ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity.

(1) random and (2) systematic errors.

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.

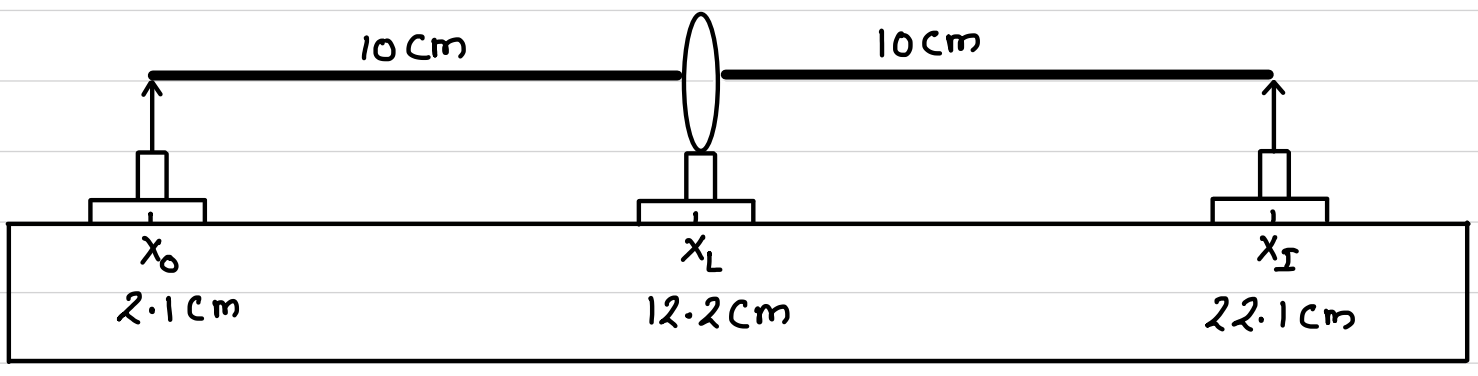
2. Systematic errors occur due to error in the procedure, or miscalibration of the instrument etc. Such errors have same size and sign for all the measurements. Such errors can be determined.

Example of Systematic error →

(i) Zero error → initial reading of machine is not zero

(ii) bench error → occurs in experiments involve bench like optical bench

Actual Reading = observed Reading − zero error or bench error



$B.E_1 = 10.1 - 10 = 0.1\ cm$

$B.E_2 = 9.9 - 10 = -0.1\ cm$

Random error →

$X = x \pm \Delta x$ → value of x lies between $x - \Delta x$ and $x + \Delta x$

Δx → Absolute error

x → main value

$\frac{\Delta x}{x}$ → fractional error or relative error

$\frac{\Delta x}{x} \times 100\%$ → percentage error

Δx →
- 1st — Given in question like $(2 \pm .1)$ m
- 2nd — many value of x is given then Δx is Calculated by Average method
- 3rd — Δx = least Count of device

Table

12, 13, 12, 14, 13, 18, 12, 13

	Value x	(deviation) $\Delta x = x - x_A$	$\lvert\Delta x\rvert$	
1.	12	−0.8	0.8	
2.	13	+0.2	0.2	$x = 12.8 \pm 0.6$
3.	12	−0.8	0.8	
4.	14	+1.2	1.2	
5.	13	+0.2	0.2	
	$\Sigma x = 64$		$\Delta x_A = \frac{3.2}{5}$	
	$x_A = 12.8$ ↑ Avg.		$= .64$	

Mathematical operations

(i) Addition / Subtraction

$$X = x \pm \Delta x \quad , \quad Y = y \pm \Delta y$$

$$X + Y = (x+y) \pm (\Delta x + \Delta y)$$

$$X - Y = (x-y) \pm (\Delta x + \Delta y)$$

Ex. $X = 10 \pm 0.1$ $\quad Y = 6 \pm 0.1$

$$X+Y = 16 \pm 0.2 \quad , \quad X-Y = 4 \pm 0.2$$

(ii) Product / division / Power

$$gz^2 = \frac{17xy^3}{t^4}$$

$$\Rightarrow \log g + 2\log z = \log 17 + \log x + 3\log y - 4\log t$$

$$\Rightarrow \frac{2dz}{z} = \frac{dx}{x} + \frac{3dy}{y} - \frac{4dt}{t}$$

$$\Rightarrow \frac{2\Delta z}{z} = \frac{\Delta x}{x} + \frac{3\Delta y}{y} - \frac{4\Delta t}{t}$$ for fractional change

$$\frac{2\Delta z}{z} = \frac{\Delta x}{x} + \frac{3\Delta y}{y} + \frac{4\Delta t}{t}$$ for error Calculation

