



Differentiation

Exercise 6.6



RISEING STAR ACADEMY

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Chapter 6

Differentiation

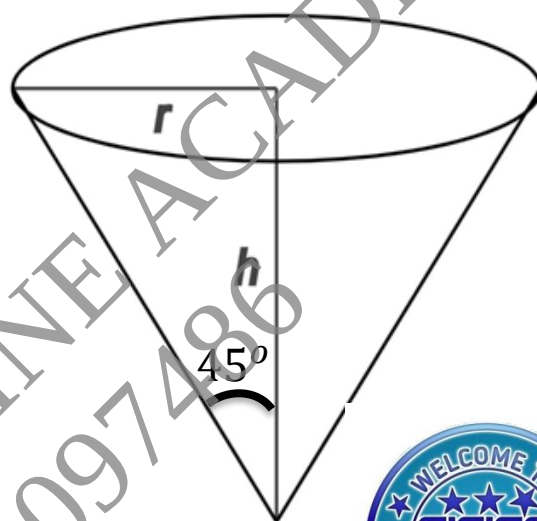
Exercise 6.6

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Question # 1

A container in the shape of a hollow cone of semi-vertical angle 45° is held with its vertex pointing downwards. Water is poured into the container at the rate of $6 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate at which the depth of the water in the cone is increasing when the depth of the water is 4 cm.



Solution:

Given Rate of change of Volume $= \frac{dV}{dt} = 6 \text{ cm}^3 \text{ s}^{-1}$

Height or depth of water = 4 cm

Let 'r' be the radius of base and 'h' be the height of the cone

$$\text{Volume of Cone} = V = \frac{1}{3} \times \pi \times r^2 \times h$$

$$\text{Now, } \tan 45^\circ = \frac{r}{h} \Rightarrow 1 = \frac{r}{h} \Rightarrow r = h$$

$$\text{Volume of Cone} = V = \frac{1}{3} \times \pi \times h^2 \times h$$

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Now, we will differentiate Volume with respect to height



$$\frac{dV}{dt} = \frac{1}{3} \times \pi \times 3h^2 \times \frac{dh}{dt}$$

$$6 = \frac{1}{3} \times \pi \times 3(4)^2 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{18}{48\pi} = 0.1194 \text{ cm/sec}$$

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Question # 2

Air is pumped into a spherical balloon at the rate of $20 \text{ cm}^3 \text{ s}^{-1}$. When the radius of the balloon is 10 cm, calculate

- (a) The rate of change of the radius
- (b) The rate of change of the surface area

Solution:

Given

Rate of change of Volume = $20 \text{ cm}^3 \text{ s}^{-1}$

Radius of balloon = $r = 10 \text{ cm}$

Let 'r' be the radius of spherical balloon.

$$\text{Volume} = V = \frac{4}{3}\pi r^3$$

$$(a) \quad V = \frac{4}{3}\pi r^3$$

Differentiating with respect to 'r'

$$\frac{dV}{dt} = \frac{4}{3}\pi \left(3r^2 \frac{dr}{dt} \right)$$

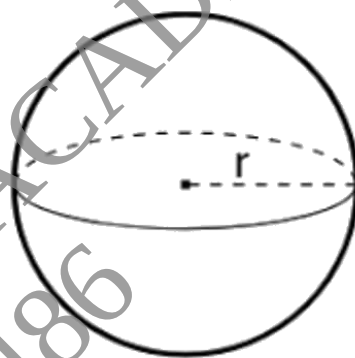
$$20 = 4\pi \times (10)^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{20}{400\pi} = 0.0159 \text{ cm/s}$$

$$(b) \quad \text{Surface Area of Sphere} = A = 4\pi r^2$$

Differentiating with respect to 't'

$$\frac{dA}{dt} = 4\pi(2r) = 8\pi r \frac{dr}{dt} = 8\pi(10)(0.0159) = 4 \text{ cm}^2 \text{ s}^{-1}.$$



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Question # 3

A cube is expanding in such a way that its volume is

changing at a rate of $216 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of change of the edge when the edge is 3 cm.

