic.

1. Solve the equation

$$\sin \frac{5}{2} \theta = -\frac{1}{2} \text{ for } 0^\circ \leq \theta \leq 180^\circ$$

2. Given that $\sin \theta = \frac{2}{3}$ and θ is an acute angle find:

(a) $\tan \theta$ giving your answer in surd form

(b) $\sec^2 \theta$

3. Solve the 1

equation $2 \sin^2(x-30^\circ) = \cos 60^\circ$ for $-180^\circ \leq x \leq 180^\circ$

4. Given that $\sin(x+30^\circ) = \cos 2x^\circ$ for $0^\circ, 0^\circ \leq x \leq 90^\circ$ find the value of x . Hence find the value of $\cos^2 3x^\circ$.

5. Given that $\sin a = \frac{1}{\sqrt{5}}$ where a is an acute angle find, without using $\sqrt{5}$ Mathematical tables

(a) $\cos a$ in the form of $\frac{a}{b}$, where a and b are rational numbers

(b) $\tan(90^\circ - a)$.

6. Give that x° is an angle in the first quadrant such that $8 \sin^2 x + 2 \cos x - 5 = 0$

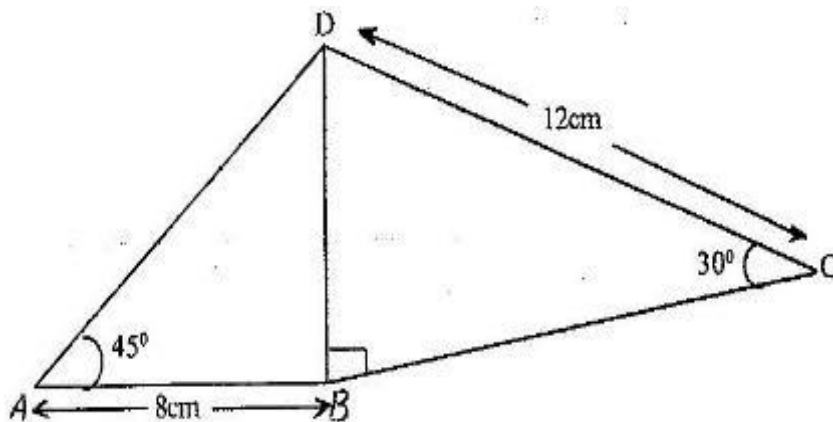
Find:

a) $\cos x$

b) $\tan x$

7. Given that $\cos 2x^\circ = 0.8070$, find x when $0^\circ \leq x \leq 360^\circ$

8. The figure below shows a quadrilateral ABCD in which $AB = 8 \text{ cm}$, $DC = 12 \text{ cm}$, $\angle BAD = 45^\circ$, $\angle CBD = 90^\circ$ and $\angle BCD = 30^\circ$.



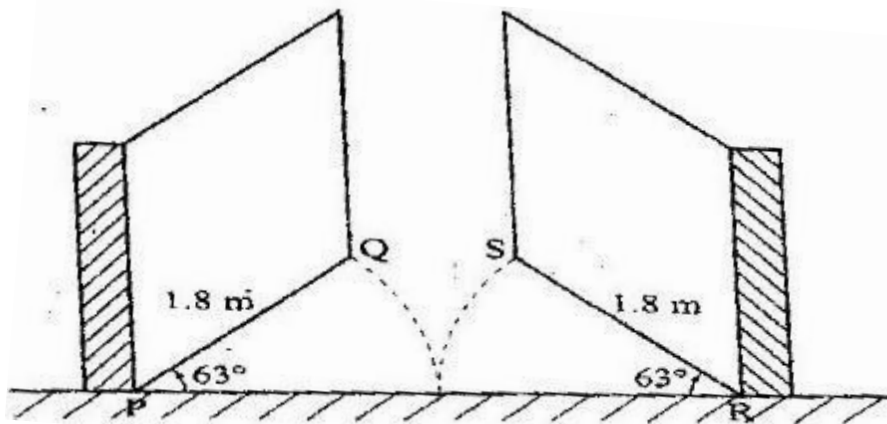
Find:

