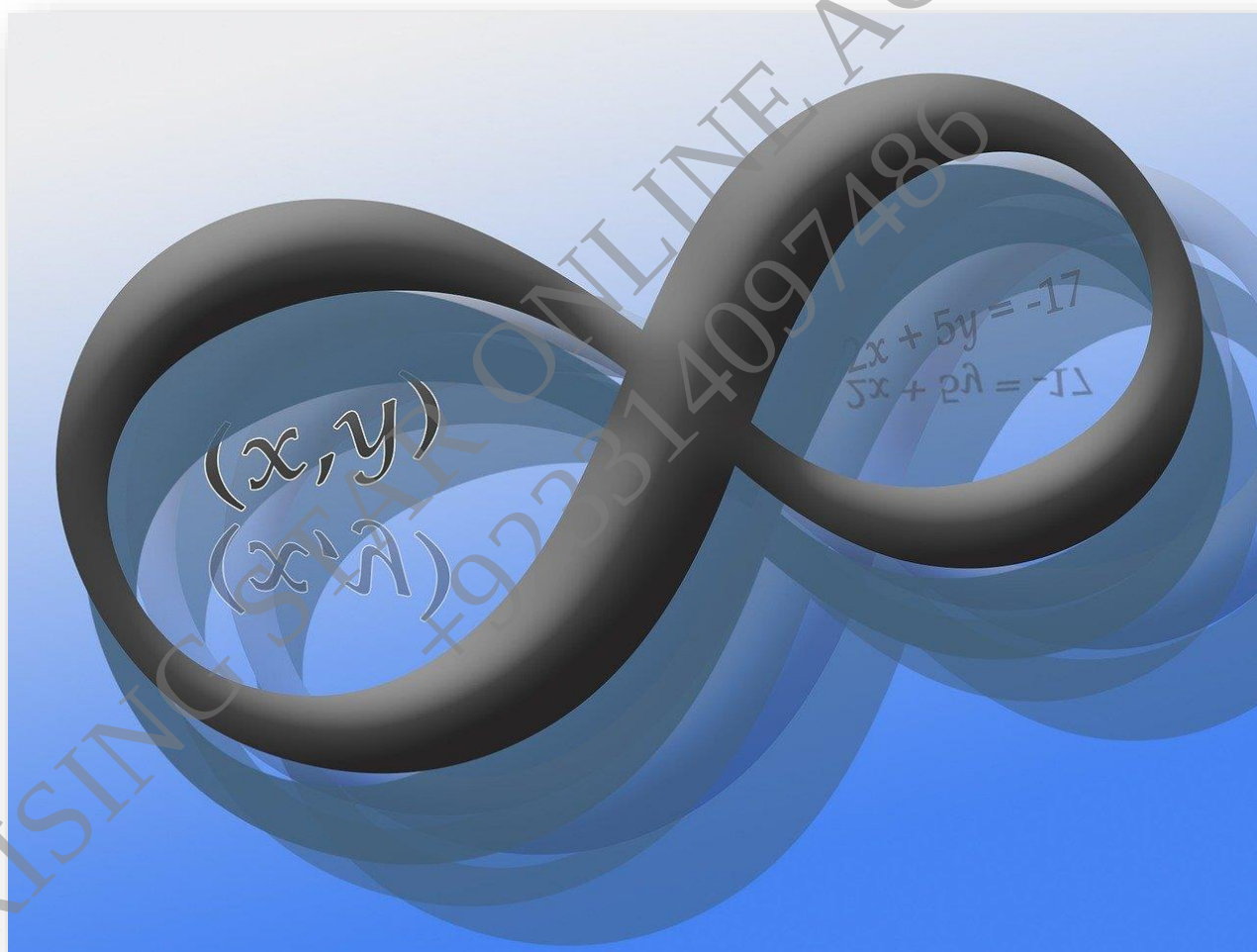


Mathematics Chapter 5

Limits and Continuity

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Question # 1

Evaluate each of the following limits:

(a) $\lim_{x \rightarrow +\infty} \frac{x+1}{x-3}$

Solution:

$$\lim_{x \rightarrow +\infty} \frac{x+1}{x-3}$$

Dividing by highest power of x

$$\lim_{x \rightarrow +\infty} \frac{x+1}{x-3} = \lim_{x \rightarrow +\infty} \frac{\frac{x+1}{x}}{\frac{x-3}{x}} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x}{x} - \frac{3}{x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{1 + \frac{1}{x}}{1 - \frac{3}{x}} = \frac{1 + \frac{1}{+\infty}}{1 - \frac{3}{+\infty}} = \frac{1 + 0}{1 - 0} = 1$$

