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Electrical Circuits & Networks

(Theory with worked out Examples, Self and Classroom Practice Questions)

PSUs Electrical Engineering

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Electrical Circuits & Networks Syllabus

Kirchhoff's laws, mesh and nodal analysis, network theorems, sinusoidal steady state analysis of single phase and three phase circuits, resonance, transient response of RL,RC,RLC circuits for different inputs, two-port networks, Two element network synthesis

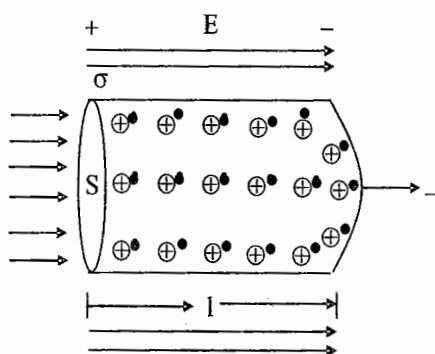
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Georg Simon Ohm
(1789 – 1854)

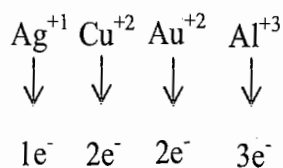
1.1 Mechanism of Electrical Energy Flow through the Conductor and the Ohm's Law



$\oplus \Rightarrow \text{Ag}^+$ ion, immobile, larger in size

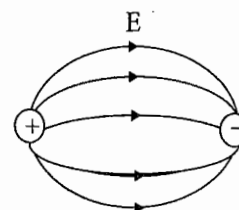
$\bullet \Rightarrow \text{Free electron}$

- The four best conductors are



- Silver is the best conductor because mobility of the free electron in a silver conductor is several times to that of other conductors, so its conductivity is very high i.e. the conductivity of any material is mainly a function of free electrons mobility within that material.
- Generally in any conductor, there are 10^{18} to 10^{23} atoms/unit volume (i.e. unit cell) and hence, there are 10^{18} to 10^{23} free electrons/unit volume in a silver conductor i.e. every conductor is very rich in free electrons.

- Whenever generators are connected to the transmission lines (basically conductors), electrical energy will be launched axially, which results in axial electric field (a perpendicular magnetic field) and hence, axial current density along the length of the conductor.



An electric dipole

- When a conductor is subjected to an axial electric field, the force exerted by the field on each free electron is given by

$$\vec{F} = \vec{E} \cdot e \text{ (N)}$$

$$e = -1.6 \times 10^{-19} \text{ C}$$

- Since 'e' is negative, the direction of force is in the direction opposite to that of 'E' and hence, there exists a net electron motion (i.e. charge motion) in the direction opposite to that of 'E'.
- The amount of charge flowing is given by

$$q = ne \text{ C}$$

$$\text{i.e. } q(t) = n(t) \cdot e \text{ C}$$

n = The number of free electrons crossing a reference cross section area 'S', a variable quantity due to large number of free electrons within the conductor (so called random variable)

- The time rate of flow of these electric charges (i.e. free electrons) is nothing but the electric current i.e.

$$i = \frac{dq}{dt} \text{ A}$$

Georg Simon Ohm found that there is a direct proportionality between the potential difference (voltage) applied across a conductor and the resultant electric current. This relationship is known as Ohm's law.



Since 'q' is negative, the current direction is opposite to that of charge motion (i.e. free electrons motion) i.e. in the direction of 'E'.

- The current per unit cross section area is nothing but the current density resulted within the conductor i.e.

$$J = \frac{i}{S} \left(\frac{A}{m^2} \right)$$

Since 'S' is a scalar, the direction of 'J' is in the direction of 'i' i.e. in the direction of 'E'.

$$\vec{F} = \vec{E} \cdot e (N)$$

$$q = n \cdot e (C)$$

$$i = \frac{dq}{dt} (A)$$

$$J = \frac{i}{S} \left(\frac{A}{m^2} \right)$$

Observation:

- As $E \uparrow \Rightarrow F \uparrow \Rightarrow n \uparrow \Rightarrow q \uparrow \Rightarrow i \uparrow \Rightarrow J \uparrow$ linearly
- As $E = 0 \Rightarrow F = 0 \Rightarrow n = 0 \Rightarrow q = 0 \Rightarrow i = 0 \Rightarrow J = 0$.
i.e., all the free electrons are at rest.

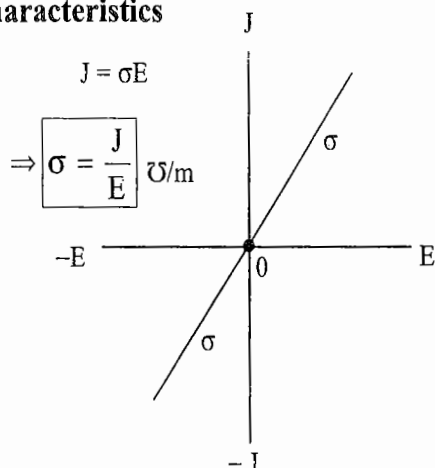
Definition of Ohm's Law:

According to Ohm, there exists a linear relation between the applied electric field (E) and the resulted current density (J) by

$J \propto E \Rightarrow J = \sigma E \rightarrow$ Basic Ohm's law in Field theory form

σ = Conductivity of the material i.e. conductor.

J - E Characteristics



Limitation:

Ohm's law i.e a linear J-E relation is valid only when the proportionality constant σ is kept constant i.e temperature is kept constant.

Example: 1.1

What happens to the conductivity σ of all the conductors, if the temperature increases beyond room temperature?

Sol: When the temperature increases from room temperature (i.e. $27^{\circ}C$ or $300^{\circ}K$), free electrons will acquire extra thermal energy at higher temperatures, which results increase in collisions among the free electrons and hence, net free electron mobility (i.e. carrier mobility) falls, so conductivity decreases.

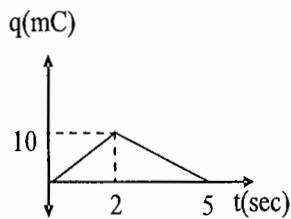
Example: 1.2

Why voltage drop occurs in any practical conductor whenever it carries electrical energy?

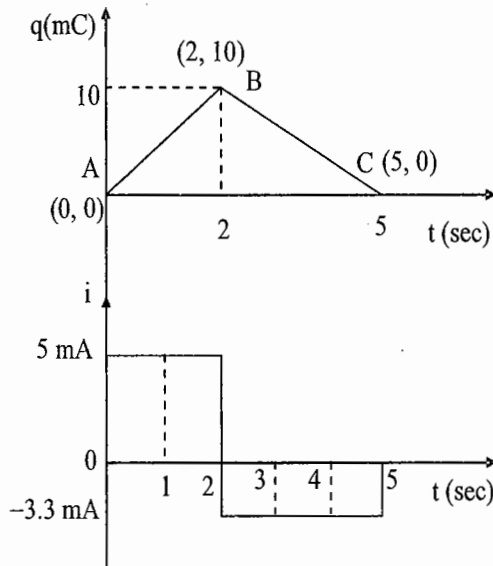
Sol: As 'E' increases, there exists an increase in collisions between the free electrons and immobile positive ions (larger in size), which results fall in the drift velocity ($v_d = \mu E$ m/s, μ = average mobility of free electrons in $m^2/V-S$, E = external electric field in V/m) and hence loss in kinetic energy ($K.E = \frac{1}{2} m v_d^2$ (J), $m = 9.11 \times 10^{-31}$ kg), this loss in energy will be dissipated in the form of heat energy, which results in voltage drop across the conductor.

Example: 1.3

The charge delivered by a constant voltage source is shown. Determine the current supplied by the source at a) $t = 1$ sec b) $t = 3$ sec



Sol:



$$i = \frac{dq}{dt}$$

$$\text{Slope of AB} = \frac{10}{2} = 5 \text{ mA} \quad [t = 1 \text{ to } 2 \text{ sec}]$$

$$\text{Slope of BC} = \frac{10-0}{2-5} = -3.33 \text{ mA} \quad [t = 2 \text{ to } 5 \text{ sec}]$$

- Amount of power dissipated within the conductor

$$P = J \cdot E \text{ (A/m}^2 \cdot \text{V/m)}$$

$$P = \sigma E \cdot E \frac{\text{AV}}{\text{m}^3}$$

$$P = \sigma E^2 \text{ W/m}^3$$

P = Power density i.e. power dissipated per unit volume (unit cube) within the conductor due to resistance effect of the conductor.

Due to this effect, the conductor gets heated up.

Observation

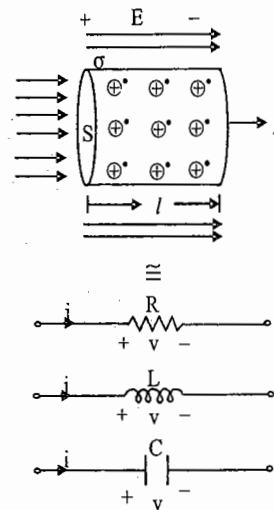
- As $E = 0 \Rightarrow \mu = 0$ (since all the electrons are at rest)

$$\Rightarrow V_d = 0 \Rightarrow K.E = 0 \Rightarrow P = 0 \text{ W/m}^3$$

Or

- As $E = 0 \Rightarrow J = 0 \Rightarrow P = J \cdot E = 0 \text{ W/m}^3$

So, when no energy flow through the conductor $\Rightarrow$ no voltage drop across the conductor



- Since $E = \frac{V}{\ell}$ and $J = \frac{i}{S}$, as ' ℓ ' and ' S ' are scalars, the drop polarities are same as electric field ' E ' polarities and the current direction is same as current density ' J ' direction.

Conclusion

Whenever a conductor is carrying electrical energy, currents through all the three passive lumped R , L , C will always flow from Positive (+ve) to Negative (-ve) terminals (i.e. from High level to Low level).

Historical

- Actually, the opposition (by immobile positive ions) for electrical energy flow (i.e. free electron motion) through the conductor is continuous or distributive in nature, but practically, we can approximate this single distributive opposition into three lumped



oppositions called resistance (R), inductance (L) and capacitance (C) for lower frequencies (i.e. up to 1MHz).

- Magnitudes of these lumped R, L, C are function of length (l), crosssection area (S) and conductivity (σ) of the conductor.
- Interconnection of these lumped elements (R, L, C) is nothing but a lumped electric circuit. Network theory is used to analyse only these lumped electric circuits at lower frequencies (i.e. up to 1MHz).
- At higher frequencies (i.e. above 1MHz), we cannot approximate distributive elements into lumped elements and hence, no lumped electric circuit, so no Network theory. Here, Field theory approach is used directly on distributive conductors.
- Field theory approach of solving distributive electric networks is valid for all the frequencies starting from zero (i.e. 0 Hz $\Rightarrow$ D-C).

1.1.1 Getting Started:

There are two Scenarios we will encounter in Network theory

Scenario1: In the presence of Active Sources

In the presence of sources, all the three passive lumped R, L, C will always absorb energy. Whenever they are absorbing energy, currents through them will always flow from Positive (+ve) to Negative (-ve) terminals (i.e. from High level to Low level).

Scenario2: In the absence of Active Sources

In the absence of sources, stored energy in the memory elements (L and C) will be delivered to the memoryless resistances. Whenever L and C elements are delivering energy, currents through them will always flow from Negative (-ve) to Positive (+ve) terminals (i.e. from Low level to High level). As the resistor always absorbs energy, current through it will always flow from Positive (+ve) to Negative (-ve) terminals.

Example: 1.4

What is the basic difference between Network and Circuit?

Sol: Circuit $\Rightarrow$ Loop or Mesh i.e. a closed path. Interconnection of circuits is nothing but a network.

Eg: Network is like a building, where as the circuits are like rooms in a building.

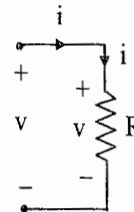
1.2 Passive Lumped R, L, C Elements and Ohm's Law

1.2.1 Resistance (R)

Since $J = \sigma E$

$$\frac{i}{S} = \sigma \frac{v}{l}$$

$$v = \frac{l}{\sigma S} i$$



$v = Ri \rightarrow$ Ohm's law in Circuit theory form

where $R = \frac{l}{\sigma S} = \text{Ohm} = \Omega = \text{Resistance parameter of the conductor.}$

Limitation

- Ohm's law i.e. a linear voltage and current relation is valid only when the proportionality constant 'R' is kept constant i.e. the temperature is kept constant.
- As temperature increases $\Rightarrow l \uparrow, S \uparrow$,
 $\frac{l}{S} = \text{almost constant, } \sigma \downarrow \text{ so } R \uparrow.$
- $R_t = R_0 (1 + \alpha t)$
Where, $\alpha =$ temperature coefficient in $^{\circ}\text{C}$, positive for all the conductors.



t = temperature in $^{\circ}\text{C}$.

So, $t \uparrow \Rightarrow R_t \uparrow$ for all the conductors.

$$\bullet \quad R = \frac{l}{\sigma S} \Rightarrow \sigma = \frac{l}{RS} = \frac{m}{\Omega \cdot \text{m}^2}$$

$$= (\Omega \cdot \text{m})^{-1} = \Omega^{-1} \text{m}^{-1} = \text{U/m or S/m.}$$

$$\bullet \quad \text{Resistivity or Specific resistance} = \rho$$

$$= \frac{1}{\sigma} = \frac{RS}{l} = \frac{\Omega \cdot \text{m}^2}{\text{m}} = (\Omega \cdot \text{m}).$$

Since $V = Ri$

$$i = \frac{V}{R}$$

$$\boxed{i = Gv} \rightarrow 3^{\text{rd}} \text{ form of Ohm's law}$$

$$\text{where } G = \frac{1}{R} = \text{mho} = \text{U}$$

= Conductance parameter of the conductor.

$$\text{Since } i = \frac{dq}{dt}$$

$$\boxed{v = R \frac{dq}{dt}} \rightarrow 4^{\text{th}} \text{ form of Ohm's law}$$

Electric power (P)

$$P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt}$$

$$\boxed{P = VI} \text{ (W) = Instantaneous power.}$$

$$\text{i.e. } p(t) = v(t) \cdot i(t) \text{ (W)}$$

Electrical energy (W)

$$\text{Since } \frac{dW}{dt} = P \Rightarrow dW = P dt$$

$$\boxed{W = \int P dt} \text{ (J)}$$

For R:

$$P = v \cdot i = R \cdot i \cdot i$$

$$\Rightarrow \boxed{P = i^2 R} \text{ (W) = Instantaneous power.}$$

$$\text{i.e., } p(t) = i^2(t) \cdot R \text{ (W)}$$

$$P = V \cdot I = V \cdot \frac{V}{R}$$

$$\boxed{P = \frac{V^2}{R}} \text{ (W) = Instantaneous power.}$$

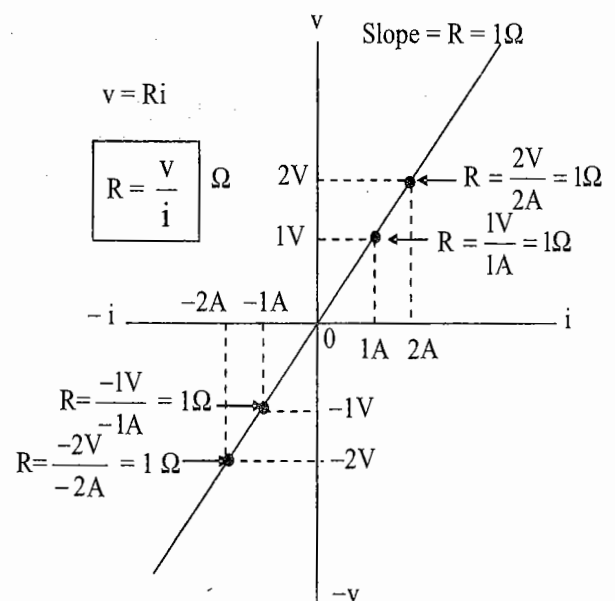
$$\text{i.e. } p(t) = \frac{v^2(t)}{R} \text{ (W)}$$

$$W = \int p dt \text{ (J)}$$

$$\boxed{W = \int i^2 R dt} \text{ (J)}$$

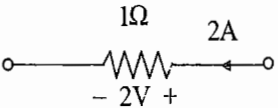
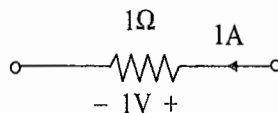
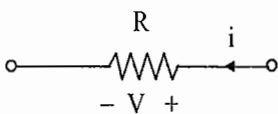
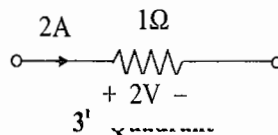
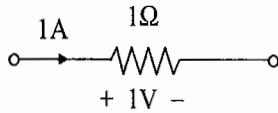
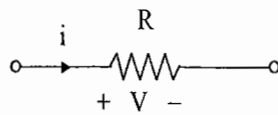
$$\text{and } \boxed{W = \int \frac{V^2}{R} dt} \text{ (J)}$$

v-i Characteristics





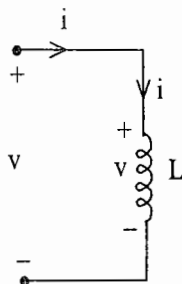
1st Quadrant



Observation

So from the characteristics, resistor is a linear, passive, bilateral and time-invariant in $v - i$ plane (these are possible only at a constant temperature, where length (l), crosssection area (S) and conductivity (σ) are kept constant i.e. 'R' is kept constant).

1.2.2 Inductance (L)



- When a conductor is wound like a spring, then it is said to be a coil. The coil will exhibit ideally inductance (L) and practically some leakage resistance (R), which is modelled in series with the inductance.
- When a time varying current is flowing through the coil, a time varying magnetic flux will be produced (by Faraday's law). The total flux produced is

$$\psi = N\phi \text{ (Wb)}$$

Where, N = the number of turns

ϕ = flux per turn

$$\text{i.e., } \psi(t) = N \cdot \phi(t) \text{ (Wb)}$$

- This total flux produced is proportional to the current flowing through the coil

$$\text{i.e } \psi \propto i \Rightarrow \psi = Li$$

Where, L = Inductance parameter of the coil.

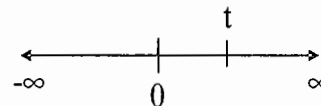
- The voltage drop across the coil is

$$v = \frac{d\psi}{dt} \text{ | By Faraday's law.}$$

$$v = \frac{d}{dt}(Li)$$

$$v = L \frac{di}{dt} \text{ V}$$

$$di = \frac{1}{L} v \cdot dt$$



$$i = \frac{1}{L} \int_{-\infty}^t v \, dt \text{ A}$$

$$p = v \cdot i = L \frac{di}{dt} \cdot i$$

$$p = Li \frac{di}{dt} \text{ (W)} = \text{Instantaneous power.}$$



$$W = \int p \, dt \, (J)$$

$$W = \int Li \left(\frac{di}{dt} \right) dt \, (J)$$

$$p = Li \frac{di}{dt} = \frac{d}{dt} \left(\frac{1}{2} Li^2 \right)$$

$$\text{Since } W = \int p \, dt \, (J)$$

$$W = \int \frac{d}{dt} \left(\frac{1}{2} Li^2 \right) dt$$

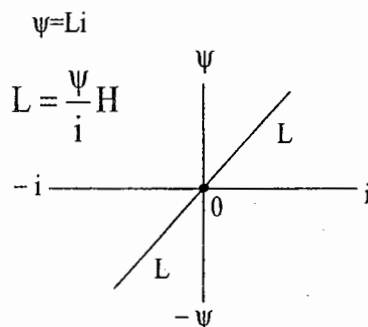
$$W = \frac{1}{2} Li^2 \, (J)$$

= Instantaneous energy

= Total energy.

i.e., the energy stored in an inductor at any instant will depend on the current flowing through the inductor at that instant. This is the total energy stored by the inductor from infinite past ($-\infty$) to the present time 't'.

ψ - i Characteristic



Observation

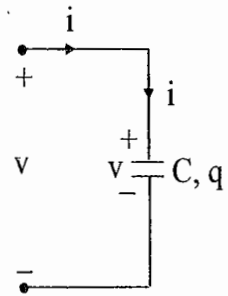
So from the characteristics, inductor is a linear, passive, bilateral and time-invariant in ψ - i plane (these are possible only at a constant temperature, where length (l), crosssection area (S) and conductivity (σ) are kept constant i.e. 'L' is kept constant).

1.2.3. Capacitance (C)

$$i = \frac{dq}{dt} \, A$$

$$q \propto v$$

$$q = Cv$$



Where,

C = Capacitance parameter of the conductor.

$$i = \frac{d}{dt} (Cv)$$

$$i = C \frac{dv}{dt} \, A$$

$$dv = \frac{1}{C} i \, dt \Rightarrow v = \frac{1}{C} \int_{-\infty}^t i \, dt \, V$$

$$p = v \cdot i$$

$$= v \cdot C \frac{dv}{dt}$$

$$p = Cv \frac{dv}{dt} \, (W) = \text{Instantaneous power.}$$

$$W = \int p \, dt \, (J)$$

$$W = \int Cv \left(\frac{dv}{dt} \right) dt \, (J)$$

$$W = \int Cv \left(\frac{dv}{dt} \right) dt \, (J)$$

$$p = Cv \frac{dv}{dt} = \frac{d}{dt} \left(\frac{1}{2} Cv^2 \right)$$

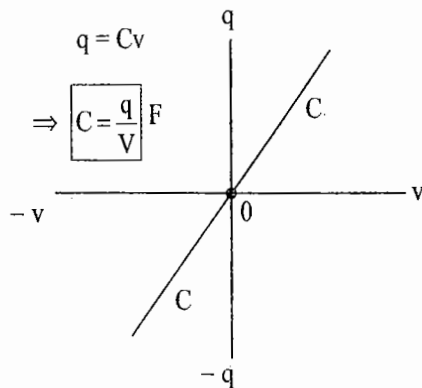
$$\text{Since } W = \int p \, dt \, (J)$$

$$W = \frac{1}{2} Cv^2 \, (J)$$

W = Instantaneous energy = Total energy.



q - v Characteristic



Observation

So from the characteristics, capacitor is a linear, passive, bilateral and time-invariant in $q - v$ plane (these are possible only at a constant temperature, where length (l), cross section area (S) and conductivity (σ) are kept constant i.e. 'C' is kept constant)

1.2.4 Relation between voltage and current in L and C elements

L: $v = L \frac{di}{dt}$

$$v_1 = L \frac{di_1}{dt} \quad \text{----- (1)} \quad v_1 \leftarrow i_1$$

$$v_2 = L \frac{di_2}{dt} \quad \text{----- (2)} \quad v_2 \leftarrow i_2$$

$$v = L \frac{d}{dt}(i_1 + i_2) \quad v_1 + v_2 \leftarrow i_1 + i_2$$

$$v = L \frac{di_1}{dt} + L \frac{di_2}{dt}$$

$$v = v_1 + v_2$$

So, the relation between voltage and current in an inductor is linear and hence

$$v = L \frac{di}{dt} \rightarrow 5^{\text{th}} \text{ form of Ohm's law}$$

Similarly

$$i = \frac{1}{L} \int_{-\infty}^t v dt \rightarrow 6^{\text{th}} \text{ form of Ohm's law}$$

C: $i = C \frac{dv}{dt}$

$$i_1 = C \frac{dv_1}{dt} \quad i_1 \leftarrow v_1$$

$$i_2 = C \frac{dv_2}{dt} \quad i_2 \leftarrow v_2$$

$$i = C \frac{d}{dt}(v_1 + v_2) \quad i_1 + i_2 \leftarrow v_1 + v_2$$

So, the relation between voltage and current in a capacitor is linear and hence

$$i = C \frac{dv}{dt} \rightarrow 7^{\text{th}} \text{ form of Ohm's law}$$

Similarly

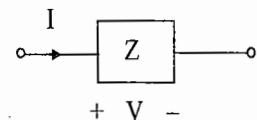
$$v = \frac{1}{C} \int_{-\infty}^t i dt \rightarrow 8^{\text{th}} \text{ form of Ohm's law}$$

Another Method:

$$Z = Z_R = R \Omega$$

$$= Z_L = j \omega L \Omega$$

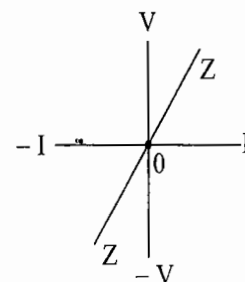
$$Z_C = \frac{1}{j \omega C} \Omega$$



- In a.c. domain, Ohm's law can be written as

$$V \propto I$$

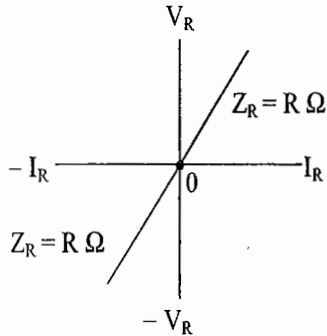
$$V = ZI$$





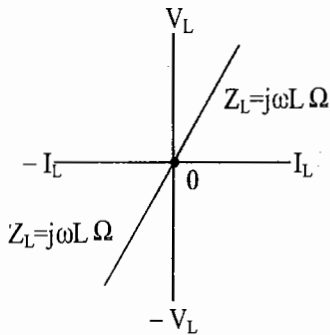
- So $V \propto I$, provided 'Z' is kept constant.

R: $V_R = Z_R I_R \mid Z_R = R \Omega$



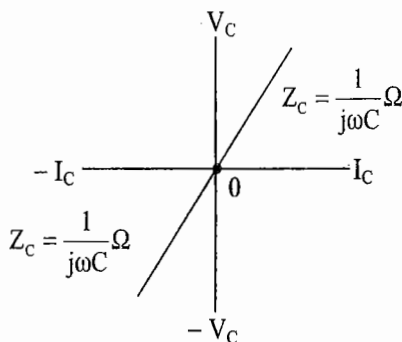
So $V_R \propto I_R$ only when 'R' is kept constant
i.e. temperature is constant.

L: $V_L = Z_L I_L \mid Z_L = j\omega L \Omega$



So $V_L \propto I_L$ only when 'L' is kept constant
i.e. temperature is constant and also at a
constant source frequency ' ω ', where Z_L is
constant.

C: $V_C = Z_C I_C \mid Z_C = \frac{1}{j\omega C} \Omega$



So $V_C \propto I_C$ only when 'C' is kept constant
i.e. temperature is constant and also at a
constant source frequency ' ω ', where Z_C is
constant.

1.2.5 Relation between Network theory and Electromagnetic theory

$$W_L = \frac{1}{2} Li^2 \text{ (J) \&}$$

$$i = \int_1 H \cdot dl \mid W_H = \frac{1}{2} \mu H^2 \left(\frac{J}{m^3} \right)$$

= Energy density i.e. energy stored per unit
volume (unit cube) in magnetic field
within the conductor due to the
inductance effect of the conductor.

$$W_C = \frac{1}{2} Cv^2 \text{ (J) \&}$$

$$v = \int_1 E \cdot dl \mid W_E = \frac{1}{2} \epsilon E^2 \left(\frac{J}{m^3} \right)$$

= Energy density i.e. energy stored per unit
volume (unit cube) in electric field
within the conductor due to the
capacitance effect of the conductor.

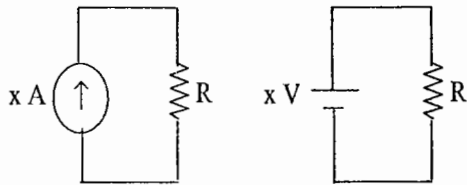
Observation

So, due to resistance, inductance and capacitance
effects of the conductor, energy loss will occur in
the conductor i.e. only part of the transmitted
source energy will be received by the load.

Example: 1.5

When a resistor R is connected to a current
source, it consumes a power of 18W. When the
same R is connected to a voltage source having
same magnitude as the current source, the power
absorbed by R is 4.5W. The magnitude of the
current source & value of R are

Sol:



$$x^2 R = 18 \text{ W}$$

$$\frac{x^2}{R} = 4.5$$

$$(R \cdot 4.5) R = 18 \text{ W}$$

$$R^2 = \frac{180}{45}$$

$$R = \sqrt{\frac{180}{45}} = 2 \Omega$$

$$x^2 \times 2 = 18$$

$$x^2 = 9 \Rightarrow x = 3$$

1.3 Types of Elements

There are ten types of elements (i.e. five pairs) as follows

1. Linear and Non-linear
2. Active and Passive
3. Bilateral and Unilateral
4. Distributed and Lumped
5. Time invariant and Time varying

Definitions

- A two terminal element is said to be linear if for all time 't', its characteristics is a straight line through the origin, otherwise it is said to be nonlinear.
- An element is said to be active if it delivers a net amount of energy to the outside world, otherwise it is said to be passive.
- An element is said to be bilateral if it offers same impedance for the different directions of same current flow, otherwise it is said to be unilateral.

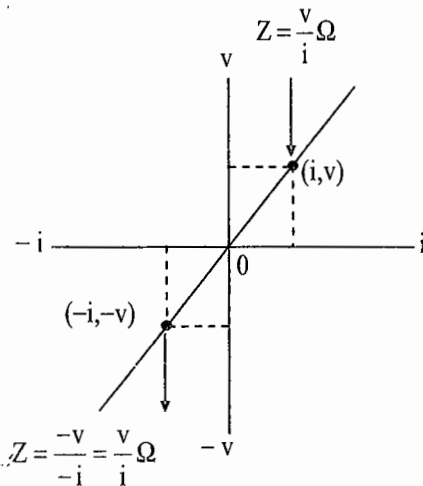
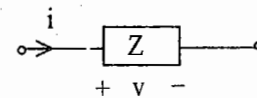
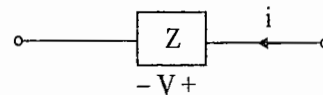


Fig. A bilateral element characteristic



(i, v)



(-i, -v)

In other words, for a bilateral element, if (i, v) is on the characteristic then (-i, -v) must also be on the characteristic, then only same voltage for the different directions of same current flow and hence same impedance i.e., for a Bilateral element, symmetry w.r.t origin exists.

- An element is said to be time invariant if for all time 't', its characteristics do not change with time, otherwise it is said to be time varying.

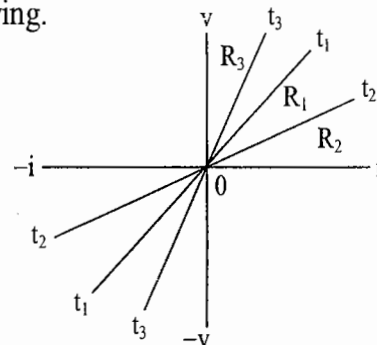


Fig. A time varying element characteristics

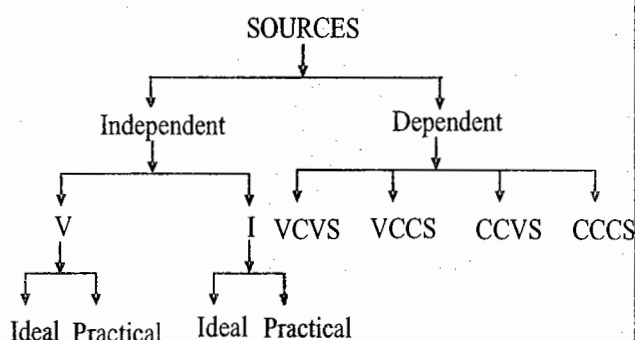
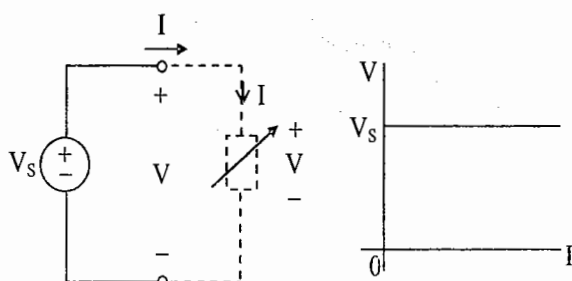
**Observation**

The above characteristics also represent a linear, passive and bilateral element.

Note: The resistors, inductors and capacitors are said to be passive if and only if $R \geq 0$, $L \geq 0$ and $C \geq 0$ (i.e. nonnegative slope of the v-i characteristics), otherwise they are said to be active i.e. $R < 0$, $L < 0$ and $C < 0$ (i.e. negative slope of the v-i characteristics).

1.4 Sources

Sources are classified as shown below

**1.4.1 Independent sources****1.4.1.1 Ideal Voltage Source**

Note:

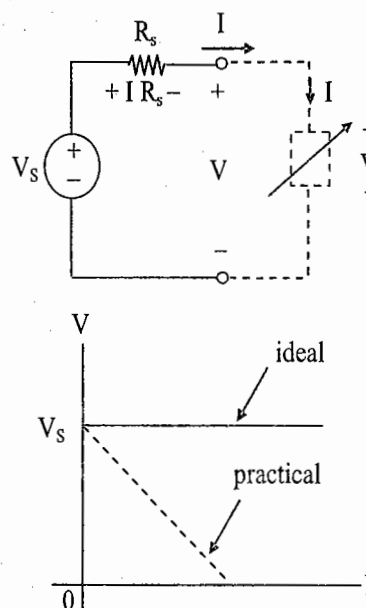
- Energy delivery i.e. current delivery in any source is always from the positive terminal i.e. current flows from negative to positive terminals.
- For the flow of current (i.e. free electron motion) always closed path is necessary.

i.e., $V = V_s$ for all 'I'

Property:

In an ideal voltage source, load voltage is independent of the load current drawn.

Note: Inherently all the sources are nonlinear in nature, since their voltage and current relation is non-linear. They are basically active and unilateral elements. D.C means time invariant and a.c means time varying elements.

1.4.1.2 Practical Voltage Source

$$\text{By KVL} \Rightarrow V_s - IR_s - V = 0$$

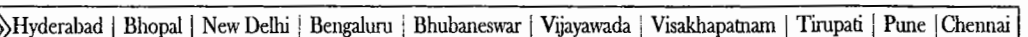
$$\Rightarrow \boxed{V = V_s - IR_s}$$

Property

In a practical voltage source, load voltage is a function of load current drawn.

Observation

When $I = 0\text{A}$, then $\boxed{V = V_s}$





makes the network a linear, an active and a bilateral.

- Controlled sources are said to be sources i.e. active elements in the presence of at least one independent source, then only controlled variables are non zero and hence their magnitudes are non zero.

1.5 Kirchhoff's Laws

There are two laws proposed by Kirchhoff

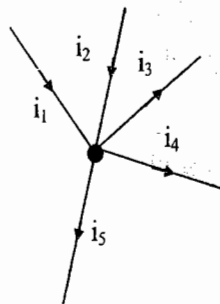
KCL: It is always defined at a node.

There are two types of nodes in network theory

- Simple Node:** It is an inter connection of only two branches.
- Principle Node:** It is an inter connection of at least three branches.

Definition

"In a lumped electric circuit, for any of its nodes and at any time 't', the algebraic sum of branch currents leaving the node is zero"



By KCL $\Rightarrow \sum \text{Leaving currents} = 0$

$$\Rightarrow -i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$$\Rightarrow i_1 + i_2 = i_3 + i_4 + i_5$$

i.e., sum of entering currents is equal to sum of leaving currents.

Since

$$i = \frac{dq}{dt} \Rightarrow \frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{dq_3}{dt} + \frac{dq_4}{dt} + \frac{dq_5}{dt}$$

At a given time 't', $\frac{d}{dt}$ is same for all branches

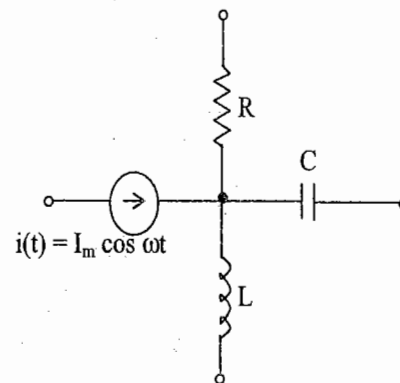
$$\Rightarrow q_1 + q_2 = q_3 + q_4 + q_5$$

i.e., sum of entering charges is equal to sum of leaving charges.

Properties

- KCL applies to any lumped electric circuit, it does not matter whether the elements are linear, non-linear, active, passive, time varying, time-invariant etc. i.e. KCL is independent of the nature of the elements connected to the node.

Ex:



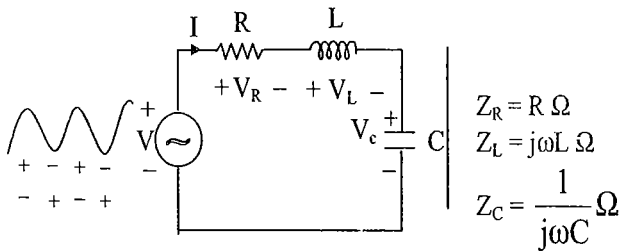
Here, R, L, C are linear, passive, bilateral, time-invariant elements and $i(t)$ is a non-linear, active, unilateral, time varying element.

- Since there is no accumulation of charge at any node, KCL expresses conservation of charge at each and every node in a lumped electric circuit.

KVL: It is always defined in a loop or mesh i.e. in a closed path.

Definition

"In a lumped electric circuit, for any of its loops and at any time 't', the algebraic sum of branch voltages around the loop is zero"



By KVL $\Rightarrow \sum \text{Branch voltages} = 0$

$$\Rightarrow V - V_R - V_L - V_C = 0$$

$$\Rightarrow \boxed{V = V_R + V_L + V_C}$$

Since $V = \frac{W}{q}$ and $i = \frac{dq}{dt}$ in series current is same and hence charge flow is same through all the elements.

$$\frac{W}{q} = \frac{W_R}{q} + \frac{W_L}{q} + \frac{W_C}{q}$$

$$\boxed{W = W_R + W_L + W_C}$$

Properties

1. KVL is independent of nature of the elements (linear, non-linear, active, passive etc.) present in a loop.
2. KVL expresses conservation of energy in every loop of a lumped electric circuit.

1.6 Nodal and Mesh Analysis Techniques

- KCL + Ohm's Law = Nodal Analysis
- KVL + Ohm's Law = Mesh Analysis

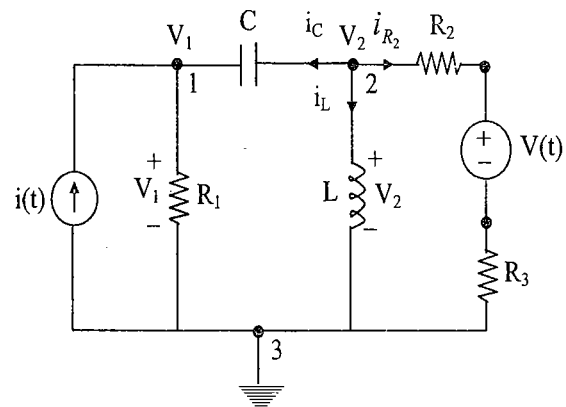
Conclusions

1. The above techniques are network solving procedures, valid only for lumped electric circuits (where KCL and KVL are valid) and

that too at a constant temperature (where Ohm's law is valid).

2. Nodal and Mesh procedures are independent of each other, since KCL and KVL are independent of each other.
3. Ohm's law is a function of nature of the elements (eight different forms of Ohm's law), where as Kirchhoff's laws are independent of nature of the elements.
4. Ohm's law is defined across an element, that element can be lumped (R, L, C) or distributed ($J = \sigma E$), whereas Kirchhoff's laws are defined to only for lumped electric circuits (one is at a node and another is in a loop or mesh).
5. Ohm's law is not applicable for active elements like Sources (Generators), since the voltage and current relation is non-linear. It is applicable only for linear passive elements like R, L, C, where the voltage and current relation is linear.

1.6.1 Nodal Analysis



Steps:

1. Identify number of nodes.
2. Assign node voltages w.r.t. the ground node, whose voltage is always equal to zero.
3. By using KCL first and Ohm's law next, write the nodal equations.

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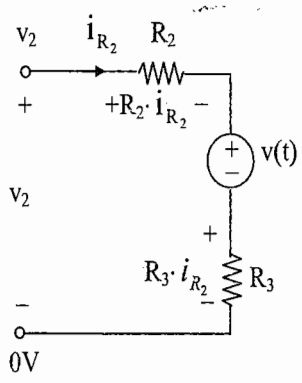
Node 2:

$$\begin{pmatrix} v_2 > v_1 \\ v_2 > 0 \\ v_2 > v(t) \end{pmatrix}$$

$$i_C + i_L + i_{R_2} = 0 \text{ (By KCL)}$$

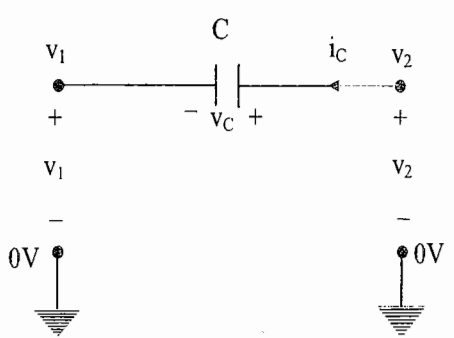
$$C \frac{d}{dt} (v_2 - v_1) + \frac{1}{L} \int_{-\infty}^t v_2 \cdot dt + \left(\frac{v_2 - v(t)}{R_2 + R_3} \right) = 0$$

(by Ohm's law) $\rightarrow$ ①



$$\text{By KVL} \Rightarrow v_2 - R_2 \cdot i_{R_2} - v(t) - R_3 \cdot i_{R_2} = 0$$

$$\Rightarrow i_{R_2} = \left(\frac{v_2 - v(t)}{R_2 + R_3} \right) \text{ A}$$



$$i_c = C \frac{dv_c}{dt} \text{ (by Ohm's law, where current flows from + to -)}$$

$$\text{By KVL} \Rightarrow v_1 + v_c - v_2 = 0 \Rightarrow v_C = v_2 - v_1$$

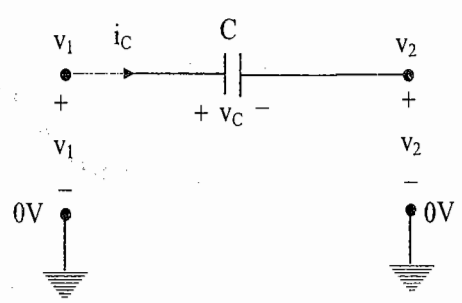
$$\text{So, } i_C = C \frac{d}{dt} (v_2 - v_1)$$

Node 1:

$$\begin{pmatrix} v_1 > 0 \\ v_1 > v_2 \end{pmatrix}$$

$$\text{Nodal} \Rightarrow -i(t) + \frac{v_1}{R_1} + C \frac{d}{dt} (v_1 - v_2) = 0 \rightarrow \text{②}$$

(by KCL and Ohm's law simultaneously)



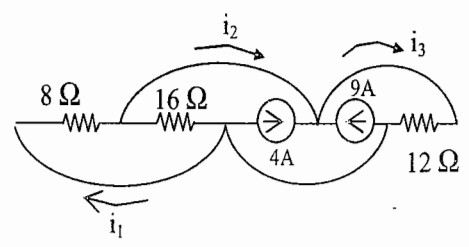
$$i_c = C \frac{dv_c}{dt} \text{ (by Ohm's law, where current flows from + to -)}$$

$$\text{By KVL} \Rightarrow v_1 - v_c - v_2 = 0 \Rightarrow v_C = v_1 - v_2$$

$$\text{So, } i_C = C \frac{d}{dt} (v_1 - v_2)$$

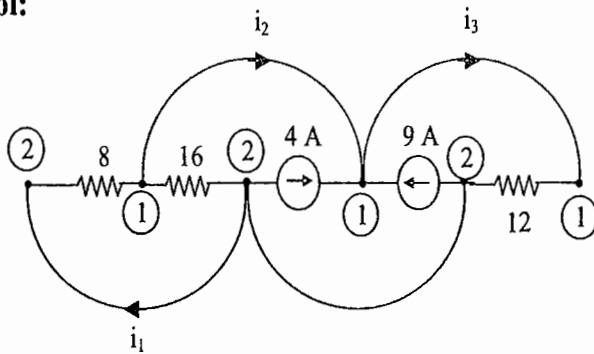
Example: 1.6

What is the value of i_1 ?





Sol:



Apply KCL at Node 1

$$\frac{V_1 - V_2}{8} + i_2 + (-i_2) - 4 - 9 + i_3$$

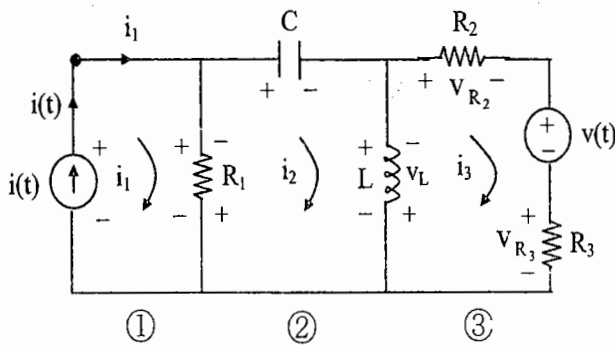
$$-i_3 + \frac{V_1 - V_2}{16} + \frac{V_1 - V_2}{12} = 0$$

$$\frac{13}{48}(V_1 - V_2) = 13$$

$$\therefore V_1 - V_2 = 48 \rightarrow (1)$$

$$i_1 = \frac{V_2 - V_1}{8} = \frac{-48}{8} = -6A$$

1.6.2 Mesh Analysis

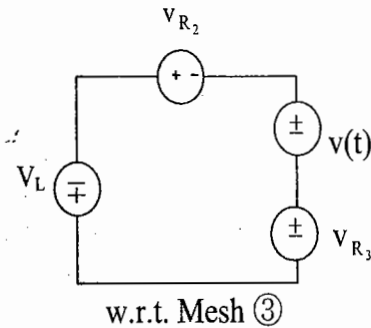


Steps:

1. Identify number of Meshes.
2. Assign mesh currents in the clockwise direction.
3. By using KVL first and Ohm's law next write the mesh equations.

Mesh 3: $\begin{pmatrix} i_3 > i_2 \\ i_3 > i_1 \end{pmatrix}$

Note: In a single branch, the branch current is equal to the mesh current and in a common branch, the branch current is equal to the difference of mesh currents.



$$V_{R_2} = R_2 i_3$$

$$V_{R_3} = R_3 i_3$$

$$V_L = L \frac{d}{dt}(i_3 - i_2)$$

$$-V_L - V_{R_2} - v(t) - V_{R_3} = 0 \text{ (by KVL)}$$

$$-L \frac{d}{dt}(i_3 - i_2) - R_2 \cdot i_3 - v(t) - R_3 \cdot i_3 = 0$$

$$\text{(by Ohm's law)} \rightarrow (1)$$

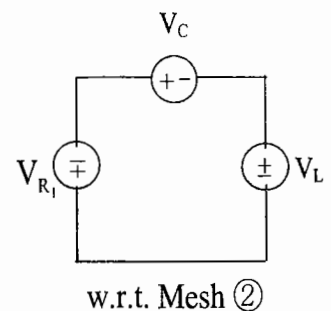
Mesh 2: $\begin{pmatrix} i_2 > i_3 \\ i_2 > i_1 \end{pmatrix}$

$$V_C = \frac{1}{C} \int_{-\infty}^t i_2 \cdot dt$$

$$V_L = L \frac{d}{dt}(i_2 - i_3)$$

$$V_{R_1} = R_1 \cdot (i_2 - i_1)$$

$$-V_{R_1} - V_C - V_L = 0$$



(b)

-I

(b)

Mesh

By KV

So, i_1

same

Proof

By K

Exam

Find

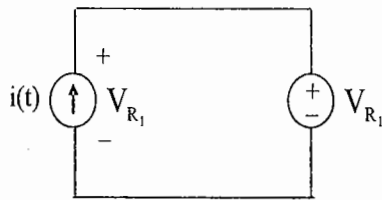


(by KVL)

$$-R_1(i_2 - i_1) - \frac{1}{C} \int_{-\infty}^t i_2 \cdot dt - L \frac{d}{dt} (i_2 - i_3) = 0$$

(by Ohm's law) $\rightarrow$ ②

Mesh 1: $\begin{pmatrix} i_1 > i_2 \\ i_1 > i_3 \end{pmatrix}$



w.r.t. Mesh ①

$$V_{R_1} = R_1 (i_1 - i_2)$$

By KVL $\Rightarrow V_{R_1} - V_{R_1} = 0 \Rightarrow \boxed{0 = 0}$

So, $i_1 = i(t) \rightarrow$ (3) (as both currents are in the same direction)

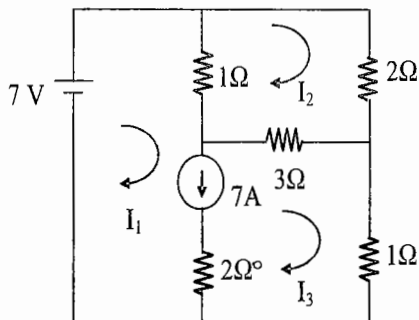
Proof:

By KCL $\Rightarrow -i(t) + i_1 = 0$

$$\Rightarrow i_1 = i(t) \rightarrow (3)$$

Example: 1.7

Find the current in the $3\text{-}\Omega$ resistor.



Sol: Meshes 1 and 3 will form a supermesh.

Writing current equation for supermesh,

$$I_1 - I_3 = 7 \quad \dots\dots\dots (i)$$

Applying KVL to the other path of the supermesh,

$$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - I_3 = 0$$

$$-I_1 + 4I_2 - 4I_3 = -7 \quad \dots\dots\dots (ii)$$

Applying KVL to Mesh 2;

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$I_2 - 6I_2 + 3I_3 = 0 \quad \dots\dots\dots (iii)$$

Solving equations (i), (ii) and (iii)

$$I_1 = 9 \text{ A}$$

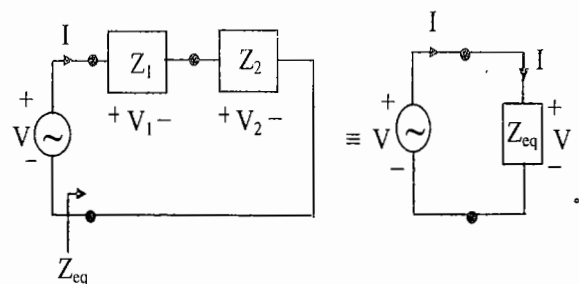
$$I_2 = 2.5 \text{ A}$$

$$I_3 = 2 \text{ A}$$

$$\begin{aligned} \text{Current through the } 3\text{-}\Omega \text{ resistor} &= I_2 - I_3 \\ &= 2.5 - 2 \\ &= 0.5 \text{ A} \end{aligned}$$

1.7 Equivalent Circuits w.r.t Passive R, L, Cs

- Two elements are said to be in series only when currents through the elements are same and two elements are said to be in parallel only when voltages across the elements are same.
- Impedances in series and admittances in parallel we can add.





$$Z = Z_R = R \Omega \Rightarrow Z_L = j \omega L \Omega$$

$$= Z_C = -\frac{1}{j \omega C} \Omega$$

$$Z_{eq} = Z_1 + Z_2$$

$$R : R_{eq} = R_1 + R_2$$

$$L : L_{eq} = L_1 + L_2$$

$$C : \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2};$$

$$\text{if } C_1 = C_2 = C \Rightarrow \boxed{C_{eq} = \frac{C}{2}}$$

Voltage Division Principle

$$V = Z_{eq} \cdot I \quad (\text{by Ohm's law})$$

$$I = \frac{V}{Z_{eq}} = \frac{V}{Z_1 + Z_2}$$

$$V_1 = Z_1 \cdot I \quad (\text{by Ohm's law})$$

$$\boxed{V_1 = \frac{V \cdot Z_1}{Z_1 + Z_2}}$$

$$V_2 = Z_2 \cdot I \quad (\text{by Ohm's law})$$

$$\boxed{V_2 = \frac{V \cdot Z_2}{Z_1 + Z_2}}$$

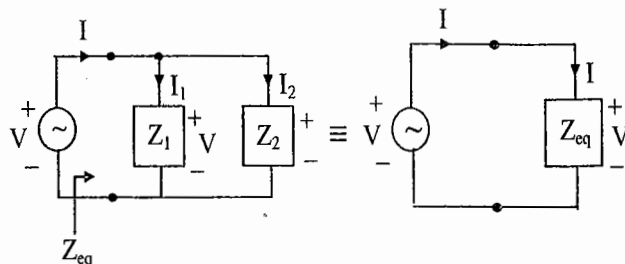
Check

$$\text{By KVL} \Rightarrow V - V_1 - V_2 = 0 \Rightarrow \boxed{V_1 + V_2 = V}$$

$$R : V_1 = \frac{V \cdot R_1}{R_1 + R_2}; \quad V_2 = \frac{V \cdot R_2}{R_1 + R_2}$$

$$L : V_1 = \frac{V \cdot L_1}{L_1 + L_2}; \quad V_2 = \frac{V \cdot L_2}{L_1 + L_2}$$

$$C : V_1 = \frac{V \cdot C_2}{C_1 + C_2}; \quad V_2 = \frac{V \cdot C_1}{C_1 + C_2}$$



$$Y_{eq} = Y_1 + Y_2$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

$$R : \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}; \text{ if } R_1 = R_2 = R$$

$$R_{eq} = \frac{R}{2}$$

$$L : \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$C : C_{eq} = C_1 + C_2$$

Current Division Principle

$$V = Z_{eq} \cdot I \quad (\text{by Ohm's law})$$

$$\Rightarrow \boxed{V = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \cdot I}$$

$$I_1 = \frac{V}{Z_1} \quad (\text{by Ohm's law})$$

$$\boxed{I_1 = \frac{I \cdot Z_2}{Z_1 + Z_2}}$$

$$I_2 = \frac{V}{Z_2} \quad (\text{by Ohm's law})$$

$$\boxed{I_2 = \frac{I \cdot Z_1}{Z_1 + Z_2}}$$

Check

$$\text{By KCL} \Rightarrow -I + I_1 + I_2 = 0$$

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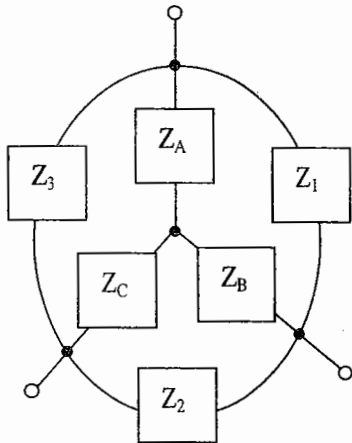
$$\Rightarrow I_1 + I_2 = I$$

$$R: I_1 = \frac{I \cdot R_2}{R_1 + R_2}; \quad I_2 = \frac{I \cdot R_1}{R_1 + R_2}$$

$$L: I_1 = \frac{I \cdot L_2}{L_1 + L_2}; \quad I_2 = \frac{I \cdot L_1}{L_1 + L_2}$$

$$C: I_1 = \frac{I \cdot C_1}{C_1 + C_2}; \quad I_2 = \frac{I \cdot C_2}{C_1 + C_2}$$

1.7.1 Star (Y) – Delta (Δ) Transformation and vice versa



Let Δ- network be given i.e. Z_1, Z_2 and Z_3 are known

$$Z_A = \frac{Z_1 \cdot Z_3}{Z_1 + Z_2 + Z_3} = \left(\frac{\text{Product}}{\text{Total}} \right)$$

$$Z_B = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2 + Z_3}; \quad Z_C = \frac{Z_2 \cdot Z_3}{Z_1 + Z_2 + Z_3}$$

Observation

If $Z_1 = Z_2 = Z_3 = Z$, then $Z_A = Z_B = Z_C$

$$= \frac{Z}{3}$$

So, for the balanced case, Δ to Y transformation will reduce the impedance by 3 times.

Let Y - network be given i.e. Z_A, Z_B and Z_C are known

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A \cdot Z_B}{Z_C} \right)$$

$$= \text{sum} + \left(\frac{\text{Their Product}}{\text{Remaining}} \right)$$

$$Z_2 = Z_B + Z_C + \left(\frac{Z_B \cdot Z_C}{Z_A} \right)$$

$$Z_3 = Z_C + Z_A + \left(\frac{Z_C \cdot Z_A}{Z_B} \right)$$

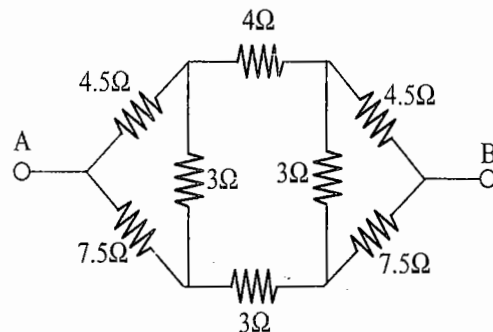
Observation

If $Z_A = Z_B = Z_C = Z$, then $Z_1 = Z_2 = Z_3 = 3Z$.

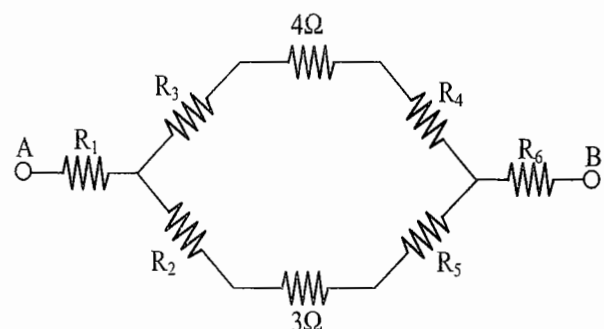
So for the balanced case, Y to Δ transformation will increase the impedance by 3 times.

Example: 1.8

Find an equivalent resistance between A and B.



Sol: Converting the two delta networks formed by resistors $4.5\Omega, 3\Omega$ and 7.5Ω into equivalent star networks, we have

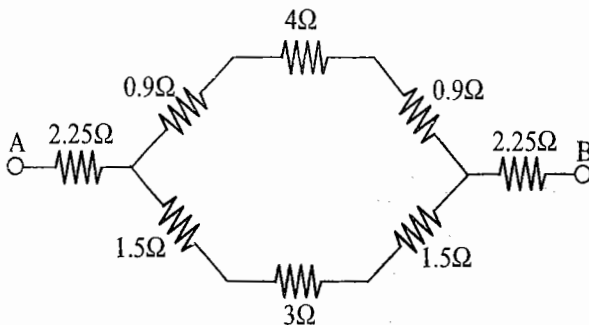


$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25 \Omega$$

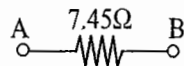
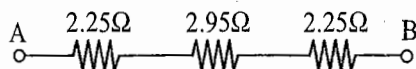
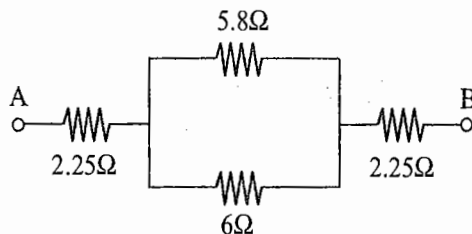
$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5 \Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9 \Omega$$

The simplified network is shown in Figure

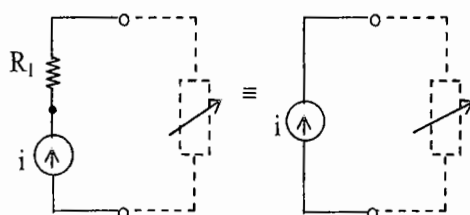


The network can be simplified as follows :



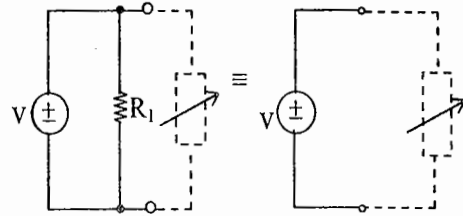
$$R_{AB} = 7.45 \Omega$$

1.8 Equivalent Circuits w.r.t Active Sources



Here $R_1 \neq \infty$,
since violation of KCL.