(iii) every other mathematical function → direct differentiate

Like $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{v^2}dv + \frac{1}{u^2}du = -\frac{1}{f^2}df$

$$\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$$

Similarly $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}$, $x = d\sin\theta$

main value → main value

Q.1 $\ell = (1 \pm .02)\ m$ $\quad g = \pi^2 (1 \pm .01)\ m/s^2$

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2\ sec.$$

$$\frac{\Delta T}{T} = \frac{\Delta \ell}{2\ell} + \frac{\Delta g}{2g} \Rightarrow \frac{\Delta T}{2} = \frac{.02}{2\times 1} + \frac{.01\pi^2}{2\pi^2}$$

$$\Delta T = 0.03$$

$$T = 2 \pm .03\ sec.$$

Q.2 $R_1 = (3 \pm .1)\ \Omega$ $\quad R_2 = (6 \pm .1)\ \Omega$ find R_{eq} in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = 2\Omega$$

$$\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \Rightarrow \frac{\Delta R_{eq}}{4} = \frac{.1}{9} + \frac{.1}{36} \Rightarrow \Delta R_{eq} =$$

Q.3 $x = a \sin\theta$ $\quad a = (4 \pm .1)mm$, $\theta = 30^\circ \pm 1^\circ$

$$x = 4 \sin 30^\circ = 2\ mm$$

$$\Delta x = a\cos\theta(\Delta\theta) + \sin\theta(\Delta a)$$

$$\Delta x = 4\cos 30^\circ\left(1^\circ \times \frac{\pi}{180^\circ}\right) + \sin 30^\circ(.1) =$$

$$\frac{\Delta x}{x} = \cot\theta\,\Delta\theta + \frac{\Delta a}{a} \qquad \{\ \ln x = \ln a + \ln\sin\theta\ \}$$

Round off

→ for round off of Single digit

> 5 → add 1 in next digit
< 5 → no effect
$= 5$ → add 1 if next digit is odd
→ no effect if next digit is even

→ for 2 digit Compare with 50 and So on

Ex.

3.153 → 3.15

3.153 → 3.2

3.15 → 3.2

3.25 → 3.2

3.25 → 3.0

3.253 → 3.3

3.35 → 3.4

3.35 → 3.0

Significant digit →

1. All nonzero digits (1-9) are to be counted as significant.
2. Zeros that have any nonzero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (2) above are NOT be considered significant digits.

4. No change in Significant digits due to unit change

Ex. 00400 km → 3

8902 → 4

0.04 → 1

1.04 → 3

3 km → 1

3.000 km → 4

008 km → 1

1.0400 → 5

0.40 → 2

$9\,km = 9 \times 10^3\,m = 0.9 \times 10^4\,m = 9 \times 10^5\,cm$

$(9\,km \neq 9000\,m \neq 9.0 \times 10^3)$

9.000 km = 9000 m → 9×10^3 m ✗

→ 9.000×10^3 m ✓

Mathematical operation

(i) Add / Subtract → first round off all inputs according to Last Common place then execude operation

Ex. $21.7 + 1.372 + 0.52 + 0.007$

$$\begin{array}{r} 21.7 \\ 1.372 \\ .52 \\ .007 \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 21.7 \\ 1.4 \\ .5 \\ .0 \\ \hline 23.6 \end{array}$$

(ii) Any other mathematical function → first execute operation then round off answers according to least Significant digits input

Ex. $\dfrac{15.6 \times 3.1}{3.0} = 16.12 \longrightarrow 16$

(15.6: ↓3, 3.1: ↓2, 3.0: ↑2)

