

Integrals

Exercise

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HiLCoE

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Workout Each Problem

Q. 1: Evaluate each of the following integrals

- a. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$
- b. $\int \frac{x^3 - 4x^2 + 3x - 7}{x^4} dx$
- c. $\int \frac{x^2 - 4}{x + 1} dx$
- d. $\int \frac{(x + 1)^2}{x\sqrt{x}} dx$
- e. $\int \frac{x^4}{x + 3} dx$
- f. $\int \frac{x^6 - 1}{x^3 + 1} dx$

► Answer to Question 1

Q. 2: Use Integration by Substitution on the following integrals

a. $\int \frac{6x - 8}{3x^2 - 8x + 5} dx$

b. $\int \frac{(\ln x)^2}{3x} dx$

c. $\int 6x \sin(x^2 + 1) dx$

d. $\int \frac{\sec^2 x}{5 + \tan x} dx$

e. $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$

f. $\int \frac{e^x}{e^x + 1} dx$

g. $\int x^2(x^3 + 1)^4 dx$

h. $\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx$

► Answer to Question 2

Answer to Q. 1

$$a. \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx = \int x^{\frac{1}{2}} dx - \int x^{\frac{-1}{2}} dx$$

Answer to Q. 1

$$\begin{aligned} a. \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx &= \int x^{\frac{1}{2}} dx - \int x^{\frac{-1}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c \end{aligned}$$

Answer to Q. 1

$$\begin{aligned} a. \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx &= \int x^{\frac{1}{2}} dx - \int x^{\frac{-1}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{2}{3} x^{\frac{3}{2}} - 2\sqrt{x} + c \end{aligned}$$

$$b. \int \frac{x^3 - 4x^2 + 3x - 7}{x^4} dx = \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{3}{x^3} - \frac{7}{x^4} \right) dx$$

$$\begin{aligned} b. \int \frac{x^3 - 4x^2 + 3x - 7}{x^4} dx &= \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{3}{x^3} - \frac{7}{x^4} \right) dx \\ &= \int \frac{1}{x} dx - \int \frac{4}{x^2} dx + \int \frac{3}{x^3} dx - \int \frac{7}{x^4} dx \end{aligned}$$

$$\begin{aligned}
 b. \int \frac{x^3 - 4x^2 + 3x - 7}{x^4} dx &= \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{3}{x^3} - \frac{7}{x^4} \right) dx \\
 &= \int \frac{1}{x} dx - \int \frac{4}{x^2} dx + \int \frac{3}{x^3} dx - \int \frac{7}{x^4} dx \\
 &= \ln |x| + \frac{4}{x} - \frac{3}{2x^2} + \frac{7}{3x^3} + c
 \end{aligned}$$

$$c. \int \frac{x^2 - 4}{x + 1} dx = \int (x - 1) dx - \int \frac{3}{x + 1} dx$$

$$\begin{aligned} c. \int \frac{x^2 - 4}{x + 1} dx &= \int (x - 1) dx - \int \frac{3}{x + 1} dx \\ &= \frac{x^2}{2} - x - 3 \ln |x + 1| + c \end{aligned}$$

$$\begin{aligned} d. \int \frac{(x+1)^2}{x\sqrt{x}} dx &= \int \frac{x^2 + 2x + 1}{x\sqrt{x}} dx \\ &= \int \sqrt{x} dx + 2 \int \frac{1}{\sqrt{x}} dx + \int \frac{1}{x\sqrt{x}} dx \end{aligned}$$

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 d. \int \frac{(x+1)^2}{x\sqrt{x}} dx &= \int \frac{x^2 + 2x + 1}{x\sqrt{x}} dx \\
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 &= \frac{x^2}{2} - x - 3 \ln |x+1| + c
 \end{aligned}$$

$$e. \int \frac{x^4}{x+3} dx = \int \left(x^3 - 3x + 3 - \frac{27}{x+3} \right) dx$$

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$$f. \int \frac{x^6 - 1}{x^3 + 1} dx = \int \frac{(x^3 - 1)(x^3 + 1)}{(x^3 + 1)} dx$$

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 &= \int (x^3 - 1) dx \\
 &= \frac{x^4}{4} - x + c
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► Question 2

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$$\int \frac{(\ln x)^2}{3x} dx = \frac{1}{3} \int \frac{(\ln x)^2}{x} dx$$

b. Put $u = \ln x$ and then $du = \frac{1}{x} dx$.

This gives

$$\begin{aligned} \int \frac{(\ln x)^2}{3x} dx &= \frac{1}{3} \int \frac{(\ln x)^2}{x} dx \\ &= \frac{1}{3} \int u^2 du \end{aligned}$$

b. Put $u = \ln x$ and then $du = \frac{1}{x} dx$.

This gives

$$\begin{aligned}\int \frac{(\ln x)^2}{3x} dx &= \frac{1}{3} \int \frac{(\ln x)^2}{x} dx \\ &= \frac{1}{3} \int u^2 du \\ &= \frac{u^3}{9} + c \\ &= \frac{(\ln x)^3}{9} + c\end{aligned}$$

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e. Put $u = 1 + \sqrt{x}$ and then $du = \frac{1}{2\sqrt{x}}dx$.

This gives

$$\begin{aligned}\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx &= 2 \int \frac{1}{u} du \\ &= 2 \ln |u| + c \\ &= 2 \ln(1 + \sqrt{x}) + c\end{aligned}$$

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$$\begin{aligned}\int x^2(x^3 + 1)^4 dx &= \frac{1}{3} \int u^4 du \\ &= \frac{u^5}{15} + c\end{aligned}$$

g. Put $u = x^3 + 1$ and then $du = 3x^2 dx$.

This gives

$$\begin{aligned}\int x^2(x^3 + 1)^4 dx &= \frac{1}{3} \int u^4 du \\ &= \frac{u^5}{15} + c \\ &= \frac{(x^3 + 1)^5}{15} + c\end{aligned}$$

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$$\begin{aligned} \int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx &= \int \cos u \, du \\ &= 2 \sin u + c \end{aligned}$$

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$$\begin{aligned} \int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx &= \int \cos u \, du \\ &= 2 \sin u + c \\ &= 2 \sin(e^{\sqrt{x}}) + c \end{aligned}$$