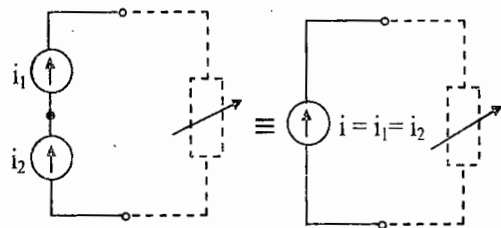
So, a resistor in series with an ideal current source can be neglected in the analysis i.e. the load current is independent of R_1 value. We cannot omit this resistor in power calculations, since $i^2 \cdot R_1 \neq 0$.



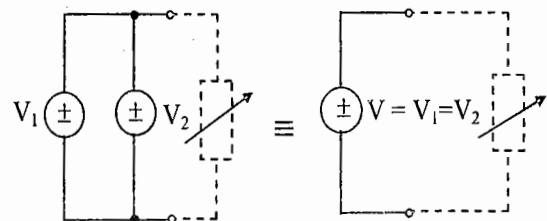
Here $R_1 \neq 0$, since
violation of KVL.

So, a resistor in parallel with an ideal voltage source can be neglected in the analysis i.e. the load voltage is independent of R_1 value. We cannot omit this resistor in power calculations,

$$\text{since } \frac{V^2}{R_1} \neq 0.$$



$$\text{By KCL} \Rightarrow -i_2 + i_1 = 0 \Rightarrow i_1 = i_2$$



$$\text{By KVL} \Rightarrow v_1 - v_2 = 0 \Rightarrow v_1 = v_2$$

So, two ideal current sources are connected in series only when their magnitudes are equal, otherwise violation of KCL, which results in the unstability of the system and hence, physical connection is not possible i.e. the circuit does not exist. Similarly, for

pa
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V₁

V₂

B

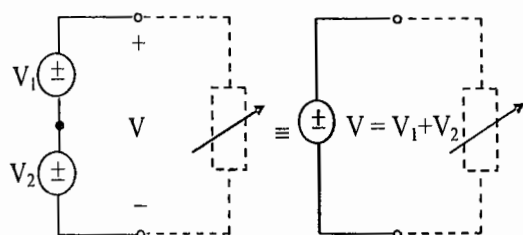
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By K

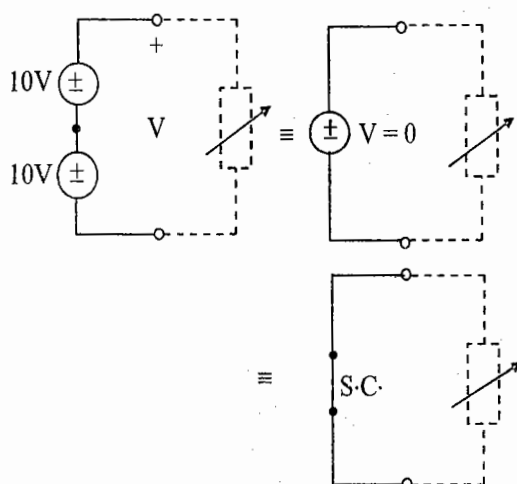


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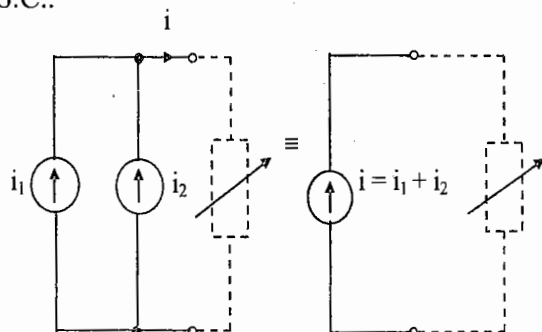
parallel connection of two ideal voltage sources, violation of KVL.



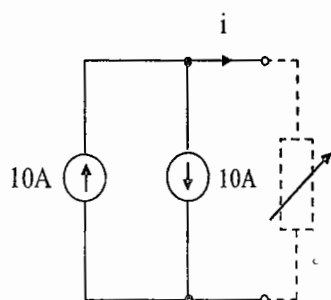
By KVL $\Rightarrow V_2 + V_1 - V = 0 \Rightarrow V = V_1 + V_2$



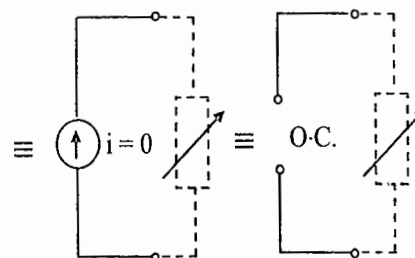
By KVL $\Rightarrow 10 - 10 - V = 0 \Rightarrow V = 0 \Rightarrow$ S.C..



By KCL $\Rightarrow -i_1 - i_2 + i = 0 \Rightarrow i = i_1 + i_2$



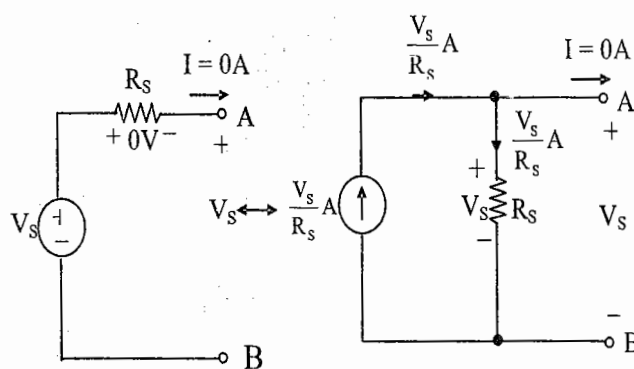
voltage
i.e. the
ue. We
lations,



By KCL $\Rightarrow -10 + 10 + i = 0 \Rightarrow i = 0 \Rightarrow$ O.C.

1.9 Source Transformation Technique

- It is a simplification technique, which eliminates extra nodes present in the network and it is applicable only for **practical sources**.
- It is impossible to convert an ideal voltage source into its equivalent current source and vice versa, since violation of KVL and KCL (as there is no resistance in the circuit).



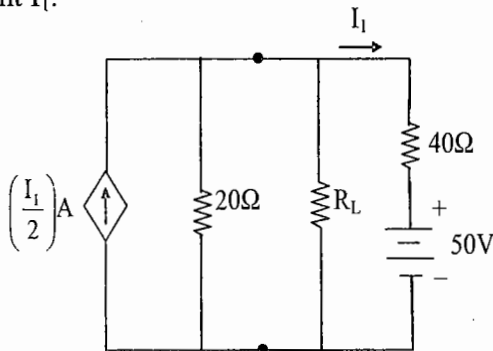
- The above circuits are equal only w.r.t. performance, but the elements and connections point of view, they are not equal.
- Source transformation is applicable even for dependent sources, provided the controlled variable **must be outside the branches**, where the source transformation is applied i.e. under any circumstances, controlled variable branch should not be disturbed, otherwise the controlled source magnitude will effect.

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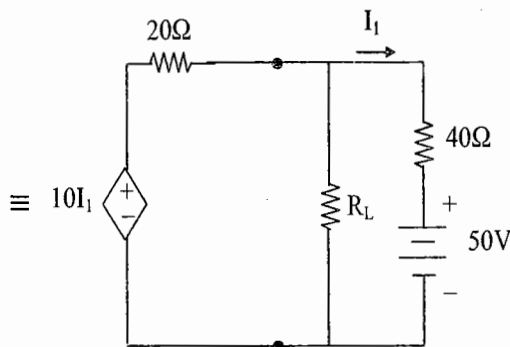


Example: 1.9

Use source transformation technique to determine current I_1 .



Sol:



1.10 Power Calculations by Tellegen's Theorem

Definition

- In an arbitrary network, algebraic sum of powers at any given instant is zero. i.e. the power delivered by some elements is equal to the power absorbed by remaining elements present in the network.

Mathematically,
$$\sum_{K=1}^b V_K i_K = 0$$

Where b = total number of branches.

- When current enters at the -ve terminal of an element, then that element will deliver

the power, otherwise it will absorb the power.

- Sources can deliver power or they can absorb power (in order to protect KCL and KVL in the network i.e. to ensure the stability of the network), whereas the passive elements will always absorb power, since current will enter at the positive terminal in the respective R , L , C (as the sources are present here).

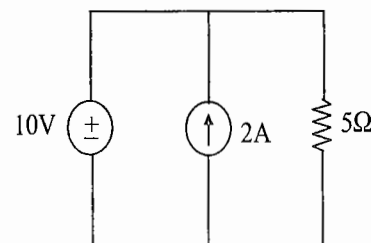
Properties:

- This theorem depends on the voltage and current product in an element, but not on the type of element (linear, non-linear, active, passive etc.) i.e. independent of the nature of the elements (like KCL and KVL)
- Tellegen's theorem expresses conservation of power (energy) in every lumped electric circuit.

Note: While verifying Tellegen's theorem, do not disturb the original network given for evaluating voltages and currents in each and every element of the network (i.e. Source transformation technique is not applied on the network).

Example: 1.10

Verify Tellegen's Theorem for the circuit shown below.



Sol: $P_{2A} = 12W$ (delivered)

$P_{2\Omega} = 8W$ (absorbed)

$P_{2V} = 2W$ (absorbed)

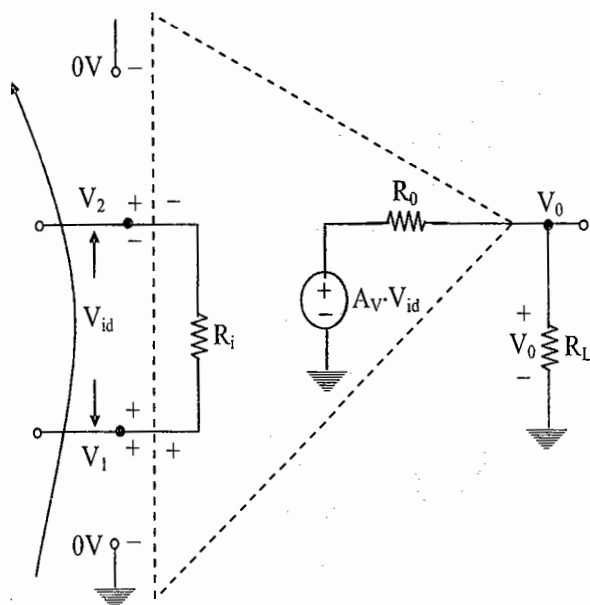
$P_{2V} = 2W$ (absorbed)

Since $P_{del} = P_{abs}$; Tellegen's theorem is satisfied



1.11 Network Theory Application to Operational Amplifiers

An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation and integration.

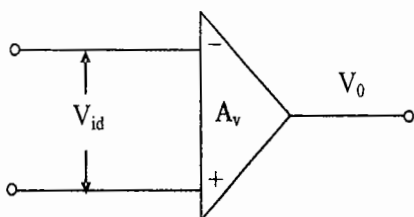


Electrical equivalent of a practical op amp

By writing KVL at the input side,

$$\Rightarrow V_1 - V_{id} - V_2 = 0 \Rightarrow V_{id} = V_1 - V_2$$

For any open loop voltage amplifier

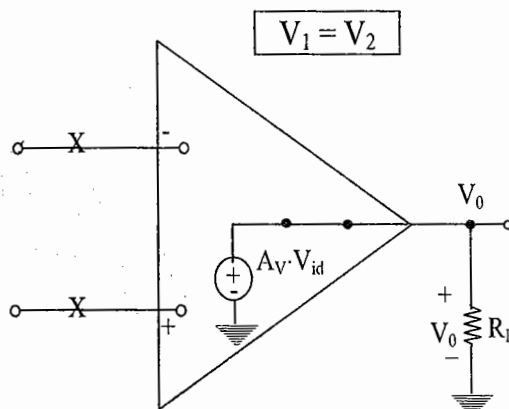


$$V_0 = A_v \cdot V_{id} \Rightarrow V_{id} = \frac{V_0}{A_v}$$

- For an ideal op amp, $A_v = \infty \Rightarrow V_{id} = 0$
 $\Rightarrow V_1 - V_2 = 0 \Rightarrow V_1 = V_2$

i.e. for an ideal op amp, voltage at the non-inverting terminal is equal to voltage at the inverting terminal.

- For an ideal op amp, $R_i = \infty$ and $R_0 = 0$



Since $R_i = \infty$, input current accepted by the ideal op amp is always equal to zero and hence, it is possible to write the nodal equations at the input side of an ideal op amp.

As $R_0 = 0 \Rightarrow$ there exists an ideal voltage source at the output side of an ideal op amp and the current through it can be any value i.e. un-known, it is impossible to write the nodal equation at the output side of an ideal op amp.

Class Room Practice Questions



LEVEL-1 Questions

01. In the circuit shown in the given Fig., the current I through R_L is (IES-EE-94)

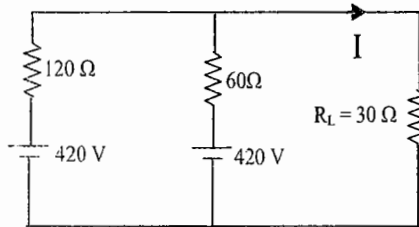
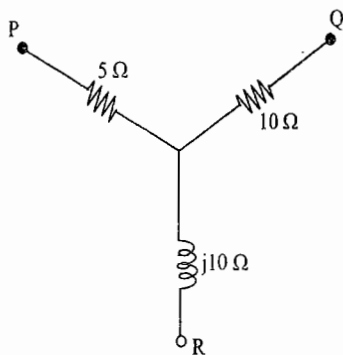


Fig.

- (a) 2 A (b) zero
(c) -2 A (d) 6 A
02. Two coupled coils with $L_1=L_2=0.6H$ have a coupling coefficient of $K=0.8$. The turns ratio $\frac{N_1}{N_2}$ is (IES-EE-94-13)
- (a) 4 (b) 2 (c) 1 (d) 0.5

03. In the delta equivalent of the given star-connected circuit $\bar{Z}_{QR}$ is equal to (IES-EE-95)



- (a) 40 Ω (b) $(20 + j10) \Omega$
(c) $5 + j\left(\frac{10}{3}\right) \Omega$ (d) $(10 + j30) \Omega$
04. Two inductive coils with self-inductances L_1 and L_2 are magnetically coupled in series

opposing and in parallel aiding respectively. The mutual inductance between the coils is M . The equivalent inductances in the two cases are respectively (IES-EE-96)

- (a) $L_1 + L_2 + 2M, \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$
(b) $L_1 + L_2 - 2M, \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$
(c) $L_1 + L_2 - 2M, \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$
(d) $L_1 + L_2 + 2M, \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

05. The coupling between two magnetically coupled coils is said to be ideal if the coefficient of coupling is (IES-EE-96)

- (a) zero (b) 0.5
(c) 1 (d) 2

06. For the network shown in the Fig., what is the voltage across the current source I ? (IES-EE-08)

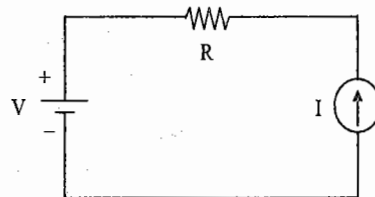
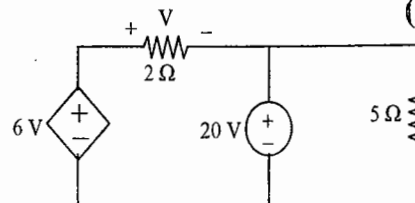


Fig.

- (a) $V - RI$ (b) $V + RI$
(c) Zero (d) $RI - V$

07. What is the current through the 2Ω resistance for the circuit as shown below? (IES-EE-09)

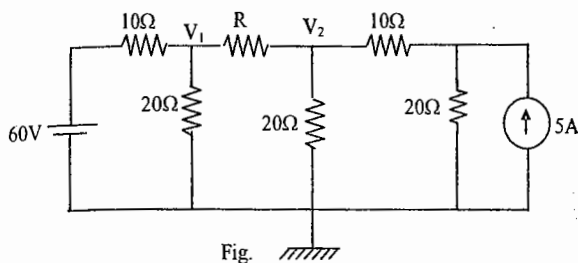


Fig



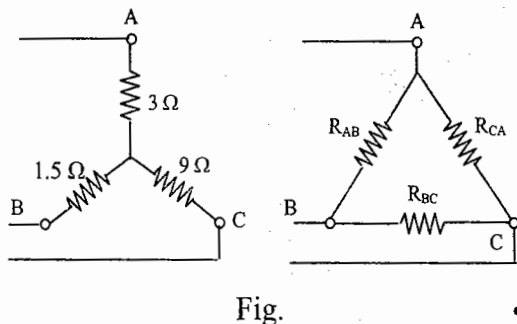
- (a) 5 A (b) 4 A
(c) 3 A (d) 2 A

08. In the below circuit, $V_1 = 40$ V when R is 10Ω . When R is zero, the value of V_2 will be (IES-EC-00)



- (a) 40 V (b) 30 V
(c) 20 V (d) 10 V

09. For the equivalent circuit shown in the given figure, the values of R_{AB} and R_{BC} are respectively (IES-EC-01)



- (a) 5 Ω and 15 Ω (b) 15 Ω and 30 Ω
(c) 30 Ω and 5 Ω (d) 20 Ω and 35 Ω

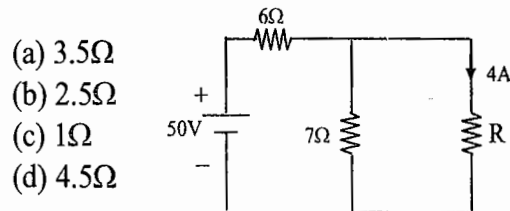
10. $\sqrt{\frac{L}{C}}$ has the dimension of (IES-EC-02)

- (a) time (b) capacitance
(c) inductance (d) resistance

11. A capacitor of capacitance C is charged by connecting it to a battery of e.m.f E . The capacitor is now disconnected and reconnected to the battery with the polarity reversed. The heat developed in the connecting wires is (IES-EC-12)

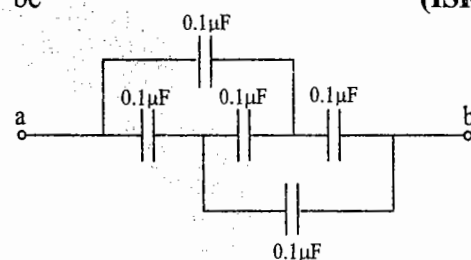
- (a) $0.5 CE^2$ (b) CE^2
(c) $2 CE^2$ (d) $3 CE^2$

12. The value of resistance R shown in the given figure is (ISRO-06)



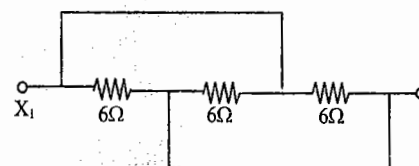
- (a) 3.5 Ω
(b) 2.5 Ω
(c) 1 Ω
(d) 4.5 Ω

13. The equivalent capacitance across 'ab' will be (ISRO-06)



- (a) 0.2 μF (b) 0.1 μF
(c) 0.5 μF (d) 0

14. Three resistors of 6Ω each are connected as shown in the following fig. The equivalent resistance between X_1 and X_2 is (ISRO-07)



- (a) 2 Ω (b) 4 Ω
(c) 8 Ω (d) 12 Ω

15. An inductor of inductance 0.1H, carrying current of 6 amps will store energy of (ISRO-09)

- (a) 6 Joules (b) 36 Joules
(c) 1.8 Joules (d) 3.6 Joules

16. An ideal voltage source will charge an ideal capacitor (HMWS)

- (a) in infinite time (b) exponentially
(c) instantaneously (d) linearly

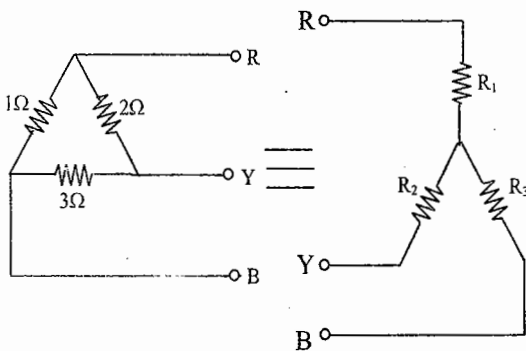


17. A series circuit has 3Ω , 10Ω and 20Ω and 2VDC in series. If 10Ω resistor is replaced by open circuit then current in the circuit is
(EPDCL-12)

(a) zero (b) increased
(c) decreased (d) constant

18. An ideal current source has zero
(a) internal conductance (SPDCL-12)
(b) internal resistance
(c) voltage on no load
(d) ripple

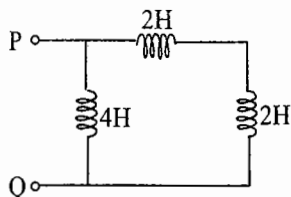
19. Three resistances of 1 ohm , 2 ohms and 3 ohms are connected in delta. These resistances are to be replaced by star connection as shown in the figure below, maintaining the same terminal conditions.



The value of highest resistance in star will be (SPDCL-12)

(a) $\frac{1}{4}\text{ ohm}$ (b) $\frac{1}{3}\text{ ohm}$
(c) $\frac{1}{2}\text{ ohm}$ (d) 1 ohm

20. The equivalent inductance of the below given circuit at the terminals P – Q is (SPDCL-12)



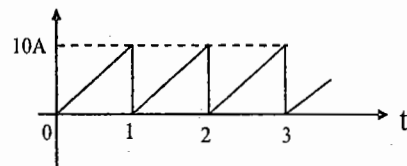
(a) 4H (b) 2H
(c) 6H (d) 8H

LEVEL-2 Questions

01. The number of turns of a coil having a time constant T are doubled. Then the new time constant will be (IES-EE-92)

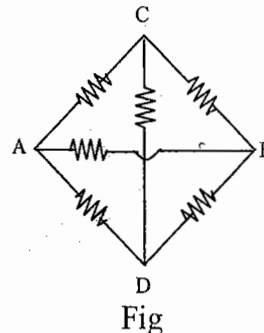
(a) T (b) $2T$
(c) $4T$ (d) $T/2$

02. Current having wave form shown is flowing in a resistance of 10 ohms . The average power is (IES-EE-92)



(a) $\frac{1000}{1}\text{ W}$ (b) $\frac{1000}{2}\text{ W}$
(c) $\frac{1000}{3}\text{ W}$ (d) $\frac{1000}{4}\text{ W}$

03. When all the resistances in the circuit are of one ohm each, the equivalent resistance across the points A and B will be (IES-EE-92)



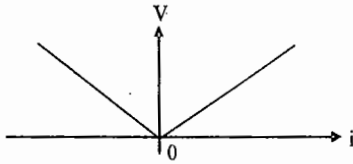
(a) 1Ω (b) 0.5Ω
(c) 2Ω (d) 1.5Ω

04. Two coupled coils connected in series have an equivalent inductance of 16 mH or 8 mH depending on the interconnection. Then the mutual inductance M between the coils is (IES-EE-94)

(a) 12 mH (b) $8\sqrt{2}\text{ mH}$
(c) 4 mH (d) 2 mH



05. The $v - i$ characteristic of an element is shown in the figure given below. The element is (IES-EE-02)



- (a) non-linear, active, non-bilateral
(b) linear, active, non-bilateral
(c) non-linear, passive, non-bilateral
(d) non-linear, active, bilateral

06. In the circuit shown below, the value of V_S is 0, when $I = 4$ A. The value of I when $V_S = 16$ V, is (IES-EE-03)

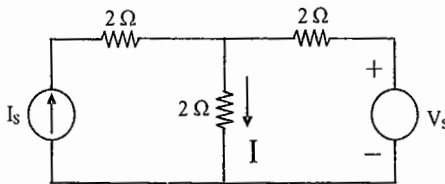
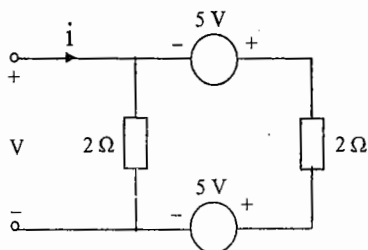


Fig.

- (a) 6 A (b) 8 A
(c) 10 A (d) 12 A

07. Consider the following circuit: Which one of the following statements is correct? (IES-EE-04)



- (a) passive and linear
(b) active and linear
(c) passive and non-linear
(d) active and non-linear

08. The black-box, N contains resistors and independent sources. If $I = 3$ A and 1.5 A for $R = 0$ and 2Ω , respectively, then what is the value of I for $R = 1 \Omega$? (IES-EE-05)

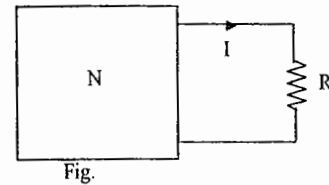


Fig.

- (a) 1 A (b) 2 A
(c) 3 A (d) 4 A

09. The current in a coil changes from 5A to 1A in 0.4 second. The induced voltage is 40 V. The self inductance in henry is (IES-EE-12)
- (a) 1 (b) 2 (c) 4 (d) 10

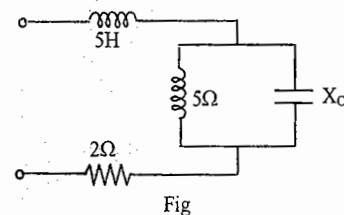
10. For a series and a parallel circuit, the equivalent total value of certain parameter X is given by

$$X_e = X_1 + X_2 + X_3 + X_4 + \dots + X_n$$

Where X_i is the i th value of the parameter and X_e is the equivalent value, and n is the number of elements. The parameter X can be (IES-EE-12)

- (a) resistance (b) current
(c) voltage (d) power

11. What will be the value of X_C so that the circuit is non inductive? (IES-EC-91)



Fig

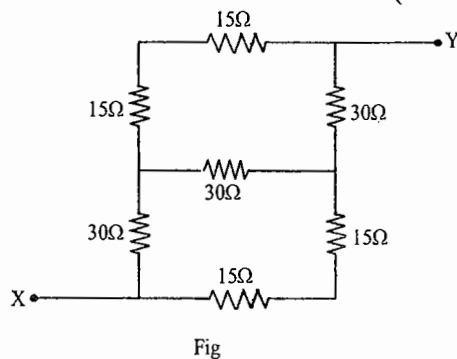
- (a) 2.5 (b) 5.0
(c) 10.0 (d) 20.0

12. A dc supply of 35 V is connected across 600Ω resistance in series with an unknown resistance. The voltmeter having a resistance of $1.2 \text{ k}\Omega$ is connected across 600Ω resistance which reads 5 V. Then the unknown resistance should be (IES-EC-92)

- (a) 50Ω (b) 1.7Ω
(c) 7.2Ω (d) $2.4 \text{ k}\Omega$

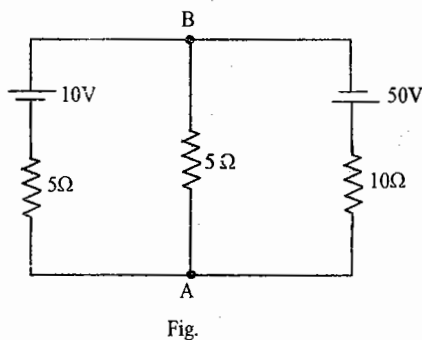


13. The equivalent resistance between the terminal points X and Y of the circuit shown is
(IES-EC-94)



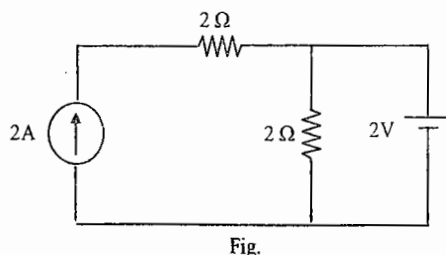
- (a) 150 ohms (b) 45 ohms
(c) 55 ohms (d) 30 ohms

14. For the circuit shown in the given figure, the voltage V_{AB} is
(IES-EC-95)



- (a) 6 V (b) 10 V
(c) 25 V (d) 40 V

15. The total power consumed in the circuit shown in the Fig., is
(IES-EC-97)



- (a) 10 W (b) 12 W
(c) 16 W (d) 20 W

16. For the circuit given, what is the expression for the voltage v ?
(IES-EC-05)

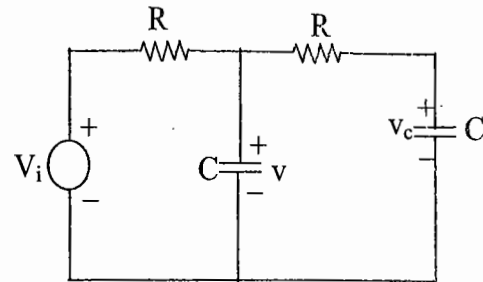


Fig.

- (a) $\dot{V}_i + v_c$ (b) v_c
(c) $RC \frac{dv_c}{dt} - v_c$ (d) $RC \frac{dv_c}{dt} + v_c$

17. The current through the branch AB in the Fig. shown is
(IES-EC-11)

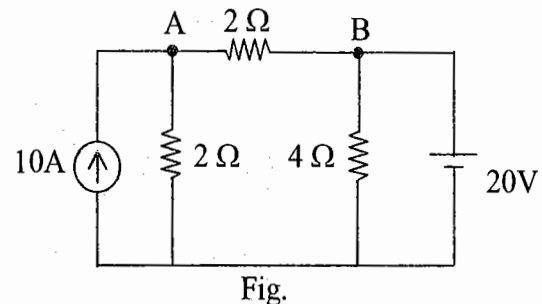


Fig.

- (a) 10 A, from A to B
(b) 10 A, from B to A
(c) 0
(d) 20 A, from B to A

18. The resistance of a parallel circuit consisting of two resistors is 12Ω . One of the resistance wires breaks and the effective resistance becomes 18Ω . The resistance of the broken wire is
(ISRO-2006)

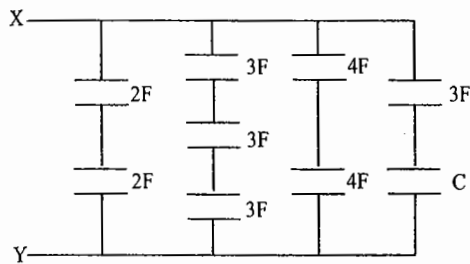
- (a) 48Ω (b) 18Ω
(c) 36Ω (d) 24Ω

19. An inductance of 1H is realized using air core with 100 turns. What will be the inductance if the number of turns are doubled
(ISRO -2009)

- (a) 1H (b) 0.5H
(c) 2H (d) None of these



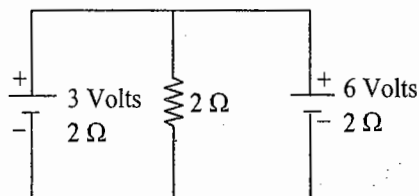
20. What is the value of C such that equivalent capacitance across $x - y$ is $5F$



(ISRO -2011)

- (a) $\frac{2}{3} F$ (b) $\frac{3}{2} F$
(c) $3F$ (d) $2F$

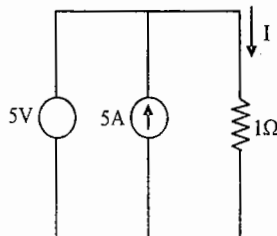
21. In the given circuit, the current flowing in the 2Ω resistor is (AEE-2012)



- (a) $1 A$ (b) $1.5 A$
(c) $0.8 A$ (d) $2 A$

22. The value of current I flowing in the 1Ω resistor in the circuit shown will be (HMWS)

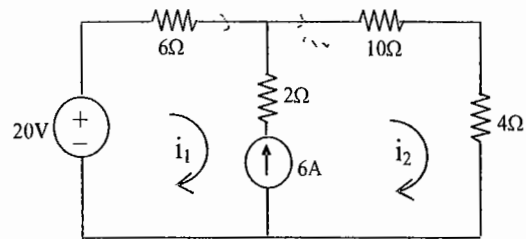
- (a) $10A$
(b) $6A$
(c) $5A$
(d) zero



23. Two resistances R_1 and R_2 gives combined resistance of 4.5 ohms when in series and 1 ohm when in parallel. The resistances are (VSP-2011)

- (a) 2 ohms and 2.5 ohms
(b) 4 ohms and 0.5 ohms
(c) 1.5 ohms and 3 ohms
(d) 5 ohms and 0.5 ohms

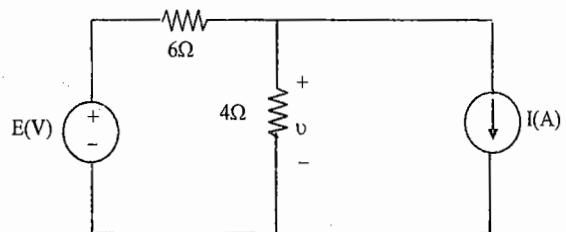
24. In the circuit shown which of the following statements is NOT correct?



(GENCO-2012)

- (a) The circuit has a super mesh
(b) $i_2 = i_1 + 6$
(c) $-20 + 6i_1 + 14i_2 = 0$
(d) $-20 + 6i_1 + 2(i_1 - i_2) = 0$

25. In the circuit shown. The voltage across 4Ω resistance 'v' can be expressed as (GENCO-2012)



- (a) $0.4E - 0.6 I$ (b) $0.6 E - 0.4 I$
(c) $0.4E - 2.4 I$ (d) $0.4E + 2.4 I$

Solutions

LEVEL-1

01. Ans: (d)

Sol:

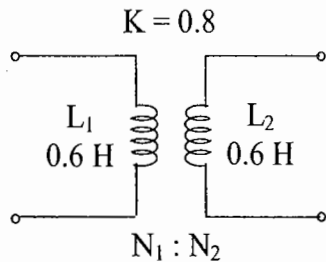
$$\frac{420 - 30I}{120} + \frac{420 - 30I}{60} = I$$

$$3.5 - 0.25 I + 7 - 0.5 I = I$$

$$10.5 = 1.75 I, I = \frac{10.5}{1.75} = 6 A$$



02. Ans: (c)



03. Ans: (d)

$$\bar{Z}_{QR} = \frac{5 \times 10 + j10 \times 10 + j10 \times 5}{5}$$

$$= \frac{50 + j150}{5} = 10 + j30 \Omega$$

04. Ans: (c)

Sol: For series opposing

$$L_{\text{equ}} = L_1 + L_2 - 2M$$

For parallel aiding

$$L_{\text{equ}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

05. Ans: (c)

Sol: The coupling between two magnetically coupled coils is said to be ideal if the coefficient of coupling is 1.

06. Ans: (b)

Sol: Voltage across the current source

$$= V + \text{Voltage across } R = V + RI$$

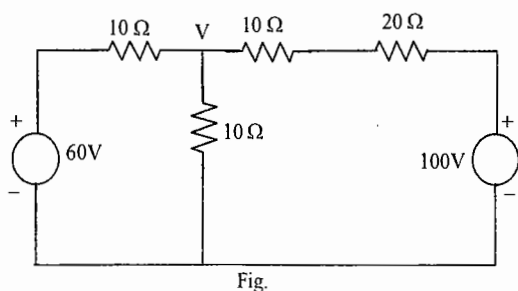
07. Ans: (d)

Sol: $6V - V = 20 \text{ Volts} \Rightarrow V = 4 \text{ Volts}$

$$I_{2\Omega} = \frac{4}{2} = 2 \text{ A}$$

08. Ans: (a)

Sol:



When $R = 0$, $V_1 = V_2 = V$ as shown in Fig.

Writing the nodal equations

$$\frac{V - 60}{10} + \frac{V}{10} + \frac{V - 100}{30} = 0$$

$$\frac{3V - 180 + 3V + V - 100}{30} = 0$$

$$7V = 280, \quad V = 40V$$

09. Ans: (a)

$$R_{AB} = \frac{(3 \times 9) + (9 \times 1.5) + (1.5 \times 3)}{9} = \frac{45}{9}$$

$$= 5 \Omega$$

$$\text{Similarly, } R_{BC} = \frac{45}{3} = 15 \Omega$$

10. Ans: (d)

Sol: $\sqrt{\frac{L}{C}}$ has the dimensions of resistance.

11. Ans: (c)

Sol: The heat developed in the connected wires is $2CE^2$

12. Ans: (a)

$$\text{Sol: } R_{\text{eq}} = 6 + \frac{7R}{R + 7}$$

$$I_{\text{total}} = \frac{50}{6 + \frac{7R}{R + 7}}$$

$$I_R = (I_{\text{total}}) \left(\frac{7}{R + 7} \right)$$

$$4 = \frac{50}{6 + \frac{7R}{R + 7}} \left(\frac{7}{R + 7} \right)$$

$$4 = \frac{50(7)}{13R + 42}$$

$$\therefore 52R + 168 = 350$$

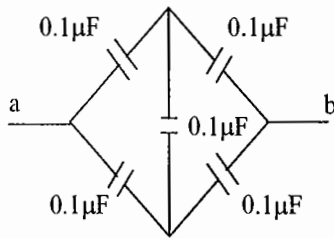
$$R = 3.5 \Omega$$

13. Ans: (b)

Sol: The circuit can be redrawn as

itions

Fig.

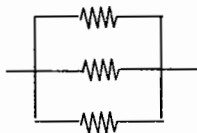


Under balanced condition

$$C_{eq} = 0.05\mu F + 0.05\mu F = 0.1\mu F$$

14. Ans: (a)

Sol: Circuit can be redrawn as



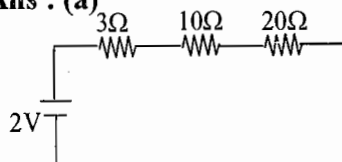
$$\begin{aligned} R_{eq} &= 6||6||6 \\ &= 2\Omega \end{aligned}$$

15. Ans: (c)

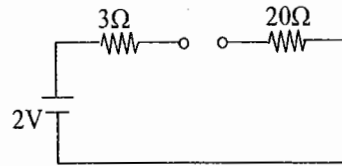
Sol: Energy stored by Inductor = $\frac{1}{2}LI^2$
 $= \frac{1}{2}(0.1)(6)^2 = 1.8 \text{ J}$

16. Ans: (c)

17. Ans : (a)



Sol: When 10Ω is replaced by open circuit



Current through circuit is zero because no closed path for current flow

18. Ans: (a)

19. Ans: (d)

Sol: $R_1 = \frac{1 \times 2}{6} = \frac{1}{3} \Omega$

$$R_2 = \frac{1 \times 3}{6} = \frac{1}{2} \Omega$$

$$R_3 = \frac{2 \times 3}{6} = 1\Omega$$

20. Ans: (b)

Sol: $L_{eq} = 4 \parallel (2+2) = 2 \text{ H}$

Level-2 Key

- | | | | | |
|---------|---------|---------|---------|---------|
| 01. (b) | 02. (c) | 03. (b) | 04. (d) | 05. (a) |
| 06. (b) | 07. (a) | 08. (b) | 09. (c) | 10. (d) |
| 11. (a) | 12. (d) | 13. (d) | 14. (a) | 15. (b) |
| 16. (d) | 17. (c) | 18. (c) | 19. (d) | 20. (b) |
| 21. (b) | 22. (c) | 23. (c) | 24. (d) | 25. (c) |



Leon Charles Thevenin
(1857 – 1926)



Necessity of Theorems:

In a complicated network with several sources, nodes and meshes, if the response in a single element is desired, then network theorems are used.

2.1 Superposition Theorem (SPT)

Definition:

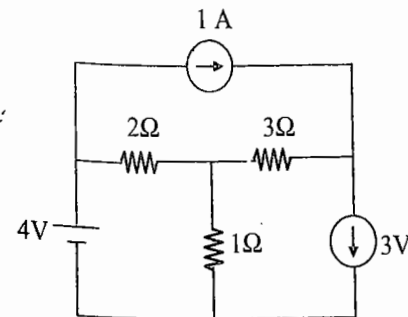
- In a linear network with several independent sources, the response in a particular branch when all the sources are acting simultaneously is equal to the linear sum of individual responses calculated by taking one independent source at a time.
- All the ideal voltage sources are eliminated from the network by shorting the sources, all the ideal current sources are eliminated from the network by opening the sources and do not disturb the dependent sources present in the network.

Properties

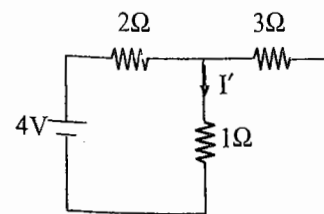
- This theorem is applicable only to linear networks i.e. the networks with R, L, C, transformer and linear controlled sources as elements.
- The presence of dependent sources makes the network an active and hence SPT is used for both active as well as passive networks.

Example: 2.1

Find the current in the 1-Ω resistor using superposition theorem.

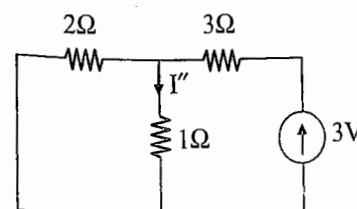


Sol: Step I: When the 4-V source is acting alone



$$I' = \frac{4}{2+1} = 1.33\text{A} (\downarrow)$$

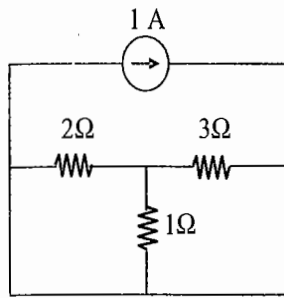
Step II: When the 3-A source is acting alone



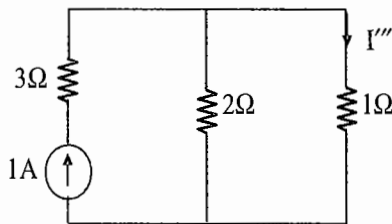
By current – division formula,

$$I'' = 3 \times \frac{2}{1+2} = 2\text{A} (\downarrow)$$

Step III: When the 1-A source is acting alone



The circuit can be redrawn as shown:



By current – division formula

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A} (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I''' \\ = 1.33 + 2 + 0.66 = 4 \text{ A} (\downarrow)$$

Homogeneity Principle

It is the principle obeyed by all the linear networks

Definition: In a linear network, if the excitation is multiplied with a constant 'K', then the responses in all other branches of the network are also multiplied with the same constant K.

Example: 2.2

Find all branch currents if source voltage is increased to 60 volts in the circuit below.

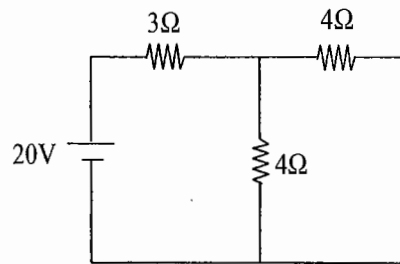
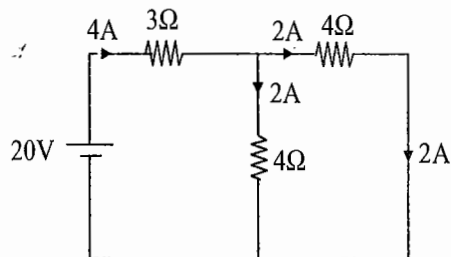
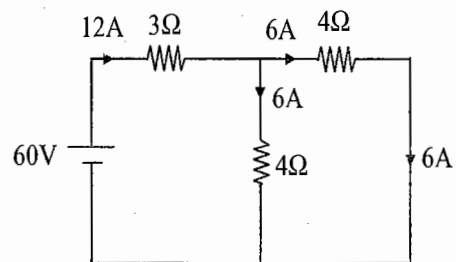


Fig.

Sol: First calculating the current in each branch.



So here, the excitation is multiplied by '3' and hence the responses are also proportionally multiplied by '3'.

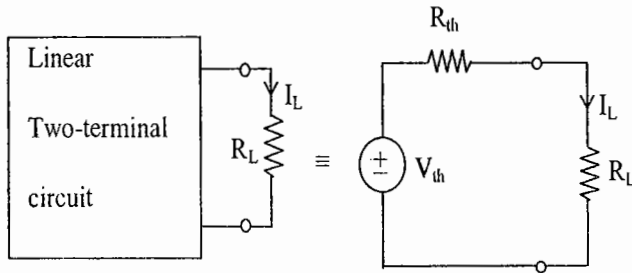


Note: When multiple sources are present, Superposition theorem to be applied first and later Homogeneity principle i.e., whenever Homogeneity principle is to be applied always ensure only one source in the network.

2.2 Thevenin's Theorem

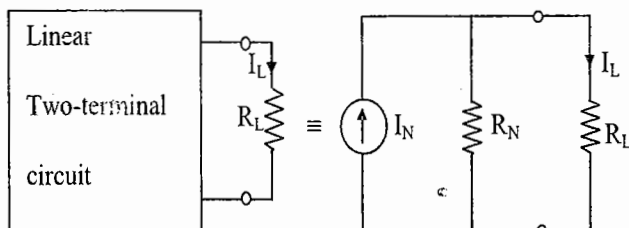
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th} , where V_{th} is the open circuit voltage at the load terminals and R_{th} is the input

or equivalent resistance at the load terminals when the independent sources are turned off. i.e killed.



2.3 Norton's Theorem

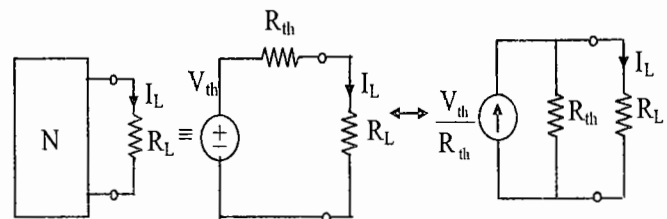
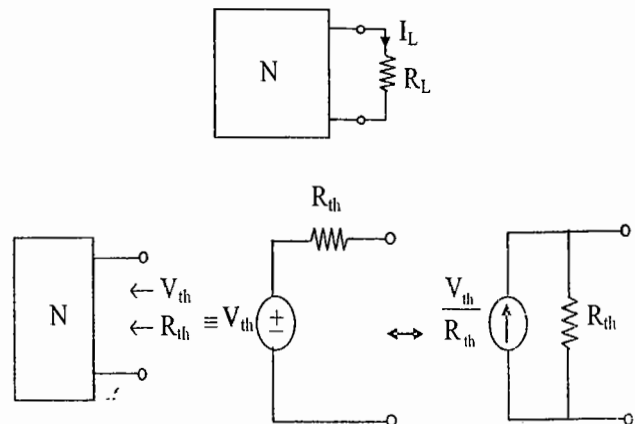
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short circuit current through the load terminals and R_N is the input or equivalent resistance at the load terminals when the independent sources are turned off. i.e killed.



Properties

- These theorems are applicable only for linear networks i.e. networks with R,L,C, transformer and linear controlled sources as elements.
- Presence of dependent sources makes the network active and hence these theorems are used for both active as well as passive networks.

Procedure



By ohm's law, $I_L = \left(\frac{V_{th}}{R_{th} + R_L} \right) A$

$$I_L = \frac{\frac{V_{th}}{R_{th}} \cdot R_{th}}{R_{th} + R_L} = \left(\frac{V_{th}}{R_{th} + R_L} \right) A$$

Cases in Thevenin's theorem (Norton's theorem) based on Sources

Case 1: All are independent sources: V_{th} , R_{th}

Case 2: At least one independent and one dependent source: V_{th} , I_{SC}

$$\Rightarrow R_{th} = \frac{V_{th}}{I_{SC}} \Omega \Rightarrow I_{SC} = \frac{V_{th}}{R_{th}} = I_N$$

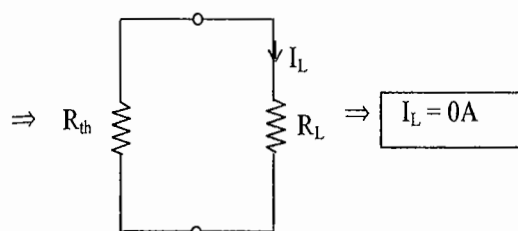
Case 3: All are dependent sources:

$$V_{th} = 0V, I_{SC} = 0A, \text{ but } R_{th} \neq 0$$

i.e.,

Exam
Find
Them

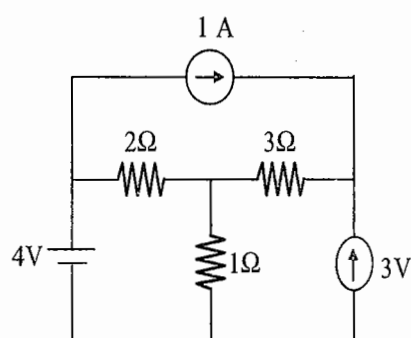
Sol:



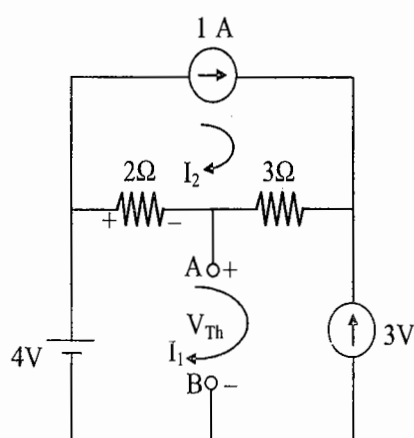
i.e., Thevenin's equivalent = Norton's equivalent

Example: 2.3

Find the current through the $1\text{-}\Omega$ resistor using Thevenin's Theorem.



Sol: Step I : Calculation of V_{Th}



Removing the $1\text{-}\Omega$ resistor from the network,

Writing the current equation for Meshes 1 and 2

$$I_1 = -3 \text{ A}$$

$$I_2 = 1 \text{ A}$$

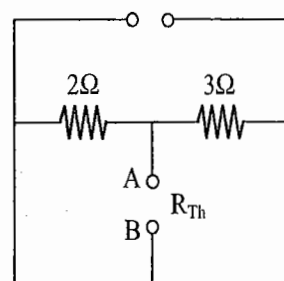
Writing V_{Th} equation ,

$$4 - 2(I_1 - I_2) - V_{Th} = 0$$

$$V_{Th} = 4 - 2(-3 - 1) = 4 - 2(-4) = 12 \text{ V}$$

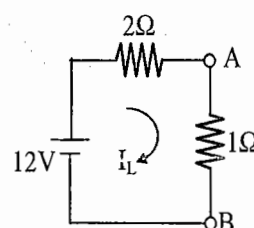
Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,



$$R_{Th} = 2\Omega$$

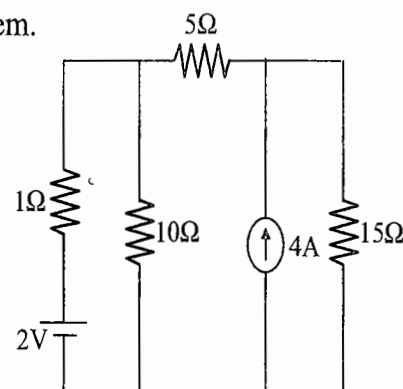
Step III : Calculation of I_L



$$I_L = \frac{12}{2+1} = 4 \text{ A}$$

Example: 2.4

Find the current through the $10\text{-}\Omega$ resistor using Norton's theorem.



Sol: Step I: Calculation of I_{SC}

Applying KVL to Mesh 1,
 $2 - I_1 = 0 \dots\dots\dots (1)$

$$I_1 = 2$$

Meshes 2 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_3 - I_2 = 4 \dots\dots\dots (2)$$

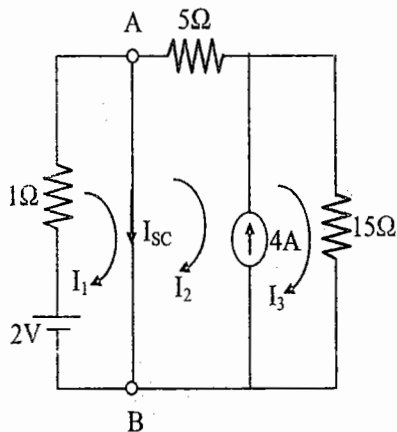
Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \dots\dots\dots (3)$$

Solving equations (1), (2) and (3),

$$I_1 = 2 \text{ A}, I_2 = -3 \text{ A and } I_3 = 1 \text{ A}$$

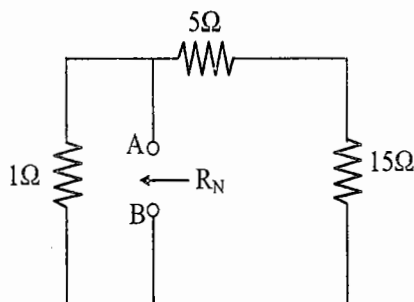
$$I_{SC} = I_1 - I_2 = 2 - (-3) = 5 \text{ A}$$



Step II:

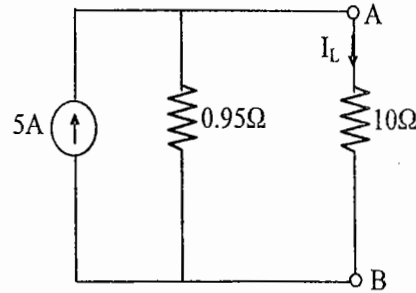
Calculation of R_N

Replacing the voltage source by a short circuit and current source by an open circuit.



$$R_N = 1 \parallel (5+15) = 0.95 \Omega$$

Step III : Calculation of I_L



$$I_L = 5 \times \frac{0.95}{10 + 0.95} = 0.43 \text{ A}$$

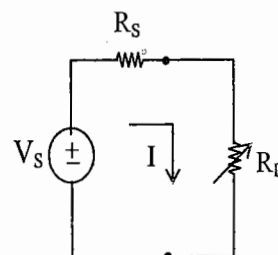
2.4 Maximum Power Transfer Theorem (MPTT)

Properties

- This theorem is applicable only for linear networks i.e. the networks with R,L,C, transformer and linear controlled sources as elements.
- The presence of dependent sources makes the network active and hence, MPTT is used for both active as well as passive networks.
- This theorem is applicable only when load is variable, otherwise (i.e load is fixed) choose the minimum internal impedance of the source, which results in maximum current through the fixed load and hence, a maximum power dissipation across the load.

Under Variable Load Conditions

2.4.1 R_S and R_L :

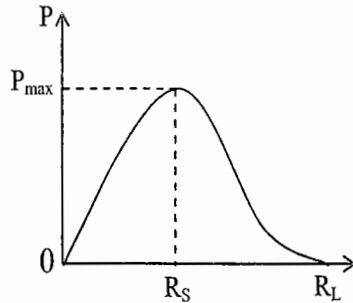




$$I = \frac{V_s}{R_s + R_L}$$

$$P = I^2 \cdot R_L \quad W = V \cdot I = R_L I \cdot I$$

$$P = \frac{V_s^2 \cdot R_L}{(R_s + R_L)^2} (W)$$



$$\frac{dP}{dR_L} = \frac{V_s^2 [(R_s + R_L)^2 \cdot 1 - R_L \cdot 2(R_s + R_L)]}{[(R_s + R_L)^2]^2}$$

$$\text{For MPT} \Rightarrow \frac{dP}{dR_L} = 0$$

$$\Rightarrow R_L = R_s \quad \Omega$$

$$P_{\max} = P_{R_L=R_s} = \frac{V_s^2}{4R_L} W = \frac{V_s^2}{4R_s} W$$

$$P_{\text{del}} = P_{\text{abs}} \text{ (By Tellegen's Theorem)}$$

$$= I^2 \cdot R_s + I^2 R_L \big|_{R_L=R_s}$$

$$= \frac{V_s^2}{2R_L} W = \frac{V_s^2}{2R_s} W$$

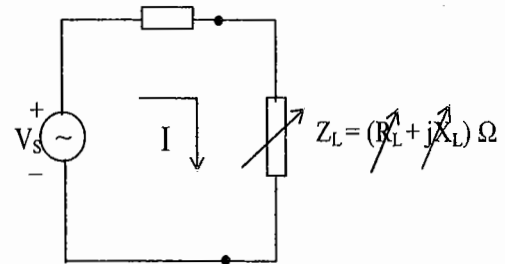
$$\eta = \frac{\text{Useful Power}}{\text{Total Power}} = \frac{4R_L}{\frac{V_s^2}{2R_L}} = \frac{1}{2} = 0.5$$

$$\% \eta = 50\%$$

So the efficiency of Maximum Power Transfer Theorem is atmost 50%.

2.4.2 Z_s and Z_L

$$Z_s = (R_s + jX_s) \Omega$$



$$I = \frac{V_s}{R_s + R_L + j(X_s + X_L)}$$

$$|I| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

$$P = |I|^2 \cdot R_L (W)$$

$$P = \frac{V_s^2 \cdot R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} (W)$$

2.4.2.1 Only ' R_L ' is a variable

$$\frac{dP}{dR_L} = \frac{V_s^2 [(R_s + R_L)^2 + (X_s + X_L)^2 \cdot 1 - R_L \cdot 2(R_s + R_L)]}{[(R_s + R_L)^2 + (X_s + X_L)^2]^2}$$

$$\text{or MPT} \frac{dP}{dR_L} = 0$$

$$\Rightarrow R_L = \sqrt{R_s^2 + (X_s + X_L)^2} \quad \Omega$$

$$P_{\max} = P \bigg|_{R_L = \sqrt{R_s^2 + (X_s + X_L)^2}}$$

2.4.2.2 Only ' X_L ' is variable

$$\frac{dP}{dX_L} = \frac{V_s^2 [(R_s + R_L)^2 + (X_s + X_L)^2 \cdot 0 - R_L \cdot 2(X_s + X_L)]}{[(R_s + R_L)^2 + (X_s + X_L)^2]^2}$$

$$\text{For MPT, } \frac{dP}{dX_L} = 0$$

$$\Rightarrow X_S + X_L = 0$$

$$P_{\max} = P \mid_{X_S + X_L = 0}$$

2.4.2.3 Both R_L & X_L are varied simultaneously

In this case, both the conditions

$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$ & $X_S + X_L = 0$ are valid at a time

$$\Rightarrow R_L = R_S$$

$$X_L = -X_S$$

$$\text{So, } Z_L = R_L + jX_L = R_S - jX_S$$

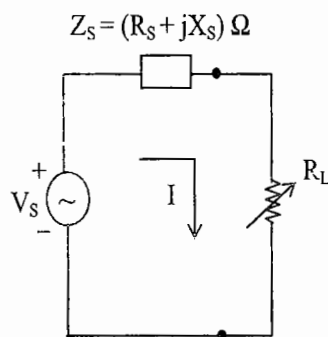
$$Z_L = Z_S^* \Omega$$

$$P_{\max} = P \mid_{R_L = R_S}$$

$$X_L = -X_S$$

$$= \frac{V_S^2}{4R_L} (\text{W}) = \frac{V_S^2}{4R_S} (\text{W})$$

2.4.3 Z_S and R_L :



It is a special case of 2.4.2.1 with $X_L = 0$.

i.e $Z_L = R_L$ only when $X_L = 0$

$$\text{So for MPT } R_L = \sqrt{R_S^2 + (X_S + X_L)^2} \mid_{X_L = 0}$$

$$R_L = \sqrt{R_S^2 + X_S^2} \Omega$$

$$R_L = |R_S + jX_S|$$

$$R_L = |Z_S| \Omega$$

$$P_{\max} = P \mid_{R_L = \sqrt{R_S^2 + X_S^2}}$$

$$\% \eta < 50\% \text{ (nearer to 50\%)}$$

2.5 Reciprocity Theorem

In a linear, passive and bilateral network, the ratio of response to excitation is constant even though the source is interchanged from input terminals to output terminals.