$\dfrac{15.6 \times 3.1}{3} = 16.12 \longrightarrow 2 \times 10^1$ $\neq 20$

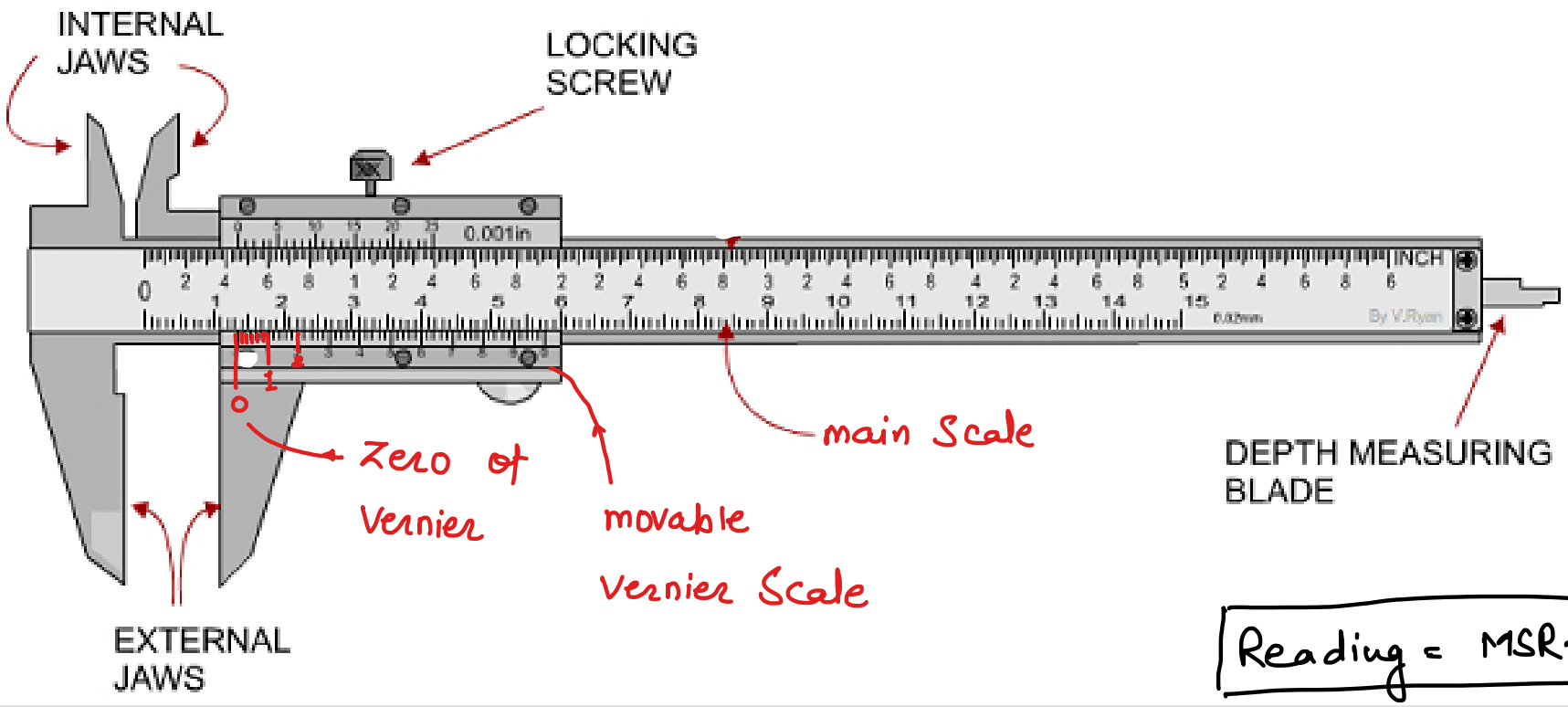
$(5)^2 = 25 \longrightarrow 2 \times 10^1$

$(5.0)^2 = 25 \rightarrow 25$

$\sqrt{10} = 3.162 \longrightarrow 3.2$

Experiments →

Vernier Caliper →



Reading = MSR + VSR

Observed Reading (OR) = Main Scale reading (MSR) + vernier Scale reading (VSR)

MSR → if Zero of vernier Scale is between x^{th} and $(x+1)^{th}$ division of main Scale then

MSR = x (MSd) → main Scale division

VSR → if y^{th} division of vernier Scale Concide with any main Scale division then

VSR = y (LC)

Let m divisions of MS are equal to n divisions of VS

$m(msd) = n(vsd) \Rightarrow vsd = \frac{m}{n}(msd)$

$LC = msd - vsd$

LC is minimum when m and n are differ by 1

Ex.1 Calculate least count (LC = msd - vsd)