(a) The length of BD

(b) The size of the angle ADB

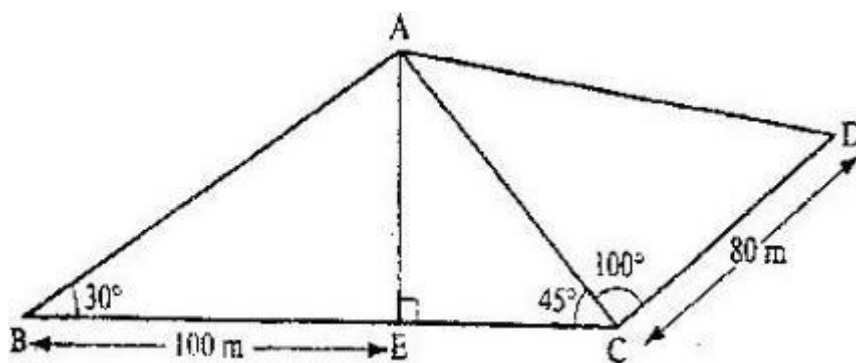
The diagram below represents a school gate with double shutters. The shutters are such opened through an angle of 63° .

The edges of the gate, PQ and RS are each 1.8 m



Calculate the shortest distance QS, correct to 4 significant figures

10...The figure below represents a quadrilateral piece of land ABCD divided into three triangular plots. The lengths BE and CD are 100m and 80m respectively. Angle $\angle ABE = 30^\circ$, $\angle ACE = 45^\circ$ and $\angle ACD = 100^\circ$



(a) Find to four significant figures:

(i) The length of AE

(ii) The length of AD

(iii) The perimeter of the piece of land

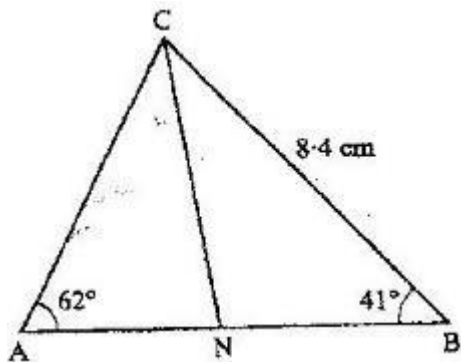
(b) The plots are to be fenced with five strands of barbed wire leaving an entrance of 2.8 m wide to each plot. The type of barbed wire to be used is sold in rolls of lengths 480m. Calculate the number of rolls of barbed wire that must be bought to complete the fencing of the plots.

11. Given that x is an acute angle and $\cos x = \frac{2\sqrt{5}}{5}$, find without using mathematical

5

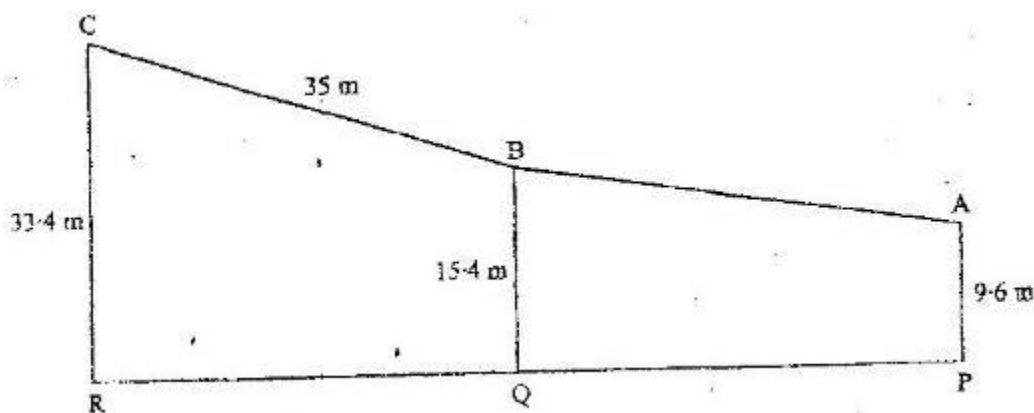
tables or a calculator, $\tan (90 - x)^\circ$.

