

Integrals

Exercise

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HiLCoE

Winter 2020

Q. 3: Use Integration by Parts on the following integrals

- a. $\int e^x(x^2 + x + 1)dx$
- b. $\int x^3 \ln x dx$
- c. $\int x^3 e^x dx$
- d. $\int x^3 \cos x dx$
- e. $\int (1 - x^2) \ln x dx$
- f. $\int x^2 4^{3x} dx$
- g. $\int e^x(\sin x + \cos x) dx$

► Answer to Question 3

Q. 4: Use Integration by Partial Fraction Decomposition.

a. $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

b. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$

c. $\int \frac{x^2+1}{x^2(x+2)} dx$

d. $\int \frac{x^2+x-5}{x^2+x-6} dx$

e. $\int \frac{x^2-x-1}{x^3+x^2-6x} dx$

f. $\int \frac{x^3+x+1}{x^2-1} dx$

g. $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

h. $\int \frac{12x^2-2x-9}{(4x^2-1)(x+3)} dx$

Q. 5: Evaluate the following integrals

a. $\int \frac{e^x}{e^{2x} - 4} dx$

b. $\int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx$

c. $\int \sin^3 x dx$

d. $\int \cos^2 x \sin^3 x dx$

e. $\int \frac{1 - \tan x}{1 + \tan x} dx$

► Answer to Q. 5

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Answer to Q. 3

a. In the lecture We have found that $\int xe^x dx = xe^x - e^x + c_1$ and $\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c_2$ Thus,

$$\int e^x(x^2 + x + 1)dx = \int x^2 e^x dx + \int xe^x dx + \int e^x dx$$

Answer to Q. 3

a. In the lecture We have found that $\int xe^x dx = xe^x - e^x + c_1$ and $\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c_2$ Thus,

$$\begin{aligned}\int e^x(x^2 + x + 1)dx &= \int x^2 e^x dx + \int xe^x dx + \int e^x dx \\ &= x^2 e^x - 2xe^x + 2e^x + xe^x - e^x + e^x + c, \quad c =\end{aligned}$$

b. Put

$$u = \ln x, \quad dv = x^3 dx$$

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b. Put

$$u = \ln x, \quad dv = x^3 dx$$

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Then,

$$\begin{aligned} \int x^3 \ln x dx &= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 dx \\ &= \frac{x^4}{4} \ln x - \frac{x^4}{16} + c \end{aligned}$$

c. Put

$$\begin{aligned}u &= x^3, & dv &= e^x dx \\ du &= 3x^2 dx, & v &= e^x.\end{aligned}$$

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Then,

$$\begin{aligned}\int x^3 e^x dx &= x^3 e^x - 3 \int x^2 e^x dx \\ &= x^3 e^x - 3(x^2 - 2x + 2)e^x + c\end{aligned}$$

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We also need to find $\int x^2 \sin x dx$.

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We also need to find $\int x^2 \sin x dx$. Put $u = x^2$ $dv = \sin x dx$
and $du = 2x dx$ $v = -\cos x$.

Then,

$$\begin{aligned} \int x^2 \sin x dx &= -x^2 \cos x + 2 \int x \cos x dx \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x \end{aligned}$$

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Combining the above, we evaluate $\int x^3 \cos x dx$. Put,

$$u = x^3, \quad dv = \cos x dx$$

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Then,

$$\begin{aligned} \int x^3 \cos x dx &= x^3 \sin x - 3 \int x^2 \sin x dx \\ &= x^3 \sin x + x^2 \cos x - 2x \sin x - 2 \cos x + c \end{aligned}$$

e. $\int (1 - x^2) \ln x dx = \int \ln x dx - \int x^2 \ln x dx.$

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$$u = \ln x, \quad dv = x^2 dx \quad \text{and} \quad du = \frac{1}{x} dx, \quad v = \frac{x^3}{3}.$$

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$$\int x^2 \ln x dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

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$$\begin{aligned} \int x^2 \ln x dx &= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx \\ &= \frac{x^3}{3} \ln x - \frac{x^3}{9} \end{aligned}$$