(i) 10 VSD = 9 MSD 1 MSD = 1 mm

(ii) 20 VSD = 19 MSD 1 MSD = 1 mm

(iii) 30 VSD = 29 MSD 1 MSD = 1°

Sol^n

(i) $1\ VSD = \frac{9}{10}\ MSD = 0.9\ mm$

$LC = 1 - 0.9 = 0.1\ mm$

(ii) $1\ VSD = \frac{19}{20}\ MSD = \frac{19}{20}\ mm$

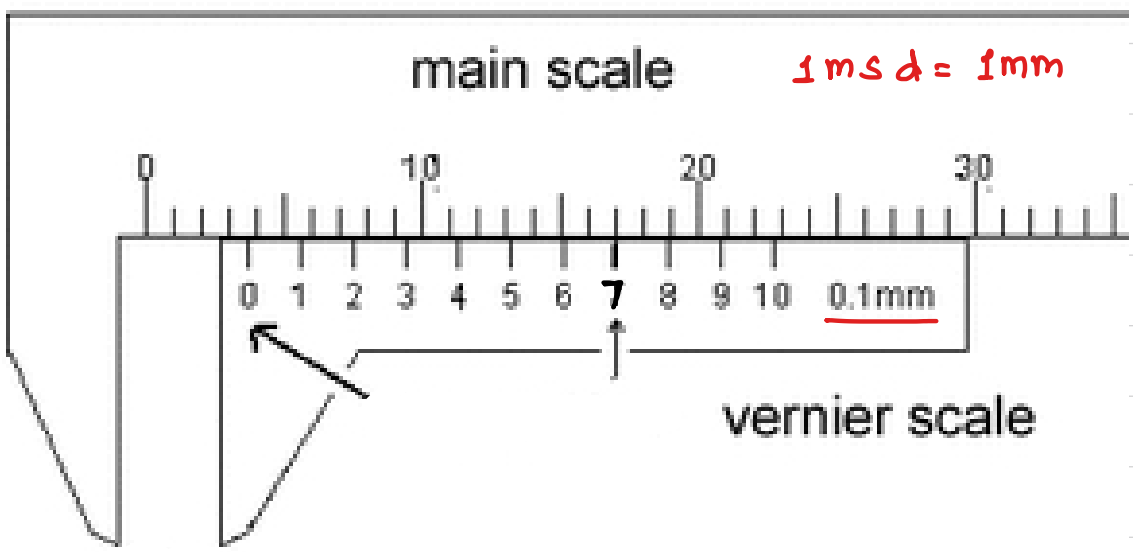
$LC = 1 - \frac{19}{20} = \frac{1}{20}\ mm = 0.05\ mm$

(iii) $1\ VSD = \frac{29}{30}\ MSD = \frac{29}{30}^{\circ}$

$LC = 1^{\circ} - \frac{29}{30}^{\circ} = \frac{1}{30}^{\circ}$

10 vsd = 9 msd

Ex.



Reading = MSR + VSR

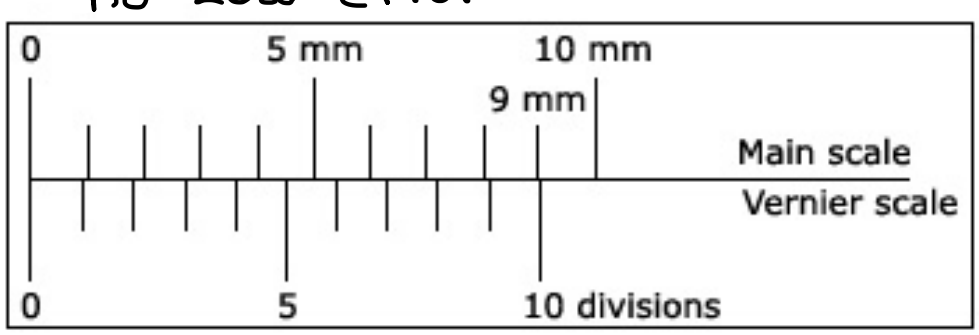
MSR → 3(1mm) = 3mm , VSR = 7(0·1) = 0·7 mm

OR = 3 + 0·7 = 3·7 mm

Zero Error (Z.E.) →

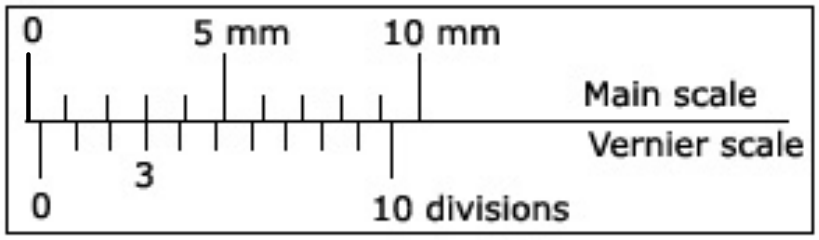
When both jaws are closed and zero of main Scale and Vernier are not Concide

no Zero error



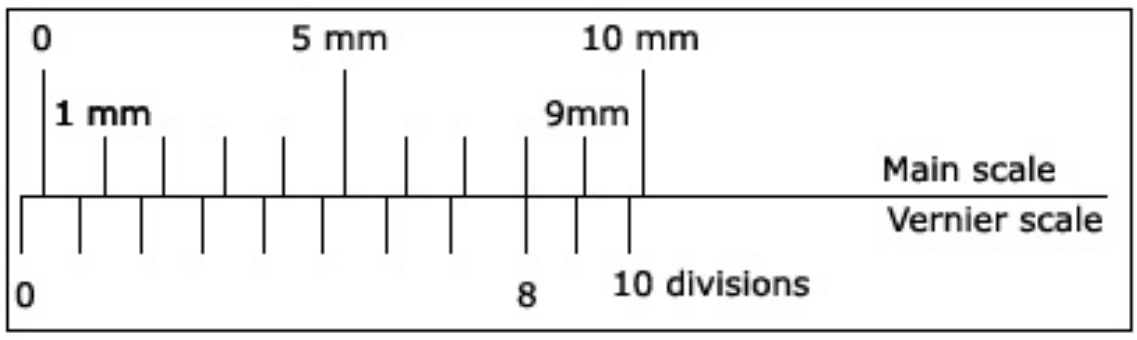
Z.E. = 0

Positive Error



Z.E. = 0 + 3(0.1) = 0.3

Nagative Error



Z.E. = −1 + 8(.1) = −0.2

+ive Zero error → Zero of vernier is in right of zero of main Scale

$$Z.E = 0 + y(L.C.)$$

−ive Zero error → Zero of vernier is in left of zero of main Scale

$$Z.E = -(msd) + y(L.C.)$$

Q. Vernier Caliper → 10 vsd = 9 msd , 1 msd = 1 mm

✓ When both jaws are closed Zero of vernier Scale is in Right of main scale zero with 7th division Concide