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(b) $\lim_{x \rightarrow +\infty} \frac{2x^2 - 5}{x + 1}$

Solution:

$$\lim_{x \rightarrow +\infty} \frac{2x^2 - 5}{x + 1}$$

Dividing by highest power of x^2

$$\begin{aligned} &= \lim_{x \rightarrow +\infty} \frac{\frac{2x^2}{x^2} - \frac{5}{x^2}}{\frac{x}{x^2} + \frac{1}{x^2}} = \lim_{x \rightarrow +\infty} \frac{2 - \frac{5}{x^2}}{\frac{1}{x} + \frac{1}{x^2}} = \frac{2 - \frac{5}{+\infty^2}}{\frac{1}{+\infty} + \frac{1}{+\infty^2}} = \frac{2 - 0}{0 + 0} = \frac{2}{0} \\ &= +\infty \end{aligned}$$

(c) $\lim_{x \rightarrow +\infty} \frac{\sqrt{1+x^2}}{3-x}$

Solution:

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{1+x^2}}{3-x}$$

Dividing by highest power of x

$$\begin{aligned} &= \lim_{x \rightarrow +\infty} \frac{\sqrt{1+x^2}}{3-x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{1}{x^2} + \frac{x^2}{x^2}}}{\frac{3}{x} - \frac{x}{x}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{1}{x^2} + 1}}{\frac{3}{x} - 1} = \frac{\sqrt{\frac{1}{+\infty} + 1}}{\frac{3}{+\infty} - 1} \\ &= \frac{1}{-1} = -1 \end{aligned}$$





(d) $\lim_{x \rightarrow +\infty} \sqrt{1+x^2} - x$

Solution:

Applying Rationalization

$$\begin{aligned} & \lim_{x \rightarrow +\infty} \sqrt{1+x^2} - x \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{\sqrt{1+x^2} - x}{1} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{1+x^2})^2 - x^2}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{1+x^2 - x^2}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{1+x^2} + x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2} + \frac{x^2}{x^2} + \frac{x}{x}}} \\ &= \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2} + 1 + 1}} = \frac{\frac{1}{+\infty}}{\sqrt{\frac{1}{+\infty} + 1 + 1}} \\ &= \frac{0}{\sqrt{0+1+1}} = \frac{0}{1} = 0 \end{aligned}$$

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Question # 2

Evaluate each of the following limits:

(a) $\lim_{x \rightarrow -\infty} \sqrt{1+x^2} - x$

Solution:

Applying Rationalization

$$\begin{aligned} & \lim_{x \rightarrow -\infty} \sqrt{1+x^2} - x \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{1+x^2} - x}{1} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow -\infty} \frac{(\sqrt{1+x^2})^2 - x^2}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow -\infty} \frac{1+x^2 - x^2}{\sqrt{1+x^2} + x} \\ &= \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{1+x^2} + x} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2} + \frac{x^2}{x^2} + \frac{x}{x}}} \\ &= \lim_{x \rightarrow -\infty} \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2} + 1 + 1}} = \frac{\frac{1}{-\infty}}{\sqrt{\frac{1}{-\infty} + 1 + 1}} \end{aligned}$$

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$$= \frac{0}{\sqrt{0+1}+1} = \frac{0}{1} = 0$$

(b) $\lim_{x \rightarrow -\infty} \frac{x-x^3}{2x^2+1}$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x-x^3}{2x^2+1} &= \lim_{x \rightarrow -\infty} \frac{\frac{x}{x^3} - \frac{x^3}{x^3}}{\frac{2x^2}{x^3} + \frac{1}{x^3}} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x^2} - 1}{\frac{2}{x} + \frac{1}{x^3}} \\ &= \frac{\frac{1}{(-\infty)^2} - 1}{\frac{2}{-\infty} + \frac{1}{(-\infty)^3}} = \frac{0-1}{0+0} = -\frac{1}{0} = -\infty \end{aligned}$$

(c) $\lim_{x \rightarrow -\infty} \frac{3x-1}{\sqrt{2+x^2}}$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x-1}{\sqrt{2+x^2}} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x}{x} - \frac{1}{x}}{\sqrt{\frac{2}{x^2} + \frac{x^2}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{3 - \frac{1}{x}}{\sqrt{\frac{2}{x^2} + 1}} \\ &= \frac{3 - \frac{1}{-\infty}}{\sqrt{\frac{2}{(-\infty)^2} + 1}} = \frac{3}{\sqrt{1}} = 3 \end{aligned}$$