Solution:

Given

Rate of change of

volume = $216 \text{ cm}^3 \text{ s}^{-1}$

Length of side = 3 cm

$$\frac{da}{dt} = ?$$

Let the edge of the cube be $a \text{ cm}$. Then

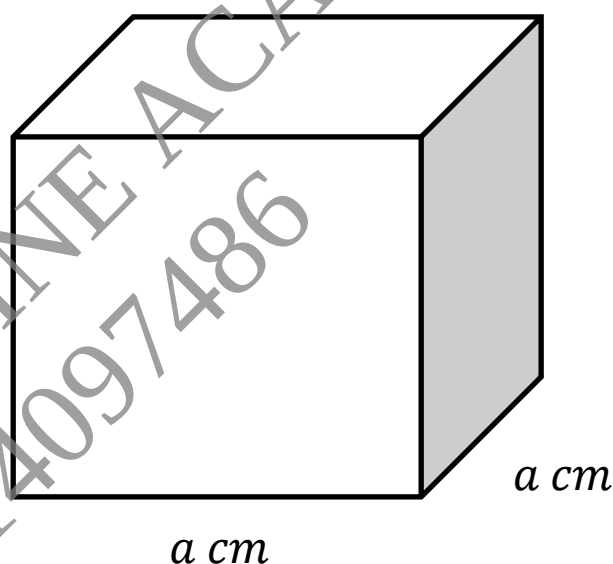
$$\text{Volume} = V = a^3$$

So,

$$\frac{dV}{dt} = 3a^2 \cdot \frac{da}{dt}$$

$$216 = 3(3)^2 \cdot \frac{da}{dt}$$

$$\frac{da}{dt} = \frac{216}{27} = 8 \text{ cm/sec}$$



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Question # 4

The area of a square is increasing at a rate of $12 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the side of the square when the length of the side is 3 cm.

Solution:

Given

Rate of change of Area of Square $= \frac{dA}{dt} = 12 \text{ cm}^3 \text{ s}^{-1}$

Length of side of square $= a = 3 \text{ cm}$

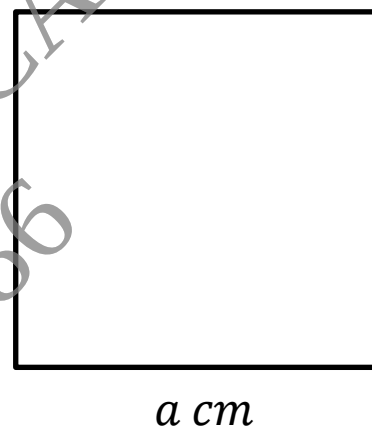
Rate of change of increase in side of square $= \frac{da}{dt} = ?$

$$A = a^2$$

$$\frac{dA}{dt} = 2a \cdot \frac{da}{dt}$$

$$12 = 2(3) \cdot \frac{da}{dt}$$

$$\frac{da}{dt} = \frac{12}{6} = 2 \text{ cm/s}$$



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Question # 5

The side of a square is increasing at the rate of $4 \text{ cm}^3 \text{ s}^{-1}$.
Find the rate of increase of the area when the
length of the side is 5 cm.

Solution:

Given

Rate of change of side of a square = $\frac{da}{dt} = 4 \text{ cm}^3 \text{ s}^{-1}$ $a \text{ cm}$

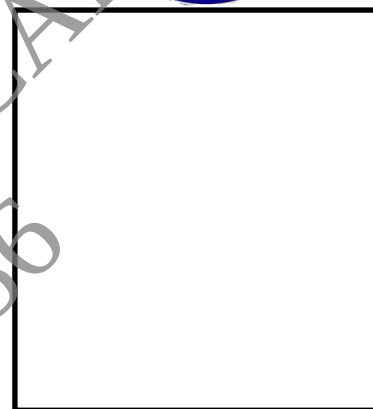
Length of the side of a square = $a = 5 \text{ cm}$

Rate of change of Area = $\frac{dA}{dt} = ?$

$$\text{Area} = A = a^2$$

$$\frac{dA}{dt} = 2a \cdot \frac{da}{dt}$$

$$\frac{dA}{dt} = 2(5) \cdot (4) = 40 \text{ cm}^2 \text{ s}^{-1}$$



$a \text{ cm}$

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Question # 6

Find the coordinates of the stationary points of each of the following curves and determine the nature of the stationary points.

(a) $y = x^2 + 3x + 1$

Solution:

$$y = x^2 + 3x + 1$$

Differentiating with respect to 'x'

$$\frac{dy}{dx} = 2x + 3$$

For Stationary Point, put $\frac{dy}{dx} = 0$

$$2x + 3 = 0$$

$$x = -\frac{3}{2}$$

For coordinates of SP

$$\Rightarrow y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + 1 = \frac{9}{4} - \frac{9}{2} + 1 = -\frac{5}{4}$$

So, the coordinates are $\left(-\frac{3}{2}, -\frac{5}{4}\right)$

For Nature of roots, take the 2nd Order derivative

$$\frac{d^2y}{dx^2} = 2 > 0$$

Nature of roots is **Minimum**.