✓ When A Cylinder is placed b/w jaws, Vernier Zero is in left of 12.4 cm with 5th division Concide

find diameter and Radius of Cylinder

Sol^n L.C. = 0.1 mm $\left\{ 1\,\text{vsd} = \frac{9}{10}\,\text{msd} = 0.9\,\text{mm} \quad LC = 1 - 0.9 = 0.1\,\text{mm} \right\}$



Z.E. = 0 + 7(0.1) = 0.7

OR = 123 + 5(0.1) = 123.5

AR = 123.5 − 0.7 = 122.8 mm = 12.28 cm

$d = (12.28 \pm .01)$ cm

$R = \frac{d}{2} = (6.14 \pm .005)$ cm

Q. for vernier caliper → 10 VSD = 11 MSD , 1 MSD = 1 mm

Zero of vernier is b/w 8.4 cm and 8.5 cm with 3rd division Concide find Reading (R)

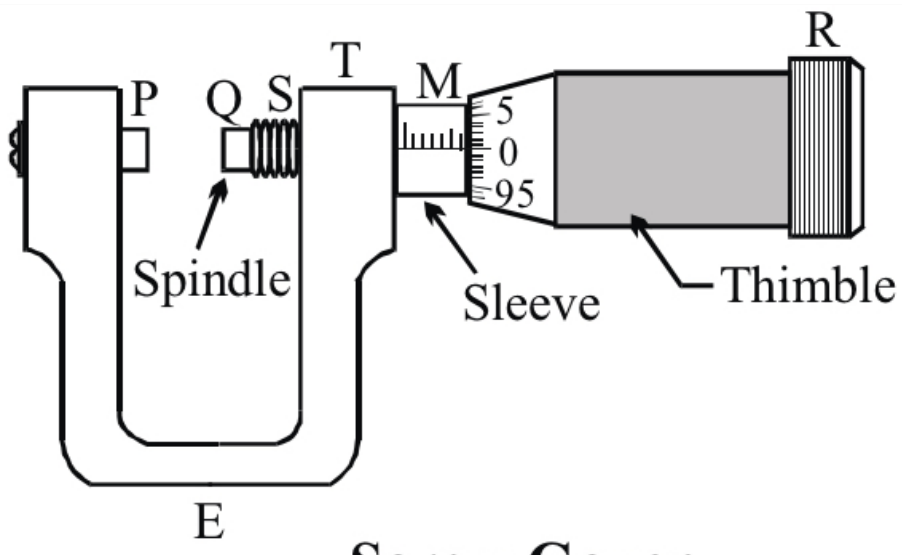
Sol^n $1\ VSD = \frac{11}{10}\ MSD = 1.1\ mm$