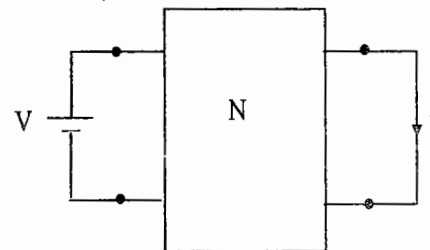


Fig (a)

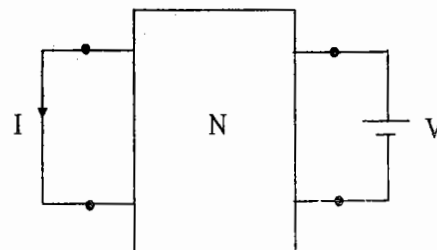


Fig (b)

$$\text{i.e., } \frac{I}{V} = \text{constant} \quad \frac{I_1}{V_1} = \frac{I_2}{V_2}$$

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Properties

- This theorem is applicable only for linear, passive and bilateral networks, i.e. networks with R, L, C and Transformer as elements, so called reciprocal Networks.
- The presence of the dependent sources makes the network active and hence the reciprocity theorem is not applicable, so called the Non-Reciprocal Network.

Example: 2.5

Verify reciprocity principle in finding current I_1 shown in figure (b) by using the data given in figure (a) below

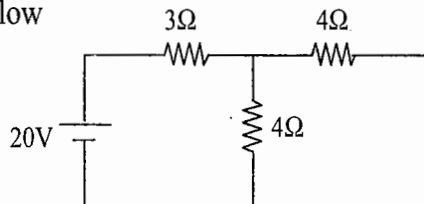


Fig (a)

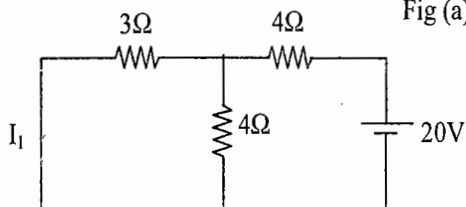
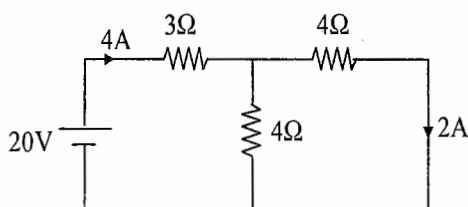
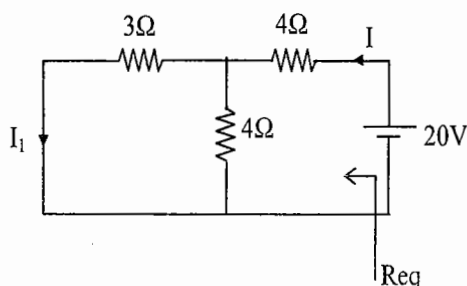


Fig (b)

Sol: First calculating the current in each branch of figure (a) given



Now interchange the position of excitation and then determining the response as below:

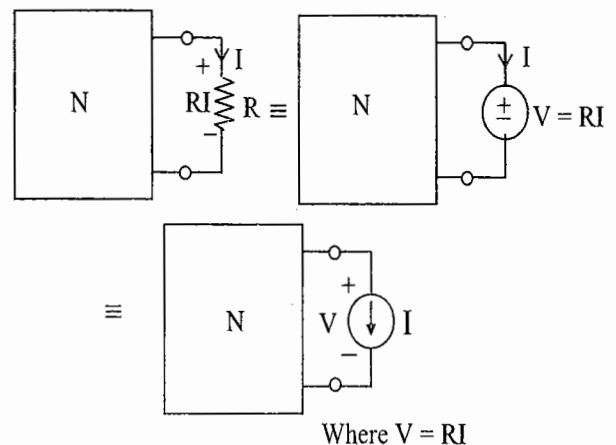


$$R_{eq} = 4 + 3 \parallel 4 = 4 + \frac{3 \times 4}{7} = \frac{40}{7} \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{20}{\frac{40}{7}} = \frac{7}{2} A \quad I_1 = \frac{7}{4+3} \cdot 4 = 2A$$

2.6 Substitution Theorem

- Any branch in a linear network can be substituted by a different branch without disturbing the voltages and currents in the entire network, provided the new branch has the same set of terminal voltage and current as the original network.
- In a linear network any passive element can be equivalently substituted by an ideal voltage source or an ideal current source, provided all the other branch currents and voltages are kept constant. This is possible only when the original passive element and the substituted active sources absorb same power.



$$P_R = P_V = P_I = V \cdot I = RI \cdot I = I^2 \cdot R \text{ (abs.)}$$

Properties

- This theorem is applicable only for linear networks i.e. the networks with R, L, C,

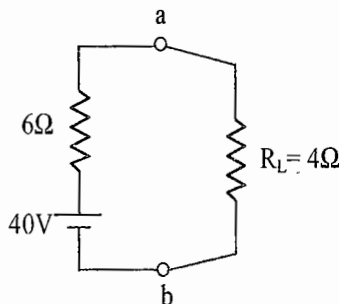


transformer and linear controlled sources as elements.

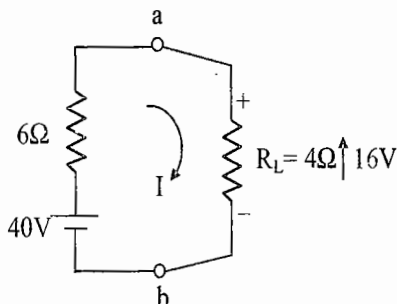
- The presence of dependent sources makes the network active and hence substitution theorem is used for both active as well as passive networks.

Example: 2.6

Use substitution theorem to represent the load resistance R_L for the circuit shown below atleast in four different ways.



Sol: For any bilateral network, if the voltage across any branch along with the current through that branch are known, this branch can be replaced by any combination of elements so that the voltage across the chosen branch and the current through the chosen are identical with those of the previous branch. Let us find the branch equivalence of R_L

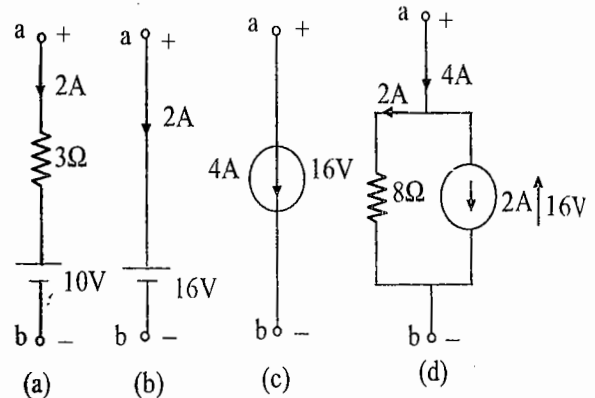


From figure, it can be written as

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{40}{6 + 4} = 4 \text{ A}$$

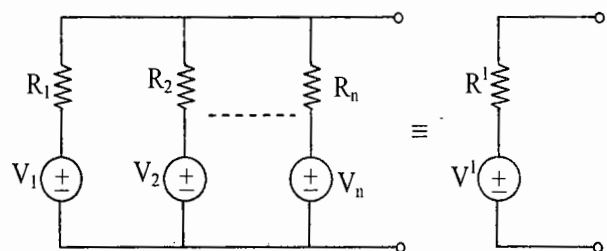
$$V_{ab} = IR_L = 4 \times 4 = 16 \text{ V}$$

The equivalents of the a-b branch (R_L) are shown below in four different ways by using ohms Law.



2.7 Millman's Theorem (or) Parallel Generator Theorem

By Millman's theorem, we can replace 'n' number of parallel Thevenin's equivalents into a single Thevenin's equivalent.



$$V^1 = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

$$\frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$\Rightarrow G^1 = G_1 + G_2 + \dots + G_n$$

$$\Rightarrow R^1 = \frac{1}{G^1} = \frac{1}{G_1 + G_2 + \dots + G_n}$$

Note: In the above case, if polarity of the source ' V_2 ' is reversed, then ' V_2 ' is replaced by ' $-V_2$ ' in the expression V^1 .

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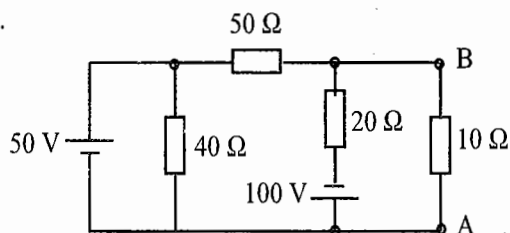


Properties

- This theorem is applicable only for linear networks i.e. the networks with R, L, C, transformer and linear controlled sources as elements.
- The presence of dependent sources makes the network active and hence Millman's theorem is used for both active as well as passive networks.

Example: 2.7

In the network shown in Fig., using Millman's theorem, or otherwise find the voltage between A and B.



Sol: The given circuit is shown in Fig. 1, where 40Ω across 50V can be deleted.

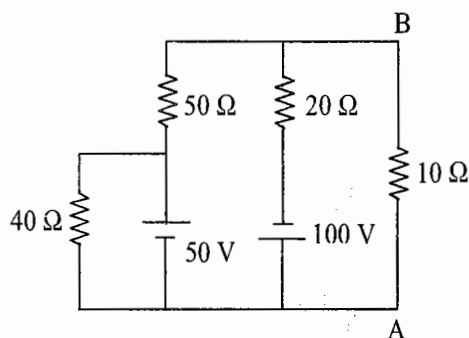


Fig. 1

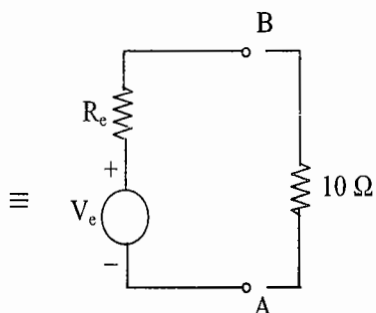


Fig. 2

V_{BA} is found by reducing the given circuit to the left of BA into a single voltage

source, V_e and a series resistance, R_e by using Millman's theorem.

The equivalent circuit is shown in Fig. 2

$$V_e = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} = \frac{\frac{50}{50} - \frac{100}{20}}{\frac{1}{50} + \frac{1}{20}}$$

$$= \frac{-4 \times 100}{7} = -\frac{400}{7} \text{ V}$$

$$R_e = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{50} + \frac{1}{20}} = \frac{100}{7} \Omega$$

$$V_{BA} = -\frac{400}{7} \times \frac{10}{\frac{100}{7} + 10}$$

$$= -\frac{400}{7} \times 10 \times \frac{7}{170} = -\frac{400}{17} \text{ V}$$

2.8 Principle of Duality

- Two linear circuits are said to be duals of one another if they are described by the same characterizing equations with dual quantities interchanged.
- Network and its dual are same only with respect to the performance, but the elements and connections point of view, they are not equal.

Dual Pairs

Resistance R	$\leftrightarrow$	Conductance G
Inductance L	$\leftrightarrow$	Capacitance C
Impedance Z	$\leftrightarrow$	Admittance Y
Voltage v	$\leftrightarrow$	Current i
Voltage source	$\leftrightarrow$	Current source
Node	$\leftrightarrow$	Mesh
Series path	$\leftrightarrow$	Parallel path
Open circuit	$\leftrightarrow$	Short circuit
KCL	$\leftrightarrow$	KVL
Thevenin	$\leftrightarrow$	Norton



Star network $\leftrightarrow$ Delta network

$R i(t) \leftrightarrow G v(t)$

$L \frac{di(t)}{dt} \leftrightarrow C \frac{dv(t)}{dt}$

$\frac{1}{C} \int i(t) dt \leftrightarrow \frac{1}{L} \int v(t) dt$

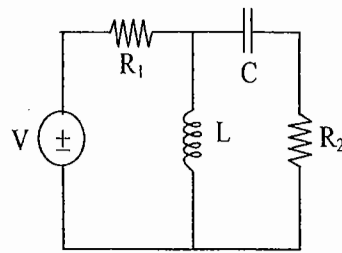
- The number of mesh equations in the original network is equal to the number of nodal equations in its dual network and vice versa.
- 'N' nodal equations represents (N+1) principle nodes.

The following steps are involved in constructing the dual of a network :

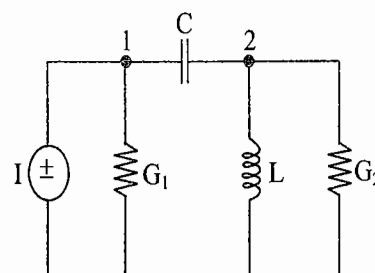
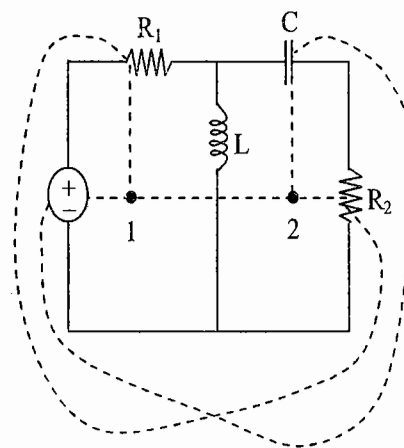
- Place a node inside each mesh of the given network. These internal nodes correspond to the independent nodes in the dual network.
- Place a node outside the given network. The external node corresponds to the datum node in the dual network.
- Connect all internal nodes in the adjacent mesh by dashed lines crossing the common branches. Elements which are the duals of the common branches will form the branches connecting the corresponding independent node in the dual network.
- Connect all internal nodes to the external node by dashed lines corresponding to all external branches. Duals of these external branches will form the branches connecting independent nodes and the datum node.
- A clockwise current in a mesh corresponds to a positive polarity (with respect to the datum node) at the dual independent node.
- A voltage rise in the direction of a clockwise mesh current corresponds to a current flowing towards the dual independent node.

Example: 2.8

Draw the dual of the network shown in Figure.



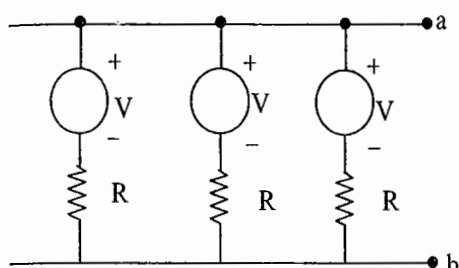
- Sol:**
- Place a node inside each mesh.
 - Place a node outside the mesh which will correspond to the datum node.
 - Connect two internal nodes through a dashed line. The element which is dual of the common branch (here capacitance) will form the branch connecting the corresponding independent node in the dual network.
 - Connect all internal nodes to the external node by dashed lines crossing all the branches. The dual of these branches will form the branches connecting the independent node and datum.



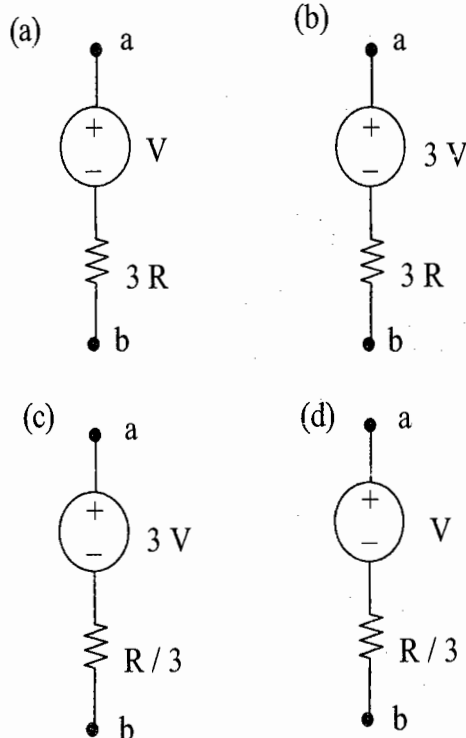
Class Room Practice Questions

LEVEL-1 Questions

01. The equivalent circuit of the following circuit is (IES-EE-92)



Fig



02. If the combined generator and line impedance is $(5 + j 10)$ ohm, then for the maximum power transfer to a load impedance from a generator of constant generator voltage, the load impedance is given by which one of the following (IES-EE-02)

- (a) $(5 + j 10) \Omega$ (b) $(5 - j 10) \Omega$
(c) $(5 + j 5) \Omega$ (d) 5Ω

03. Superposition theorem is not applicable for (IES-EE-02)

- (a) voltage calculations
(b) bilateral elements
(c) power calculations
(d) passive elements

04. A voltage source having an internal impedance of $(8 + j6)$ ohms supplies power to a resistive load. What should be the load resistance for maximum power transferred to it? (IES-EE-04)

- (a) 8 ohms (b) 6 ohms
(c) 10 ohms (d) $\sqrt{10}$ ohms

05. A load is connected to an active network. At the terminals to which the load is connected, $R_{th} = 10\Omega$ and $V_{th} = 60V$. Then maximum power supplied to the load is (IES-EE-10)

- (a) 360 W (b) 90 W
(c) 60 W (d) 10 W

06. The maximum power will be transferred from a voltage source to a load when (IES-EE-12)

- (a) the source impedance is half that of the load impedance
(b) the source impedance is equal to that of the load impedance
(c) the source impedance is twice that of the load impedance
(d) both source and load impedances must be zero

07. A source having internal impedance of $(9 + j12)\Omega$ is to deliver maximum power to a resistive load. The load resistance should be (IES-EC-10)

- (a) 9Ω (b) 12Ω
(c) 15Ω (d) 21Ω



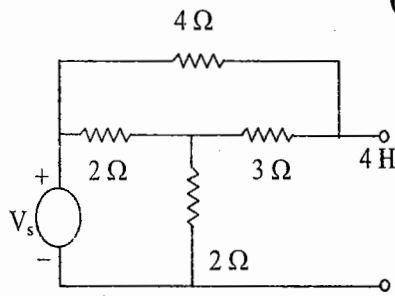
08. The maximum power that a 12V DC source with an internal resistance of 2Ω can supply to a resistive load is **(IES-EC-12)**

(a) 72W (b) 48W
(c) 24W (d) 18W

09. Kirchhoff's current and voltage laws apply for **(AEE-2012)**

(a) resistive circuits only
(b) linear circuits only
(c) nonlinear circuits only
(d) both linear and nonlinear circuits

10. In the circuit of fig below R_{th} is given by **(HMWS)**



(a) 5Ω (b) 2Ω
(c) 4Ω (d) 6Ω

11. Tellegen's theorem is applicable to **(HMWS)**
(a) linear networks only
(b) non-linear network only
(c) both linear and non-linear networks
(d) neither linear nor non-linear networks

12. The Nodal analysis is primarily based on the application of **(EPDCL-12)**

(a) Ohm's law
(b) Kirchhoff's current law
(c) Kirchhoff's voltage law
(d) Both (1) and (2)

13. Maximum power transfer takes place at an efficiency of **(SPDCL-12)**

(a) 25% (b) 75%
(c) 50% (d) 100%

14. Superposition theorem requires as many circuits to be solved as there are

(a) sources **(SPDCL-12)**
(b) nodes
(c) sources + nodes
(d) sources + nodes + meshes

LEVEL-2 Questions

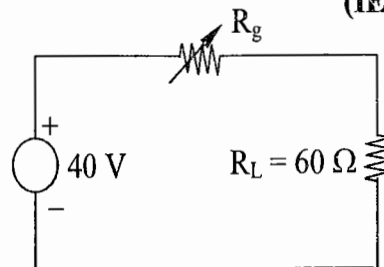
01. An ac source of voltage E_s and an internal impedance of $Z_s = (R_s + jX_s)$ is connected to a load of impedance $Z_L = (R_L + jX_L)$. Consider the following conditions in this regard.

1. $X_L = X_s$, if only X_L is varied.
2. $X_L = -X_s$, if only X_L is varied.
3. $R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$, if only R_L is varied.
4. $|Z_L| = |Z_s|$, if the magnitude alone of Z_L is varied, keeping the phase angle fixed.

Among these conditions, those which are to be satisfied for maximum power transfer from the source to the load would include **(IES-EE-93)**

(a) 2 and 3 (b) 1 and 3
(c) 1, 2 and 4 (d) 2, 3 and 4

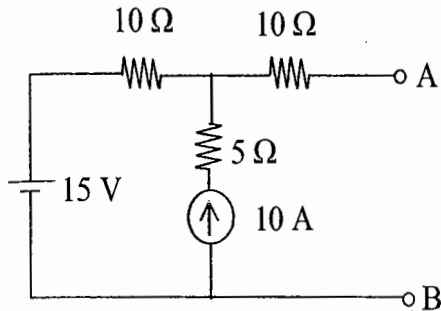
02. If R_g in the circuit shown in the given figure is variable between 20Ω and 80Ω then the maximum power transferred to load R_L will be **(IES-EE-95)**



(a) 15 W (b) 13.33 W
(c) 6.67 W (d) 2.4 W

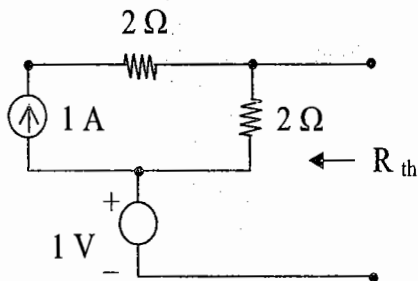


03. In the network shown in the given figure, the Thevenin source and the impedance across terminals A - B will be respectively (IES-EE-99)



- (a) 15 V and 13.33 Ω
(b) 50 V and 15 Ω
(c) 115 V and 20 Ω
(d) 100 V and 25 Ω

04. The Thevenin's equivalent resistance R_{th} for the given network is (IES-EE-01)



- (a) 1 Ω (b) 2 Ω
(c) 4 Ω (d) Infinity

05. Applying Norton's Theorem, the Norton's equivalent circuit to the left of the terminals a and b in the below circuit is having equivalent current source (I_N) and equivalent resistance (R_N) as (IES-EE-10)

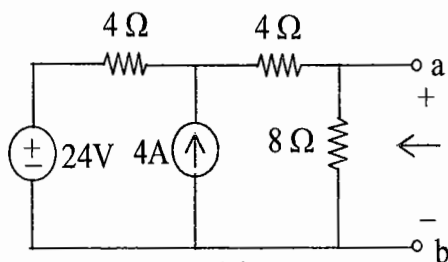


Fig.

- (a) $I_N = 5$ A ; $R_N = 4$ Ω
(b) $I_N = 4$ A ; $R_N = 6$ Ω
(c) $I_N = 9$ A ; $R_N = 1.6$ Ω
(d) $I_N = 4$ A ; $R_N = 3$ Ω

06. In the lattice network, find the value of R for the maximum power transfer to the load. (IES-EC-92)

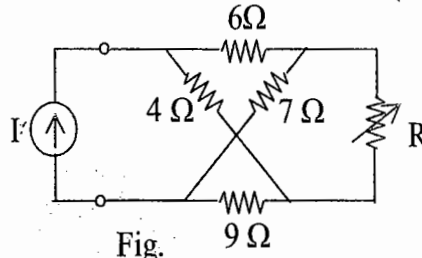


Fig.

- (a) 5 (b) 6.5 (c) 8 (d) 9

07. In the network shown in the given figure current $i = 0$ when $E = 4$ V, $I = 2$ A and $i = 1$ A when $E = 8$ V, $I = 2$ A. The Thevenin voltage and the resistance looking into the terminals AB are (IES-EC-96)

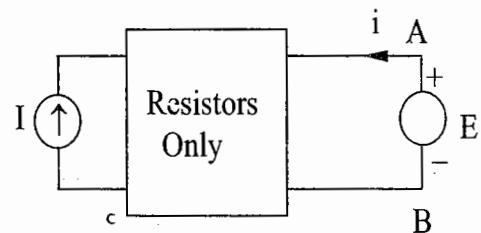


Fig.

- (a) 4 V, 2 Ω (b) 4 V, 4 Ω
(c) 8 V, 2 Ω (d) 8 V, 4 Ω

08. Which one of the following is applicable to any network-linear or non-linear, active or passive, time-varying or invariant-as long as Kirchhoff's laws are not violated?

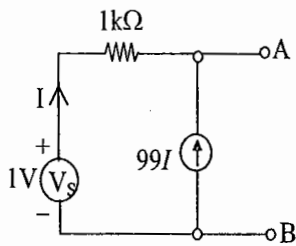
(IES-EC-09)

- (a) Tellegen's theorem
(b) Reciprocity theorem
(c) Maximum power transfer theorem
(d) Superposition theorem



09. Thevenin's equivalent resistance as seen from the terminals AB for the circuit is

(IES-EC-12)



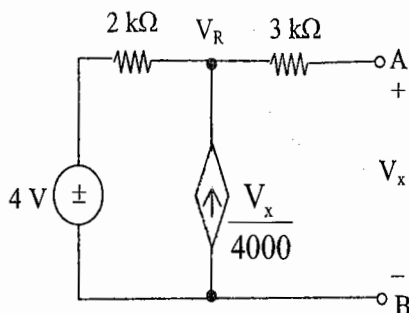
- (a) $1k\Omega$ (b) 10Ω
(c) 100Ω (d) $10k\Omega$

10. A voltage source V with internal impedance $R + jX$ is driving such a load as to have maximum power transfer. Power transfer to load is given by

(HMWS)

- (a) $\frac{V^2}{R}$ (b) $\frac{V^2}{2R}$
(c) $\frac{V^2}{4R}$ (d) $\left(\frac{V}{R + X}\right)^2 R$

11. For the circuit shown in figure. The Thevenin's voltage and resistance at terminals A and B respectively, are.



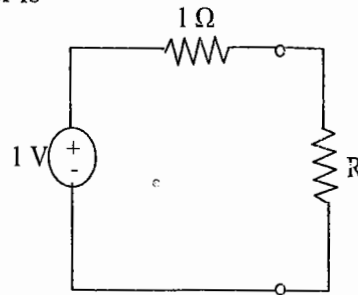
(JTO)

- (a) 8 V and $5k\Omega$
(b) 8 V and $10k\Omega$
(c) 4V and $5k\Omega$
(d) 4 V and $10k\Omega$

12. For the circuit shown in figure a value of R (in Ω) to which the voltage source would

- deliver 50% of the maximum deliverable power is

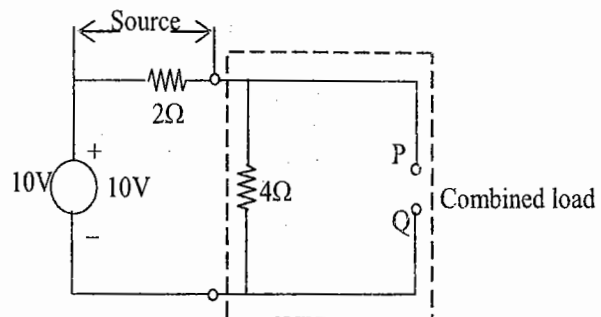
(JTO)



- (a) 1 (b) 0.5
(c) 0.25 (d) $3 - \sqrt{8}$

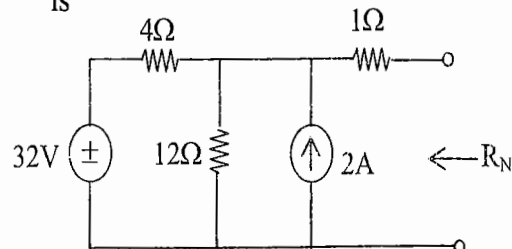
13. The resistance which should be connected between terminals P and Q for maximum transfer of power from source to the combined load is

(SPDCL-12)



- (a) Zero Ω (b) 2Ω
(c) 4Ω (d) infinity ohms

14. The Norton's resistance of the circuit shown is



(GENCO-12)

- (a) 17Ω (b) 3Ω
(c) 4Ω (d) 0.9Ω

15.

16.

01.

a

b

c

02.

So



15. Kirchhoff's current law is based on the law of (GENCO-12)
 (a) conservation of energy
 (b) conservation of charge
 (c) conservation of momentum
 (d) conservation of mass
16. Kirchhoff's voltage and current laws cannot be applied at (SAIL-13)
 (a) low frequency
 (b) constant temperature
 (c) linear circuit
 (d) High frequency

Solutions

LEVEL-1

01. Ans: (d)

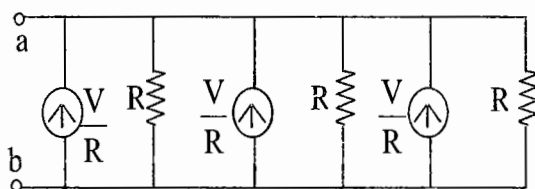


Fig.1

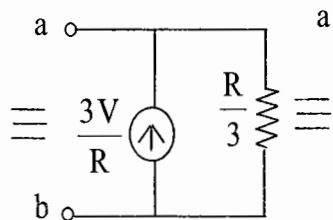


Fig.2

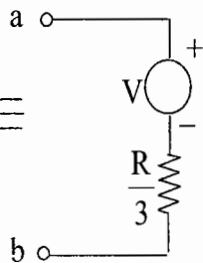


Fig.3

The equivalent circuit across the terminals a, b is obtained by simplifying the circuit as shown in Fig. 1, Fig. 2 and Fig. 3.

02. Ans: (b)

Sol: For maximum power transfer,
 $Z_L = Z^* = (5 - j 10) \Omega$

03. Ans: (c)

Sol: Superposition theorem is not applicable for power calculations as Power is proportional to V^2 or I^2 .

04. Ans: (c)

Sol: $R_L = \sqrt{R_s^2 + X_s^2} = \sqrt{64 + 36} = 10 \Omega$

05. Ans: (b)

Sol:

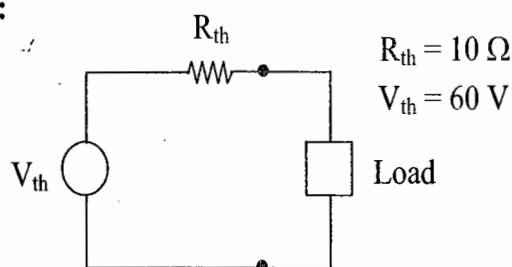


Fig. 1

Referring to Fig. 1.

Maximum power is supplied to the load, when load is $R_L = R_{th} = 10 \Omega$

$$\therefore I_L = \frac{60}{10+10} = 3 \text{ A}$$

$$\therefore \text{Maximum power} = I_L^2 R_L = 90 \text{ W}$$

06. Ans: (b)

Sol: When the source impedance is equal to load impedance maximum power will transfer to load.

07. Ans: (c)

Sol: $Z_i = (9 + j 12) \Omega$

For maximum power transfer, resistive load,

$$R_L = \sqrt{R_i^2 + X_i^2} = \sqrt{9^2 + 12^2} = 15 \Omega$$

08. Ans: (d)

Sol: The maximum power that can be delivered

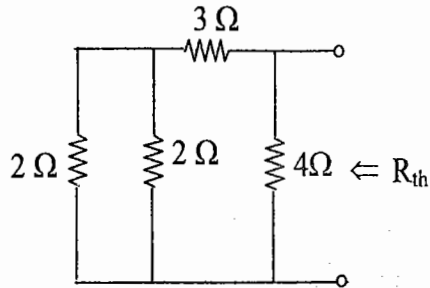
$$\text{is } P_{\max} = \frac{V_s^2}{4R_L} = \frac{(12)^2}{4(2)} = 18 \text{ Watts}$$



09. Ans: (b)

10. Ans: (b)

Sol: R_{th} is the resistance seen through load terminals when all Independent sources are replaced by their internal resistance



$$R_{th} = 4 \parallel (3+1) = 2\ \Omega$$

11. Ans: (c)

12. Ans: (d)

13. Ans: (c)

14. Ans: (a)

Level-2 Key

01. (d) 02. (a) 03. (c) 04. (b) 05. (a)

06. (b) 07. (b) 08. (a) 09. (c) 10. (c)

11. (a) 12. (a) 13. (c) 14. (c) 15. (b)

16. (d)

3

Chapter

Transient Circuit Analysis



Pierre-Simon-Laplace
(1749-1827)

3.1 Laplace Transform Application for Solving Steady State AC Circuits

f(t)	F(s)	f(t)	F(s)
i(t)	I(s)	$\delta(t)$	1
v(t)	V(s)	u(t)	1/s
R	R	r(t)	1/s ²
L	SL	p(t)	1/s ³
C	$\frac{1}{SC} \Omega$	$e^{-at}u(t)$	$\frac{1}{s+a}$
		ku(t)	k/s

$$\frac{V(s)}{I(s)} = Z(s) \Omega$$

$$\Rightarrow V(s) = Z(s).I(s) \Rightarrow I(s) = \frac{V(s)}{Z(s)}$$

$$\frac{I(s)}{V(s)} = Y(s) \mathcal{U} \Rightarrow V(s) = \frac{I(s)}{Y(s)}$$

$$\Rightarrow I(s) = Y(s).V(s)$$

$$Y(s) = \frac{1}{Z(s)} \mathcal{U}$$

Z(s)	Y(s)
$R \Omega$	$\frac{1}{R} \mathcal{U}$
$SL \Omega$	$\frac{1}{SL} \mathcal{U}$
$\frac{1}{SC} \Omega$	$SC \mathcal{U}$

We can add impedances in series and admittances in parallel.

Example: 3.1

Find out the Laplace transform of the shifted gate pulse shown below.

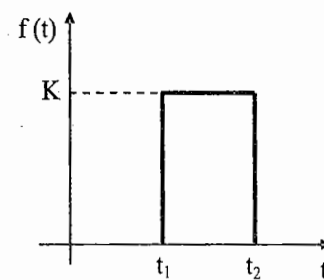
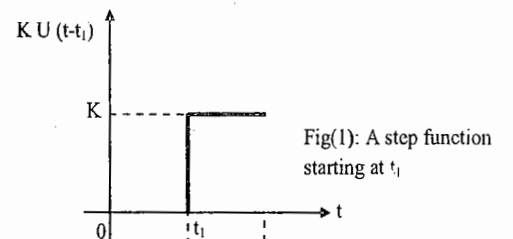
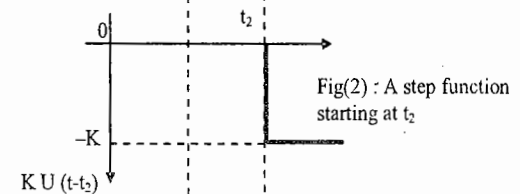


Fig: A shifted gate function

Sol: This shifted gate function can be synthesized with the help of two step functions as follows:



Fig(1): A step function starting at t₁



Fig(2): A step function starting at t₂

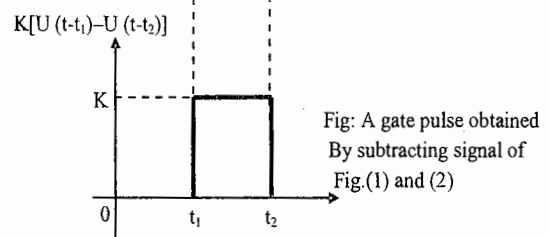


Fig: A gate pulse obtained By subtracting signal of Fig.(1) and (2)

Pierre-Simon, marquis de Laplace was a French mathematician and astronomer formulated Laplace's equation, and pioneered the Laplace transform which appears in many branches of mathematical physics.



The above waveforms show that the gate pulse can be represented as,

$$f(t) = K[u(t-t_1) - u(t-t_2)] \dots$$

$$\ell[f(t)] = \ell\{K[u(t-t_1) - u(t-t_2)]\}$$

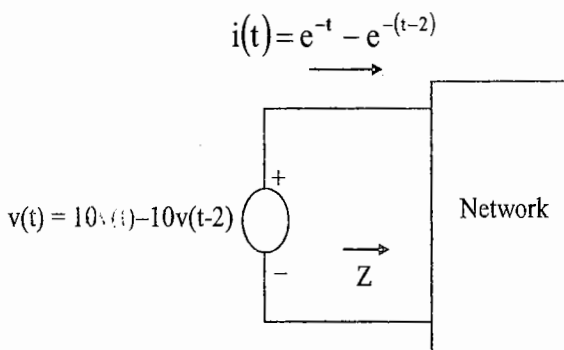
$$= K\{\ell[u(t-t_1)] - \ell[u(t-t_2)]\}$$

$$= K\left\{\frac{1}{s}e^{-st_1} - \frac{1}{s}e^{-st_2}\right\} \text{ by equation ..}$$

$$= \frac{K}{s}(e^{-st_1} - e^{-st_2})$$

Example: 3.2

Figure shows an impedance whose voltage and current are as shown.



Find out the impedance Z of the network using laplace transform method.

Sol: Let us first evaluate the Laplace transforms of $v(t)$ and $i(t)$.

$$V(t) = 10u(t) - 10u(t-2)$$

By taking Laplace transform of above equation using standard transform tables, we get,

$$\begin{aligned} V(s) &= 10 \cdot \frac{1}{s} - 10 \cdot \frac{1}{s} e^{-2s} \\ &= \frac{10}{s}(1 - e^{-2s}) \end{aligned}$$

The current is given as, $i(t) = e^{-t} - e^{-(t-2)}$

Taking Laplace transform of above equation, we get,

$$I(s) = \frac{1}{s+1} - \frac{1}{s+1} e^{-2s} = \frac{1 - e^{-2s}}{s+1}$$

The driving point impedance $Z(s)$ is given as,

$$\begin{aligned} Z(s) &= \frac{V(s)}{I(s)} \\ &= \frac{\frac{10}{s}(1 - e^{-2s})}{\frac{1 - e^{-2s}}{s+1}} = 10 \cdot \frac{s+1}{s} = 10\left(1 + \frac{1}{s}\right) \\ &= 10 + \frac{10}{s} = 10 + \frac{1}{\frac{1}{10}s} \end{aligned}$$

We know that Laplace transform of R i.e., the value is unchanged and Laplace transform of capacitor is $\frac{1}{sC}$. Thus, the

above impedance appears to be series connection of resistor and capacitor. i.e.,

$$Z(s) = R + \frac{1}{sC} ; R = 10\Omega \text{ and } C = \frac{1}{10}F$$

The circuit will be,

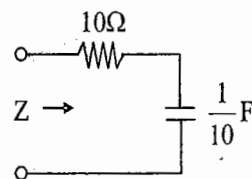
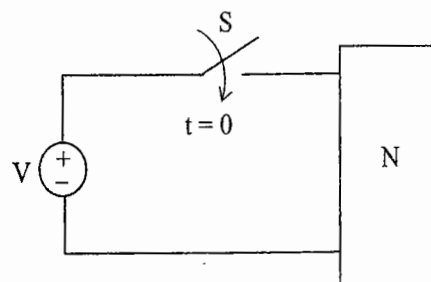


Fig: Impedance of the network

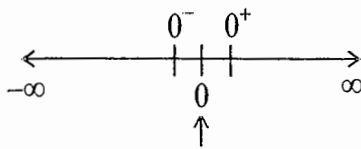
3.2 Introduction to Transient Analysis



Intro

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3.2.1



$$0^- = -0.000 \dots\dots 01$$

$$0^+ = +0.000 \dots\dots 01$$

$$0^+ - 0^- = 0.000 \dots\dots 02$$

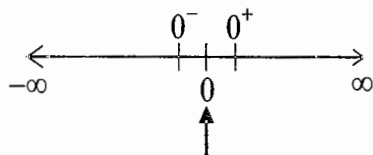
$$= 0.0$$

i.e. within zero time

Introduction:

- Transients in the system is because of the presence of energy storing elements called the inductor and the capacitor. Since the energy stored in a memory element cannot change instantaneously i.e. within zero time, (due to integrations of energy variables i_L and V_C) L and C elements will oppose sudden changes in the system, which results in instability of the system due to severe oscillations.
- When the network consists of only resistances (a memoryless system), no transients occur in the system at the time of switching, since the resistor can accommodate any amount of voltages and currents ($v = Ri$ or $i = v/R$).
- The transient effects are more severe for d.c. as compared to a.c. and the transient-free condition is possible only for a.c. excitations (at zero crossings of sine or cosine).

3.2.1 Behaviour of L and C elements at $t = 0^+$ and as $t \rightarrow \infty$ instants



$$Z_L = sL \Omega$$

$$Z_C = \frac{1}{sC} \Omega$$

$$t = 0^+ \Rightarrow s = \infty \Rightarrow Z_L = \infty \Rightarrow L \rightarrow \text{O.C.}$$

$$Z_C = 0 \Rightarrow C \rightarrow \text{S.C.}$$

$$t \rightarrow \infty \Rightarrow s \rightarrow 0 \Rightarrow Z_L \rightarrow 0 \Rightarrow L \rightarrow \text{S.C.}$$

$$\Rightarrow Z_C \rightarrow \infty \Rightarrow C \rightarrow \text{O.C.}$$

A long time after the switching action ($t \rightarrow \infty$) is the steady state. In steady state, the inductor behaviour is a short circuit behaviour and the capacitor behaviour is an open circuit behaviour.

Steady state or Drunk State

Whenever an independent D.C. source is connected to the network for a long time (i.e. ideally infinite amount of time and practically up to 5 time constants), then the network is said to be in steady state. In steady state, the energy stored in the network is maximum and constant. So energy stored in the memory elements is maximum and constant.

$$\text{i.e. } \frac{1}{2} Li_L^2 = \text{Maximum \& Constant}$$

$$\Rightarrow i_L = \text{Maximum \& Constant}$$

So, inductor will act as a constant current source in steady state.

$$\text{Since } V_L = L \frac{di_L}{dt} \Rightarrow v_L \rightarrow 0 \Rightarrow L \rightarrow \text{S.C.}$$

$$\text{Similarly } \frac{1}{2} C v_C^2 = \text{Maximum \& Constant.}$$

$$\Rightarrow v_C = \text{Maximum \& Constant}$$

So, capacitor will act as a constant voltage source in steady state.

$$\text{Since } i_C = C \frac{dv_C}{dt} \Rightarrow i_C \rightarrow 0 \Rightarrow C \rightarrow \text{O.C.}$$

Observation:

The above discussion is valid only for dc excitations / sources.

What about AC excitations / sources?

Ans: For AC, during the positive slope of the period, both L and C elements are charging

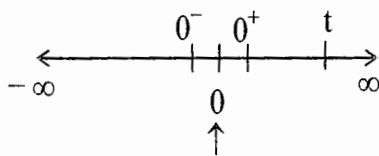


[i.e., storing the energy] and during the negative slope of the period, they are discharging [i.e. delivering the energy], so the net energy observed over the period is zero. And hence, L and C elements are always present for a.c excitations / sources

Blind approach:

- In steady state, $Z_L = j\omega L \Omega$ and $Z_C = \frac{1}{j\omega C} \Omega$
- For A.C, $\omega \neq 0$. And hence $Z_L = j\omega L \Omega (\neq 0)$ and $Z_C = \frac{1}{j\omega C} \Omega (\neq \infty)$ i.e. The impedances are finite and hence, L and C elements are always present for a.c excitations / sources.
- For d.c, $\omega = 0$ and hence
 $Z_L = 0 \Omega \Rightarrow L \rightarrow S.C$
 $Z_C = \infty \Omega \Rightarrow C \rightarrow O.C$
 i.e., The L&C elements will dies for d.c excitations / sources in steady state.

3.2.2. Inductor current at $t = 0^-$ and at $t = 0^+$ instants



$$\begin{aligned} \underline{L:} \quad i_L(t) &= \frac{1}{L} \int_{-\infty}^t V_L(t).dt \\ &= \frac{1}{L} \int_{-\infty}^{0^-} V_L(t).dt + \frac{1}{L} \int_{0^-}^t V_L(t).dt \\ &= i_L(0^-) + \frac{1}{L} \int_{0^-}^t V_L(t).dt \end{aligned}$$

$$\text{At } t = 0^+ \Rightarrow i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} V_L = 0.dt$$

$$i_L(0^+) = i_L(0^-) \text{ A}$$

$$E_L(0^+) = E_L(0^-) \text{ J}$$

So, inductor current cannot change instantaneously i.e. with zero time for all the practical excitations. Similarly the energy.

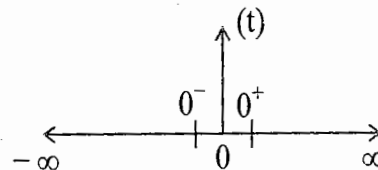
$$\text{If } v_L(t) = \delta(t), \text{ then } i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} \delta(t).dt$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \text{ A}$$

$$E_L(0^+) > E_L(0^-) \text{ J}$$

So, inductor current can change instantaneously i.e. within zero time for impulse voltage across it. Similarly the energy.

More Specifically for impulse:



$$V_L(t) = \delta(t) = 0 \text{ for } -\infty \leq t < 0^-$$

$$\Rightarrow i_L(0^-) = \frac{1}{L} \int_{-\infty}^{0^-} V_L(t).dt = 0 \text{ A}$$

$$i_L(0^+) = \frac{1}{L} \text{ A}$$

$$\begin{aligned} \text{So, } E_L(0^-) &= 0 \text{ J \& } E_L(0^+) = \frac{1}{2} \cdot L \cdot i_L^2(0^+) \\ &= \frac{1}{2} \cdot L \cdot \left(\frac{1}{L}\right)^2 \end{aligned}$$

$$E_L(0^+) = \frac{1}{2L} \text{ J}$$





3.2.3. Capacitor voltage at $t = 0^-$ and at $t = 0^+$ instants

$$C: V_C(t) = \frac{1}{C} \int_{-\infty}^t i_C \cdot dt = V_C(0^-) + \frac{1}{C} \int_{0^-}^t i_C \cdot dt$$

At $t = 0^+ \Rightarrow V_C(0^+) = V_C(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i_C \cdot dt = 0$

$$V_C(0^+) = V_C(0^-) \text{ V}$$

For all practical inputs

$$E_C(0^+) = E_C(0^-) \text{ J}$$

More Specifically for impulse:

$$i_C = \delta(t) = 0 \text{ for } -\infty \leq t < 0^-$$

$$\Rightarrow V_C(0^-) = \frac{1}{C} \int_{-\infty}^{0^-} i_C \cdot dt = 0 \text{ V}$$

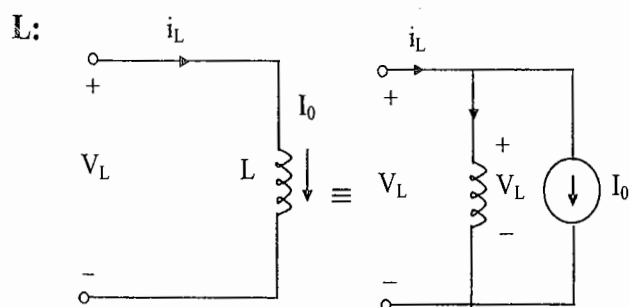
$$V_C(0^+) = \frac{1}{C} \text{ V}$$

So, $E_C(0^-) = 0 \text{ J}$ & $E_C(0^+)$

$$= \frac{1}{2} \cdot C \cdot V_C^2(0^+) = \frac{1}{2} \cdot C \cdot \left(\frac{1}{C}\right)^2$$

$$E_C(0^+) = \frac{1}{2C} \text{ J}$$

3.2.4 Equivalent Circuits:

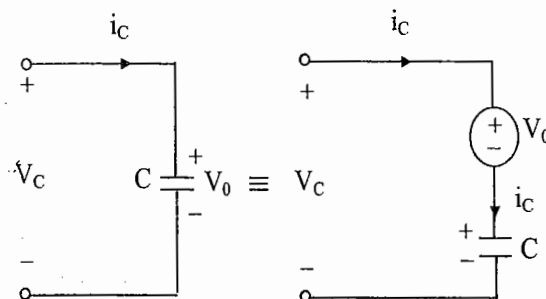


$$i_L = \frac{1}{L} \int_{-\infty}^t V_L \cdot dt \Rightarrow i_L = i_L(0) + \frac{1}{L} \int_{0^-}^t V_L \cdot dt \quad | i_L(0) = I_0$$

Proof: By KCL,

$$-i_L + \frac{1}{L} \int_{0^-}^t V_L \cdot dt + I_0 = 0 \Rightarrow i_L = I_0 + \frac{1}{L} \int_{0^-}^t V_L \cdot dt$$

C:



$$V_C = \frac{1}{C} \int_{-\infty}^t i_C \cdot dt$$

$$V_C = V_C(0) + \frac{1}{C} \int_{0^-}^t i_C \cdot dt \quad | V_C(0) = V_0$$

Proof: By KVL

$$\Rightarrow V_C - V_0 - \frac{1}{C} \int_{0^-}^t i_C \cdot dt = 0$$

$$\Rightarrow V_C = V_0 + \frac{1}{C} \int_{0^-}^t i_C \cdot dt$$

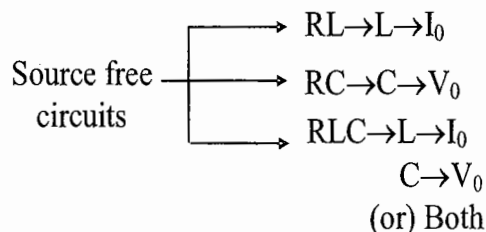
• Classification of Transients:

- i) D.C Transients
- ii) A.C Transients

3.3. DC Transients

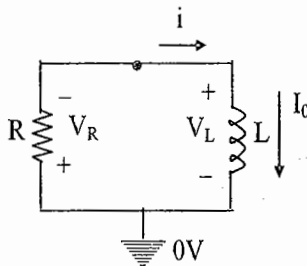
3.3.1 Source free circuits

(i.e. without independent sources)



Property: In all the source free circuits, stored energies in the memory elements are maximum at $t = 0$ and as a function of time, these stored energies will be delivered to the memoryless resistances in an exponentially decayed manner and hence, the energy present in the network at steady state ($t \rightarrow \infty$) is **zero**.

3.3.1.1 Sourcefree RL circuit



For $t \geq 0$;

By KVL

$$-V_R - V_L = 0$$

$$-R \cdot i - L \cdot \frac{di}{dt} = 0 \Rightarrow \frac{di}{dt} + \frac{R}{L} i = 0$$

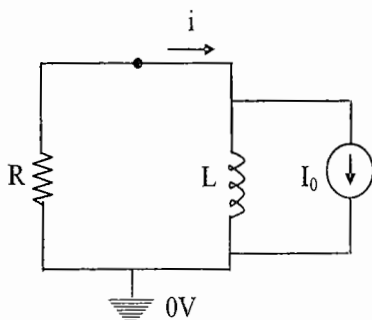
$$\text{Let } D = \frac{d}{dt} \Rightarrow \left(D + \frac{R}{L} \right) i = 0$$

$$\begin{aligned} \text{Characteristic equation } \Rightarrow D + \frac{R}{L} &= 0 \\ \Rightarrow D &= -\frac{R}{L} \end{aligned}$$

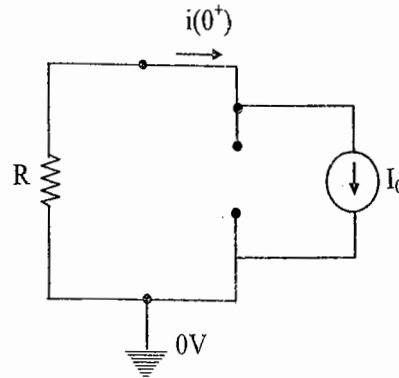
It is like $S = \alpha$

$$i(t) = K e^{\alpha t} \text{ for } 0 \leq t \leq \infty$$

$$= K e^{-\frac{R}{L} t} \text{ A for } 0 \leq t \leq \infty$$



For $t \geq 0$



At $t = 0^+$: A resistive circuit

$$\text{By KCL } \Rightarrow -i(0^+) + I_0 = 0$$

$$\Rightarrow i(0^+) = I_0$$

$$\text{So, } t = 0^+ \Rightarrow i(0^+) = k e^{-0} \Rightarrow k = i(0^+) = I_0$$

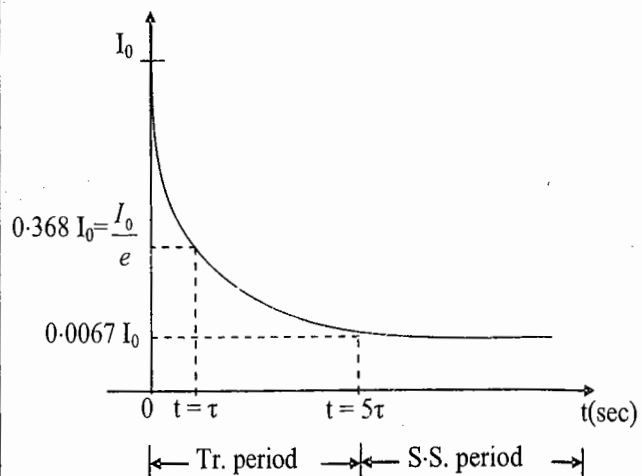
$$\Rightarrow i(t) = I_0 e^{-\frac{R}{L} t} \text{ A for } 0 \leq t \leq \infty$$

$$\Rightarrow i(t) = I_0 e^{-\frac{t}{\tau}} \text{ A for } 0 \leq t \leq \infty$$

$$\tau = \frac{L}{R} = \text{sec} = \text{Time constant of the circuit}$$

$$V_L(t) = L \frac{di(t)}{dt} \text{ by Ohm's law}$$

$i(t)$, Amp



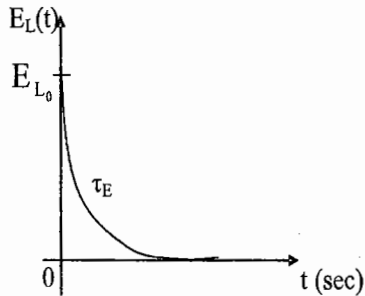
At $t = 5\tau$,

$$i(t) = I_0 e^{-5} = \frac{I_0}{e^5} = 0.0067 I_0 \approx 0.0$$



$$E_L(t) = \frac{1}{2} \cdot L \cdot i^2(t) (J) = \frac{1}{2} \cdot L \cdot \left(I_0 e^{-\frac{t}{\tau_i}} \right)^2$$

$$= E_{L_0} e^{-\frac{t}{\tau_E}} (J) \text{ for } 0 \leq t \leq \infty$$

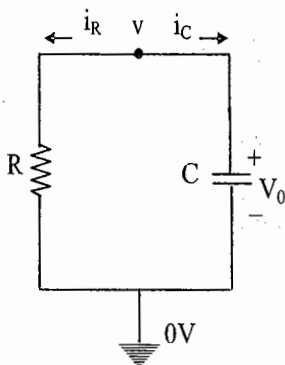


$$\tau_E = \frac{\tau_i}{2} = \frac{L}{2R} \text{ sec}$$

So the energy decay is 2 times faster than the current decay.

3.3.1.2. Sourcefree RC circuit

For $t \geq 0$

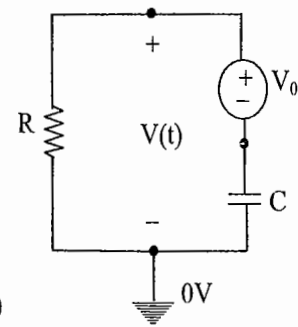


By KCL

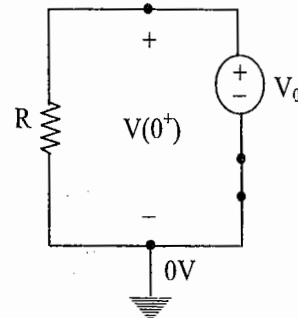
$$\Rightarrow i_R + i_C = 0 \Rightarrow \frac{V}{R} + C \frac{dV}{dt} = 0$$

$$\Rightarrow \frac{dV}{dt} + \frac{V}{RC} = 0 \Rightarrow (D + \frac{1}{RC})V = 0$$

$$\Rightarrow V(t) = K e^{-\frac{t}{RC}} V \text{ for } 0 \leq t \leq \infty$$



For $t \geq 0$



At $t = 0^+$; A resistive circuit

$$\Rightarrow V(0^+) = V_0$$

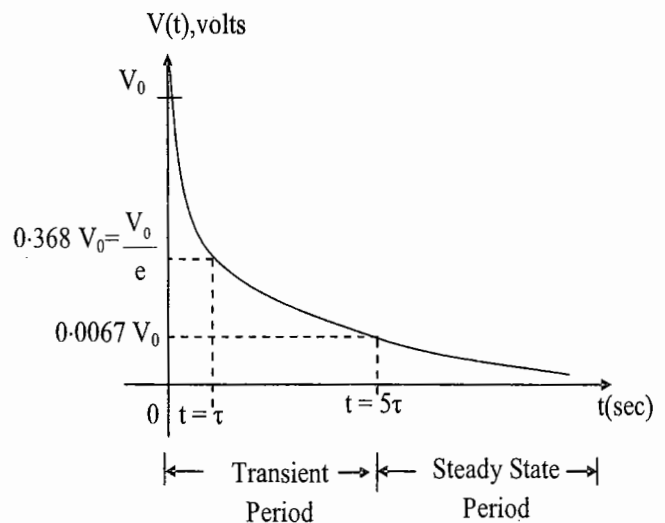
$$\text{So, } t = 0^+ \Rightarrow V(0^+) = K e^{-0} \Rightarrow K = v(0^+) = V_0$$

$$V(t) = V_0 e^{-\frac{t}{RC}} V \text{ for } 0 \leq t \leq \infty$$

$$V(t) = V_0 e^{-\frac{t}{\tau}} V \text{ for } 0 \leq t \leq \infty$$

$\tau = RC = \text{Sec} = \text{Time constant of the circuit}$

$$i_C = C \frac{dv(t)}{dt} \text{ by Ohm's law}$$

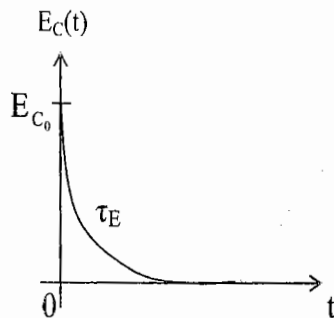




$$\text{At } t = 5\tau, v(t) = V_0 e^{-5} = \frac{I_0}{e^5} = 0.0067 V_0 \cong 0.0$$

$$E_C(t) = \frac{1}{2} \cdot C \cdot v^2(t) \text{ (J)} = \frac{1}{2} \cdot C \cdot \left(V_0 e^{-\frac{t}{\tau_v}} \right)^2$$

$$= E_{C_0} e^{-\frac{t}{\tau_E}} \text{ (J) for } 0 \leq t \leq \infty$$



$$\tau_E = \frac{\tau_v}{2} = \frac{RC}{2} \text{ sec}$$

So the energy decay is 2 times faster than the voltage decay.

sec = sec

$$\tau_i = \tau_v \Rightarrow \frac{L}{R} = RC$$

$$\frac{L}{CR^2} = 1 = \text{Unitless}$$

$$\frac{CR^2}{L} = 1 = \text{Unitless},$$

$$CR^2 = L = \text{Henry}$$

$$\frac{L}{R^2} = C = \text{Farad},$$

$$\frac{L}{RC} = R = \text{Ohm}$$

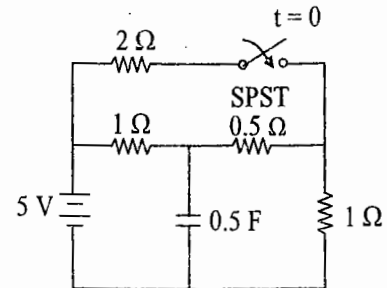
$$\frac{L}{C} = R^2 = \text{Ohm}^2,$$

$$\sqrt{\frac{L}{C}} = R = \text{Ohm}$$

Example: 3.3

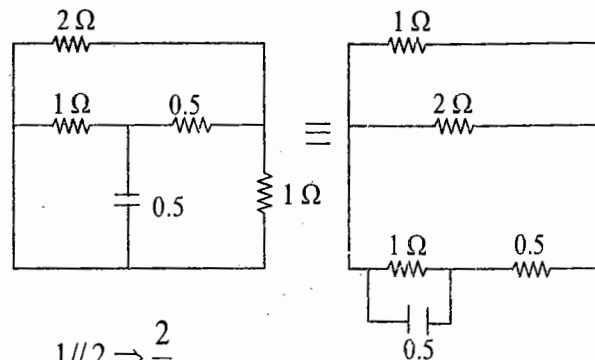
SPST is closed at $t = 0$. What is the time constant of the circuit.

