EUEE Questions On Vectors and Transformation of the Plane

1. Line 1 passes through (0,5) and (-5,0). What is the angle between the y – axis and Line 1 in radian measure?

   A. $\frac{\pi}{4}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{3}{4}\pi$

2. Which of the following is a vector that lies on the line through (0,0) and (2,4)?

   A. $\vec{u} = (2,1)$ B. $\vec{u} = (-1,-2)$ C. $\vec{u} = (\frac{1}{2},-2)$ D. $\vec{u} = (-2,-6)$

3. Let i and j be the standard unit vectors in the directions of positive x- axis and positive y – axis, respectively, and $\overline{AB}$ be a vector from the point A(-1,0) to the point B(2,2). If $\overline{Y}$ = 3 $\overline{AB}$ -3i + 2j, then the unit vector in the direction of $\overline{V}$ is equal to

   A. $\left(\frac{3}{5},\frac{4}{5}\right)$ B. $\left(\frac{3}{5},\frac{4}{5}\right)$ C. $\left(\frac{-3}{5},\frac{4}{5}\right)$ D. $\left(\frac{3}{5},\frac{4}{5}\right)$

4. Which of the following is a vector equation of the line tangent of the circle $x^2 + y^2 + 2x - 7 = 0$ at (1,2)?

   A. (x,y) = (0,3) + λ (-1,2)
   B. (x,y) = (1,2) + λ (2,-1)
   C. (x,y) = (0,3) + λ (1,-1)
   D. (x,y) = (1,3) + λ (-1,2)

5. Let be the line whose equation is 2x – y = 10. Which one of the following is the equation of the following is the equation of the image of after a reflection in the line y = 2x – 5 followed by a rotation through the angel of 90° about the origin?

   A. X + 2y = 0
   B. 2x + y = 0
   C. x + 2y = 5
   D. x-2y=5

6. A line given by a vector equation r(t) = (0,3) +t(1,1) is tangent to a circle at point (0,3). If the radius of the circle is $\sqrt{2}$, which one of the following is the center of the circle?

   A. (1,4) B. (1,-4) C. (-1,2) D. (1,2)

7. If $\vec{u} = (-3, x) and \vec{V} = (x, y-2)$ are vectors, what is the value of y so that $\vec{u} + \vec{v} = 3\vec{u} - \frac{1}{2}\vec{v}$?

   A. $\frac{2}{3}$ B. $-\frac{10}{3}$ C. -4 D. $-\frac{22}{3}$

8. Which one of the following is necessarily true?

   A. If $\|\vec{A}\| = \|\vec{B}\|$, then $\vec{A} = \vec{B}$.

   B. $\|K\vec{A}\| = K\|\vec{A}\|$, for any real number k.

   C. If $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A}\vec{B}$ = 0

   D. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A}.\vec{u} \equiv \|\vec{A}\|$

9. If $\vec{A}$ is perpendicular to $\vec{B}$, what is the cosine of the angle between $\vec{A}$ and $\vec{A} - \vec{B}$?

   A. $\frac{\|\vec{A} - \vec{B}\|}{\|\vec{A}\|}$ B. $\frac{\|\vec{A}\|}{\|\vec{A} - \vec{B}\|}$ C. $\frac{\|\vec{A} - \vec{B}\|}{\|\vec{B}\|}$ D. $\frac{\|\vec{B}\|}{\|\vec{A} - \vec{B}\|}$

10. What is the image of the ellipse whose equation is $2(x+2)^2+(y-1)^2=2$ under a translation that takes (2,1) to (4,0) followed by a rotation of 90°?

    A. $X^2 + 2y^2 = 2$ C. $2(x-4)^2 + y^2 = 2$

    B. $2x^2 + y^2 = 2$ D. $(x-4)^2 + 2y^2=2$

11. *Suppose* $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$ *and* $\vec{B}$ *is a vector in space such that* $|\vec{B}| = \vec{A}.\vec{B}$. *If* $\vec{U}$ *is the unit vector in the direction of* $\vec{B}$, *then* $|\vec{A} + \vec{U}|^2$ *is equal to:*

    A. 16 B. 12 C. 10 D. 14

12. *What is the image of the line given by (x,y) = (-1,0) + t(3,6),t$\in$R, under the translation that takes (1,0) to (0,1) following by the reflection about the line y = 2x?*

A. y = 2x + 3 *B. y = 2x – 3*

C. y = 2x + 6 *D. y = 2x – 5*

13. *Suppose* $\vec{A} = 3\vec{i} - 4\vec{j}$ *and* $\vec{B}$ *is a vector in the xy=plane such that the angle between* $\vec{A}$ *and* $\vec{B}$ *is* $\frac{\pi}{3}$ *. If* $\vec{U}$ *is the unit vector in the direction of* $\vec{B}$ *, then* $\vec{A}(\vec{A} - 2\vec{U})$ *is equal to:*