12. In the figure below $\angle A = 62^\circ$, $\angle B = 41^\circ$, $BC = 8.4$ cm and CN is the bisector of $\angle ACB$.



Calculate the length of CN to 1 decimal place.

13. In the diagram below PA represents an electricity post of height 9.6 m. BB and RC represents two storey buildings of heights 15.4 m and 33.4 m respectively. The angle of depression of A from B is 5.5° While the angle of elevation of C from B is 30.5° and $BC = 35\text{m}$.



- Calculate, to the nearest metre, the distance AB
- By scale drawing find,
 - The distance AC in metres
 - $\angle BCA$ and hence determine the angle of depression of A from C

More questions

1. Solve the equation: (2 mks)

$$\sin \frac{5}{2}X = -\frac{1}{2} \text{ for } 0^\circ \leq X \leq 180^\circ$$

2. (a) Complete the table below, leaving all your values correct to 2 d.p. for the functions $y = \cos x$ and $y = 2\cos(x + 30)^\circ$ (2 mks)

X°	0°	60°	120°	180°	240°	300°	360°	420°	480°	540°
$\cos X$	1.00			-1.00		0.50				
$2\cos(x+30)$	1.73		-1.73		0.00					

- (b) For the function $y = 2\cos(x+30)^\circ$

State:

- The period (1 mk)
- Phase angle (1 mk)

(c) On the same axes draw the waves of the functions $y = \cos x$ and $y = 2\cos(x+30^\circ)$ for $0^\circ \leq x \leq 540^\circ$. Use the scale 1cm rep 30° horizontally and 2 cm rep 1 unit vertically
(4 mks)

(d) Use your graph above to solve the inequality $2\cos(x+30^\circ) \leq \cos x$ (2 mks)

3. Find the value of x in the equation.

$$\cos(3x - 180^\circ) = \frac{\sqrt{3}}{2} \quad \text{in the range } 0^\circ \leq x \leq 180^\circ \quad (3 \text{ marks})$$

4. Given that $\tan \theta = \frac{11}{60}$ and θ is an acute angle, find without using tables $\cos(90^\circ - \theta)$
(2mks)

5. Solve for θ if $-\frac{1}{4} \sin(2x + 30^\circ) = 0.1607$, $0^\circ \leq \theta \leq 360^\circ$ (3mks)

6. Given that $\cos \theta = \frac{5}{13}$ and that $270^\circ \leq \theta \leq 360^\circ$, work out the value of $\tan \theta + \sin \theta$ without using a calculator or mathematical tables. (3 marks)

7. Solve for x in the range $0^\circ \leq x \leq 180^\circ$
(4mks)
 $-8 \sin^2 x - 2 \cos x = -5$.

8. If $\tan x^\circ = \frac{12}{5}$ and x is a reflex angle, find the value of $5\sin x + \cos x$ without using a calculator or mathematical tables

9. Find θ given that $2 \cos 3\theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$

10. Without a mathematical table or a calculator, simplify: $\cos 300^\circ \times \sin 120^\circ$ giving your answer in

$\cos 330^\circ - \sin 405^\circ$ rationalized surd form.

11. Express in surds form and rationalize the denominator.

$$\frac{1}{\sin 60^\circ \sin 45^\circ - \sin 45^\circ}$$

12. Simplify the following without using tables;
 $\tan 45^\circ + \cos 45^\circ \sin 60^\circ$

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CHAPTER FOURTY FOUR

SURDS

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