- (a) 26/7
- (b) 7/26
- (c) 7/13
- (d) none



Ans: (b)

Sol: At $t = 0$



$$1//2 \Rightarrow \frac{2}{3}$$

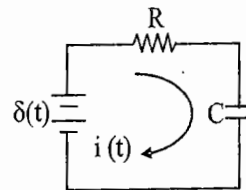
$$\frac{2}{3} + 0.5 = \frac{7}{6}$$

$$R_{eq} = \frac{7}{6} // 1\Omega \Rightarrow R_{eq} = \frac{7}{13}$$

$$\text{Time constant} = R_{eq} C = \frac{7}{13} \cdot \frac{1}{2} = \frac{7}{26}$$

Example: 3.4

The circuit is shown, find $i(t)$ if the impulse voltage is applied to the circuit.

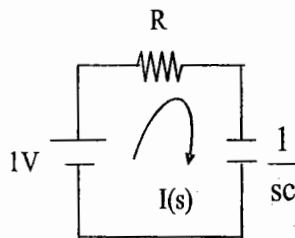




- (a) $i(t) = \frac{1}{R[1 - (1/RC)e^{-t/RC}]} u(t)$
- (b) $i(t) = \frac{1}{R[e^{-t/RC}]} u(t)$
- (c) $i(t) = \frac{1}{R[\delta(t) - (1/RC)e^{-t/RC}]} u(t)$
- (d) none

Ans: (c)

Sol:



$$I(s) = \frac{1}{R + \frac{1}{sC}} = \frac{sC}{sRC + 1}$$

$$I(s) = \frac{\frac{sC}{RC}}{s + \frac{1}{RC}} = \frac{\frac{s}{R}}{s + \frac{1}{RC}}$$

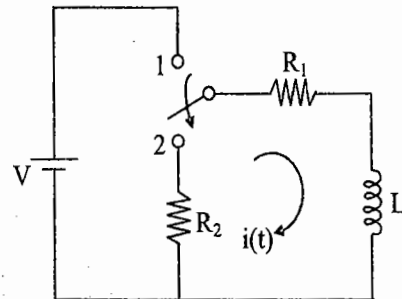
$$I(s) = \frac{1}{R} \left(\frac{s + \frac{1}{RC} - \frac{1}{RC}}{s + \frac{1}{RC}} \right)$$

$$= \frac{1}{R} \left(1 - \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \right)$$

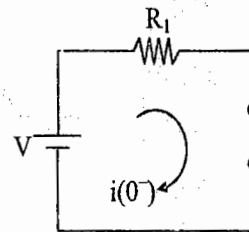
$$i(t) = \frac{1}{R} \left(\delta(t) - \frac{1}{RC} e^{-t/RC} \right)$$

Example: 3.5

In the network of figure, the switch is initially at the position 1. On the steady state having reached, the switch is changed to the position 2. Find current $i(t)$.



Sol: At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.



$$i(0^-) = \frac{V}{R_1}$$

Since the inductor does not allow sudden change in current,

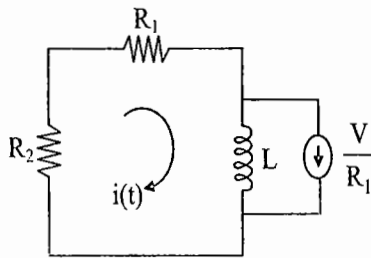
$$i(0^+) = \frac{V}{R_1}$$

Writing KVL equation for $t > 0$,

$$(R_1 + R_2)i + L \frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{(R_1 + R_2)}{L} i = 0$$

The solution of this differential equation is given by



$$i(t) = K e^{-\left(\frac{R_1 + R_2}{L}\right)t}$$

$$\text{At } t = 0, i(0) = \frac{V}{R_1}$$

$$\frac{V}{R_1} = K e^0 = K$$

$$i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1 + R_2}{L}\right)t} \text{ for } t > 0$$

Example: 3.6

The capacitor in the circuit as shown below is initially charged to 12 V with S_1 and S_2 open. S_1 is closed at $t = 0$ while S_2 is closed at $t = 3$. The waveform of the capacitor current is represented by

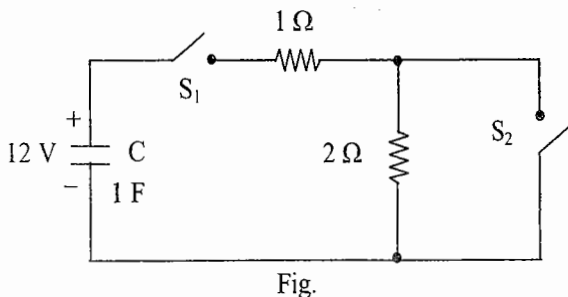


Fig.

Sol: For $0 < t < 3$, the circuit is shown in Fig.1.

$$i(0_+) = \frac{12}{3} = 4 \text{ A}$$

$$\tau = RC = 1 \times 3 = 3 \text{ sec}$$

$$i(t) = 4 e^{-\frac{t}{3}}$$

$$i(3_-) = 4 e^{-1} = 1.48$$

$$v_C(3_-) = v_C(3_+) = 4 e^{-1} \times 3 = 4.44$$

At $t = 3$ S_2 is closed. 2Ω is SC.

Current jumps to a higher value and then decays to zero quickly with a reduced time constant.

For $3 < t < 8$, the circuit is shown in Fig. 2.

$$i_C(3_+) = \frac{v_C(3_+)}{1} = 4.44$$

$$i_C(\infty) = 0$$

$i_C(t)$ for $0 < t < \infty$ is shown in Fig. 3

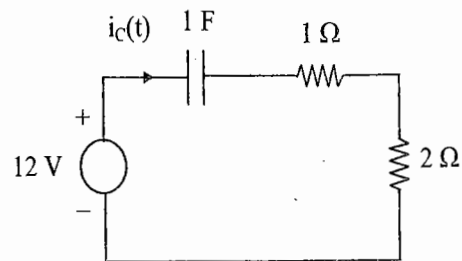


Fig.1

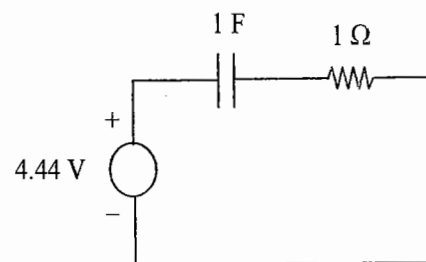


Fig.2

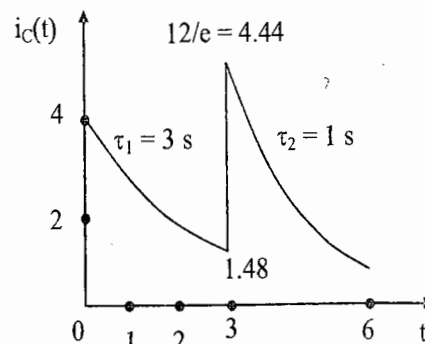


Fig. 3



3.3.2. With Source Circuits & using Laplace Transform Analysis

3.3.2.1 Behaviour of inductor and capacitor with sources, initial ($t = 0^+$) and final ($t \rightarrow \infty$) conditions

$$\text{At } t \rightarrow 0^+ \Rightarrow L = \text{O.C} \Rightarrow C = \text{S.C}$$

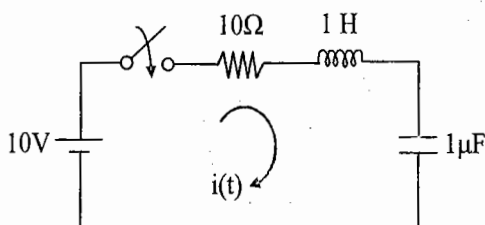
$$\text{As } t \rightarrow \infty \Rightarrow L = \text{S.C} \Rightarrow C = \text{O.C}$$

Property: At these two instants, L and C elements will lose their significance and hence the nature of the circuit is **resistive**.

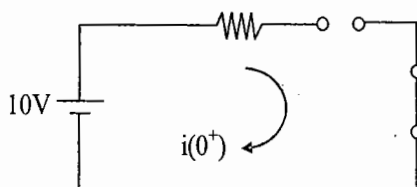
Example: 3.7

In the network shown in figure, the switch is closed. Assuming all initial conditions as zero,

find i , $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.



Sol:



At $t = 0^-$

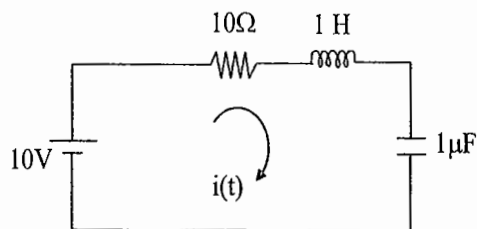
$$i(0^-) = 0 ; v_c(0^-) = 0$$

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

$$i(0^+) = 0$$

$$v_c(0^+) = 0$$

Writing KVL equation for $t > 0$



$$10 - 10i - 1 \frac{di}{dt} - \frac{1}{10 \times 10^{-6}} \int_0^t i dt = 0$$

$$10 = 10i + \frac{di}{dt} + \frac{1}{10 \times 10^{-6}} \int_0^t i dt \dots\dots\dots(i)$$

At $t = 0^+$

$$10 = 10i(0^+) + \frac{di}{dt}(0^+) + 0$$

$$\frac{di}{dt}(0^+) = 10 \text{ A/s}$$

Differentiating the equation (i), we get

$$0 = 10 \frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{1}{10 \times 10^{-6}} i$$

At $t = 0^+$

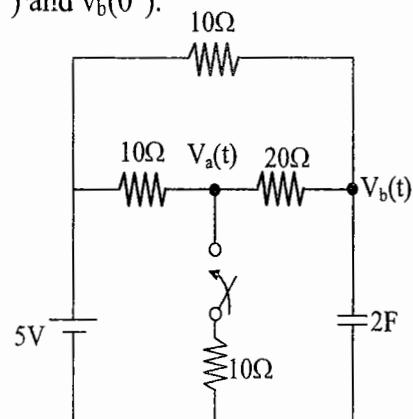
$$0 = 10 \frac{di}{dt}(0^+) + \frac{d^2i}{dt^2}(0^+) + \frac{1}{10^{-5}} i(0^+)$$

$$\frac{d^2i}{dt^2}(0^+) = -10 \times 10 = -100 \text{ A/s}^2$$

Example: 3.8

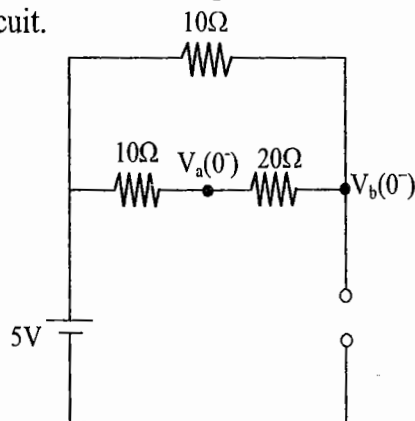
In the accompanying figure is shown a network in which a steady state is reached with switch open.

At $t = 0$, switch is closed. Determine $v_a(0^-)$, $v_a(0^+)$, $v_b(0^-)$ and $v_b(0^+)$.





Sol: At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.



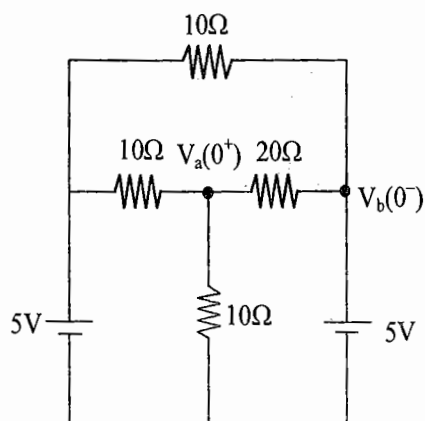
$$V_a(0^-) = 5 \text{ V}$$

$$V_b(0^-) = 5 \text{ V}$$

At $t = 0^+$, the capacitor acts as a voltage source of 5V.

$$V_b(0^+) = 5 \text{ V}$$

Writing KCL equation at $t = 0^+$.



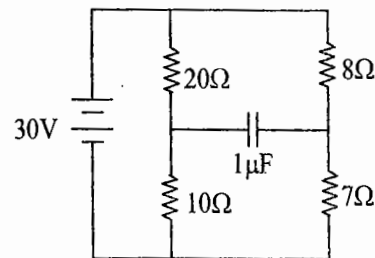
$$\frac{v_a(0^+) - 5}{10} + \frac{v_a(0^+)}{10} + \frac{v_a(0^+) - 5}{20} = 0$$

$$0.25 v_a(0^+) = 0.75$$

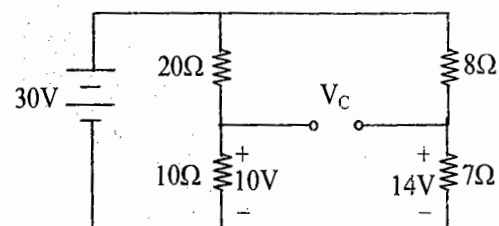
$$v_a(0^+) = 3 \text{ V}$$

Example: 3.9

The energy stored in the capacitor is



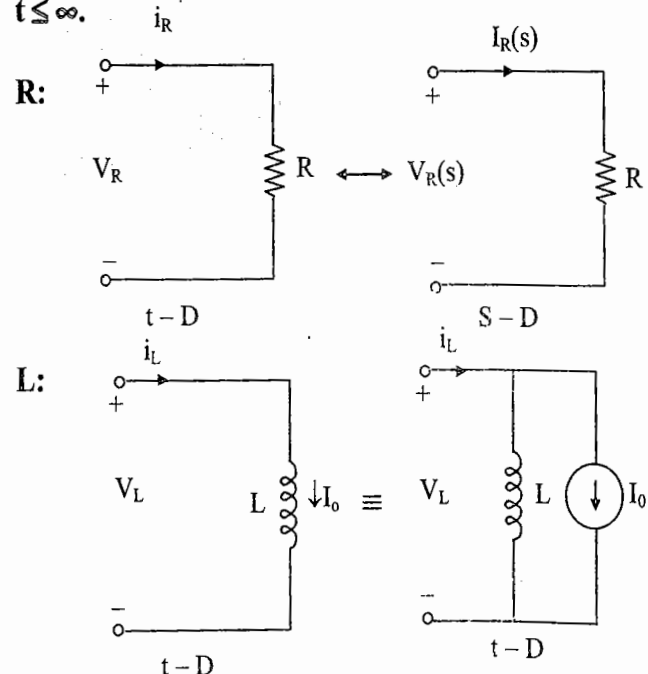
Sol: Network at steady state,

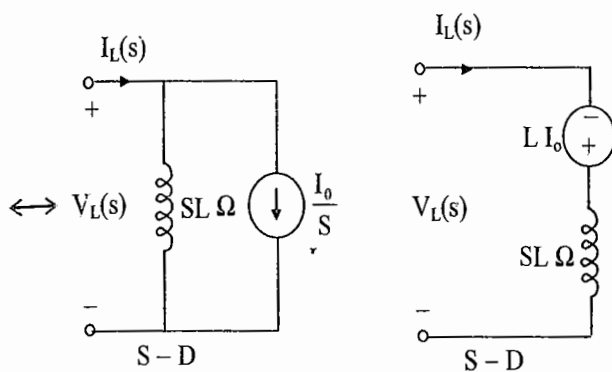


$$V_C = 4 \text{ V}$$

$$\begin{aligned} \text{Energy stored} &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times 1 \times (4)^2 \mu\text{J} = 8 \mu\text{J} \end{aligned}$$

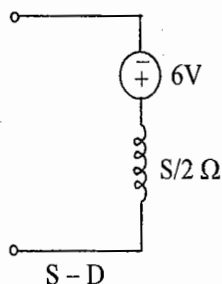
3.3.2.2 With sources, Laplace transforms approach of solving transient problems for $0 \leq t \leq \infty$.





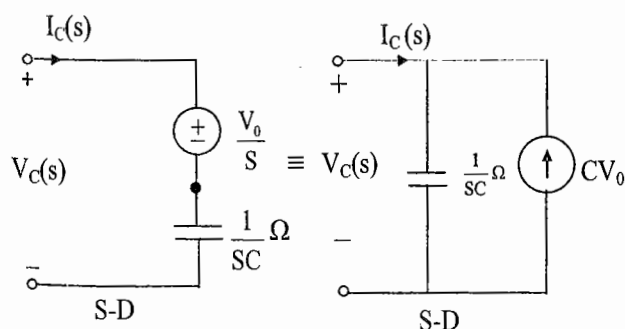
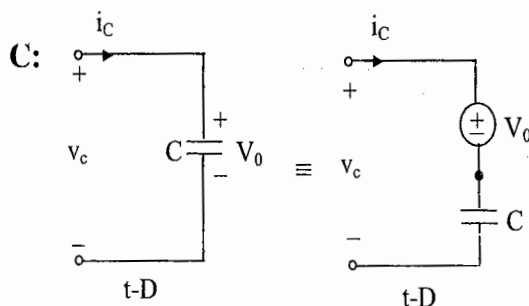
Example: 3.10

The network below represents the equivalent of an inductor in Laplace domain. Determine initial current through it



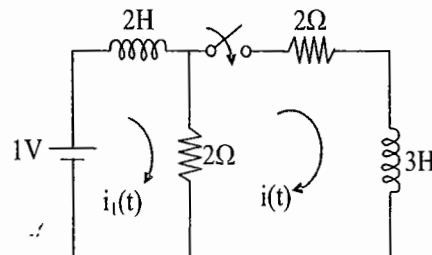
Ans: Since $LI_0 = 6V$ and $L = \frac{1}{2} H$

$$\text{So } I_0 = \frac{6}{L} \Rightarrow \frac{6}{1/2} = 12 A$$

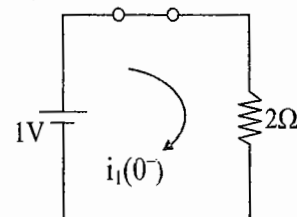


Example: 3.11

In the network shown in figure, the switch is closed at $t = 0$, the steady-state being reached before $t = 0$. Determine current through inductor of 3 H.



Sol: At $t = 0^-$, steady-state condition is reached. Hence, the inductor of 2H acts as a short circuit.



$$i_1(0^-) = \frac{1}{2} A$$

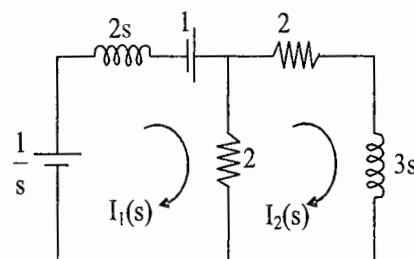
$$i_2(0^-) = 0$$

Since current through the inductor cannot change instantaneously,

$$i_1(0^+) = \frac{1}{2} A$$

$$i_2(0^+) = 0$$

For $t > 0$, the transformed network is shown in figure. Applying KVL to Mesh 1,





$$\frac{1}{s} - 2sI_1(s) + 1 - 2(I_1(s) - I_2(s)) = 0$$

$$(2 + 2s)I_1(s) - 2I_2(s) = 1 + \frac{1}{s}$$

Applying KVL to Mesh 2,

$$-2(I_2(s) - I_1(s)) - 2I_2(s) - 3sI_2(s) = 0$$

$$-2I_1(s) + (4 + 3s)I_2(s) = 0$$

By Cramer's Rule,

$$I_2(s) = \frac{\begin{vmatrix} 2+2s & 1+1/s \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2+2s & -2 \\ -2 & 4+3s \end{vmatrix}} = \frac{\frac{2}{s}(s+1)}{(2s+2)(4+3s)-4}$$

$$= \frac{s+1}{s(2s^2+7s+2)} = \frac{s+1}{3s\left(s+\frac{1}{3}\right)(s+2)}$$

$$= \frac{\frac{1}{3}(s+1)}{s(s+2)\left(s+\frac{1}{3}\right)}$$

By partial - fraction expansion,

$$I_2(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+\frac{1}{3}}$$

$$A = \frac{\frac{1}{3}(s+1)}{\left(s+2\right)\left(s+\frac{1}{3}\right)} \bigg|_{s=0} = \frac{1}{2}$$

$$B = \frac{\frac{1}{3}(s+1)}{s\left(s+\frac{1}{3}\right)} \bigg|_{s=-2} = -\frac{1}{10}$$

$$C = \frac{\frac{1}{3}(s+1)}{s(s+2)} \bigg|_{s=-\frac{1}{3}} = -\frac{2}{5}$$

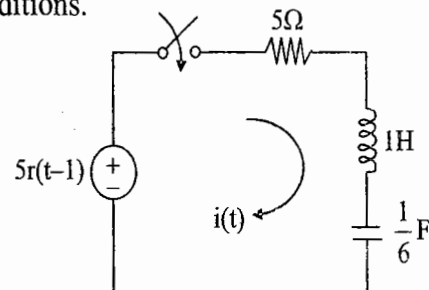
$$I_2(s) = \frac{1}{2s} - \frac{1}{10(s+2)} - \frac{2}{5\left(s+\frac{1}{3}\right)}$$

Taking inverse Laplace transform,

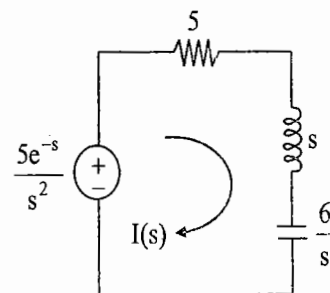
$$i_2(t) = \frac{1}{2} - \frac{1}{10}e^{-2t} - \frac{2}{5}e^{-\left(\frac{1}{3}\right)t} \quad \text{for } t > 0$$

Example: 3.12

For the network shown, determine the current $i(t)$ when the switch is closed at $t=0$ with zero initial conditions.



Sol: The transformed network is shown in figure



Applying KVL to the Mesh for $t > 0$,

$$\frac{5e^{-s}}{s^2} - 5I(s) - \frac{6}{s}I(s) = 0$$

$$5I(s) + sI(s) + \frac{6}{s}I(s) = \frac{5e^{-s}}{s^2}$$

$$I(s) = \frac{5e^{-s}}{s(s^2+5s+6)} = \frac{5e^{-s}}{s(s+3)(s+2)}$$





By partial -fraction expansion,

$$\frac{1}{s(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$A = \left. \frac{1}{(s+3)(s+2)} \right|_{s=0} = \frac{1}{6}$$

$$B = \left. \frac{1}{s(s+2)} \right|_{s=-3} = \frac{1}{3}$$

$$C = \left. \frac{1}{s(s+3)} \right|_{s=-2} = -\frac{1}{2}$$

$$I(s) = 5e^{-s} \left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)} \right]$$

$$= \frac{5e^{-s}}{6s} + \frac{5e^{-s}}{3s+3} - \frac{5e^{-s}}{2s+2}$$

Taking inverse Laplace transform,

$$i(t) = \frac{5}{6}u(t-1) + \frac{5}{3}e^{-3(t-1)}u(t-1) - \frac{5}{2}e^{-2(t-1)}u(t-1)$$

for $t > 0$

3.4 Phasor Representation and its Importance in Steady State AC Circuit Analysis

A **Phasor** is a complex number that represents amplitude and phase of a sinusoid. It is defined **only for the cosinusoidal signals**. All sinusoidal signals are converted into cosinusoids by **subtracting 90° from phase**.

As a complex quantity, a phasor may be expressed in exponential form, polar form or rectangular form. Since a phasor has magnitude and phase ("direction"), it behaves as a vector.

$$V(t) = V_m \cos(\omega t + \phi)$$

$$= \text{R.P} \left[V_m e^{j(\omega t + \phi)} \right]$$

$$= \text{R.P} \left[V_m e^{j\phi} \cdot e^{j\omega t} \right]$$

$$= \text{R.P} \left[V_m e^{j\omega t} \right]$$

$V = V_m e^{j\theta} \rightarrow$ Exponential form of phasor representation

$= V_m \angle \theta \rightarrow$ Polar form of the phasor representation

$= V_m (\cos \phi + j \sin \phi) \rightarrow$ Rectangular, form of Phasor representation

$$i(t) = I_m \cos(\omega t + \theta)$$

$$I = I_m \angle \theta \rightarrow \text{Polar}$$

$$= I_m e^{j\theta} \rightarrow \text{Expo.}$$

$$= I_m (\cos \theta + j \sin \theta) \rightarrow \text{Rect.}$$

$$\text{eg: } V(t) = 10 \cos(2t + 30^\circ) \text{ V}$$

$$V = 10 \angle 30^\circ \rightarrow \text{Polar}$$

$$= 10e^{j30^\circ} \rightarrow \text{Expo.}$$

$$= 10(\cos 30^\circ + j \sin 30^\circ) \rightarrow \text{Rect.}$$

$$= 10 \left(\frac{\sqrt{3}}{2} + j \frac{1}{2} \right) \text{ i.e., phasor is a}$$

representation of complex number

$$\text{eg: } i(t) = 10 \sin(2t + 30^\circ) \text{ A}$$

$$= 10 \cos(2t + 30^\circ - 90^\circ) \text{ A}$$

$$= 10 \cos(2t - 60^\circ) \text{ A}$$

$$I = 10 \angle -60^\circ$$

$$= 10 e^{-j60^\circ}$$

$$= 10 (\cos 60^\circ - j \sin 60^\circ)$$

$$= 10 \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right)$$

$$\boxed{\frac{V}{I} = Z \Omega}$$

$$V = Z.I \Rightarrow I = \frac{V}{Z}$$

$$\boxed{\frac{I}{V} = Y \text{ u}}$$



$$I = Y.V \Rightarrow V = \frac{I}{Y}$$

$$\boxed{Y = \frac{1}{Z}} U$$

$$\underline{t-D} \leftrightarrow \underline{P-D}$$

$$i(t) \leftrightarrow I$$

$$v(t) \leftrightarrow V$$

$$R \leftrightarrow R\Omega$$

$$L \leftrightarrow j\omega L \Omega \quad (s=j\omega)$$

$$\leftrightarrow \frac{1}{j\omega C} \Omega$$

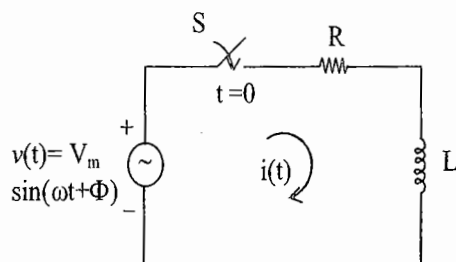
Note:

1. Analysis of a.c in steady state is generally carried out in the phasor domain i.e. KCL, KVL, Ohm's law, Nodal, Mesh & Source transformation are written only in the phasor domain.
2. Addition and subtraction operations of phasors are carried out only in the rectangular form, whereas division and multiplication operations of phasors are carried out either in polar or in exponential form.

3.5 A.C. Transients

3.5.1 Steady State Response & Transient - Free condition for RL, RC and RLC Circuits

3.5.1.1 Series RL circuit

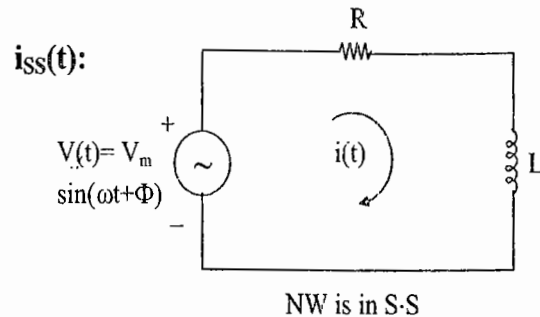


$$C.S = C.F + P.I$$

= Natural response + Forced response.
= Response up to 5τ + Response after 5τ
= Transient response + Steady state response

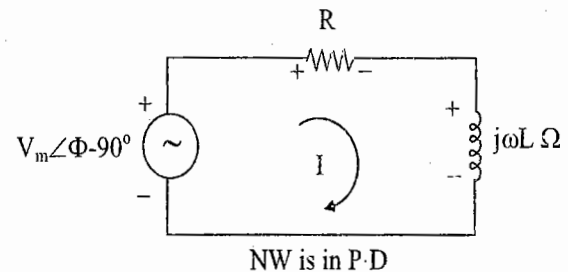
$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$i(t) = K e^{-\frac{t}{\tau}} + i_{ss}(t) = K e^{-\frac{R}{L}t} + i_{ss}(t)$$



Method (1): By Phasor approach

Transform above network into phasor domain.



$$V = Z \cdot I \text{ By ohm's law in P.D}$$

By KVL in P-D

$$\Rightarrow V_m \angle \Phi - 90^\circ - R \cdot I - j\omega L \cdot I = 0$$

$$\Rightarrow I = \frac{V_m \angle \Phi - 90^\circ}{R + j\omega L} = \frac{V_m \angle \Phi - 90^\circ}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\Rightarrow I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle \Phi - \tan^{-1}\left(\frac{\omega L}{R}\right) - 90^\circ$$

$$= \alpha \angle \beta = \alpha e^{j\beta}$$

$$i(t) = R.P [I e^{j\omega t}] A$$

$$= \alpha \cos(\omega t + \beta)$$

$$= \alpha \cos(\omega t + \Phi - \tan^{-1} \frac{\omega L}{R} - 90^\circ)$$



$$= \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \Phi - \tan^{-1} \frac{\omega L}{R}\right) A$$

$$= i_{ss}(t)$$

Method (2): By LTA

$$V(t) = V_m \sin(\omega t + \Phi)$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R + sL}$$

$$H(j\omega) = \frac{1}{R + j\omega L}$$

$$= \frac{1}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1} \frac{\omega L}{R}$$

$$\Rightarrow I(s) = H(s) \cdot V(s)$$

$$\Rightarrow i(t) = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \cdot V_m \sin\left(\omega t + \Phi - \tan^{-1} \frac{\omega L}{R}\right) A$$

$$= i_{ss}(t)$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= K e^{-Rt/L} + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \Phi - \tan^{-1} \frac{\omega L}{R}\right)$$

$$i(0^-) = 0A = i(0^+)$$

$$\Rightarrow 0 = K e^{-0} + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(0 + \Phi - \tan^{-1} \frac{\omega L}{R}\right)$$

$$\Rightarrow K = \frac{-V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\Phi - \tan^{-1} \frac{\omega L}{R}\right) \ll 1$$

As $K \ll 1$, the transient effects are less severe for a.c.

$$V_L(t) = L \frac{di(t)}{dt} \text{ By Ohm's law}$$

$$\text{Suppose } \Phi - \tan^{-1} \frac{\omega L}{R} = 0 \Rightarrow K = 0$$

$$\Rightarrow i_{tr}(t) = 0 \Rightarrow i(t) = i_{ss}(t)$$

i.e. A transient-free response.

So, condition for transient-free response at $t = 0$ is

$$\Phi = \tan^{-1} \frac{\omega L}{R}$$

$$\text{i.e. } (\omega t + \Phi)|_{t=0} = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

Note:

- So, if the total phase of excitation at the time of switching is equal to $\tan^{-1} \left(\frac{\omega L}{R} \right)$ then no transients will result in the system at the time of switching for sinusoidal excitation.

- If the switch is closed at $t = t_0$, then the condition for the transient free response at $t = t_0$ is

$$\omega t_0 + \Phi = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

- If the excitation is $V(t) = V_m \cos(\omega t + \Phi)$, then sine is replaced by cosine in the steady state response and hence 'K' is a function of cosine.

- Suppose, $\Phi - \tan^{-1} \frac{\omega L}{R} = \frac{\pi}{2}$
 $\Rightarrow K = 0 \Rightarrow i_{tr}(t) = 0$
 $\Rightarrow i(t) = i_{ss}(t)$

i.e. A transient-free response

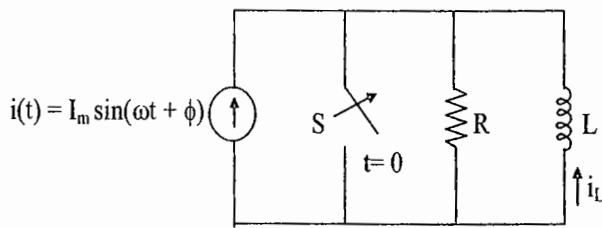
$$\Phi = \tan^{-1} \left(\frac{\omega L}{R} \right) + \frac{\pi}{2} \text{ at } t = 0$$

$$\omega t_0 + \Phi = \tan^{-1} \left(\frac{\omega L}{R} \right) + \frac{\pi}{2} \text{ at } t = t_0$$

- In the above cases if 'L' is replaced by 'C' then $\tau = \frac{L}{R}$ should be replaced by $\tau = RC$



3.5.1.2 Parallel RL Circuit



$i_{Lss}(t)$ by LTA:

$$\frac{I_L(S)}{I(S)} = H(S) = \left(\frac{\text{desired L.T}}{\text{excitation L.T}} \right)$$

$$I_L(S) = \frac{I(S) \cdot R}{R + SL}$$

(By current division in s - domain)

$$\frac{I_L(S)}{I(S)} = H(S) = \frac{R}{R + SL}$$

$$H(j\omega) = \frac{1}{1 + \frac{j\omega L}{R}} = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i_L(t) = i_{Ltr}(t) + i_{Lss}(t)$$

$$= Ke^{-Rt/L} + \frac{I_m}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} \sin(\omega t + \Phi - \tan^{-1} \frac{\omega L}{R})$$

$$i_L(0^-) = 0 = i_L(0^+)$$

$$K = \frac{-I_m}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} \sin\left(\Phi - \tan^{-1} \frac{\omega L}{R}\right) \ll 1$$

$$v_L(t) = L \frac{di_L(t)}{dt} \text{ by Ohm's law}$$

$$\text{Suppose } \Phi - \tan^{-1} \frac{\omega L}{R} = 0$$

$$\Rightarrow K = 0 \Rightarrow i_{Ltr}(t) = 0$$

$$\Rightarrow i_L(t) = i_{Lss}(t)$$

i.e., A transient - free response

$$\Phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ at } t = 0$$

$$\omega t_0 + \Phi = \tan^{-1} \frac{\omega L}{R} \text{ at } t = t_0$$

(same as series RL circuit)

If $i(t) = I_m \cos(\omega t + \Phi)$, then

$$\Phi = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2} \text{ at } t = 0$$

$$\omega t_0 + \Phi = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2} \text{ at } t = t_0$$

In the above cases if 'L' is replaced by 'C', then

$\tau = \frac{L}{R}$ should be replaced by $\tau = RC$.

Note: Transient - free condition is not possible for networks with both the energy storing elements i.e. for RLC networks.

Reason:

$$S_1, S_2 = \alpha \pm j\beta$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= e^{\alpha t} (K_1 \cos \beta t + K_2 \sin \beta t) + i_{ss}(t)$$

Here by applying initial conditions $i_L(0^+)$ and $v_C(0^+)$, K_1 is a function of sine and K_2 is a function of cosine (vice versa), hence no time will satisfy both K_1 and K_2 simultaneously to zero and so the transient term is always present in the complete response.

Obs: So, the transient - free time (t_0 value) in RL and RC circuits with A.C excitation will depend

on the
on the
the n
not o
or I_m

Exan

For
respo

$v_s(t)$

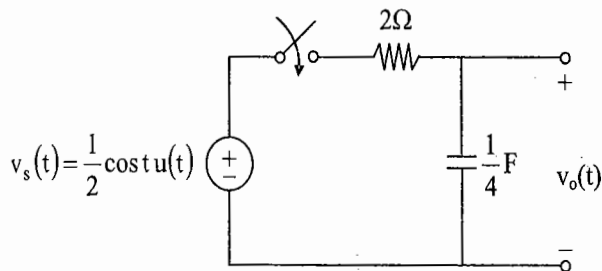
Sol:



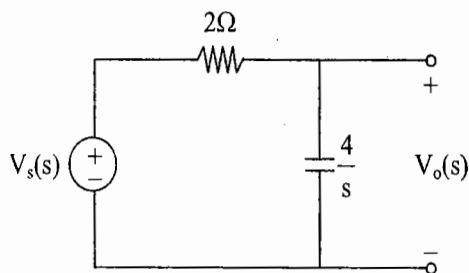
on the source frequency (ω), its initial phase (Φ), on the circuit constants (R , L , C values) and on the nature of the excitation (sine or cosine), but not on the maximum value of the excitation (V_m or I_m).

Example: 3.13

For the network shown in figure, find the response $v_o(t)$.



Sol: The transformed network is shown in figure



$$V_s(s) = \frac{1}{2} \frac{s}{s^2 + 1}$$

By voltage-division formula,

$$V_o(s) = V_s(s) \times \frac{\frac{4}{s}}{2 + \frac{4}{s}}$$

$$= \frac{2V_s(s)}{s+2} = \frac{s}{(s^2+1)(s+2)}$$

By partial - fraction expansion ,

$$V_o(s) = \frac{As+B}{s^2+1} + \frac{C}{s+2}$$

$$s = (As+B)(s+2) + C(s^2+1)$$

$$s = s^2(A+C) + s(2A+B) + (2B+C)$$

Comparing coefficient of s^2 , s and s^0 , we have

$$A + C = 0$$

$$2A + B = 1$$

$$2B + C = 0$$

Solving the equations, we get

$$A = 0.4, B = 0.2, C = -0.4$$

$$V_o(s) = \frac{0.4s + 0.2}{s^2 + 1} - \frac{0.4}{s+2}$$

$$= \frac{0.4s}{s^2+1} + \frac{0.2}{s^2+1} - \frac{0.4}{s+2}$$

Taking the inverse Laplace transform,

$$i(t) = 0.4 \cos t + 0.2 \sin t - 0.4e^{-2t} \text{ for } t > 0$$

Class Room Practice Questions

LEVEL-1 Questions

01. A square wave is fed to an RC circuit. Then
(IES-EE-92)

- (a) voltage across R is square and across C is not square
- (b) voltage across C is not square and across R is not square
- (c) voltage across both R and C is square
- (d) voltage across both R and C is not square

02. If the unit step response of a network is $(1 - e^{-\alpha t})$ then its impulse response will be
(IES-EE-97)

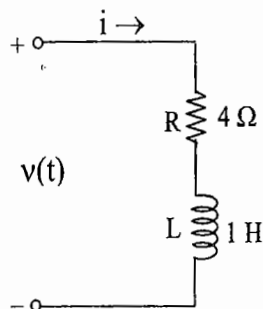
- (a) $\alpha e^{-\alpha t}$
- (b) $\frac{1}{\alpha} e^{-t/\alpha}$
- (c) $\frac{1}{\alpha e^{-t/\alpha}}$
- (d) $(1 - \alpha)^{-\alpha t}$

03. A system function has a pole at $s = 0$ and a zero at $s = -1$. The constant multiplier is unity. For an excitation $\sin t$, what is the steady-state response?
(IES-EE-06)

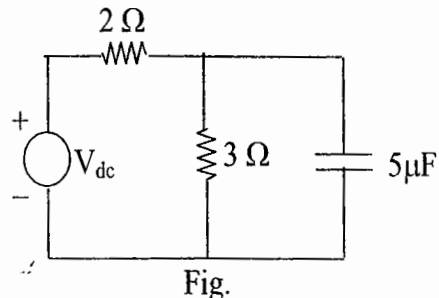
- (a) $\sqrt{2} \sin(t + 45^\circ)$
- (b) $\sqrt{2} \sin(t - 45^\circ)$
- (c) $\sin(t - 45^\circ)$
- (d) $\sin t$

04. Consider an L-R circuit in which a current $i = 5e^{-2t}$ A is flowing. The voltage across the R-L circuit is given by
(IES-EE-12)

- (a) $20e^{-2t}$ V
- (b) $-10e^{-2t}$ V
- (c) $10e^{-2t}$ V
- (d) $5e^{-2t}$ V



05. The time constant associated with the capacitor changing in the circuit shown in Fig. is
(IES-EC-96)



- (a) $6 \mu s$
- (b) $10 \mu s$
- (c) $15 \mu s$
- (d) $25 \mu s$

06. Time constants of R-L and R-C circuits are respectively:
(IES-EC-11)

$R = 1 \Omega$; $L = 1$ H and $C = 1$ F

- (a) 1 sec and 1 sec
- (b) 1 sec and 2 sec
- (c) 2 sec and 3 sec
- (d) 2 sec and 4 sec

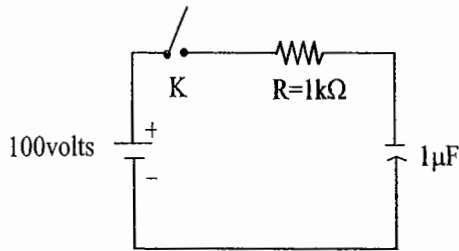
07. Time constant of a series R-L circuit equals
(ISRO-2006)

- (a) L/R second
- (b) LR second
- (c) L^2R
- (d) LR^2

08. A dc voltage V is applied at time $t = 0$ to a series RC circuit. The steady state current is
(FRO)

- (a) $\frac{V}{R}$
- (b) $\frac{V}{C}$
- (c) $\frac{V}{\sqrt{R^2 + C^2}}$
- (d) Zero

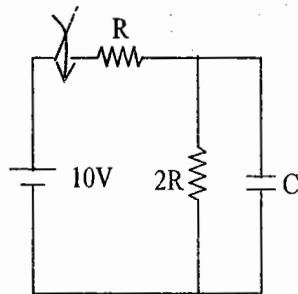
09. For the network shown below, the switch K is closed at $t = 0$. With capacitor uncharged, the value of current at $t = 0^+$ is
(HMWS)



- (a) zero amp (b) 0.1 amp
(c) 0.01 amp (d) 0.001 amp

10. The time constant of the network shown in figure is

- (a) $2RC$
(b) $3RC$
(c) $\frac{RC}{2}$
(d) $\frac{2RC}{3}$



LEVEL-2 Questions

01. Consider the following data :

(IES-EE-93)

1. Input applied for $t < t_0$
2. Input applied for $t \geq t_0$
3. State of the network at $t = t_0$
4. State of the network at $t < t_0$

Among these, those needed for determining the response of a linear network for $t > t_0$ would include

- (a) 1, 3 and 4 (b) 2, 3 and 4
(c) 2 and 3 (d) 2 and 4

02. Two coils having equal resistances but different inductances are connected in series. The time constant of the series combination is the

(IES-EE-93)

- (a) sum of the time constant of the individual coils
(b) average of the time constant of individual coils

- (c) geometric mean of the time constants of the individual coils
(d) product of the time constants of the individual coils

03. The response of an initially relaxed linear circuit to a signal V_s is $e^{-2t} u(t)$. If the signal is changed to $\left(V_s + 2 \frac{dV_s}{dt} \right)$ the

response would be (IES-EE-94)

- (a) $-4e^{-2t} u(t) + \delta(t)$
(b) $-3e^{-2t} u(t) + 2\delta(t)$
(c) $4e^{-2t} u(t) - \delta(t)$
(d) $5e^{-2t} u(t) + 2\delta(t)$

04. A unit impulse voltage is applied at $t = 0$ to the R-L circuit shown below while $i(0^-) = 1$ A. What is the expression for $i(t)$?

(IES-EE-05)

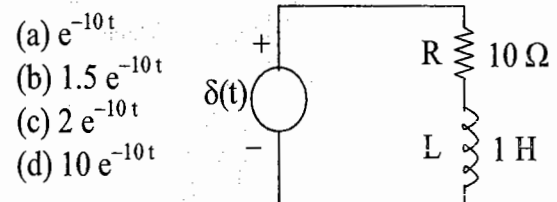


Fig.

- (a) e^{-10t}
(b) $1.5 e^{-10t}$
(c) $2 e^{-10t}$
(d) $10 e^{-10t}$

05. In the circuit shown in the Fig. below, if $i_s = u(t)$ A, then what are the initial and steady-state voltages across the capacitor ?

(IES-EE-06)

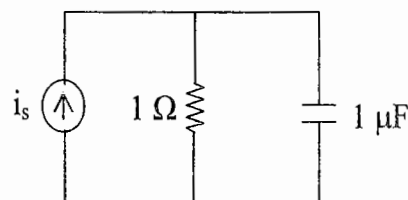


Fig.

- (a) 1 V and 1 V, respectively
(b) 1 V and 0, respectively
(c) 0 and 1 V, respectively
(d) 0 and 0, respectively



06. In the circuit, if $V_C(0) = 25$ V, the expression for $V_C(t)$ for $t > 0$ is

(IES-EE-11)

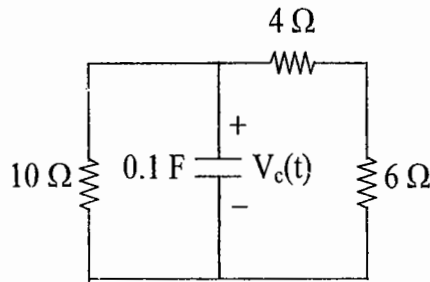


Fig.

- (a) $V_C(t) = 20 e^{-0.4t}$ V
(b) $V_C(t) = 25 e^{-0.4t}$ V
(c) $V_C(t) = 20 e^{-2.5t}$ V
(d) $V_C(t) = 25 e^{-2t}$ V

07. For the circuit shown below different time constants are given:

(IES-EC-92)

1. 0.5×10^{-3} ms
2. 2×10^{-3} ms
3. 0.25×10^{-3} ms
4. 10^{-3} ms

What are the charging and discharging time constants respectively?

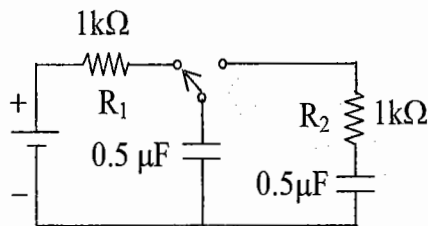


Fig.

- (a) 1, 2
(b) 2, 3
(c) 1, 3
(d) 2, 4

08. A step function voltage is applied to an RLC series circuit having $R = 2\Omega$, $L = 1$ H and $C = 1$ F. the transient current response of the circuit would be

(IES-EC-95)

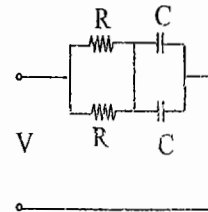
- (a) over damped
(b) critically damped

- (c) underdamped
(d) over, under or critically damped depending upon the magnitude of the step voltage

09. The time-constant of the network shown in the figure is

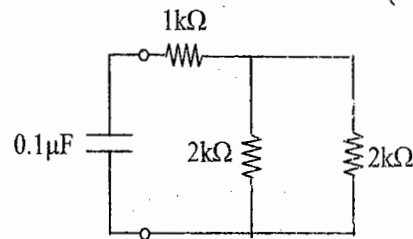
(ISRO -2007)

- (a) CR
(b) CR^2
(c) $\frac{CR}{4}$
(d) $\frac{CR}{2}$



10. The time constant for the circuit shown below is

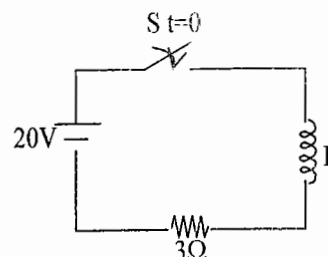
(ISRO -2009)



- (a) 0.2 microsecond
(b) 0.8 millisecond
(c) 0.4 millisecond
(d) 0.2 millisecond

11. For the circuit shown in the figure, the switch is closed at $t = 0$. After some time when the current in the inductor was 6A, the rate of change of current through the inductor was 4A/sec. The value of the inductor is

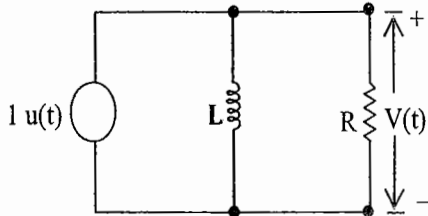
(AEE-2012)





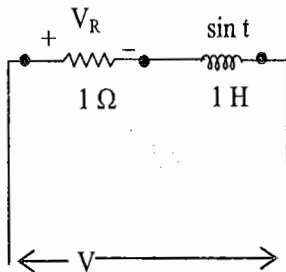
- (a) 0.5H (b) 1.5H
(c) 1.0H (d) Indeterminate

12. In the circuit of fig given $v(t)$ for $t > 0$ is given by (AEE-2012)



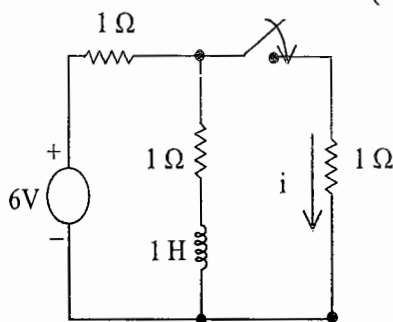
- (a) $IR e^{-LT/R}$ (b) $IR e^{-RT/L}$
(c) $IR (1 - e^{-RT/L})$ (d) $IR(1 - e^{-LT/R})$

13. In the circuit of series RL given in fig., V is given by (AEE-2012)



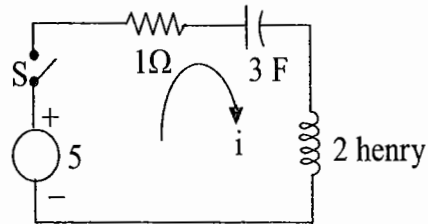
- (a) $2 \sin t$ (b) $2 \cos t$
(c) $\sin(t + 45^\circ)$ (d) $\sqrt{2} \sin(t + 45^\circ)$

14. In the circuit of fig shown steady state has been reached with the switch open the switch is closed at $t=0$ the current I at $t=0^+$ is (AEE-2012)



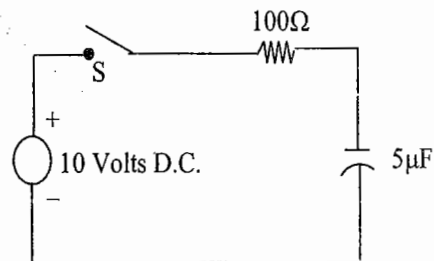
- (a) $3/2$ A (b) 9A
(c) 2A (d) 4A

15. The current $i(0^+)$ and $\frac{di}{dt}$ in the circuit given below are



- (a) 0, 0 (b) 0, $\frac{3}{2}$
(c) $\frac{3}{2}$, 0 (d) $\frac{3}{2}$, $\frac{3}{2}$

16. In the given R - C circuit, the current reaches its maximum value (SPDCL-12)



- (a) after 50 μ .sec. of turning on the switch S
(b) after 100 μ .sec. of turning on the switch S
(c) after 1000 μ .sec. of turning on the switch S
(d) immediately after turning on the switch S

17. A capacitor C at time $t = 0^+$ with initial charge Q_0 acts as (SPDCL-12)
(a) short circuit
(b) open circuit
(c) current source
(d) voltage source



Solutions

LEVEL-1

01. Ans: (d)

Sol: $V_C(t)$ rises exponentially during the rising, charging half cycle and falls during the discharging, falling portion of the adjacent half cycle. $V_R(t)$ is also not square.

02. Ans: (a)

Sol: $u(t) \longrightarrow 1 - e^{-\alpha t}$

$$\delta(t) = \frac{d}{dt} u(t) \longrightarrow \frac{d}{dt} (1 - e^{-\alpha t})$$

$$= \alpha e^{-\alpha t}, t > 0$$

$$= 0, t < 0$$

03. Ans: (b)

Sol: $H(s) = \frac{1(s+1)}{s}, H(j\omega)$

$$= -j \frac{(1+j\omega)}{\omega} = \frac{\omega - j}{\omega}, \omega = 1$$

$$H(j1) = 1 - j$$

$$|H(j1)| = \sqrt{2}, \angle H(j1) = -45^\circ$$

$$\text{Steady state response} = 1 |H(j1)| \sin$$

$$(t + \angle H(j1)) = \sqrt{2} \sin(t - 45^\circ)$$

04. Ans: (c)

Sol: $V_R = 5e^{-2t} \times 4 = 20e^{-2t} V$

$$V_L = L \frac{di}{dt} = 1 \left[\frac{d}{dt} (5e^{-2t}) \right]$$

$$= -10e^{-2t}$$

$$V(t) = V_R + V_L$$

$$= 10e^{-2t} V$$

05. Ans: (a)

Sol: $\tau = RC, R = 2 \parallel 3 = \frac{6}{5} \Omega, C = 5 \times 10^{-6} F$

$$\tau = \frac{6}{5} \times 5 \times 10^{-6} = 6 \mu \text{sec}$$

06. Ans: (a)

Sol: Time constant

$$= \frac{L}{R} = 1 \text{ sec, for R-L circuit}$$

$$= CR = 1 \text{ sec for RC circuit}$$

07. Ans: (a)

08. Ans: (d)

Sol: Under steady state capacitor acts as a open circuit

$\therefore$ current is zero

09. Ans: (b)

Sol: Capacitor opposes sudden change in voltage

$\therefore$ At $t = 0^+$ it acts as a short circuit

$$\text{At } t = 0^+, i = \frac{V}{R} = \frac{100}{10^3} = 0.1 A$$

10. Ans: (d)

Sol: Equivalent resistance seen through capacitor terminals after switch operation,

$$R_{eq} = (2R) \parallel (R) = \frac{2R}{3}$$

$$\therefore \text{Time constant} = R_{eq} C = \frac{2RC}{3}$$

Level-2 Key

01. (d) 02. (b) 03. (b) 04. (c) 05. (c)

06. (d) 07. (c) 08. (b) 09. (a) 10. (a)

11. (a) 12. (b) 13. (d) 14. (c) 15. (a)

16. (d) 17. (a)



AC Circuit Analysis

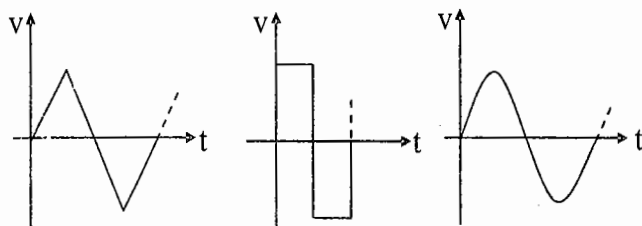


Alessandro Volta
(1745 – 1827)

4.1 Average and RMS Values of Periodic Signals

Introduction:

An alternating waveform changes its magnitude and direction periodically. Figure shows various AC waveforms.



Many times, alternating voltages and currents are represented by a sinusoidal waveforms.

A sinusoidal voltage can be represented as

$$\begin{aligned} v &= V_m \sin \theta \\ &= V_m \sin \omega t \\ &= V_m \sin 2\pi ft \\ &= V_m \sin \frac{2\pi}{T} t \end{aligned}$$

Terms related with alternating Quantity:

- 1. Waveform:** A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Figure shows a few waveforms.
- 2. Cycle:** One complete set of positive and negative values of an alternating quantity is termed as cycle.
- 3. Frequency:** The number of cycles per second of an alternating quantity is known as

frequency. It is denoted by f and is expressed in hertz(Hz) or cycles per second (c/s).

- 4. Time period:** The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by T and is expressed in seconds.

$$T = \frac{1}{f}$$

- 5. Amplitude:** The maximum positive or negative value of an alternating quantity is called the amplitude.
- 6. Phase:** The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.
- 7. Phase difference:** This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier as compared to the other quantity.

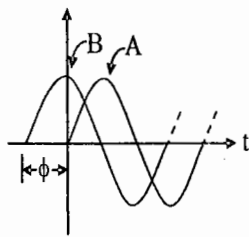
A lagging alternating quantity is one which attains its maximum or zero value later than the other quantity.

A plus (+) sign when used in connection with the phase difference denotes 'lead', whereas a minus(-) sign denotes 'lag'.

$$V_A = V_m \sin \omega t$$

$$V_B = V_m \sin(\omega t + \phi)$$

Here, quantity B leads A by a phase angle ϕ .



Let $x(t)$ be the periodic signal of period 'T',
then $x(t \pm T) = x(t)$

Average Value = DC Value

$$= X_{avg} = X_{dc} = \frac{1}{T} \int_0^T x(t) dt$$

$$= \frac{\text{Area of the signal over one period}}{\text{Period}}$$

$$\text{RMS Value} = X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

$$\text{Peak Factor} = \frac{x(t)_{max}}{x(t)_{avg}} = \frac{X_{max}}{X_{avg}}$$

$$\text{Form Factor} = \frac{X_{rms}}{X_{avg}}$$

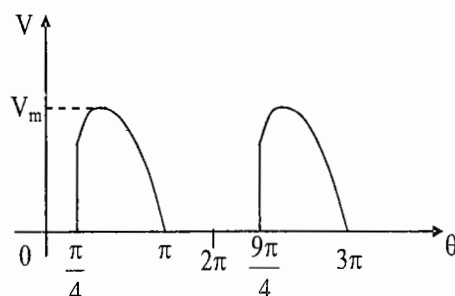
- The average and value of sine or cosine function of any phase and frequency is zero, and its rms value is $\frac{\text{Maximum value}}{\sqrt{2}}$
- If $f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots + b_1 \sin \omega t + b_2 \sin 2\omega t + \dots$, then the RMS value of $f(t)$

$$f(t)_{RMS} = \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 + \dots + b_1^2 + b_2^2 + \dots)}$$

Example: 4.1

Find the average value and rms value of the waveform shown in figure.