A. 20 *B. 5* *C. 15* *D. 30*

14. *Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points?*

A. 4x + 3y + 19 = 0 *C. 2y = 5x + 43*

B. 3x + 4y + 14 = 0 *D. 2x = y – 50*

15. *If* ℓ *is the line passing through (0,2) and parallel to* $V = \vec{i} + 3\vec{j}$ *, which one of the following is true about* ℓ *and the circle $(x-2)^2 + (y-1)^2=5$?*

A. ℓ *is tangent to the circle at (0,2)*

B. ℓ *is tangent to the circle at some point p, where p ≠ (0,2)*

C. Intersects the circle at two distinct points.

D. The distance between ℓ *and the center of the circle is greater than* $\sqrt{5}$

16. *If a translation T take the circle $x^2 + y^2 – 2x + 6y +3 = 0$ into the circle whose equation is $(x+2)^2 + (y-4)^2=7$, then what is the image of the origin under T?*

A. (-3,7) *B. (1,2)* *C. (1,-3)* *D. (-2,4)*

17. *If A(-2,3), B=)3,1) and C is any other point on the plane, then which one of the following is the coordinate from of* $\overrightarrow{AC} - \overrightarrow{BC}$ *?*

A. (-5,2) *B. (5,-2)* *C. (1,4)* *D. (-1,-4)*

18. *What is the equation of a line that passes through the point (-1,2) and parallel to a the vector (1,-1)?*

A. 2x – y = 1 *B. x + y – 1 = 0*

C. x – 2y = 3 *D. y – 2x + 1 = 0*

19. If $\vec{A} = 4\vec{i} - 3\vec{j}$ and $\vec{u}$ is a unit vector such that $|\vec{A} + \vec{u}|^2 = 27$, then the cosine of the angle between $\vec{A}$ and $\vec{u}$ is equal to _____

A. 0.1 B. 0.2 C. 0.3 D. 0.4

20. If A=(1,-2), B=(-3,2) and $\vec{V}$ is a position vector such $2\vec{V} + \overrightarrow{AB} = \vec{0}$, then $\vec{V}$ is equal to _____.

A. (2,0) B. (-1,0) C. (-2,2) D. (2,-2)

21. What is the image of the ellipse $(x-1)^2+4y^2=1$ under the translation that takes (1,1) to (0,2) followed by the reflection through the x-axis?

A. $x^2+4(y-1)^2=1$ C. $x^2+4(y+1)^2=1$

B. $4x^2+(y-1)^2=1$ D. $4x^2+(y+1)^2=1$

22. If $\vec{V} = \overrightarrow{AB} + 3\overrightarrow{BA}$, where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$?

A. $6\overrightarrow{AB}$ B. $-6\overrightarrow{AB}$ C. $12\overrightarrow{AB}$ D. $-12\overrightarrow{AB}$

23. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$ and $|\vec{A}| = 4$, then $\vec{A} \cdot \vec{u}$ is equal to _____.

A. $\frac{1}{4}$ B. 4 C. $\frac{1}{2}$ D. 2

24. If $\vec{A}$ and $\vec{B}$ are parallel vectors with opposite directions and $|\vec{B}| = |2\vec{A}|$, is equal to:

A. $\vec{A}$ B. $-\vec{A}$ C. $-3\vec{A}$ D. $3\vec{A}$

25. The image of a figure with vertices A(1, 2), B(3, 6), C(-1, 2), and D(-2,-2) after reflection across the x-axis is:

A. $A'(1,-2), B'(-3,-6), C'(1,-2), and D'(2,2)$

B. $A'(-1,-2), B'(-3,6), C'(1,-2), and D'(2,-2)$

C. $A'(1,-2), B'(3,-6), C'(-1,-2), and D'(-2,2)$

D. $A'(1,-2), B'(3,6), C'(-1,2), and D'(-2,-2)$

26. What is the value k, for which the two vectors $\vec{u} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2k \\ -5 \\ 4 \end{pmatrix}$ are perpendicular?

A. 4 B. -4 C. 3 D. -3

27. Which one of the following is the equation of the image of l after being translated by the vector $u = (2, -1)$ followed by a rotation through 45° about the origin?

A. $y = \sqrt{2}x$ B. $y = \sqrt{2}$ C. $x = 0$ D. $y = -2\sqrt{2}x$

28. In order to measure the height of a tower, suppose a surveyor takes two sightings from a transit 1meter high which are positioned d meters apart on the same ground level as in the figure below. If the first measured angle of elevation is α and the second is β (see, the figure), then what is the height of the tower (in meter) in terms of α, β and d ?