$L.C. = 1 - 1.1 = -0.1\ mm$

Reading $R = 84 + (10-3)(0.1) = 84.7\ mm$

$R = 85 + 3(-0.1) = 84.7\ mm$

Screw gauge →



Screw Gauge

Observed Reading = MSR + CSR

MSR → if last visible main scale division is x then

$$MSR = x\,(Msd)$$

CSR → if y^{th} division of Circular Scale is Concide with main Scale line

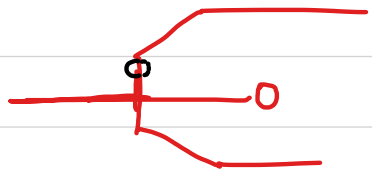
$$CSR = y\,(L.C.)$$

$$L.C. = \frac{\text{Pitch}}{\text{no. of divisions of Circular Scale}}$$

Pitch → distance moved by Thimble in one Complete rotation

Generally pitch and msd are same but they may be different if given in question

Zero error →



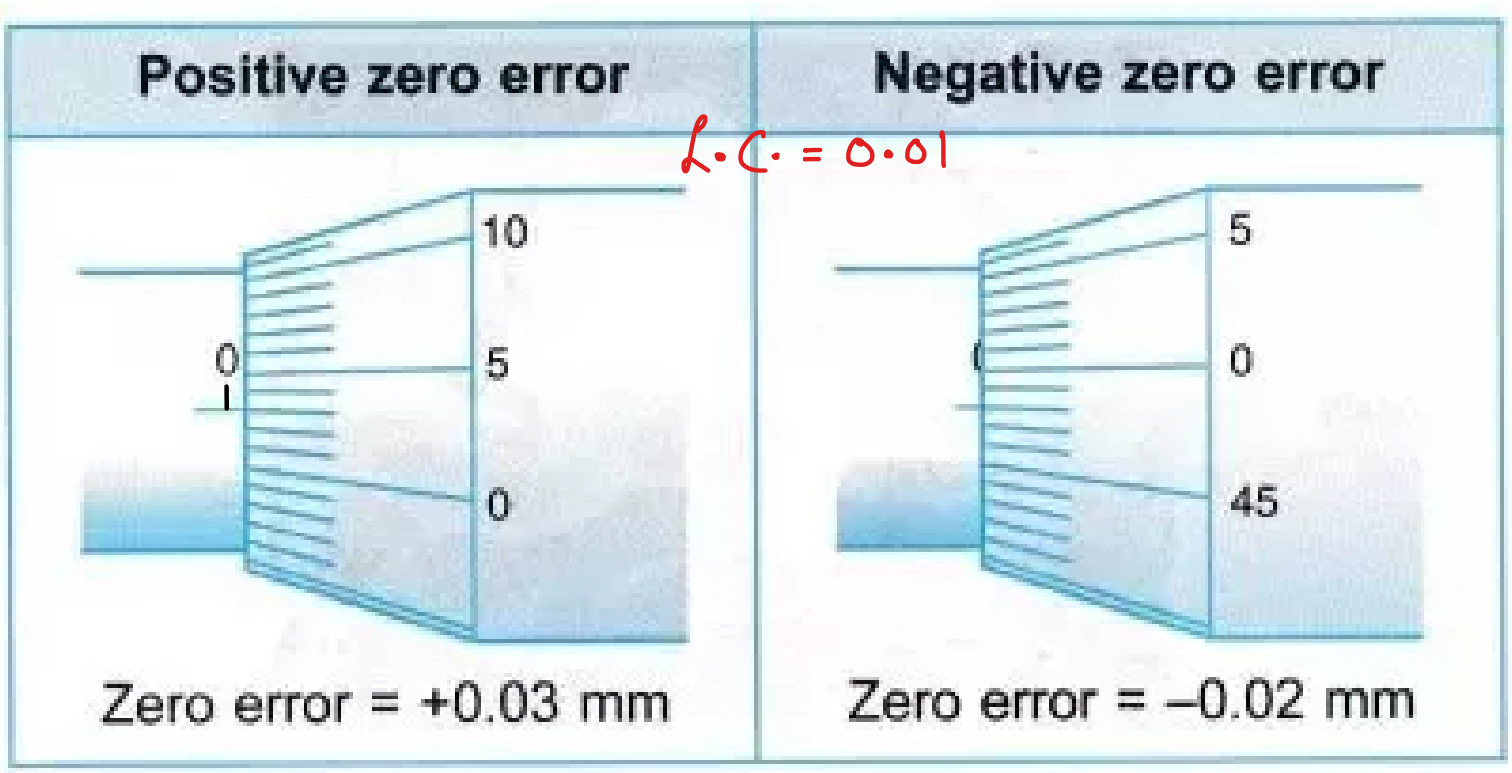
no Z.E.

+ive Zero error → When Jaws are closed and Zero of main Scale is clearly visible

$$Z.E. = 0 + y(L.C.)$$

-ive Zero error → When Jaws are closed and Zero of main Scale is not visible or barely visible