Sol:



$$v = 0 \quad 0 < \theta < \pi/4$$

$$= V_m \sin \theta \quad \pi/4 < \theta < \pi$$

$$= 0 \quad \pi < \theta < 2\pi$$

$$V_{avg} = \frac{1}{2\pi} \int_0^{2\pi} v(\theta) d\theta$$

$$= \frac{1}{2\pi} \int_{\pi/4}^{\pi} V_m \sin \theta d\theta$$

$$= \frac{V_m}{2\pi} (-\cos \theta)_{\pi/4}^{\pi}$$

$$= \frac{V_m}{2\pi} = (1 + 0.707) = 0.272 V_m$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2(\theta) d\theta}$$

$$= \sqrt{\frac{1}{2\pi} \int_{\pi/4}^{\pi} V_m^2 \sin^2 \theta d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \int_{\pi/4}^{\pi} \sin^2 \theta d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \int_{\pi/4}^{\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta}$$

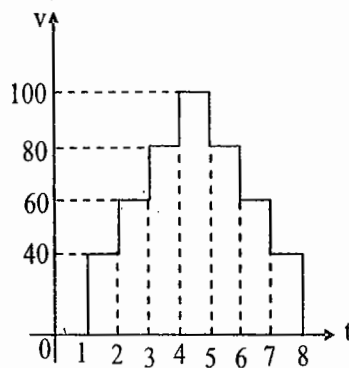
$$= \sqrt{\frac{V_m^2}{2\pi} \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right)_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4} \right)}$$

$$= \sqrt{0.227 V_m^2} = 0.476 V_m$$

Example: 4.2

Find the rms and average value of the waveform shown in figure.



Sol: $V_{avg} = \frac{0 + 40 + 60 + 80 + 100 + 80 + 60 + 40}{8}$

$$= 57.5 \text{ V}$$

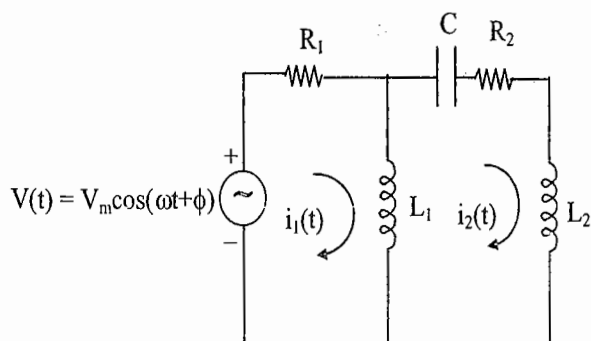
$$V_{rms} = \sqrt{\frac{0^2 + (40)^2 + (60)^2 + (80)^2 + (100)^2 + (80)^2 + (60)^2 + (40)^2}{8}}$$

$$= 64.42 \text{ V}$$

4.2 Sinusoidal Steady State Analysis (S³ A) by Using Phasors

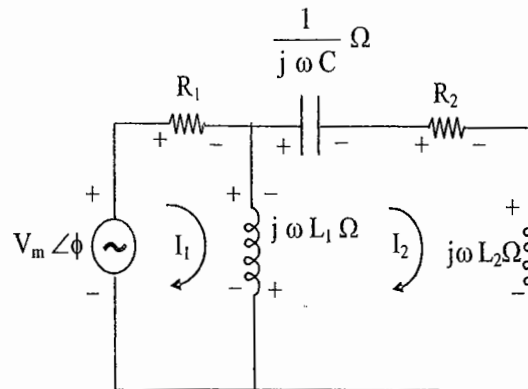
Steady state indicates absence of transients. It is achieved practically after 5 time constants of switching action and analysis in steady state is generally carried out by using phasors.

Procedure:



N/W is in S.S.

Transform the above network into phasor domain



N/W is in P.D

$$V = Z \cdot I$$

By KVL in P - D

$$V_m \angle \phi - R_1 I_1 - j\omega L_1 (I_1 - I_2) = 0 \dots\dots (1)$$

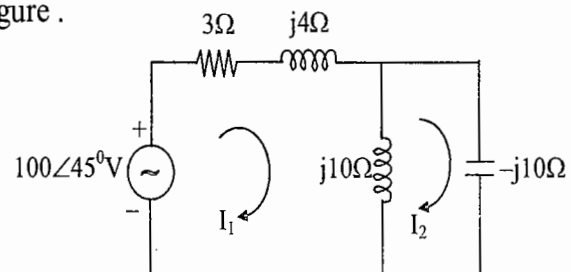
$$\frac{-1}{j\omega C} \cdot I_2 - R_2 \cdot I_2 - j\omega L_2 \cdot I_2 - j\omega L_1 \cdot (I_2 - I_1) = 0 \dots\dots (2)$$

$$I_1 = \frac{\Delta_1}{\Delta} \quad i_1(t) = R \cdot P \left[I_1 \cdot e^{j\omega t} \right] A$$

$$I_2 = \frac{\Delta_2}{\Delta} \quad i_2(t) = R \cdot P \left[I_2 \cdot e^{j\omega t} \right] A$$

Example: 4.3

Find mesh currents I_1 and I_2 in the network of figure .



Sol: Applying KVL to Mesh 1,

$$100 \angle 45^\circ - (3 + j4) I_1 - j10(I_1 - I_2) = 0$$

$$(3 + j14) I_1 - j10I_2 = 100 \angle 45^\circ \dots\dots (1)$$

Applying KVL to Mesh 2,

$$-j10(I_2 - I_1) + j10(I_2) = 0$$

$$j10I_1 = 0$$



$$I_1 = 0$$

Substituting I_1 in equation (1) we get ,

$$-j10I_2 = 100 \angle 45^\circ$$

$$I_2 = \frac{100 \angle 45^\circ}{10 \angle -90^\circ} = 10 \angle 135^\circ \text{ A}$$

Example: 4.4

A two-element series circuit is connected across an ac source given by $e = 200\sqrt{2} \sin(314t + 20^\circ)$. The current in the circuit then is found to be $i = 10\sqrt{2} \cos(314t - 25^\circ)$. Determine the parameters of the circuit.

$$\text{Sol: } i = 10\sqrt{2} \cos(314t - 25^\circ) \\ = 10\sqrt{2} \sin(314t - 25^\circ + 90^\circ)$$

$$i = 10\sqrt{2} \sin(314t + 65^\circ)$$

$$\text{Also, } e = 200\sqrt{2} \sin(314t + 20^\circ)$$

$$\text{Current lead emf by } (65^\circ - 20^\circ) = 45^\circ$$

$$\text{Power factor} = \cos 45^\circ = 0.707 \text{ (leading)}$$

$$\text{Now, } Z = \frac{V_m}{I_m} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20 \Omega$$

$$R = Z \cos \phi = 20 (0.707) = 14.14 \Omega \text{ and}$$

$$X_C = Z \sin \phi = 20 (0.707) = 14.14 \Omega$$

$$X_C = \frac{1}{2\pi f C} = 14.14 \Rightarrow C = \frac{1}{2\pi f \times 14.14}$$

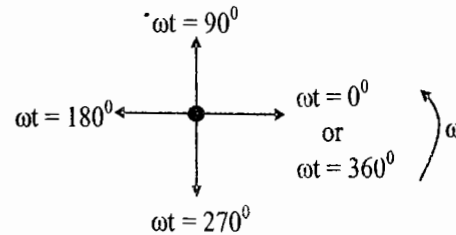
$$= \frac{1}{2 \times 3.14 \times 50 \times 14.14} = 2.2523 \times 10^{-4} \text{ F}$$

$$= 225.23 \mu\text{F}$$

4.3 Phasor Diagrams

- A graphical or pictorial representation of phasors is known as **phasor diagram**.
- Phasor diagram is a pictorial representation of all the phase voltages and their respective currents in a network.

- Phasor is a **rotating vector**, which rotates in the **anti-clockwise direction** with angular frequency ' ω ' in the time domain.



$$Z_R = R \angle 0^\circ \Omega$$

$$Z_L = j \omega L \Omega = j X_L \Omega = X_L \angle 90^\circ \Omega /$$

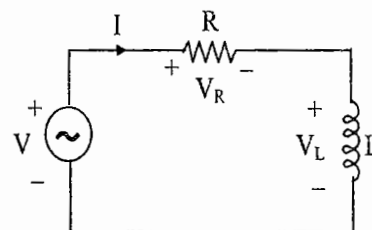
$$\text{where } X_L = \omega L$$

$$Z_C = \frac{1}{j \omega C} \Omega = -j \cdot \frac{1}{\omega C}$$

$$= -j X_C = X_C \angle -90^\circ \Omega /$$

$$\text{where } X_C = \frac{1}{\omega C}$$

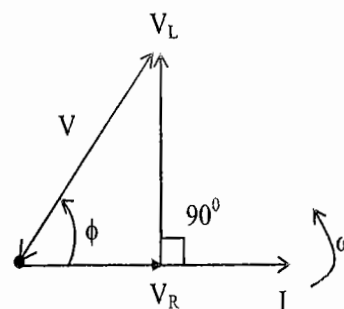
4.3.1 Series RL Circuit



N/W is in S.S.

$$V_R = I \cdot R$$

$$V_L = I \cdot Z_L = I \cdot X_L \angle 90^\circ$$





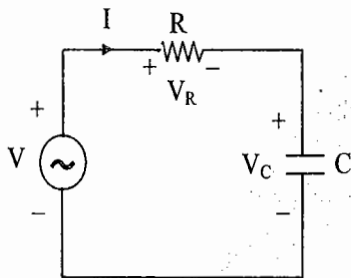
$$V = \sqrt{V_R^2 + V_L^2}$$

$$\phi = \tan^{-1} \left(\frac{V_L}{V_R} \right) = \text{Impedance Angle}$$

= Admittance Angle

$$\cos \phi = \text{pf} = \left(\frac{V_R}{V} \right) (\text{lag}) \text{ i.e. "I" lags "V"}$$

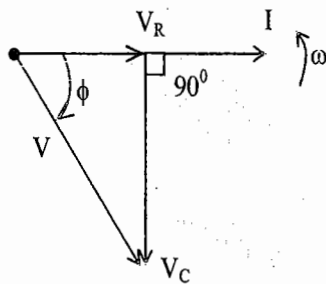
4.3.2 Series RC Circuit



N/W is in S.S.

$$V_R = I \cdot R$$

$$V_C = I \cdot Z_C = I \cdot X_C \angle -90^\circ$$



$$V = \sqrt{V_R^2 + V_C^2}$$

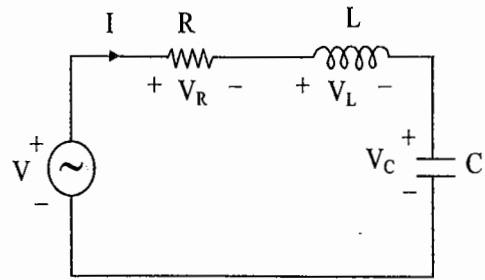
$$\phi = \tan^{-1} \left(\frac{V_C}{V_R} \right) = \text{Impedance Angle}$$

= Admittance Angle

$$\cos \phi = \text{Pf} = \left(\frac{V_R}{V} \right) (\text{lead})$$

i.e. "I" leads "V"

4.3.3 Series RLC Circuit



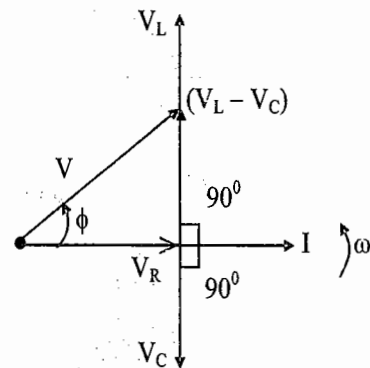
N/W is in S.S.

$$V_R = I \cdot R$$

$$V_L = I \cdot X_L \angle 90^\circ$$

$$V_C = I \cdot X_C \angle -90^\circ$$

(i) $V_L > V_C$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

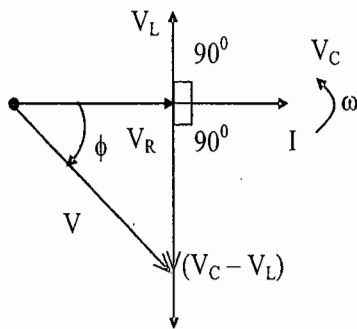
= Impedance Angle

= Admittance Angle

$$\cos \phi = \text{P.f} = \left(\frac{V_R}{V} \right) (\text{lag})$$

i.e. "I" lags "V"

(ii) $V_C > V_L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{V_C - V_L}{V_R} \right)$$

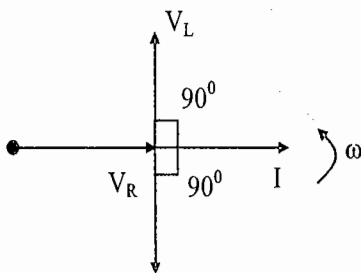
= Impedance Angle

= Admittance Angle

$$\cos \phi = \text{p.f.} = \left(\frac{V_R}{V} \right) (\text{lead})$$

i.e. "I" leads "V"

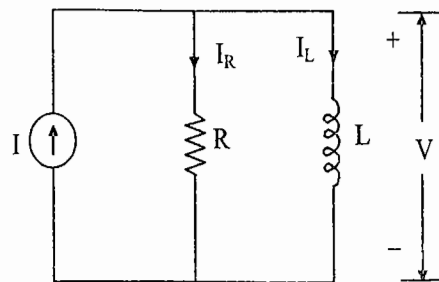
(iii) $V_L = V_C$



$$\phi = 0^\circ \Rightarrow \text{Unity power factor}$$

In the above phasor diagram, inductor and capacitor voltages are equal in magnitude and opposite in phase. So they cancel each other and this is called "**resonance**".

4.3.4 Parallel RL Circuit

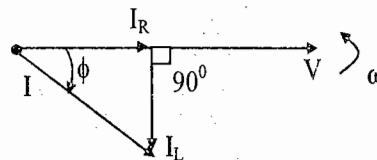


N/W is in S.S.

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{Z_L}$$

$$\Rightarrow I_L = \frac{V}{X_L} \angle -90^\circ$$



$$I = \sqrt{I_R^2 + I_L^2}$$

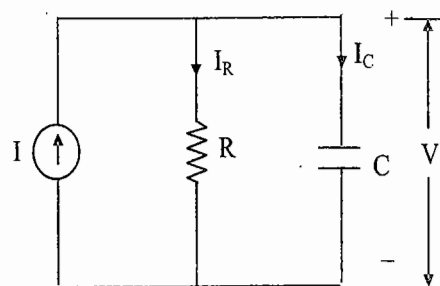
$$\phi = \tan^{-1} \left(\frac{I_L}{I_R} \right) = \text{Impedance Angle}$$

= Admittance Angle

$$\cos \phi = \text{p.f.} = \left(\frac{I_R}{I} \right) (\text{lag})$$

i.e. "I" lags "V"

4.3.5 Parallel RC Circuit



N/W is in S.S.

4.3.6

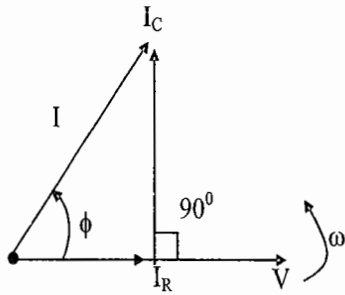




$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{Z_C}$$

$$\Rightarrow I_C = \frac{V}{X_C} \angle 90^\circ$$



$$I = \sqrt{I_R^2 + I_C^2}$$

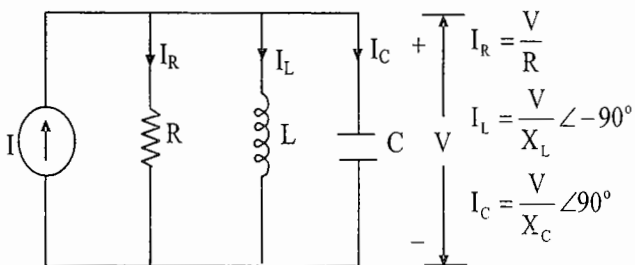
$$\phi = \tan^{-1} \left(\frac{I_C}{I_R} \right) = \text{Impedance Angle}$$

$$= \text{Admittance Angle}$$

$$\cos \phi = \text{p.f} = \left(\frac{I_R}{I} \right) \text{ (lead)}$$

i.e "I" leads "V"

4.3.6 Parallel RLC Circuit



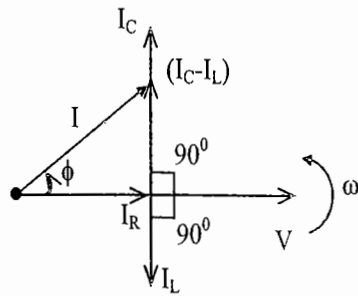
N/w is in S.S.

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

(i) $I_C > I_L$



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

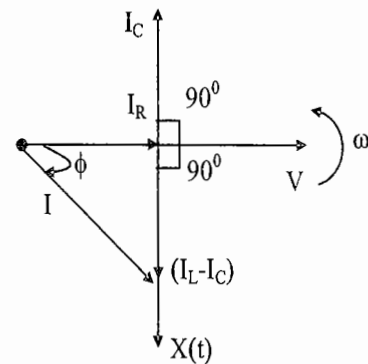
= Impedance Angle

= Admittance Angle

$$\cos \phi = \text{p.f} = \left(\frac{I_R}{I} \right) \text{ (lead)}$$

i.e "I" leads "V"

(ii) $I_L > I_C$



$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{I_L - I_C}{I_R} \right)$$

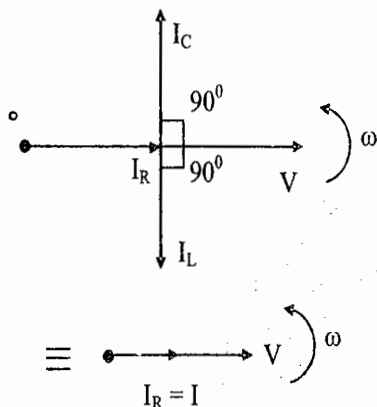
= Impedance Angle

= Admittance Angle

$$\cos\phi = \text{p.f.} = \left(\frac{I_R}{I} \right) (\text{lag})$$

i.e. "I" lags "V"

(iii) $I_L = I_C$



$$\phi = 0^\circ \Rightarrow \text{U.P.f.}$$

So the circuit is under "resonance".

4.4 Power Calculations

4.4.1. Power relations in AC circuits

For a DC circuit, the power is given by

$$P_{dc} = VI = I^2R = \frac{V^2}{R}$$

The power at any instant in an AC circuit is called instantaneous power. It is defined by the product of voltage and current at that instant, i.e.

$$P = vi$$

As the voltage and current are not constant in an AC circuit, the power value will also change with time. Therefore, the average power taken over a period T is

$$P = \frac{1}{T} \int_0^T vi \, dt$$

$$\text{Also, } P = \frac{1}{2\pi} \int_0^{2\pi} vi \, dt$$

The average power is also called the active power or true power and is measured in watts (W). The bigger units are kilowatts (kW) and megawatts (MW).

4.4.2. Power in Pure resistive circuits

The voltage and current in a pure resistive circuit are in phase and may be respectively represented by

$$v = V_m \sin \omega t$$

$$\text{and } i = I_m \sin \omega t$$

The instantaneous power is given by

$$P = vi$$

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{1}{2} V_m I_m (1 - \cos^2 \omega t)$$

\therefore The active power is given by

$$P = \frac{1}{T} \int_0^T vi \, dt$$

$$= \frac{V_m I_m}{2T} \int_0^T (1 - \cos^2 \omega t) \, dt$$

$$= \frac{1}{2} V_m I_m \left(\because \int_0^T \cos^2 \omega t \, dt = 0 \right)$$

$$= \left(\frac{V_m}{\sqrt{2}} \right) \left(\frac{I_m}{\sqrt{2}} \right) = V_{rms} I_{rms} \, W$$

The power waveform is positive throughout the cycle. This is because voltage and current are in phase. The power flows only in one direction. It is consumed in R and appears in the form of heat.

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4.4.3. Power in pure inductive circuits

The voltage leads the current by 90° in a pure inductive circuit. Let the voltage and current at any instant be respectively represented by

$$v = V_m \sin(\omega t + 90^\circ)$$

$$\text{and } i = I_m \sin \omega t$$

The instantaneous power is given by

$$p = vi$$

$$= V_m I_m \sin(\omega t + 90^\circ) \sin \omega t$$

$$= \frac{1}{2} V_m I_m (2 \cos \omega t \sin \omega t)$$

$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Active power (or) Average Power

$$P_L = \frac{1}{2T} \int_0^T V_m I_m \sin 2\omega t dt$$

$$= \frac{1}{2T} V_m I_m \frac{1}{2\omega} (-\cos 2\omega t)_0^T$$

$$= -\frac{V_m I_m}{4\omega T} (\cos 2\omega T - \cos 0)$$

$$= -\frac{V_m I_m}{4\omega T} (\cos 4\pi - \cos 0)$$

$$= -\frac{V_m I_m}{4\omega T} (1 - 1) = 0$$

Therefore, in a pure inductive circuit, the active power supplied over a complete cycle average out to zero.

Reactive power: The power supplied to a reactance (inductive or capacitive) is termed reactive power. Unlike active power denoted by symbol P , the reactive power is denoted by symbol Q and given the name volt-amp-reactive

(var). Let us now see how the reactive power is calculated.

The peak value of instantaneous power is given by

$$Q_L = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

For a pure inductive circuit,

$$V_L = V_{rms} = X_L I_L$$

$$\therefore Q_L = V_L I_L = I_L^2 X_L$$

$$\text{Also, } Q_L = \frac{V_L^2}{X_L}$$

Here Q_L is the rate of change of reactive energy between the load and the source. By convention, Q_L is taken as positive and is called the lagging reactive power.

4.4.4. Power in pure capacitive circuits:

The voltage lags the current by 90° in a pure capacitive circuit. Let the voltage and current at any instant t be represented respectively by

$$v = V_m \sin \omega t$$

$$\text{and } i = I_m \sin(\omega t + 90^\circ)$$

$\therefore$ The instantaneous power can be written as

$$p = vi = \frac{1}{2} V_m I_m 2 \sin \omega t \sin(\omega t + 90^\circ)$$

$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Active power (or) Average Power

$$P_C = \frac{1}{T} \int_0^T p dt = \frac{V_m I_m}{2T} \int_0^T \sin 2\omega t dt$$

$$= -\frac{V_m I_m}{4\omega T} (\cos 2\omega t)_0^T$$



$$= -\frac{V_m I_m}{4\omega T} (1-1) = 0$$

Reactive power

The peak value of instantaneous power can be written as

$$P_c = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

For a purely capacitive circuit,

$$V_{rms} = V_c = X_c I_c$$

$$Q_c = V_c I_c = I_c^2 X_c$$

or
$$Q_c = \frac{V_c^2}{X_c}$$

Here, Q_c is the rate of change of reactive power between the load and the source. By convention, Q_c is taken as negative and is called the leading reactive power.

4.4.5. Power in a general series circuit

For a passive ac circuit, let the voltage and current at any instant to be given by

$$v = V_m \sin \omega t$$

$$\text{and } i = I_m \sin(\omega t - \phi)$$

Where ϕ is the phase difference between the voltage and the current. If ϕ is -ve, current leads the voltage.

The instantaneous power p is given by

$$\begin{aligned} p &= vi \\ &= V_m I_m \sin \omega t \sin(\omega t - \phi) \\ &= \frac{1}{2} V_m I_m (2 \sin \omega t \sin(\omega t - \phi)) \\ &= \frac{1}{2} V_m I_m (\cos \phi - \cos(2\omega t - \phi)) \end{aligned}$$

The average power P is given by

$$\begin{aligned} P_L &= \frac{1}{2T} \int_0^T \sin \omega t \, dt \\ &= \frac{1}{T} \int_0^T \frac{V_m I_m}{2} (\cos \phi - \cos(2\omega t - \phi)) \, dt \\ &= \frac{V_m I_m}{2T} \left(\int_0^T \cos \phi \, dt - \int_0^T \cos(2\omega t - \phi) \, dt \right) \\ &= \frac{V_m I_m}{2T} \times \cos \phi \times T \left(\because \int_0^T \cos(2\omega t - \phi) \, dt = 0 \right) \\ &= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi = V_{rms} I_{rms} \cos \phi \end{aligned}$$

The product of V_{rms} and I_{rms} is called the apparent power and P is called the true or real power.

$$\therefore P = \text{Apparent power} \times \cos \phi$$

or

$$\text{True(real) power} = \text{Apparent power} \times \text{Power factor}$$

That is, the product of rms values of voltage and current with the cosine of the angle between them (i.e., the power factor) is called the real power (or true power) denoted by symbol P .

Reactive power: We can now generalize our previous definition and state that the product of rms values of voltage and current with the sine of the angle between them is called the reactive power Q .

$$\therefore Q = V_{rms} I_{rms} \sin \phi$$

The reactive power is also called the wattless power or quadrature power. The following table represents the different expressions of power in ac series circuits:

Circuit

R

L

C

RL

RC

RLC

Power

True

Power

and

Reactive

Power

Sin

is

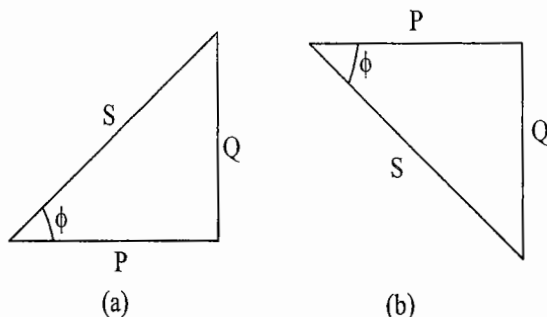


Circuit	Power factor	Real power $P = V_{rms} I_{rms} \cos \phi$	Reactive power $Q = V_{rms} I_{rms} \sin \phi$	Apparent power $S = \sqrt{P^2 + Q^2}$
R	1	$V_{rms} I_{rms}$	0	$V_{rms} I_{rms}$
L	0	0	$V_{rms} I_{rms} (\text{lag})$	$V_{rms} I_{rms}$
C	0	0	$V_{rms} I_{rms} (\text{lead})$	$V_{rms} I_{rms}$
RL	$\cos \phi (\text{lag})$	$V_{rms} I_{rms} \cos \phi$	$V_{rms} I_{rms} \sin \phi (\text{lag})$	$V_{rms} I_{rms}$
RC	$\cos \phi (\text{lead})$	$V_{rms} I_{rms} \cos \phi$	$V_{rms} I_{rms} \sin \phi (\text{lead})$	$V_{rms} I_{rms}$
RLC	$\cos \phi (\text{lag})$	$V_{rms} I_{rms} \cos \phi$	$V_{rms} I_{rms} \sin \phi (\text{lag})$	$V_{rms} I_{rms}$
	$\cos \phi (\text{lead})$	$V_{rms} I_{rms} \cos \phi$	$V_{rms} I_{rms} \sin \phi (\text{lead})$	$V_{rms} I_{rms}$
	$\cos \phi = 1$ (in phase)	$V_{rms} I_{rms}$	0	$V_{rms} I_{rms}$

Power Triangle:

The geometrical representation of the apparent power S , the average (true power / real power) P and the reactive power Q is called the power triangle. Figure (a) shows the power triangle for an inductive load.

Similarly, the power triangle for a capacitive load is shown in figure (b).



Complex Power:

The powers S , P and Q of a power triangle can be obtained from the product VI^* , where I^* is the conjugate of I . It is called the complex power S .

For a capacitive load, let

$$V = V_{rms} e^{j\theta} \quad \text{and} \quad I = I_{rms} e^{+j(\theta+\phi)}$$

$$S = VI^*$$

$$= V_{rms} e^{j\theta} I_{rms} e^{-j(\theta+\phi)}$$

$$= V_{rms} I_{rms} e^{-j\phi}$$

$$= V_{rms} I_{rms} (\cos \phi - j \sin \phi)$$

$$= V_{rms} I_{rms} \cos \phi - j V_{rms} I_{rms} \sin \phi$$

$$= P - jQ$$

Similarly, for an inductive load, let

$$V = V_{rms} e^{j\theta} \quad \text{and} \quad I = I_{rms} e^{+j(\theta+\phi)}$$

$$S = VI^*$$

$$= V_{rms} e^{j\theta} I_{rms} e^{-j(\theta-\phi)}$$

$$= V_{rms} I_{rms} e^{+j\phi}$$

$$= V_{rms} I_{rms} (\cos \phi + j \sin \phi)$$

$$= V_{rms} I_{rms} \cos \phi + j V_{rms} I_{rms} \sin \phi$$

$$= P + jQ$$

By convention, $+Q$ indicates lagging reactive power, while $-Q$ indicates leading reactive power. The inductive load absorbs the lagging reactive power and the capacitive load supplies the lagging reactive power. In other words, the inductive load supplies the leading reactive power; whereas, the capacitive load absorbs the leading reactive power.

Conclusions:

Case (i)

$$\text{Let } v(t) = V_m \cos(\omega t + \alpha) \text{ V}$$

$$i(t) = I_m \cos(\omega t + \beta) \text{ A}$$



(a) True Power = Active Power = Real Power
 $= P = V_{rms} \cdot I_{rms} \cdot \cos\phi \text{ (W)}$

$$\phi = (\alpha \sim \beta)$$

= The angle between voltage and current

(b) Reactive Power = Imaginary Power = Q
 $= V_{rms} \cdot I_{rms} \cdot \sin\phi \text{ (VAR)}$

(c) Complex Power = Apparent Power = S
 $= V_{rms} \cdot I_{rms} \text{ (VA)}$
 $S = \sqrt{P^2 + Q^2} \text{ (VA)}$

Case (ii)

Let $Z = R \pm jX$

$i(t) = I_m \cos(\omega t + \beta) \text{ A}$

$\Rightarrow I_{rms}^2 \cdot Z = I_{rms}^2 \cdot R \pm j I_{rms}^2 \cdot X$

$\Rightarrow S = P \pm jQ$

$|S| = \sqrt{P^2 + Q^2}$

$\angle S = \pm \tan^{-1} \left(\frac{Q}{P} \right)$

$P = I_{rms}^2 \cdot R \text{ (W)} = P_{avg}$

$Q = I_{rms}^2 \cdot X \text{ (VAR)} = Q_{avg}$

Case (iii)

Let $Z = R \pm jX$

$v(t) = V_m \sin(\omega t + \alpha) \text{ V}$

$\Rightarrow Y = \frac{1}{Z} = G \mp jB$

$\Rightarrow V_{rms}^2 \cdot Y = V_{rms}^2 \cdot G \mp j V_{rms}^2 \cdot B$

$\Rightarrow S = P \mp jQ$

$|S| = \sqrt{P^2 + Q^2}$

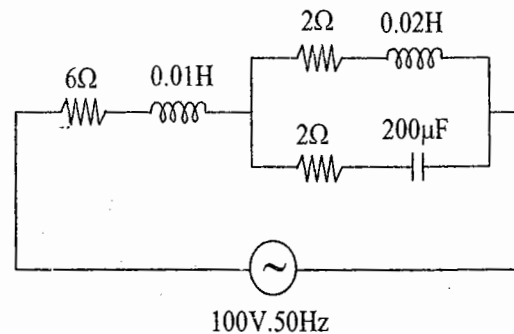
$\angle S = \mp \tan^{-1} \left(\frac{Q}{P} \right)$

$P = V_{rms}^2 \cdot G \text{ (W)} = P_{avg}$

$Q = V_{rms}^2 \cdot B \text{ (VAR)} = Q_{avg}$

Example: 4.5

Determine the current in the circuit of figure. Also, find the power consumed as well as pf.



Sol: $X_{L1} = 2\pi \times 50 \times 0.01 = 3.14 \Omega$

$X_{L2} = 2\pi \times 50 \times 0.02 = 6.28 \Omega$

$X_C = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92 \Omega$

$\bar{Z}_1 = 6 + j3.14 \Omega$

$\bar{Z}_2 = 4 + j6.28 \Omega$

$\bar{Z}_3 = 2 - j15.92 \Omega$

$\bar{Z} = \bar{Z}_1 + \frac{\bar{Z}_2 \times \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$

$= (6 + j3.14) + \frac{(4 + j6.28)(2 - j15.92)}{(4 + j6.28) + (2 - j15.92)}$

$= 17.27 \angle 30.75^\circ \Omega$

$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{100 \angle 0^\circ}{17.27 \angle 30.75^\circ} = 5.79 \angle -30.75^\circ \text{ A}$

$P = VI \cos \phi = 100 \times 5.79 \times \cos(30.75^\circ)$

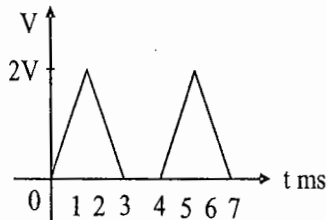
$= 497.94 \text{ W}$

$\text{pf} = \cos \phi = \cos(30.75^\circ) = 0.86 \text{ (lagging)}$



Example: 4.6

Find the average power delivered to a 10Ω resistor with a voltage across it as shown in the figure.



Sol: $V_{rms} = \frac{1}{4} \int_0^4 (V)^2 dt$

$$= 2 \times \frac{1}{4} \int_0^{1.5} \left(\frac{3}{4}t\right)^2 dt = \frac{1}{2} \int_0^{1.5} \frac{9}{16} t^2 dt$$

$$= \frac{1}{2} \times \frac{9}{16} \times \frac{t^3}{3} \Big|_0^{1.5}$$

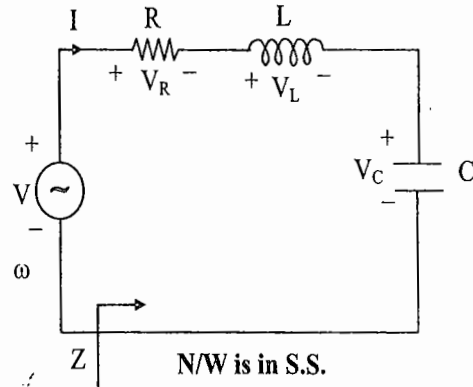
$$V_{rms} = \frac{3}{2 \times 16} \times (1.5)^3 = 0.316$$

$$P = \frac{(V_{rms})^2}{R} = \frac{(0.316)^2}{10} = 100\text{mW}$$

4.5 Concept of Resonance or Frequency Domain Analysis of RLC Circuits

Resonance in electric circuits is because of the presence of both energy storing elements called inductor and capacitor. At a fixed frequency ω_0 , L and C elements will exchange energy freely as a function of time, which results in sinusoidal oscillations either across inductor or capacitor.

4.5.1 Series RLC circuit – Series Resonance



$$Z = Z_R + Z_L + Z_C$$

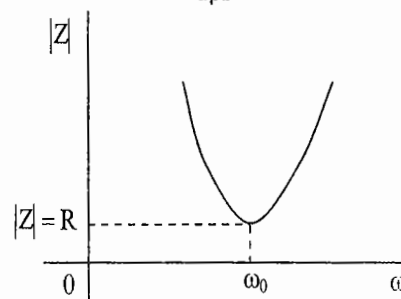
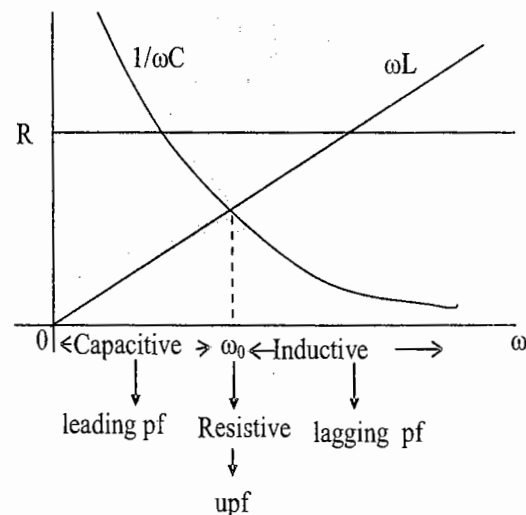
$$Z(j\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

i.e., different sources with same network offers different impedances.

Since $V = Z \cdot I$,

In series, current flowing through all the elements is same.

As Z is more $\Rightarrow V$ is more $\Rightarrow$ that nature of the circuit.





$$\omega = \omega_0 \Rightarrow \frac{1}{\omega_0 C} = \omega_0 L$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$\Rightarrow Z(j\omega_0) = R$$

$$\text{Since, } I = \frac{V}{Z} \Rightarrow \boxed{I = \frac{V}{R}}$$

= Response at resonance.

Here, $Z_L + Z_C = 0 \Rightarrow Z = R \Rightarrow$ L-C combination is like a short circuit

$$\omega = \omega_0$$

$$V_R = IR \quad V_L = I \cdot Z_L = \frac{V}{R} \cdot R = \frac{V}{R} \cdot j\omega_0 L$$

$$\Rightarrow \boxed{V_R = V} \Rightarrow \boxed{V_L = QV \angle 90^\circ}$$

$$\text{Where, } Q = \frac{\omega_0 L}{R}$$

$$V_C = I Z_C = \frac{V}{R} \cdot \frac{1}{j\omega_0 C} \Rightarrow \boxed{V_C = QV \angle -90^\circ}$$

$$\text{Where, } Q = \frac{1}{\omega_0 CR}$$

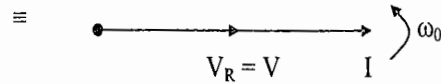
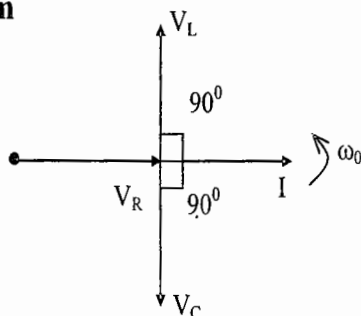
Check:

$$\begin{aligned} \text{By KVL in P.D } \Rightarrow V_R + V_L + V_C \\ = V + jQV - jQV = V \end{aligned}$$

Observation:

Here $V_L + V_C = 0 \Rightarrow$ LC combination is like a short circuit. (with respect to source).

Phasor Diagram



$$\phi = 0^\circ \Rightarrow \text{upf}$$

Freq Response:

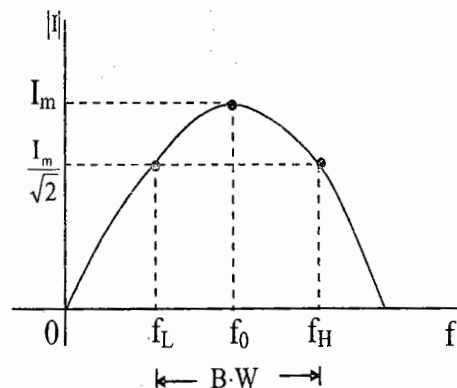
(i.e response magnitude vs frequency)

$$I = \frac{V}{Z} \Rightarrow |I| = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\omega = 0 \Rightarrow |I| = 0$$

$\omega = \omega_0 \Rightarrow |I| = \frac{V}{R} = I_m$ ($\because$ LC combination is like a short circuit)

$$\omega = \infty \Rightarrow |I| = 0$$



$$B \cdot W = f_H - f_L = \frac{f_0}{Q}$$

$$\text{Where } Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$\sqrt{f_H f_L} = f_0$$

- At **resonance**, impedance reaches a minimum value and hence, current reaches a maximum value, so called **acceptor circuit**.
- Since voltages across L and C elements are magnified by 'Q' times (This amplification is due to stored energies in L and C elements),

Osc

Let

Sinc

Sinc

To
at 1

i.e
cc
sc





the series RLC circuit at **resonance** is also called as **voltage magnification circuit**.

Oscillation phenomenon

Let $v(t) = V_m \cos \omega_0 t$

$\omega = \omega_0 \Rightarrow Z = R$

Since, $i(t) = \frac{v(t)}{Z} \Rightarrow i(t) = \frac{V_m}{R} \cos \omega_0 t$

$= i_R(t) = i_L(t) = i_C(t)$ (from basics)

Since periodic current through the resistor

$P_{avg} = I_{rms}^2 R$ (W)

$$= \left(\frac{\frac{V_m}{R}}{\sqrt{2}} \right)^2 \cdot R = \frac{V_m^2}{2R} \text{ (W)}$$

$v_C(t) = \frac{1}{C} \int i_C(t) dt = \frac{V_m}{\omega_0 RC} \sin \omega_0 t$

Total energy stored in the circuit in steady state at resonance is

$W(t) = W_L(t) + W_C(t)$

$= \frac{1}{2} \cdot L \cdot i_L^2(t) + \frac{1}{2} \cdot C \cdot v_C^2(t)$

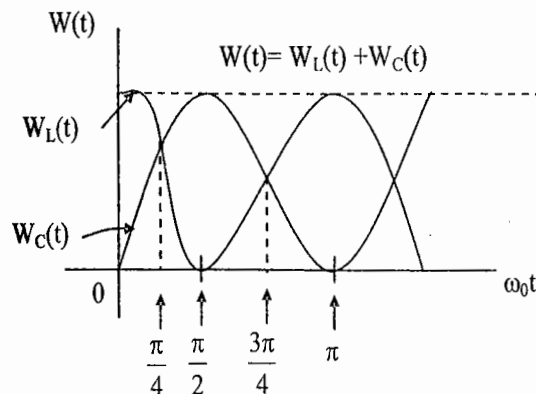
$= \frac{1}{2} \cdot L \cdot \frac{V_m^2}{R^2} \cos^2 \omega_0 t + \frac{1}{2} \cdot C \cdot \frac{V_m^2}{R^2 \omega_0^2 C^2} \sin^2 \omega_0 t$

$= \frac{1}{2} \cdot L \cdot \frac{V_m^2}{R^2} \text{ (J)}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 L = \frac{1}{C}$

$\Rightarrow \omega_0^2 C = \frac{1}{L}$

i.e., The total energy stored in the circuit is constant as the L & C elements are not seeing the source at resonance.



$Q = \frac{\omega_0 \times \text{maximum energy stored in L or C at resonance}}{\text{Average power dissipated at resonance}}$

$$Q = \frac{\omega_0 \times \frac{1}{2} L \frac{V_m^2}{R^2}}{\frac{1}{2} \frac{V_m^2}{R}} = \frac{\omega_0 L}{R} = \frac{\omega_0^2 L}{\omega_0 R} = \frac{1}{\omega_0 CR}$$

$\left(\because \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 \cdot L = \frac{1}{C} \right)$

$$\Rightarrow Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

So Q factor is a function of only circuit constants. As 'Q' is more $\Rightarrow$ circuit is said to be more selective and oscillations produced are of high quality. i.e. clean in nature (Like crystal oscillator)

$\rightarrow \text{As } R \rightarrow 0 \Rightarrow Q \rightarrow \infty \Rightarrow \text{B.W} \rightarrow 0$

$\Rightarrow f_H - f_L \rightarrow 0 \Rightarrow f_H = f_L = f_0$

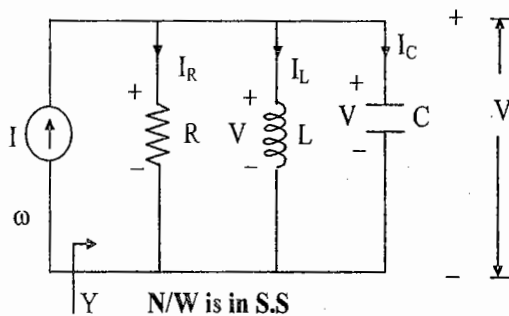
i.e., the circuit operation approaches to a single frequency called the **resonant frequency**, which is impossible to achieve physically, since violation of KVL in the circuit i.e. the voltage source gets short circuited.

So, for the physical existence of the circuit, a minimum R is to be maintained in the circuit (for maximum Q) and this **minimum** resistance is called as the **critical resistance** of the circuit, where the damping ratio $\xi = 1$.

$$\xi = \frac{1}{2Q} = 1 \Rightarrow Q = \frac{1}{2} \Rightarrow \frac{1}{R_{cr}} \sqrt{\frac{L}{C}} = \frac{1}{2}$$

$$\Rightarrow R_{cr} = 2\sqrt{\frac{L}{C}} \Omega = R_{min}$$

4.5.2 Parallel RLC Circuit - Parallel Resonance



Since $I = Y.V$

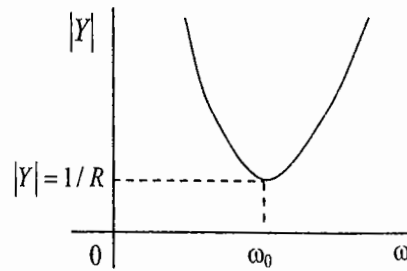
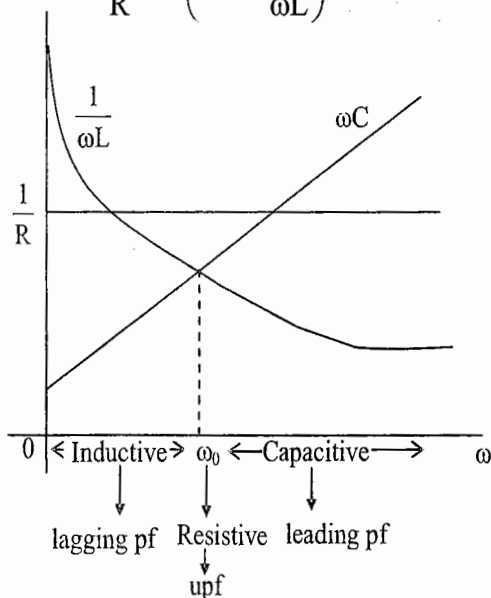
In parallel, the voltage is same across all the elements.

As 'Y' is more $\Rightarrow$ I is more $\Rightarrow$ That nature of the circuit.

$$Y = Y_R + Y_L + Y_C$$

$$= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$\Rightarrow Y(j\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$



$$\omega = \omega_0 \Rightarrow \frac{1}{\omega_0 L} = \omega_0 C,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \Rightarrow Y(j\omega_0) = \frac{1}{R}$$

Since $V = \frac{I}{Y} \Rightarrow \boxed{V = IR}$

Response at resonance

$$\omega = \omega_0$$

$$I_R = \frac{V}{R} = \frac{IR}{R} \Rightarrow \boxed{I_R = I}$$

$$I_L = \frac{V}{Z_L} = \frac{IR}{j\omega_0 L} \Rightarrow \boxed{I_L = QI \angle -90^\circ}$$

Where $Q = \frac{R}{\omega_0 L}$

$$I_C = \frac{V}{Z_C} = \frac{IR}{\frac{1}{j\omega_0 C}} \Rightarrow \boxed{I_C = QI \angle 90^\circ}$$

Where $Q = \omega_0 CR$

Check:

By KCL in P.D $\Rightarrow I_R + I_L + I_C$

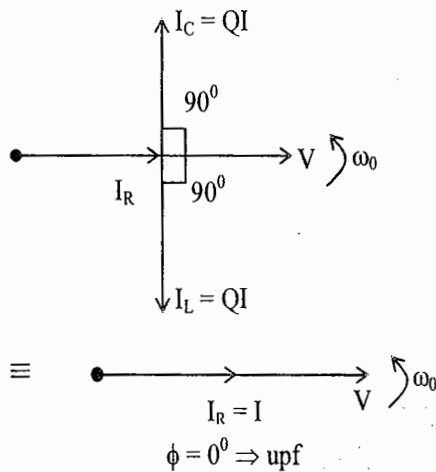
$$= I - jQI + jQI = I$$

Obs:

Here, $I_L + I_C = 0 \Rightarrow$ LC - combination is like an open circuit. (with respect to source)



Phasor Diagram



i.e., AC circuit behaves like DC circuit at resonance.

Frequency response

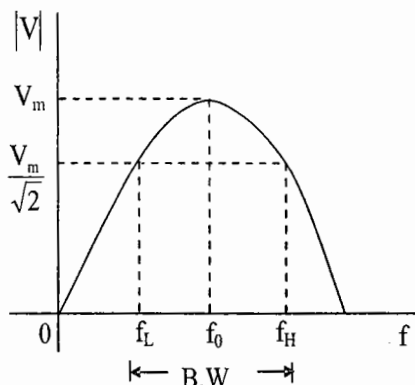
$$V = \frac{I}{Y}$$

$$|V| = \left| \frac{I}{Y} \right| = \frac{I}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2}}$$

$$\omega = 0 \Rightarrow |V| = 0$$

$$\omega = \omega_0 \Rightarrow |V| = IR = V_m$$

$$\omega = \infty \Rightarrow |V| = 0$$



$$B.W = f_H - f_L = \frac{f_0}{Q}$$

$$\text{Where } Q = \frac{R}{\omega_0 L} = \omega_0 CR$$

$$\sqrt{f_H f_L} = f_0$$

- At resonance, admittance reaches a minimum value (i.e., the impedance reaches a maximum value) and hence the circuit accepts less current ($I = Y.V$), so called the **rejector circuit**.
- Since current through L & C elements are magnified by Q times (This amplification also due to the stored energies in L and C elements), parallel RLC circuit at resonance is also called as **current magnification circuit**.

Oscillation phenomenon

$$\text{Let } i(t) = I_m \sin \omega_0 t$$

$$\omega = \omega_0 \Rightarrow Y = \frac{1}{R}$$

$$\begin{aligned} \text{Since } v(t) &= \frac{i(t)}{Y} = R I_m \sin \omega_0 t \\ &= v_R(t) = v_C(t) = v_L(t) \end{aligned}$$

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \text{ (W)}$$

$$= \frac{\left(\frac{R I_m}{\sqrt{2}} \right)^2}{R} = \frac{1}{2} I_m^2 R \text{ (W)}$$

$$i_L(t) = \frac{1}{L} \int v_L(t) dt = -\frac{R I_m}{\omega_0 L} \cos \omega_0 t$$

Total energy stored in the circuit at steady state of resonance is

$$W(t) = W_L(t) + W_C(t)$$



$$\begin{aligned}
 &= \frac{1}{2} L i_L^2(t) + \frac{1}{2} C v_C^2(t) \\
 &= \frac{1}{2} L \frac{R^2 I_m^2}{\omega_0^2 L^2} \cos^2 \omega_0 t + \frac{1}{2} C R^2 I_m^2 \sin^2 \omega_0 t \\
 &= \frac{1}{2} C R^2 I_m^2 (J)
 \end{aligned}$$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega_0^2 L = \frac{1}{C}$$

$$\Rightarrow \omega_0^2 C = \frac{1}{L}$$

$$\begin{aligned}
 Q &= \omega_0 \times \frac{\frac{1}{2} C R^2 I_m^2}{\frac{1}{2} I_m^2 R} \\
 &= \omega_0 C R = \frac{\omega_0^2 C R}{\omega_0} = \frac{R}{\omega_0 L}
 \end{aligned}$$

$$\Rightarrow Q = \omega_0 C R = \frac{R}{\omega_0 L} = R \sqrt{C/L}$$

$$\rightarrow \text{As } R \rightarrow \infty \Rightarrow Z \rightarrow \infty \Rightarrow Y \rightarrow 0 \Rightarrow Q \rightarrow \infty$$

$$\Rightarrow B.W \rightarrow 0 \Rightarrow f_H - f_L \rightarrow 0 \Rightarrow f_H = f_L = f_0$$

i.e. the circuit operation approaches a single frequency called the **resonant frequency**, which is impossible to achieve physically, since violation of KCL in the circuit i.e. the current source gets open circuited.

So, for the physical existence of the circuit, a maximum R is to be maintained (for maximum Q) and this maximum resistance is called as the **critical resistance** of the circuit, where the damping ratio $\xi = 1$.

$$\xi = \frac{1}{2Q} = 1$$

$$Q = \frac{1}{2} \Rightarrow R_{cr} \sqrt{C/L} = \frac{1}{2}$$

$$\Rightarrow R_{cr} = \frac{1}{2} \sqrt{\frac{L}{C}} \Omega = R_{max}$$

4.6 Locus or Circle Diagrams

Locus diagrams are useful for analysis and designing of the circuit. Eg: Filters, Induction machines, Synchronous machines.

With respect to practical application, it is possible to develop possible following locus diagrams:

- i) Current locus diagram
- ii) Voltage locus diagram
- iii) Impedance locus diagram
- iv) Admittance locus diagram

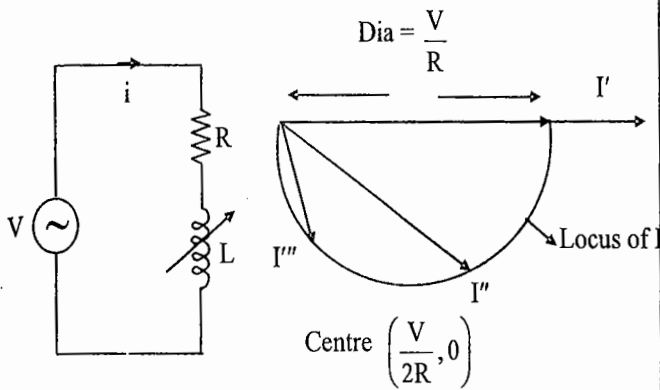
The path traced by terminals of current vectors by varying either any one of the circuit elements (or) by varying source frequency is called as current locus.

In the circuit shown below, as the inductance is varied, the locus of current i in the circuit is plotted below.

If $L = 0$, circuit is purely resistive and i is in phase with V and its magnitude is (V/R) shown as diameter.

For a small value of inductance L present in the circuit it is an R-L load where current lags voltage by impedance angle (i phasor rotates clockwise).

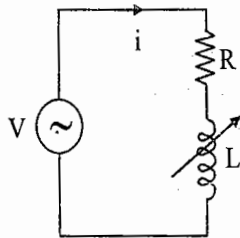
As L value increases, the current ' i ' further lags as shown in the figure below.



Also in the above network, by keeping all the elements constant and by varying source frequency also, current locus diagram of same shape is obtained.

Example: 4.7

Develop current locus for the circuit shown:



Sol: $R = 0$, $Z = X_L$

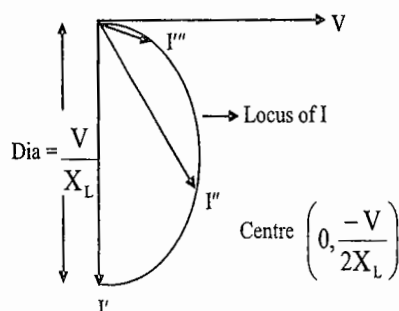
$$\therefore I = \frac{V}{X_L}$$

$$\theta = 90^\circ$$

$$R \uparrow Z \uparrow I \downarrow$$

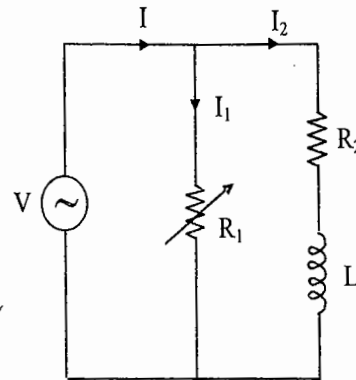
$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) \downarrow$$

$$R \cong \infty, Z \cong \infty, \therefore I = 0$$



Example: 4.8

Develop current locus of I_1 and I for the circuit shown.



Sol: $R_1 = 0.1\Omega$

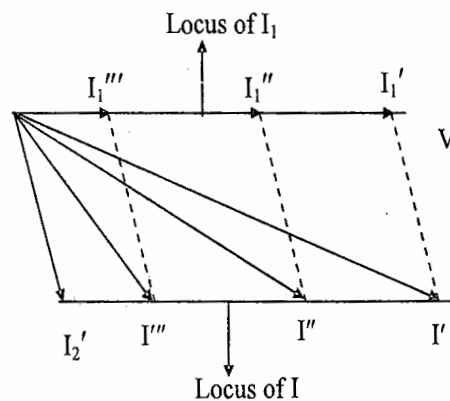
$$I_1 = \frac{V}{R_1}$$

$$\theta_1 = 0$$

$$I = I_1 + I_2$$

$$R_1 \uparrow I_1 \downarrow, \theta_1 = 0, I_2 = \text{constant}$$

$$I \downarrow, R_1 \cong \infty, I_1 = 0, I = I_2$$

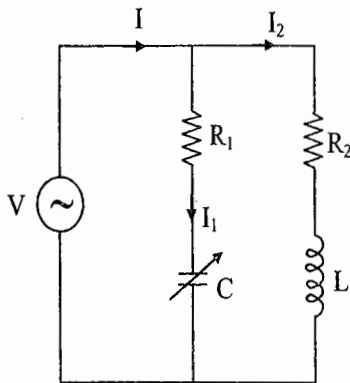


When power factor angle is constant, the shape of the current locus is straight line.

When PF angle is variable, the shape of current locus is semi circle.

Example: 4.9

Develop current locus of I_1' and I for the circuit shown.



Sol: $X_C = \frac{1}{2\pi fC}$

$X_C = 0$;

$Z_1 = R_1$

$I_1 = \frac{V}{R_1}$

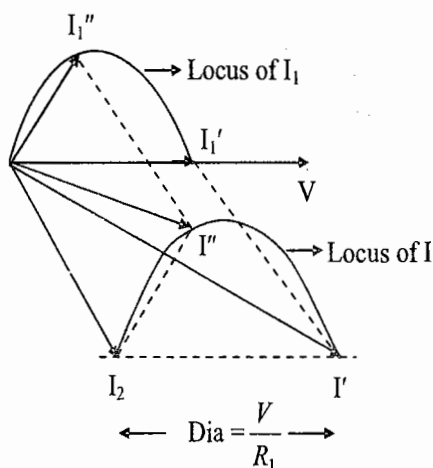
$\theta_1 = 0$

$I_1 = I_1 + I_2$

$X_C \uparrow \uparrow Z_1 = \sqrt{R^2 + X_C^2}$

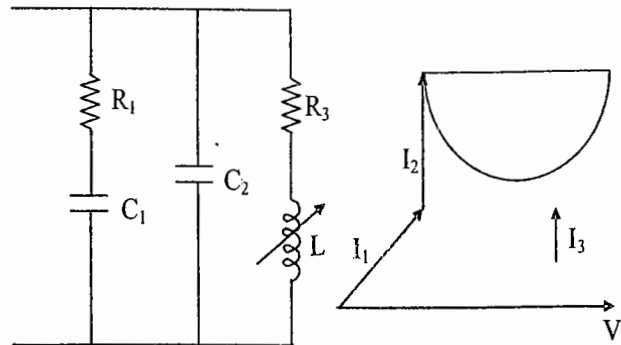
$I_1 \downarrow \theta_1 = \tan^{-1} \left(\frac{-X_C}{R} \right) \uparrow$

$X_C \cong \infty$, $Z_1 \cong \infty$, $I_1 = 0$, $I_1 = I_2$



Example: 4.10

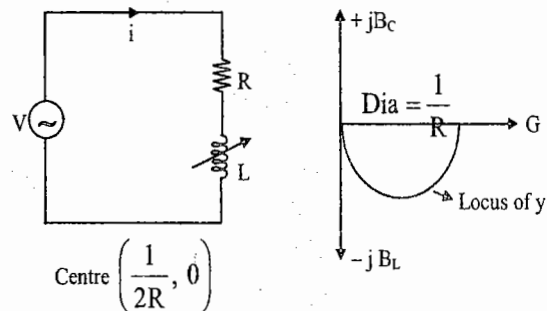
Design a network for given current locus diagram.