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(b) $y = -x^2 + 2x - 3$

Solution:

$$y = -x^2 + 2x - 3$$

Differentiating with respect to 'x'

$$\frac{dy}{dx} = -2x + 2$$

For Stationary Point, put $\frac{dy}{dx} = 0$

$$-2x + 2 = 0$$

$$x = 1$$

For coordinates of SP

$$\Rightarrow y = -(1)^2 + 2(1) - 3 = -1 + 2 - 3 = -2$$

So, the coordinates are (1, -2)

For Nature of roots, take the 2nd Order derivative

$$\frac{d^2y}{dx^2} = -2 < 0$$

Nature of roots is **Maximum**.



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(c) $y = 3x^3 - 3x^2 - 8x + 5$

Solution:

$$y = 3x^3 - 3x^2 - 8x + 5$$

Differentiating with respect to 'x'

$$\frac{dy}{dx} = 9x^2 - 6x - 8$$

For Stationary Point, put $\frac{dy}{dx} = 0$

$$9x^2 - 6x - 8 = 0$$

$$9x^2 - 12x + 6x - 8 = 0$$

$$3x(3x - 4) + 2(3x - 4) = 0$$

$$(3x - 4)(3x + 2) = 0$$

$$x = \frac{4}{3} ; x = -\frac{2}{3}$$

For coordinates of SP, at $x = \frac{4}{3}$

$$\Rightarrow y = 3\left(\frac{4}{3}\right)^3 - 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 5 = -\frac{35}{9}$$

For coordinates of SP, at $x = -\frac{2}{3}$

$$\Rightarrow y = 3\left(-\frac{2}{3}\right)^3 - 3\left(-\frac{2}{3}\right)^2 - 8\left(-\frac{2}{3}\right) + 5 = \frac{73}{9}$$

So, the coordinates are $\left(\frac{4}{3}, -\frac{35}{9}\right)$ & $\left(-\frac{2}{3}, \frac{73}{9}\right)$

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For Nature of roots, take the 2nd Order derivative

$$\frac{d^2y}{dx^2} = 18x - 6$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=\frac{4}{3}} = 18\left(\frac{4}{3}\right) - 6 = 18 > 0 \quad \text{Minimum}$$

$$\frac{d^2y}{dx^2} = 18x - 6$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=-\frac{2}{3}} = 18\left(-\frac{2}{3}\right) - 6 = -18 < 0 \quad \text{Maximum}$$

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(d) $y = 4x(x^2 - 12)$

Solution:

$$y = 4x(x^2 - 12)$$

$$y = 4x^3 - 48x$$

$$\frac{dy}{dx} = 12x^2 - 48$$

For Stationary Point, put $\frac{dy}{dx} = 0$

$$12x^2 - 48 = 0$$

$$12(x^2 - 4) = 0$$

$$(x - 2)(x + 2) = 0$$

$$x = 2 ; x = -2$$

For coordinates of SP, at $x = 2$

$$\Rightarrow y = 4(2)^3 - 48(2) = 32 - 96 = -64$$

For coordinates of SP, at $x = -2$

$$\Rightarrow y = 4(-2)^3 - 48(-2) = -32 + 96 = 64$$

So, the coordinates are $(2, -64)$ & $(-2, 64)$

For Nature of roots, take the 2nd Order derivative

$$\frac{d^2y}{dx^2} = 24x$$

$$\left[\frac{d^2y}{dx^2} \right]_{x=2} = 24(2) = 48 > 0 \quad \text{Minimum}$$



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$$\frac{d^2y}{dx^2} = 24x$$

$$\left[\frac{d^2y}{dx^2} \right]_{x=-2} = 24(-2) = -48 < 0 \quad \text{Maximum}$$



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(e) $y = x^3 - 3x^2 + 3x - 7$

Solution:

$$y = x^3 - 3x^2 + 3x - 7$$

$$\frac{dy}{dx} = 3x^2 - 6x + 3$$

For Stationary Point, put $\frac{dy}{dx} = 0$

$$3x^2 - 6x + 3 = 0$$

$$3(x^2 - 2x + 1) = 0$$

$$(x^2 - 2x + 1) = 0$$

$$(x - 1)^2 = 0$$

Taking Square root on both sides

$$x = 1$$

For coordinates of SP, at $x = 1$

$$\Rightarrow y = (1)^3 - 3(1)^2 + 3(1) - 7 = -6$$

So, the coordinates are $(1, -6)$

For Nature of roots, take the 2nd Order derivative

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=1} = 6(1) - 6 = 0 \quad \text{Inflection Point}$$