$$Z.E. = -Msd + y(L.C.)$$



$Z.E. = 0 + 3(.01)$

$= 0.03$ mm

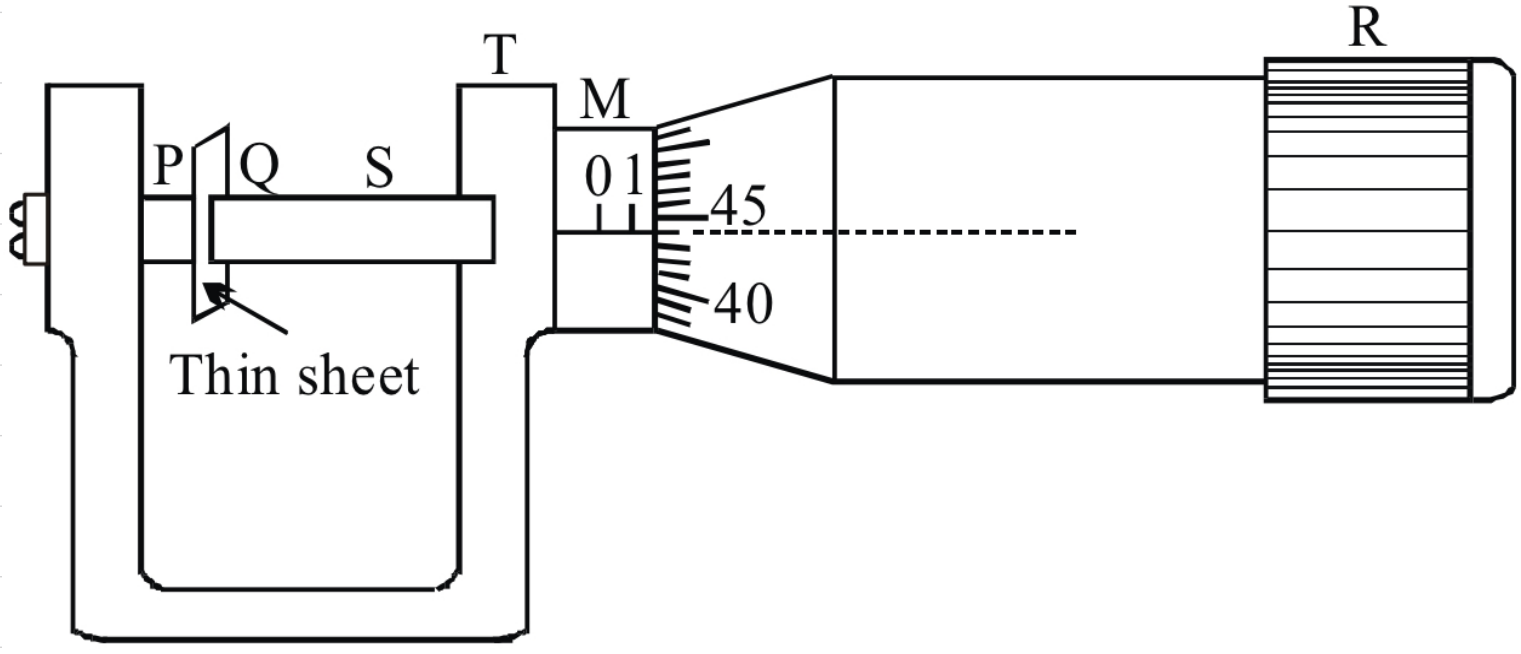
$Z.E. = -1 + (48)(.01)$

$= -0.02$ mm

Q. find Reading (i) 1 msd = 1 mm , Pitch = 1 mm
Circular divisions = 100

(ii) 1 msd = 1 mm , Pitch = 2 mm
Circular divisions = 100

(iii) 1 msd = 0.5 mm , Pitch = 0.5 mm
Circular divisions = 50



Soln (i) $LC = \frac{1}{100} = 0.01$ mm

$R = 1 + 44(0.01) = 1.44$ mm

(ii) $LC = \frac{2}{100} = 0.02$ mm

$R = 1 + 44(.02) = 1.88$ mm

(iii) $LC = \frac{0.5}{50} = 0.01$

$R = 0.5 + 0.01(44) = 0.94$ mm

Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad g = \frac{4\pi^2 l}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \longrightarrow \frac{2\Delta T_0}{T_0}$$