Semicircle downward = Inductor

Semicircle upward = Capacitor

Admittance Locus Diagram:

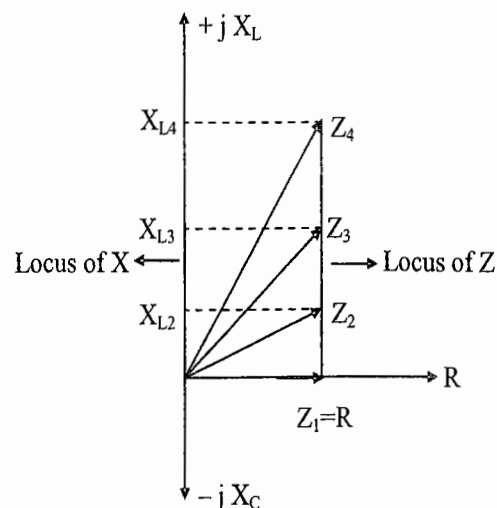


Impedance Locus Diagram:

$X_L = 0$, $X_L \uparrow$

$Z = R$, $Z \uparrow$

$\theta = 0$, $\theta = \tan^{-1} \left(\frac{X_L}{R} \right) \uparrow$



01.

02.

03.

04.

05.

Class Room Practice Questions

LEVEL-1 Questions

01. An alternating current source having voltage $E = 10 \sin(\omega t + \pi/3)$ is connected in an a.c. circuit. If the current drawn from the circuit varies as $I = 5 \sin(\omega t - \pi/3)$, the impedance of the circuit will be

(IES-EE-92)

- (a) 22 ohm (b) 16 ohm
(c) 30.8 ohm (d) none of the above.

02. In a series R - L - C circuit excited by a voltage $e = E \sin \omega t$, where, $LC < \frac{1}{\omega^2}$

(IES-EE-92)

- (a) Current lags the applied voltage
(b) Current leads the applied voltage
(c) Current is in phase with the applied voltage
(d) Voltages across L and C are equal.

03. A high Q coil has (IES-EE-92)

- (a) Large bandwidth (b) High losses
(c) Low losses (d) Flat response

04. When Q factor of a circuit is high, then

(IES-EE-92)

- (a) Power factor of the circuit is high
(b) Impedance of the circuit is high
(c) Bandwidth is large
(d) None of line

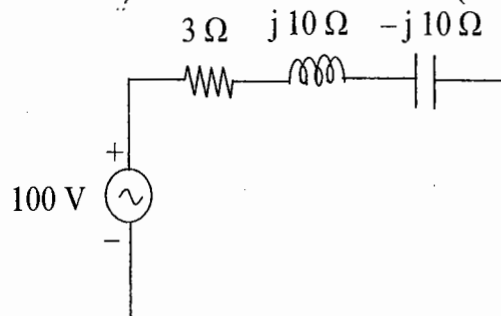
05. An RLC series circuit has f_1 and f_2 as the half power frequencies and f_0 as the resonance frequency. The Q - factor of the circuit is given by (IES-EE-94)

- (a) $\frac{f_1 + f_2}{2f_0}$ (b) $\frac{f_1 - f_0}{f_2 - f_0}$
(c) $\frac{f_0}{f_2 - f_1}$ (d) $\frac{f_2 + f_1}{f_0}$

06. If the source of 200 V rms supplies active power of 600 W and reactive power of 800 VAR. The rms current drawn from the source is (IES-EE-99)

- (a) 10 A (b) 5 A
(c) 3.75 A (d) 2.5 A

07. The reactive power drawn from the source in the network shown in the given figure is (IES-EE-99)



- (a) 300 VAR (b) 200 VAR
(c) 100 VAR (d) zero

08. For a series RLC circuit, the power factor at the lower half power frequency is (IES-EE-99)

- (a) 0.5 lagging (b) 0.5 leading
(c) unity (d) 0.707 leading

09. If the resonant frequency of the circuit shown in Figure - I is 1 kHz, the resonant frequency of the circuit shown in Figure - II will be (IES-EE-00)

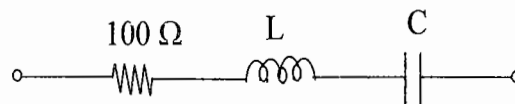


Figure - I

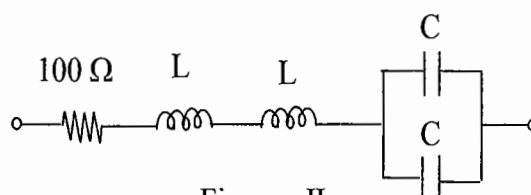
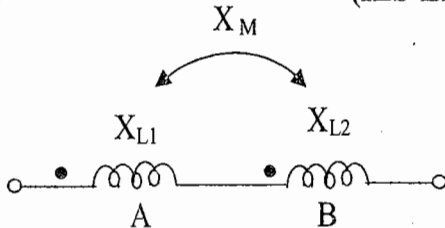


Figure - II

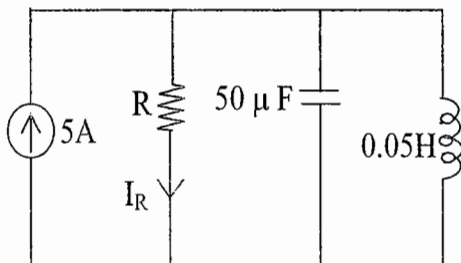


- (a) 4 kHz (b) 2 kHz
(c) 0.5 kHz (d) 0.25 kHz

10. The equivalent reactance of two coils A and B connected as in the given figure is given by **(IES-EE-00)**



- (a) $X_{L1} + X_{L2} - 2 X_M$
(b) $X_{L1} + X_{L2} + X_M$
(c) $X_{L1} + X_{L2} - X_M$
(d) $X_{L1} + X_{L2} + 2 X_M$
11. An imperfect capacitor is represented by a capacitance C in parallel with a resistance R . The value of its dissipation factor $\tan \delta$ is **(IES-EE-02)**
- (a) $\omega C R$ (b) $\omega^2 C R$
(c) $\frac{1}{\omega^2 C R}$ (d) $\frac{1}{\omega C R}$
12. In the given circuit, at resonance, I_R in Amperes is equal to **(IES-EE-02)**



- (a) 0 (b) 10 (c) 5 (d) 0.5
13. In a series R-L-C circuit, the maximum voltage across the capacitor occurs at a frequency **(IES-EE-06)**
- (a) Double the resonant frequency
(b) Equal to resonant frequency

- (c) $\sqrt{2}$ times the resonant frequency
(d) Below the resonant frequency

14. When is a series RLC circuit overdamped? **(IES-EE-07)**

- (a) $\frac{R^2}{4L^2} = \frac{1}{LC}$ (b) $\frac{R^2}{4L} < \frac{1}{C}$
(c) $\frac{R^2}{4L} > \frac{1}{C}$ (d) $\frac{R^2}{4C^2} = \frac{1}{LC}$

15. In a series resonance circuit, at resonance, selectivity Q is equal to **(IES-EE-12)**

- (a) $\frac{1}{R\sqrt{LC}}$ (b) $\frac{1}{R}\sqrt{C}$
(c) $\frac{1}{R}\sqrt{\frac{L}{C}}$ (d) $\frac{1}{R}\sqrt{LC}$

16. Consider the following statements regarding the properties of an R-L-C series circuit under resonance: **(IES-EC-09)**

1. Current in the circuit is in phase with applied voltage.
 2. Voltage drop across capacitor C and inductance L are equal in magnitude
 3. Voltage across the capacitor is equal in magnitude to the applied voltage.
 4. Current in the circuit is maximum.
- Which of the above statements is/are correct?

- (a) 1 only (b) 1, 2 and 4
(c) 2 and 4 (d) 1, 3 and 4

17. At 3-dB frequencies, current in the series RLC circuit equal current at resonance multiplied by **(ISRO-06)**

- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2\sqrt{2}}$

18. A
A
ir
(
(

19. V
re
b
(
(

20. V
v
c
(
(

21. A
c

(
(

22. I
(

(
(

23. ?



18. A series RLC circuit resonates at 1000kHz. At frequency of 995kHz, the circuit impedance is (ISRO-06)

(a) Resistive (b) minimum
(c) inductive (d) capacitive

19. When L is doubled and C is halved, the resonance frequency of series tuned circuit becomes (ISRO-06)

(a) Doubled (b) Halved
(c) One quarter (d) Unchanged

20. With fixed value capacitor C and variable voltage V across it, the energy stored in the capacitor is (ISRO-07)

(a) CV^2 (b) $0.5CV^2$
(c) $2CV^2$ (d) CV

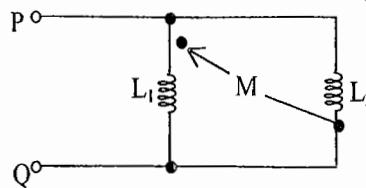
21. A dc voltage V is applied to a series RL circuit. The steady state current (ISRO-07)

(a) V/R (b) V/L
(c) $\frac{V}{\sqrt{R^2 + L^2}}$ (d) zero

22. In RLC circuits, the current at resonance is
(a) maximum in parallel resonance and inimum in series resonance
(b) maximum in series resonance and minimum in parallel resonance
(c) maximum in both series and parallel resonances
(d) minimum in both series and parallel resonances

23. For an RLC series circuit, current at series resonance is
(a) maximum at leading pf
(b) maximum at lagging pf
(c) maximum at unity pf
(d) minimum at unity pf

24. The equivalent inductance of the circuit between terminals P and Q is equal to (SPDCL-12)



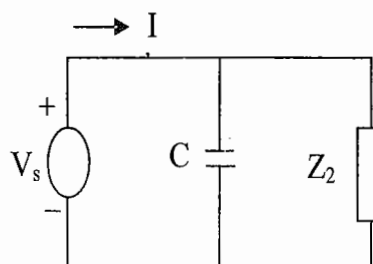
(a) $\frac{(L_1 + L_2 + 2M)}{(L_1 L_2 - M^2)}$
(b) $\frac{(L_1 + L_2 - 2M)}{(L_1 L_2 - M^2)}$
(c) $\frac{(L_1 L_2 - M^2)}{(L_1 + L_2 + 2M)}$
(d) $\frac{(L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)}$

25. The impedance of a two element series circuit is represented by $(20-j10)\Omega$ at a certain frequency. If the frequency is doubled; the new value of impedance is (GENCO-12)

(a) $(20-j5)\Omega$ (b) $(40-j20)\Omega$
(c) $(10-j10)\Omega$ (d) $(20-j20)\Omega$

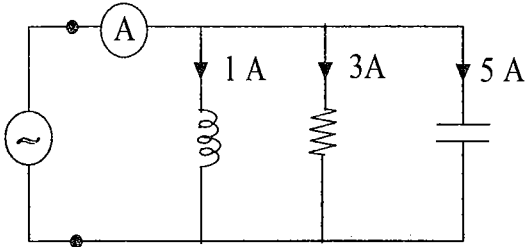
LEVEL-2 Questions

01. In the circuit shown in the figure, $V_s = V_m \sin 2t$ and $Z_2 = 1+j$. The value of C is chosen such that the current I is in phase with V_s . The value of C (in farads) is (IES-EE-93)



- (a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$
(c) 2 (d) 4

02. The current read by the ammeter A in the ac circuit shown in the given figure is
(IES-EE-95)



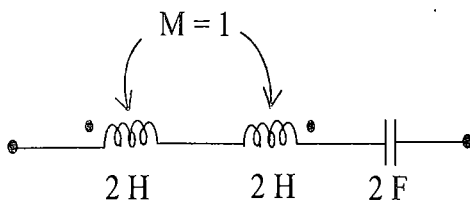
- (a) 9 A (b) 5 A
(c) 3 A (d) 1 A

03. Consider the following statements with respect to a series R - L - C circuit under resonance condition: (IES-EE-95)

1. All the applied voltage appears across R
2. There is no voltage across either L or C.
3. The voltage across L and C are equal to their maximum values.

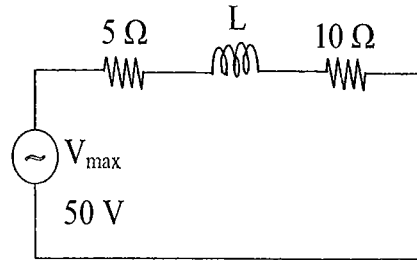
- (a) 1 alone is correct
(b) 2 alone is correct
(c) 1 and 3 are correct
(d) 1 and 2 are correct

04. The resonant frequency of the series circuit shown in the given figure is (IES-EE-95)



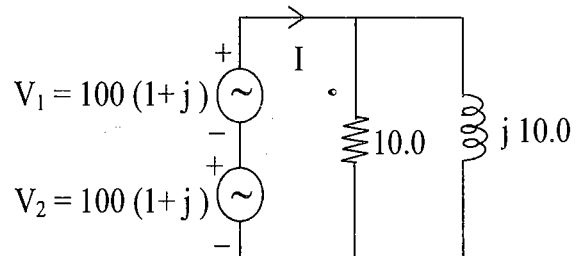
- (a) $\frac{1}{4\pi}$ Hz (b) $\frac{1}{4\pi\sqrt{2}}$ Hz
(c) $\frac{1}{2\pi\sqrt{2}}$ Hz (d) $\frac{1}{4\pi\sqrt{3}}$ Hz

05. In the circuit shown in the given fig., if the power consumed by the 5 ohm resistor is 10 W, then the power factor of the circuit is
(IES-EE-96)



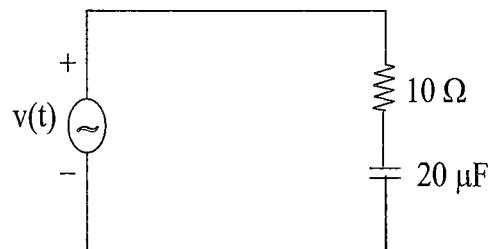
- (a) 0.8 (b) 0.6
(c) 0.5 (d) zero

06. The phase angle of the current 'I' with respect to the voltage V_1 in the circuit shown in the figure is
(IES-EE-97)



- (a) 0° (b) $+45^\circ$
(c) -45° (d) -90°

07. In the given RC circuit, the current $i(t) = 2 \cos 5000 t$ A. (IES-EE-02)



The applied voltage $v(t)$ is

- (a) $28.28 \cos (5000 t - 45^\circ)$ V
(b) $28.28 \cos (5000 t + 45^\circ)$ V
(c) $28.28 \sin (5000 t - 45^\circ)$ V
(d) $28.28 \sin (5000 t + 45^\circ)$ V



08. For a parallel RLC resonant circuit, the damped frequency is $\sqrt{8}$ r/s and bandwidth is 2 r/s. What is its resonant frequency?

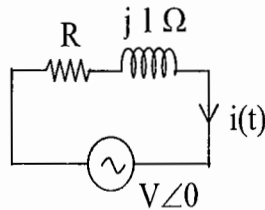
(IES-EE-04)

- (a) 2 (b) $\sqrt{7}$
(c) $\sqrt{10}$ (d) 3

09. For the network shown, if the current $i(t) = \sqrt{2} \sin(\omega t - 30^\circ)$, then what is the value of R?

(IES-EE-08)

- (a) 1 Ω
(b) 3 Ω
(c) $\sqrt{3}$ Ω
(d) $3\sqrt{3}$ Ω



10. What is the approximate steady state current in the above circuit?

(IES-EC-09)

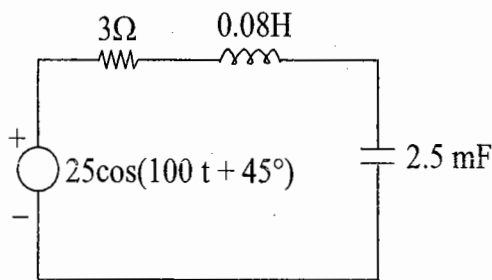
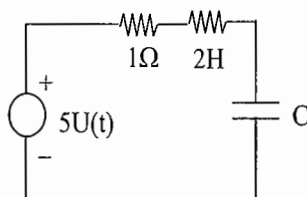


Fig.

- (a) 50 A (b) 25 A
(c) 5 A (d) 1 A

11. The value of C which gives the critical damping in the given circuit is

(ISRO-06)



- (a) 2F (b) 4F
(c) 8F (d) 1F

12. A series RLC circuit resonates at 3MHz and has 3-dB bandwidth of 10 kHz. The Q of the circuit at resonance

(ISRO-06)

- (a) 30 (b) $\frac{300}{\sqrt{2}}$ (c) 300 (d) $300\sqrt{2}$

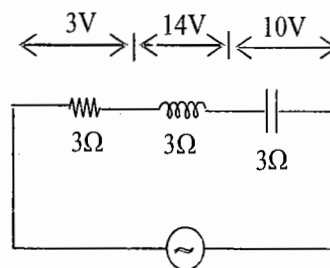
13. In a series resonant circuit, with the increase in L

(ISRO-06)

- (a) Resonant frequency will decrease
(b) Bandwidth will decrease
(c) Q will increase
(d) All of these

14. The source in the circuit shown is a sinusoidal source. The supply voltages across various elements are marked in the figure. The input voltage is

(ISRO -07)

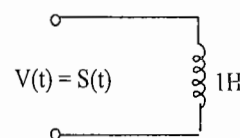


- (a) 10V (b) 5V
(c) 27V (d) 24V

15. When a unit impulse voltage is applied to an inductor of 1H, the energy supplied by the source is

(ISRO-08)

- (a) ∞
(b) 1 J
(c) $\frac{1}{2}$ J
(d) 0



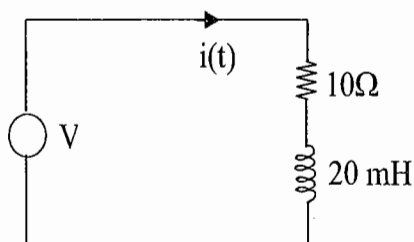
16. A coil has self resistance of 5 Ω and inductance of 1 μ H. The value of Q at 1MHz frequency is

(ISRO -09)

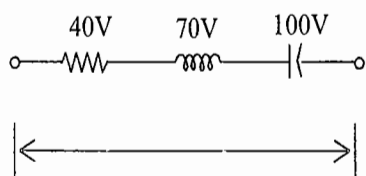
- (a) 200 (b) 100
(c) 40 (d) None of these



17. In the circuit given below, the current is $i(t) = 4 \sin(500t)$ amps. The applied voltage in volts
(FRO)/(SPDCL-12)



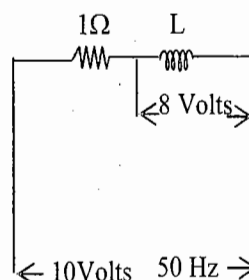
- (a) $40 \sin(500t)$
 (b) $56.56 \sin(500t + 45^\circ)$
 (c) $40 \cos(500t)$
 (d) $56.56 \cos(500t + 45^\circ)$
18. The value of current at resonance in a series RLC circuit is affected by the value of
 (a) R (b) L
 (c) C (d) R, L, and C
19. Two coupled inductors $L_1 = 0.2$ H and $L_2 = 0.45$ H have coefficient of coupling $k = 0.8$. The mutual inductance M is
 (a) 0.2 H (b) 0.24 H
 (c) 0.42 H (d) 0.8 H
20. The supply voltage $|V|$ in diagram below is



- (a) 210V (b) 70V
 (c) 50V (d) 230V
21. An alternating voltage has a periodic time $2T$. The voltage for a time $\frac{1}{3}T$ is 50V, for

$\frac{1}{6}T$, is 20V and zero for $\frac{1}{2}T$. Calculate the rms value and the average value of the voltage

- (a) 30, 20 (b) 35, 20
 (c) 20, 35 (d) 25, 35
22. A RLC series circuit resonant at frequency of f_0 . If all the component values are doubled the new resonant frequency is
(VSP011)
 (a) $2f_0$ (b) f_0
 (c) $0.5f_0$ (d) $0.25f_0$
23. The peak of conductance curve of a series RLC circuit occurs at
(EPDCL-12)
 (a) zero frequency
 (b) below the resonant frequency
 (c) resonant frequency
 (d) above the resonant frequency
24. For the circuit given below, the current through 1 ohm resistor will be
(EPDCL-12)



- (a) 2 amps (b) 4 amps
 (c) 6 amps (d) 8 amps

01.
Sol:

02.
Sol:

03.
Sol:

04.
Sol:

05.
Sol:



Solutions

LEVEL-1

01. Ans: (d)

Sol: Phasor, $E = 10 \angle \pi/3$

Phasor, $I = 5 \angle -\pi/3$

$$Z = \frac{E}{I} = 2 \angle 2\pi/3$$

02. Ans: (b)

Sol: When $\omega = \omega_0 =$ Resonant frequency

$$= \sqrt{\frac{1}{LC}}, Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R$$

If $LC < \frac{1}{\omega^2}$, $\omega < \sqrt{\frac{1}{LC}}$ ($= \omega_0$), then the

reactance of Z is negative and circuit is capacitive and current leads the applied voltage.

03. Ans: (c)

Sol: $Q = \frac{\omega_0 L}{R}$

If Q is high, R is less (low loss) and bandwidth is small as $BW = \frac{f_0}{Q}$.

04. Ans: (b)

Sol: For series circuit,

$$Z = R + j\omega L = R \left(1 + j \frac{\omega L}{R}\right) = R(1 + jQ)I$$

mpedance of the circuit is high.

05. Ans: (c)

Sol: $BW = \frac{f_0}{Q}$, $Q = \frac{f_0}{BW} = \frac{f_0}{f_2 - f_1}$, where f_2

and f_1 are the upper and lower half power frequencies respectively.

06. Ans: (b)

Sol: $V I \cos \phi = 600$

$$V I \sin \phi = 800$$

$$\tan \phi = \frac{4}{3}, \sec^2 \phi = \frac{25}{9}, \sec \phi = \frac{5}{3},$$

$$\cos \phi = \frac{3}{5} = 0.6$$

$$I = \frac{600}{V \cos \phi} = \frac{600}{200 \times 0.6} = 5 \text{ A}$$

07. Ans: (d)

Sol: The reactive power is zero as $Z = 3\Omega$ and the reactance is zero.

08. Ans: (d)

Sol: $\frac{I}{V_s} = \frac{1}{R} \frac{V_R}{V_s} = \frac{1}{R} \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$

At lower half-power frequency, ω_1

$$\frac{I}{V_s} = \frac{1}{R} \frac{1}{1 - j1}$$

$\therefore I$ leads V_s by 45°

$\therefore \text{PF} = \cos \phi = \cos 45^\circ = 0.707$ leading.

09. Ans: (c)

Sol: $f_{01} = \frac{1}{2\pi\sqrt{LC}} = 10^3$

$$f_{02} = \frac{1}{2\pi\sqrt{2L \times 2C}}$$

$$= \frac{1}{2} f_{01} = 500 \text{ Hz or } 0.5 \text{ kHz}$$

10. Ans: (d)

Sol: The coupling is aiding.

$$L_{\text{equ}} = L_1 + L_2 + 2M$$

$$j\omega L_{\text{equ}} = j\omega L_1 + j\omega L_2 + 2j\omega M$$

$$\therefore X_{\text{equ}} = X_{L_1} + X_{L_2} + 2X_M$$



11. Ans: (d)

12. Ans: (c)

$$\text{Sol: } Y = \frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right)$$

$$= \frac{1}{R} \quad \text{if } \omega_0 = \frac{1}{\sqrt{LC}}$$

At resonance, parallel LC is open. $I_R = I_S = 5A$

13. Ans: (b)

Sol: $V_C(\max) = Q V_S$ if $Q > 1$ at

$$\omega = \omega_{\text{res}}$$

$$= V_S \quad \text{if } Q < 1 \text{ at } \omega = 0$$

14. Ans: (c)

Sol: Poles are given by

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\text{When } \left(\frac{R}{L} \right)^2 > \left(\frac{4}{LC} \right) \text{ or } \frac{R^2}{4L} > \frac{1}{C},$$

poles are distinct, real and negative and the circuit is overdamped.

15. Ans: (c)

$$\text{Sol: } Q = \frac{\omega_0 L}{R}$$

$$\text{Where } \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$\therefore Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

16. Ans: (b)

Sol: For a series RLC circuit,

$$\bar{I} = \frac{\bar{V}}{Z}, \quad Z = R + j \left(\omega L - \frac{1}{\omega C} \right)$$

At resonance,

$$\omega L = \frac{1}{\omega C}, \quad \bar{I} = \frac{\bar{V}}{R} \text{ (maximum)},$$

$\bar{I}$ and $\bar{V}$ are in phase, $\bar{V}_L = -\bar{V}_C$

17. Ans: (b)

18. Ans: (d)

Sol: Series resonance circuit is capacitive before resonant frequency & is Inductive afterwards. Also it is resistive at resonant frequency.

19. Ans: (d)

Sol: Resonance frequency of series circuit =

$$\frac{1}{\sqrt{LC}}$$

$$\text{Now } L = 2L \text{ \& } C = \frac{C}{2}$$

New, Resonance frequency

$$= \frac{1}{\sqrt{(2L)\left(\frac{C}{2}\right)}} = \frac{1}{\sqrt{LC}}$$

20. Ans: (b)

21. Ans: (a)

Sol: Inductor acts as a S.C. under steady state

$$\therefore I_{s.s} = \frac{V}{R}$$

22. Ans: (b)

Sol: In RLC circuits, current is maximum for series circuit and minimum for parallel ckt at resonance condition.

23. Ans: (c)

Sol: (i) At resonance condition, circuit is resistive hence unity pf

(ii) For series resonance circuit, current is maximum at resonance

24.
Sol:

25.
Sol



24. Ans: (c)

Sol: Current is entering in dotted terminals of one coil and leaving from dotted terminals of other coil. If means both coils are connected in opposite polarity

$$\therefore L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

25. Ans: (a)

Sol: For a given impedance, real part represents resistance of network which is independent of frequency.

Imaginary part represent reactance which depends on frequency

$$\text{When } j\left(\omega L - \frac{1}{\omega C}\right) > 0$$

$\Rightarrow$ circuit is inductive and reactance is proportional to frequency also reactance is positive

$$\text{When } j\left(\omega L - \frac{1}{\omega C}\right) < 0$$

Circuit is capacitive and reactance is inversely proportional to frequency also reactance is negative.

For given impedance $(20-j10)\Omega$, circuit is capacitive as reactance is negative

$$\therefore \text{Reactance} \propto \frac{1}{\text{frequency}}$$

$$\therefore \text{New impedance} = (20-j5)\Omega$$

Level-2 Key

01. (a)	02. (b)	03. (a)	04. (a)	05. (b)
06. (c)	07. (a)	08. (d)	09. (c)	10. (c)
11. (c)	12. (c)	13. (d)	14. (b)	15. (c)
16. (d)	17. (b)	18. (a)	19. (b)	20. (c)
21. (c)	22. (c)	23. (c)	24. (c)	



Introduction

A **port** is a pair of nodes across which a device can be connected. The voltage is measured across the pair of nodes and the current going into one node is the same as the current coming out of the other node in the pair. These pairs are entry (or exit) points of the network.

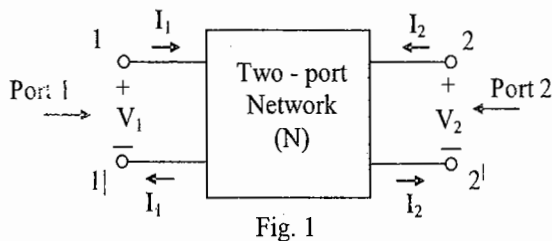


Fig. 1

So, a network with two input terminals and two output terminals is called a **four-terminal network** or a **two-port network**.

In a two-port network, there are two voltage variables and two current variables. According to the choice of input and output ports, these voltage and current variables can be arranged in different equations, giving rise to different port parameters.

Relationships of Two-Port variables

In order to describe the relationships among the port voltages and currents of an n -port network, ' n ' number of linear equation is required. However, the choice of two independent and two dependent variables is dependent on the particular application.

So, from four variables, we can select two variables as independent variables in $4C_2 =$

$6(n_{C_2})$ different ways and hence, six sets of two port parameters for a given network

Types of parameters as mentioned below:

1. Open-circuit impedance parameters (Z-parameters)
2. Short-circuit admittance parameters (Y-parameters)
3. Transmission or chain parameters (T-parameters or ABCD- parameters)
4. Inverse transmission parameters (T' -parameters)
5. Hybrid parameters (h-parameters)
6. Inverse hybrid parameters (g-parameters)

Symmetrical Network

A symmetrical network is a network which is divided into two equal halves, with each half as a mirror image of the other.

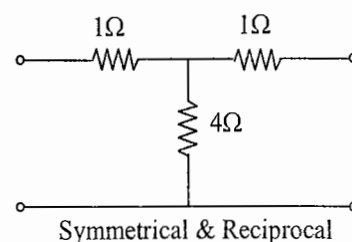
In other words, if the network looks same from both the ports, then it is said to be Symmetrical, otherwise it is said to be Asymmetrical.

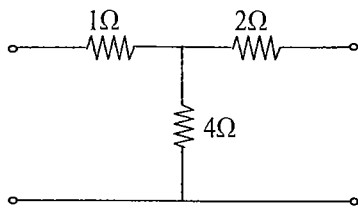
Reciprocal Network

If the network obeys Reciprocity theorem, then it is said to be reciprocal, otherwise it is said to be non-reciprocal.

All the passive networks are always reciprocal and all the active networks are always non-reciprocal.

Eg:





Asymmetrical & Reciprocal

The relation between the four variables V_1 , I_1 , V_2 and I_2 can be described as a set of **two** equations in **six** ways by taking two of them as dependent variables, which depend upon the other two as

independent variables. The four coefficients of the independent variables in the equations are known as the two-port parameters describing the network.

The six sets of two-port parameters are indicated in the following table. The parameters can be calculated from the circuit, which is obtained after implementing either a short circuit ($V_2 = 0$ or $I_2 = 0$) or an open circuit ($I_1 = 0$ or $I_2 = 0$) as shown in Fig. 2(a), 2(b), 2(c) and 2(d)

Name of the parameters	Express	In terms of	Equation	Associated Matrix	Condition for reciprocity, symmetry
Open-circuit Impedance (z)	V_1 V_2	I_1 and I_2 I_1 and I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$	$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$Z_{12} = Z_{21}$ $Z_{11} = Z_{22}$
Short-circuit Admittance (y)	I_1 I_2	V_1 and V_2 V_1 and V_2	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$	$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$Y_{12} = Y_{21}$ $Y_{11} = Y_{22}$
Transmission (T) (A, B, C, D)	V_1 I_1	V_2 and I_2 V_2 and I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$AD - BC = 1$ $A = D$
Inverse Transmission (T') (A', B', C', D')	V_2 I_2	V_1 and I_1 V_1 and I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$	$[T'] = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$	$A'D' - B'C' = 1$ $A' = D'$
Hybrid (h)	V_1 I_2	I_1 and V_2 I_1 and V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$h_{12} = -h_{21}$ $\Delta_h = 1$
Inverse hybrid (g)	I_1 V_2	V_1 and I_2 V_1 and I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$g_{12} = -g_{21}$ $\Delta_g = 1$

Transmission parameters are also called as chain parameters or general circuit parameters.

The symbol Δ indicates the determinant. For example $\Delta_h = \det [h] = h_{11} h_{22} - h_{12} h_{21}$

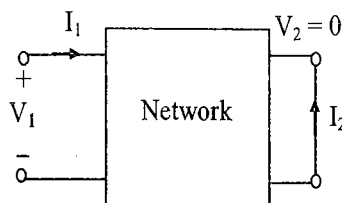


Fig. 2(a)

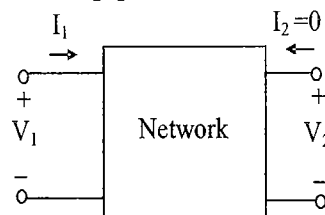


Fig. 2(b)

Port 2 is SC for calculating y_{11} , y_{21} , h_{11} , h_{21} , B & D

Port 2 is OC for calculating z_{11} , z_{21} , g_{11} , g_{21} , A & C

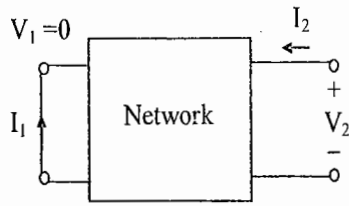


Fig. 2(c)

Port 1 is SC for calculating y_{12} , y_{22} , g_{12} , g_{22} , B' & D'

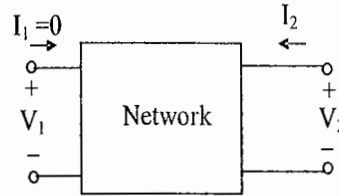


Fig. 2(d)

Port 1 is OC for calculating z_{12} , z_{22} , h_{12} , h_{22} , A' & C'

The parameters of one set can be expressed in terms of parameters of any other set by appropriately using the two sets of equations. For example

$$[y] = [z]^{-1}, [T'] = [T]^{-1}, [g] = [h]^{-1} \rightarrow (1)$$

5.1 Open Circuit Impedance Parameters (Z-Parameters)

The impedance parameters represent the relation between the voltages and the currents in the two-port network.

The impedance parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ or, } \begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}$$

In this matrix equation, it is easily seen without even expanding the individual equations, that

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \text{Driving point impedance at Port-1}$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \text{Transfer impedance}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \text{Transfer impedance}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \text{Driving point impedance at Port-2}$$

It can be seen that Z-parameters correspond to the *driving point* and *transfer* impedances at each port with the other port having zero current (i.e., open circuit). Thus, these parameters are referred to as the open-circuit parameters.

Example: 5.1

Determine the Z-parameters for the network shown in fig 1.1

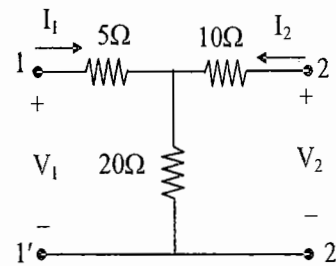


Fig. 1.1

Sol: We consider two situations:

(a) When $I_1 = 0$, i.e., Port-1 is open circuited

In this case, no current will flow through the 5-Ω resistor.

By KVL in the right mesh, we get $10I_2 + 20I_2 - V_2 = 0$



$$\therefore Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = 30\Omega$$

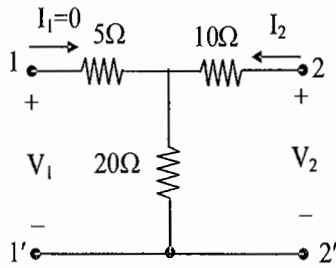


Fig 1.1 (a) When $I_1=0$

From fig 1.1 (a) we get $V_1 = 20I_2$

$$\therefore Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 20\Omega$$

- (b) When $I_2 = 0$, i.e., Port-2 is open circuited
In this case, no current will flow through the 10-Ω resistor.

By KVL in the left mesh, we get
 $5I_1 + 20I_1 - V_1 = 0$

$$\therefore Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = 25\Omega$$

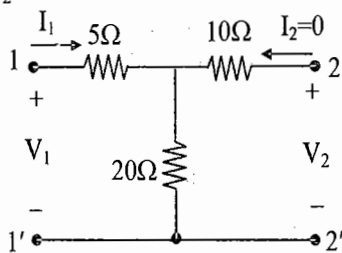


Fig 1.1 (b) When $I_2=0$

From fig 1.1 (b), we get $V_2 = 20I_1$

$$\therefore Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = 20\Omega$$

Therefore, the z-parameters of the network

$$\text{are } [Z] = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix} \Omega$$

5.2 Short Circuit Admittance Parameters (Y-Parameters)

The admittance parameters represent the relation between the currents and the voltages in the two-port network.

The admittance parameter matrix may be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{ or, } \begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}$$

The parameters Y_{11} , Y_{12} , Y_{21} , Y_{22} can be defined in a similar manner, with either V_1 or V_2 on short circuit.

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \text{Driving point admittance at Port-1}$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \text{Transfer admittance}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \text{Transfer admittance}$$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \text{driving point admittance at Port-2}$$

It can be seen that the Y-parameters correspond to the *driving point* and *transfer* admittances at each port with the other port having zero voltage (i.e., short circuit). Thus, these parameters are also referred as the short circuit parameters.

Example: 5.2

Find the y-parameters for the network shown in fig 1.2

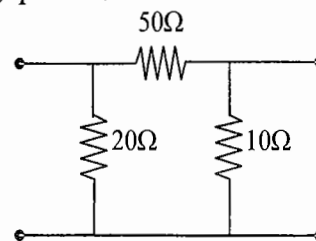


Fig 1.2

Sol: We consider two situations:

When $V_1 = 0$, i.e., Port -1 is short circuited

In this case, no current will flow through the $20\text{-}\Omega$ resistor. The modified circuit is shown in fig 1.4(a)

By KCL at the node 2,

$$\frac{V_2 - 0}{10} + \frac{V_2 - 0}{50} = I_2$$

$$\therefore y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{1}{10} + \frac{1}{50} = 0.12\Omega$$

Also, from fig 1.2 (a) we get $I_1 = \frac{0 - V_2}{50}$

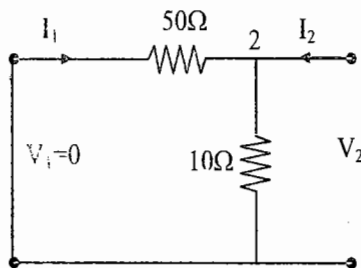


Fig1.2 (a) When $V_1=0$

$$\therefore y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = -\frac{1}{50} = -0.02\Omega$$

When $V_2 = 0$, i.e., Port-2 is short circuited

In this case, no current will flow through the $10\text{-}\Omega$ resistor. The modified circuit is shown in fig 1.2 (b)

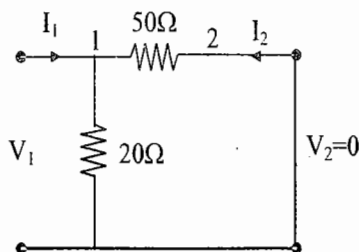


Fig1.2 (b) When $V_2=0$

By KCL at the node 1,

$$\frac{V_1 - 0}{20} + \frac{V_1 - 0}{50} = I_1$$

$$\therefore y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{1}{20} + \frac{1}{50} = 0.07\Omega$$

Also, from fig 1.4 (b) we get $I_2 = \frac{0 - V_1}{50}$

$$\therefore y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = -\frac{1}{50} = -0.02\Omega$$

Therefore, the y-parameters of the network

$$\text{are } [y] = \begin{bmatrix} 0.07 & -0.02 \\ -0.02 & 0.12 \end{bmatrix} \Omega$$

5.3 Transmission Parameters (ABCD-Parameters)

The ABCD parameters represent the relation between the input quantities and the output quantities in the two-port network. They are thus voltage-current pairs.

However, as the quantities are defined as an input-output relation, the output current is marked as going out rather than as coming into the port.

The transmission parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ or, } V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

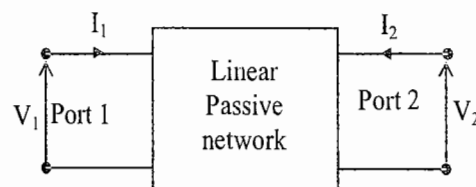


Fig : Two-port current and voltage variables for calculation of transmission line parameters

The parameters A,B,C,D can be defined in a similar manner with either Port 2 on short circuit or Port 2 on open circuit



$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \text{open-circuit reverse voltage gain}$$

$$B = \left. \frac{V_1}{I_2} \right|_{V_2=0} = \text{Short-circuit transfer impedance}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \text{open-circuit transfer admittance}$$

$$D = \left. \frac{I_1}{I_2} \right|_{V_2=0} = \text{Short-circuit reverse current gain}$$

These parameters are known as transmission parameters as in a transmission line, the currents enter at one end and leave at the other end, and we need to know a relation between the sending-end quantities and the receiving-end quantities.

Example: 5.3

For the network shown in fig 1.3, determine the ABCD parameters.

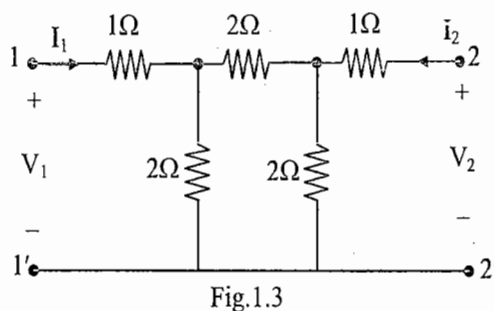


Fig.1.3

Sol: The ABCD-parameter equations are

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

For the network shown in fig 1.3, we convert the delta consisting of the resistances of 2 Ω each into its equivalent star so that the circuit becomes as shown in fig.1.3(a) and fig.1.3(b).

$$r_1 = r_2 = r_3 = \frac{2 \times 2}{2 + 2 + 2} = \frac{2}{3} \Omega$$

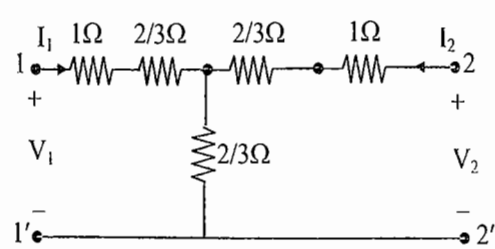


Fig 1.3 (a)

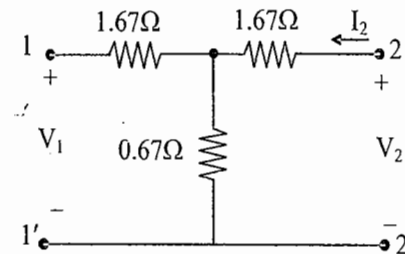


Fig 1.3 (b)

To find the ABCD parameters, we consider two situations:

When $V_2 = 0$, i.e., Port – 2 is short-circuited.

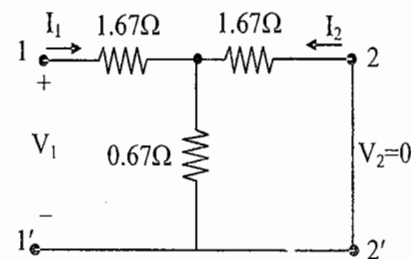


Fig 1.3 (c)

As shown in fig 1.3 (c), by KVL we get,

$$1.67I_1 + 0.67(I_1 + I_2) = V_1$$

or, $2.33I_1 + 0.67I_2 = V_1$

and, $0.67(I_1 + I_2) + 1.67I_2 = 0$

or, $I_1 = -\frac{2.33}{0.67}I_2 = -3.5I_2$

$$\therefore D = \left| -\frac{I_1}{I_2} \right|_{V_2=0} = 3.5$$

Putting this value in the first equation, we get,

$$2.33 \times (-3.5)I_2 + 0.67I_2 = V_1$$

$$\Rightarrow B = \left. -\frac{V_1}{I_2} \right|_{V_2=0} = 7.5\Omega$$

When $I_2 = 0$ i.e., Port-2 is open-circuited
Here, no current will flow through the right side of the 1.67Ω resistance.

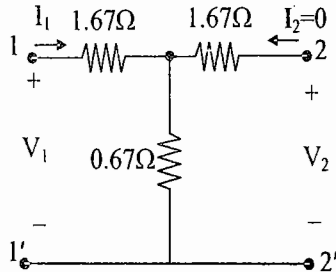


Fig 1.3 (d)

By KVL, we get, $V_1 = (1.67 + 0.67)I_1 = 2.33I_1$
and, $V_2 = 0.67I_1$

$$\therefore C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \frac{1}{0.67} = 1.5 \text{ } \bar{U}$$

$$\therefore A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{2.33I_1}{0.67I_1} = 3.5$$

Therefore, the ABCD parameter of the network are $A = 3.5$; $B = 7.5 \Omega$; $C = 1.5 \bar{U}$; and $D = 3.5$

5.4 Inverse Transmission Parameters (A'B'C'D'-Parameters)

The inverse A'B'C'D' parameters represent the inverse relation between the input quantities and the output quantities in the two-port network. They are also voltage-current pairs.

Here also, the output current is marked as going out rather than as coming into the port as shown in fig 1.5. the inverse transmission parameters matrix may be written as

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \text{ or, } \begin{aligned} V_2 &= A'V_1 - B'I_1 \\ I_2 &= C'V_1 - D'I_1 \end{aligned}$$

The parameters A' , B' , C' , D' can be defined in a similar manner with either Port 1 on short circuit or Port 1 on open circuit.

$$A' = \left. \frac{V_2}{V_1} \right|_{I_1=0} = \text{open-circuit voltage gain}$$

$$B' = \left. -\frac{V_2}{I_1} \right|_{V_1=0} = \text{Short-circuit transfer impedance}$$

$$C' = \left. \frac{I_2}{V_1} \right|_{I_1=0} = \text{Open-circuit transfer admittance}$$

$$D' = \left. -\frac{I_2}{I_1} \right|_{V_1=0} = \text{Short-circuit current gain}$$

5.5 Hybrid Parameters (h-Parameters)

The hybrid parameters represent a mixed or hybrid relation between the voltage and the current in the two-port network.

The hybrid parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \text{ or, } \begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}$$

The h-parameters can be defined in a similar manner and are commonly used in some electronic circuit analysis.

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = \text{Short-circuit input impedance}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \text{Open-circuit reverse voltage gain}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = \text{Short-circuit current gain}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \text{Open-circuit output admittance}$$

As the h-parameters are dimensionally mixed, they are also named mixed parameters. Transistor circuit models are generally represented by these parameters, as the input impedance (h_{11}) and the short-circuit current gain (h_{21}) can be easily measured by making the output short-circuited.

hybrid-model of the CE configuration of transistor

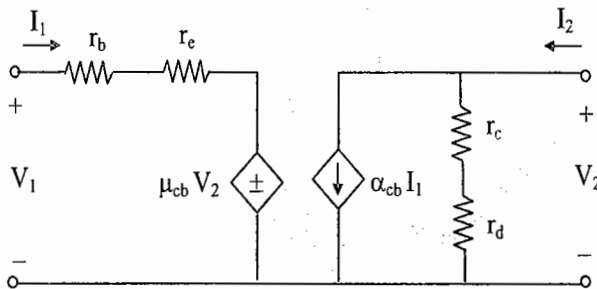


Fig. (7)

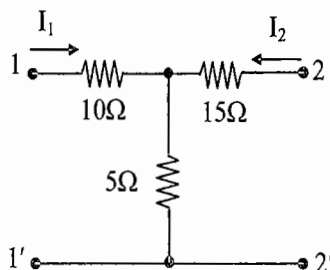
Model of the CE connected transistor

$$[h] = \begin{bmatrix} r_b + r_e & \mu_{bc} \\ \alpha_{cb} & 1/(r_c + r_d) \end{bmatrix}$$

Neither Reciprocal nor Symmetric
(Contains dependent sources)

Example: 5.4

Find the hybrid parameters for the network shown in below figure



Sol: By KVL, $15I_1 + 5I_2 = V_1$ (i)

$$5I_1 + 20I_2 = V_2 \text{ (ii)}$$

Thus, the z-parameters are $z_{11} = (5 + j10)\Omega$

$$z_{12} = z_{21} = 5\Omega \quad z_{22} = (5 + j15)\Omega$$

The hybrid parameters equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{and} \quad I_2 = h_{21}I_1 + h_{22}V_2$$

From equation (ii), we get,

$$I_2 = -\frac{5}{20}I_1 + \frac{V_2}{20} = -\frac{1}{4}I_1 + \frac{1}{20}V_2 \text{(iii)}$$

Putting this value of I_2 in equation (i), we

$$\text{get, } 15I_1 + 5\left[-\frac{1}{4}I_1 + \frac{V_2}{20}\right] = V_1$$

$$\Rightarrow V_1 = \frac{55}{4}I_1 + \frac{1}{4}V_2 \text{(iv)}$$

Comparing equation (iii) and (iv) with the standard equations of h-parameters, we get,

$$h_{11} = \frac{55}{4}\Omega; h_{12} = \frac{1}{4}; h_{21} = -\frac{1}{4}; h_{22} = \frac{1}{20} \text{ } \Omega$$

5.6 Inverse hybrid Parameters (g-Parameters)

The inverse hybrid parameters also represent a mixed or hybrid relation between the voltages and the currents in the two-port network.

The inverse hybrid parameter matrix may be written as

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \quad \text{or,} \quad \begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned}$$



The g-parameters can be defined in a similar manner and are commonly used in some electronic circuit analysis.

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} = \text{Open-circuit input admittance}$$

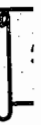
$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0} = \text{Short-circuit reverse current gain}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0} = \text{Open-circuit voltage gain}$$

$$g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0} = \text{Short-circuit output impedance}$$

Interrelation Between Two-Port Network Parameters

[Z]	$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$	$\begin{pmatrix} \frac{y_{22}}{\Delta y} & -\frac{y_{21}}{\Delta y} \\ -\frac{y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{pmatrix}$	$\begin{pmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{pmatrix}$	$\begin{pmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{pmatrix}$
[Y]	$\begin{pmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{pmatrix}$	$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$	$\begin{pmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{pmatrix}$
[ABCD]	$\begin{pmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{pmatrix}$	$\begin{pmatrix} -\frac{y_{22}}{\Delta y} & -\frac{1}{\Delta y} \\ \frac{y_{21}}{\Delta y} & -\frac{y_{11}}{\Delta y} \end{pmatrix}$	$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$	$\begin{pmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ \frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{pmatrix}$
[h]	$\begin{pmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{pmatrix}$	$\begin{pmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{pmatrix}$	$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$



5.7 Interconnection of 2-port Networks

Series connection:

This is illustrated in Fig. (1). Here, the individual Z – parameter matrices get added.

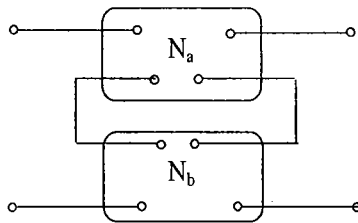


Fig. (1)

$$(Z) = (Z_a) + (Z_b) \dots\dots\dots (1)$$

For this relation to be true, the nature of the original networks should not be altered by series connection.

Parallel connection:

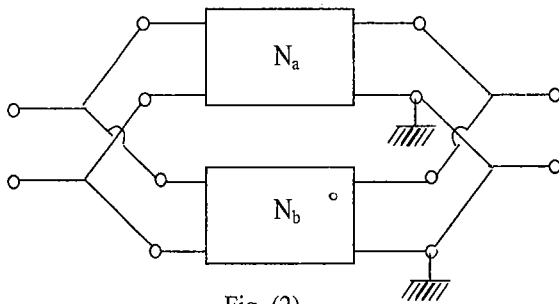


Fig. (2)

This is illustrated in Fig. 2 with 2-port networks having common ground terminal.

For this case, the individual Y-parameter matrices get added to give the Y-parameter matrix of the overall network.

$$(Y) = (Y_a) + (Y_b) \dots\dots\dots (2)$$

The individual Y-parameters also get added :

$$Y_{11} = Y_{11a} + Y_{11b}, \text{ etc.}$$

This relation is true only when the nature of the two networks is not altered by parallel connection. Fig.2(a) shows a Twin –T network, which is illustrated in Fig 2(b) as the parallel connection of two 2 – port (3 terminal) T-networks. Similarly Fig 3(a) shows a bridged –T network, which is illustrated in Fig. 3(b) as the parallel connection of a T-network and a π -network with only a series element, Z_a .

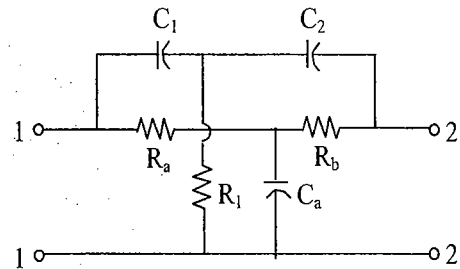


Fig. 2(a)

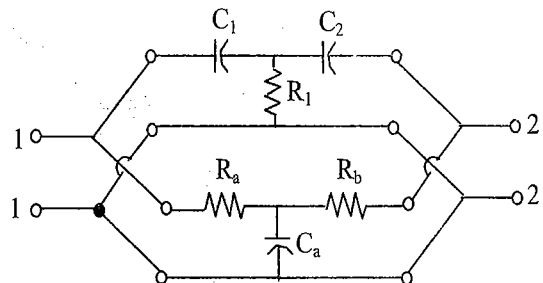


Fig. 2(b)

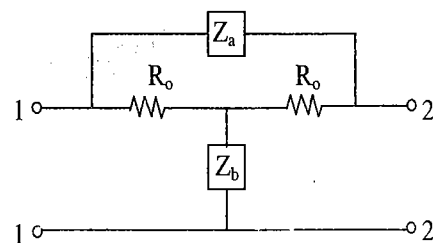


Fig. 3(a)

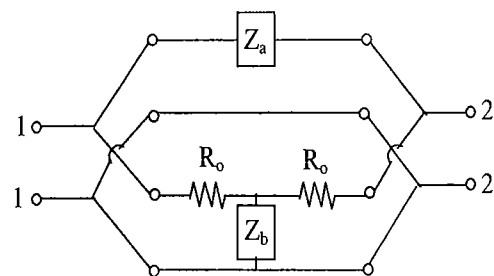


Fig. 3(b)

Cascade (chain or tandem) connection:

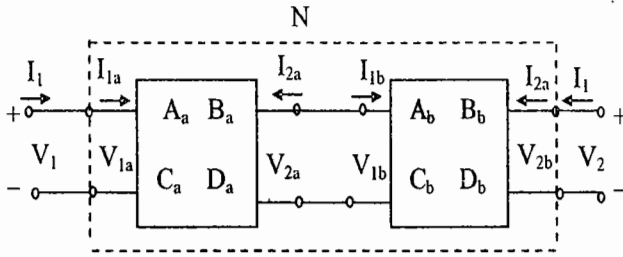


Fig. (4)

Fig. (4) illustrates this connection. Here, the individual T- matrices get multiplied.

$$(T) = (T_a) \times (T_b) \dots\dots\dots (3)$$

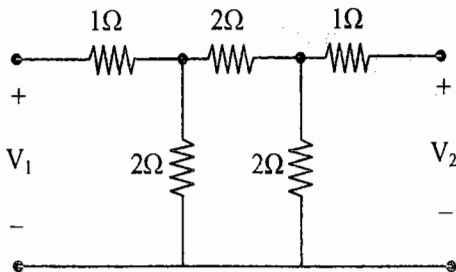
Which is valid irrespective of whether the original networks have a common ground terminal or not.

Note that $A \neq A_a A_b$, etc.

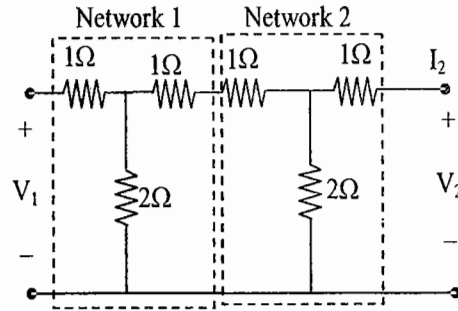
$$A = A_a A_b + B_a C_b, \text{ etc.}$$

Example: 5.5

Find the transmission parameters for the network shown in fig considering two networks connected in cascade.



Sol: The network of fig can be considered to be the cascade connection of two – port network as shown in fig



We know that for cascade connection, the overall transmission parameter matrix is the product of the individual transmission parameter matrices

We find the transmission parameter matrix of the individual sections.