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$$\begin{aligned} \int x^2 \ln x dx &= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx \\ &= \frac{x^3}{3} \ln x - \frac{x^3}{9} \end{aligned}$$

$$\text{Thus, } \int (1 - x^2) \ln x dx = x \ln x - x - \frac{x^3}{3} \ln x + \frac{x^3}{9} + c.$$

f. $\int x^2 4^{3x} dx = \int x^2 (64)^x dx$

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Imitating an example in the lecture,

$$\int x^2 4^{3x} dx = \frac{1}{\ln(64)} x^2 (64)^x - \frac{2}{\ln^2(64)} x (64)^x + \frac{2}{\ln^3(64)} (64)^x + c.$$

g. Using the last example in our lecture on integration by parts,
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$$\int e^x \cos x dx = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x \quad (1)$$

g. Using the last example in our lecture on integration by parts, with $\alpha = \beta = 1$,

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For $\int e^x \sin x dx$, put, $u = e^x$ $dv = \sin x dx$ and
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 $du = e^x dx$ $v = -\cos x$. Then,

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx$$

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For $\int e^x \sin x dx$, put, $u = e^x$ $dv = \sin x dx$ and
 $du = e^x dx$ $v = -\cos x$. Then,

$$\begin{aligned} \int e^x \sin x dx &= -e^x \cos x + \int e^x \cos x dx \\ &= -e^x \cos x + \frac{1}{2}e^x \sin x + \frac{1}{2}e^x \cos x \end{aligned}$$

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For $\int e^x \sin x dx$, put, $u = e^x$ $dv = \sin x dx$ and
 $du = e^x dx$ $v = -\cos x$. Then,

$$\begin{aligned} \int e^x \sin x dx &= -e^x \cos x + \int e^x \cos x dx \\ &= -e^x \cos x + \frac{1}{2}e^x \sin x + \frac{1}{2}e^x \cos x \\ &= \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x \end{aligned} \quad (2)$$

g. Using the last example in our lecture on integration by parts, with $\alpha = \beta = 1$,

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For $\int e^x \sin x dx$, put, $u = e^x$ $dv = \sin x dx$ and
 $du = e^x dx$ $v = -\cos x$. Then,

$$\begin{aligned} \int e^x \sin x dx &= -e^x \cos x + \int e^x \cos x dx \\ &= -e^x \cos x + \frac{1}{2}e^x \sin x + \frac{1}{2}e^x \cos x \\ &= \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x \end{aligned} \quad (2)$$

Equations (1) and (2) give $\int (e^x \sin x + e^x \cos x) dx = e^x \sin x + c$.

► Question 4

Answer to Q. 4

a. Write, $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}.$

Answer to Q. 4

a. Write, $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$.

Adding and equating like terms gives the system of equations

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Adding and equating like terms gives the system of equations

$$\begin{cases} A_1 + A_2 + A_3 = 0 \\ 5A_1 + 4A_2 + 3A_3 = -1 \\ 6A_1 + 3A_2 + 2A_3 = 0 \end{cases}.$$

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a. Write, $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$.

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Solving the system gives

$$A_1 = \frac{1}{2}, A_2 = -2, A_3 = \frac{3}{2} \text{ (Please check!).}$$

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a. Write, $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$.

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$$\int \frac{x}{(x-1)(x-2)(x-3)} dx = \frac{1}{2} \int \frac{1}{x-1} dx - 2 \int \frac{1}{x-2} dx + \frac{3}{2} \int \frac{1}{x-3} dx$$

Answer to Q. 4

a. Write, $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$.

Adding and equating like terms gives the system of equations

$$\begin{cases} A_1 + A_2 + A_3 = 0 \\ 5A_1 + 4A_2 + 3A_3 = -1 \\ 6A_1 + 3A_2 + 2A_3 = 0 \end{cases}.$$

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$$A_1 = \frac{1}{2}, A_2 = -2, A_3 = \frac{3}{2} \text{ (Please check!)}. \text{ Thus,}$$

$$\begin{aligned} \int \frac{x}{(x-1)(x-2)(x-3)} dx &= \frac{1}{2} \int \frac{1}{x-1} dx - 2 \int \frac{1}{x-2} dx + \frac{3}{2} \int \frac{1}{x-3} dx \\ &= \frac{1}{2} \ln |x-1| - 2 \ln |x-2| + \frac{3}{2} \ln |x-3| + C \end{aligned}$$

b. Write, $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A_1}{x + 1} + \frac{A_2}{(x + 1)^2} + \frac{A_3}{x + 3}.$