A. $\frac{\tan\alpha\tan\beta}{(\tan\alpha-\tan\beta)d}+1$

B. $\left(\frac{\tan\alpha-\tan\beta}{\tan\alpha\tan\beta}\right)d+1$

C. $\left(\frac{\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}\right)d+1$

D. $\left(\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}\right)d+1$

29. If the dot products of a vector $\vec{A}$ with the vectors $i-j+k, 2i+j-3k$ and $i+j+k$ are 4,0 and 2 , respectively, what is $\vec{A}$?

A. $\vec{A} = (2,1,1)$ B. $\vec{A} = (-2,1,-1)$ C. $A = (-2,-1,1)$ D. $\vec{A} = (2,-1,1)$

30. What is the image of the circle $x^2+y^2-4x-6y+12=0$ when it is reflected with respect to the line $y=-x$?

A. $(x+3)^2+(y+2)^2=1$

B. $(x+2)^2+(y+3)^2=1$

C. $(x-2)^2+(y-3)^2=1$

D. $(x-3)^2+(y-2)^2=1$

31. If the image of the line $2x-3y=7$ under a translation is $2x-3y=0$, which one of the following is a translation vector of the translation line?

A. $u=(2-1)$ B. $u=(-2,1)$ C. $u=(-1,2)$ D. $u=(t,-2)$

32. What is the standard equation of the line passing through the point $(2,3)$ and parallel to the line given by $\begin{cases} x=1+2\lambda \\ y=-2-\lambda \end{cases}, \lambda \in R$?

A. $\frac{x-2}{-1}=\frac{1-3}{2}$ B. $\frac{x-2}{2}=\frac{y-3}{-1}$ C. $\frac{x-1}{2}-\frac{1-3}{-2}$ D. $\frac{x-1}{1}=\frac{y-3}{-2}$

33. Let L be the line given by the vector equation $(x,y)=(1,1)+t(\sqrt{3},1), t \in R$ What is the equation of the image of L after being rotated 15° about $(1,1)$ and then translated by vector $\mathbf{u}=(-1,1)$?

A. $x-y=2$ B. $\sqrt{3}x-y=2$ C. $-x+y=2$ D. $-x+\sqrt{3}y=1$

34. Ship A and B depart from the same point at the same time on the course $N60^\circ E$ and $N40^\circ E$, respectively. If the speed of Ship A is $20\ km$ per hour and the speed of Ship B is $30\ km$ per hour, what is the distance between the two ships just after 30 minutes of their departure?

[You may take: $\cos(40^\circ)=0.77,\ \cos(20^\circ)=0.94,\ \sin(20^\circ)=0.34$]

A. $\sqrt{43}\ km$ B. $\sqrt{40}\ km$ C. $\sqrt{50}\ km$ D. $\sqrt{53}\ km$

35. When the plane is rotated 45° about the point $(1,-2)$, then what would be the image of the point $(2,4)$?

A. $\left(1-\frac{\sqrt{2}}{2},-2+\frac{\sqrt{2}}{2}\right)$ B. $\left(\frac{5\sqrt{2}}{2},\frac{7\sqrt{2}}{2}\right)$ C. $\left(1-\frac{5\sqrt{2}}{2},-2+\frac{7\sqrt{2}}{2}\right)$ D. $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

36. If $\mathbf{u} = 2\vec{j} - \vec{k}$ and $\mathbf{v} = \vec{i} - 8\vec{j} + 3\vec{k}$, then what is the unit vector in the direction of $5\mathbf{u} + \mathbf{v}$?

A. $\vec{i} + 2\vec{j} - 2\vec{k}$ B. $\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$ C. $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$ D. $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}$

37. Which of the following is NOT true about the scalar product of vectors?
   A. The scalar product of non-zero parallel vectors is zero.
   B. The scalar product is commutative.
   C. The scalar product of perpendicular vectors is zero.
   D. The scalar product is distributive over addition of vectors.

38. Which one of the following is NOT true about transformation?
   A. Translations move angles to congruent angles.
   B. Rotations move triangles in to congruent triangles.
   C. Reflection is not a rigid motion.
   D. Rotation is a rigid motion.

39. What is the image of the ellipse $\frac{(x-3)^2}{6} + \frac{(y-2)^2}{4} = 1$ when it is rotated through 90° about the origin?

A. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{6} = 1$ B. $\frac{(x+2)^2}{6} + \frac{(y-3)^2}{4} = 1$

C. $4(x+3)^2 + 6(y-2)^2 = 24$ D. $6(x+2)^2 + 4(y-3)^2 = 24$

40. Consider the circle given by $x^2 + y^2 = 2$ and the line l given by parametric vector equation $(x, y) = (2,0) + t(-1,1)$. Which of the following is true?
   A. l is tangent to the circle at $(1/2, \sqrt{7}/2)$.
   B. l intersects with the circle at two distinct points.
   C. l and the circle have no common point.
   D. The distance from the center of the circle to l is $\sqrt{2}$.