For each section, the z-parameters are given as

$$z_{11} = (1 + 2) = 3\Omega; \quad z_{12} = z_{21} = 2\Omega;$$

$$z_{22} = (1 + 2) = 3\Omega$$

By the interrelationship between z-parameters and transmission parameters, we get

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$$

$$\therefore A = \frac{z_{11}}{z_{21}} = \frac{3}{2}; \quad B = \frac{\Delta z}{z_{21}} = \frac{3 \times 3 - 2^2}{2} = \frac{5}{2}\Omega;$$

$$C = \frac{1}{z_{21}} = \frac{1}{2} \text{ U} ; \quad D = \frac{z_{22}}{z_{21}} = \frac{3}{2}$$

Therefore, the transmission parameter matrix of the network of fig (a) is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} \times \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & \frac{15}{2} \\ \frac{3}{2} & \frac{7}{2} \end{bmatrix}$$

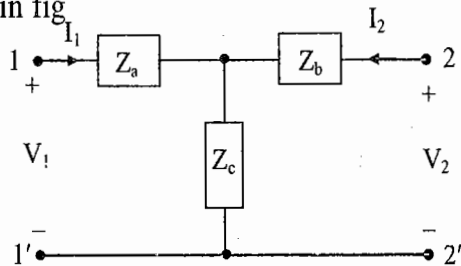
5.8 Typical networks and Parameters

We consider six typical two-port networks

1. T networks
2. π network
3. Ladder network
4. Lattice network
5. Bridge-T network
6. Parallel-T or twin-T or notch-filter network

5.8.1 T or Star or Y network

The configuration of a typical T-network is shown in fig



By KVL equations for the two meshes, we get

$$(Z_a + Z_b)I_1 + Z_c I_2 = V_1$$

$$\text{and } Z_c I_1 + (Z_b + Z_c)I_2 = V_2$$

Thus, the z-parameters are

$$Z_{11} = (Z_a + Z_b); Z_{12} = Z_{21} = Z_c; Z_{22} = (Z_b + Z_c)$$

Rearranging,

$$Z_a = (Z_{11} - Z_{12}); Z_b = (Z_{22} - Z_{12}) Z_c = Z_{12} + Z_{22}$$

From the inter-relationship, we get for T-network, the transmission parameters as

$$A = \frac{Z_{11}}{Z_{21}} = \left(1 + \frac{Z_a}{Z_c}\right); B = \frac{\Delta Z}{Z_{21}} = \left(Z_a + Z_b + \frac{Z_a Z_b}{Z_c}\right);$$

$$C = \frac{1}{Z_{12}} = \frac{1}{Z_c}; D = \frac{Z_{22}}{Z_{12}} = \left(1 + \frac{Z_b}{Z_c}\right)$$

$$\text{Conversely, } Z_a = \frac{(A-1)}{C}; Z_b = \frac{(D-1)}{C}; \text{ and}$$

$$Z_c = \frac{1}{C} \text{ and the y-parameters,}$$

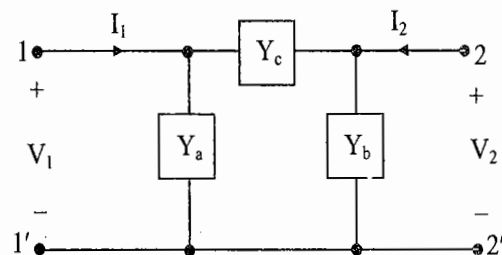
$$y_{11}' = \frac{Z_{22}}{\Delta Z} = \frac{Z_b + Z_c}{Z_a Z_b + Z_b Z_c + Z_c Z_a};$$

$$y_{12} = y_{21} = -\frac{Z_{12}}{\Delta Z} = \frac{-Z_c}{Z_a Z_b + Z_b Z_c + Z_c Z_a};$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{Z_a + Z_c}{Z_a Z_b + Z_b Z_c + Z_c Z_a};$$

5.8.2 π or Delta Network

The configuration of a typical π - network is shown in fig



By KCL equations at the two nodes, we get

$$(V_1 - V_2)Y_c + V_1 Y_a = I_1 \Rightarrow (Y_a + Y_c)V_1 - Y_c V_2 = I_1$$

$$V_2 Y_b + (V_2 - V_1)Y_c = I_2 \Rightarrow -Y_c V_1 + (Y_b + Y_c)V_2 = I_2 \text{ Thus}$$

, the y-parameters are

$$y_{11} = (Y_a + Y_b); y_{12} = y_{21} = -Y_c; y_{22} = (Y_b + Y_c)$$

Rearranging,

$$Y_a = (y_{11} + y_{12}); Y_b = (y_{22} + y_{12}) Y_c = -y_{12} = -y_{21}$$

From the inter-relationship we get for π -

network, the transmission parameters as

$$A = -\frac{y_{22}}{y_{21}} = \left(1 + \frac{Y_b}{Y_c}\right); B = -\frac{1}{y_{21}} = \frac{1}{Y_c};$$

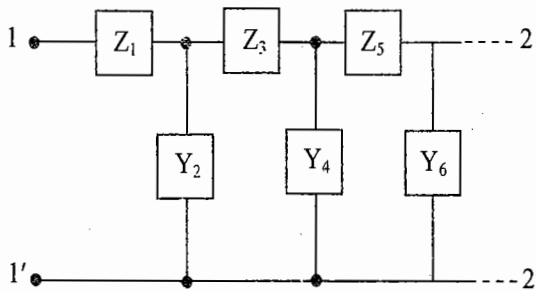
$$C = -\frac{\Delta y}{y_{21}} = \left(Y_a + Y_b + \frac{Y_a Y_b}{Y_c}\right); D = -\frac{y_{11}}{y_{21}} = \left(1 + \frac{Y_a}{Y_c}\right)$$

Conversely, $Y_a = \frac{(D-1)}{B}$; $Y_b = \frac{(A-1)}{B}$; and

$$Y_c = \frac{1}{B}$$

5.8.3 Ladder Network

The configuration of a typical ladder – network is shown in fig:



The series arms are indicated by their impedances $Z_1, Z_3, Z_5, \dots$ and the shunt arms are indicated by their admittances $Y_2, Y_4, Y_6, \dots$

In order to find the driving point impedance at Port -1 we start computation at Port -2 with Y_6 ; i.e., inverting Y_6 , combining with Z_5 , inverting the sum, and so on.

Thus, the driving point impedance at Port 1-1' is given as

$$Z_{11} = Z_1 + \frac{1}{Y_2 + \frac{1}{Z_3 + \frac{1}{Y_4 + \frac{1}{Z_5 + \frac{1}{Y_6}}}}}$$

Note: This equation is known as continued fraction.

In order to find the transfer function, we again start at the output port and then proceed backward, applying KCL and KVL where necessary.

5.8.4 Lattice Network

A Lattice network forms the basis of design of most four- terminal networks like attenuators, filters etc.

It consists of two identical impedances in series arm and two identical impedances in shunt arm as shown in fig (a)

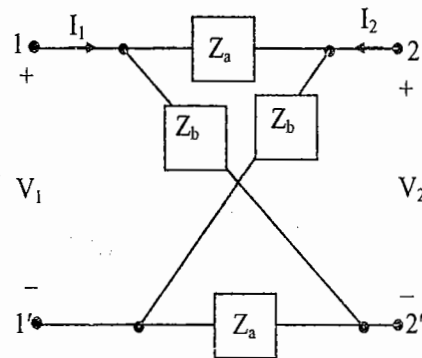


Fig (a)

Here, Z_a are the series arms and Z_b are the diagonal or shunt arms.

To find the z-parameters, we redraw the networks as shown in fig (b)

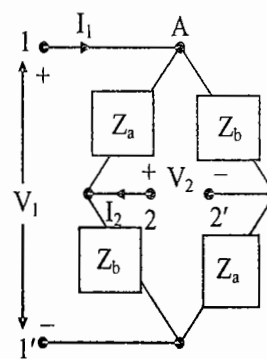


Fig (b)

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Also,

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$$\therefore Z_{21}$$

$$\therefore Z_a$$

5.8.5

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$$Y_{jk}$$



Assuming $I_2 = 0$, the current $I_1 =$ enters the bridge at the point A and divides equally between the two arms.

$$\therefore \frac{I_1}{2} Z_a + V_2 = \frac{I_1}{2} Z_b \Rightarrow V_2 = I_1 \left(\frac{Z_b - Z_a}{2} \right)$$

$$\therefore Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \left(\frac{Z_b - Z_a}{2} \right)$$

Also,

$$V_1 = \frac{I_1}{2} (Z_a + Z_b) \Rightarrow z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \left(\frac{Z_b + Z_a}{2} \right) \text{ as}$$

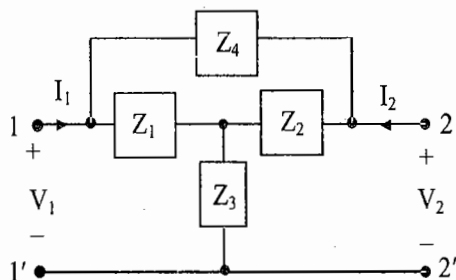
the network is reciprocal and symmetrical

$$\therefore z_{21} = z_{12} = \left(\frac{Z_b - Z_a}{2} \right) \text{ and } z_{11} = z_{22} = \left(\frac{Z_b + Z_a}{2} \right)$$

$$\therefore Z_a = (z_{11} - z_{12}) \text{ and } Z_b = (z_{11} + z_{12})$$

5.8.5 Bridge- T Network

The configuration of a typical bridge-T network is shown in fig



In this case, the current in the output depends on the voltages and a number of nodes instead of one; thus resulting in a number of simultaneous equations.

Therefore, for such networks, analyzing either on node basis or on loop basis, the network function is expressed as a quotient of determinants.

For loop basis, the admittance function,

$$Y_{jk} = \frac{\Delta_{kj}}{\Delta}, \text{ where } \Delta \text{ is the loop basis system}$$

determinant and Δ_{kj} is the cofactor. Here, Y 's will be y -parameters if the output is shorted.

For node basis, the impedance function,

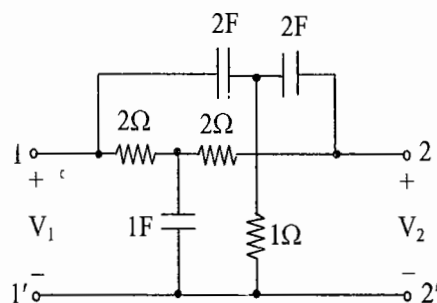
$$Z_{jk} = \frac{\Delta'_{kj}}{\Delta'}, \text{ where } \Delta' \text{ is the loop basis system}$$

determinant and Δ'_{kj} is the cofactor. Here, Z 's will be z -parameters if the output is open-circuited.

$$\text{Also, } G_{21} = \frac{V_2}{V_1} = \frac{\Delta'_{21}}{\Delta'_{11}} \alpha_{21} = \frac{I_2}{I_1} = \frac{\Delta_{21}}{\Delta_{11}}$$

5.8.6 Parallel-T or Twin - T or Notch-Filter Network

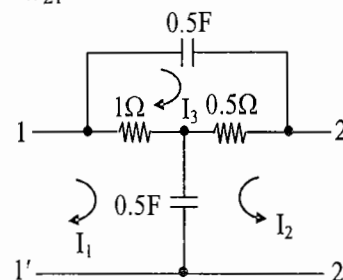
The configuration of typical twin-T network is shown in fig



In this case also, the current in the output depends on the voltages and a number of nodes instead of one; thus, resulting in a number of simultaneous equations. We solve the network in the similar process as in the bridge-T network.

Example: 5.6

For the bridge-T RC network, find the transfer admittance Y_{21}



Sol: By KVL:

$$\left(1 + \frac{2}{s}\right)I_1 + \frac{2}{s}I_2 - I_3 = V_1$$

$$\frac{2}{s}I_1 + \left(\frac{1}{2} + \frac{2}{s}\right)I_2 + \frac{1}{2}I_3 = V_2$$

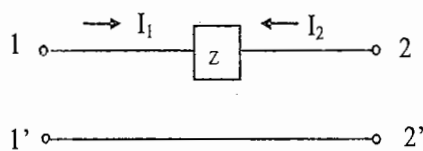
$$-I_1 + \frac{1}{2}I_2 + \left(\frac{3}{2} + \frac{2}{s}\right)I_3 = 0$$

$$\therefore \Delta = \begin{vmatrix} \left(1 + \frac{2}{s}\right) & \frac{2}{s} & -1 \\ \frac{2}{s} & \left(\frac{1}{2} + \frac{2}{s}\right) & \frac{1}{2} \\ -1 & \frac{1}{2} & \left(\frac{3}{2} + \frac{2}{s}\right) \end{vmatrix} = \frac{s+6}{s^2}$$

$$\Delta_{12} = (-1)^{1+2} \begin{vmatrix} \frac{2}{s} & \frac{1}{2} \\ -1 & \left(\frac{3}{2} + \frac{2}{s}\right) \end{vmatrix} = -\frac{s^2 + 6s + 8}{2(s+6)}$$

5.8.7. For Series and Shunt elements.

For Series element ($I_1 = -I_2$)



Z-parameters are not defined for this network

($\because$ If $I_2 = 0$ then I_1 also Zero)

Y-Parameters can be defined by

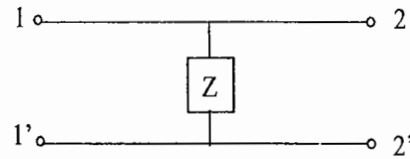
$$[Y] = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$$

ABCD parameters can be defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

Reciprocal and Symmetric

For Shunt element ($V_1 = V_2$)



Y-parameters are not defined for this network

($\because$ If $V_2 = 0$ then V_1 also Zero)

Z-Parameters can be defined by

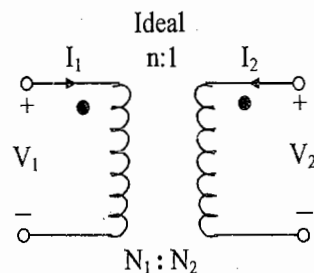
$$[Z] = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

ABCD parameters can be defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

Reciprocal and Symmetric

For an Ideal Transformer (i.e., $K = 1$)



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{1}{n} = \frac{I_1}{-I_2}$$

Reciprocal Symmetric if $n = 1$

Provided " $-I_2$ " is the leaving current from port (2),

i.e. the current delivered to the load.

$$[T] = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \quad [h] = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix}$$

Neither Z-parameters nor Y-parameters are defined for this network.

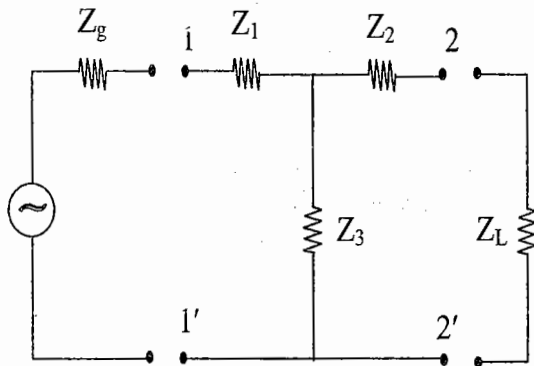
Class Room Practice Questions

LEVEL-1 Questions

01. When a number of 2 – port networks are connected in cascade, the individual (IES-EE-95)

(a) Z_{oc} matrices are added
 (b) Y_{sc} – matrices are added
 (c) Chain matrices are multiplied
 (d) h – matrices are multiplied

02. An impedance match is desired at the 1 – 1' port of the two – port network shown in the given figure. The match will be obtained when Z_g equals (IES-EE-96)



(a) $Z_1 + Z_3$
 (b) $Z_1 + \frac{Z_3(Z_2 + Z_L)}{Z_2 + Z_3 + Z_L}$
 (c) $\frac{Z_1(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}$
 (d) Z_1

03. For a two – port network to be reciprocal, it is necessary that (IES-EE-98)

(a) $z_{11} = z_{22}$ and $y_{21} = y_{12}$
 (b) $z_{11} = z_{22}$ and $AD - BC = 0$
 (c) $h_{21} = -h_{12}$ and $AD - BC = 1$
 (d) $y_{21} = y_{12}$ and $h_{21} = h_{12}$

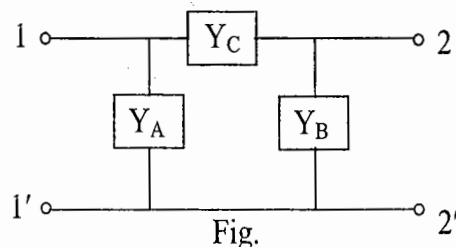
04. Two two – port networks with transmission parameters A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 respectively are cascaded. The transmission parameter matrix of the cascaded network will be (IES-EE-98)

(a) $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} + \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$
 (b) $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$
 (c) $\begin{bmatrix} A_1 & A_2 & B_1 & B_2 \\ C_1 & C_2 & D_1 & D_2 \end{bmatrix}$
 (d) $\begin{bmatrix} (A_1 A_2 + C_1 C_2) & (A_1 A_2 - B_1 D_1) \\ (C_1 A_2 - D_1 C_2) & (C_1 C_2 + D_1 D_2) \end{bmatrix}$

05. The h parameters h_{11} and h_{22} are related to z and y parameters as (IES-EE-01)

(a) $h_{11} = z_{11}$ and $h_{22} = 1/z_{22}$
 (b) $h_{11} = z_{11}$ and $h_{22} = y_{22}$
 (c) $h_{11} = 1/y_{11}$ and $h_{22} = 1/z_{22}$
 (d) $h_{11} = 1/y_{11}$ and $h_{22} = y_{22}$

06. For the two-port network as shown below, Y_{12} is equal to (IES-EE-03)



(a) $Y_A + Y_B$ (b) $Y_C + \left(\frac{Y_A Y_B}{Y_A + Y_B} \right)$
 (c) $-Y_C$ (d) Y_C

07. For the two port network shown below, what is the voltage transfer function $V_2(s)/V_1(s)$? (IES-EE-05)

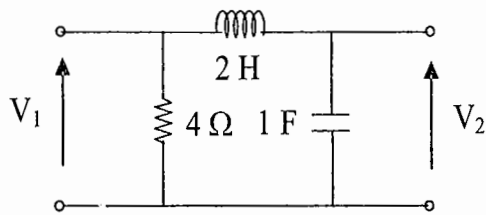


Fig.

- (a) $\frac{s}{(1+2s^2)}$ (b) $\frac{s}{(1+2s)}$
(c) $\frac{1}{(1+2s^2)}$ (d) $\frac{1}{(1+2s)}$

08. The 2-port network of Fig. A has open circuit impedance parameters given by matrix (IES-EC-91)

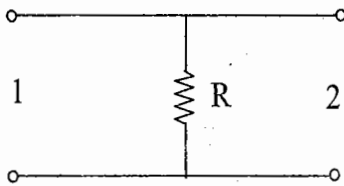


Fig.

- (a) $\begin{bmatrix} R & R \\ R & R \end{bmatrix}$ (b) $\begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$
(c) $\begin{bmatrix} \infty & R \\ R & \infty \end{bmatrix}$ (d) $\begin{bmatrix} R & \infty \\ \infty & R \end{bmatrix}$

09. The impedance matrices of two, two- port networks are given by (IES-EC-00)

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix}$$

If these two networks are connected in series, the impedance matrix of the resulting two-port network will be

- (a) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (b) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$
(c) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$ (d) indeterminate

10. Consider the following statements associated with two-port networks: (IES-EC-07)

1. $Z_{12} = Z_{21}$ 2. $Y_{12} = Y_{21}$
3. $h_{12} = h_{21}$ 4. $AD - BC = 1$

Which of the statements given above are correct?

- (a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

11. With respect to transmission parameters, which one of the following is correct? (IES-EC-09)

- (a) A and B are dimensionless
(b) B and C are dimensionless
(c) A and D are dimensionless
(d) B and D are dimensionless

12. If the connection of two two-ports is such that the transmission matrix of the overall network is the product of the transmission matrices of the individual networks, what type of connection is it? (IES-EC-09)

- (a) Series connection
(b) Cascade connection
(c) Parallel connection
(d) None of the above

13. In a two-port network, hybrid parameters express

- (a) V_1 and V_2 in terms of I_1 and I_2
(b) I_1 and I_2 in terms of V_1 and V_2
(c) V_1 and I_1 in terms of V_2 and I_2
(d) V_1 and I_2 in terms of V_2 and I_1

14. In a two port network, open circuit impedance parameters express (SPDCL-12)

- (a) V_1, V_2 in terms of I_1, I_2
(b) I_1, I_2 in terms of V_1, V_2
(c) V_1, I_1 in terms of V_2, I_2
(d) V_1, I_2 in terms of V_2, I_1



15. Admittance is the reciprocal of
(SPDCL-12)

- (a) impedance (b) inductance
(c) susceptance (d) reactance

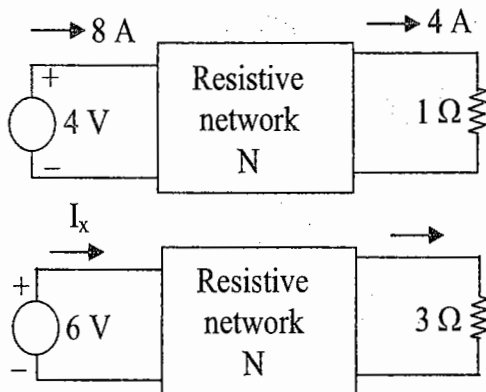
16. Impedance

$Z_1 = 20 \angle 50^\circ \Omega$ and $Z_2 = 10 \angle 30^\circ \Omega$. Then $\frac{Z_1}{Z_2}$ is
(SPDCL-12)

- (a) $2 \angle 80^\circ \Omega$ (b) $2 \angle 50^\circ \Omega$
(c) $2 \angle 30^\circ \Omega$ (d) $2 \angle 20^\circ \Omega$

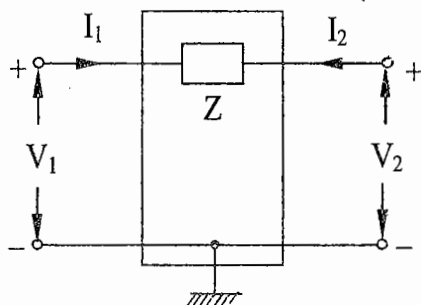
LEVEL-2 Questions

01. The figure shows two different sets of input and output variables for the same two – port resistive network N. I_x is (IES-EE-93)



- (a) 12 A (b) 8 A
(c) 4 A (d) $\frac{3}{2}$ A

02. The short – circuit admittance matrix of the network shown in the given figure is (IES-EE-94)



- (a) $\begin{pmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ \frac{1}{Z} & \frac{1}{Z} \end{pmatrix}$
(c) $\begin{pmatrix} \frac{1}{Z} & \frac{1}{Z} \\ -1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{Z} & 1 \\ -1 & \frac{1}{Z} \end{pmatrix}$

03. Consider the following statements :
For a bilateral network,

1. $A = D$
2. $Z_{12} = Z_{21}$
3. $h_{12} = -h_{21}$

Of these statements

(IES-EE-94)

- (a) 1, 2 and 3 are correct
(b) 1 and 2 are correct
(c) 1 and 3 are correct
(d) 2 and 3 are correct

04. A two – port network is defined by the relations $I_1 = 2V_1 + V_2$ and $I_2 = 2V_1 + 3V_2$. Then Z_{12} is (IES-EE-95)

- (a) -2Ω (b) -1Ω
(c) $-\frac{1}{2} \Omega$ (d) $-1/4 \Omega$

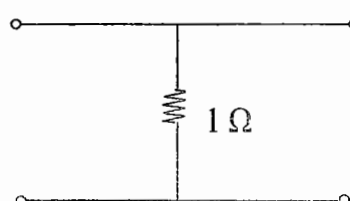
05. With the usual notation, a two – port resistive network satisfies the condition

$$A = D = \frac{3}{2}B = \frac{4}{3}C$$

The Z_{11} of the network is (IES-EE-95)

- (a) $5/3$ (b) $4/3$
(c) $2/3$ (d) $1/3$

06. The Z and h parameters of the network shown in the given figure will be (IES-EE-96)





- (a) $Z = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$; $h = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
 (b) $Z = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$; $h = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$
 (c) $Z = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$; $h = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$
 (d) $Z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $h = \begin{bmatrix} 0 & -1 \\ 2 & 1/2 \end{bmatrix}$

07. For a two – port symmetrical bilateral network, if $A = 3$ and $B = 1 \Omega$, the value of parameter C will be (IES-EE-00)

- (a) 4 S (b) 6 S
(c) 8 S (d) 16 S

08. Two identical T-sections, such as one shown below are connected in series. What is the y_{11} of the combination? (IES-EE-04)

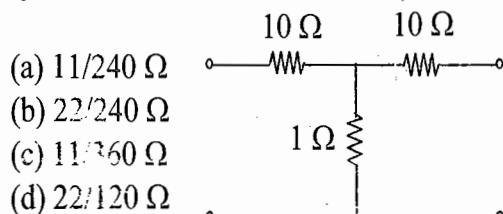


Fig.

- (a) $11/240 \Omega$
(b) $22/240 \Omega$
(c) $11/60 \Omega$
(d) $22/120 \Omega$

09. If the Z-parameters for the T-network as shown below are $Z_{11} = 40 \Omega$, $Z_{22} = 50 \Omega$ and $Z_{12} = Z_{21} = 30 \Omega$, then what are the values of Z_1 , Z_2 and Z_3 ? (IES-EE-09)

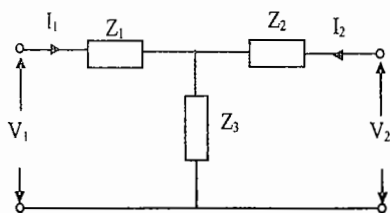


Fig.

- (a) 10 Ω , 20 Ω and 30 Ω
(b) 20 Ω , 30 Ω and 20 Ω
(c) 30 Ω , 40 Ω and 10 Ω
(d) 40 Ω , 50 Ω and 10 Ω

10. A two – port network is symmetrical if (IES-EC-95)

- (a) $z_{11} z_{22} - z_{12} z_{21} = 1$
(b) $AD - BC = 1$
(c) $h_{11} h_{22} - h_{12} h_{21} = 1$
(d) $y_{11} y_{22} - y_{12} y_{21} = 1$

11. Which one of the following pairs is correctly matched? (IES-EC-95)

- (a) Symmetrical two – port network
: $AD - BC = 1$
(b) Reciprocal two – port network
: $z_{11} = z_{12}$
(c) Inverse hybrid parameter
: A, B, C, D
(d) Hybrid parameter
: $(V_1, I_2), (I_1, V_2)$

12. Which one of the following is the transmission matrix for the network shown in the figure given below? (IES-EC-06)

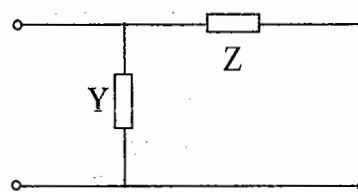


Fig.

- (a) $\begin{bmatrix} 1 & 1+yz \\ y & z \end{bmatrix}$ (b) $\begin{bmatrix} 1+yz & z \\ y & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & z \\ y & 1+yz \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1+yz \\ z & y \end{bmatrix}$

13. Which of the following are the conditions for a two port passive network to be a reciprocal one? (IES-EC-06)

1. $z_{12} = z_{21}$ 2. $y_{12} = y_{21}$ 3. $h_{12} = -h_{21}$

Select the correct condition from the code given below:

- (a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3



14. When two 2-port networks are connected in parallel, it is convenient to use
(SPDCL-12)

- (a) open circuit impedance parameters
- (b) short circuit admittance parameters
- (c) transmission parameters
- (d) inverse hybrid parameters

15. The two-port parameter h_{21} is called
(GENCO-12)

- (a) open-circuit output admittance
- (b) short-circuit input impedance
- (c) open-circuit reverse voltage gain
- (d) short-circuit forward current gain

Solutions

LEVEL-1

01. Ans: (c)

Sol: 2 – port NW's in cascade :

Individual chain matrices (ABCD matrices) are multiplied to get the overall chain matrix.

02. Ans: (b)

Sol: Impedance match is obtained when $Z_g = Z_1 + [Z_3 \parallel (Z_2 + Z_L)]$

$$= Z_1 + \frac{Z_3(Z_2 + Z_L)}{Z_2 + Z_3 + Z_L}$$

03. Ans: (c)

Sol: For a reciprocal NW :

$$z_{12} = z_{21}, y_{21} = y_{12}, h_{12} = -h_{21}, \\ AD - BC = 1$$

04. Ans: (b)

Sol: $[T] = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

$$= \begin{bmatrix} A_1 A_2 + B_1 C_2 & A_1 B_2 + B_1 D_2 \\ C_1 A_2 + D_1 C_2 & C_1 B_2 + D_1 D_2 \end{bmatrix}$$

05. Ans: (c)

Sol: $h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = \frac{1}{\left. \frac{I_1}{V_1} \right|_{V_2=0}} = \frac{1}{y_{11}}$
 $h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{\left. \frac{V_2}{I_2} \right|_{I_1=0}} = \frac{1}{z_{22}}$

06. Ans: (c)

Sol: $Y_{12} = - \left. \frac{I_1}{V_2} \right|_{V_1=0} = -Y_C$

07. Ans: (c)

Sol: $\frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{s}}{2s + \frac{1}{s}} = \frac{1}{2s^2 + 1}$

08. Ans: (a)

Sol: From the given network $V_1 = V_2$

Keep $I_2 = 0$ then $V_1 = R I_1$

$$Z_{11} = \frac{V_1}{I_1} = R, \quad Z_{21} = \frac{V_2}{I_1} = R$$

Keep $I_1 = 0$ then $V_2 = R I_2$

$$Z_{12} = \frac{V_1}{I_2} = R, \quad Z_{22} = \frac{V_2}{I_2} = R$$

$$[Z] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

09. Ans: (b)

Sol: For series connection

$$[Z] = [Z_A] + [Z_B] \\ = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix} = \begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$$

10. Ans: (d)

Sol: A 2-port network is reciprocal



$$Z_{12} = Z_{21}, Y_{12} = Y_{21}, AD - BC = 1, h_{12} = -h_{21}, g_{12} = -g_{21}$$

11. Ans: (c)

Sol: $V_1 = A V_2 - B I_2$

$$I_1 = C V_2 - D I_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0}, D = - \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

A and D are dimensionless, whereas B is in ohms and C is in mhos.

12. Ans: (b)

Sol: For cascade connection,

$$[T] = [T_1] [T_2]$$

13. Ans: (d)

Sol: $V_1 = h_{11} I_1 + h_{12} V_2$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

14. Ans: (a)

Sol: open circuit impedance parameters means Z-parameters only

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

15. Ans: (a)

Sol: Admittance is reciprocal of impedance
conductance is reciprocal of resistance
susceptance is reciprocal of reactance.

16. Ans: (d)

Sol: $\frac{Z_1}{Z_2} = \frac{20 \angle 50^\circ}{10 \angle 30^\circ} = \frac{20}{10} \angle (50^\circ - 30^\circ)$
 $= 2 \angle 20^\circ \Omega$

Level-2 Key

01. (b)	02. (a)	03. (d)	04. (d)	05. (b)
06. (b)	07. (c)	08. (a)	09. (a)	10. (c)
11. (d)	12. (c)	13. (a)	14. (b)	15. (d)

6

Chapter

Three Phase Circuits



Galileo Ferraris
(1847 - 1897)

6.1 Poly Phase System

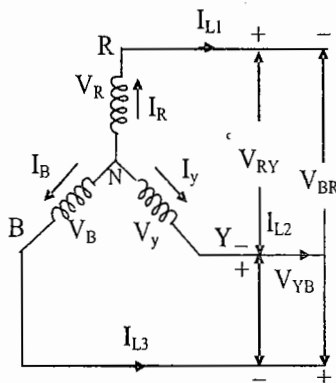
- More than one phase:
2, 3, 4, 5, 6, 7, 8, 9..... phases
- 3, 6, 9, 12.....phases i.e., multiples of '3', are used in industrial applications.

Main advantage of 3-ph system are

Smaller in Size
Constant output power
Greater output power
Superior motor performance
Ease in parallel operation
Reliability
Interconnection of system is possible in (Y & Δ).

- Mutual phase shift between each phase is $\frac{360^\circ}{3} = 120^\circ$

6.2 STAR Connected Balanced Three Phase Supply



From the figure,

$$I_{L1} = I_{L2} = I_{L3} = I_R = I_Y = I_B$$

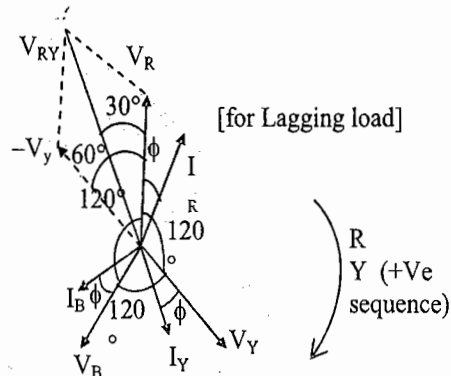
i.e., $I_{ph} = I_L$ in Y-connection

$$V_{RN} = V_{BN} = V_{YN} = V_{ph} \text{ (Phase Voltage)}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \text{ (Line Voltage)}$$

$$V_{RY} = V_R - V_Y; V_{YB} = V_Y - V_B; V_{BR} = V_B - V_R$$

Phasor diagram considering phase voltages as reference



$$V_R = V_Y = V_B = V_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_L = \sqrt{3} V_{ph}$$

With Respect to Phase voltage,

- Line voltage is leading by an angle of 30°
- For inductive load, phase displacement between line voltage and line current is $(30^\circ + \phi)$
- For resistive load, phase displacement between line voltage and line current is 30°
- For capacitive load, phase displacement between line voltage and line current is $(30^\circ - \phi)$

$$V_{RY} = V_L \angle 0^\circ$$

$$V_{YB} = V_L \angle -120^\circ$$

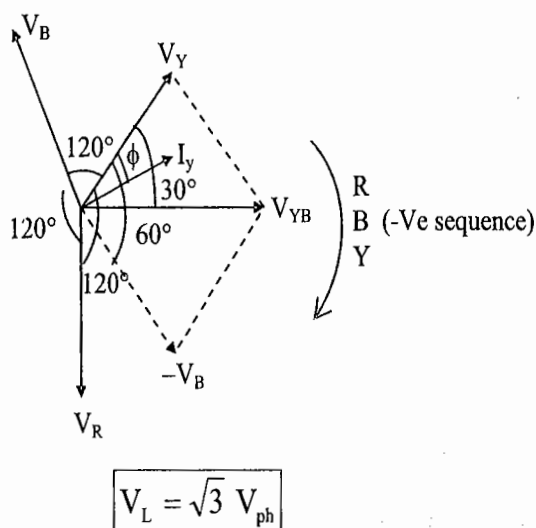
$$V_{BR} = V_L \angle -240^\circ$$

$$\text{(or)} V_L \angle +120^\circ$$

$$V_R = V_{ph} \angle -30^\circ$$

$$V_Y = V_{ph} \angle -120^\circ - 30^\circ$$

$$V_B = V_{ph} \angle +120^\circ - 30^\circ$$



With respect to Phase Voltage

1. Line voltage is lagging by an angle of 30°
2. For inductive load, phase displacement between line voltage and line current is $(30^\circ - \phi)$
3. For resistive load, the phase displacement between line voltage and line current is 30°
4. For capacitive load, the phase displacement between line voltage and line current is $(30^\circ + \phi)$

$$\begin{aligned} V_{RY} &= V_L \angle 0^\circ & V_R &= V_{ph} \angle +30^\circ \\ V_{YB} &= V_L \angle +120^\circ & V_Y &= V_{ph} \angle +120^\circ + 30^\circ \\ V_{BR} &= V_L \angle -120^\circ & V_B &= V_{ph} \angle -120^\circ + 30^\circ \\ & \text{(or)} & &= V_L \angle +240^\circ \end{aligned}$$

Power in three phase star connected system :

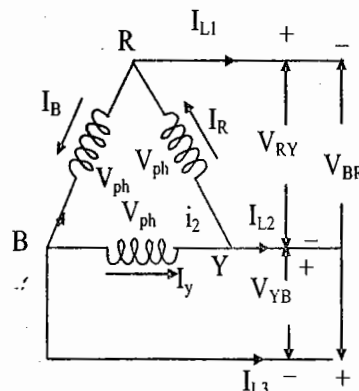
$$S = 3 V_{ph} I_{ph} = 3 \cdot \frac{V_L}{\sqrt{3}} I_L$$

$$\Rightarrow S = \sqrt{3} V_L I_L \quad \text{VAs}$$

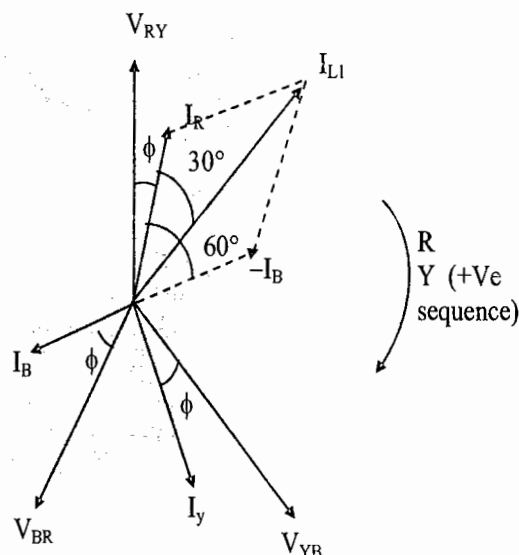
$$P = \sqrt{3} V_L I_L \cos \phi \quad \text{Watts}$$

$$Q = \sqrt{3} V_L I_L \sin \phi \quad \text{VARs}$$

6.3 DELTA connected balanced three phase supply



$$\begin{aligned} V_{RY} &= V_{YB} = V_{BY} = V_L \\ \Rightarrow V_L &= V_{ph} \\ I_{L1} &= I_R - I_B; \quad I_{L2} = I_Y - I_R; \\ I_{L3} &= I_B - I_Y; \end{aligned}$$



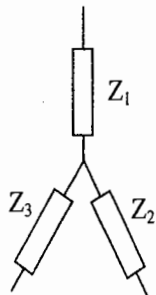
1. In +ve phase-sequence. of Δ - connected system, with respect to phase current, line current lags by an angle of 30°
2. In -ve phase sequence of Δ -connection system, with respect to phase currents, line currents leads by 30°
3. In +ve phase sequence, the phase displacement between line voltage and line current is $L \rightarrow 30^\circ + \phi$, $R \rightarrow 30^\circ$, $C \rightarrow 30^\circ - \phi$



4. In -ve phase sequence, the phase displacement between line voltage and line current is $L \rightarrow 30^\circ - \phi$, $R \rightarrow 30^\circ$, $C \rightarrow 30^\circ + \phi$

6.4 Types of load and interconnection of three phase circuits

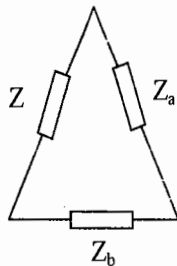
Star connected load



$$Z_1 = Z_2 = Z_3 \Rightarrow \text{Balanced}$$

$$Z_1 \neq Z_2 \neq Z_3 \Rightarrow \text{unbalanced}$$

Delta connected load



$$Z_a = Z_b = Z_c \Rightarrow \text{Balanced}$$

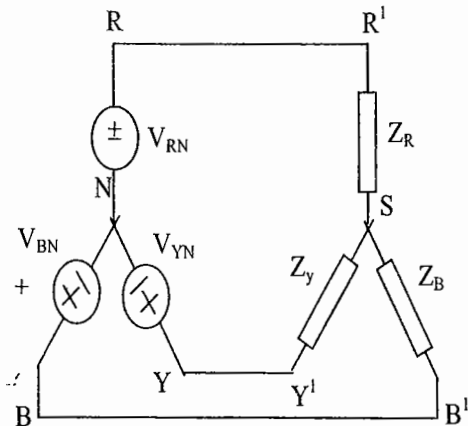
$$Z_a \neq Z_b \neq Z_c \Rightarrow \text{unbalanced}$$

Interconnection between Source – Load :

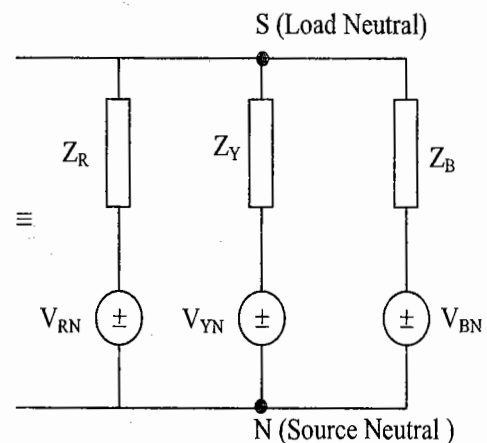
Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

- System source is always balanced
- Potential at neutral point of source is always zero, whether the load is balanced (or) unbalanced.
- Load neutral is considered as floating point neutral with respect to unbalanced load.

3- ϕ , 3-wire:-



By applying Millman's theorem, the above circuit can be simplified to



Potential difference between source neutral (N) and load neutral (S) is

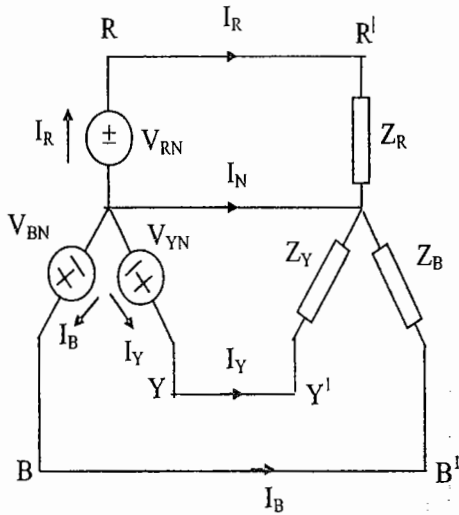
$$V_{SN} = \frac{\frac{V_{RN}}{Z_R} + \frac{V_{YN}}{Z_Y} + \frac{V_{BN}}{Z_B}}{\frac{1}{Z_R} + \frac{1}{Z_Y} + \frac{1}{Z_B}}$$

$$V_{SN} = \frac{V_{RN} \cdot Y_R + V_{YN} \cdot Y_Y + V_{BN} \cdot Y_B}{Y_R + Y_Y + Y_B}$$

$$V_{NS} = -V_{SN}$$



3- ϕ , 4-wire:



In 3- ϕ , 4-wire system

Neutral current

$$I_N = -(I_R + I_Y + I_B)$$

$$I_R = \frac{V_{RN}}{Z_R}; I_Y = \frac{V_{YN}}{Z_Y}; I_B = \frac{V_{BN}}{Z_B}$$

6.5 Measurement of 3- ϕ power

- 3- wattmeter method used for balanced or unbalanced loads.
- 2 - wattmeter method used for balanced or unbalanced loads.
- 1-wattmeter method used for balanced load only

3- ϕ , 3- wattmeter method :

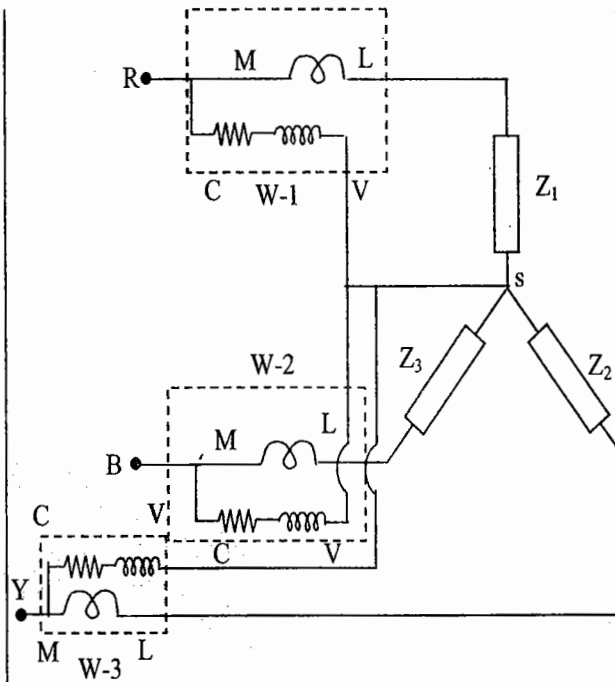
In a commercial wattmeter, we can identify the terminal connections as

M $\rightarrow$ Mains

L $\rightarrow$ Load

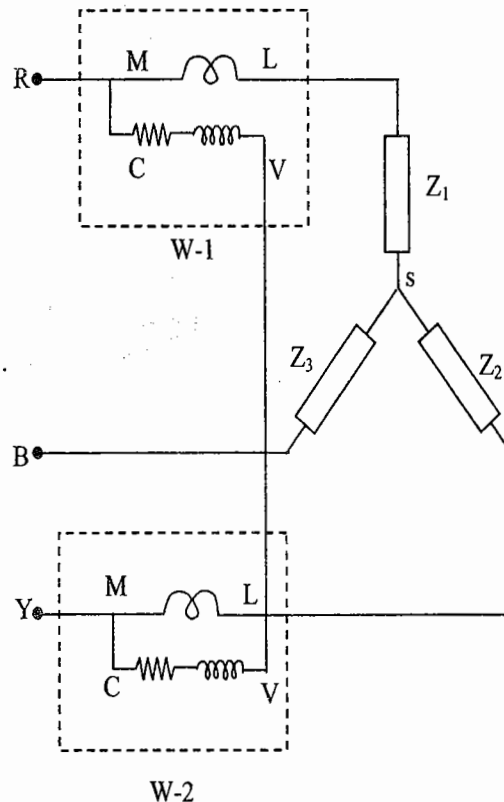
C $\rightarrow$ Common

V $\rightarrow$ Voltage



Total three phase power = $W_1 + W_2 + W_3$

3- ϕ , 2- wattmeter method :



Wa

(i)

(ii)

(iii)

Wa

(i)

(ii)

(iii)



Wattmeter W_1 :

- (i) Current measured by current coil of wattmeter - 1 = I_R
- (ii) Voltage measured by pressure coil of wattmeter - 1 = $V_{RB} = V_R - V_B$
- (iii) The phase displacement between ' I_R ' and ' V_{RB} ' is equal to
 - $(30^\circ - \phi) \rightarrow$ Inductive
 - $30^\circ \rightarrow$ Resistive
 - $(30^\circ + \phi) \rightarrow$ capacitive

Wattmeter W_2 :

- (i) Current measured by current of wattmeter - 2 is = I_Y
- (ii) Voltage measured by pressure coil of wattmeter - 2
 $V_{YB} = V_Y - V_B$
- (iii) The phase displacement between ' I_Y ' and ' V_{YB} ' is
 - $(30^\circ + \phi) \rightarrow$ Inductive
 - $30^\circ \rightarrow$ Resistive
 - $(30^\circ - \phi) \rightarrow$ Capacitive

So, $W_1 = I_R V_{RB} \cos(30^\circ - \phi)$

$W_2 = I_Y V_{YB} \cos(30^\circ + \phi)$

For balanced load

$$I_R = I_Y = I_B = I_L$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$W_1 + W_2 = V_L I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)]$$

So, total active power

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$$

also $W_1 - W_2 = V_L I_L \sin \phi$

But reactive power $Q = \sqrt{3} V_L I_L \sin \phi$

$$Q = \sqrt{3} [W_1 - W_2]$$

$$\Rightarrow \frac{Q}{P} = \frac{\sqrt{3} V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} = \frac{\sqrt{3} (W_1 - W_2)}{W_1 + W_2}$$

$$\Rightarrow \tan \phi = \frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)}$$

Power factor angle

$$\phi = \tan^{-1} \left[\frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)} \right]$$

Effect of power factor on wattmeter reading:

$$W_1 = V_L I_L \cos(30^\circ - \phi)$$

$$W_2 = V_L I_L \cos(30^\circ + \phi)$$

Case (i): if $\phi = 0$,

$$W_1 = V_L I_L \cos 30^\circ$$

$$W_2 = V_L I_L \cos 30^\circ$$

$$W_1 = W_2$$

If P.F. angle is 0° , both wattmeter readings are equal

Case (ii): if $\phi = 30^\circ$

$$W_1 = V_L I_L$$

$$W_2 = \frac{V_L I_L}{2}$$

$$W_2 = \frac{W_1}{2}$$

If P.F. angle is 30° , one wattmeter readings is half of the other.

Case (iii): if $\phi = 60^\circ$

$$W_1 = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = 0$$

If P.F. angle is 60° , one of the wattmeter leading is zero

Case (iv): if $\phi = 90^\circ$

$$W_1 = \frac{V_L I_L}{2}$$

$$W_2 = -\frac{V_L I_L}{2}$$



$$W_2 = -W_1$$

If P.F. angle is 90° , one wattmeter reading is negative.

Example: 6.1

The line voltages V_{ab} , V_{bc} , V_{ca} of 3- ϕ , 100 V, ACB system can be represented as

- (a) $100 \angle -120$, $100 \angle 0$, $100 \angle 120$
- (b) $100 \angle 120$, $100 \angle 0$, $100 \angle -120$
- (c) $100 \angle 240$, $100 \angle 0$, $173 \angle -240$
- (d) None

Ans: (a)

Sol:

+ve (sequence)	-ve (sequence)
0°	0°
-120°	$+120^\circ (-240^\circ)$
$+120^\circ$	$-120^\circ (+240^\circ)$

Example: 6.2

The phase voltage V_a , V_b , V_c of a 3- ϕ , 173 V ABC system can be represented as

- (a) $100 \angle 90$, $100 \angle -30$, $100 \angle -150$
- (b) $100 \angle -90$, $100 \angle -210$, $100 \angle 30$
- (c) $173 \angle 90$, $173 \angle -150$, $73 \angle -30^\circ$
- (d) $100 \angle 90^\circ$, $100 \angle -150$, $100(X) \angle -30^\circ$

Ans: (a)

Sol: since positive sequence ABC

$$\begin{aligned} & -30^\circ \\ & -30^\circ - 120^\circ = -150^\circ \\ & -30^\circ + 120^\circ = 90^\circ \end{aligned}$$

So,

$$\begin{aligned} & -30^\circ \\ & -150^\circ \\ & +90^\circ \end{aligned} \downarrow$$

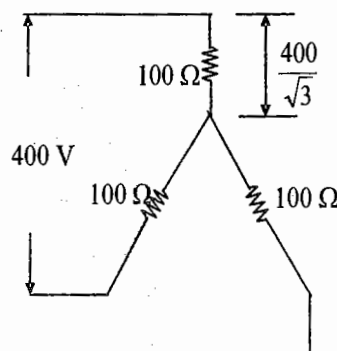
Example: 6.3

Three 100Ω non-inductive resistors are connected in

- (a) Star
 - (b) Delta
- across a 400V, 50Hz supply
- Calculate power taken from supply in each case.
- (c) In the event of one of the resistance gets open circuited, then what would be the value of total power taken from mains in each of the above two cases.

Sol:

(a) Star:



$$P = 3 \cdot V_{ph} I_{ph}$$

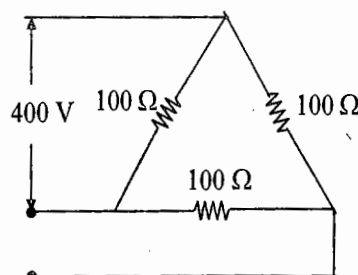
$$400$$

$$I_{ph} = \frac{\frac{400}{\sqrt{3}}}{100}$$

$$\Rightarrow p = (3)(V_{ph})(I_{ph}) = (3)\left(\frac{400}{\sqrt{3}}\right)\left(\frac{\frac{400}{\sqrt{3}}}{100}\right)$$

$$P_Y = 1600 \text{ W}$$

(b) Delta



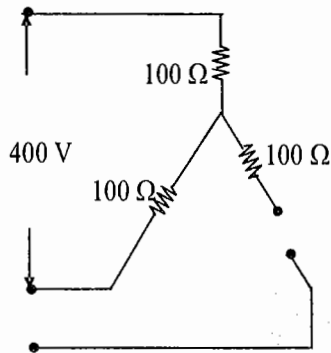
$$I_{ph} = \frac{400}{100} = 4$$



$$P = (3) (V_{ph}) (I_{ph})$$

$$P_{\Delta} = (3) (400) (4) = 4800 \text{ W}$$

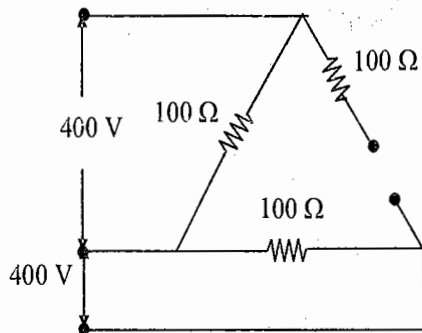
- (c) If one of the resistance in star connection gets open



$$P = \frac{(400)^2}{(100 + 100)} = \frac{400 \times 400}{200}$$

$$P_{\text{open-Y}} = 800 \text{ W}$$

- If one of the resistance in delta connection gets open



$$P = \frac{(400)^2}{100} + \frac{(400)^2}{100}$$

$$P_{\text{open-}\Delta} = 3200 \text{ W}$$

Conclusions :

$$P_{\Delta} = 3.P_Y$$

$$P_{\text{open}}(Y) = \frac{P_Y}{2}$$

$$P_{\text{open}}(\Delta) = \frac{2}{3} P_{\Delta}$$

Example: 6.4

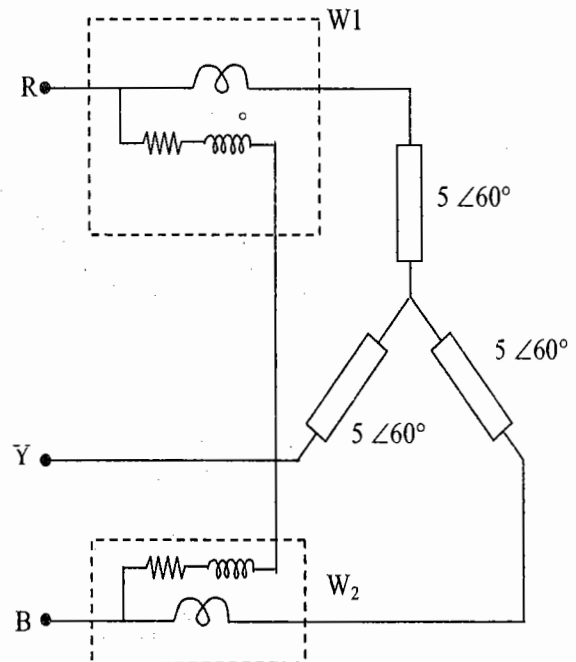
The minimum wattmeters required to measure 3- ϕ , 3-Wire balanced (or) unbalanced power is
(a) 1 (b) 2 (c) 3 (d) 4

Ans: (b)

Example: 6.5

A L-L input voltage to the 50 Hz A.C. circuit shown below is 100V rms, assuming that phase sequence is R-Y-B, wattmeters should read

- (a) 80, 866 (b) 500, 500
(c) 0, 1000 (d) 2500, 750



Ans: (c)

$$\text{Sol: } W_1 = V_L I_L \cos(30^\circ + \phi)$$

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$\therefore$ the load is balanced and impedance angle is 60° (i.e. $\phi = 60^\circ$)

$$\text{So, } W_1 = (100) \left(\frac{100\sqrt{3}}{5} \right) \cos(30 + 60) = 0$$

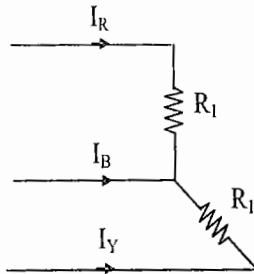
$$W_2 = (100) \left(\frac{100/\sqrt{3}}{5} \right) \cos(30 - 60)$$



$$= (100) \left(\frac{100}{5\sqrt{3}} \right) \frac{\sqrt{3}}{2} = 1000 \text{ W}$$

Example: 6.6

For a 3- ϕ circuit shown in figure, the ratio of I_R , I_Y and I_B is



- (a) 1:1: $\sqrt{3}$ (b) 1: 1: 2
(c) 1: 1: 0 (d) 1:1: $\frac{\sqrt{3}}{2}$

Ans: (a)

Sol: $I_R = I_{ph}$

$$I_Y = I_{ph}$$

$$I_B = \sqrt{3} I_R = \sqrt{3} I_Y$$

$$\therefore I_B^2 = I_R^2 + I_R^2 + 2I_R^2 \cos \frac{120}{2}$$

$$I_B = \sqrt{3} I_R = \sqrt{3} I_{ph}$$

$$\text{In this case, } I_R : I_Y : I_B = I_{ph} : I_{ph} : \sqrt{3} I_{ph} \\ = 1:1:\sqrt{3}$$

$$I_R = I_Y$$

' I_B ' is the phasor sum of I_R and I_Y at angle 120°

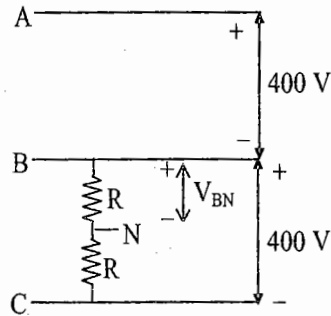
Example: 6.7

Two resistors of equal value are connected in series across the lines 'B' and 'C' of a symmetrical 3-phase system. What is the magnitude of voltage between line A and the junction of resistors.

- (a) $\frac{400}{\sqrt{3}}$ (b) $\frac{400}{\sqrt{3}}$
(c) $\frac{200}{\sqrt{3}}$ (d) $100\sqrt{3}$

Ans: (c)

Sol:



$\therefore$ Given that symmetrical 3- ϕ and 400 V system

$$V_{AN} = ? \quad V_{AN} = V_{AB} + V_{BN}$$

$$V_{AB} = 400 \angle 0^\circ$$

$$V_{BN} = \frac{V_{BC}}{2} = \frac{400 \angle -120^\circ}{2} = 200 \angle -120^\circ$$

$$V_{AN} = 400 \angle 0^\circ + 200 \angle -120^\circ$$

$$|V_{AN}| = 200\sqrt{3} \text{ Volts}$$

Class Room Practice Questions

LEVEL-1 Questions

01. A balanced delta connected load has an impedance of $9\angle 30^\circ$ ohms per phase. What is the impedance per phase of its equivalent star? (IES-EE-04)
(a) $27\angle 30^\circ \Omega$ (b) $27\angle 90^\circ \Omega$
(c) $3\angle 30^\circ \Omega$ (d) $3\angle 20^\circ \Omega$
02. A 3-phase, 4-wire system supplies power to a balanced star-connected load. The current in each phase is 15A. The current in the neutral wire will be (IES-EE-12)
(a) 15 A (b) 45 A
(c) 8.66 A (d) 0 A
03. The power expression in 3-phase circuit in terms of line voltage V_L , line current I_L and power factor of the load is $\sqrt{3} V_L I_L \cos \phi$, where ϕ is the angle between (SPDCL-12)
(a) line voltage and line current
(b) line voltage and phase current
(c) phase voltage and line current
(d) phase voltage and phase current

LEVEL-2 Questions

01. A 3-phase delta connected symmetrical load consumes P watt of power from a balanced supply. If the same load is connected in star to the same supply, then what is the power consumption? (IES-EE-05)
(a) $P/3$ (b) P
(c) $\sqrt{3} P$ (d) $3 P$
02. A 3-phase, 3-wire supply feeds a star connected load consisting of 3 equal resistors. If one of the resistors is to be

removed, then what is the reduction in power as compared to the original power?

(IES-EE-07)

- (a) 25 % of the original power
(b) 33.3 % of the original power
(c) 50 % of the original power
(d) 66.6 % of the original power
03. The voltage phasor of a circuit is $10\angle 15^\circ$ V and the current phasor is $2\angle -45^\circ$ A. The active and reactive powers in the circuit are
(a) 10W and 17.32 VAR
(b) 5W and 8.66 VAR
(c) 200W and 60VAR
(d) $20\sqrt{2}$ W and $10\sqrt{2}$ VAR
04. For an ac circuit $v = V_m \sin(\omega t + \alpha)$ and $i = I_m \sin(\omega t + \beta)$. The phase angle between voltage and current waveform is (APPSC-12)
(a) $\alpha + \beta$ or $\alpha - \beta$
(b) $\alpha - \beta$ or $\beta - \alpha$
(c) $\beta - \alpha$ or $\beta + \alpha$
(d) $\alpha - \beta$
05. The voltage from A to N in a balanced three-phase circuit is $231\angle -30^\circ$. If the phase sequence is positive, what is the value of V_{BC} ? (APPSC-12)
(a) $400\angle -30$ (b) $400\angle -90$
(c) $400\angle 120$ (d) $400\angle -150$



Solutions

LEVEL-1

01. Ans: (c)

$$\text{Sol: } Z_p(\text{star}) = \frac{9 \angle 30^\circ \ 9 \angle 30^\circ}{27 \angle 30^\circ} = 3 \angle 30^\circ \Omega$$

02. Ans: (d)

$$\text{Sol: } I_n = 15 \angle 0^\circ + 15 \angle -120^\circ + 15 \angle -240^\circ \\ = 0$$

03. Ans: (d)

Sol: Power factor angle ϕ is angle between phase voltage and phase current.