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b. Write, $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A_1}{x + 1} + \frac{A_2}{(x + 1)^2} + \frac{A_3}{x + 3}.$

Adding and equating like terms gives the system of equations

$$\begin{cases} A_1 + A_3 = 0 \\ 4A_1 + A_2 + 2A_3 = 3 \\ 3A_1 + 3A_2 + A_3 = -2 \end{cases}.$$

b. Write, $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A_1}{x + 1} + \frac{A_2}{(x + 1)^2} + \frac{A_3}{x + 3}$.

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$$\begin{cases} A_1 + A_3 = 0 \\ 4A_1 + A_2 + 2A_3 = 3 \\ 3A_1 + 3A_2 + A_3 = -2 \end{cases}.$$

Solving the system gives

$$A_1 = \frac{11}{4}, A_2 = \frac{-5}{2}, A_3 = \frac{-11}{4} \text{ (Please check!).}$$

b. Write, $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A_1}{x + 1} + \frac{A_2}{(x + 1)^2} + \frac{A_3}{x + 3}.$

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$$\int \frac{3x - 2}{(x + 1)^2(x + 3)} dx = \frac{11}{4} \int \frac{1}{x + 1} dx - \frac{5}{2} \int \frac{1}{(x + 1)^2} dx - \frac{11}{4} \int \frac{1}{x + 3} dx$$

b. Write, $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A_1}{x + 1} + \frac{A_2}{(x + 1)^2} + \frac{A_3}{x + 3}$.

Adding and equating like terms gives the system of equations

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Solving the system gives

$$A_1 = \frac{11}{4}, A_2 = \frac{-5}{2}, A_3 = \frac{-11}{4} \text{ (Please check!)}. \text{ Thus,}$$

$$\begin{aligned} \int \frac{3x - 2}{(x + 1)^2(x + 3)} dx &= \frac{11}{4} \int \frac{1}{x + 1} dx - \frac{5}{2} \int \frac{1}{(x + 1)^2} dx - \frac{11}{4} \int \frac{1}{x + 3} dx \\ &= \frac{11}{4} \ln |x + 1| - \frac{5}{2} \ln(x + 1)^2 - \frac{11}{4} \ln |x + 3| + \end{aligned}$$

c. Write, $\frac{x^2 + 1}{x^2(x + 2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x + 2}$.

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Adding and equating like terms gives the system of equations

$$\begin{cases} A_1 + A_3 = 1 \\ 2A_1 + A_2 = 0 \\ 2A_2 = 1 \end{cases}.$$

c. Write, $\frac{x^2 + 1}{x^2(x + 2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x + 2}$.

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$$\begin{cases} A_1 + A_3 = 1 \\ 2A_1 + A_2 = 0 \\ 2A_2 = 1 \end{cases}.$$

Solving the system gives

$$A_1 = -\frac{1}{4}, A_2 = \frac{1}{2}, A_3 = \frac{5}{4} \text{ (Please check!).}$$

c. Write, $\frac{x^2 + 1}{x^2(x + 2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x + 2}$.

Adding and equating like terms gives the system of equations

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$$A_1 = -\frac{1}{4}, A_2 = \frac{1}{2}, A_3 = \frac{5}{4} \text{ (Please check!)}. \text{ Thus,}$$

$$\int \frac{x^2 + 1}{x^2(x + 2)} dx = -\frac{1}{4} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x^2} dx + \frac{5}{4} \int \frac{1}{x + 2} dx$$

c. Write, $\frac{x^2 + 1}{x^2(x + 2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x + 2}$.