$T \rightarrow$ Time period

Let n oscillations take T_0 time with clock of least Count ΔT_0

then $T = T_0/n$ and $\Delta T = \Delta T_0/n$

for good results, n should be high ✓
ΔT_0 should be low
θ_0 should be low ✓

$\theta_0 \rightarrow$ angular amplitude

Searl's experiment → To Calculate young's modulus of wire

$$\text{Stress} = Y \times \text{Strain} = \frac{Mg}{A} = Y\frac{l}{L}$$

$$\boxed{Y = \frac{4MgL}{\pi d^2 l}}$$

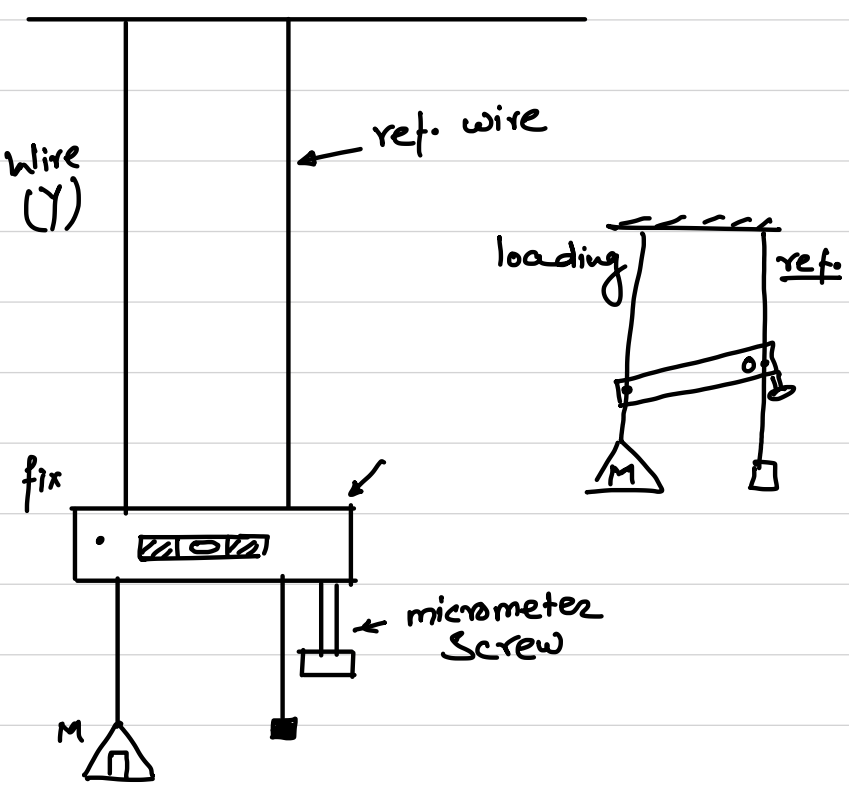
$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

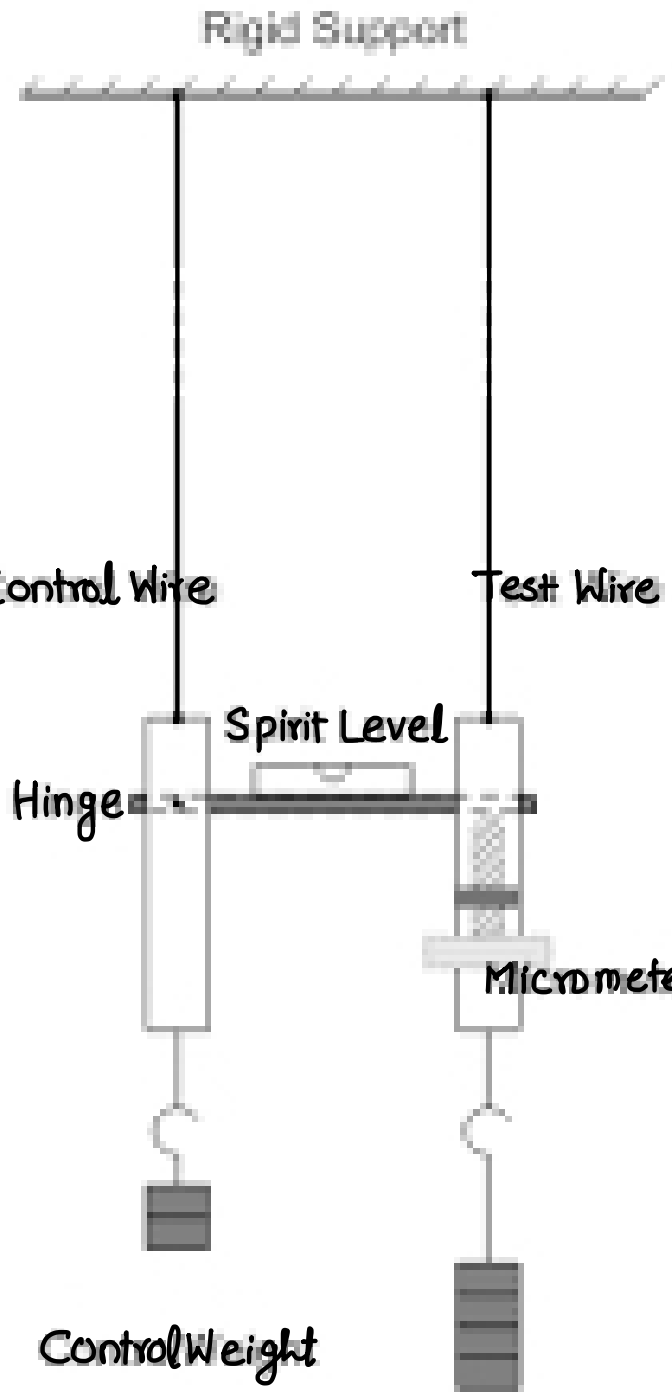
$d \rightarrow$ diameter (Reading of Screw gauge)

$\Delta d \rightarrow$ LC of Screw gauge

$l \rightarrow$ Change in length

$\Delta l \rightarrow$ least Count of micrometer





for resonance tube

$$V = 2f(l_2 - l_1) \quad \text{and} \quad e = \frac{l_2 - 3l_1}{2}$$

$$\frac{\Delta V}{V} = \frac{\Delta f}{f} + \frac{\Delta(l_2 - l_1)}{l_2 - l_1} = \frac{\Delta f}{f} + \frac{\Delta l_2 + \Delta l_1}{l_2 - l_1}$$