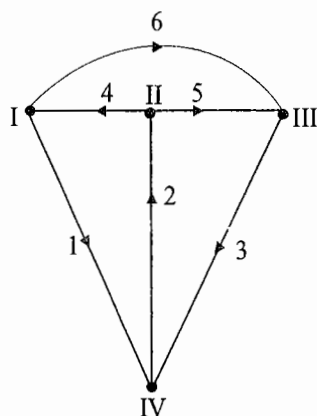
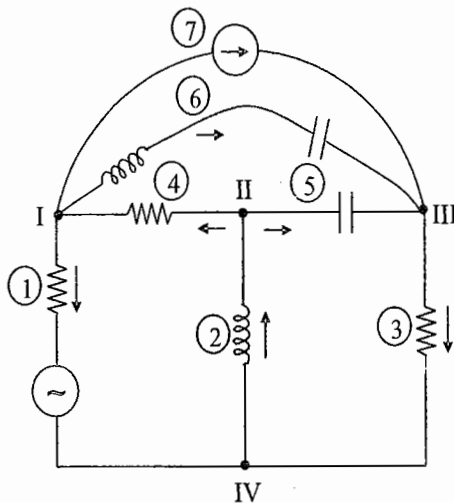
Level-2 Key

01. (b) 02. (c) 03. (a) 04. (b) 05. (c)



Introduction

Graph Theory is geometrical representation of given electrical network by replacing all the passive elements by line segments (branches or edges) and ideal voltage sources are replaced by short circuits and all the ideal current sources are replaced by open circuits.



A linear oriented graph

Observation

Graph branches are less than or equal to the network branches.

- The number of nodes or vertices = $n = 4$
- The number of edges or branches = $b = 6$

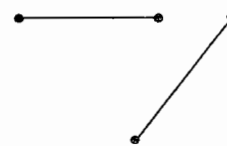
Complete Graph or Standard Graph

Def: In a complete graph between any pair of nodes **only one branch** is connected for all the combinations.

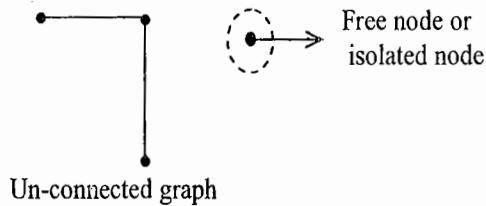
The minimum number of edges to make a graph complete with 'n' nodes is ${}^nC_2$.

Node	Graph	Edges
2		$1 = \left(\frac{2(2-1)}{2} \right)$
3		$3 = \left(\frac{3(3-1)}{2} \right)$
4		$6 = \left(\frac{4(4-1)}{2} \right)$
5		$10 = \left(\frac{5(5-1)}{2} \right)$

Connected Graph



Connected graph



In a connected graph all the nodes are connected by atleast one branch, otherwise it is said to be unconnected.

Sub Graph

It is a graph with less number of branches (least one branch) as compared to the original graph.

Tree of a Graph

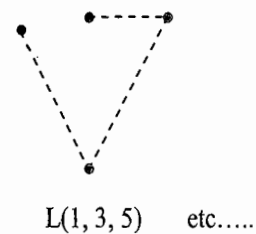
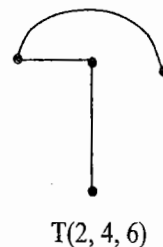
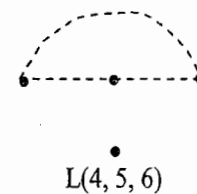
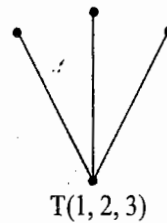
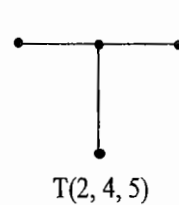
Def: Tree is a connected sub graph, which connects all the nodes without any closed loop.

- Tree branches are called twigs
- The number of tree branches of any graph = $(n-1)$
- For the present graph there are $(4-1) = 3$ tree branches
- The number of possible trees of a complete graph with 'n' nodes = $n^{(n-2)}$
- For the present complete graph there are $4^{(4-2)} = 4^2 = 16$ possible trees.

Co - tree or the complimented tree

Def: It is a tree formed with all the removed branches from the original graph in order to construct a tree.

- Co tree branches are called links or chords
- The number of links of any graph = $L = b - (n-1) = L = (b-n+1)$
- For the present graph there are $6 - 4 + 1 = 3$ links.



i.e., The remaining 13 trees at their respective Co-Trees.

Obs:

So, the co-tree may consist a loop in its representation.

Tree + Co-tree = Original graph i.e., for any graph both trees and co-trees are always equal.

7.1 Incidence Matrix (A_I)

Let the branch currents be $i_1, i_2, \dots, i_6$ for the branches 1, 2, ...6 respectively

By writing the KCL equations at every node

$$\text{I} \Rightarrow i_1 - i_4 + i_6 = 0$$

$$\text{II} \Rightarrow -i_2 + i_4 + i_5 = 0$$

$$\text{III} \Rightarrow i_3 - i_5 - i_6 = 0$$



$$IV \Rightarrow -i_1 + i_2 - i_3 = 0$$

$$\begin{array}{l} I \\ II \\ III \\ IV \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ -1 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

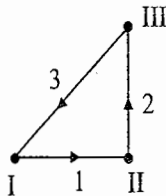
$\Downarrow$
(A_I)

Properties

1. For a given graph the incidence matrix is unique and its order is $(n \times b)$
2. The rank of incidence matrix is equal to rank of the graph $= (n-1)$

For the present graph rank is $(4-1) = 3$

3. The determinant of the complete incidence matrix of a closed loop is always equal to zero.



$$[A_I] = \begin{bmatrix} I & 1 & 2 & 3 \\ II & -1 & 1 & 0 \\ III & 0 & -1 & 1 \end{bmatrix}_{3 \times 3}$$

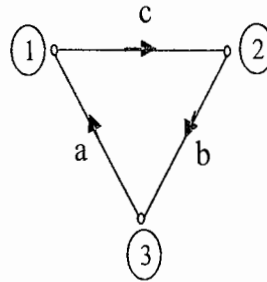
$$|A_I| = 0 \text{ so rank} = r \neq n \neq 3$$

$$\Rightarrow \text{rank} = r = n-1 = 3-1 = 2$$

The determinant of incidence Matrix of a closed loop graph is 0.

Incidence matrix of a closed loop graph is of order $n \times n$.

Ex:



$$[A] = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\det |A| = -1(-(-1)) + 1(1) = -1 + 1 = 0.$$

If the incidence Matrices of '2' independent network graph are identical, they said to obey the principle of Isomorphism.

7.2 Tie-Set matrix (B_f)

Tie-Set matrix (B_f) is also called Fundamental loop matrix (or) Fundamental circuit matrix.

Tie-Set:

Def: It is a closed path which contains only one link together any number of twigs.

Let the branch voltages be $v_1, v_2, \dots, v_6$ for the branches 1, 2, ...6 respectively.

f-loops or f-ckts or tie sets

These are the minimum number of loop or mesh equations required to solve the corresponding network of a given graph.

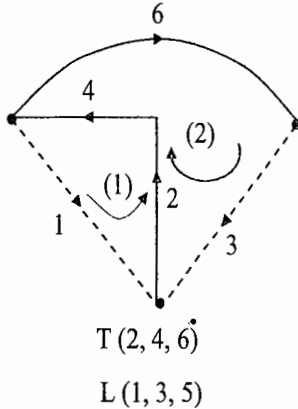
Steps

1. Select a tree.
2. By adding one link at a time to the existing tree will result one f- loop at a time.



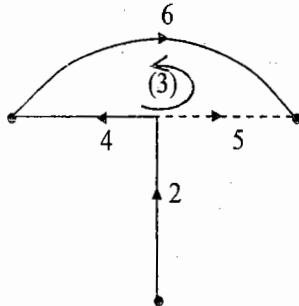
3. Select the f-loop current direction as in the link branch direction.

Since every f-loop consists of only one link and hence unique loop current direction.



$$f_{l_1} \Rightarrow v_1 + v_2 + v_4 = 0$$

$$f_{l_2} \Rightarrow v_3 + v_2 + v_4 + v_6 = 0$$



$$f_{l_3} \Rightarrow v_5 - v_6 - v_4 = 0$$

$$f_{l_1} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Downarrow (B_f)$

Properties

1. Rank of Tie-set matrix = (b-n+1)
For the present matrix rank is (6-4+1) = 3

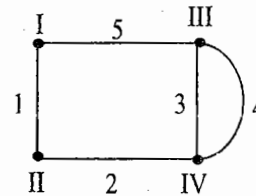
2. Since every link will result one Tie set at a time, for any graph the number of f-loops always equal to number of links = (b-n+1)
3. Every fundamental loop consists of only one link (Unique) in its representation.
4. Since every tree will result one Tie-set matrix at a time, for any graph the number of f-loop matrices are always equal to the number of trees.

For the complete graph there are $n^{(n-2)}$ f-loop matrices.

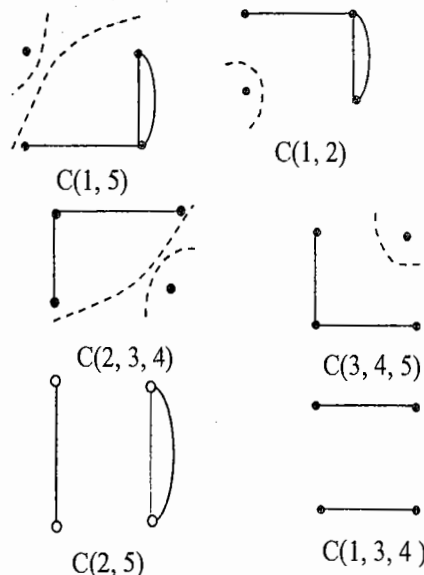
7.3 Cut-Set matrix (Q_c)

Cut Set

Def: It contains only one twig together any number of links which may cut and divide the network into two parts.



Obs: It is not a complete graph.





$C(2, 3, 5) \times$

$C(2, 4, 5) \times$

- The removal of (2, 5) branches itself gives the cut-set, there is no need of removing 3 or 4 and hence improper cut sets.

$C(1, 2, 5) \times$



- The removal of (1, 2, 5) branches divides the graph into three isolated parts, so it is not at all a cut set.

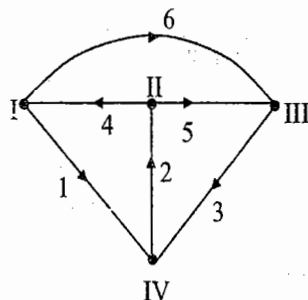
Std. graph

$C(1, 2, 3)$

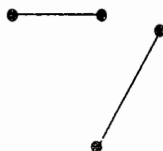
$C(1, 4, 6)$

$C(2, 4, 5)$

$C(3, 5, 6)$

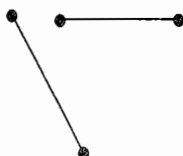


- $C(1, 2, 5, 6)$



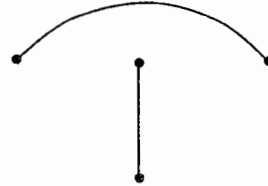
The graph after cut set i.e. the remaining branches

- $C(2, 3, 4, 6)$



The graph after cut set i.e. the remaining branches

$C(1, 3, 4, 5)$



The graph after cut set i.e. the remaining branches

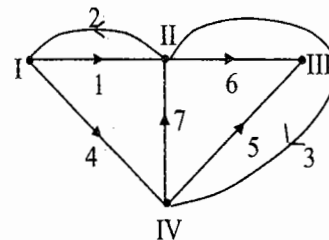
f-Cut Sets

These are the minimum number of nodal equations required to solve the corresponding network of a given graph.

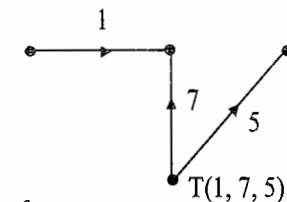
Steps

- Select a tree
- By removing one tree branch at a time will result one f-cut set at a time.
- Select the f-cut set direction as in the tree branch direction.

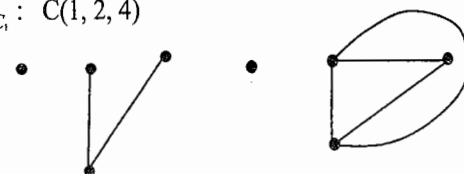
Since every f-cut set consists of only one tree branch and hence unique f-cut set direction.



Obs: It is not a complete graph.



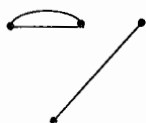
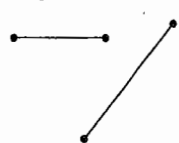
$f_C: C(1, 2, 4)$





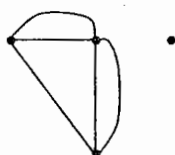
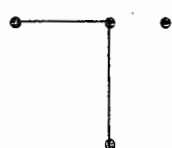
The graph after The f-cut set i.e the remaining branches.

$$f_{C_1} : C(7, 4, 6, 3)$$



The graph after the f-cutset
i.e., the remaining branches

$$f_{C_2} : C(5, 6)$$



The graph after the f-cutset
i.e., the remaining branches

f-Cut set matrix (Q_C)

$$\begin{matrix} f_{C_1} \\ f_{C_2} \\ f_{C_3} \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} (3 \times 7) \\ (n-1) \times b \end{matrix}$$

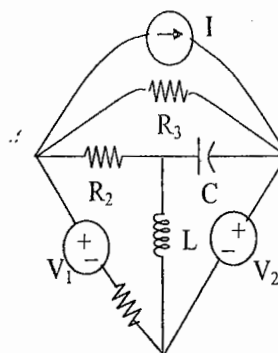
Properties

1. The rank of the f-cut set matrix = $(n-1)$
For the present matrix rank is $(4-1) = 3$
2. Since, the removal of one tree branch will result one f-cut set at a time, for any graph the number of f-cut sets are always equal to the number of tree branches i.e. twigs = $(n-1)$.
3. Every f-cut set consist of only one tree branch in its representation.
4. Since every tree will result one f-cut set matrix (Q_C) at a time, for any graph the number of Q_C matrices are always equal to the number of trees.

5. For the complete graph there are $n^{(n-2)}$ f-cut set matrices.

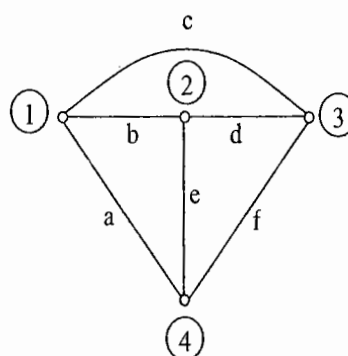
Example: 7.1

Construct the correct graph of the network shown below.



Sol: Voltage Source = S.C

Current Source = O.C



Node (n) = 4

Branch (b) = 6

Mesh (m) → independent loops

$$m = 3 \rightarrow [6 - (4-1)]$$

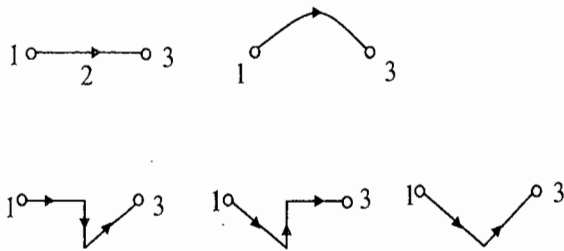
$$\text{Loop } (l) = 3 + 4 = 7 \quad \therefore l_1 = [b - (n-1)]$$

Example: 7.2

Number of possible path for the above graph from node (1) to node (3) is 5.

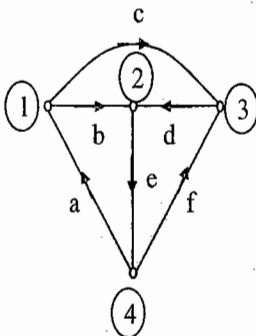


Sol:



Example: 7.3

Construct the complete incidence Matrix for the oriented graph shown below.



Sol:

$$[A] = \begin{matrix} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} & \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{matrix}_{4 \times 6}$$

The algebraic sum of the elements of every column vertically is zero.

Example: 7.4

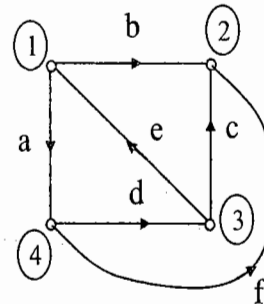
Construct the oriented graph of a network where incidence matrix is given as

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$$

Sol: The given is an $[A_r]$, So make it (complete $[A]$)

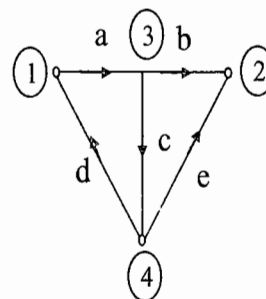
$$\begin{matrix} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$\therefore$ The network graph has nodes = 4, edges = 6



Example: 7.5

The number of possible trees from network graph shown below is _____ & draw them separately.



Sol: Just connected graph

$$\text{Number of trees} = \det |[A_r][A_r]^T|$$



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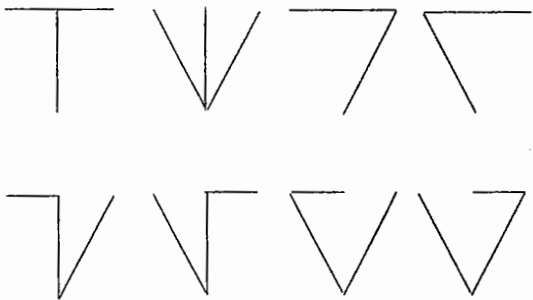
A
S

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[A_r][A_r]^T = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

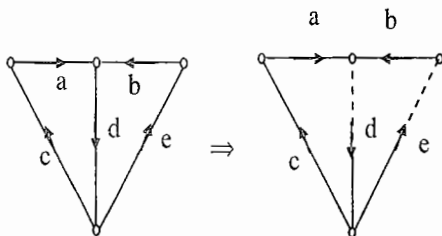
$$\det |[A_r][A_r]^T| = 2(6 - 1) - (2) \\ = 10 - 2 = 8 \text{ (Possible terms)}$$

These trees are



Example: 7.6

Construct the Tie-Set Matrix & write the equilibrium equation in standard KVL form for network graph shown below by considering a, b, c as tree branches.



Sol: Number of f.loops = $5 - (4 - 1) = 2$

$$f l_1 = a, c, d \rightarrow i_1$$

$$f l_2 = a, b, c, e \rightarrow i_2$$

$$[M] = \begin{matrix} & a & b & c & d & e \\ \begin{matrix} i_1 \\ i_2 \end{matrix} & \begin{bmatrix} +1 & 0 & -1 & +1 & 0 \\ +1 & +1 & -1 & 0 & +1 \end{bmatrix}_{2 \times 5} \end{matrix}$$

Equilibrium Equations:

Let $j_a, j_b, j_c, j_d, j_e \rightarrow$ Branch currents

$V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages

Set - I:

KVL $[\rightarrow]$

$$[M][V_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}_{(2 \times 5)} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5 \times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2 \times 1}$$

Set - II:

Relation between 'j' & 'i' $[\downarrow]$

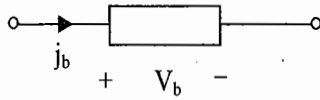
$$[M]^T [I_i] = [J_b]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5 \times 2} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5 \times 1}$$

$$\begin{bmatrix} i_1 + i_2 \\ i_2 \\ -i_1 - i_2 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}$$

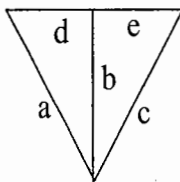


Note: In graph theory if graph is given, we consider every edge as a load set of reference (by default)



Example: 7.7

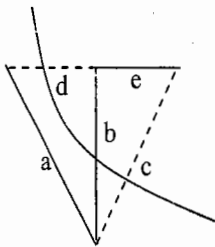
Which of following set of branches represents a proper cut set for network graph shown below.



- (a) a c d (b) b d c
(c) a c e (d) c d e

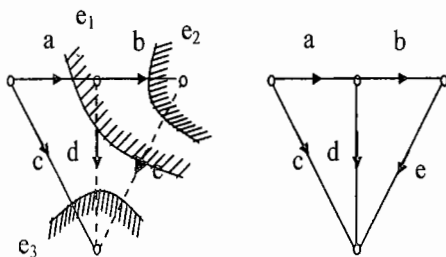
Ans: (b)

Sol:



Example: 7.8

Construct the cut-set matrix & write the equilibrium equation in standard KCL form for the oriented graph shown below by considering branches a, b, c as tree branches.



Sol: Number of f-cut sets = 4 - 1 = 3

$$f c_1 = a, b, e \rightarrow e_1$$

$$f c_2 = b, e \rightarrow e_2$$

$$f c_3 = c, d, e \rightarrow e_3 \downarrow$$

$$[C] = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & +1 \end{bmatrix} \end{matrix}_{3 \times 5}$$

Let,

$j_a, j_b, j_c, j_d, j_e \rightarrow$ Branch currents

$V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages

Set - I:

KCL [$\rightarrow$]

$$[C][J_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}_{(3 \times 5)} \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

$$\begin{bmatrix} j_a - j_d - j_e \\ j_b - j_e \\ j_c + j_d + j_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Set - II:

Relation between 'V' & 'e' [$\downarrow$]

$$[C]^T [e_i] = [V_b]$$

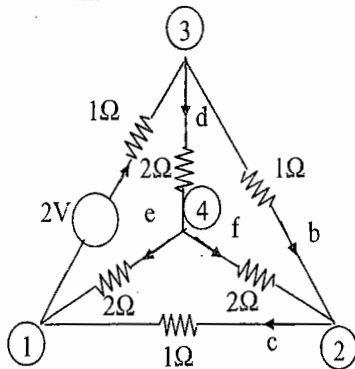


$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix}_{5 \times 3} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5 \times 1}$$

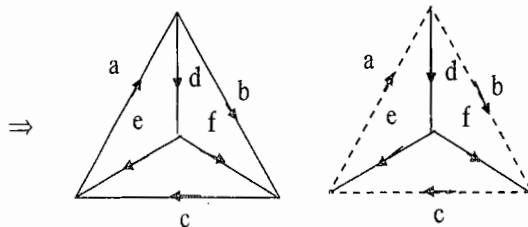
$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ -e_1 + e_3 \\ -e_1 - e_2 + e_3 \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}$$

Example: 7.9

Solve the network the find all the branch currents and voltages by writing the equations in standard KVL & KCL form.



Sol:



01. TIE SET:

$$[M] [Z_b] [M]^T [I] = [M] [V_s] - [M] [Z_b] [I_s]$$

$$[M] = \begin{bmatrix} i_1 & 1 & 0 & 0 & 1 & 1 & 0 \\ i_2 & 0 & 1 & 0 & -1 & 0 & -1 \\ i_3 & 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}_{3 \times 6}$$

$$[Z_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6 \times 6}$$

$$[I] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_{3 \times 1} \quad [V_s] = \begin{bmatrix} +2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1}$$

LHS:

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}_{6 \times 3} = \begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

RHS:

$$[M] [V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

Final LHS

Final RHS

$$\begin{bmatrix} 5i_1 - 2i_2 - 2i_3 \\ -2i_1 + 5i_2 - 2i_3 \\ -2i_1 - 2i_2 + 5i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \text{ Find RMS}$$

on solving, we get

$$i_1 = \frac{6}{7}A, i_2 = \frac{4}{7}A, i_3 = \frac{4}{7}A$$

Then, branch currents can be calculated by

$$[M]^T [I_b] = [I_s]$$

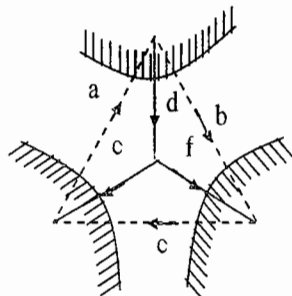
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 6/7 \\ 4/7 \\ 4/7 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \\ j_f \end{bmatrix}$$

Final Ans:

$$j_a = \frac{6}{7}, j_b = \frac{4}{7}, j_c = \frac{4}{7}, j_d = \frac{2}{7}, j_e = \frac{2}{7}, j_f = 0$$

02. Cut-Set:

$$[C] [Y_b] [C]^T [e_t] = [C] [I_s] - [C] [Y_b] [V_s]$$



$$[C] = \begin{bmatrix} e_1 & -1 & 1 & 0 & 1 & 0 & 0 \\ e_2 & -1 & 0 & 1 & 0 & 1 & 0 \\ e_3 & 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix}_{3 \times 6}$$

$$[Y_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}_{3 \times 6}$$

$$[e_t] = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} \quad [V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1}$$

LHS:

$$\begin{bmatrix} -1 & 1 & 0 & 1/2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1/2 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5/2 & 1 & 1 \\ 1 & 5/2 & -1 \\ 1 & -1 & 5/2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

RHS:

$$- [C] [Y_b] [V_s]$$

$$- \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} +2 \\ +2 \\ 0 \end{bmatrix}$$

Final LHS

Final RHS



$$\begin{bmatrix} 2.5e_1 + e_2 + e_3 \\ e_1 + 2.5e_2 - e_3 \\ e_1 - e_2 + 2.5e_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

On solving, we get

$$e_1 = 4/7 \text{ V}$$

$$e_2 = 4/7 \text{ V}$$

$$e_3 = 0 \text{ V}$$

then, branch voltages can be calculated by

$$[C]^T [e_t] = [V_b]$$

$$\begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4/7 \\ 4/7 \\ 0 \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix}$$

Final Answers

$$V_a = \frac{-8}{7} \text{ V}, V_b = \frac{4}{7} \text{ V}, V_c = \frac{4}{7} \text{ V}, V_d = \frac{4}{7} \text{ V}$$

$$V_e = \frac{4}{7} \text{ V}, V_f = 0 \text{ V}$$

Class Room Practice Questions

LEVEL-1 Questions

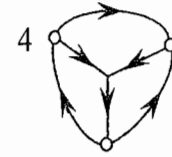
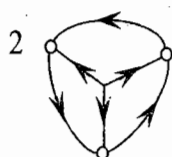
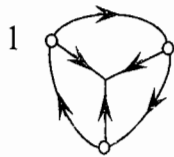
01. A network having n nodes and b branches will have (IES-EC-91)

(a) $(b + n)$ links
(b) $b - n + 1$ links
(c) $b + n - 1$ links
(d) $b + n + 1$ links

02. A network has 10 nodes and 17 branches in all. The number of node pair voltages would be (IES-EC-91)

(a) 7 (b) 9
(c) 10 (d) 45

03. Which of the following oriented graphs have the same fundamental loop matrix?



Select the correct answer using the *codes* given below: (IES-EC-93)

Codes:

(a) 1 and 2 (b) 2 and 3
(c) 1, 3 and 4 (d) 2, 3 and 4

04. For a network of 11 branches and 6 nodes, what is the number of independent loops? (IES-EC-05)

(a) 4 (b) 5
(c) 6 (d) 11

05. What is the total number of trees for the graph shown? (IES-EC-06)

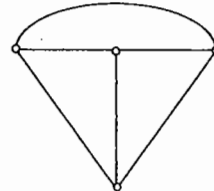


Fig.

(a) 4 (b) 8
(c) 12 (d) 16

06. In a network with twelve circuit elements and five nodes, what is the minimum number of mesh equations? (IES-EC-09)

(a) 24 (b) 12
(c) 10 (d) 8

Solutions

LEVEL-1

01. Ans: (b)

Sol: No. of tree branches = $(n - 1)$
 $\therefore$ No. of co-tree branches = No. of links
 $=$ No. of chords = 1
 $= b - (n - 1) = b - n + 1$

02. Ans: (d)

Sol: $n = 10$, $b = 17$

$$\text{No. of node pair voltage} = \sum_{i=1}^9 i = 45$$

Note that the number of fundamental cutsets =
 Number of tree branches = Number of
 independent nodal equations = $n - 1 = 9$

03. Ans: (a)

Sol: The orientation in graphs (1) and (2) is exactly opposite i.e., the current directions in graph (2) are exactly opposite to those in graph (1).



In such a case the fundamental loop matrix is same for both graphs.

04. Ans: (c)

Sol: No. of tree branches = $6 - 1 = 5$

No. of independent loops = No. of links = $11 - 5 = 6$

05. Ans: (d)

Sol: The graph is a complete graph

2 Number of trees = n^{n-2}

Where n = number of nodes.

$$= 4^{4-2} = 16$$

06. Ans: (d)

Sol: No. of tree branches

$$= 5 - 1 = 4$$

Minimum no. of mesh equations

$$= 12 - 4 = 8$$



Synthesis of Passive Networks

Introduction

- For a given network and a source of excitation, we can determine the response of the network. This is known as analysis of electrical networks. In network analysis, the structure of the network is known.
- In network synthesis, the excitation as well as the response of the network is known but the structure of the network is not known.
- By synthesis, we can find out the structure of the network. The network synthesis does not give a unique solution. For a particular response we can get a number of network structures. The network must be causal and stable
- For a network to be realizable, it must be stable, i.e. for a bounded input there should be bounded output. For stability in frequency domain, the poles of the system must lie in the left-half plane or on the $j\omega$ -axis, i.e. simple poles on the $j\omega$ -axis are permissible. Let the transfer function to be synthesized be given as below.

$$H(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

- The order of the numerator 'n' cannot exceed the order of the denominator 'm' by more than unity. For $n > (m + 1)$, multiple poles will be present and that is not acceptable.
- To synthesize the network, the required elements are linear passive and time-invariant and are inherently causal. We will check the

physical realizability of $H(s)$ in terms of the following conditions.

1. No poles of $H(s)$ should lie in the right-half of s-plane.
 2. There should not be any multiple poles of $H(s)$.
 3. The degree of numerator should not be more than unity with respect to that of the denominator.
- The denominator polynomial of the transfer function becomes the Hurwitz polynomial. The objective of this chapter is to introduce synthesis of passive networks.

8.1 Restriction on the Location of the Poles and Zeros in the S-Plane

We consider a network function $F(s) = \frac{N(s)}{D(s)}$

1. The coefficients in the polynomials $N(s)$ and $D(s)$ must be real and positive.
2. Complex and imaginary poles must be conjugate.
3. The real part of all poles and zeros must either be negative or zero. Thus, all poles and zeros must lie on the left half of the s-plane or on the imaginary axis.
4. If the real part of any pole or zero is zero then that pole or zero must be simple, not repeated. This includes the origin.
5. Polynomials $N(s)$ and $D(s)$ must not have missing terms between those of highest and lowest order unless all even terms or all odd terms are missing.



6. The degree of $N(s)$ and $D(s)$ may differ by zero or one only.
7. The lowest degree of $N(s)$ and $D(s)$ may differ by one at most.

8.1.1 A function $F(s) = p(s)/q(s)$ is positive real if

1. $F(s)$ is real for real s
i.e., $F(\sigma)$ is real for $s = \sigma$
2. $\operatorname{Re} F(s) \geq 0$ for $\operatorname{Re} s = \sigma \geq 0$

8.1.2 Necessary Conditions For Driving Point Functions

1. The coefficients in the polynomials $N(s)$ and $D(s)$ must be real and positive.
2. Complex and imaginary poles must be conjugate.
3. The real part of all poles and zeros must either be negative or zero. Thus, all poles and zeros must lie on the left half of the s -plane or on the imaginary axis.
4. If the real part of any pole or zero is zero then that pole or zero must be simple, not repeated. This includes the origin.
5. Polynomials $N(s)$ and $D(s)$ must not have missing terms between those of highest and lowest degree unless all odd terms are missing.
6. The degree of $N(s)$ and $D(s)$ may differ by zero or one only.
7. The lowest degree of $N(s)$ and $D(s)$ may differ by one at most.

8.1.3 Test for necessary and sufficient conditions

1. $F(s)$ is real for real s or $\operatorname{Arg} F(s) = 0$ or π when $\operatorname{Arg} s = 0$.
2. i. $F(s)$ has no poles in the right half plane.
ii. Imaginary axis poles of $F(s)$ are simple; residues evaluated at these poles are real and positive.
iii. $\operatorname{Re} Y(j\omega) \geq 0, 0 \leq \omega \leq \infty$

Example 8.1

Check whether (i) $F(s) = (s+4)/(s^2+5s+3)$,
(ii) $(s+2)/(s^2+3)$ is PR,

Sol: For $F(s) = (s+a)/(s^2+bs+c)$ to be PR,

$a, b, c \geq 0$, & $b \geq a$

(i) $(s+4)/(s^2+5s+3)$ is PR as it satisfies the above conditions.

(ii) However the function $(s+2)/(s^2+3)$ is not PR because $b < a$

Example 8.2

Check whether function

$$F(s) = \frac{(s^2 + a_1s + a_0)}{(s^2 + b_1s + b_0)} \text{ is PR}$$

Sol: The given function will be PR for

$$a_1 b_1 \geq (\sqrt{a_0} - \sqrt{b_0})^2$$

8.2 Hurwitz Polynomials

- Any polynomial $P(s)$ is said to be Hurwitz, if the following conditions are satisfied.
 1. $P(s)$ is real when s is real.
 2. The real parts of the roots of $P(s)$ should be zero or negative.
- The Hurwitz polynomial

$$P(s) = a_0s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_{2-1}s + a_n$$

Where $P(s)$ has the following properties.

1. The coefficients of s must be positive.
2. The roots of both the even and odd parts of $P(s)$ must lie on the $j\omega$ -axis.



3. All the quotients during the continued fraction expansion of the ratio of even to odd parts or odd to even parts must be positive.
4. If the polynomial $P(s)$ obeys the conditions of Hurwitz, the polynomial must be Hurwitz to within a multiplicative factor $W(s)$ such that

$$P(s) = W(s) P_1(s)$$

5. If the ratio of $P(s)$ and its derivative gives a continued fraction expansion with all positive coefficients, $P(s)$ is said to be Hurwitz.

8.3 Positive Real Functions

- The driving point impedance $Z(s)$ or driving point admittance $Y(s)$ of a single-port network can be represented by

$$H(s) = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m} = \frac{P(s)}{Q(s)}$$

- The function $H(s)$ will be called a positive real function if it satisfies the following conditions:
 1. $F(s)$ will be real if s is real.
 2. $Q(s)$ is Hurwitz polynomial.
 3. If $H(s)$ has poles on real axis, the poles must be simple and the residues must be real and positive.
 4. $\text{Real}[H(j\omega)] \geq 0$ for all value of ω .
- The properties of positive real functions are given below:
 1. Both $P(s)$ and $Q(s)$ are Hurwitz polynomial and they may contain factors of the form $s^2 + \omega^2$.
 2. The highest and lowest powers of $P(s)$ and $Q(s)$ differ by unity.

3. If $H(s)$ is a positive real function, the reciprocal of $H(s)$ must be a positive real function.
4. The sum of positive real functions is a positive real function too. But the difference of positive real functions may not be a positive real function.

8.4 Basic Philosophy of Synthesis

- For a given positive real function, it must be decomposed into smaller functions and each of them must be a positive real function. Here each function represents a single element R , L or C , or a suitable parallel combination of R , L and C . Let $Z(s)$ be represented by

$$Z(s) = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}$$

It is possible to decompose the function $Z(s)$ by any of the following methods or a combination of these where the methods are given by

Step – I: Removal of a pole at infinity,

Step – II: Of a pole at origin,

Step – III: Removal of conjugate imaginary poles, and

Step – IV: Removal of a constant.

Note: Depending upon the given network function, the above steps are applicable.

Step – I: Removal of a Pole at Infinity:

If the order of the numerator of $Z(s)$ is greater than that of denominator $Z(s)$ by 1, i.e.

$n - m = 1$, we can take out the term $\frac{a_0}{b_0} s$ by

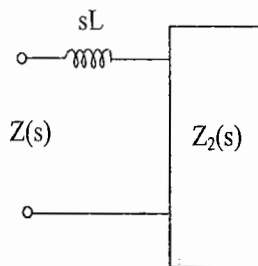
dividing the numerator by the denominator. We can write that,

$$Z(s) = \frac{a_0 s}{b_0} + Z_2(s)$$

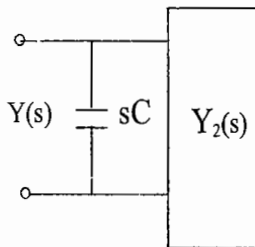


Where $Z_2(s)$ is the reminder of $Z(s)$: In the above equation $\frac{a_0}{b_0}s$ indicates that an indicator should

be taken out and this is a series inductor. On the other hand, if the function is an admittance, removal of a pole at infinity will indicate that a capacitor will be taken out and it will be a shunt capacitor.



(a) impedance network



(b) Admittance network

Step- II: Removal of a Pole at Origin:

To remove a pole at origin, it should be possible to take s common from the denominator polynomial of $Z(s)$. Here $Z(s)$ will be represented by

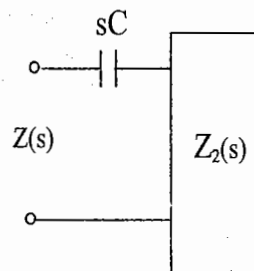
$$Z(s) = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}$$

$$= \frac{K_0}{s} \cdot \frac{c_0 s^{n-1} + c_1 s^{n-2} + \dots + c_{n-1}}{b_{m-1} + b_{m-2} s + \dots + b_1 s^{m-2} + b_0 s^{m-1}}$$

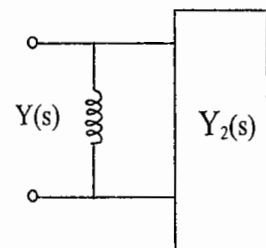
or $Z(s) = Z_1(s) Z_2(s)$

$$Z_1(s) = \frac{K_0}{s}$$

Here, $Z_1(s)$ represents a capacitor in case of $Z(s)$ as shown in Figure (a) whereas $Z_1(a)$ will be represent an inductor in case of $Y(s)$ as shown in figure (b).



(a) impedance network



(b) Admittance network

Step-III: Removal of Conjugate Imaginary Poles:

If $Z(s)$ has poles at $s = \pm j\omega_1$, $Z_1(s)$ written as

$$Z_1(s) = (s + j\omega_1)(s - j\omega_1) = s^2 + \omega_1^2$$

factors the denominator polynomial. Hence $Z(s)$ is given by

$$Z(s) = \frac{P(s)}{(s^2 + \omega_1^2) Q_1(s)} \quad (\text{or})$$

$$Z(s) = \frac{2K_1 s}{s^2 + \omega_1^2} + Z_2(s)$$

If L and C are in parallel, we can write that

$$\frac{\frac{sL}{sC}}{sL + \frac{1}{sC}} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}} = \frac{2K_1 s}{s^2 + \omega_1^2}$$

Therefore, $C = \frac{1}{2K_1}$ and

$$L = \frac{2K_1}{\omega_1^2}$$

Hence $\frac{2K_1 s}{s^2 + \omega_1^2}$ represents the parallel combination of L and C in series with $Z_2(s)$.

If the given function be an admittance, we can

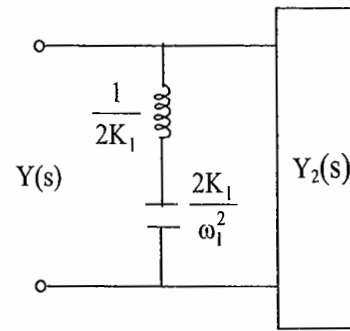
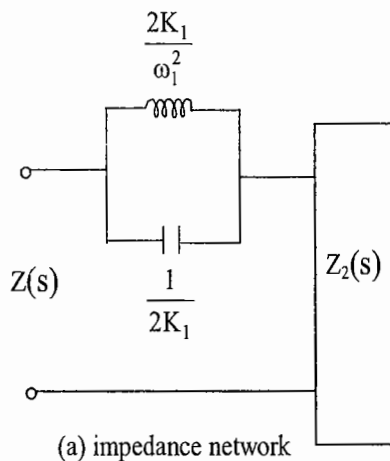
write that $Y(s) = \frac{2K_1 s}{s^2 + \omega_1^2} + Y_2(s)$

In the above equation $\frac{2K_1 s}{s^2 + \omega_1^2}$ represents a shunt across the terminals with L and C in series.

$$\begin{aligned} \therefore Y_1(s) &= \frac{1}{sL + \frac{1}{sC}} = \frac{Cs}{LCs^2 + 1} \\ &= \frac{\frac{s}{L}}{s^2 + \frac{1}{LC}} = \frac{2K_1 s}{s^2 + \omega_1^2} \end{aligned}$$

$$\therefore L = \frac{1}{2K_1} \text{ and } C = \frac{2K_1}{\omega_1^2}$$

The above two representations are shown in figure.



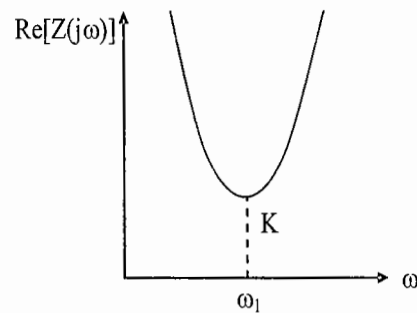
(b) Admittance network

Step – IV: Removal of a Constant

The plot of $\text{Re}[Z(j\omega)]$ versus ω . That $\text{Re}[Z(j\omega)]$ is minimum at $\omega = \omega_1$ and it is equal to K. We can write that $\text{Re}[Z(j\omega)]$ is minimum at $\omega = \omega_1$ and it is equal to K. We can write that

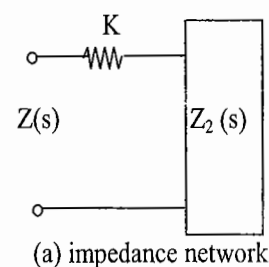
$$Z(s) = K + Z_2(s)$$

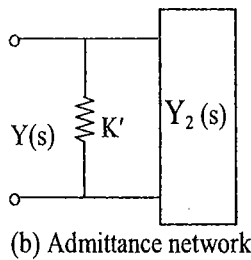
Where K represents the resistance. If the original function be $Y(s)$, K will represent conductance and $Y(s)$ can be written as



$$Y(s) = K' + Y_2(s)$$

Where $K' = \frac{1}{K}$ represents the conductance. The above two representations.





8.5 Driving point synthesis of one-port Networks with two types of Elements

Evaluation of the elements of a network from its driving points impedance or admittance (i.e., Immittance) functions is known as driving point synthesis. We discuss here the properties and synthesis of driving point immittance function of one-port networks with two passive elements. The three different networks are.

1. LC network
2. RL network, and
3. RC network

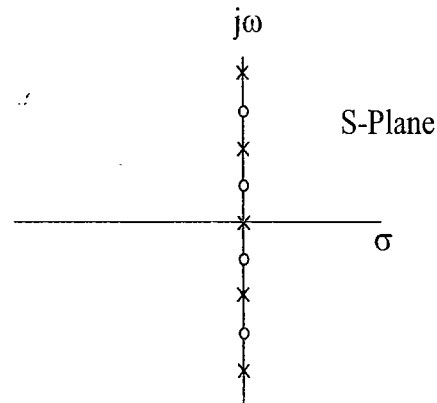
8.5.1 Properties of LC Driving Point Functions

1. Driving -point immittance function $[Z_{LC}(s)]$ or $[Y_{LC}(s)]$ is the ratio of even to odd or odd to even polynomials.
2. Since both $E_i(s)$ and $O_i(s)$ are Hurwitz, they have only imaginary roots. Thus, the poles and zeros of the immittance function are on the imaginary ($j\omega$) axis.
3. The poles and zeros of the immittance function interlace on the $j\omega$ -axis, i.e., the poles and zeros alternate on the imaginary axis.
4. The highest as well as the lowest powers of the numerator and denominator polynomials must differ by unity.
5. There must be either a zero or pole at origin ($s=0$) and infinity ($s = \infty$)
6. The residues of the poles are real and positive.

7. The slope of the reactance (or Susceptance) curve is positive, $\frac{d}{d\omega} [X(\omega) \text{ or } B(\omega)] > 0$

for $-\infty < \omega < \infty$

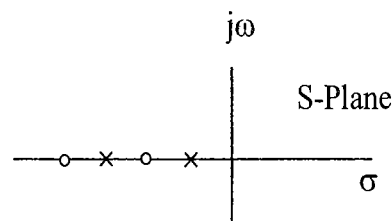
$$\text{Ex: } Z(S) = \frac{(S^2 + 1)(S^2 + 3)}{S(S^2 + 2)(S^2 + 4)}$$



8.5.2 Properties of RC impedance and RL Admittance Functions:

1. Poles and zeros lie on the negative real axis, and they alternate.
2. The residues of the poles must be real and positive.
3. The critical frequencies nearest to the origin or at the origin must be a pole, whereas, the critical frequency nearest to infinity or at infinity must be a zero.

$$\text{Ex: } Z(S) = \frac{(S+2)(S+4)}{(S+1)(S+3)}$$

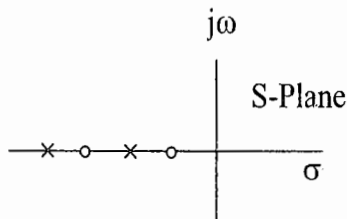




8.5.3 Properties of RL Impedance and RC Admittance Functions

1. Poles and zeros lie on the negative real axis, and they alternate.
2. The residues of the poles must be real and positive.
3. The critical frequency nearest to the origin or at the origin must be a zero; whereas, the critical frequency nearest to infinity or at infinity must be a pole.

Ex: $Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$



Example:8.3

The impedance function

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)} \text{ is}$$

Sol:

LC driving point function

Example:8.4

The lowest and the highest critical frequencies of R-C driving-point impedance are respectively

Sol: A pole and a zero

Example:8.5

The driving point immittance function of a one port network is $F(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$. Then the function is

Sol: RL – admittance function

8.6 Methods of Synthesis

The network synthesis does not provide a unique solution to a given problem. For a given positive real function $F(s)$, the network can be synthesized by any one of the following two methods:

1. Foster forms
2. Cauer forms

Foster Forms:

The networks synthesized by partial function expansion methods are called Foster type of networks.

Foster form I:

It provides series impedance network realization $Z(s)$.

Example:8.6

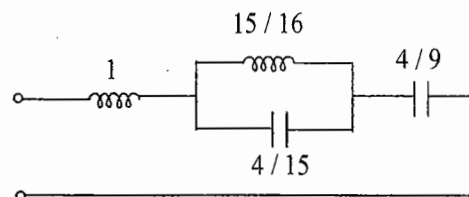
$$Z(s) = \frac{(s^2+1)(s^2+9)}{(s^2+4)}$$

Sol: Partial Fraction Expansion

$$= K_1 s + \frac{K_2}{s} + \frac{K_3 s}{s^2 + 4}$$

$$K_1 = \frac{Z(s)}{s} \Big|_{s \rightarrow \infty} = 1, \quad K_2 = s \cdot Z(s) \Big|_{s \rightarrow 0} = \frac{9}{4}, \quad K_3 = \frac{s^2 Z(s)}{s^2 + 4} \Big|_{s^2 \rightarrow -4}$$

$$= \frac{s^2 + 4}{s} Z(s) = \frac{15}{4}$$



FOSTER – I FORM



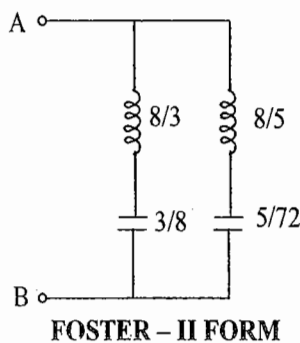
Foster form II:

It provides parallel admittance network realization $Y(s)$.

Example:8.7

$$Y(s) = \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 9)}$$

Sol: Partial Fraction Expansion



$$= \frac{K_1 s}{(s^2 + 1)} + \frac{K_2 s}{(s^2 + 9)}$$

$$K_1 = \lim_{s^2 \rightarrow -1} \frac{s^2 + 1}{s} Y(s) = \frac{3}{8}$$

$$K_2 = \lim_{s^2 \rightarrow -9} \frac{s^2 + 9}{s} Y(s) = \frac{5}{8}$$

Cauer Forms:

The network synthesized by continued fraction expansion methods are called Cauer type of network. By this technique, the synthesized network is of a ladder type. Cauer forms are of two types.

Cauer – I

The first Cauer form of a network realization is obtained by repeated removal of poles at infinity ($s = \infty$), which corresponds to arrange the

numerator and denominator of the network function in descending powers of 's' and then to find the continued fraction expansion.

Example:8.8

$$Z(s) = s^4 + 10s^2 + 9$$

Sol:

$$\frac{s^3 + 4s}{s^4 + 10s^2 + 9} \left(s \right)$$

$$\frac{s^3 + 4s}{s^4 + 10s^2 + 9}$$

$$6s^2 + 9 \left(s^3 + 4s \right) \left(\frac{1}{6} s \right)$$

$$s^3 + \frac{3}{2} s$$

$$\frac{5}{2} s \left(6s^2 + 9 \right) \left(\frac{12}{5} s \right)$$

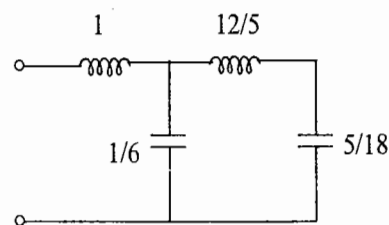
$$6s^2$$

$$9 \left(\frac{5}{2} s \right) \left(\frac{5}{18} s \right)$$

$$\frac{5}{2} s$$

$$0$$

Continued Fraction Expansion, $N(s)$ and $D(s)$ are arranged in descending order



CAUER – I FORM

Cauer – II

The second Cauer form of network realization is obtained by repeated removal of poles at origin ($s = 0$), which corresponds to arrange the numerator and denominator of the network function in



ascending powers of 's' and then to find the continued fraction expansion.

It may be mentioned here that in both the forms of the networks, Foster and Cauer, the number of elements are one greater than the number of internal critical frequencies, which are the poles and zeros of the network function excluding those at $s = 0$ and $s = \infty$. Both the forms of network give the minimum number of elements for a specified network function. For this reason, Foster and Cauer forms of network are termed canonical network (canonic means that the network contains the minimum possible number of elements).

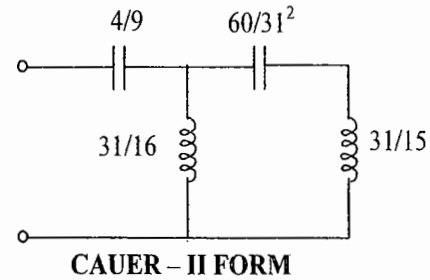
Example:8.9

$$Z(s) = s^4 + 10s^2 + 9$$

Sol: Continued Fraction Expansion, $N(s)$ and

$D(s)$ arranged in ascending order

$$\begin{array}{r} 4s + s^3 \bigg) 9 + 10s^2 + s^4 \bigg(s \\ \underline{9 + \frac{9}{4}s^2} \\ \frac{31}{4}s^2 + s^4 \bigg) 4s + s^3 \bigg(\frac{16}{31}s \\ \underline{4s + \frac{16}{31}s^3} \\ \frac{15}{31}s^3 \bigg) \frac{31}{4}s^2 + s^4 \bigg(\frac{(31)^2}{60s} \\ \underline{\frac{31}{4}s^2} \\ s^4 \bigg) \frac{15}{31}s^3 \bigg(\frac{15}{31}s \\ \underline{\frac{15}{31}s^3} \\ 0 \end{array}$$



Thus, there are four basic canonic forms of networks while dealing with the synthesis:

- | | |
|-----------------------|--------------------|
| (a) Foster I form | (b) Foster II form |
| (c) Cauer I form, and | (d) Cauer II form |

Note:

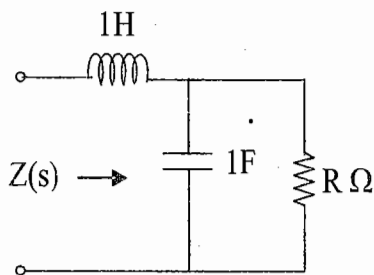
- Poles and Zeros are given the common name, critical frequencies.
- P's & Z's at $s = 0$ and $s = \infty$ are classified as external critical frequencies. All other P's & Z's form the internal critical frequencies.
- For a L-C driving point immittance function, a pair of imaginary conjugate frequency is considered as a single internal critical frequency.
- Minimum number of elements required is equal to the number of internal critical frequencies plus 1.
- F-I, F-II, C-I, C-II are the realizations with minimum number of elements. Hence they are known as canonic networks.

Class Room Practice Questions

LEVEL-1 Questions

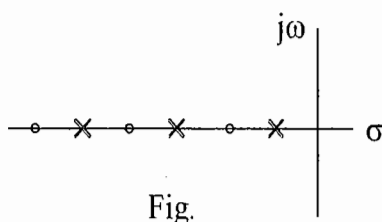
01. For a system to be stable. (IES-EE-93)
- all poles and zeros of the transfer function must be in the right half of s - plane
 - all poles and zeros of the transfer function must be on the imaginary axis
 - all the poles of the transfer function must be in the left half of the s - plane
 - all the zeros of the transfer function must be in the left half of the s - plane

02. The driving-point impedance of the circuit shown in the Fig. is (IES-EE-93)



- $\frac{s^2 + Rs}{1 + Rs}$
- $\frac{Rs^2 + s + R}{Rs + 1}$
- $\frac{s^2(R+1) + 3}{1 + Rs}$
- $\frac{1}{(1 + Rs)^2}$

03. The pole-zero configuration of an impedance function is given in the Fig. The network is (IES-EE-94)



- R-L realizable
- R-C realizable
- L-C realizable
- R-L-C realizable

04. The reciprocal of a network function is (IES-EE-95)
- an immittance function, if the original function is an immittance function
 - a transfer function, if the original function is a transfer function
 - never an immittance function
 - never a transfer function

05. $F(s) = \frac{(s+1)(s+3)}{s(s+2)}$ represents an

(IES-EE-95)

- RC impedance
- RC admittance
- RC impedance and an RL admittance
- RL admittance

06. A Hurwitz polynomial has

(IES-EE-96)

- zeros only in the left half of the s - plane
- poles only in the left half of the s - plane
- zeros anywhere in the s - plane
- poles on the $j\omega$ axis only

07. The lowest and the highest critical frequencies of R-C driving-point impedance are respectively (IES-EE-12)

- a zero and a pole
- a pole and a pole
- a zero and a zero
- a pole and a zero



LEVEL-2 Questions

01. Consider the following properties:

1. $Z(s)$ is real when s is real
2. $\frac{1}{Z(s)}$ is not necessarily positive real
3. All coefficients of $Z(s)$ are real and non-negative
4. No poles or zeros of $Z(s)$ are in the right half of the s -plane

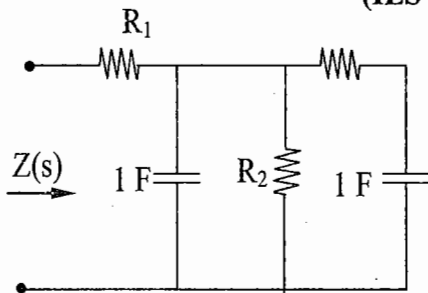
Those properties which are satisfied by a positive real network function $Z(s)$ would include

(IES-EE-93)

- | | |
|----------------|----------------|
| (a) 1, 2 and 3 | (b) 2, 3 and 4 |
| (c) 1, 2 and 4 | (d) 1, 3 and 4 |

02. For the network shown in the given figure $Z(0) = 3 \Omega$ and $Z(\infty) = 2 \Omega$. The values of R_1 and R_2 will respectively be

(IES-EE-94)



- | | |
|-----------------------------|-----------------------------|
| (a) 2Ω , 1Ω | (b) 1Ω , 2Ω |
| (c) 3Ω , 2Ω | (d) 2Ω , 3Ω |

03. For an all-pass function zeros are in the

(IES-EE-96)

- (a) right half plane (RHP) and poles in the left half plane (LHP)
- (b) LHP and poles in LHP
- (c) LHP and poles in RHP
- (d) RHP and poles in RHP

04. The network function

$$F(s) = \frac{(s+2)}{(s+1)(s+3)}$$
 represents an

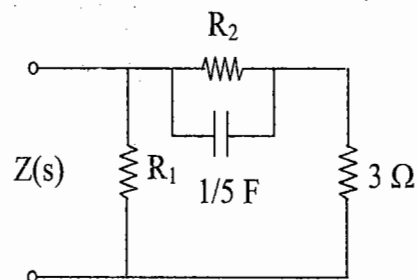
(IES-EE-98)

- (a) RC impedance
- (b) RL impedance
- (c) RC impedance and an RL admittance
- (d) RC admittance and an RL impedance

05. Consider the following circuit :

In the below circuit $Z(s) = 2$ as $s \rightarrow \infty$ and $Z(s) = 3$ as $s \rightarrow 0$. What are, respectively the values of R_1 and R_2 ?

(IES-EE-04)



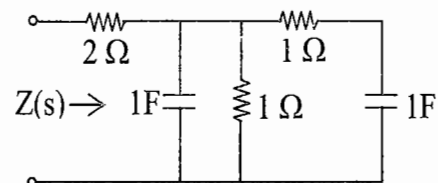
- | | |
|-----------------------------|-----------------------------|
| (a) 6Ω , 3Ω | (b) 4Ω , 2Ω |
| (c) 3Ω , 6Ω | (d) 6Ω , 2Ω |

06. The realization of the driving point impedance function

$$Z(s) = \frac{(\alpha s^2 + 7s + 3)}{(s^2 + 3s + \beta)}$$
 is shown in the

circuit given below : What are, respectively the values of α and β ?

(IES-EE-04)



- | | |
|----------|----------|
| (a) 5, 2 | (b) 2, 5 |
| (c) 2, 3 | (d) 2, 1 |



Solutions

LEVEL-1

01. Ans: (c)

Sol: For a system to be stable, all the poles of the transfer function must be in the left half of the s - plane.

02. Ans: (b)

$$\begin{aligned} \text{Sol: } Z(s) &= 1s + \frac{R \times \frac{1}{s}}{R + \frac{1}{s}} \\ &= s + \frac{R}{Rs+1} = \frac{Rs^2 + s + R}{Rs+1} \end{aligned}$$

03. Ans: (b)

Sol: The pole-zero configuration of $Z(s)$ is P Z P Z P Z alternating on the negative real axis with pole nearest the origin and zero nearest ∞ . This $Z(s)$ is R-C realizable.

04. Ans: (a)

Sol: Immittance is common name used to indicate either an Impedance or admittance. Hence the reciprocal of a NW function is an Immittance

function, if the original function is an Immittance function.

05. Ans: (c)

$$\text{Sol: } F(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

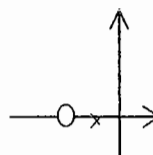
It has P-Z-P-Z structure with P at origin. $F(s)$ represents Z_{RC} as well as Y_{RL} .

06. Ans: (b)

Sol: A Hurwitz polynomial has poles only in the left half of the s - plane

07. Ans: (d)

Sol:



The location of pole, zero for a R-C driving - point impedance function

Level-2 Key

01. (d) 02. (a) 03. (a) 04. (c) 05. (a)
06. (d)