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$$A_1 = -\frac{1}{4}, A_2 = \frac{1}{2}, A_3 = \frac{5}{4} \text{ (Please check!)}. \text{ Thus,}$$

$$\begin{aligned} \int \frac{x^2 + 1}{x^2(x + 2)} dx &= -\frac{1}{4} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x^2} dx + \frac{5}{4} \int \frac{1}{x + 2} dx \\ &= -\frac{1}{4} \ln |x| - \frac{1}{2x} + \frac{5}{4} \ln |x + 2| + c \end{aligned}$$

d. Use long division, and write, $\frac{x^2 + x - 5}{x^2 + x - 6} = 1 + \frac{A_1}{x - 2} + \frac{A_2}{x + 3}$.

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$$\begin{cases} A_1 + A_2 = 0 \\ 3A_1 - 2A_2 = 1 \end{cases}.$$

d. Use long division, and write, $\frac{x^2 + x - 5}{x^2 + x - 6} = 1 + \frac{A_1}{x - 2} + \frac{A_2}{x + 3}$.

Adding and equating like terms gives the system of equations

$$\begin{cases} A_1 + A_2 = 0 \\ 3A_1 - 2A_2 = 1 \end{cases}.$$

Solving the system gives $A_1 = -\frac{1}{5}, A_2 = -\frac{1}{5}$ (*Please check!*).

d. Use long division, and write, $\frac{x^2 + x - 5}{x^2 + x - 6} = 1 + \frac{A_1}{x - 2} + \frac{A_2}{x + 3}$.
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Solving the system gives $A_1 = -\frac{1}{5}, A_2 = -\frac{1}{5}$ (*Please check!*).

Thus,

$$\int \frac{x^2 + x - 5}{x^2 + x - 6} dx = \int dx + \frac{1}{5} \int \frac{1}{x - 2} dx - \frac{1}{5} \int \frac{1}{x + 3} dx$$

d. Use long division, and write, $\frac{x^2 + x - 5}{x^2 + x - 6} = 1 + \frac{A_1}{x - 2} + \frac{A_2}{x + 3}$.
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Solving the system gives $A_1 = -\frac{1}{5}, A_2 = -\frac{1}{5}$ (*Please check!*).

Thus,

$$\begin{aligned} \int \frac{x^2 + x - 5}{x^2 + x - 6} dx &= \int dx + \frac{1}{5} \int \frac{1}{x - 2} dx - \frac{1}{5} \int \frac{1}{x + 3} dx \\ &= x + \frac{1}{5} \ln |x - 2| - \frac{1}{5} \ln |x + 3| + c \end{aligned}$$

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$$A_1 = \frac{1}{6}, A_2 = -\frac{1}{12}, A_3 = \frac{3}{4} (\text{Please check!}).$$

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h. Write, $\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{A_1}{2x - 1} + \frac{A_2}{2x + 1} + \frac{A_3}{x + 3}.$

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$$\begin{cases} 2A_1 + 2A_2 + 4A_3 = 12 \\ 7A_1 + 5A_2 = -2 \\ 3A_1 - 3A_2 - A_3 = -9 \end{cases}.$$

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Thus,

$$\int \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} dx = - \int \frac{1}{2x - 1} dx + \int \frac{1}{2x + 1} dx + 3 \int \frac{1}{x + 3} dx$$

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► Question 5

Answer to Q. 5

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$\int \frac{e^x}{e^{2x} - 4} dx = \int \frac{1}{u^2 - 4} du$. Next apply integration by partial fraction decomposition on the result. you get,

$$\int \frac{e^x}{e^{2x} - 4} dx = \frac{1}{4} \ln(e^x - 2) - \frac{1}{4} \ln(e^x + 2) + c.$$

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$\int \frac{-u}{(u+1)(u-2)} du$. Next apply integration by partial fraction decomposition on the result. you get,

$$\int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx = \frac{1}{3} \ln(1 + \cos x) - \frac{2}{3} \ln |\cos x - 2| + c.$$

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 $\int \sin^3 x dx = \int \sin x(1 - \cos^2 x) dx = -\cos x - \int \cos^2 x \sin x dx.$

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 $\int \sin^3 x dx = \int \sin x(1 - \cos^2 x) dx = -\cos x - \int \cos^2 x \sin x dx$.
Next put $u = \cos x$ and using substitution evaluate, to get
 $\int \sin^3 x dx = -\cos x + \frac{\cos^3 x}{3} + c$

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Put $u = \cos x + \sin x$ and complete.