



Differentiation

Exercise 6.1

RISING STAR ACADEMY

+92-331-4097486

BY SIR HAROON BASHIR

Chapter 6

Differentiation

Exercise 6.1

Question # 1

Differentiate each of the following:

(a) x^3

Solution: $\frac{d}{dx}(x^3) = 3x^2$

(b) $\sqrt[3]{x}$

Solution: $\frac{d}{dx}(\sqrt[3]{x}) = \frac{d}{dx}(x^{\frac{1}{3}}) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$

(c) $3x^{-5}$

Solution: $\frac{d}{dx}(3x^{-5}) = -15x^{-6}$

(d) $\frac{1}{5x}$

Solution:

$$\frac{d}{dx}\left(\frac{1}{5x}\right) = \frac{1}{5} \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{1}{5} \frac{d}{dx}(x^{-1}) = -\frac{1}{5}x^{-2}$$



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(e) $\frac{1}{x}$

Solution: $\frac{d}{dx} \left(\frac{1}{x} \right) = \frac{d}{dx} (x^{-1}) = -1x^{-2}$

(f) π

Solution: $\frac{d}{dx} (\pi) = 0$



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Question # 2

Find $\frac{dy}{dx}$ for each of the following:

(a) $y = 6x^5 + 4x^3 - 3x^2 + 2x - 7$

Solution:

Differentiating with respect to x on both sides

$$\frac{d}{dx}(y) = \frac{d}{dx}(6x^5 + 4x^3 - 3x^2 + 2x - 7)$$

$$\frac{dy}{dx} = 30x^4 + 12x^2 - 6x + 2$$



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(b) $y = x + \frac{3}{\sqrt{x}} + \frac{2}{\sqrt{x^3}}$

Solution:

Differentiating with respect to x on both sides

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x + \frac{3}{\sqrt{x}} + \frac{2}{\sqrt{x^3}}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + 3 \cdot x^{-\frac{1}{2}} + 2 \cdot x^{-\frac{3}{2}}\right)$$

$$\frac{dy}{dx} = 1 - \frac{3}{2} \cdot x^{-\frac{3}{2}} - 3 \cdot x^{-\frac{5}{2}}$$



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(c) $y = \left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^2$

Solution:

Differentiating with respect to x on both sides

$$\frac{d}{dx}(y) = \frac{d}{dx} \left(\left(\frac{1}{\sqrt{x}}\right)^2 + (\sqrt{x})^2 - 2 \times \frac{1}{\sqrt{x}} \times \sqrt{x} \right)$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{1}{x} + x - 2 \right)$$

$$\frac{dy}{dx} = -\frac{1}{x^2} + 1$$

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Question # 3

Differentiate the following with respect to x

(a) $\frac{2x^2+4x}{x}$

Solution:

Let

$$y = \frac{2x^2+4x}{x}$$

$$y = 2x + 4$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} (2x + 4) = 2$$



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(b) $\frac{x^2 - 6x + 4}{x}$

Solution:

Let $y = \frac{x^2 - 6x + 4}{x}$

$$y = x - 6 + \frac{4}{x}$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(x - 6 + \frac{4}{x} \right) = 1 - \frac{4}{x^2}$$



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(c) $\frac{4x^3 - 5x - 3}{2x}$

Solution:

Let $y = \frac{4x^3 - 5x - 3}{2x}$

$$y = 2x^2 - \frac{5}{2} - \frac{3}{2x}$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(2x^2 - \frac{5}{2} - \frac{3}{2} \cdot \frac{1}{x} \right) = 4x + \frac{3}{2} \cdot \frac{1}{x^2}$$



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Question # 4

Differentiate each of the following with respect to x

(a) $-\frac{3}{8x^4} - 6x + 7$

Solution:

Let $y = -\frac{3}{8x^4} - 6x + 7$

$$y = -\frac{3}{8} \cdot \frac{1}{x^4} - 6x + 7$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(-\frac{3}{8} \cdot \frac{1}{x^4} - 6x + 7 \right)$$

$$\frac{dy}{dx} = \frac{3}{2} \cdot \frac{1}{x^5} - 6$$



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(b) $3x^4 + 5x^3 - \frac{4}{\sqrt{x}}$

Solution:

Let $y = 3x^4 + 5x^3 - \frac{4}{\sqrt{x}}$

$$y = 3x^4 + 5x^3 - 4 \cdot x^{-\frac{1}{2}}$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(3x^4 + 5x^3 - 4 \cdot x^{-\frac{1}{2}} \right)$$

$$\frac{dy}{dx} = 12x^3 + 15x^2 + 2 \cdot x^{-\frac{3}{2}}$$

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(c) $\frac{3x^2 - 4t^2}{3}$



Solution:

Let $y = \frac{3x^2 - 4t^2}{3}$

$$y = x^2 - \frac{4}{3}t^2$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^2 - \frac{4}{3}t^2 \right) = 2x + 0 = 2x$$

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(d) $\frac{3\sqrt{x}-4}{7}$

Solution:

Let $y = \frac{3\sqrt{x}-4}{7}$

$$y = \frac{3}{7}\sqrt{x} - \frac{4}{7}$$

Differentiating both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{3}{7}\sqrt{x} - \frac{4}{7} \right)$$

$$\frac{dy}{dx} = \frac{3}{7} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - 0$$

$$\frac{dy}{dx} = \frac{3}{14} \cdot \frac{1}{\sqrt{x}}$$

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Question # 5

Find the coordinated of the point on the curve $y = x^3 - 3x^2 + 6x + 2$ at which the gradient is 3.

Solution:

$$y = x^3 - 3x^2 + 6x + 2$$

Differentiate both sides with respect to 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 6x + 2)$$

$$\frac{dy}{dx} = 3x^2 - 6x + 6$$

Given that $\frac{dy}{dx} = 3$

$$3 = 3x^2 - 6x + 6$$

$$3x^2 - 6x + 6 - 3 = 0$$

$$3x^2 - 6x + 3 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1$$



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Put $x = 1$ in curve to get 'y'.

$$y = (1)^3 - 3(1)^2 + 6(1) + 2 = 1 - 3 + 6 + 2 = 6$$

So, the coordinates are (1, 6)

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Question # 6

Calculate the gradient of the tangent to the curve at the given point

(a) $y = \sqrt{x}(2 - x), x = 9$

Solution:

$$y = \sqrt{x}(2 - x)$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}(2 - x)) = \frac{d}{dx}(2\sqrt{x} - x\sqrt{x}) = \frac{d}{dx}\left(2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} - \frac{3}{2}\sqrt{x}$$

Put $x = 9$ to find the Gradient

$$\frac{dy}{dx} = \frac{1}{\sqrt{9}} - \frac{3}{2}\sqrt{9} = \frac{1}{3} - \frac{3}{2} \cdot 3 = \frac{1}{3} - \frac{9}{2} = -\frac{25}{6}$$

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(b) $y = \frac{6-4x}{x}, \quad x = -2$

Solution:

$$y = \frac{6-4x}{x}$$

$$y = \frac{6}{x} - 4$$

$$y = 6 \cdot x^{-1} - 4$$

Differentiating both sides with respect to 'x'

$$\frac{dy}{dx} = \frac{d}{dx} (6 \cdot x^{-1} - 4)$$

$$\frac{dy}{dx} = -6 \cdot x^{-2} = -\frac{6}{x^2}$$

